Aghion And Howitt’s Basic Schumpeterian Model Of Growth Through Creative Destruction: A Geometric Interpretation

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ABSTRACT

The present paper takes a geometric approach to characterize the competitive forces behind innovation and dynamic general equilibria determination in the model of growth through creative destruction constructed by Aghion and Howitt (1992). All can be comprehended intuitively from the geometric presentation. While Aghion and Howitt’s original presentation of the basic model was essentially analytical, often with fairly intricate mathematics focusing on stationary equilibria with positive growth, the geometric presentation taken here has the benefit of making what in the original paper was a bundle of mathematical notation more comprehensible intuitively.

KEYWORDS: Endogenous growth, innovation, creative destruction, general equilibrium.

1. INTRODUCTION

Aghion and Howitt (1992) develop a model of growth through creative destruction in which vertical innovations constitute the underlying source of growth, and the innovation process or research competition is modeled as in the patent-race literature. Each innovation consists of the invention of a new intermediate good, and a successful innovator obtains a patent which it can use to monopolize the intermediate sector. Equilibrium is determined by a forward-looking difference equation, according to which the amount of research in any period depends upon the expected amount of research next period. The basic model of the paper is presented in Section 2, and Section 3 derives the functional relationship between research in two successive periods that defines equilibrium in the economy (see also Aghion and Howitt, 1988).

This basic model (also referred to as the “quality-ladder” model) was constructed with the purpose of bringing Schumpeter’s theory of development back into the mainstream of macroeconomic theory. Following Schumpeter, the model assumes that individual innovations are sufficiently important to affect the entire economy. Besides, it is well known that the general equilibrium theory that has dominated the mainstream assumes among other things that technology is given, and thus the neoclassical growth models assumed technological progress to be exogenous. Moreover, the source of the intertemporal, functional relationship above is creative destruction. That is, the (rational) expectation of more future research discourages current research by threatening to destroy the monopoly rents created by current research. Based on Schumpeter’s idea of creative destruction, the model assumes a factor of obsolescence, according to which better products render previous ones obsolete.
The basic model of Aghion and Howitt (1992) is sketched in Chapter 2 of a most important book of endogenous growth theory by Aghion and Howitt (1998). The most immediate extensions of the basic model are addressed in the last section of Chapter 2 (technology transfers and cross-country convergence). The book explores several other dimensions in which the Schumpeterian paradigm can be fruitfully applied and developed. In the following chapters, the basic Schumpeterian framework is in fact extended and generalized in many different directions to address a broader range of issues related to growth (unemployment, business cycles, market structure, income distribution and wage inequality, and so on).

The present paper takes a geometric approach to characterize the competitive forces behind innovation and dynamic general equilibria determination in the model of growth through creative destruction constructed by Aghion and Howitt (1992). All can be comprehended intuitively from the geometric presentation. While Aghion and Howitt’s original presentation of the basic model was essentially analytical, often with fairly intricate mathematics focusing on stationary equilibria with positive growth, the geometric presentation taken here has the benefit of making what in the original paper was a bundle of mathematical notation more comprehensible intuitively.

The main contributions of a geometric approach in the economic literature have been to deliver simplicity and transparency to formal theory, especially when verbal explanations of economic ideas and concepts seem convoluted and unintuitive, and even to correct in some formal modeling cases significant interpretational errors. Many textbooks in economic theory use graphs and tables abundantly, therefore enabling authors to depict complex interactions simply. In economics, a picture truly is worth a thousand words.

A brief review of literature on the role of graphs as powerful economic tools next includes the presentation of a macroeconomic framework and its four-quadrant graph to start with. Sinclair (1983) develops a general equilibrium model of the aggregate economy where technological progress is assumed to be exogenous. Work has been undertaken using this platform to identify the employment implications of technological progress. In Sinclair’s presentation, a four-quadrant diagram depicting the market for a composite good, the aggregate production function, the labor market, and a graph of the relationship between the price level and the real wage is the key (see Figures 7.1, 7.6 and 7.7). In turn technological progress by raising the productivity of all inputs will shift the aggregate production function outwards, will raise the marginal productivity of labor at any given level of employment, and by inducing firms to produce more output at any price level, will shift outwards the supply curve for a homogenous, all-purpose final good produced in the economy. The relationship between technological progress and unemployment in macroeconomics models has been previously analyzed in depth by Sinclair (1981). The impact of improved disembodied technology on the demand for labor can be found in Sinclair by varying technology parameters in the production function to reflect the various types of technological progress considered: pure labor or capital augmenting, or Hicks-neutral.

A four-quadrant graph is also present in a microeconomics textbook of reference to explain why the average-variable-cost curve is typically U-shaped. For that purpose, Figure 7.12 in Chacholiades (1986) shows how to derive an important relationship between the average variable cost and the average physical product of labor. And the derivation of the short-run marginal cost curve is left to the interested reader. For that purpose, Figure 7.12 should be amended. Note carefully that the shape of the marginal cost depends on the behavior of the
marginal physical product. Using different sorts of graphs, Scherer (1972) extends and corrects Nordhaus’s pioneering theory of optimal patent life. The geometric presentation taken in Scherer proves an important tool to find bluntly the socially optimal patent life and to comprehend intuitively a comparative statics result regarding the optimal patent life and the curvature of the invention possibility function. While the balancing of the marginal social benefit against the marginal social cost necessary to find the social optimum is shown in one figure, the latter result is illustrated with a new diagram consisting of three panels displaying invention possibility functions with increasingly sharp curvatures. The rest of this short paper is organized as follows. Section 2 below presents a geometric interpretation of the basic Schumpeterian model. Section 3 contains brief concluding remarks.

2. A GEOMETRIC INTERPRETATION OF THE BASIC MODEL

In this presentation, Figure 1 is the key to understand the working of the basic model of Aghion and Howitt and to determine any equilibrium it possesses. In this diagram, all axes of all four spaces represent positive quantities.

Starting with the intermediate sector in the third quadrant, $\tilde{x}(\omega_t)$ is the demand for manufacturing labor during interval $t$, in which the employment $x_t$ of skilled labor in manufacturing is a decreasing function of the productivity-adjusted wage rate $\omega_t$. The subscript $t = 0, 1, 2$ and so on refers to the interval starting with the $t^{th}$ innovation and
ending just before the \((t + 1)^n\). Thus a period in the model is the time between two successive innovations.

All markets are perfectly competitive except for intermediate goods. It is assumed that each innovation creates an economy-wide monopoly in the production of intermediate goods. The intermediate producer that uses the \(t^\text{th}\) innovation is thought of as being the \(t^\text{th}\) successful innovator in the economy. The consumption good is produced using the intermediate good. Let \(x_t\) be the flow of the intermediate good produced by the monopolist. It is assumed that the production of a unit of intermediate good requires one unit of labor, so that the unit cost of the intermediate good is the wage rate. Thus, as stated above, \(x_t\) also equals employment of skilled labor in manufacturing. It is assumed that the final (or consumption) good sector is competitive, so that the price \(p_t\) at which the \(t^\text{th}\) innovator can sell the flow \(x_t\) of intermediate input must equal the marginal product of the intermediate input, which in turn is the inverse demand curve facing an intermediate monopolist charging the price \(p_t\). The intermediate monopolist’s objective is to maximize the expected present value of profits over the current interval \(t\). And the monopolist’s choice of output \(x_t\) is given by the first-order condition (2.6) in Aghion and Howitt (1992): 

\[
x_t = \frac{\omega_t}{\tau_t}.
\]

The fourth quadrant depicts the labor market. The curve in this quadrant corresponds to the equilibrium condition for the labor market in Aghion and Howitt (1992), which is called the labor market condition \(L\) in Aghion and Howitt (1998), and is shown as a straight line with slope minus one in the \((x_t, n_t)\) space. The labor market is assumed to be competitive, and society has two uses for its fixed stock of skilled labor, \(N\). It can produce intermediate goods, one for one, and it can be used in research. That is, equation \((L)\) is 

\[
N = x_t + n_t
\]

where \(n_t\) is the current amount of labor used in research. It is also assumed a perfectly informed and flexible world, in which the price variable rapidly equilibrates the labor market. That is, the productivity-adjusted wage rate \(\omega_t\) is the solution to the labor market clearing condition \((L)\): 

\[
N = \bar{x}(\omega_t) + n_t.
\]

We need to complete the description of the model with the introduction of the research sector. The curve in the second quadrant depicts the arbitrage condition defined by equation \((A)\) in Aghion and Howitt (1998), which is a particular case of condition (2.10) in Aghion and Howitt (1992). The curve corresponding to \((A)\) is shown as an inverse relationship between the expected amount of research next period \(n_{t+1}\) and the growth-adjusted wage rate \(\omega_t\). It is assumed that the research sector is competitive, with any individual being free to engage in research activities. The arbitrage equation \((A)\) determines the amount of labor devoted to research activities and reflects this free allocation of labor between manufacturing and research, as the value of an hour in manufacturing must also be the wage rate paid to skilled workers in research.

Following Aghion and Howitt (1998) Chapter 2, for geometric convenience and simplicity, we shall restrict attention here to the “linear” research technology case. The Poisson arrival rate of innovations in the economy at any instant is accordingly a linear function of the flow of skilled labor used in research, \(n\). In this case the position of the curve corresponding to condition \((A)\) in the second quadrant is independent of the current amount of research \(n_t\). However, almost all of the analysis in Aghion and Howitt (1992) is conducted under the more general research technology hypothesis. Both conditions for a research firm’s optimization problem (2.10) and the functional relationship (3.2) in Aghion and Howitt (1992) are thus defined in terms of the general research technology. In this case any
A member of a family of curves depicting the arbitrage condition may be derived by allowing \( n_t \) to vary.

This arbitrage condition (A) governs the dynamics of the economy over its successive innovations. Observe that the negative slope of the curve corresponding to (A) reflects the sum of two elements, the influence of a creative destruction effect, and the impact of a general equilibrium effect, with implications for the slope of the functional relationship between research in two successive periods in the first quadrant. A higher level of research \( n_{t+1} \) tomorrow will both imply (i) a higher rate of creative destruction, that is a higher Poisson arrival rate of the next innovation and hence shortening the expected lifetime of the monopoly to be enjoyed by the next innovator and (ii) higher future wages \( \omega_{t+1} \), as indicated by arrows in spaces \((x_t, n_t)\) and then \((x_t, \omega_t)\) both of which with current period now being re-expressed in terms of interval \( t + 1 \), and hence lessening the flow of profit to be appropriated by the next innovator. This in turn will lower the discounted expected payoff of the \((t + 1)^{th}\) innovation and will discourage current research \( n_t \). The basic model is now completely described by both the arbitrage condition (A) and the labor market clearing equation (L).

In the first quadrant we have the \( \psi(n_{t+1}) \) curve showing a negative correlation between current and future research in equilibrium. Equilibrium in the economy is determined by the forward-looking difference equation (3.2) \( n_t = \psi(n_{t+1}) \). A perfect foresight equilibrium (PFE) is defined as a sequence \( \{n_t\}_0^\infty \) satisfying (3.2) for all \( t \geq 0 \). In this quadrant, the sequence \( \{n_0, n_1, \ldots\} \) constructed from the clockwise spiral starting at \( n_0 \) constitutes a PFE.

Observe on this regard that at point B the current level of research is \( n_0 \) and the next level is \( n_1 \). Moving horizontally to the right towards the 45º-line in the first quadrant and then vertically downward, we register \( n_1 \), which is the horizontal coordinate of point C on the \( \psi(n_{t+1}) \) curve. Thus this first quadrant basically corresponds to Aghion and Howitt (1998) Figure 2.3. Moreover the analysis by Aghion and Howitt focuses on stationary equilibria with positive growth. A stationary equilibrium corresponds to a PFE with \( n_i \) constant. It is defined as the solution to \( \hat{n} = \psi(\hat{n}) \), or equivalently as the intersection between the \( \psi(n_{t+1}) \) curve and the 45º-line in the first quadrant. There exists a unique stationary equilibrium, \( \hat{n} \), as illustrated in Figure 1. And given that the model is fully characterized by both conditions (A) and (L), a stationary equilibrium is simply defined as the stationary solution to system (A) and (L).

Figure 1 illustrates such a relationship between conditions (A) and (L) and the forward-looking difference equation \( n_t = \psi(n_{t+1}) \), and shows also how to derive the \( \psi(n_{t+1}) \) curve from the curves corresponding to (A) and (L), the demand curve for manufacturing labor in the intermediate sector being embedded in the latter. From the graphical representations given in the second, third, and fourth quadrants, we derive the \( \psi(n_{t+1}) \) curve in the first quadrant. For any arbitrary point on the \( \psi(n_{t+1}) \) curve, such as B, we complete rectangle \( B_1B_2B_3B \). By completing the rectangle we determine perfectly both coordinates \( (n_0, n_1) \) of point B in the first quadrant. The justification for this geometric procedure is rather simple and therefore is omitted here. In this manner we can determine as many points as we please on the \( \psi(n_{t+1}) \) curve.

A particular case of the dynamic general equilibrium model can be introduced to illustrate how different but related markets which are simultaneously in equilibrium interact in the economy. A strategic monopsony effect has been ignored until this point in the description of the basic model, by assuming that intermediate firms take as given the wage of skilled
labor and the amount of research. Now to deal with the strategic monopsony effect, assume that the intermediate firm takes into account its influence on the amount of current research and thereby the expected lifetime of its monopoly. By increasing its demand $x_t$ of skilled labor more than the short-run profit maximizing amount, the monopolist can raise the wage rate that must also be paid to skilled workers in research, as indicated by arrows designating the directions of change in equilibrium quantity and price variables in the $(x_t, \omega_t)$ space. The effect is to reduce the equilibrium amount of current research $n_t$ and consequently to decrease the Poisson arrival rate of the $(t+1)^{st}$ innovation, as indicated by arrows in the $(x_t, n_t)$ space.

Combining conditions for the intermediate monopolist’s and a research firm’s optimization problems (2.5), (2.7), (2.10), (2.12) together with the equilibrium condition $N = x_t + n_t$ in Aghion and Howitt (1992) yields condition (3.1). Condition (3.1) determines the functional relationship (3.2) $n_t = \psi(n_{t+1})$. While the left-hand side of condition (3.1) defines the “marginal cost of research,” $c(n_t)$, the right-hand side defines the “marginal benefit of research,” $b(n_{t+1})$. The amount of current research depends negatively upon future research through two known effects, corresponding to the two places in which $n_{t+1}$ enters the expression for the marginal benefit of research: the creative destruction effect and a general equilibrium effect.

The analysis can then be carried off entirely within an adequate equilibrium framework such as the following “reduced-form” diagram. The economic intuition underlying the graph is clear and easy to follow.

![Diagram](image)

FIGURE 2:
A reduced-form diagram of the competitive forces behind innovation and dynamic general equilibria

Note that the functional relationship depicted in the first quadrant can be derived from a geometrical exercise of the kind presented earlier, by allowing $n_t$ to take a range of values. In this manner we can determine as many points as we please on the $\psi(n_{t+1})$ curve.
Moreover the analysis by Aghion and Howitt focuses on stationary equilibria with positive growth. As Figure 2 shows, it is assumed that \( c(0) < b(0) \); then \( \hat{n} > 0 \). In this case growth is positive because innovations arrive at a positive Poisson rate. As \( c(n) \) is strictly increasing and \( b(n_{t+1}) \) is strictly decreasing, the functional relationship \( \psi(n_{t+1}) \) is a strictly decreasing function wherever it is positive-valued. Instead, if \( c(0) \geq b(0) \) then \( \hat{n} = 0 \) and there is no growth, because the Poisson arrival rate of innovation is zero. In this case the functional relationship \( \psi(n_{t+1}) \) is identically zero.

3. CONCLUSIONS

The paper has presented a geometric interpretation which suffices to characterize the heart of a competitive research sector and dynamic general equilibria determination in the model of endogenous growth through creative destruction constructed by Aghion and Howitt (1992).

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