Theoretical Analysis Regarding a Zero Lower Bound on Nominal Interest Rates

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1. Introduction

The object of this paper is to explore theoretical issues relating to the idea that there exists a zero lower bound on nominal interest rates and to the possibility that such a bound might interfere with the conduct of monetary policy in an environment of low inflation. The possibility of such an impediment has been mentioned over the years by Vickrey (1954), Phelps (1972), Okun (1981), and Summers (1991); recently it has been analyzed quantitatively by Fuhrer and Madigan (1997), Rotemberg and Woodford (1997), Orphanides and Wieland (1998), Wolman (1998), Reifschneider and Williams (1999), and possibly others. There has been little explicit theoretical analysis, however, the main exception that I am aware of being some work in progress by Woodford (1999).

Interest in the subject of a zero lower bound—which will be abbreviated below as ZLB—has been greatly enhanced in recent years by the success that central banks have had in reducing average inflation rates to the range of 1-3 percent (per annum), and by the failure of Japanese stabilization policy to prevent a prolonged macroeconomic slump in which short-term nominal interest rates[1] have fallen to figures approximating zero. Many writers have suggested, especially in the journalistic literature, that such a situation leaves a central bank helpless to provide macroeconomic stimulus. This point of view has been contested by Goodfriend (1997), Krugman (1999), Meltzer (1999), and others.

The discussion below takes up a number of distinct issues and utilizes a variety of analytical models. It begins in Section 2 with a simple but explicit analysis of the source of a possible ZLB on interest rates. Next, Section 3 introduces an argument to the effect that most formal analysis has overstated the restrictiveness of the ZLB by failing to recognize forces that tend to raise steady-state real rates of interest when maintained (i.e., policy target) inflation rates are lowered. Section 4 returns to models with inflation-invariant steady-state real rates and reconsiders the popular practice of analyzing monetary policy in models with no monetary variables. It is argued that neglect of monetary variables is theoretically inappropriate, but probably not quantitatively important. The main analysis, concerning the effectiveness of monetary policy in a ZLB situation, is put forth in Sections 5 and 6. The first of these argues that if the one-period nominal interest rate is for some reason fixed at zero (or some other value), there is nevertheless a route for monetary stabilization policy operating via the foreign exchange market.

Then in Section 6 the quantitative importance of this stabilization approach is investigated by means of a structural macroeconomic model developed and utilized previously. Section 7 takes up a somewhat esoteric topic concerning dynamic stability analysis and expresses disagreement with some alarming views recently put forth. Finally, Section 8 provides a concluding overview. Because of the variety of topics considered, different models are used from section to section. Unfortunately, some accompanying changes in notation are needed, to which the reader should be alert.

2. The Source of a Zero Lower Bound

Let us begin with an elementary but explicit analysis of the theoretical basis for the common-sense belief that nominal interest rates cannot be negative. For this purpose it will be useful to consider an extremely simple general equilibrium model that abstracts from uncertainty and sticky prices, both of which will be introduced later in the paper. Thus we imagine an economy populated by a large number of identical (but independently acting) households, a typical one of which seeks at time 1 to maximize the objective function

$$u(c_1) + \beta u(c_2) + \beta^2 u(c_3) + \ldots$$

subject to a sequence of constraints for t = 1,2,….

$$f(n_t,k_t) - tx_t = c_t + k_{t+1} - (1-\delta)\, k_t + (1+\pi_t)m_{t+1} - m_t + (1+\pi_t)\,(1+R_t)^{-1}\, b_{t+1} - b_t + w_t\,(n_t - 1) + \psi(c_t,m_t). \qquad (1)$$

Here c_t denotes consumption[2] during period t while $y_t = f(n_t, k_t)$ is the output produced by the household from inputs of labor (n_t) and the services of capital (k_t), in accordance with the well-behaved production function $f(n_t, k_t)$. The economy should be thought of as one in which there are many distinct goods; households specialize in production but choose consumption bundles c_t that include many differentiated goods. As has become well-known, the formulation shown can then be justified by assuming that c_t is a CES index of the various goods while P_t indexes the money price of one consumption bundle and is appropriately related to the prices of the distinct goods.[3] In (1), tx_t reflects lump-sum taxes net of transfers from the government, $\pi_t = (P_{t+1}-P_t)/P_t$; m_t is real money balances held by the household at the start of t; and b_{t+1} is the number of bonds purchased in t, each for the price $(1 + R_t)^{-1}$, and redeemed in t + 1 for one unit of money.

The final term in (1), $\psi(c_t,m_t)$, reflects the transaction-facilitating properties of money, i.e., the economy's medium of exchange (MOE). Over a range corresponding to normal conditions the function ψ has partial derivatives satisfying $\psi_1(c_t, m_t) > 0$ and $\psi_2(c_t,m_t) < 0$, but at very low inflation rates the latter inequality may not hold, as will be discussed shortly. Thus the assumption is that the act of acquiring the many-good bundles consumed during t requires an expenditure of resources on transaction services, in addition to the purchase price of the goods. The magnitude of this expenditure increases with c_t, but is decreased—at least up to a point—by larger holdings of real money balances, which make it less likely that their holder will have to resort to barter or hastily-negotiated credit arrangements to effect desired purchases.[4] There are, of course, other analytical devices for representing this transaction-facilitating property of the MOE. Quite common are cash-in-advance and money-in-utility-function specifications, and I have often promoted the "shopping time" approach that expresses transaction requirements in terms of time rather than tangible resources. By and large, the messages conveyed by most of these devices are the same.[5] But the resource transaction-cost variant represented in (1) is somewhat cleaner analytically and so will be utilized throughout most of the present paper.

Assuming that f( ), u( ), and ψ( ) are such that interior solutions are obtained, the household's first-order conditions for optimality in the problem stated above include, for t = 1,2…:

(2) $$u'(c_t) - \lambda_t[1+\psi_1(c_t,m_t)] = 0$$

(3) $$f_1(n_t,k_t) - w_t = 0$$

(4) $$-\lambda_t + \beta\lambda_{t+1} [f_2(n_{t+1}, k_{t+1}) + 1-\delta] = 0$$

(5) $$-\lambda_t(1+\pi_t) + \beta\lambda_{t+1}[1 - \psi_2(c_{t+1},m_{t+1})] = 0$$

(6) $$-\lambda_t(1+\pi_t)(1+R_t)^{-1} + \lambda_{t+1} \beta = 0$$

Here λ_t is the Lagrange multiplier attached to constraint (1). There are also transversality conditions (TCs) pertaining to the household's choice problem. Presuming them to hold, equations (1) – (6) determine optimal time paths for c_t, m_{t+1}, k_{t+1}, n_t, b_{t+1}, and λ_t in response to market- or policy-determined values of w_t, π_t, tx_t, and R_t.

For general equilibrium, we have, in addition to relations (1) – (6), the following:

(7) $n_t = 1$

(8) $m_t = M_t/P_t$

(9) $g_t - tx_t = (1+\pi_t)\, m_{t+1} - m_t + (1+\pi_t)\, (1+R_t)^{-1} b_{t+1} - b_t$

(10) $\pi_t = (P_{t+1}-P_t)/P_t$

Here (7) and (8) are market-clearing conditions, (9) is the identity that reflects the government's budget constraint, and (10) is the inflation definition mentioned above—all expressed in per-household terms where relevant. We assume that the government—a combination of a central bank that issues high-powered money M_t and a fiscal authority—exogenously determines time paths for the variables g_t, b_t, and M_t. Then the model's 10 equations (plus TCs) determine time paths for c_t, m_t, k_t, n_t, λ_t, tx_t, w_t, R_t, P_t, and π_t.

If we were to append the Fisher identity

(11) $1 + r_t \equiv (1+R_t)\, (1+\pi_t)^{-1}$,

then equations (4) and (6) would imply

(12) $1 + r_t = f_2(n_{t+1},k_{t+1}) - \delta + 1$,

i.e., that the real rates of return on bonds and capital are equalized. If the model featured uncertainty regarding tastes or technology, then the differing risk characteristics of bonds and capital would introduce a stochastic differential in these returns. We now ask, is there anything in the equations governing market equilibrium in the system at hand that is suggestive of a ZLB on the value of R_t? For an answer, we combine (5) and (6) to obtain

(13) $1 + R_t = 1 - \psi_2(c_{t+1},m_{t+1})$,

which says that the interest rate R_t equals –1 times the partial derivative of transaction costs with respect to real money balances. The latter may be described as the marginal service yield from holding money balances; thus our condition (13) can also be written as

(14) $$R_t = -P_{t+1} \frac{\partial \psi(c_{t+1}, m_{t+1})}{\partial M_{t+1}}.$$

This equality is similar to (24) on p. 32 of Friedman (1969), under the assumption that bonds provide no non-pecuniary services to their holders.

From the foregoing it is apparent that any bounds pertaining to R_t are going to be decisively influenced by limits on $\psi_2(c_t, m_t)$. If $\psi_2 <0$ strictly, then we would have the implication $R_t>0$. But it seems rather implausible that such would be the case. One would expect to have $\psi_{22}>0$ over an extended range, so that the marginal service yield on m_t decreases as the quantity of money held grows larger. But at some point, say $m^*(c_t)$, real money holdings would be so large in relation to spending—e.g., ten times as large as annual spending flows!—that additional money holdings would not provide any extra services. Then $-\psi_2$ would fall to zero at $m = m^*(c)$. Suppose then, that $\psi(c_t, m_t)$ is such that the relationship between $-\psi_2$ and m_t is (for given c_t) as depicted in Figure 1A. Then with $\psi_2(c_t,m_t) \leq 0$, condition (13) implies $R_t \geq 0$. In this way we obtain, via formal analysis, a ZLB on the one-period interest rate.

Continuing this line of thought, however, it seems apparent that $\psi(c_t,m_t)$ in (1) should be regarded as reflecting transaction services together with any storage costs associated with money. If the economy's MOE were metallic coins or uncoined bullion, then storage costs would clearly be relevant. But even with paper money, which we are presuming to be relevant for the analysis at hand, one can imagine that stocks of money balances could be so large that storage costs at the margin would become non-negligible.[6] In that case Figure 1B would be relevant, and a ZLB on R_t would not be implied. To the author, this case seems most relevant.[7] It is, nevertheless, rather difficult to imagine that such storage costs could permit R_t to be negative by more than a few basis points in any currently-conceivable circumstances.

The main point of the foregoing discussion is that the presence or absence of a ZLB on (short term) nominal interest rates depends upon the properties of the function or constraint of the model that represents the transaction-facilitating properties of the economy's MOE, together with any storage costs necessitated by stocks of the MOE. Strictly speaking, then, it seems to be logically unsatisfactory to discuss the topic of a ZLB in the context of a model that takes no explicit position concerning the properties of $\psi(c_t, m_t)$ or some analogous function that depicts the transaction and storage cost properties of the MOE.

Before proceeding, it will be useful to prepare some background for the next topic by noting one of the steady-state properties of the system at hand. With no population growth and no technical progress, a steady state equilibrium must have constant values of c_t, y_t, k_t, r_t, and m_t so the rates of growth of P_t and M_t must be equal: the inflation rate must equal the rate of growth of the money stock. But furthermore (4) implies that

$$\lambda = \beta\lambda[f_2(n,k) + 1-\delta] \tag{15}$$

holds in a steady state, so that

$$1/\beta = 1 +\rho = 1 + f_2(n,k) - \delta, \tag{16}$$

i.e., that $r = f_2(n,k) - \delta = \rho$. Of course the latter implies via (11) that

$$1 + R = (1+\rho)\,(1+\pi). \tag{17}$$

If then the rate of money growth μ and inflation were negative and greater in absolute value than ρ, a negative value of R would be implied. Suppose, then, that the central bank were to destroy money at a constant rate $-\mu$ larger than ρ and that the nature of $\psi(\quad)$ is such that $\psi_2(\ ) \leq 0$ so that there is a ZLB on R. It would then appear to be the case that no steady-state equilibrium is possible, when $-\mu>\rho$.

The property just derived should be regarded, in my judgment, as a weakness of the model at hand—a defect due to the model's assumption that the steady-state equilibrium value of r is totally independent of the ongoing inflation rate. This superneutrality property is a useful approximation for thinking about macro-monetary issues, but is a rather special property of models with time-separable, infinite-horizon utility functions for the household agents. Not all well-known models possess this property, however, as we shall review in the next section.

3. Real-Rate Effects of Inflation

The present section considers the possibility that a permanent reduction in the central bank's inflation target—a fall in the long-run average inflation rate—may not sharply reduce the "policy buffer" between the average level of R_t and the ZLB because of an increase in the steady-state real rate, r.[8] The reason is that a decreased pecuniary yield differential between capital and money may induce wealth-holders to allocate a larger fraction of their wealth to money and less to capital. As this occurs, the marginal product of capital will tend to rise, carrying the real yield on paper assets along with it. We need to consider whether such effects can be of quantitative importance.

Effects of this type cannot obtain in the Sidrauski-Brock model of Section 2, of course, but can in models in which individuals have finite lifetimes. For our formal analysis, let us for simplicity adopt an overlapping generations (OG) setup in which agents live for only two periods, keeping in mind that such periods must be thought of as (say) 25 years in duration.

Notation for our model is similar, but not identical, to that of McCallum (1987). Thus c_t denotes consumption when young, and x_{t+1} consumption when old, of an individual born in period t. Such an individual's utility function when young is $u(c_t, x_{t+1})$, which we specialize to the separable form

(18) $$u(c_t,x_{t+1}) = v(c_t) + \beta v(x_{t+1}).$$

Here the discount factor must be recognized to pertain to a period of 25 years. Thus for an annual rate of time preference of 0.025, we would have $\beta = (1.025)^{-25} = 0.5394$.

When young, individuals supply one unit of labor inelastically and earn a real wage of w_t. Old individuals cannot work, but can and do operate production processes using capital goods (obtained from their savings when young) and hired labor of youths. Let $y_t = f(n_t, k_t)$ denote output in t of an old producer who has k_t units of capital to use and hires n_t young workers.

Young individuals can hold their savings $w_t – c_t$ in the form of capital, bonds, or money. Let k_{t+1}, b_{t+1}, and ξ_t denote capital, bond, and real-money holdings at the end of t. Then the young person's budget constraint is

(19) $$w_t = c_t + k_{t+1} + b_{t+1} (1+r_t)^{-1} + \xi_t,$$

and when old in t+1 this person will be constrained by

(20) $$x_{t+1} = f(n_{t+1},k_{t+1}) + (1-\delta)k_{t+1} – w_{t+1} n_{t+1} + b_{t+1} + \xi_t P_t/P_{t+1} + tr_{t+1} - \psi(x_{t+1},m_{t+1}).$$

Here $tr_{t+1} = -tx_{t+1}$ denotes lump-sum transfers to an old person in t + 1, while money holdings are

(21) $$m_{t+1} = \xi_t P_t/P_{t+1} + tr_{t+1}$$

and $\psi(x_{t+1}, m_{t+1})$ represents transaction costs of consuming in old age. The function ψ has the same general interpretation as in Section 2, although it is more strained by the extreme length of a period in the present setting.[9]

Maximization of (18) subject to (19), (20), and (21) yields the following first-order optimality conditions for a young individual:

(22) $\quad v'(c_t) = \beta v'(x_{t+1})\,[f_2(n_{t+1},k_{t+1}) + 1\text{-}\delta]/[1+\psi_1(x_{t+1},m_{t+1})]$

(23) $\quad f_1(n_{t+1}, k_{t+1}) = w_{t+1}$

(24) $\quad v'(c_t) = \beta v'(x_{t+1})\,(P_t/P_{t+1})[1 - \psi_2(x_{t+1}, m_{t+1})]/[1+\psi_1(x_{t+1},m_{t+1})]$

(25) $\quad r_t = f_2(n_{t+1}, k_{t+1}) - \delta.$

These plus (19), (20), and (21) determine the individual's choices of c_t, x_{t+1}, k_{t+1}, n_{t+1}, b_{t+1}, m_{t+1}, and ξ_t given exogenous (to the individual) values of w_t, w_{t+1}, tr_t, P_t/P_{t+1}, and r_t.

We assume that population growth proceeds at the rate ν, so that $1 + \nu$ is the number of young persons per old person in each period. Then for general equilibrium, we must have for each t = 1,2,…

(26) $\quad n_t = 1 +\nu,$

(27) $\quad M_t/P_t = (1+\nu)\,\xi_t,$

where M_t = money supply per old person in t, after transfers, and the government budget identity

(28) $\quad P_t(g_t + tr_t) = (1+\nu)M_t - M_{t-1} + (1+\nu)P_tb_{t+1}(1+r_t)^{-1} - P_tb_t.$

Assuming that the government sets time paths for g_t (government purchases), M_t, and b_t, the ten listed equations determine equilibrium paths for the variables c_t, x_t, k_{t+1}, n_t, m_t, ξ_t, P_t, tr_t, w_t, and r_t (t = 1,2,…).[10]

In a steady-state equilibrium, we have constant values for c, x, m, ξ, k, w, r, b, $\pi = (P_{t+1}/P_t)\text{-}1$, tr, and $n = 1+\nu$. In this context, $r = f_2(1+\nu, k)-\delta$ and $w = f_1(1+\nu,k)$. But determination of the value of k is not independent of m and P_t/P_{t+1}. For simplicity, let us eliminate g and b from the model. Then the relevant set of conditions determining k, c, x, and m is

(29) $\quad v'(c) = \beta v'(x)[f_2(1+\nu,k) + 1\text{-}\delta] / [1+\psi_1(x,m)]$

(30) $\quad v'(c) = \beta v'(x)[1\text{-}\psi_2(x,m)] / [1+\psi_1(x,m)](1+\pi)$

(31) $\quad f(1+\nu,k) + (1\text{-}\delta)k = x + (1+\nu)c + (1+\nu)k + \psi(x,m)$

(32) $\quad f_1(1+\nu,k) = c + k + m/(1+\nu).$

Here equations (29), (31), and (32) include the four endogenous variables c, x, k, and m. Therefore their values cannot be determined without use of (30), where the inflation rate π appears. If we treat π as exogenous—determined by the average growth rate of M—then the monetary authority's choice of π will typically affect the steady-state value of k and therefore r.

Our objective now is to see whether the effect of π on r is quantitatively large enough to be of policy significance. Thus we need to calibrate the model at hand. For the production function we take $y = A\, n^{.64}\, k^{.36}$ and choose A to yield a realistic value of k/y. In annual terms the latter would be about 3 so, in our setup with 25-year periods, we need k/y in the range of about 0.1-0.2. In equilibrium, $n = 1+\nu$ so if we take population growth to be one percent per year we obtain $n = (1.01)^{25} = 1.2824$. We also want the real rate of interest to be around 2.0 – 4.0 percent per year. It turns out that together these requirements suggest a value of A = 20. For the 25-year depreciation rate, we use $\delta = 0.90$, which implies that about 10 percent of gross output goes to depreciation. For our utility function, an intertemporal elasticity of substitution of 0.25 seems appropriate,[11] so we specify $u(c) = (1-\theta)^{-1}c^{1-\theta}$ with $\theta = 4$, which implies $u'(c) = c^{-4}$.

Turning now to the transaction cost function $\psi(x,m)$, let us suppose that the cost per unit of purchases declines with m/x, according to

$$\psi(x,m)/x = a_1(x/m)^{a_2} \qquad a_1, a_2 > 0. \tag{33}$$

From a relation analogous to (13) we have that the elasticity of money demand with respect to the nominal interest rate equals $-1/(1+a_2)$. To yield a conservatively small value of 0.2, we then set $a_2 = 4$. Finally, to keep the ratio of m to k (or m to y) realistically small, we specify $a_1 = 0.1 \times 10^{-8}$. With these values, we consider annualized inflation rates of 10, 5, 2, 1, 0, -2, -5, and -10 percent. In terms of our 25-year periods, these imply values of π as shown in Table 1. Steady state values of c, x, k, and m are also reported in Table 1 for these alternative inflation rates. It will be observed that real money balances rise and the capital stock falls as inflation rates are reduced toward zero, and then on into the negative range. The real rate of interest—the marginal product of capital net of depreciation—is shown in the final column in annualized percentage terms. It rises quite slowly with reduced inflation, but climbs more significantly as deflation ranges are encountered. Indeed, in the model at hand, it seems to rise enough to keep the nominal interest rate positive in all cases, even with substantial deflation. That finding accords with the type of effect that this section was designed to investigate.

In quantitative terms, however, Table 1 results are unlikely to provide much reassurance to policymakers concerned with the issue raised by Summers, Okun, and others, which has to do with cyclical stabilization, not the potential infeasibility of sizeable steady-state deflation rates. In that regard, our quantitative results suggest that the increase in the steady-state real interest rate associated with a reduction in inflation from 2 percent to 0 percent (per year) would be negligible. Accordingly, we henceforth ignore the effects of inflation on the average real rate of interest, and return to models of the Sidrauski-Brock type in which the steady-state real rate r is invariant to alternative maintained inflation rates.

4. The Role of Monetary Variables in Policy Analysis

At this point we resume the main line of argument. Much practical monetary policy analysis during recent years has been conducted, as is well known, in models that include no monetary variables whatsoever. Instead, they consist of the three following components: (i) an IS-type relation (or set of relations) that specifies how interest rate movements affect aggregate demand and output; (ii) a price adjustment equation (or set of equations) that specifies how inflation behaves in response to the output gap and to expectations regarding future inflation; and (iii) a monetary policy rule that specifies each period's settings of an interest-rate instrument. These settings are typically made in response to recent or predicted values of the economy's inflation rate and its output gap—as, e.g., in the case of a Taylor rule. Examples of such analytical work, stemming from conferences held by NBER and the Sveriges Riksbank (in collaboration with IIES of Stockholm University), are presented in Taylor (1999) and in the June 1999 issue of the Journal of Monetary Economics (vol. 43, no. 3). This practice of conducting monetary policy analysis in models with no monetary variables is of particular interest in situations in which interest rates are close to a ZLB. But before turning to that case, it will be useful to consider the absence of monetary variables from a more general perspective.

As a point of reference, let us write out a simple specification of the IS-AS-MP type under discussion.[12] Symbols are basically the same as in Section 2, but in addition we let $\bar{y}_t$ be the natural-rate value of y_t, i.e., the value that would prevail in the absence of any price stickiness in the economy, and define $\bar{p}_t$ as the associated price level. Also, let v_t and e_t represent shocks to spending and monetary policy behavior. We suppose that $\bar{y}_t$ is generated exogenously, influenced perhaps by the shock v_t. Then the schematic model is given by the following three equations:

(34) $\log y_t = b_0 + b_1(R_t - E_t\Delta\log p_{t+1}) + b_2(\log g_t – E_t \log g_{t+1}) + E_t y_{t+1} + v_t$

(35) $\log p_t – \log p_{t-1} = (1-\alpha)(\log \bar{p}_{t-1} - \log p_{t-1}) + E_{t-1}(\log \bar{p}_t - \log \bar{p}_{t-1})$

(36) $R_t = E_{t-1}\Delta\log p_{t+1} + \mu_0 + \mu_1(E_{t-1}\Delta\log p_{t+1} - \pi^*) + \mu_2(\log y_t – \log \bar{y}_t) + e_t$.

Here (36) is a Taylor-style (1993a) policy rule, with a forward-looking flavor, and (35) is a particular price adjustment specification that will be discussed below in Section 5. For present purposes, our concern is with the IS-type relationship (34) about which we ask: can it be given an adequate theoretical foundation?

In fact, a reasonably satisfactory justification has become quite well-known from a number of papers, including Kerr and King (1996), Rotemberg and Woodford (1997), McCallum and Nelson (1999c), and Clarida, Gali, and Gertler (1999), among others. It can be outlined briefly as follows. Consider the optimizing model presented in Section 2 and note that equations (4) and (12) can be combined to yield

(37) $\lambda_t = \beta\lambda_{t+1}(1+r_t)$,

where λ_t is the shadow value (in utility units) of a unit of output in t while r_t is the real rate of interest. Suppose then that the transaction-cost function $\psi(c_t,m_t)$ is separable, so that its first partial derivative with respect to c_t can be written as $\psi_1(c_t)$. Then using (2) for λ_t we can substitute into (37) and obtain

(38) $u'(c_t)/[1+\psi_1(c_t)] = \beta u'(c_{t+1})(1+r_t)/[1+\psi_1(c_{t+1})]$.

Now the latter is a relationship that determines a household's choice of c_t in response to r_t and its expectations regarding c_{t+1}. Taking a log-linear approximation, then, we can obtain[13]

(39) $\log c_t = b_0' + E_t\log c_{t+1} + b_1' r_t$

where $b_1' < 0$. In the literature, derivations such as the foregoing have usually been presented in models in which the transaction-facilitating property of money is expressed by including m_t as an argument of the utility function, rather than in the manner involving our transaction-cost approach. But the basic idea is the same. And in either case, a disturbance term will appear on the right-hand side of (39) if there is a serially-correlated preference shock appearing appropriately in the utility function.

Next, armed with (39) we make use of the economy's overall resource constraint. A log-linear approximation is written as

(40) $\log y_t = \omega_1 \log c_t + \omega_2 \log i_t + \omega_3 \log g_t$

where i_t denotes investment in period t. The weights ω_j sum to 1.0 and reflect average shares of the three components. We substitute (39) into (40) for c_t and solve out $E_t c_{t+1}$ using (40), thereby obtaining

(41) $\log y_t = b_0 + b_1 r_t + b_2(i_t - E_t i_{t+1}) + b_3(g_t - E_t g_{t+1}) + E_t y_{t+1} + v_t$.

The latter is the "expectational IS function" that we set out to justify. It might be mentioned that applications have often ignored the investment and government spending terms. In the case of investment, that practice is rationalized by treating capital as a fixed constant (e.g., Rotemberg and Woodford (1997)) or by treating log investment as an exogenous random walk (McCallum and Nelson (1999c)).[14] If assumed exogenous, government spending can be included easily. Thus we end up with a relation basically equivalent to (34).

At this point let us return our attention to the system (34), (35), (36). If g_t is excluded, or assumed exogenous, then the system is complete in the sense that y_t, p_t, and R_t are the only endogenous variables. To append a money demand function, which could be derived in the model of Section 2 by solving (13) for m_{t+1}, would be redundant. The only role of such a function would be to describe the path of the nominal money stock M_t that would be necessary to support the R_t policy rule (36). Including this relation in the model would therefore have no effect on time paths of the variables y_t, p_t, and R_t.

But of course it should be clear that this conclusion depends upon the absence of any term involving real money balances in the expectational IS function (34). And that absence depends upon the assumption, inserted provisionally three paragraphs ago, that the transaction-cost function $\psi(c_t, m_t)$ is separable in c_t and m_t. An obvious task, then, is to reconsider that crucial assumption. But my own position has already been introduced in Section 3, where it is suggested that a plausible specification for $\psi(\ )$ would be of the form

(42) $\psi(c_t,m_t) = c_t a_1(c_t/m_t)^{a2}$

with $a_{1,}$ $a_2 > 0$. This function is clearly not separable so the issue becomes one involving quantitative magnitudes. Is the role of real money balances in the IS function likely to be quantitatively important?[15] To approach that question, let us see how the IS function would be specified under the assumption that (42) is the relevant specification for transaction costs. Then equation (2) can be written (assuming that $u'(c_t) = c_t^{-\theta}$) as

(43) $\lambda_t = c_t^{-\theta} /[1 + (1+a_2)a_1(c_t/m_t)^{a2}]$

and a log-linear approximation would be

(44) $\log \lambda_t = -\theta \log c_t - \phi(\log c_t - \log m_t)$,

provided that ϕ/a_2 is small relative to 1.0, where $\phi = a_1(1+a_2)a_2(c/m)^{a2}$.[16] Substitution of (44) into the log of (37) followed by rearrangement yields

(45) $\log c_t = E_t\log c_{t+1} + (\theta+\phi)^{-1}[\phi(\log m_t - E_t\log m_{t+1}) - r_t - \log \beta]$ + disturbance.

Clearly, then, combination of the latter with (40) would result in an IS function like (41) but including an additional term, equal to

(46) $[(c/y)\phi/(\phi+\theta)](\log m_t - E_t \log m_{t+1})$.

In sum, we have found that non-separability of $\psi(c,m)$ implies that a term involving real money balances appears in the expectational IS function based on optimizing analysis, and if the form of $\psi(c,m)$ is as given in (42) then the additional term can be approximated by expression (46). It is clearly of interest, then, to obtain an idea of the magnitude of the attached coefficient, i.e., the term in brackets in (46).

To do so we again draw on the implication that the money demand equation in the model under discussion would be of the form (13), which we now write as

(47) $R = a_1\, a_2\, (c_t/m_t)^{1+a2}$.

Then an assumed money demand elasticity with respect to R_t of –0.2 would again suggest a value of a_2 = 4. To calibrate a_1, let us express R and c/m in units pertaining to annual time periods. Then for R a value of 0.05 would be reasonably appropriate and for c/m a value of 5.[17] These choices yield $a_1 = 0.05/4(5)^5 = 4 \times 10^{-6}$. Also let $\theta = 2.5$. Thus we have $\phi = 20(4 \times 10^{-6})(5)^4 = 0.05$ and $\phi/(\phi+\theta) = 0.05/(0.05 + 2.5) =$

0.0199. Consequently, the coefficient attached to $\log m_t - E_t \log m_{t+1}$ in the IS function is estimated by our calibration exercise to be smaller than 0.02. Of course there are numerous uncertainties and approximations involved, but the figure obtained seems to be too small to justify any confidence that the effect of real money terms in the IS function would be economically sizeable, contingent upon our basic model specification.

Woodford (1999, Sect. 3.1) suggests a considerably larger number (approximately 0.1) for the comparable slope coefficient, primarily because he assumes a much larger value for the intertemporal elasticity of substitution in consumption.[18] Nevertheless, he concludes that there is no prospect from this source for escape from a liquidity trap situation—one with R_t at a ZLB—because approximations such as those used above break down in the vicinity of satiation with monetary transaction services. Woodford's reasoning seems to be correct, but we will consider the liquidity trap issue more generally in the next section.

First, however, it should be noted that some analysts—most notably Meltzer (1999)—argue that monetary variables cannot legitimately be ignored in policy analysis because they are related to market outcomes in a manner that does not work through a real-money-balance term in an IS function. Instead, Meltzer argues that the relevant transmission process involves adjustment in the relative prices of assets that are not recognized in simple models such as the ones used here (and by Woodford (1999)). Of course Meltzer is correct to say that it is a gross simplification of reality to pretend that economies include only two assets.[19] Whether monetary policy can be used to systematically influence the relevant relative asset prices is, however, an open question.[20] Also, modelling of the relevant transmission process is both necessary and difficult. For one such relative price it does seem clear, however, that there are systematic effects of monetary policy that are both relevant and comprehensible. That argument will be spelled out in the following section.

5. Stabilizing Monetary Policy in a Liquidity Trap

Probably the most contentious and important topic under consideration is the idea that the potential stabilizing powers of monetary policy can be nullified by the occurrence of a "liquidity trap," i.e., a situation in which the central bank's usual policy instrument R_t cannot be lowered past a prevailing ZLB (or possibly some negative lower bound as suggested in Figure 1B). The purpose of the present section is to argue, by means of an expository model, that even in a liquidity trap there is scope for monetary stabilization policy provided that the economy is internationally open—as all actual economies are.[21] Then the argument will be evaluated quantitatively in an optimizing model in Section 6.

We begin by specifying a schematic model designed for illustrative purposes. The macroeconomic structure will consist of an open-economy IS sector, with no real-money terms included, a price adjustment relation, and a monetary policy rule. For the IS sector we have

$$y_t = E_t y_{t+1} + b_0 + b_1(R - E_t\Delta p_{t+1}) + b_2(x_t - E_t x_{t+1}) + v_t \tag{48}$$

$$x_t = c_1(s_{t-1} - p_{t-1}) + c_2 y_{t-1}. \tag{49}$$

Here (48) is an expectational IS function of the type described above, in which now y_t denotes the log of output, p_t the log of the price level, and x_t the log of net exports.[22] (Please note the <u>change in notation</u> relative to Sections 2 and 4!) The disturbance term v_t, taken for simplicity to be white noise, reflects taste shocks. As suggested by optimizing analysis, $b_1<0$ and $b_2>0$. In the real interest rate term, R has been written without a subscript so as to reflect the hypothesized liquidity trap situation, i.e., that the one-period nominal interest rate R_t is held fixed over time by some force not explicitly modeled but presumed to reflect a ZLB or some such constraint. Relation (49) represents effects of relative prices and incomes on net exports. We treat foreign prices and income as constant, so $s_t - p_t$ represents the modeled economy's (log) real exchange rate, s_t being the log of the domestic price of foreign exchange. We presume, as is quite standard, that $c_1>0$ and $c_2<0$.[23] A one-period lag is assumed for simplicity, but distributed-lag effects would not fundamentally alter the model. It is necessary, in the present simplified setup, that the effect of $s_t - p_t$ on x_t not be entirely contemporaneous, for reasons discussed below in footnote 25. (No such assumption will be used, however, in the more complete model of Section 6.)

Next, regarding price adjustment behavior we posit that

$$p_t - p_{t-1} = (1-\alpha)(\bar{p}_{t-1} - p_{t-1}) + E_{t-1}(\bar{p}_t - \bar{p}_{t-1}), \tag{50}$$

where $\bar{p}_t$ represents the price that would be market-clearing in the absence of nominal stickiness. With $0<\alpha<1$ price level stickiness is implied, however. McCallum and Nelson (1999a) show that (50) is equivalent to the Barro-Grossman-Mussa-McCallum "P-bar" model, which is one of the few sticky-price formulations that implies satisfaction of the natural rate hypothesis of Lucas (1972). They also show that (50) is equivalent (assuming demand function log-linearity) to the condition

(50′) $$E_{t-1}\tilde{y}_t = \alpha\tilde{y}_{t-1},$$

where $\tilde{y}_t = y_t - \bar{y}_t$ with $\bar{y}_t$ representing the (log) natural-rate value of output, i.e., the value that would prevail with fully flexible prices. Relation (50′) can be used instead of (50) in a model, analytical or numerical, to significantly facilitate the analysis.

For simplicity suppose that $\bar{y}_t = \bar{y}$. Then equations (48)-(50) contain the endogenous variables y_t, x_t, s_t, and p_t. We close the system with the following monetary policy rule:

(51) $$s_t - s_{t-1} = \mu_0 - \mu_1(\Delta p_t - \pi^*) - \mu_2 E_{t-1}\tilde{y}_t + e_t$$

where $\mu_1, \mu_2 > 0$. Thus when inflation is low and/or expected output is below its natural-rate value, the rate of depreciation of the exchange rate Δs_t is increased. This is accomplished via central bank purchases of foreign exchange at a pace more rapid than is normal. Such an action reflects expansionary monetary policy, conducted in accordance with the rule (51), designed to stabilize Δp_t toward its target value π^* and $\tilde{y}_t$ toward zero (y_t toward $\bar{y}_t$). In (51), e_t represents the unsystematic "shock" component of monetary policy, which we take to be white noise. The constant term μ_0 is set equal to the average real rate of interest, which is $-b_0/b_1$.

The MSV rational expectations solution to the system (48)-(51) can be obtained as follows.[24] Write (49) in first difference form $\Delta x_t = c_1(\Delta s_{t-1} - \Delta p_{t-1}) + c_2(y_{t-1} - y_{t-2})$, and substitute into (48) in place of $E_t\Delta x_{t+1}$. That step yields[25]

(52) $$y_t = E_t y_{t+1} + b_1(R - E_t\Delta p_{t+1}) - b_2 E_t[c_1(\Delta s_t - \Delta p_t) + c_2(y_t - y_{t-1})] + v_t.$$

Then (50), (51), and (52) comprise a system for which the MSV solution is of the form

(53) $$y_t = \phi_{10} + \phi_{11}y_{t-1} + \phi_{12}v_t + \phi_{13}e_t$$

(54) $$\Delta p_t = \phi_{20} + \phi_{21}y_{t-1} + \phi_{22}v_t + \phi_{23}e_t$$

(55) $$\Delta s_t = \phi_{30} + \phi_{31}y_{t-1} + \phi_{32}v_t + \phi_{33}e_t.$$

Thus we have $Ey_{t+1} = \phi_{10} + \phi_{11}y_t$, $E_t\Delta p_{t+1} = \phi_{20} + \phi_{21}y_t$, and $E_t\Delta s_{t+1} = \phi_{30} + \phi_{31}y_t$ with y_t given by (53). Substitution into (50)-(52) and application of the undetermined coefficients procedure indicates that the solution values (ignoring constants) are as follows, where $\Phi \equiv 1-\alpha + b_1\phi_{21} + b_2c_2$:

(56) $$\phi_{11} = \alpha \qquad \phi_{12} = 1/\Phi \qquad \phi_{13} = -b_2c_1/\Phi$$

$$\phi_{21} = \frac{b_2c_2(1-\alpha) - b_2c_1\alpha\mu_2 - \alpha(1-\alpha)}{b_1 - c_1b_2(1-\mu_1)} \qquad \phi_{22} = 0 \qquad \phi_{23} = 0$$

$$\phi_{31} = \alpha\mu_2 + \mu_1\phi_{21} \qquad \phi_{32} = 0 \qquad \phi_{33} = 1$$

Unfortunately, it is not possible to determine the sign of ϕ_{21} or therefore Φ on the basis of the qualitative specification given above. Nevertheless, it can be seen from (56) that as $\mu_1 \rightarrow \infty$, i.e., as the strength of policy response to $\Delta p_t - \pi^*$ increases without bound, $\phi_{21} \rightarrow 0$ and therefore the variance of $\Delta p_t - \pi^*$ goes to zero. Also, as $\mu_2 \rightarrow \infty$, $\phi_{21} \rightarrow \pm\infty$, so $1/\Phi \rightarrow 0$ causing the variability of y_t relative to $E_{t-1}y_t = \alpha y_{t-1}$ to approach zero. So the exchange-rate-based stabilization rule (51) possesses policy effectiveness. The extent to which this is quantitatively significant will be explored, in a somewhat larger model that is realistically calibrated, in the following section.

Before turning to that exploration it will be appropriate to provide some additional discussion concerning the nature of policy rule (51). First, is such a rule feasible? Is it possible, that is, for a central bank to control an economy's nominal exchange rate under liquidity trap conditions, with (domestic) agents satiated with the transaction-facilitating services of money and a short-term nominal interest rate equal to zero? In that regard it must be noted that, in a model of the type under discussion, the left-hand-side variable in the policy rule is not literally an instrument but rather an indicator variable. Assuming that the model applies to quarterly or monthly time periods, that is, the value of Δs_t on the left-hand-side of (51) can be viewed as an intermediate "operating target" to be obtained by day-to-day or hour-by-hour manipulation of other tools (e.g., open-market purchases) serving literally as the central bank's instrument. The issue, then, is whether a central bank can, based on virtually continuous observation of its exchange rate s_t, push it in the desired direction? There are limits to how far a central bank can reduce

s_t, i.e., appreciate its currency, since it will always hold at most a finite stock of foreign exchange reserves. But depreciation, i.e., upward movement of s_t, is the crucial requirement in the situation under discussion. And it seems clear that there would be no economic limit to the upward movement of s_t that could be engineered by central bank purchases (with high powered money) of foreign exchange.[26]

The idea that s_t can be used as an instrument variable (in the relevant sense) is not a new one. For a number of years, for example, economists associated with the Reserve Bank of New Zealand (RBNZ) used s_t as the instrument variable in analytical descriptions of RBNZ policy; see, e.g., Grimes and Wong (1994) or Hansen and Margaritis (1993). Alternatively, Ball (1999), Gerlack and Smets (1996), and others have described the use by several countries of a "monetary conditions index" as an instrument variable. There are various definitions of a monetary conditions index (MCI), but those that I have seen all feature measures that in some fashion combine a short-term interest rate and an exchange rate. One plausible, dimensionally coherent definition would be

$$mci_t = \omega R_t - (1-\omega)\Delta s_t. \tag{57}$$

Clearly, in a ZLB situation this mci_t measure would reduce to use of a Δs_t instrument, as specified in (51). Thus there seems to be significant practical evidence of two types, as well as a priori reasoning, to support the hypothesis that use of a Δs_t instrument is feasible. Nevertheless, more discussion will be provided, immediately.

In the model presented above, there are two non-standard features. The first is that R_t is held fixed at $R_t = 0$; that feature is imposed so as to address the issues concerned with monetary policy in a ZLB situation. The second feature is that the model apparently does not include a relationship reflecting uncovered interest parity (UIP). In that regard, most analysts (including myself) would normally include UIP as one component of an open-economy macroeconomic model—despite the existence of mountains of empirical evidence that are, at least on the surface, strongly inconsistent with UIP on a quarter-to-quarter basis.[27] So how is UIP avoided here? The answer is as follows.

It is well known that, to be consistent with the data, UIP relations must include a discrepancy term, typically referred to as a risk premium. Thus UIP in empirical models is typically expressed as

$$R_t - R_t^* = E_t \Delta s_{t+1} + \xi_t, \tag{58}$$

where the risk premium ξ_t has a large variance relative to shock terms and furthermore is serially correlated.[28] Recently it has been common practice to treat ξ_t as generated exogenously, but there are theoretical reasons for believing that it would be related to the relative amounts of outside domestic and foreign nominal liabilities outstanding. For example, a hypothesis widely entertained during the 1970s might be expressed as

(59) $$\xi_t = \lambda[B_t - (B_t^* - s_t)] + \zeta_t$$

where B_t and B_t^* are logs of domestic and foreign government debt (including base money) and ζ_t is an exogenous stochastic shock term. Substituting and recognizing that lags could be involved, we then write

(60) $$R_t - R_t^* = (E_t s_{t+1} - s_t) + \lambda (L)[B_t - B_t^* + s_t] + \zeta_t,$$

which is similar to equations prominent in several older writings of Dornbusch (e.g., 1980, p. 169, and 1987, p.7). This "portfolio balance" hypothesis has receded from its earlier prominence because empirical studies by Frankel (1982, 1984), Dooley and Isard (1983), and others failed to find empirical support. But it seems implausible to believe that no such relation obtains in fact, i.e., that ξ_t is totally unaffected by the $B_t - B_t^*$ variable. And if such a relation does obtain, then our procedure above is fully justified. For (60) indicates that even with $R_t = R$, s_t can be affected by purchases of foreign exchange since they alter the value of $B_t - B_t^*$. Yet the precise specification of relation (60) need not be known, and the relation need not be included in the model, for exactly the same reason that money demand functions are not needed in analyses that presume use of an interest rate instrument. Thus appending (60) to the model (48)-(51) would have no effect on the implied behavior of Δp_t, x_t, y_t, or Δs_t; it would merely specify the magnitude of open-market purchases of foreign exchange needed to implement the Δs_t policy rule (51).

Still another way of expressing the argument is as follows. Suppose that policy rule (51) is relevant but the economy is not in a liquidity trap. Then let strict UIP be included as part of the model and note that R_t is determined endogenously. In that determination of R_t, the UIP relation plays a major role—one might say that UIP is the "proximate determinant" of R_t. Next suppose that the economy in question has a fixed nominal exchange rate. Then $E_t\Delta s_{t+1} = 0$ in all periods so the UIP condition implies that the home-foreign interest rate differential is constant over time; in that case the home country's central bank has no influence on R_t, not even temporarily. Most practical analysts would not, however, accept that conclusion. Instead they would view this lack of influence over R_t as a medium-term tendency, and would contend that on a month-to-month or quarter-to-quarter basis the central bank can influence R_t, keeping it temporarily high or low relative to the relevant foreign rate. But that contention implies that strict UIP does not hold on a period-to-period basis. Instead the home-foreign interest differential can be temporarily influenced by policy actions of the central bank as suggested by formulation (60).[29] Some evidence supportive of this position has been provided by Stockman (1992).

6. Quantitative Application

Our objective now is to provide quantitative support for the position developed in the previous section, viz., that a policy feedback rule with an exchange rate instrument can provide macroeconomic stabilization in a situation in which interest rate manipulation is infeasible because of a ZLB. The basic research strategy is to adopt a quantitative open-economy macroeconomic model, alter the policy rule so as to use Δs_t rather than R_t as the left-hand-side instrument or indicator variable, and impose the constraint that $R_t \equiv 0$. The latter step requires that some relationship in the model be ignored to avoid over-determination of the endogenous variables; the relationship that we ignore is UIP.

The model to be used here as a starting point is the small-scale, open economy, quarterly model based on explicit optimizing analysis that is developed by McCallum and Nelson (1999b). It has been utilized subsequently—together with an additional variant—by McCallum (1999a); the next three paragraphs constitute an adaptation of descriptive material taken from the last-mentioned paper.

Basing one's analysis on the assumption of explicit optimizing behavior by the modeled individuals in a general equilibrium setting is obviously not sufficient—and perhaps not necessary—for the creation of a structural model that is specified with reasonable accuracy relative to economic reality. The optimizing general equilibrium approach can be very helpful in this respect, however, since it eliminates potential internal logical inconsistencies that are possible when this source of intellectual discipline is absent. The model at hand, henceforth termed the M-N model, has a simple basic structure since it depicts an economy in which all individuals are infinite-lived and alike. As with many recent models designed for policy analysis, it assumes that goods prices are "sticky," i.e., adjust only slowly in response to changes in conditions. It differs from many previous efforts in this genre, however, in three ways. First, the gradual price-adjustment specification satisfies the strict version of the natural-rate hypothesis. Second, the modeled economy is open to international trade of goods and securities. And, third, individuals' utility functions do not feature time-separability, but instead depart in a manner that reflects habit formation.

This last feature is specified as follows. A typical agent desires at t to maximize $E_t(U_t + \beta U_{t+1} + \ldots)$, where the within-period measure U_t is specified as

(61) $$U_t = \exp(v_t)(\sigma/(\sigma-1))[C_t/C_{t-1}^{h}]^{(\sigma-1)/\sigma} + (1-\gamma)^{-1}[M_t/Pt]^{1-\gamma}.$$

Here C_t is a CES consumption index, M_t/P_t is real domestic money balances, v_t is a stochastic preference shock, and h is a parameter satisfying $0 \leq h < 1$. With h = 0, preferences feature intertemporal separability, but with h>0 there exists "habit formation" that makes consumption demand less volatile.

The open-economy aspect of the model is one in which produced goods may be consumed in the home economy or sold abroad. Imports are exclusively raw materials, used as inputs in a production process that combines these materials and labor according to a CES production function. Capital accumulation is not modeled endogenously, but securities are traded internationally. The relative price of imports in terms of domestic goods, i.e., the real exchange rate, affects the demand for exports and imports, the latter in a explicit maximizing fashion. Nominal exchange rates and the home country one-period nominal interest rate are related in the M-N model by a version of uncovered interest parity that realistically includes a stochastic and highly variable "risk premium" term (as in Taylor (1993b) and many multi-country econometric models). That relationship is not included, however, in the present application.

Price adjustments conform to the P-bar model, mentioned above, but with capacity output $\bar{y}_t$ now treated as a variable that depends upon raw material inputs and the state of technology, the latter driven by an exogenous stochastic shock that enters production in a labor-augmenting fashion.[30] As mentioned above, price adjustment behavior implies $E_{t-1}\tilde{y}_t = \alpha\tilde{y}_{t-1}$, so application of the unconditional expectation operator yields $E\tilde{y}_t = \alpha E\tilde{y}_t$ and with $\alpha \neq 0$ this implies $E\tilde{y}_t = 0$ regardless of the monetary policy rule employed. This strict natural-rate property is not a feature of the Calvo-Rotemberg or Fuhrer-Moore models of price adjustment. Indeed, there are very few sticky-price models that have the natural rate property, the only other one that I know of being Gray-Fischer style nominal contracts that imply limited persistence of $\tilde{y}_t$ magnitudes.

The foregoing paragraphs should provide the reader with a broad qualitative overview of the basic M-N model. Quantitatively, the model is calibrated by reference to empirical relationships estimated in various studies with U.S. data.[31] In terms of openness, a crucial consideration in the present context, the U.S. economy is of course quite similar to Japan or to Euroland (i.e., the members of the European monetary union). For a complete description of the model, the reader may consult McCallum and Nelson (1996b), with an additional price-adjustment variant described in McCallum (1999a).[32]

Our objective now is to combine the M-N model with policy rule (51),[33] generate rational expectations solutions, and then characterize the effects of monetary policy on the behavior of inflation and the output gap $\tilde{y}_t$. In considering policy effects, we shall devote some attention to the unsystematic (shock) component e_t, but will place more emphasis on the systematic part of policy behavior since in practice it accounts for most of the variability of policy instruments.[34] In this analysis use will be made of impulse response functions and also stochastic simulations. In these simulations, all constant terms are set to equal zero—a standard practice in work of this type—so the standard deviation of Δp_t can be interpreted as the root-mean-square-error (RMSE) value of $\Delta p_t - \pi^*$ and the standard deviation of $\tilde{y}_t$ as the RMSE value of $y_t - \bar{y}_t$. In all cases, the reported magnitudes are mean values (of standard deviations) averaged over 100 replications, with each run pertaining to a sample period of 200 quarters (after 53 start-up periods are discarded). Calculation of the RE solutions are conducted using the algorithm of Paul Klein (1997).

A first set of results is presented in Table 2. There for each μ_1, μ_2 combination, the three reported values are standard deviations of Δp_t, $\tilde{y}_t$, and Δs_t, respectively. Going down each column we see that increases in the feedback policy coefficient μ_1 serve to decrease the variability of inflation around its (implicit) target value. Similarly, in each row we see that increases in μ_2 typically decrease the variability of $\tilde{y}_t$, although not strongly in the region $0 < \mu_2 < 1$. Simultaneously, increases in μ_2 serve to increase the variability of inflation over most of the range considered. Thus it is clear that the systematic component of monetary policy is relevant for inflation and output gap stabilization in the ZLB situation under analysis, much as is the case with more familiar policy rule studies.

A more graphic way to represent the stabilizing effects of the policy rule is by means of impulse response functions. Several figures presented below plot responses of y_t (not $\tilde{y}_t$), p_t, Δp_t, q_t, s_t, and R_t to a unit realization of various shocks appearing in the system.[35] In Figure 2A, responses to a unit realization of the v_t taste shock (see equation (61)) are reported for policy rule parameter values of $\mu_1 = 1$ and $\mu_2 = 1$. In Figure 2B the experiment is the same except that μ_1 is increased to 10, reflecting a much stronger monetary policy reaction to departures of inflation from its target level. A comparison of the lower left-hand panels of these two figures shows that the response of inflation to the shock is greatly muted by the stronger policy reaction represented in the second case. Also, the middle right-hand panels reveal clearly the stronger reaction of the Δs_t instrument in this case.

Some readers may be surprised by the negative response of p_t to a positive realization of v_t. There is no clear-cut reason to believe that anything is logically amiss in the model, for the behavior of p_t to a real shock in any dynamic optimizing framework depends in subtle ways on details of the specification.[36] There are, however, some aspects of the model at hand that are not fully consistent with the time series properties of important macroeconomic variables. Most prominent of the failures, perhaps, is the rather small amount of inflation persistence in the basic M-N model.[37] In McCallum (1999a), this problem is attacked by replacing the P-bar price adjustment relation (53) with the following:

(62) $$\Delta p_t = 0.5\, E_t\, \Delta p_{t+1} + 0.5\, \Delta p_{t-1} + \alpha_1 \tilde{y}_t + u_t.$$

The latter, which is similar but not identical to the specification of Fuhrer and Moore (1995), [38] imparts a good bit of persistence to the inflation process as can be seen readily from the lower left-hand panels in parts A and B of Figure 3.

Table 3 and Figures 3A and 3B report results analogous to those presented previously for the M-N model with the P-bar price adjustment relation. In the version with (62) replacing (53) the qualitative conclusions are much the same: the variability of π_t and $\tilde{y}_t$ is smaller with larger values of μ_1 and μ_2, respectively. The figures in Table 3 suggest that inflation variability is quite weakly responsive to μ_1, but the main reason for this finding is that a sizeable fraction of the inflation variability is directly due to the presence of the u_t shock term in the price adjustment rule (62). That component of the variance of Δp_t is only slightly affected by policy. The standard deviation of the output gap, by contrast, appears slightly more responsive to μ_2 in Table 3 than in Table 2.

Figures 3A and 3B present impulse response functions for cases analogous to those in Figures 2A and 2B, i.e., cases with $\mu_1 = 1$ and $\mu_1 = 10$, respectively ($\mu_2 = 1.0$ in both cases). The lower left-hand panels show that the muting of Δp_t responses to this particular taste shock is quite slight, although definitely perceptible. The exchange rate (instrument) reactions are, of course, much larger in the part B panels Of most interest in these figures, probably, are the inflation responses—for two reasons. First, inflation now rises in response to a positive v_t realization, in contrast with Figure 2. Second, the shape of the impulse response function suggests that there is considerable persistence of inflation in the model at hand—which in fact there is.

To conclude this section, let us turn to the unsystematic component of monetary policy—i.e., e_t shocks. The results are shown in Figures 4 and 5, the former pertaining to the basic M-N model and the latter to the variant with price-adjustment equation (62). Part A of each figure has $\mu_1 = 1.0$, $\mu_2 = 1.0$ and part B has $\mu_1 = 10.0$, $\mu_2 = 1.0$. In these figures we see that the strength of policy reaction to $\Delta p_t - \pi^*$ has a major effect on the responses of both inflation and also the output gap, with larger values of μ_1 reducing Δp_t responses sharply and y_t responses considerably. Again, incidentally, inflation persistence shows up as a property of the model with (62). All in all, our quantitative results support the proposition that monetary policy can be effectively stabilizing even with R_t frozen in a liquidity trap.

7. Issues Regarding Dynamic Analysis

In this section the object is to consider some slightly esoteric issues concerning dynamic analysis. This discussion is included because several writers—e.g., Benhabib, Schmitt-Grohe, and Uribe (1998), Krugman (1999), and Reifschneider and Williams (1999)—have suggested that recognition of the existence of a ZLB has drastic effects on the dynamic properties of models that include interest-rate policy rules such as the Taylor rule. It is my own belief that these particular effects represent theoretical curiosa that are not relevant for practical policy analysis, even granting the possibility of a ZLB-induced liquidity trap.[39]

The argument here will be conducted in the context of a simple example. To maintain some continuity in the face of the various topics considered in this paper, let us adopt the model of Section 4, but simplified by elimination of government purchases and stochastic shocks.[40] Also, we now use notation such that y_t and p_t represent <u>logs</u> of output and the price level. Finally, and merely for simplicity, we let $\mu_2 = 0$ in the Taylor rule, making it one of the inflation-targeting variety. With those amendments, the model (34)-(36) can be written as

(63) $$y_t = E_t y_{t+1} + b_0 + b_1 (R_t - E_t \Delta p_{t+1})$$

(64) $$\Delta p_t = (1-\alpha)(\bar{p}_{t-1} - p_{t-1}) + E_{t-1}(\bar{p}_t - \bar{p}_{t-1})$$

(65) $$R_t = -b_0/b_1 - \mu_1 \pi^* + (1+\mu_1)\Delta p_t.$$

Thus we have, as in Section 4, an expectational IS function consistent with optimizing behavior, a price adjustment relation that features some inflation persistence yet satisfies the natural-rate hypothesis, and a policy rule that is designed to stabilize inflation around the target value π^*.

Before seeking a rational expectations solution, we again express (64) as

(53'') $$E_{t-1} y_t = \alpha y_{t-1}$$

and combine (63) with (65) as follows:

(66) $$y_t = E_t y_{t+1} + b_1[(1+\mu_1)\Delta p_t - \mu_1 \pi^* - E_t \Delta p_{t+1}].$$

In this system (53″),(66) there is only one relevant state variable, y_{t-1}, so the unique “bubble-free” or “fundamentals” MSV solution will be of the form

(67) $y_t = \phi_{10} + \phi_{11}y_{t-1}$

(68) $\Delta p_t = \phi_{20} + \phi_{21}y_{t-1}$

and it is clear from (53") that $\phi_{10} = 0$ with $\phi_{11} = \alpha$.[41] Substitution of $E_t y_{t+1} = \alpha(\alpha y_{t-1})$ and $E_t\Delta p_{t+1} = \phi_{20} + \phi_{21}(\alpha y_{t-1})$ into (66), followed by application of the undetermined coefficients (UC) logic, yields the following solution for inflation:

(69) $\Delta p_t = \pi^* + [\alpha(1-\alpha)/b_1(1+\mu_1-\alpha)]\ y_{t-1}$.

Thus Δp_t equals π^* on average and would fluctuate around that value if stochastic shocks were included in the system.

Suppose, however, that in obtaining a solution the analyst specified that Δp_{t-1} is a relevant state variable, even though it appears nowhere in the system (63), (53″), (65). Then instead of (68) we would have

(70) $\Delta p_t = \phi_{20} + \phi_{21}y_{t-1} + \phi_{22}\Delta p_{t-1}$.

Again (53") would imply that $y_t = \alpha y_{t-1}$, but application of the UC procedure would now imply that the solution value for ϕ_{22} is either 0 or $1+\mu_1$. Thus for Δp_t we would obtain either the same solution as before, equation (69), or else

(71) $\Delta p_t = -\mu_1\pi^* - [(1-\alpha)/b_1]\ y_{t-1} + (1+\mu_1)\Delta p_{t-1}$.

The latter gives π^* as the steady-state value of inflation (when $y_t = 0$ and $\Delta p_t = \Delta p$), but with $\mu_1 > 0$ as suggested by Taylor the dynamic behavior of Δp_t would be explosive. If the system "begins" with $\Delta p_{t-1} > \pi^*$ inflation will increase explosively; if the initial value is less than π^* then it will approach $-\infty$, according to (71).[42]

In the absence of a ZLB, $\Delta p_t \to -\infty$ would be ruled out as a solution path in a complete version of the model because it would violate a transversality condition necessary for optimizing behavior. But with recognition of a ZLB, it becomes apparent that inflation cannot behave as specified by (71) when the ZLB is encountered. Instead, the outcome is that Δp_t approaches the negative value $b_0/b_1 = -\bar{r}$, which corresponds to $R_t \to 0$. Thus the Taylor rule has, in this case, failed to stabilize inflation around its target value.[43] For a graphical representation, see Figure 6, which is—so as to permit a two-dimensional diagram—drawn for the special case with complete price flexibility (i.e., $y_t = 0$). If the system begins with an initial inflation rate below π^*, it will approach $-\bar{r}$ in an oscillatory fashion. In the absence of the ZLB, by contrast, inflation would approach $-\infty$ according to (71) so a transversality condition that ruled out such a path would lead the analyst back to (69).

My own conclusion is quite different. It is that the last ZLB solution is not economically relevant.[44] It is a bubble solution that results from designating Δp_{t-1} as a relevant state variable even though it does not appear in the system (i.e., is in my terminology a redundant state variable). It is my belief that emphasis on such bubble or non-fundamental solutions constitutes a perversion of the original objectives of rational expectations analysis.[45] But in any event it can be noted that the MSV solution (69), which clearly is a RE solution to the model at hand, is entirely well behaved so long as $\pi^* > b_0/b = -\bar{r}$. In Figure 6 this solution implies that Δp_t is determined at point A in each period. With $y_t = 0$, there are no dynamic adjustments—which is natural since there are no shocks and no relevant state variables other than y_{t-1} (which equals zero in Figure 6, though not in the more general case considered algebraically).

Furthermore, it should also be noted that the non-MSV solution (71) implies that inflation explodes toward $+\infty$ if the system "begins" with a value above π^*, since fully-developed models typically include no transversality condition that would preclude such behavior. (Again see Figure 6.) Thus if one is inclined to doubt the stabilizing property of Taylor rules, or interest-instrument rules for inflation targeting, then this doubt should logically exist without any regard to ZLB considerations!

The foregoing analysis is not specific to the model utilized, but applies rather generally (I believe) to models with optimizing IS functions and either flexible prices or forward-looking price adjustment specifications.[46] In particular, it would apply to price adjustment relations of the Calvo-Rotemberg type or the more general form (62). I have not used either of these in the foregoing example for expositional convenience: the former gives a MSV solution with no relevant state variables—a case that is expositionally confusing as well as dull—while the latter leads to a cubic expression for the counterpart of the coefficient ϕ_{22} in (70) and is therefore difficult to work with analytically (although the analysis is quite manageable in numerical systems such as those of Section 6).

8. Conclusion

We conclude with a brief overview. The present paper has explored a number of distinct theoretical issues that are relevant to recent discussions regarding the possibility of a zero lower bound (ZLB) on nominal interest rates and the implications of such a bound for monetary policy in regimes with low inflation. First, the paper seeks to spell out an explicit theoretical rationale for the idea that a ZLB may exist and indicates that its validity depends upon the assumption that it is costless at the margin to store money (the economy's medium of exchange). It is argued that the foregoing assumption is probably not correct, strictly speaking, so that negative interest rates are possible. But the quantitative extent of the phenomenon is almost certainly very small. Second, an investigation is conducted of the extent to which the absence of superneutrality will lead to an increase in the steady-state real rate of interest as steady-state inflation is reduced (and turned negative) by sustained policy. The conclusion based on a quantitative overlapping generations model is that this effect is unlikely to be of much importance in the context of stabilization issues, although it is of considerable theoretical relevance as it suggests that real rates would rise sufficiently to keep an economy's steady-state nominal rate positive even with sizeable rates of deflation. Next, the analysis returns to models with the property of real-interest invariance to maintained inflation and explores the suitability of the common practice of conducting monetary policy analysis in models with no monetary variables. It is argued that this practice is almost certainly unjustified in a strict sense, but again the quantitative magnitude of the omitted effects is estimated to be very small.

The most important analysis, from the perspective of current policy issues, is that of Sections 5 and 6. In the former it is shown analytically that even if short run nominal interest rates are fixed at zero, there nevertheless exits a route for monetary policy actions to exert stabilizing effects on inflation and output (relative to capacity). This route, available in any economy that is open to foreign trade of goods and securities, works by a policy rule that adjusts the rate of depreciation of the exchange rate, acting in the role of an instrument variable, so as to meet stabilization objectives. The analysis presumes that strict uncovered interest parity does not prevail on a period by period basis, a presumption for which there is much empirical justification. Then in Section 6 the quantitative magnitude of this stabilization strategy is investigated by means of simulations with a small but complete macroeconomic model, one that is designed to be consistent with optimizing analysis and calibrated to U.S. quarterly data. The results suggest that the extent of stabilization that can be obtained by this exchange-rate approach is substantial.

Finally, recent warnings concerning some alarming theoretical results, obtained with Taylor-style policy rules in optimizing models that recognize the existence of a ZLB, are reconsidered. It is argued that these anomalous and undesirable effects obtain only when non-fundamental “bubble” solutions are considered despite the existence of fundamental solutions. Consequently, it is suggested—but not established conclusively—that the empirical relevance of such effects is highly dubious. Furthermore, if bubble solutions are considered then undesirable outcomes occur even if there is no ZLB.

Table 1

Effects of Inflation on Steady-State Real Interest Rate

Infl.,% pa	π	c	x	k	m	r, % pa
10	9.835	16.39	16.47	6.781	0.178	3.86
5	2.386	16.33	17.21	6.719	0.239	3.89
2	0.641	16.28	17.44	6.672	0.288	3.90
1	0.283	16.26	17.49	6.648	0.308	3.91
0	0.000	16.23	17.53	6.617	0.333	3.93
-2	-0.397	16.14	17.57	6.510	0.408	3.97
-5	- .723	13.72	16.19	3.947	1.958	5.26
-10	- .928	5.86	9.70	0.462	3.261	11.11

Table 2

Simulation Results with Basic Model
Standard Deviations of Δp_t, $\tilde{y}_t$, and Δs_t

	Value of μ_2				
Value of μ_1	0.0	0.5	1.0	5.0	10.0
0.0	2.19	2.00	2.55	8.36	12.25
	1.53	1.48	1.42	0.97	0.68
	3.98	4.05	4.16	5.91	7.31
0.5	1.46	1.26	1.53	5.57	8.64
	1.59	1.52	1.44	1.09	0.81
	4.06	4.14	4.17	5.15	6.20
1.0	1.12	0.93	1.04	4.00	6.63
	1.53	1.53	1.45	1.14	0.92
	4.18	4.17	4.25	5.07	6.21
5.0	0.47	0.38	0.33	1.09	2.16
	1.59	1.53	1.53	1.36	1.24
	4.67	4.61	4.61	4.85	5.81
10.0	0.29	0.24	0.21	0.53	1.14
	1.54	1.53	1.54	1.43	1.37
	4.92	4.91	4.86	4.84	5.30

Table 3

Simulation Results in Model with Equation (62)
Standard Deviations of Δp_t, $\tilde{y}_t$, and Δs_t

	Value of μ_2				
Value of μ_1	0.0	0.5	1.0	5.0	10.0
0.0	3.56	3.68	3.78	4.55	5.16
	2.83	2.32	1.96	1.48	1.58
	4.00	4.16	4.42	8.13	15.26
0.5	3.52	3.48	3.66	4.37	4.83
	2.90	2.37	2.13	1.46	1.44
	4.29	4.38	4.66	8.35	14.59
1.0	3.42	3.45	3.39	4.20	4.60
	2.88	2.51	2.14	1.49	1.39
	4.99	5.09	5.27	8.88	14.57
5.0	3.13	3.10	3.10	3.39	3.78
	3.37	3.24	3.04	2.14	1.64
	13.72	13.56	13.42	14.34	16.81
10.0	2.82	2.92	2.85	3.02	3.30
	3.80	3.87	3.60	2.85	2.24
	23.41	23.87	23.15	21.93	22.19

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Figure 1: Alternative Specifications of Transaction Cost Function

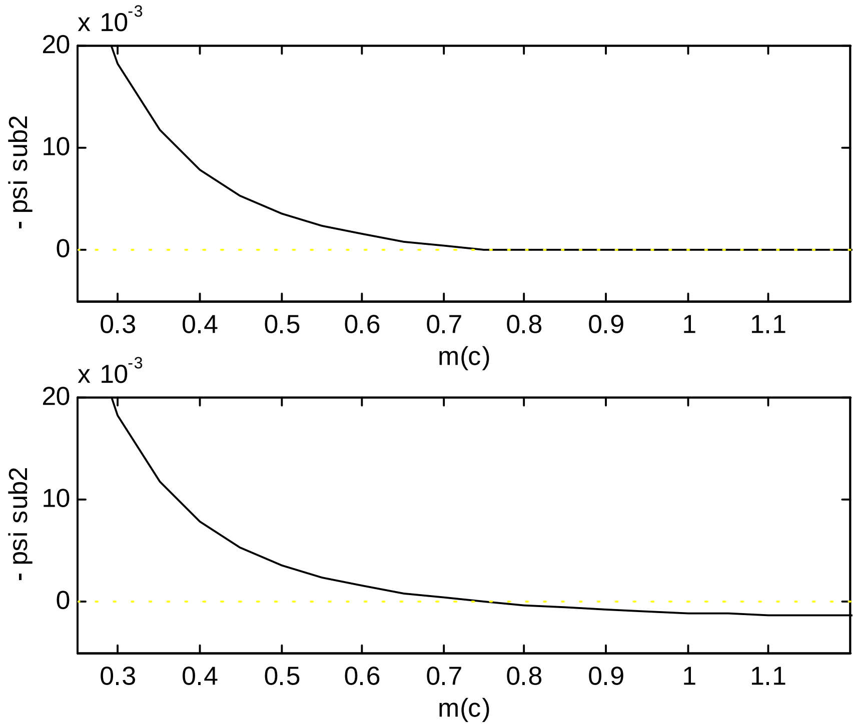
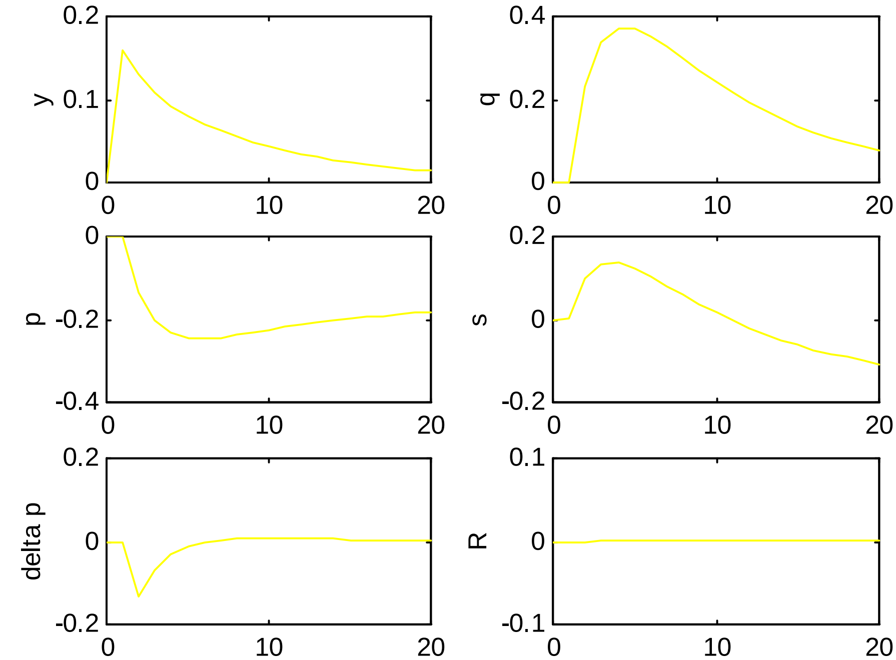
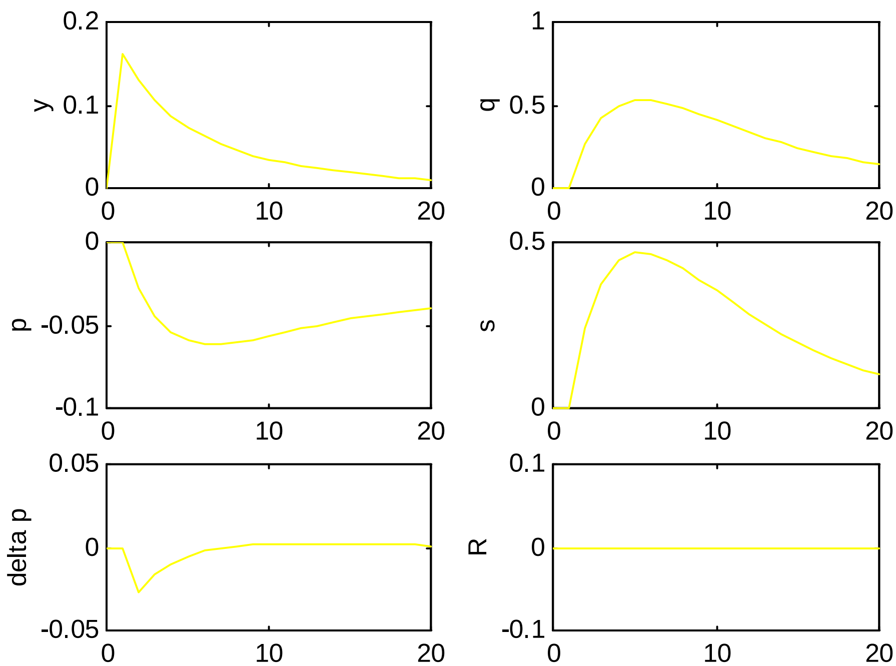


Figure 2: Impulse Responses to IS Shock, Basic Model

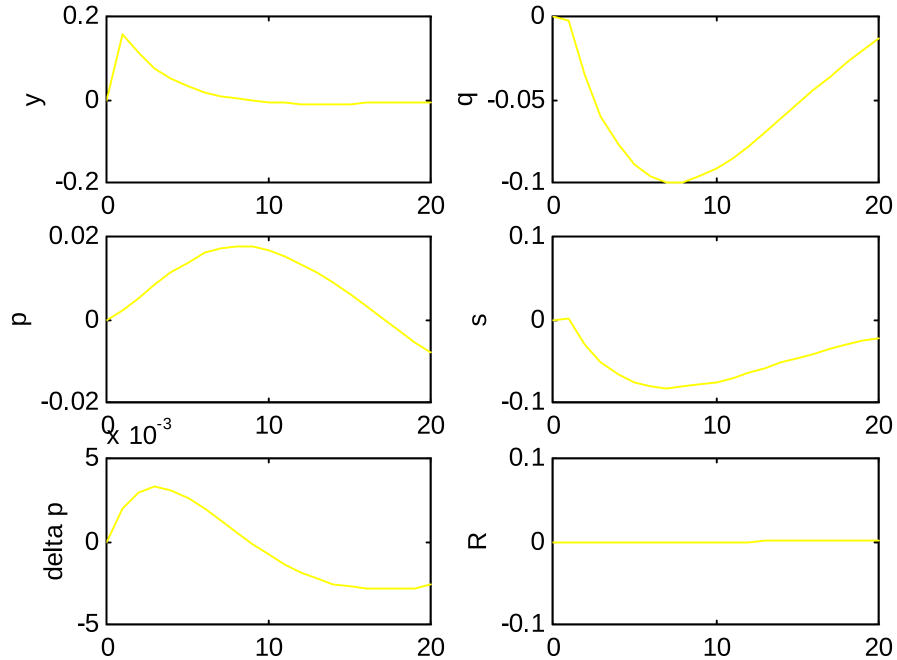


2A: Responses to Unit Shock to IS; mu1=1.0, mu2=1.0

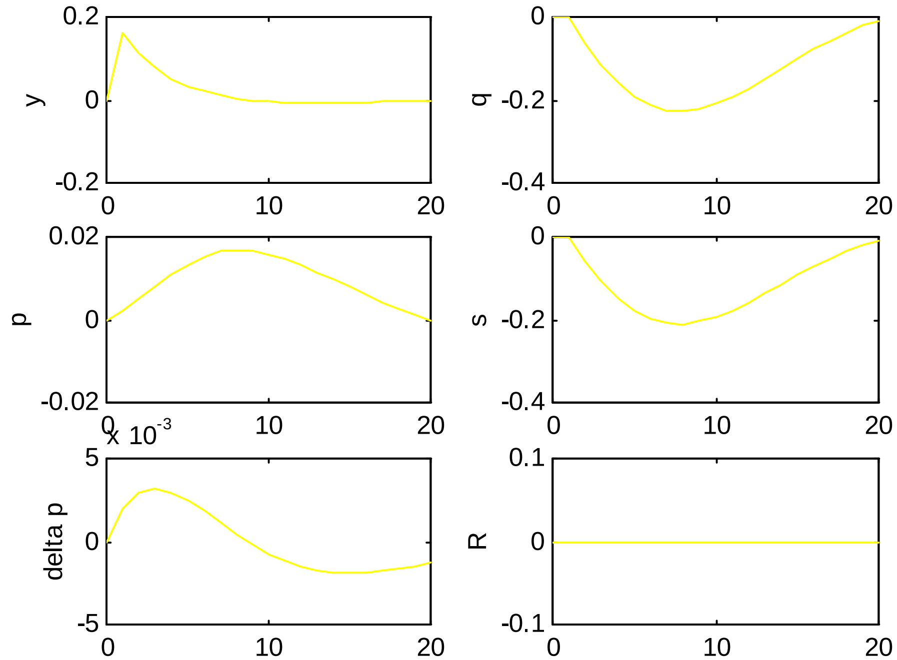


2B: Responses to Unit Shock to IS; mu1=10.0, mu2=1.0

Figure 3: Impulse Responses to IS Shock, Model with (62)

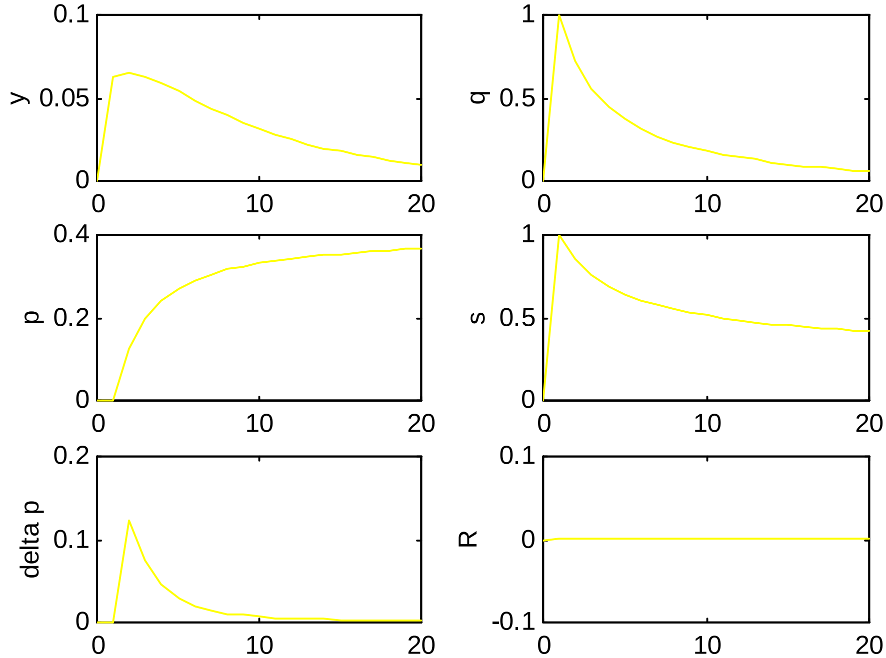


3A Responses to Unit Shock to IS, mu1=1.0, mu2=1.0

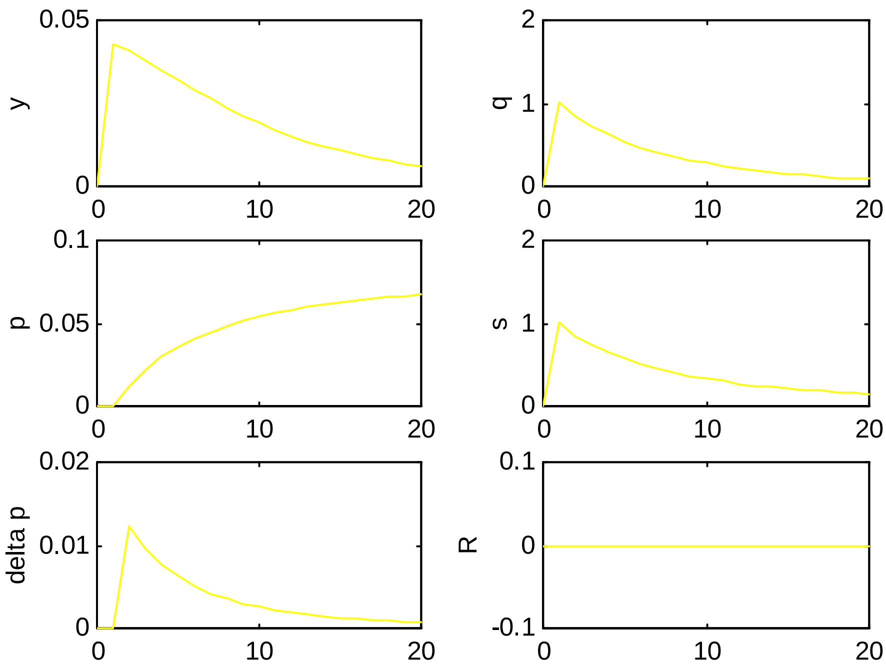


3B Responses to Unit Shock to IS, mu1=1.0, mu2=10.0

Figure 4: Impulse Responses to Policy Shock, Basic Model

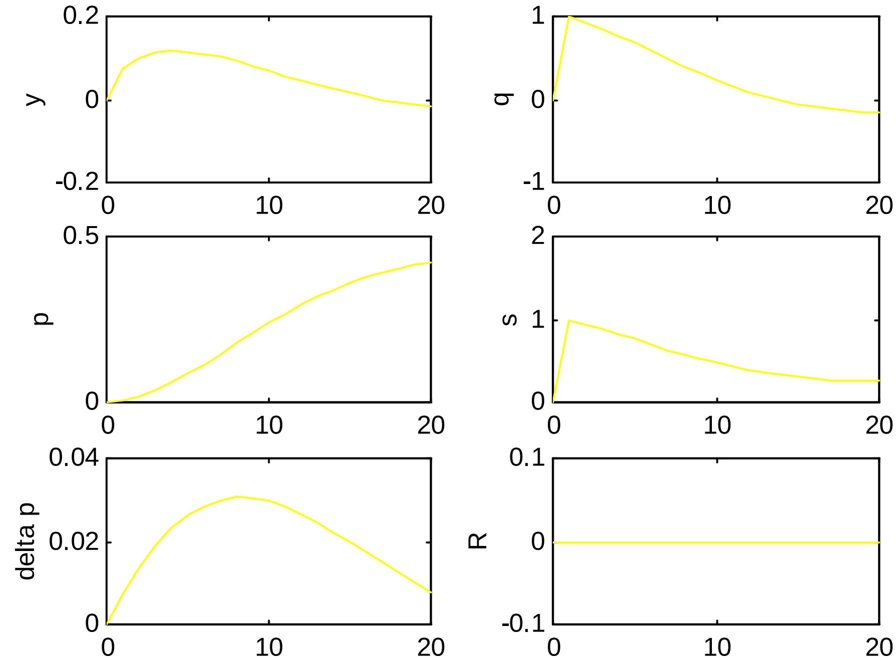


4A: Responses to Unit Shock to Policy Rule, mu1=1.0, mu2=1.0

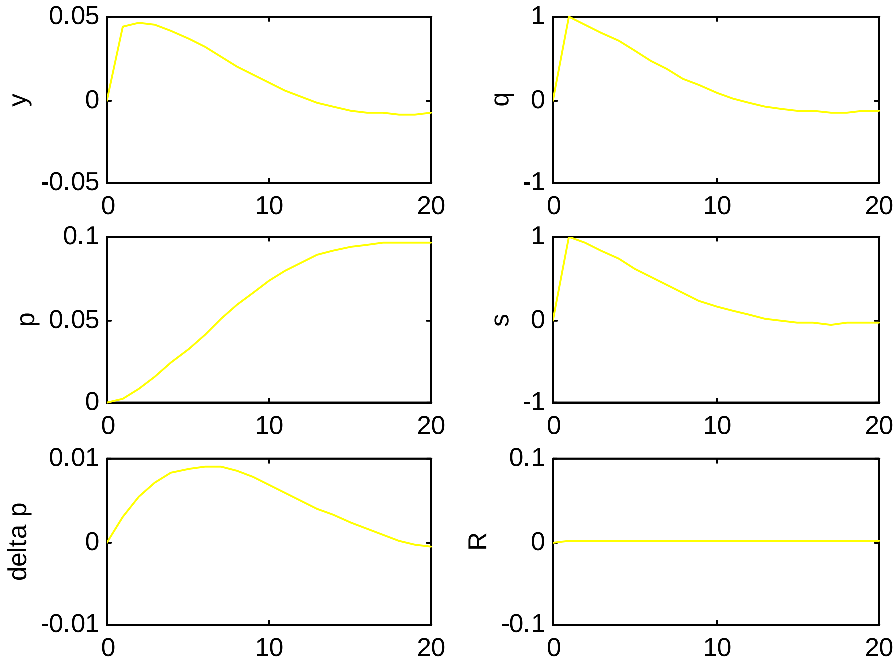


4B: Responses to Unit Shock to Policy Rule, mu1=10.0, mu2=1.0

Figure 5: Impulse Responses to Policy Shock, Model with (62)



5A: Responses to Unit Shock to Policy Rule, mu1=1.0, mu2=1.0



5B: Responses to Unit Shock to Policy Rule, mu1=10.0, mu2=1.0

Figure 6: Inflation Dynamics In Model (63)-(65)

References

[1] Henceforth, the term "interest rate" should be understood to mean nominal interest, unless the modifier "real" is included or very obviously implied.

[2] More specifically, c_t is the number of many-commodity bundles consumed during t, as discussed below.

[3] See, for example, the treatments in Rotemberg and Woodford (1997, pp. 308-310), Blanchard and Fischer (1989, pp. 376-381), or Obstfeld and Rogoff (1996, pp. 236-8, 661-5).

[4] There has been a sizeable volume of theoretical work in recent years that seeks to provide a firm micro-theoretic basis for the MOE role of money; leading examples are Kiyotaki and Wright (1989), Lacker and Schreft (1996), and Wallace (1997). The specification of $\psi(\ )$ used here is intended to serve as a reduced-form shorthand for these analyses, one that is suitable for macroeconomic (but not microeconomic) issues. The model does not explain which asset society has somehow selected as the MOE, but the discussion presumes that it is paper money issued by a governmentally sponsored central bank.

[5] An exception is that the cash-in-advance setup implies an interest-inelastic money demand function.

[6] With paper money, pure storage costs would depend upon nominal rather than real money balances. But one can construct a relationship such as that of Figure 1B nevertheless, contingent upon some assumption regarding the composition of bills by denomination. For "storage" costs that reflect insurance or guard services, the relationship would pertain to real balances in any case.

[7] For some supportive argumentation, see Thornton (1999).

[8] The policy buffer term is taken from Clouse et.al. (1999).

[9] It is natural to ask why no such shopping considerations apply to young consumers. For simplicity we assume that they must obtain their goods by barter since they have no assets when born. The cost of conducting barter exchanges should then be included also, but has been omitted for simplicity. It would appear that this omission should not have major misleading effects on the analysis that follows.

[10] In this setup, $m_t = M_t/P_t$ is not assumed but can be shown to be implied by the equilibrium conditions. Also implied is the overall resource constraint $f(n_t, k_t) + (1-\delta)k_t = x_{t+1} + (1+\nu)c_t + (1+\nu)k_{t+1} + g_t + \psi(x_t,m_t)$.

[11] Virtually identical results were obtained with an elasticity of 0.4, i.e., $\theta = 2.5$.

[12] Here AS and MP stand for aggregate supply and monetary policy, respectively.

[13] For a summary of useful approximation formulae, see Uhlig (1997).

[14] A formulation with endogenous investment, together with an analysis of the constant-capital assumption, is developed by Casares and McCallum (1999).

[15] This is apparently the way that Woodford (1999) views the issue.

[16] Here (c/m) is interpreted as a steady-state value. Similar usages appear below, e.g., in (46).

[17] For the latter, we use recent U.S. ratios of consumption to M1.

[18] That Woodford uses a money-in-the-utility-function formulation, rather than (42), seems an inessential difference.

[19] That is the case for the model of this section because (abstracting from uncertainty) capital and bonds are perfect substitutes.

[20] In Casares and McCallum (1999) the model includes bonds and capital goods that are not perfect substitutes, because of capital adjustment costs. But their relative prices are not influenced by the systematic component of monetary policy.

[21] I am indebted to Edward Nelson for encouraging me to pursue the approach developed in this section and the one to follow.

[22] More precisely, x_t is the log of exports minus the log of imports.

[23] In the next section, a model is presented in which the import component of (49) is modelled in an optimizing fashion.

[24] Here MSV stands for "minimum state variable." For a recent discussion that characterizes the MSV solution as the solution that excludes "bubble" components, together with the exposition of an algorithm that yields the unique MSV solution in a very broad class of linear models, see McCallum (1999b).

[25] Here it can be seen why a purely contemporaneous version of (49) is unsatisfactory in the present setup: it would introduce $E_t\Delta p_{t+1}$ rather than Δp_t into (52), and then Δp_t would appear nowhere in the system.

[26] Of course there might be political limits, but that is a different matter altogether, outside the scope of the present paper.

[27] A standard reference is Lewis (1995). For data averaged over long time periods, see Flood and Taylor (1997).

[28] In McCallum and Nelson (1999a), the variance is by far the largest of any exogenous disturbance and the process is an AR(1) with coefficient 0.5. These values were taken from evidence in Taylor (1993b).

[29] In this regard, see the evidence of Flood and Rose (1996) for members of the European Exchange Rate Mechanism.

[30] As mentioned above, we treat capital as exogenously determined.

[31] Notably, the value of 0.8 for h in (61) was estimated by Fuhrer (1998).

[32] One small change effected in the latter reference and utilized here is to use 0.95 rather than 1.00 for the autoregressive coefficient in AR(1) processes generating technology and foreign income shocks.

[33] For the standard deviation of the policy shock term e_t, I have used 0.01. This is much larger than is estimated for actual policy rules with an R_t instrument, but is only one-fourth as large as standard deviations of Δs_t for major economies under current policy regimes.

[34] For an argument to this effect—but presuming an interest rate instrument—see McCallum (1999a).

[35] Here and below q_t denotes the log of the real exchange rate.

[36] In the present model, the response of Δp_t to $v_t > 0$ would be positive if v_t were an AR(1) process with autocorrelation coefficient of 0.5.

[37] There is some persistence of Δp_t shown in Figure 2, which is not the case for several prominent models with sticky price levels (see Nelson (1998)), but not much.

[38] The present implementation follows McCallum (1999a) in setting $\alpha_1 = 0.0032$ and $\sigma_u = 0.02$.

[39] It should be unnecessary to mention that many accomplished theorists are likely to disagree with my views on this particular issue.

[40] This streamlining is irrelevant for the issues at hand.

[41] It should be emphasized that y_t would not be policy-invariant if the system included stochastic shocks.

[42] Here the word "begins" is put in quote marks because the MSV approach suggests that any beginning or initial value is irrelevant.

[43] Related problems are emphasized, in a limited-participated model, by Christiano and Gust (1999).

[44] Woodford (1999, Sect. 3) also argues that the solution of the previous paragraph is unlikely to prevail in actual economies, but his reasoning is different and will often lead to conclusions that do not agree with mine.

[45] The second objective, in addition to ruling out the possibility of persistently-maintained expectational errors, was to provide an objective list—dictated by the model—of relevant determinants of expectations.

[46] The specification used on pp. 7-10 of Reifschneider and Williams (1999) includes IS and price-adjustment relations that are entirely backward looking.