Securitization, Transparency and Liquidity

Marco Pagano and Paolo Volpin

3 December 2008

Abstract

We present a model in which issuers of structured bonds choose coarse and opaque ratings to enhance the liquidity of their primary market, at the cost of reducing secondary market liquidity or even causing it to freeze. The degree of transparency is inefficiently low if the social value of secondary market liquidity exceeds its private value. We analyze various types of public intervention – requiring transparency for rating agencies, providing liquidity to distressed banks or supporting secondary market prices – and find that their welfare implications are quite different. Finally, transparency is greater if issuers restrain the issue size, or tranche it so as to sell the more information-sensitive tranche to sophisticated investors only.

JEL classification: D82, G21, G18.

Keywords: securitization, rating, liquidity, subprime lending crisis, default.

Authors’ affiliations: Marco Pagano, Università di Napoli Federico II, CSEF, EIEF and CEPR; e-mail: mrpagano@tin.it. Paolo Volpin, London Business School and CEPR; e-mail: pvolpin@london.edu.

Acknowledgments: We thank Fernando Alvarez, Giovanni Dell’Ariccia, Ingolf Dittmann, Vito Gala, Denis Gromb, Gustav Loeffler, Guillaume Plantin, Daniele Terlizzese, Raman Uppal, Dimitri Vayanos and Xavier Vives for providing insightful comments. We also thank seminar participants at the EIEF Workshop on the Subprime Crisis (Rome), the International Conference on Price, Liquidity and Credit Risk (Kostanz), Amsterdam, Brunel, Erasmus University Rotterdam, IESE (Barcelona), IMF and the London Business School. The Paul Woolley Centre for the Study of Capital Market Dysfunctionality (LSE) provided a congenial environment for the first draft of this paper.
1 Introduction

The securitization of mortgage loans is now the consensus culprit in the 2007-08 sub-prime lending crisis, to the point that it can be regarded as its distinctive feature (Adrian and Shin, 2008; Brunnermeier, 2008; Gorton, 2008; and Kashyap, Rajan and Stein, 2008, among others). In particular, it is commonplace to lay a good part of the blame for the crisis on the poor transparency of the ratings that accompanied these massive securitizations (see for instance the Financial Stability Forum Report, 2008, and IMF, 2008). Not only is the reliance on ratings held responsible for the very widespread mispricing of risk (Brennan, Hein and Poon, 2008) and for reducing originators’ incentives to base lending on soft information (Rajan, Seru and Vig, 2008); but the implied information loss is seen as a source of the subsequent market illiquidity. After June 2007, the market for all structured debt securities shut down, and even money market liquidity often dried up. This illiquidity in turn created an enormous overhang of illiquid assets on banks’ balance sheets, triggering or aggravating the credit crunch (Spaventa, 2008).

However, the links between securitization, ratings and market liquidity are less than obvious. How does the opaque rating process affect the market liquidity of structured bonds? And if it does, why should issuers of such bonds choose opaque rather than transparent ratings? After all, if the secondary market is expected to be illiquid, the issue price should be lower.1 But the pre-crisis behavior of issuers and investors alike suggests instead that they both saw considerable benefits in securitization based on relatively coarse information. The fact that this process is now highlighted as a major inefficiency suggests that there is a discrepancy between the private and the social benefits of transparency in securitization. What is the source of the discrepancy, and when should it be greatest? How do different forms of pub-

1This insight is consistent with the results by Fahri, Lerner and Tirole (2008), who present a model where sellers of a product of uncertain quality buy certification services from information certifiers. In their setting, sellers always prefer certification to be transparent rather than opaque.
lic intervention compare in dealing with the problem? These questions are crucial in view of the current plans of tightening regulation of rating agencies in both the United States and Europe.

In this paper, we propose a model of the impact of transparency on the market for complex securities that addresses these issues. Issuers may wish their structured bonds to carry coarse ratings in order to expand the size and liquidity of their primary market. This is because few potential buyers are sophisticated enough to understand the pricing implications of complex information. Releasing such information would create a “winner’s curse” problem for unsophisticated investors in the issue market. This is one instance of a more general pattern: when some investors have limited ability to process information, releasing more public information may increase adverse selection and so reduce market liquidity. Incidentally, this underscores the point that the standard thesis that transparency enhances liquidity hinges on the assumption that market participants are all equally skilled at information processing.

But while uninformative ratings enhance liquidity in the primary market, they may reduce it, even drastically, in the secondary market. This is because the information that is not disclosed at the issue stage may still be uncovered by sophisticated investors later on, especially if it enables them to earn large rents in secondary market trading. So limiting transparency at the issue stage shifts the adverse selection problem to the secondary market. In choosing the degree of rating transparency, issuers effectively face a tradeoff between primary and secondary market liquidity.

Their choice of transparency will depend on the value that investors are expected to place on secondary market liquidity, and on the severity of the adverse selection problem in the primary market. If secondary market liquidity is valuable and adverse selection would not greatly damage primary market liquidity, then issuers will choose

---

2For instance, the complex information might concern the covariance between default losses and the marginal utility of consumption. Coval, Jurek and Stafford (2007) study the mispricing that arises if ratings only assess the probability of default and fail to indicate whether default is likely to occur in high-marginal-utility states. Brennan, Hein and Poon (2008) show that some mispricing arises even if ratings assess the expected default loss, rather than simply its probability.
transparent and informative ratings even at the cost of some reduction in primary market liquidity. Conversely, if investors care little about secondary market liquidity and adverse selection would greatly impair that of the primary market, then they will go for coarse and uninformative ratings.

In general, however, the degree of ratings transparency chosen by issuers will fall short of the socially optimal whenever secondary market liquidity has a social value in excess of its private one. This would be the case if a secondary market freeze triggered a cumulative process of defaults and premature liquidation of assets in the economy, for instance because banks’ interlocking debt and credit positions create a gridlock effect. In this case, the socially efficient degree of ratings transparency is higher than that chosen by the issuers of structured bonds, thus creating a rationale for regulation. Mandatory transparency is more likely to be socially desirable when secondary market liquidity is valuable and adverse selection in the primary market is not too severe. Nevertheless, such regulation does have a cost in terms of reduced liquidity at the issue stage.

We also analyze the effects of two forms of \textit{ex-post} public liquidity provision: an intervention targeted to distressed bondholders in case of market freeze, and one intended to support the CDO secondary market price. The former is \textit{ex-post} efficient but reduces the issuers’ \textit{ex-ante} incentives to opt for transparent ratings, because it lowers the costs of secondary market illiquidity associated with low transparency. This it increases the value of low-transparency as against high-transparency securities, and has the undesirable consequence of expanding the parameter region where low transparency and market freeze occur. An intervention aimed at supporting the CDO secondary market price is even more misguided, however: the liquidity injected by the government simply attracts more informed trading, and provides no relief to distressed bondholders who seek liquidity.

Finally, we show that if issuers accept a degree of “restraint” in their issue size or if they tranche the issue, the area in which transparency is privately optimal expands:
a transparent rating no longer causes adverse selection in the primary market if the issue size is such that sophisticated investors alone can buy it and priced to appeal only to them. The gain to the issuer is a smaller discount; the cost, a lower volume. Tranching is an even better way to address the problem, if the tranches are designed and priced so that sophisticated investors purchase the risky, information-sensitive tranche while unsophisticated ones buy the safe and information-insensitive one. This allows the issuer to place a larger issue than he could by limiting the overall size of an untranched issue, but it is feasible only when sophisticated investors have enough wealth to absorb the information-sensitive tranche. If this is the case, then issuers will choose transparent ratings whenever it is socially optimal, which shows that tranching also has a bright side, and is not only a tool to gain from mispricing, as is argued by Brennan et al. (2008).

This argument is akin to that of Gorton and Pennacchi (1990), that the trading losses associated with information asymmetry can be mitigated by designing securities with cash flows that are insensitive to private information and which can therefore be safely bought by uninformed investors who seek them only for their liquidity needs. An even closer argument is offered by Plantin (2004), who shows that when asset-backed securities are sold to heterogenous investors, it is optimal for the sophisticated ones to concentrate on the most junior tranches and leave more senior tranches to the unsophisticated, as this reduces adverse selection on the senior and spurs information collection on the junior tranches.3

Significantly, our model does not posit agency problems in the rating agencies. We assume that rating agencies generate and process information according to the issuer’s instructions, and cannot give the issuer any information beyond what they report to the market. In other words, in the opaque regime, they provide coarse ratings to market and issuer alike. An alternative (or possibly complementary) hypothesis is that the rating agencies can collude with issuers to hide or misreport information to

---

3The latter idea – that tranching is beneficial because it elicits information collection by sophisticated investors – is already present in Boot and Thakor (1993).
the market so as to overstate the quality of the issuers’ securities (Bolton and Freixas, 2008).

A recent report by the Securities and Exchange Commission (SEC, 2008) offers support for both views. Based on a 10-month scrutiny of the three major credit rating agencies – Fitch, Moody’s, Standard & Poor’s – in the recent turmoil in the subprime mortgage-related securities markets, the study finds that some of them appear to have suffered from conflicts of interest, but also indicates that low transparency was a critical aspect of their business. Significant components of the rating process were not disclosed, and the policies and procedures for rating CDOs were poorly documented. These practices, as we suggest here, may be determined by issuers’ choice.

The paper is organized as follows. Section 2 lays out the structure of the model. Section 3 solves for the equilibrium, and identifies the circumstances in which securitization is privately efficient. In Section 4 we determine the cases in which the socially efficient level of transparency may be higher than the privately optimal level, and consider three forms of public intervention: (i) mandating primary market transparency, (ii) liquidity provision targeted to distressed investors in case of a market freeze, and (iii) intervention to support the CDO price in the secondary market. In Section 5 we explore the implications of letting the issuer choose the size of the CDO issue or tranche it, beside the degree of transparency. Section 6 concludes.

2 The Model

An issuer owns a continuum of measure 1 of financial claims (e.g., mortgage loans) and wants to sell them because the proceeds can be invested elsewhere for a net return \( r > 0 \). The payoff of an individual claim \( i \) can be \( v_B \) or \( v_G \), where \( G \) and \( B \) stand for “good” (prime) and “bad” (subprime) loans, respectively. Good and bad loans yield a high payoff \( V_H \) with probabilities \( p + \theta \) and \( p \) respectively, and a low payoff \( V_L \) with probability \( 1 - p - \theta \) and \( 1 - p \). Therefore the expected payoff of good
claims exceeds that of bad claims by \( \theta(V_H - V_L) \).

The bad claim never pays \( V_H \) when the good one pays \( V_L \) but does pay \( V_L \) with probability \( q \) when the good one pays \( V_H \). Therefore \( \theta = (1 - p)q \). The difference between the expected payoffs depends on the parameter \( q \): in the limiting case \( q = 0 \), the two claims have identical payoffs; in the polar opposite case \( q = 1 \), the good claim always pays \( V_H \) and the bad one \( V_L \). The quality of bad claims, as measured by the probability \( p \), is assumed to be common knowledge. In contrast, the parameter \( q \) is unknown to everyone, including the issuer. The actual value of \( q \) equals \((1 + \sigma)/2\) with probability \(1/2\) and \((1 - \sigma)/2\) otherwise, so that its unconditional mean is \(1/2\) and \( \sigma \in [0,1] \) measures uncertainty about \( q \). For notational convenience, we denote the deviation of \( q \) from its mean as \( \tilde{q} \equiv q - 1/2 \).

### 2.1 Securitization

We assume that the issuer must sell these claims as a portfolio because selling them one-by-one would be prohibitively costly.\(^4\) Denoting the fraction of good claims in the portfolio by \( \lambda \), the portfolio per-claim payoff is \( v_P = \lambda v_G + (1 - \lambda) v_B \), which takes three possible values: a high value \( V_H \) if both claim types do well (which occurs with probability \( p \)); an intermediate value \( V_L + \lambda(V_H - V_L) \) if only good claims do well, which happens with probability \( (1 - p)q \); and a low value \( V_L \) if both claim types do poorly, which happens with probability \( (1 - p)(1 - q) \).\(^5\)

The portfolio is sold as a collateralized debt obligation (CDO), promising to repay a face value \( F \) ranging between the high and the intermediate payoffs in Table 1:

\[
F \in (V_L + \lambda(V_H - V_L), V_H].
\] \(^1\)

---

\(^4\)The high cost is because the payoff of each claim has an idiosyncratic random component that is known to the issuer and can be certified by the rating agency at a cost but unknown to investors. So overcoming adverse selection problems would require each individual claim to be rated by the agency - as noted, a prohibitive expense. Pooling the claims diversifies away this idiosyncratic risk, removing the need for the rating agency to perform the detailed assessment.

\(^5\)The state where good claims do poorly and bad ones do well never occurs, by the assumption made in Table 1.
Below we will show that in equilibrium issuers will set the face value to lie within this interval – indeed, they will choose it to equal $V_H$. Therefore, the CDO’s payoff $x$ is its face value $F$ if the underlying portfolio pays $V_H$; otherwise, one of the two default payoffs shown in Table 1. The loss inflicted on the CDO holders is larger when both claim types do poorly (outcome $D_1$) than in the intermediate case (outcome $D_2$).

<table>
<thead>
<tr>
<th>Asset Payoff ($v_P$)</th>
<th>CDO Payoff ($x$)</th>
<th>CDO Outcomes</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_H$</td>
<td>$F$</td>
<td>no default (ND)</td>
<td>$p$</td>
</tr>
<tr>
<td>$V_L + \lambda(V_H - V_L)$</td>
<td>$V_L + \lambda(V_H - V_L)$</td>
<td>default, small loss ($D_1$)</td>
<td>$(1 - p)q$</td>
</tr>
<tr>
<td>$V_L$</td>
<td>$V_L$</td>
<td>default, large loss ($D_2$)</td>
<td>$(1 - p)(1 - q)$</td>
</tr>
</tbody>
</table>

The quality of the portfolio, as measured by $\lambda$, is assumed to be known to the issuer but unknown to investors, who regard it as a random variable with support $[0, 1]$. If the true value of $\lambda$ were not certified by a rating agency, investors would suspect that it is very low, implying a correspondingly low price for the portfolio – an instance of Akerlof’s lemon problem. As a result, at the very least the issuer will want the rating agency to certify the true value of the fraction $\lambda$ of good loans in the portfolio. We define this situation – where only $\lambda$ is disclosed at issue – as a “low-transparency” regime.

In fact, certifying $\lambda$ does not reveal the entire distribution of the CDO’s payoffs. As shown by Table 1, to this purpose one also needs to know $q$, so as to assess how diverse the claims of the underlying portfolio are. Even though, as already explained, this parameter is unknown to issuer and investors alike, we assume that it can be ascertained by a specialized rating agency using the information available at the issue stage. For simplicity, rating agencies are assumed to do this at zero

---

6 The assumption that the issuer does not know about some price-relevant characteristics of his asset, but can learn of them from specialized intermediaries or investors is commonplace in the literature on IPOs, and motivates the book-building method for IPO sales (see Benveniste and Spindt, 1989, among others). A similar assumption is made by Dow, Goldstein, and Guembel (2007) and Hennessy (2008), who show that companies may gain information about their investment opportunities from market prices.
cost, and to disclose the value of \( q \) publicly, due to penalties or reputational costs for misreporting or selective disclosure. We define this regime as “high-transparency”. In our setting, high transparency is equivalent to disclosing the expected default loss

\[
F - V_L - \lambda q(V_H - V_L).
\]

In the low-transparency scenario, each of the ratings published by the agency (Aa1, Aa2, Aa3, A1, A2, A3, Baa1, Baa2, Baa3, etc.) corresponds to a possible value of \( \lambda \) or equivalently to the corresponding value of the expected CDO outcome. For instance, if \( \lambda \) can take 15 possible values, there are 15 possible ratings. In the high-transparency scenario, the number of possible ratings is compounded by the number of possible realizations of \( q \). Since in our setting \( q \) can take one of two values, the number of possible ratings escalates to 30. Alternatively, the rating agency can issue a bi-dimensional rating, the two dimensions being the CDO quality \( \lambda \) and the diversity of the underlying portfolio \( q \). In either case, with high transparency the rating is defined on a more finely partitioned information set.

Investors’ estimate of default losses at the issue stage depends on the informativeness of ratings. In the low-transparency scenario, no investor can assess the probability weights to be assigned to the CDO payoffs in the two default outcomes \( D_1 \) and \( D_2 \). Therefore, all investors rely on the unconditional mean of these weights and attach an equal probability \((1 - p)/2\) to each of the two default outcomes.

In the high-transparency scenario, instead, a fraction \( \mu \) of investors are sophisticated enough to draw the correct pricing implications of the realized \( q \) revealed by the rating agency. These investors can distinguish between \( D_1 \) and \( D_2 \), and therefore upon learning the realized \( q \), they will assess their respective probabilities as \((1 - p)q\) and \((1 - p)(1 - q)\). The remaining \( 1 - \mu \) investors are unsophisticated: they cannot distinguish between the two default outcomes, so they cannot interpret the information on \( q \), and will evaluate the expected loss on the CDO at its unconditional value \( F - (V_L + \lambda \frac{V_H - V_L}{2}) \). In other words, these investors are unable to process the agency’s more finely partitioned information.
Sophisticated investors are assumed to lack the wherewithal to buy the entire CDO issue. Since the price they would offer for the entire issue is the expected CDO payoff conditional on the realized \( q \), the relevant condition is that their total wealth \( A_S < \max_q E(x \mid q) \). In contrast, unsophisticated investors are sufficiently wealthy to absorb the entire issue: their wealth \( A_U > E(x) \), since their offer price for the entire CDO issue is the unconditional expectation of its payoff. As in Rock (1986), these assumptions imply that for the issue to succeed, the price of the CDO must be such as to induce participation by the unsophisticated investors.

### 2.2 Time Line

The time line is shown in Figure 1. At the initial stage 0, the fraction of good claims \( \lambda \) and the diversification parameter \( q \) are determined; the issuer learns the former but not the latter.

At stage 1, the issuer chooses either low or high transparency, the rating agency reveals the corresponding information, and the CDO is sold on the primary market at price \( P_1 \).

At stage 2, people learn whether the CDO is in default or not. At the same time, a fraction \( \pi \) of investors are hit by a liquidity shock and must decide whether to sell their stake in the secondary market (at stage 3) or else to liquidate other assets at a discount \( \Delta \) (as by fire sales of securities or recalling loans). Alternatively, \( \Delta \) may be seen as the bank’s private cost of failing to meet an obligation to its own lenders (e.g., the penalty for restructuring a loan owed to another bank). If default is announced, the sophisticated investors not hit by the liquidity shock may try to acquire costly information about \( q \) to trade on it (unless of course \( q \) was already disclosed at stage 1).

---

\( ^7 \)It is important to notice that the relevant constraint arises when \( \tilde{q} = \sigma/2 \). In fact, if \( A_S \in \{ E(x \mid \tilde{q} = -\sigma/2), E(x \mid \tilde{q} = \sigma/2) \} \), sophisticated investors can buy the entire issue if \( \tilde{q} = -\sigma/2 \) at a price \( E(v_P \mid \tilde{q} = -\sigma/2) \). If instead \( \tilde{q} = \sigma/2 \), sophisticated investors are not wealthy enough, so unsophisticated investors are needed. However, the latter cannot distinguish between the two scenarios and can only participate in both cases or in neither. Hence, adverse selection applies.
Their probability $\phi$ of discovering $q$ is increasing in the resources spent on information acquisition: they learn it with probability $\phi$ by paying a cost $C\phi$.

At stage 3, investors can trade the CDO on a secondary market, where risk-neutral and competitive market makers set bid and ask quotes so as to make zero profits. These market makers are sophisticated, in that they are able to draw the pricing implications of the realized value of $q$, if this is publicly disclosed.

At stage 4, the payoffs of the underlying portfolios and of the CDOs are realized.

### 2.3 Private and Social Value of Liquidity

As we have seen, the investors who may seek liquidity on the secondary market are “discretionary liquidity traders”: their demand for liquidity is not completely inelastic, because they can turn to an alternative source of liquidity at a private cost $\Delta$. If the hypothetical discount at which CDOs would trade were to exceed $\Delta$, these investors will refrain from liquidating their CDOs and instead resort to the alternative sources of liquidity.

However, these alternative ways of generating liquidity may entail costs for third parties as well. For instance, if the CDO holder decides to call back loans or to default on his own debts, it may force other borrowers or lenders into default and trigger a chain reaction due to the interlocking balance sheets of banks and firms. Insofar as CDO secondary market liquidity spares this cost to society, its social value exceeds its private value.

For simplicity, we model the additional value of liquidity to third parties as $\gamma \Delta$, where $\gamma \geq 0$ measures the negative externality of secondary market illiquidity. Thus the total social value of liquidity is $(1 + \gamma)\Delta$, and the limiting case $\gamma = 0$ captures a situation where market liquidity generates no externalities.
3 Equilibrium Prices and Transparency

In this setting, what degree of ratings transparency will issuers choose? In this section we solve the model by backward induction, starting with the determination of the equilibrium price at stage 3, when the secondary market for CDOs opens. We then turn to stage 2, when sophisticated investors decide whether to gather information and liquidity traders decide whether to sell their CDOs, and finally to stage 1, when issuers choose the information to be gathered and disclosed by the rating agency.

3.1 Secondary Market Price

If the CDO is known to repay its face value $F$ (outcome $ND$), its secondary market price is simply:

$$P_{3}^{ND} = E(x | ND) = F.$$

The market is perfectly liquid: if hit by liquidity shocks, investors can sell the CDO at price $P_{3}^{ND}$.

If the CDO is expected to be in default (outcome $D_1$ or $D_2$), to determine the corresponding level of the CDO price $P_{3}^{D}$ we must consider three cases, depending on the information made available to investors at stage 1.

First, in the high-transparency regime, investors and market makers learn the realization of $q$. Since market makers are sophisticated, they interpret the rating and impound the realized $q$ in their secondary market quotes. The CDO’s price at stage 3 is simply the expected value of the underlying portfolio conditional on default, which can be computed as the sum of the payoffs in $D_1$ and $D_2$ shown in Table 2, weighted by their respective probabilities $q$ and $1 - q$:

$$P_{3}^{D} = E(x | D_1 \cup D_2, q) = V_L + \lambda q(V_H - V_L).$$

In this case, the secondary market is perfectly liquid, as prices are fully revealing: liquidity traders have no transaction costs. In this case the market price is a random
variable, whose value depends on the realization of $q$ and on average is equal to:

$$E(P_D^3) = V_L + \frac{\lambda}{2}(V_H - V_L) \equiv \bar{V}_D,$$

which is the unconditional expectation of the CDO recovery value in the default states $D_1$ and $D_2$. Using this notation and recalling the definition $\tilde{q} \equiv q - 1/2$, the secondary market price in the high-transparency regime can be rewritten as the sum of the expected CDO recovery value and a zero-mean innovation:

$$P_D^3 = \bar{V}_D + \lambda \tilde{q}(V_H - V_L).$$

In the low-transparency regime, we need to distinguish between the subgame where sophisticated investors collect information on $q$ and that in which they elect not to. In the latter, all investors estimate $q$ at its expected value $1/2$, so that CDO price at stage 3 is:

$$P_D^3 = E(x \mid D_1 \cup D_2) = \bar{V}_D,$$

which is the average value of the price in the high-transparency regime. In this case too, the secondary market is perfectly liquid, since there are no informational asymmetries between investors, and again liquidity traders have no transaction costs.

In the other subgame, where a fraction $\phi > 0$ of the sophisticated investors become informed, the secondary CDO market is characterized by asymmetric information. In the default states, the market maker will set the bid price $P_D^3$ so as to recover from the uninformed investors what he loses to the informed, as in Glosten and Milgrom (1985). Suppose that investors sell whenever they suffer a liquidity shock, which happens with probability $\pi$ (we verify the validity of this assumption below). Informed investors (a fraction $\phi \mu$ of all investors) may sell even in the absence of a shock, if the bid price is above their estimate of the CDO value; that is, if $q = (1-\sigma)/2$, which occurs with probability $1/2$. To avoid dissipating their informational rents, informed traders will camouflage as liquidity traders, placing the same size orders. Hence, the frequency of an investor submitting a sell order is $\pi + \phi \mu (1-\pi)/2$. The market maker gains $\bar{V}_D - P_D^3$ when he trades with an uninformed investor, and
loses $P^D_3 - \nabla^D + \sigma \lambda (V_H - V_L)/2$ when he trades with an informed one. Hence, his zero-profit condition is
\[
\pi(\nabla^D - P^D_3) = (1 - \pi)\frac{\phi \mu}{2} \left[ P^D_3 - \nabla^D + \sigma \lambda (V_H - V_L)/2 \right],
\]
and the implied equilibrium price is
\[
P^D_3 = \nabla^D - \frac{(1 - \pi)\phi \mu}{2\pi + (1 - \pi)\phi \mu} \frac{\sigma}{2} \lambda (V_H - V_L)
\]
\[= \nabla^D - \frac{(1 - \pi)\phi \mu}{2\pi + (1 - \pi)\phi \mu} R, \tag{2}
\]
where
\[
R \equiv \frac{\sigma}{2} \lambda (V_H - V_L) \tag{3}
\]
is the rent that an informed trader extracts from an uninformed one (conditional on these both trading). As one would expect, the rent is increasing in the variance $\sigma$ of the signal gathered by informed traders and in the signal’s value $\lambda (V_H - V_L)$. The informed traders’ rent $R$ is weighted by the probability of a sell order being placed by an informed trader, $(1 - \pi)\phi \mu/[2\pi + (1 - \pi)\phi \mu]$. This expected rent translates into a discount sustained by liquidity traders in the secondary market: if hit by a liquidity shock, they must sell the CDO at a discount off the unconditional expectation of its final payoff.

### 3.2 Decision whether to Acquire Information

In the low-transparency regime, the sophisticated investors who are not hit by a liquidity shock may have the incentive to learn the realization of $q$. The cost of learning $q$ with probability $\phi$ is $C\phi$. The gain from learning $q$ equals the market makers’ expected trading loss as determined above:
\[
P^D_3 - V_L - (1 - \sigma)\lambda (V_H - V_L)/2 = \frac{2\pi}{2\pi + (1 - \pi)\phi \mu} R,
\]
where in the second step the gain is evaluated at the equilibrium price $P^D_3$ in (2). This gain accrues to informed investors only if condition (4) holds, which ensures
that unsophisticated investors trade in the secondary market, and it is obtained with probability $\frac{1}{2}$, since only when $q = (1 - \sigma)/2$ do informed investors make a profit by selling the CDO.\(^8\) Hence, the expected profit from gathering information is:

$$\frac{\phi}{2} \cdot \frac{2\pi}{2\pi + (1 - \pi)\phi \mu} R - C\phi.$$  

Assuming that in the aggregate sophisticated investors choose to gather information up to the point where these expected profits fall to zero, $\phi$ will be set at the level:

$$\phi^* = \max \left\{ \frac{\pi}{\mu(1 - \pi)} \left( \frac{R}{C} - 2 \right), 0 \right\}.$$  

Therefore, sophisticated investors acquire information – that is, choose $\phi^* > 0$ – only if $R < 2C$.\(^9\)

Note that the sophisticated investors’ decision to collect information is conditional on uninformed traders selling whenever they suffer a liquidity shock. But they will actually want to do so only if the discount does not exceed the reservation value $\Delta$ that they place on liquidity. Formally, they will sell if

$$\Delta \geq \frac{(1 - \pi)\phi \mu}{2\pi + (1 - \pi)\phi \mu} R. \quad (4)$$

When this constraint is satisfied, unsophisticated investors will participate in the secondary market even when it is not perfectly liquid. If, instead, constraint (4) is violated, unsophisticated investors will not trade, market makers will be unable to recoup their losses on trading with informed investors, and the market will freeze.

Substituting $\phi^*$ into equation (2), we obtain the stage-3 equilibrium price when the secondary market is illiquid, while substituting $\phi^*$ into the uninformed investors’ participation constraint (4), we find that these investors participate if $\Delta \geq R - 2C$.

---

\(^8\)With the same probability, sophisticated investors learn that $q = (1 + \sigma)/2$, which cannot be exploited by selling the CDO, since by assumption there are no liquidity buyers.

\(^9\)We assume that the sophisticated investors cannot acquire information about $q$ before the CDO is sold on the primary market. Hence they invest in information only just before the secondary market opens: they therefore wait until they learn whether the security is in default and whether they are not liquidity-constrained.
Notice that in this case the market for the security is inactive and the value of the portfolio to liquidity sellers is $V^D - \Delta$.

Summarizing, when default is expected at stage 3, the secondary market price will depend on the transparency regime chosen at stage 1 and on parameter values as follows:

$$P^D_3 = \begin{cases} 
V^D + \lambda q(V_H - V_L) & \text{with high transparency;} \\
V^D & \text{with low transparency, if } R \leq 2C; \\
V^D - (R - 2C) & \text{with low transparency, if } R \in (2C, 2C + \Delta]; \\
\text{none: market freeze} & \text{with low transparency, if } R > 2C + \Delta.
\end{cases} \quad (5)$$

Based on this result, we characterize the equilibrium secondary market outcome:

**Proposition 1** In the high-transparency regime, the secondary market is perfectly liquid. In the low-transparency regime, the secondary market is

(i) perfectly liquid if the expected rent from informed trading is low: $R \leq 2C$;

(ii) illiquid if the expected rent from informed trading is at an intermediate level: $R \in (2C, 2C + \Delta]$;

(iii) inactive if the expected rent from informed trading is high: $R > 2C + \Delta$.

Therefore, the secondary market’s ability to cater to liquidity sellers varies inversely with the rent that can be earned by informed investors. Since this rent is rationally anticipated when the CDO is sold on the primary market, it will translate into an illiquidity discount at the issue stage, as is shown in the next section.

### 3.3 Primary Market Price

Under the assumptions of Section 2, the equilibrium price in the primary market is such that unsophisticated investors break even in expectation, conditional on their information set and on the probability of their bids being successful:

$$P_1 = \xi E \left( x \mid \tilde{q} = \frac{\sigma}{2} \right) + (1 - \xi) E \left( x \mid \tilde{q} = -\frac{\sigma}{2} \right), \quad (6)$$
where $\xi$ is the probability that unsophisticated investors successfully bid for a high-value CDO, if sophisticated investors play their optimal bidding strategy. Of course, this probability is a function of the issuer’s disclosure policy, which determines the information set of the sophisticated investors.

### 3.3.1 Issue Price with Low Transparency

If the realization of $q$ is not disclosed, at stage 1 the two types of investor are on an equal footing in their valuation of the securities. The price is determined by the unconditional expectation of the payoff, $pF + (1 - p)V^D$, minus the expected stage-3 liquidity costs:

$$P_1 = \begin{cases} 
pF + (1 - p)V^D & \text{if } R \leq 2C; \\
pF + (1 - p)\left[V^D - \pi(R - 2C)\right] & \text{if } R \in (2C, 2C + \Delta]; \\
pF + (1 - p)\left(V^D - \pi\Delta\right) & \text{if } R > 2C + \Delta. 
\end{cases}$$

### 3.3.2 Issue Price with High Transparency

If $q$ is disclosed, the secondary market is expected to be perfectly liquid, so unsophisticated investors require no illiquidity discount on that account. But sophisticated investors have an informational advantage at the issue stage, so that the CDO is underpriced. To see this, consider that for sophisticated investors the conditional expectation of the CDO payoff is

$$E(x \mid q) = pF + (1 - p)[V^D + \lambda \tilde{q}(V_H - V_L)],$$

so that they are willing to bid and pay a price $P > E(x \mid \tilde{q} = -\sigma/2)$ if $\tilde{q} = \sigma/2$, but they place no bids if $\tilde{q} = -\sigma/2$. As a result, if $\tilde{q} = \sigma/2$ both types of investor bid: the sophisticated investors get a share of the portfolio with probability $\mu$ and the unsophisticated get it with probability $1 - \mu$. If $\tilde{q} = -\sigma/2$, unsophisticated investors get a share of the portfolio with certainty.

Thus the probability of an unsophisticated investor’s buying the CDO if $\tilde{q} = \sigma/2$
is $\xi = (1 - \mu)/(2 - \mu) < 1/2$ and, using equation (6), the issue price is

$$P_1 = pF + (1 - p) \left( \nu^O - \frac{\mu}{2 - \mu} R \right),$$

(9)

where $(1 - p)\mu R/(2 - \mu)$ is the discount required by unsophisticated traders to compensate for their winner’s curse. This price is increasing in the fraction of sophisticated investors $\mu$ and in their informational rent $R$, as both these parameters tend to exacerbate adverse selection in the primary market.

### 3.3.3 Face Value of the CDO

The issuer invests any proceeds from the sale of the CDO at a net return $r$. Hence, he will choose the face value of the CDO, $F$, so as to maximize the issue price $P_1$. Notice that $F$ enters in the same way in expressions (7) and (9) for the issue price. Hence, the choice of $F$ is independent of the choice of transparency (to be analyzed in the next section). As both expressions are strictly increasing in $F$, the issue price is maximized for $F = V_H$, which verifies the original assumption (1) about the face value of the CDO.

### 3.4 Choice of Transparency

Which regime will the issuer choose to maximize the issue price $P_1$? The answer boils down to comparing expressions (7) and (9), and is best understood graphically. Figure 2 illustrates how the issuer’s optimal choice depends on the parameters of the model. The probability of the liquidity shock $\pi$ is measured along the horizontal axis, the informational rent in the secondary market $R$ along the vertical axis. In the lowest region where $R \leq 2C$, the issuer will choose low transparency. As the profits from information do not compensate for the cost of its collection, the secondary market is perfectly liquid. Hence, the issuer’s only concern is to avoid underpricing in the primary market, which is achieved by choosing low transparency.

In the intermediate region where $R \in (2C, 2C + \Delta]$, the discount associated with high transparency is $\mu R/(2 - \mu)$, whereas the discount with low transparency is
\( \pi(R - 2C) \). Hence, the regime with low transparency dominates if \( R \left( \pi - \frac{\mu}{2-\mu} \right) < 2\pi C \). This condition is always met if \( \pi < \frac{\mu - \Delta + 2C}{2-\mu} \). In this parameter region, consequently, issuers will go for low transparency if the probability \( \pi \) of investors requiring liquidity and their reservation value of liquidity \( \Delta \) are low, and/or when there is adverse selection in the primary market (i.e. when the proportion \( \mu \) of sophisticated investors is relatively large). Intuitively, if there is little demand for secondary market liquidity and/or adverse selection seriously impedes primary market liquidity, issuers concentrate on avoiding underpricing in the primary market. So they will choose low transparency at the cost of sacrificing liquidity down the road. If this condition is not met, i.e. if \( \pi > \frac{\mu - \Delta + 2C}{2-\mu} \), the choice on transparency also depends on the magnitude of the informational rent \( R \): since the liquidity discount is increasing both in the probability of liquidation \( \pi \) and in the magnitude of the loss to informed traders \( R \), low transparency will be chosen only if a higher \( \pi \) is offset by a lower \( R \). Graphically, we must stay to the left of the curved locus \( R = \pi \Delta + 2 \pi C - \pi \frac{\mu}{2-\mu} \).

In the top region where \( R > 2C + \Delta \), if there is low transparency at the issue stage, the secondary market freezes. So the issuer will bear the expected liquidity cost \( \pi \Delta \), while saving the underpricing cost \( \frac{\mu}{2-\mu} R \). Hence low transparency is preferred if \( \pi \Delta < \frac{\mu}{2-\mu} R \), that is, in the area to the left of the upward-sloping line \( R = \pi (2 - \mu) \Delta / \mu \); to its right, high transparency is preferred. Therefore, as in the intermediate region, there too issuers choose low transparency if \( \pi \) and \( \Delta \) are low and/or \( \mu \) is high, that is, if there is low demand for secondary market liquidity and/or primary market liquidity would be low if ratings were transparent.

In conclusion, high transparency is optimal in the shaded region in Figure 2 where the probability of the liquidity shock is sufficiently great. This shaded region vanishes if \((2 - \mu) \pi \Delta < \mu (\Delta + 2C)\), since in this case the downward-sloping curve lies above the horizontal line \( 2C + \Delta \): if \( \pi \) and \( \Delta \) are sufficiently small and/or \( \mu \) sufficiently large, issuers never choose transparent ratings. Conversely, a transparency region always exists if the abscissa of its leftmost point \( A, \pi = \frac{\mu}{2-\mu} \frac{2C + \Delta}{\Delta} \), is strictly smaller.
than 1, which is equivalent to the condition \( \Delta/C > \mu/(1 - \mu) \). In line with our previous results, this condition is more likely to be met, the larger the reservation price of liquidity \( \Delta \) and the smaller the fraction of informed traders \( \mu \), but it is also more likely to be met if the cost \( C \) of gathering private information is low, so that adverse selection in the secondary market is expected to be severe.

These results are summarized in the following:

**Proposition 2** Issuers choose high transparency in the region \( R \in \left[ \frac{2\pi C(2-\mu)}{\pi(2-\mu)-\mu}, \frac{\pi \Delta(2-\mu)}{\mu} \right] \), whose magnitude is increasing in the probability of liquidation \( \pi \) and in the reservation value of liquidity \( \Delta \), and is decreasing in the fraction of sophisticated investors \( \mu \) and in their information gathering costs \( C \). This region is non-empty if and only if \( \Delta/C > \mu/(1 - \mu) \).

Based on the issuer’s optimal choice of transparency, we can now write the expression for the equilibrium CDO price in the primary market, denoting for brevity by \( \nabla \) the expected value of the portfolio \( pV_H + (1 - p)V_D \):

\[
P_1 = \begin{cases} 
  \nabla & \text{if } R \leq 2C, \\
  \nabla - (1 - p)\pi(R - 2C) & \text{if } R \in (2C, \min\{2C + \Delta, \frac{2\pi C(2-\mu)}{\pi(2-\mu)-\mu}\}], \\
  \nabla - (1 - p)\frac{\mu}{2-\mu}R & \text{if } R \in \left[ \frac{2\pi C(2-\mu)}{\pi(2-\mu)-\mu}, \frac{\pi \Delta(2-\mu)}{\mu} \right], \\
  \nabla - (1 - p)\pi \Delta & \text{if } R > \max\{2C + \Delta, \frac{\pi \Delta(2-\mu)}{\mu}\}. 
\end{cases}
\]

In (10), only the third expression corresponds to the high-transparency regime, where the price contains a discount for the winner’s curse problem in the primary market. The other three expressions show that the implications of low transparency for issue prices differ greatly depending on the parameter region: no discount (top line), a discount due to low secondary market liquidity (second line) or an even deeper discount arising from secondary market freezing (bottom line).
4 Public Policy

The shadow value of liquidity to society may exceed the private value $\Delta$ placed on liquidity by distressed investors. As we saw in Section 2.3, this point is captured by denoting the social value of stage-3 liquidity as $(1 + \gamma)\Delta$, where $\gamma$ measures the intensity of the liquidity externalities. This creates the potential for welfare-enhancing public policies, which can take the form of mandatory transparency on the primary market or else of an intervention aimed at reviving an illiquid secondary market. This intervention can in turn take two forms: it can be targeted to investors hit by the liquidity shock or aimed at supporting the price on the CDO market. In this section we illustrate the effects of these interventions on transparency and social welfare.

4.1 Mandating Transparency

Suppose the government can mandate high transparency at the issue stage: in which parameter regions is this socially efficient? The first step in answering is to define social welfare. Recall that the capital raised by the issuer is invested in some profitable new undertaking, producing a net return $r > 0$. Hence, the proceeds from securitization $P_1$ enter the social welfare weighted by $r$.

With high transparency, social welfare is

$$W = r[V - (1 - p)\frac{\mu}{2 - \mu}R],$$

showing that in this regime inefficiency only arises from adverse selection in the primary market (captured by the second term). With low transparency, welfare is

$$W = \begin{cases} 
V & \text{if } R \leq 2C; \\
V - (1 - p)\pi(R - 2C) & \text{if } R \in (2C, 2C + \Delta]; \\
V - (1 - p)\pi\gamma\Delta - (1 - p)\pi\gamma\Delta & \text{if } R > 2C + \Delta,
\end{cases}$$

showing that inefficiencies (may) arise from adverse selection in the secondary market (in the second and third expressions).
The socially optimal choice depends on the comparison between expressions (11) and (12). This is best done by comparing Figures 2 and 3: the only difference from the private choice of transparency characterized by Proposition 2 is found in the top region \((R > 2C + \Delta)\), where the secondary market freezes if there is low transparency, so that the issuer will sustain the expected liquidity cost \(r\pi\Delta\) but save the underpricing cost \(r\frac{\mu}{2-\mu}R\). However, the secondary market freeze generates an additional social cost due to the negative externality \(\gamma\pi\Delta\).

Figure 3 shows that the area where high transparency is socially optimal is larger than that where it is privately optimal. Within the top region where low transparency triggers a secondary market freeze, high transparency is socially – though not necessarily privately – preferred whenever \((r + \gamma)\pi\Delta > r\frac{\mu}{2-\mu}R\), that is, to the right of the upward-sloping line \(R = \pi\Delta\frac{(r + \gamma)(2 - \mu)}{(r\mu)}\). In other words, transparency is welfare-enhancing if secondary market liquidity has a great social value (high \(\gamma\), \(\pi\) and \(\Delta\)) and/or the supply of primary market liquidity is not severely impaired by transparent ratings (low \(\mu\) and \(R\)). Conversely, when the opposite conditions obtain, low transparency is preferred both privately and socially: if \((r + \gamma)\pi\Delta < r\frac{\mu}{2-\mu}R\), requiring transparent ratings would be detrimental.

The interesting case arises in the dark grey region defined by the condition that \(R \in \left(\frac{\pi\Delta(2-\mu)}{\mu}, \frac{(r + \gamma)\pi\Delta(2-\mu)}{r\mu}\right]\). There, high transparency is socially efficient but privately inefficient. Intuitively, in this area issuers see the underpricing in the primary market as costlier than the expected liquidity cost borne by investors (so low transparency is privately optimal) but less costly than the social harm of market freezing (so high transparency is socially optimal). In this area, making disclosure mandatory is welfare enhancing. To summarize:

**Proposition 3** Mandating high transparency increases welfare if (and only if) (i) the secondary market would otherwise be inactive \((R > 2C + \Delta)\) and (ii) the condition \(R \in \left(\frac{\pi\Delta(2-\mu)}{\mu}, \frac{(r + \gamma)\pi\Delta(2-\mu)}{r\mu}\right]\) is satisfied.
4.2 Liquidity Provision to Distressed Investors

An alternative form of policy intervention is to relieve the liquidity shortage when the secondary market freezes at $t = 3$. Assuming that the market freeze forces CDO holders hit by the liquidity shock to sell other assets at the “fire sale” discount $\Delta$, the government may target liquidity $L \leq \Delta$ to these distressed investors, for instance by purchasing their assets at a discount $\Delta - L$ rather than $\Delta$. In the limiting case $L = \Delta$, it would make their assets perfectly liquid. Alternatively, the government may acquire stakes in the equity of distressed CDO holders and thereby reduce the need for fire sales of assets. In either case, the liquidity injection reduces the reservation value of liquidity from $\Delta$ to $\Delta - L$. This has a social cost $\tau L^2 / 2$, where the parameter $\tau > 0$ captures the cost of the distortionary taxes needed to finance the added liquidity.

This modification of the model has two important consequences. First, the expectation of the liquidity injection may distort the choice of transparency *ex ante*: anticipating that the demand for liquidity will be satisfied to some extent by public intervention, issuers will be less concerned over secondary market liquidity. Second, the liquidity injection affects welfare, so it becomes important to determine its optimal size.

We start with the effect on the choice of transparency. This is easily determined by replacing the reservation value of liquidity $\Delta$ with $\Delta - L$ in Proposition 2: now high transparency is optimal in the region $R \in \left[\frac{2\pi C(2-\mu)}{\pi(2-\mu) - \mu}, \frac{\pi(\Delta-L)(2-\mu)}{\mu}\right]$, whose area is decreasing in $L$ and vanishes for $L < \Delta - C\mu/(1 - \mu)$. The reduction of the high-transparency area is illustrated in Figure 4. With $L > 0$, high transparency is only chosen in the dark-grey area, while if $L = 0$ (as in Proposition 2) it is also chosen in the light-grey area. In this area, the liquidity injection induces the issuer to choose low transparency because it lowers the reservation value of liquidity compared with Proposition 2. Formally, the injection changes the issuer’s indifference condition between low and high transparency to the locus $R = \pi(\Delta-L)(2-\mu)/\mu$, which is flatter than the corresponding line in Figure 2.
The liquidity injection also expands the area in which the secondary market freezes compared with Figure 2: the relevant condition is now \( R > 2C + \Delta - L \), so that the horizontal line above which the freeze occurs shifts downward, as illustrated in Figure 4. Precisely because the intervention reduces the cost of generating liquidity outside the CDO market, liquidity traders will shun the market in a wider range of circumstances. This result, together with the reduced incentives to issuers for transparency, indicates that the liquidity injection, though beneficial \textit{ex post}, may have perverse effects \textit{ex ante}.

This leads us to the second question: how large should be the liquidity injection \( L \) planned in case of market freeze? With high transparency, there is no role for liquidity provision as the market is perfectly liquid. The same is true if transparency is low but \( R \leq 2C + \Delta - L \), so that the market operates, albeit possibly with low liquidity. Therefore, the only relevant case is a complete market freeze, which occurs if \( R > 2C + \Delta - L \). In this region, social welfare has three components: (i) the net value of the CDO, \( r[V - \pi(1-p)(\Delta - L)] \); (ii) the negative externality \( -\gamma \pi(1-p)(\Delta - L) \); and (iii) the expected cost of distortionary taxation \( -\tau \pi(1-p)L^2/2 \). Therefore, social welfare is

\[
W = r[V - \pi(1-p)(\Delta - L)] - \gamma \pi(1-p)(\Delta - L) - \tau \pi(1-p)L^2/2
\]

and the liquidity provider chooses \( L \in [0, \Delta] \) to maximize \( W \). Maximizing this expression with respect to \( L \), the optimal liquidity injection is found to be \( L^* = \min ((r + \gamma) / \tau, \Delta) \). So if it is not set at the corner solution \( \Delta \), which eliminates the “fire sale” discount \( \Delta \) altogether, the optimal liquidity injection is increasing in the profitability of the proceeds from the CDO sale \( (r) \) and in the liquidity externality \( (\gamma) \) and decreasing in the marginal cost of taxes \( (\tau) \).

If the government can precommit to this optimal liquidity provision, and if issuers and investors have rational expectations, we can replace \( L^* \) in the condition that defines the upper bound of the high-transparency region, yielding \( R < \pi [\Delta - \min ((r + \gamma)/\tau, \Delta)] (2 - \mu)/\mu \). This condition is never satisfied when the op-
timal liquidity injection is at its maximal level $\Delta$, implying that in this case the transparency region disappears. But even when the optimal injection is an internal optimum $L^* = (r + \gamma)/\tau < \Delta$, the transparency region $R \in \left[ \frac{2\pi C(2-\mu)}{\pi(2-\mu)-\mu}, \frac{\pi(\Delta-L^*)(2-\mu)}{\mu} \right]$ is non-empty only if the liquidity injection satisfies the more binding constraint $L^* < \Delta - C\mu/[\pi(2-\mu) - \mu]$. The following proposition summarizes these results:

**Proposition 4** If expected, public liquidity provision to the investors in need of liquidity reduces the magnitude of the high-transparency region and increases that of the market freeze region. The optimal liquidity injection is $L^* = \min\{(r + \gamma)/\tau, \Delta\}$ and is consistent with high transparency if and only if it does not exceed $\Delta - C\mu/[\pi(2-\mu) - \mu]$.

### 4.3 Public Intervention in the CDO Market

In the previous section, the government was assumed to target the liquidity injection to the investors hit by a liquidity shock, since it can identify the degree of their distress. Alternatively, the government may intervene to support the market for CDOs without targeting liquidity sellers, either by replacing the market makers or subsidizing them. This was the main feature of the initial version of Paulson plan in the U.S., which envisaged “reverse auctions” aimed at buying back securitized loans from banks – a plan later replaced by an approach targeted at recapitalizing distressed banks, and thus closer to the intervention described in the previous section. In this section, we consider what would be the effect of public intervention in the CDO market.

The only change in the model occurs in the low-transparency regime. With a government subsidy $L$, market makers can now make a loss. Hence, in case of default, they set a higher price than in the basic model in Section 3:

$$P^D_3 = L + \nabla^D - (1 - \pi)\phi\mu R \frac{(1 - \pi)\phi\mu}{2\pi + (1 - \pi)\phi\mu}$$

The difference from Section 3 is the subsidy $L$ that is now transferred to investors selling in the secondary market. This may relieve investors hit by a liquidity shock.
but also increases the incentives of sophisticated investors to acquire information. The total effect of this policy depends on the balance between these effects.

First, the investors hit by a liquidity shock will be more likely to sell in the secondary market. They will do so if:

$$\Delta \geq \frac{(1 - \pi)\phi \mu R}{2\pi + (1 - \pi)\phi \mu} - L.$$ 

Second, the expected profit from gathering information is now:

$$\frac{\phi}{2} \left[ L + \frac{2\pi}{2\pi + \phi \mu (1 - \pi)} R \right] - C\phi.$$ 

Assuming as before that in the aggregate sophisticated investors gather information up to the point where these expected profits are zero, $\phi$ will be set at the level:

$$\phi^* = \max \left\{ \frac{2\pi(R + L - 2C)}{(1 - \pi)\mu(2C - L)}, 0 \right\}.$$ 

Therefore, sophisticated investors are more likely to acquire information the greater is $L$.

Replacing $\phi^*$ into the participation constraint for investors hit by a liquidity shock, we find that these investors participate if $\Delta \geq R + L - 2C - L = R - 2C$. This equation is identical to the one in Section 3. So the price in the secondary market in case of default is unchanged at $P_D^3 = V_D^D - (R - 2C)$.

Therefore injecting liquidity by supporting the market for CDOs has no effect on the equilibrium: namely, there is no change in the size of the area where the market freezes. The only consequence is an increase in the investment in information by the sophisticated investors. Intuitively, the liquidity injection is entirely absorbed by heightened informed trading, so that in equilibrium none of it reaches the liquidity traders whose distress it was intended to alleviate. Hence, if the public funds needed to provide liquidity $L$ are raised via distortionary taxes, we can conclude that:

**Proposition 5** Providing liquidity in the market for CDOs is socially inefficient.
5 Extensions

In the analysis so far, issuers have been assumed to securitize and sell a given portfolio. In this section, we explore how the results are modified if issuers are allowed to choose the size of the portfolio to be securitized, or alternatively to split it into two securities of different risk, a practice known as “tranching”.

These extensions change the model in a critical way. The basic tradeoff between the liquidity of the primary and secondary markets exists only because the sophisticated investors are not numerous and wealthy enough to buy the entire CDO issue. Because of this assumption, uninformed investors must be drawn into the primary market. As these investors cannot process information about \( q \), high transparency comes at the cost of adverse selection in the primary market.

Reducing or tranching the issue are two ways to alleviate the dearth of sophisticated capital. A large enough reduction in issue size eliminates the need for unsophisticated investors, and thus the illiquidity cost of high transparency, albeit at the cost of less revenue. Issuers can do even better by tranching the issue, creating two securities with different sensitivity to complex information, so as to induce sophisticated investors alone to buy the more information-sensitive tranche and the unsophisticated to invest in the safe one. They can thus go for high transparency without reducing the total size of the issue, provided sophisticated investors are wealthy enough as to absorb the entire information-sensitive tranche.

5.1 Reducing the Issue Size

Consider an issuer who at the beginning of stage 1 can proportionately scale down the portfolio of credits that he wishes to securitize. To reduce the number of possible cases, we assume here that \( \Delta/C < \mu/(1-\mu) \), so that without restricting issuance the issuer would always opt for low transparency in equilibrium, based on Proposition 2. We show instead that when the issue size is suitably reduced, under certain parameter
restrictions the issuer may opt for transparent ratings. The question then is whether
the issuer will ever find it worthwhile to bear the cost associated with a smaller issue.

Suppose that instead of selling the entire portfolio, the issuer can sell a fraction $s \in [0,1]$, thus rescaling the portfolio’s payoff to $sx$. Recall that sophisticated investors are assumed to be unable to buy the entire issue in both states of nature, i.e. $A_S < E(x \mid \tilde{q} = \sigma/2)$. But if the issuer can rescale, he can choose to sell a fraction $s$ such that sophisticated investors can buy it entirely, that is, $A_S \geq E(sx \mid \tilde{q} = \sigma/2)$ or $s \leq A_S/E(x \mid \tilde{q} = \sigma/2)$. In this case, there is no loss in disclosing $q$: the issue will sell (fractionally) at the price (8) computed above for the high-transparency regime and, since $F = V_H$ (as shown in Section 3.3.3), the cutoff value of $s$ is:

$$
\overline{s} = \frac{A_S}{pV_H + (1-p)[\varpi^{p} + \lambda \sigma^2(V_H - V_L)]} = \frac{A_S}{\varpi + (1-p)R},
$$

where in the second step we used definition (3). Notice that $\overline{s} < 1$ because by assumption $A_S < \varpi + (1-p)R$. The denominator of this expression is nothing but the equilibrium price $P_1$ that would obtain with transparency setting $\mu = 1$ in the third line of (10), since $\overline{s}$ is chosen precisely so as to make the fraction of sophisticated investors equal to one.

Since the issuer decides whether or not to scale down the issue to $\overline{s}$ without knowing $q$, he bases the decision on the expected issue price of the fractional portfolio, $P_1^\overline{s}$, which is a fraction $\overline{s}$ of the expected portfolio payoff:

$$
P_1^\overline{s} = \overline{s}\varpi = \frac{A_S}{\varpi + (1-p)R}\varpi.
$$

In this case, he will also choose high transparency, as just explained. Alternatively, the issuer can decide to sell the entire issue, that is, set $s = 1$, and opt for low transparency, given our assumption $\Delta/C < \mu/(1-\mu)$. In this case, the revenue from securitization will be:

$$
P_1 = \begin{cases} 
\varpi & \text{if } R \leq 2C \\
\varpi - (1-p)\varpi(R - 2C) & \text{if } R \in (2C, 2C + \Delta] \\
\varpi - (1-p)\varpi\Delta & \text{if } R > 2C + \Delta 
\end{cases}
$$
The choice turns on a comparison between this expression and the revenue \( P_1 \) from the scaled-down sale. The wealth of the sophisticated investors plays a key role. To see this, consider that the revenue \( P_s \) from the restrained sale is directly proportional to \( A_S \). Accordingly, restricting the issue size is not revenue-increasing if the wealth of the sophisticated investors falls short of the threshold

\[ \overline{A} = \left[ \overline{V} + (1-p)(2C + \Delta) \right] \left[ 1 - (1-p) \overline{\Delta} \right] . \]  

(16)

The condition \( A_S > \overline{A} \) is necessary for the smaller sale to be revenue-increasing, but not sufficient. Additional parameter restrictions must be met, as illustrated in Figure 5: choosing \( s = \overline{s} \) and high transparency is revenue-increasing for the issuer only within the grey area below the concave and above the convex curve. If \( A_S > \overline{A} \), then the two curves intersect when the probability of liquidation \( \pi \) is less than 1, so that the region is non-empty. But to be in the grey area, it is also necessary that, for any given \( R \), the value of liquidation probability \( \pi \) is high enough. Specifically, for \( R \in (2C, 2C + \Delta) \), which defines the region of market illiquidity if \( s = 1 \), issuers will opt for \( s = \overline{s} \) and high transparency only if \( \pi > (1 - \overline{s})\overline{V}/[(1-p)(R - 2C)] \). And for \( R > 2C + \Delta \), which defines the region of market freeze if \( s = 1 \), they do so only if \( \pi > (1 - \overline{s})\overline{V}/[(1-p)\Delta] \). In both cases, the intuitive rationale is clear: by curtailing issuance and choosing high transparency, the issuer makes the market liquid when it would otherwise be illiquid or inactive, and this is profitable only if traders are sufficiently likely to liquidate the CDO, that is, if the demand for liquidity is high enough.

These results, proved in the Appendix, are summarized in the following:

**Proposition 6** Reducing the size of the issue is never optimal if sophisticated investors have limited capital: \( A_S \leq \overline{A} \), defined in (16). Otherwise, it is optimal to reduce the size of the issue to \( \overline{s} = A_S/[(\overline{V} + (1-p)R)] < 1 \) and to choose high transparency if: (i) \( \pi > (1 - \overline{s})\overline{V}/[(1-p)(R - 2C)] \) for \( R \in [2C, 2C + \Delta] \) or (ii) \( \pi > (1 - \overline{s})\overline{V}/[(1-p)\Delta] \) for \( R > 2C + \Delta \).
5.2 Tranching

We now allow the issuer to split the CDO issue into two tranches: senior claims $S$ with face value $F_S$ and payoff $x^S$ and junior claims $J$ with face value $F_J$ and payoff $x^J$. In case of default, senior claims will be paid first and in total before junior claims are paid. The former claims are sold at price $P_S$, the latter at $P_J$. Does tranching increase the total proceeds?

Recall that the payoffs of the underlying asset portfolio are $V_H$ if there is no default (which occurs with probability $p$); $V_L + \lambda(V_H - V_L)$ in default with small losses (which occurs with probability $(1-p)q$); and $V_L$ otherwise. Since $V_L$ is the lowest payoff, the issuer can sell a senior claim on the portfolio’s payoff with face value $F_S \leq V_L$ at a price $P_S = F_S$, because this claim is safe. It is in the interest of the issuer to sell as much as possible of this claim. Hence, in equilibrium its face value must be $F_S = V_L$.

To maximize the proceeds from securitization, the issuer can also sell a junior claim with face value $F_J = V_H - V_L$ and a risky payoff $x^J$ that equals 0 with probability $(1-p)(1-q)$, $\lambda(V_H - V_L)$ with probability $(1-p)q$, and $V_H - V_L$ with probability $p$. So the payoff of the junior tranche depends on $q$, and has expected value of:

$$E(x^J|q) = [p + \lambda(1-p)q](V_H - V_L) = (V - V_L) + \lambda(1-p)\tilde{q}(V_H - V_L)$$

There are three possible cases, depending on the wealth of the sophisticated investors. First, if $A_S < E(x^J|\tilde{q} = -\sigma/2)$, it is so low that they cannot buy the entire junior tranche in either state of nature. In this case tranching brings no benefit, because the junior claim must be priced at a discount to attract unsophisticated investors.

Second, if their wealth is in an intermediate range, that is, $A_S \in [E(x^J|\tilde{q} = -\sigma/2), E(x^J|\tilde{q} = \sigma/2)]$, they can buy the entire junior tranche if $\tilde{q} = -\sigma/2$ at a price $E(x^J|\tilde{q} = -\sigma/2)$, but not if $\tilde{q} = \sigma/2$, so that in the good state unsophisticated investors are needed. But the latter cannot distinguish between the two scenarios.
and so only participate in either case or in neither. Hence, for the junior issue to be sold, its price will be discounted due to the adverse selection in the primary market: 

\[ P_J = E(x^J) - (1-p) \frac{\sigma}{2} \mu R \] as in (9). Again, tranching yields no benefits to the issuer.

Finally, the sophisticated investors may be wealthy enough to buy the entire junior tranche in both states of nature; that is:

\[ A_S \geq E(x^J | \tilde{q} = \sigma/2) = (V - V_L) + \lambda(1-p) \frac{\sigma}{2} (V_H - V_L) = (V - V_L) + (1-p)R, \] (17)

which shows that the condition also depends on the adverse selection rent \( R \) not being too large. If this condition holds, tranching is profitable if combined with high transparency, since investors will sort themselves into the two markets according to their degree of sophistication.\(^{10}\) Sophisticated investors will buy the junior security to exploit their superior information-processing ability, and competition between them ensures that this tranche sells at \( P_J = E(x^J | \tilde{q}) \), with no discount. At this price, unsophisticated investors will have no incentive to purchase the junior tranche and will instead self-select into the market for the senior tranche where they suffer no informational disadvantage. Hence, the issuer avoids a tradeoff in the transparency choice: if sophisticated investors are wealthy enough, issuers choose high transparency and tranching, and the entire portfolio will be correctly priced without a liquidity discount. This is outcome that is efficient both privately and socially.

In this setting, tranching is always associated with transparency: otherwise, there is no sorting of investors into the two markets, and so the proceeds are the same with and without tranching. It should be noticed that if condition (17) holds, tranching dominates the reduction of issue size, since it allows the issuer to sell the entire issue without any liquidity discount. The expected proceeds from the securitization are \( \bar{V} \) rather than \( s\bar{V} \). By the same token, even when condition (17) is violated, using the two methods jointly is revenue-efficient: the issuer will find it profitable to sell the largest amount of the junior tranche that the sophisticated investors can absorb,

\(^{10}\) We make the following tie-breaking assumption: when indifferent, sophisticated investors choose to invest in the riskier claim, whereas unsophisticated ones choose the safe claim.
and retain the rest of it, while selling the whole senior tranche to the unsophisticated investors. And in fact, this is a recurrent pattern in actual securitizations.

In conclusion, we have shown that:

**Proposition 7** The issuer will tranche the issue and opt for high transparency when sophisticated investors are sufficiently wealthy: \( A_S \geq (\overline{V} - V_L) + (1 - p)R \). This increases the proceeds from securitization and makes the market perfectly liquid, achieving the socially efficient outcome.

## 6 Conclusions

Is there a conflict between expanding the placement of complex financial instruments and preserving the transparency and liquidity of their secondary markets? Put more bluntly, is “popularizing finance” at odds with “keeping financial markets a safe place”? The subprime crisis has thrown this question for the designers of financial regulation into high relief.

The answer provided here is that indeed the conflict exists, and that it may be particularly relevant to the securitization process. Marketing large amounts of CDOs means selling them also to unsophisticated investors, who cannot process the information necessary to price them. In fact, if such information were released, it would put them at a disadvantage vis-à-vis the “smart money” that can process it. This creates an incentive for CDO issuers to negotiate with credit rating agencies a low level of transparency – that is, relatively coarse and uninformative ratings. Ironically, the elimination of some price-relevant information is functional to enhanced liquidity in the CDO new issue market.

However, low transparency at the issue stage comes at the cost of a less liquid, or even totally frozen-up, secondary market. This is because with poor transparency sophisticated investors may succeed in procuring the undisclosed information. Therefore, trading in the secondary market will be hampered by adverse selection, while
with high transparency this would not occur.

Although privately optimal, low transparency may be inefficient socially when the illiquidity of the secondary market has negative repercussions on the economy, as by triggering a spiral of defaults and bankruptcies. In these cases, regulation making greater disclosure mandatory for rating agencies is socially optimal. Our model therefore offers support for the current regulatory efforts to increase disclosure of credit rating agencies.

We also analyze the effects of two forms of ex-post liquidity injection: one targeted at distressed investors in the context of a market freeze, and another aimed at supporting the CDO price in an illiquid secondary market. It turns out that the first policy, while efficient ex post, nevertheless diminishes the issuers’ incentives to opt for transparent ratings ex ante, and also enlarges the parameter region where the market will freeze. The second type of intervention is even more misguided: the liquidity injected by the government to support the CDO market simply attracts more informed trading, so that distressed bondholders seeking liquidity do not benefit from it.

Finally, we show that in some cases regulation is not needed. First, if the demand for secondary market liquidity is strong or adverse selection in the primary market is not severe, issuers themselves will opt for transparent ratings. Second, issuers may themselves limit the size of their CDO issue and sell only to sophisticated investors. Or, even better, they may split the issue into an information-sensitive junior tranche for the sophisticates and a safe senior tranche for the unsophisticated. In both of these cases, they will find transparent ratings privately optimal, as they would not reduce primary market liquidity but would enhance that of the secondary market. Issuers, however, will opt for such policies only if sophisticated investors can absorb a large portion of the CDO. When this condition is not met, public intervention is still warranted.
Appendix

Proof of Proposition 6: If $R \leq 2C$, the optimal size is $s^* = 1$. This is because when $R \leq 2C$, there is no adverse selection problem in the secondary market.

If $R \in (2C, 2C + \Delta]$, the relevant comparison is between (14) (with $s = \bar{s}$) and $\bar{V} - (1 - p)\pi(R - 2C)$ (with $s = 1$). Equating the two expressions yields a lower bound on $\pi$ as a function of $R$: if

$$\pi > \frac{(1 - \bar{s})\bar{V}}{(1 - p)(R - 2C)},$$

(A1)

then $s = \bar{s}$ and high transparency are optimal; otherwise, $s = 1$ and low transparency are preferred. The lower bound (A1) is shown in Figure 4 as a downward sloping/convex with asymptote at $R = 2C$.

If $R > 2C + \Delta$, the comparison is (14) (with $s = \bar{s}$) and $\bar{V} - (1 - p)\pi\Delta$ (with $s = 1$). Equating these two expressions yields another lower bound on $\pi$ as a function of $R$: if

$$\pi > \frac{(1 - \bar{s})\bar{V}}{(1 - p)\Delta},$$

(A2)

then $s = \bar{s}$ and high transparency are optimal; otherwise, $s = 1$ and low transparency are preferred. The lower bound (A2) is shown in Figure 4 as an upward sloping and concave curve intersecting the horizontal axis at $\pi = (\bar{V} - A_S)/(1 - p)\Delta$ and the vertical line corresponding to $\pi = 1$ at $R = \left[\frac{A_S}{\bar{V} - (1 - p)\Delta} - 1\right] \frac{\bar{V}}{1 - p}$.

The lower bounds (A1) and (A2) have the same value at their intersection, which occurs for $R = 2C + \Delta$:

$$\pi = \left[1 - \frac{A_S}{\bar{V} + (1 - p)(2C + \Delta)}\right] \frac{\bar{V}}{(1 - p)\Delta}.$$

The area where $s = \bar{s}$ is not empty only if this value is less than 1. This is easily seen to require $A_S > \bar{A}$ as defined in (16). Therefore, if $A_S \leq \bar{A}$, $s = 1$ and low transparency are always optimal. □
References


The fraction of good claims $\lambda$ and their relative quality $q$ are determined.

- CDO issuer chooses low or high transparency.
- Rating agency reveals the corresponding information.
- Primary market opens.

Everybody learns if CDO is in default.

- Liquidity shock hits a fraction $\pi$ of investors, who decide whether to trade on the secondary market or sell other assets at discount $\Delta$.
- Sophisticated investors decide whether to invest $C\phi$ to learn $q$ with probability $\phi$.

Secondary market opens.

Payoffs of underlying security and CDOs are realized.

Figure 1: Time Line

Figure 2: Privately Optimal Choice of Transparency
Figure 3: Socially Optimal Choice of Transparency

Figure 4: Public Liquidity Provision and Private Choice of Transparency

- 37 -
Figure 5: Choice of CDO Size if $A_S > \bar{A}$