Implied Volatility and Forward Price Term Structures

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This paper discusses the relation between forward price models (FPM) of HJM-type and the so-called implied volatility term structure (VTS). We assume both forward contracts and at-the-money (ATM) options, on a same underlying, are liquidly traded in the market. In this general setup we derive a no arbitrage relationship between the volatility of forward prices and the implied VTS dynamics. For deterministic forward price volatilities we show a one to one correspondence between the functional form of forward price volatilities and that of the implied VTS. We then characterize all implied VTS parameterizations that are consistent with models that admit a finite dimensional realization (FDR) of forward prices. We finish by considering the possibility of stochastic forward price volatilities and deriving a no arbitrage drift condition that must hold for the dynamics of ATM implied VTS.

Executive Summary

Many financial institutions trade forward contracts and option contracts written on the same underlying. In order to price and hedge their forward contracts they choose a forward price model (FPM). Likewise, to price and hedge their option contracts they choose an option price model. Nonetheless, little or no effort seems to be spent analyzing possible inconsistencies between the two actions.

FPM differ in their choice of forward prices volatility so practitioners focus in choosing forward volatilities that are both appropriate to the underlying at stake and treatable. On what concerns option models, the market practice is to observe implied Black or Black-Scholes volatilities quotes from market data and choose a parameterization (or an interpolation method) to obtain an entire volatility surface. Pricing and hedging is then based on that surface. Whenever their focus is just on one strike, typically at-the-money (ATM) options as it is the most traded on the market, their task reduced to finding good models for the so-called implied volatility term structure (VTS).
In this paper we study the connection between FPM and (implied) VTS. As far we know, this relationship has not been exploited previously in the literature.

The inspiration comes from the so-called market models on implied volatility surfaces. In these models liquidly traded plain vanilla options and the underlying asset are modeled simultaneously and the ultimate goal is to find the arbitrage free dynamics for the implied volatility surface. Examples of such studies are [7], [13], [14], [10], [4], [1], [6], [5] or [15]. Here we take a different approach. Instead of modeling the underlying asset and all plain vanilla options simultaneously, we take all forward contracts and at-the-money (ATM) option contracts on a same underlying as our basic assets, and model simultaneously forward prices and ATM option prices. By choosing only ATM options as our basic assets we focus on term structure issues.

We argue that forward price term structures contain information on expected volatilities over future periods just like the implied VTS, so there should be a connection between the term structure of implied volatilities and forward prices. The goal of this paper is to formally characterize that link. Interesting questions addressed are:

1. Given a FPM, what can we say about the implied VTS?
2. Given a parameterized family of implied VTS curves, what can we say about the FPM?
3. Which VTS parameterizations are consistent with “nice” FPM? I.e., with FPM that admits a finite dimensional realization (FDR)?

The paper organization and its main contributions are as follows. In Section 2 we present the FPM framework and illustrate it with concrete FPM previously studied in the literature. On the VTS side we define exactly what we mean by a VTS an present popular industry parameterizations. In Section 3 we exploit the connection between FPM and VTS for deterministic forward price volatilities. We derive the basic arbitrage relationship and show that the popular industry parameterizations for the VTS are not consistent with treatable FPM. The notion of “treatability of FPM has to do with existence of closed-form solutions for the forward price term structure and existence of finite dimensional realizations of models of the HJM–type (see [8] or [3]). We conclude this section characterizing all the VTS parameterizations that are compatible with FPM that admit a FDR, and end up suggesting two new parameterizations for the VTS that processes the required “nice” properties. In Section 4 we deal with stochastic forward volatilities and derive a non-arbitrage drift condition that must be satisfied by the VTS dynamics. . In Section 5 summarized the results and points out directions for future research.
References