Investor Protection and Asset Prices*

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Abstract

Corporations in most countries are run by controlling shareholders, who have substantially smaller cash flow rights than their control rights in the firm. This separation of ownership and control allows the controlling shareholders to pursue private benefits at the cost of outside minority investors by diverting resources away from the firm and distorting corporate investment and payout policies. We develop a dynamic stochastic general equilibrium asset pricing model that acknowledges the implications of agency conflicts through weak investor protection on security prices. We show that countries with stronger investor protection have (i) higher market to book valuations, (ii) lower volatilities of equity returns, (iii) lower interest rates and equity risk premia, and (iv) for relative risk aversion larger than unity, lower dividend yields. These predictions are consistent with empirical findings.

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1 Introduction

Separation of corporate control from ownership is one of the main features of modern capital markets. Among its many virtues it allows the participation of small investors in the equity market, increasing the supply of funds, dissipating risks across the economy, and lowering the cost of capital for firms. Its biggest drawback is the agency conflict between controlling shareholders that run the firm, or more generally corporate insiders, and outside minority investors who have cash flow rights on the firm, but no control rights (e.g. Berle and Means (1932) and Jensen and Meckling (1976)). Minimizing such agency costs is the objective of a large body of research in corporate finance. Recent corporate scandals in the US constitute a stern reminder of the existence of these costs and of the private benefits derived by insiders even in the least suspicious market.\(^1\)

If controlling shareholders derive private benefits from control at the cost of outside minority shareholders, how do equity returns of firms with more mechanisms of corporate governance vary from those with less? To our knowledge this is the first paper to address this question. Consider for example the recently proposed merger between AmBev, a Brazilian brewer listed in both Brazil and NY, and Belgium’s Interbrew. Owners of ordinary shares in AmBev will swap them for Interbrew stock, “probably at a premium,” leaving owners of non-voting preference shares out of the deal and with what appears to be “high-priced junk” (The Economist, March 27, 2004). With efficient stock markets, prices will have already reflected the possibility of such events. In fact, there is now considerable evidence that suggests that firm value increases with the extent of protection of minority investors, and with the stock ownership of controlling shareholders if their cash flow rights exceed their control rights.\(^2\) The next obvious question is: What do the expected returns on AmBev look like?

To answer our main question, we depart from traditional production based equilibrium asset pricing models in two important ways. First, we model production in the context of an agency model where corporate governance frictions play a critical role. Second, we embed this setup into an equilibrium asset pricing model with heterogeneous risk averse agents.

Given an ownership structure and degree of investor protection, controlling sharehold-

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\(^1\)Barclay and Holderness (1989) and Dyck and Zingales (2004) among others provide evidence on private benefits of control.

\(^2\)See Claessens et al. (2002), La Porta et al. (2002), Black et al. (2003), Gompers et al. (2003), and Baek et al. (2004). See La Porta et al. (2000b) and Denis and McConnell (2003) for surveys on the investor protection literature.
ers choose investment and payout policies as well as consumption and bond holdings to maximize their lifetime utility. The investment decision trades off postponing consumption of the controlling shareholder and the increased volatility of consumption, against the *private* marginal return to capital net of depreciation, where the private return to capital includes the derived private benefits net of the cost of extracting these benefits. That increased investment generates higher volatility of consumption is a result of our assumption that capital accumulation is stochastic with shocks displaying higher volatility in times of more investment. This is a natural assumption given that firms produce some of their own capital (e.g. some equipment as well as intangible capital). Because only the benefits to investing depend directly on the corporate control frictions, the controlling shareholder chooses to overinvest in the firm relative to a world with perfect investor protection. Overinvestment is in line with Jensen’s (1986) free-cash flow and empire-building hypothesis (for evidence see, for example, Lang, Stulz, and Walking (1991) and Harford (1999)).

Consider now the way in which minority investors price the stock of this firm. Minority investors take the investment and payout decisions by the controlling shareholder as given, and solve an intertemporal consumption and portfolio choice problem a la Merton (1971). Under perfect investor protection Tobin’s $q$ is larger than unity. That is, the value of installed capital is larger than the value of to-be-installed capital as seen by the minority investors. The intuition is that the net change of capital stock over a fixed time interval is subject to both a net investment, but also innovations that are proportional to the level of investment. That is, *new* investment introduces uncertainty into the capital accumulation process. This production risk is systematic and thus is priced in equilibrium. As a result, risk averse investors view it as costly to adjust the capital stock in equilibrium. This in turn drives a wedge between the price of uninstalled capital and the price of installed capital. Under imperfect investor protection Tobin’s $q$ can be lower than one because private benefits of control are extracted from installed capital and not from new capital goods, which drives down the price of installed capital relative to new investment. Improvements in investor protection alleviate the agency conflicts and increase the value of installed capital, i.e. Tobin’s $q$. La Porta et al. (2002), Shleifer and Wolfenzon (2002), and Lan and Wang (2004) work out the same prediction relating Tobin’s $q$ to investor protection in a partial equilibrium analysis with risk neutrality. Ours is the first paper to derive the predictions of investor protection changes on Tobin’s $q$ in a general equilibrium asset pricing model with risk averse agents.

Our main result is that the excess equity return is affected in equilibrium by changes
in the degree of investor protection. Weaker investor protection increases the riskiness of the stock to minority investor and thus the risk premium they charge to hold its shares. In the model, the risk premium reflects the price attached by the minority investors to the uncertainty associated with the economy’s single factor. A positive shock to capital leads to a higher stock return and a higher dividend payment. Minority investors’ consumption increases and their marginal utility declines. This negative correlation between stock returns and marginal utility of consumption is larger in absolute terms when investor protection is weaker and the investment rate is higher. This is because the value of this correlation in equilibrium is tied to the volatility of the capital stock, and a higher investment rate makes the existing capital stock more volatile. Thus, the risk premium is larger.

This new finding is consistent with the evidence in Bekaert and Harvey (1997) and Bekaert and Urias (1999) that show that emerging markets display higher volatility of returns and larger equity risk premia. Arguably, these countries have weaker corporate governance. Similarly, Erb et al. (1996) show that expected returns, as well as volatility, are higher when country credit risk is higher. Gompers et al. (2003) find that firms with stronger shareholder rights outperformed those with weaker shareholder rights. Consistent with our prediction, Table 8 of Gompers et al. (2001) indicates that the portfolio of firms with weaker shareholder rights consistently displayed greater riskiness (i.e. higher correlation to market returns), specially in the first half of the analysis. Daouk, Lee, and Ng (2004) show that improvements in their index of capital market governance are associated with lower equity risk premia.

The model predicts that dividend yields move with the extent of investor protection in a way that depends on the coefficient of risk aversion. This is because worse investor protection leads to higher interest rates in the model thus changing the willingness of investors to save. If agents have logarithmic utility functions, the dividend yield is equal to the agents’ subjective discount rate and hence is unaffected by investor protection. This reflects the myopic nature of investors. When the coefficient of relative risk aversion is larger than one, as most estimates indicate, the substitution effect is weaker than the income effect in the inter-temporal allocation of consumption. Thus a higher interest rate that results from weaker investor protection leads to an increased demand for consumption and a higher dividend yield.

[Talk about the measured costs of imperfect investor protection.]

This paper is closely related to two strands of literature. First, it relates to the investor protection (law and finance) literature in corporate finance that links corporate
governance to firm value or Tobin’s $q$ (e.g. La Porta et al. (1999, 2002), Claessens et al. (2002)). In these studies, mostly empirical, the main goal has been to identify how different degrees of investor protection (often across countries) or of ownership by a controlling shareholder affect firm value. Second, it links to the literature that studies the asset pricing implications of corporate financing frictions. The papers in this literature that are closest to ours are Dow et al. (2003) and Albuquerque and Wang (2004). Dow et al. (2003) develop a closed-economy model in which the manager has an empire building preference (as in Jensen (1986)) and wants to invest all of the firm’s free cash flow if possible. As a result, the shareholder needs to use some of the firm’s resources to hire auditors to constrain the manager’s empire building incentives. Motivated by the large empirical literature on investor protection and ownership and managerial structure around the world, our model acknowledges that managers in firms in most countries around the world are often controlling shareholders who themselves have cash flow rights in the firm and thus do not simply burn all of the firm’s free cash flow. Rather, the controlling shareholder trades off his gain from pursuing his private benefits with the cost of seeing his share of firm value decrease due to the distortion of investment decisions. The critical determinant of this trade-off is the extent of investor protection. Hence, our model allows us to derive predictions on asset prices and investor protection.

Albuquerque and Wang (2004) generalize the setup in the current paper to a two-country world where productivity shocks follow a stationary regime switching Markov process and discuss the business cycle and exchange rate properties of such model. In another agency-cost based asset pricing model, Holmstrom and Tirole (2001) propose an equilibrium asset pricing model by assuming that the entrepreneur is able to extract private benefits from the firm and cannot promise the investors to fully return their funds. As a result, collateralizable assets that can be seized by investors when the firm is in financial trouble command a premium. In equilibrium, this generates a desire to hoard liquidity ex ante by firms in order to increase funding thus leads to a liquidity premium. Gomes et al. (2001) link costly external financing to asset prices.

The remainder of the paper is organized as follows. The next section presents the model and a theorem with the equilibrium characterization. Section 3 proves the existence of equilibrium and derives some basic results on investment. Section 4 illustrates the perfect investor protection benchmark and section 5 gives the main results for interest rates, prices and returns. Section 6 provides a calibration to the model and presents some comparative statics and section 7 quantitatively studies the costs of imperfect investor protection. In section 8 we suggest a reinterpretation of our production setup and section
9 concludes. The appendices contain the technical details and proofs the propositions in the paper.

2 The Model

The economy is populated by two types of agents, a controlling shareholder that operates the firm, and a continuum of identical minority investors of unit mass. Agents have infinite horizon and time is continuous.

Next, we describe the consumption and production sides of the economy, and the objectives and choice variables of both the controlling shareholder and the minority investors.

2.1 Setup

Production and Investment Opportunities. The firm is defined by a production technology. Let $K$ be firm’s capital stock process. We assume that $K$ evolves according to

$$dK(t) = (I(t) - \delta K(t))\, dt + \epsilon I(t)\, dZ(t),$$  

where $\epsilon > 0$, $\delta > 0$ is the depreciation rate, $Z(t)$ is a Brownian process, and $I(t)$ represents the firm’s gross investment. Gross investment is given by

$$I(t) = hK(t) - D(t) - s(t)hK(t).$$  

Gross investment $I$ equals gross output $hK$, minus the sum of dividends $D$, and the private benefits extracted by the controlling shareholder $shK$.

The production function has constant returns to scale and the capital accumulation process is stochastic with shocks proportional to gross investment $I$. This modelling device gives great tractability and has the simple interpretation that capital accumulation is part of the production process. It is also similar to the production process adopted in Cox, Ingersoll, and Ross (1985). The magnitude of private benefits extracted by the controlling shareholder depends on the degree of investor protection which we discuss below.

Next, we discuss the controlling shareholder’s objective and his decision variables. We are motivated by the large amount of empirical evidence around the world in delegating the firm’s decision making to the controlling shareholder.

\[ \text{See also section 8 below for an alternative interpretation of the production setup.} \]
Controlling Shareholder. The controlling shareholder is risk averse and has lifetime utility over consumption plans given by

$$E \left[ \int_0^\infty e^{-\rho t} u(C_1(t)) dt \right],$$

where $C_1$ denotes the flow of consumption of the controlling shareholder, and the period utility function is given by

$$u(C) = \begin{cases} \frac{1}{1-\gamma} (C^{1-\gamma} - 1) & \gamma \geq 0, \gamma \neq 1 \\ \log C & \gamma = 1 \end{cases}.$$ (4)

The rate of time preference is $\rho > 0$, and $\gamma$ is the coefficient of relative risk aversion. Throughout the paper, we use the subscripts “1” and “2” to index variables for the controlling shareholder and the minority investor, respectively.

The controlling shareholder owns a fixed fraction $\alpha < 1$ of the firm’s shares. This ownership share gives him control over the firm’s investment and payout policies. In real economies, control rights generally differ from cash flows rights: a fraction of votes higher than that of cash flow rights can be obtained by either owning shares with superior voting rights, through ownership pyramids, cross ownership, or by controlling the board. We refer readers to Bebchuk et al. (2000) for details on how control rights can differ from cash-flow rights. For now, we treat $\alpha$ as constant and non-tradable. This assumption is consistent with La Porta et al. (1999) who argue that the controlling shareholder’s ownership share is extremely stable over time, but is not needed. In section 3.3, we allow the controlling shareholder to optimize over his ownership stake and show that the no trade outcome is indeed an equilibrium.

We assume that the controlling shareholder can only invest his wealth in the risk-free asset. Let $W_1$ denote the controlling shareholder’s tradable wealth. The risk-free asset holdings of the controlling shareholder are $B_1(t) = W_1(t)$. We assume that the controlling shareholder’s initial tradable asset holding is zero, in that $W_1(0) = 0$. Therefore, the controlling shareholder’s tradable wealth $W_1(t)$ evolves as follows:

$$dW_1(t) = [r(t)W_1(t) + M(t) - C_1(t)] dt,$$

where $M(t)$ is the flow of goods which the controlling shareholder obtains from the firm.

4 Giving all the control rights to a controlling shareholder is in line with evidence provided in La Porta et al. (1999) who document for many countries that the control of firms is often heavily concentrated in the hands of a founding family.
either through dividend payments $\alpha D(t)$ or through private benefits:\footnote{See Barclay and Holderness (1989) for early work on the empirical evidence in support of private benefits of control. See also Dyck and Zingales (2004) and Bae, Kang and Kim (2002).}

\[ M(t) = \alpha D(t) + s(t)hK(t) - \Phi(s(t), hK(t)). \]  
(6)

Private benefits of control are modelled as a fraction $s(t)$ of gross output $hK(t)$, with $h > 0$ being the productivity of capital. Expropriation is costly to both the firm and the controlling shareholder and, \textit{ceteris paribus}, for the controlling shareholder pursuing private benefits is more costly when investor protection is stronger. If the controlling shareholder diverts a fraction $s$ of the gross revenue $hK(t)$, then he pays a cost

\[ \Phi(s, hK) = \frac{\eta}{2} s^2 hK. \]  
(7)

The cost function (7) is increasing and convex in the fraction $s$ of gross output that the controlling shareholder diverts for private benefits. The convexity of $\Phi(s, hK)$ in $s$ guarantees that it is more costly to divert increasingly large fractions of private benefits. For the remainder of the paper, we use the word “stealing” to mean “the pursuit of private benefits by diverting resources away from firm.” The cost function (7) also assumes that the cost of diverting a given fraction $s$ of cash from a larger firm is assumed to be higher, because a larger amount $shK$ of gross output is diverted. That is, $\partial \Phi(s, hK) / \partial K > 0$. But, the total cost of stealing the same level $shK$ is lower for a larger firm than for a smaller firm. This can be seen by re-writing the cost of stealing as $\Phi(s, hK) = \eta (shK)^2 / (2hK)$.\footnote{Note that the functional form of the stealing cost function is analogous to that of the adjustment cost function in corporate investment models such Abel (1979) and Hayashi (1982).}

Following La Porta et al. (2002), we interpret the parameter $\eta$ as a measure of investor protection. A higher $\eta$ implies a larger marginal cost $\eta shK$ of diverting cash for private benefits. In the case of $\eta = 0$, there is no cost of diverting cash for private benefits and the financing channel breaks down, because investors anticipate no payback from the firm after they sink their funds. As a result, \textit{ex ante}, no investor is willing to invest in the firm. In contrast, in the limiting case of $\eta = \infty$, the marginal cost of pursuing a marginal unit of private benefit is infinity and minority shareholders are thus fully protected from expropriation. We will show later that in this case in the equilibrium we analyze the incentives of the controlling shareholder are perfectly aligned with those of the minority investors.

In summary, the objective for the controlling shareholder is to maximize his life-time utility defined in (3) and (4), subject to the firm’s capital stock dynamics given in (1)-(2), the controlling shareholder’s wealth accumulation dynamics (5)-(6), the cost function

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(7) for the controlling shareholder to pursue his private benefits, and the transversality
condition specified in the appendix. In solving his optimization problem, the control-
ling shareholder chooses \( \{C_1(t), s(t), I(t), K(t), D(t), W_1(t) : t \geq 0\} \) and takes the
equilibrium interest rate process \( \{r(t) : t \geq 0\} \) as exogenously given.

Let \( D \) and \( K \) be the dividend and firm’s capital stock process chosen by the controlling
shareholder. Without loss of generality, we may write both the dividend and capital stock
processes as follows:

\[
\begin{align*}
  dD(t) &= \mu_D(t)D(t)dt + \sigma_D(t)D(t)dZ(t), \\
  dK(t) &= \mu_K(t)K(t)dt + \sigma_K(t)K(t)dZ(t),
\end{align*}
\]

where the drift processes \( \mu_D \) and \( \mu_K \), and the volatility processes \( \sigma_D \) and \( \sigma_K \) are chosen
by the controlling shareholder.

**Financial Assets.** Outside minority investors trade equity shares on the firm. While
the controlling shareholder chooses the dividend stream, the price of the firm’s stock is
determined in equilibrium by rational minority investors. We write the equilibrium stock
price process as follows:

\[
  dP(t) = \mu_P(t)P(t)dt + \sigma_P(t)P(t)dZ(t),
\]

where \( \mu_P \) and \( \sigma_P \) are the equilibrium drift and volatility processes for stock returns.

In addition to firm stock traded by minority investors, there is also a risk-free asset
available in zero net supply. Both minority investors and the controlling shareholder may
trade the risk-free asset. Let \( r(t) \) be the short term interest rate paid on this risk-free
asset. We determine \( r, \mu_P \) and \( \sigma_P \) simultaneously in equilibrium in Section 3.

**Minority Investors.** Minority investors have the same preferences as the controlling
shareholder does. They jointly own \( (1 - \alpha) \) of the firm’s shares and can sell or buy of these
shares in competitive markets with other minority investors at the equilibrium price \( P(t) \).
They can also invest in the risk-free asset earning interest at the equilibrium interest rate.
The minority investor’s optimization problem is a standard consumption-asset allocation
problem in the spirit of Merton (1971). Unlike Merton (1971), in our model, both the
stock price and the interest rate are endogenously determined in equilibrium.

Each minority investor accumulates his wealth as follows:

\[
  dW_2(t) = [r(t)W_2(t) - C_2(t) + \omega(t)W_2(t)\lambda(t)]dt + \sigma_P(t)\omega(t)W_2(t)dZ(t),
\]
where $\lambda(t)$ is the excess stock return inclusive of dividend payments $D(t)$, in that $\lambda(t) \equiv \mu P(t) + D(t) / P(t) - r(t)$. We use the subscript ‘2’ to denote variables chosen by minority investors, when it is necessary to differentiate the corresponding variables for the controlling shareholder. For example, in the wealth accumulation equation (11), $C_2(t)$ is the flow of consumption of the minority investor and $\omega(t)$ is the fraction of his wealth invested in the firm’s stock. The risk-free asset holdings of the minority investors are $B_2(t) = (1 - \omega(t)) W_2(t)$. Finally, each minority investor’s initial wealth is $W_2(0) > 0$.

Each minority investor chooses $\{C_2(t) , W_2(t) , \omega(t) : t \geq 0\}$ to maximize his lifetime utility function subject to his wealth accumulation dynamics (11), and the transversality condition specified in the appendix. In solving this problem, the minority investor takes as given the equilibrium dividend process, the firm’s stock price, and the interest rate.

### 2.2 Definition and Existence of Equilibrium

We are now ready to define an equilibrium in our economy and state the theorem characterizing the equilibrium.

**Definition 1** An equilibrium in which the interest rate $r$ is constant has the following properties:

(i) $\{C_1(t) , s(t) , I(t) , K(t) , D(t) , W_1(t) : t \geq 0\}$ solve the controlling shareholder’s problem for the given interest rate $r$;

(ii) $\{C_2(t) , W_2(t) , \omega(t) : t \geq 0\}$ solve the minority investor’s problem for given interest rate $r$, and stock price and dividend payout stochastic processes $\{P(t) , D(t) : t \geq 0\}$;

(iii) The risk-free asset market clears, in that $B_1(t) + B_2(t) = 0$, for all $t$;

(iv) The stock market clears for minority investors, in that $(1 - \alpha) P(t) = \omega(t) W_2(t)$, for all $t$;

(v) The consumption goods market clears, in that $C_1(t) + C_2(t) = (1 - \alpha) D(t) + M(t)$, for all $t$.

Note that a difficulty with our model is the presence of heterogeneous investors. In general, with heterogeneous investors, agents keep track of the wealth distribution in the economy $(W_1(t), W_2(t))$ besides the level of physical capital invested in the firm.
K(t). In our model though this problem is greatly simplified. First, in all equilibria with a constant interest rate, the tradable part of the controlling shareholder’s wealth $W_1(t) = 0$. Second, the wealth of the minority investors is proportional to $K(t)$. This feature significantly reduces the dimensionality of the problem from three state variables to one. The theorem to be introduced below completely characterizes the equilibrium.

Before we present the main theorem, we state the assumptions that are needed for our conjectured equilibrium.

**Assumption 1** $h > \rho + \delta (1 - \gamma)$.

**Assumption 2** $1 - \alpha < \eta$.

**Assumption 3** $2(\gamma + 1) [(1 + \psi) h - \rho - \delta (1 - \gamma)] \varepsilon^2 \leq \gamma [1 + (1 + \psi) h \varepsilon^2]^2$.

**Assumption 4** $(1 - \phi) h > i$.

**Assumption 5** $\rho + (\gamma - 1) (i - \delta) - \gamma (\gamma - 1) i^2 \varepsilon^2 / 2 > 0$.

Assumption 1 states that the firm is sufficiently productive and thus investment will be positive for risk-neutral firms under perfect investor protection. Assumption 2 ensures agency costs exist and lie within the economically interesting and relevant region. Assumptions 3 and 4 ensure positive investment and dividends, respectively. Assumption 5 gives rise to finite and positive Tobin’s $q$ and dividend yield. While we have described the intuition behind these assumptions, obviously we cannot take the intuition and implications of these assumptions in isolation. In general, these assumptions *jointly* ensure that the equilibrium exists with positive and finite net private benefits, investment rate, dividend, Tobin’s $q$.

**Theorem 1** *Under Assumptions 1-5, there exists an equilibrium with the following properties. The outside minority investors hold no risk-free asset ($B_2(t) = 0$), and only stock ($\omega(t) = 1$). Minority investors’ consumption equals their entitled dividends:*

$$C_2(t) = (1 - \alpha) D(t).$$

*The controlling shareholder holds no risk-free asset ($B_1(t) = 0$). He steals a constant fraction of gross revenue, in that*

$$s(t) = \phi \equiv \frac{1 - \alpha}{\eta}.$$  \quad (12)
The controlling shareholder’s consumption $C_1(t)$, firm’s investment $I(t)$, and firm’s dividend payout $D(t)$ are all proportional to firm’s capital stock $K(t)$, in that $C_1(t)/K(t) = M(t)/K(t) = m$, $I(t)/K(t) = i$, $D(t)/K(t) = d$, are given by

$$m = \alpha [(1+\psi) h - i] > 0,$$

$$i = \frac{1 + (1 + \psi) h \epsilon^2}{(\gamma + 1) \epsilon^2} \left[1 - \sqrt{1 - \frac{2(\gamma + 1) \epsilon^2 ((1 + \psi) h - \rho - \delta (1 - \gamma))}{\gamma [1 + (1 + \psi) h \epsilon^2]^2}}\right] > 0,$$

$$d = (1 - \phi) h - i > 0,$$

and \(\psi\) is a measure of agency costs and is given by

$$\psi = \frac{(1 - \alpha)^2}{2 \alpha \eta}.$$  

The equilibrium dividend process (8), the stock price process (10), and the capital accumulation process (9) all follow geometric Brownian motions, with the same drift and volatility coefficients, in that $\mu_D = \mu_P = \mu_K = i - \delta$ and volatility $\sigma_D = \sigma_P = \sigma_K = \delta \epsilon$, where $i$ is the constant equilibrium investment-capital ratio given in (14). The equilibrium firm value is also proportional to firm capital stock, in that $P(t) = qK(t)$, where the coefficient $q$, known as Tobin’s $q$, is given by

$$q = \left(1 + \frac{1 - \alpha^2}{2 \eta \epsilon \phi} h\right)^{-1} \frac{1}{1 - \gamma \epsilon^2 i}.$$  

The equilibrium interest rate is given by

$$r = \rho + \gamma \mu_D - \frac{\sigma_D^2}{2} \gamma (\gamma + 1).$$

The parameter \(\psi\) given in (16) summarizes the relevance of investor protection and the controlling shareholder’s cash-flow rights in the firm on firm’s investment and payout decisions. In particular, \(\psi\) is an decreasing function of the cost of stealing \(\eta\), and of the equity share of the controlling shareholder \(\alpha\).

In equilibrium, financial and real variables—price $P$, dividend $D$, controlling shareholder’s consumption $C_1$ and wealth $W_1$, firm investment $I$, minority investor’s consumption $C_2$ and wealth $W_2$—are all proportional to the firm’s capital stock $K$. That is, in our model, the economy grows stochastically on a balanced path. In order to deliver such an intuitive and analytically tractable equilibrium, the following assumptions or properties in the model are useful: (i) a constant return to scale production and capital accumulation technology specified in (1); (ii) optimal “net” private benefits linear in the firm’s
capital stock (arising from the assumptions that the controlling shareholder’s benefit of stealing is linear and his cost of stealing is quadratic); (iii) the controlling shareholder and the minority investors have preferences that are homothetic with respect to the firm’s capital stock. We think the key intuition and results of our model are robust to various generalizations. Since the economy is on a balanced growth path, in the remainder of the paper we focus primarily on scaled variables such as the investment-capital ratio \( i \) and the dividend-capital ratio \( d \).\(^7\)

In the next section, we prove Theorem 1, present the derivations of equilibrium prices and quantities, and highlight the intuition behind the construction and solution methodology of the equilibrium.

3 Equilibrium Characterization

The natural and direct way to solve for the model’s equilibria in our economy is to solve the controlling shareholder’s consumption and production decisions and the minority investor’s consumption and asset allocation problem for a general price process and to aggregate up the demands for the stock, the risk-free asset, and the consumption good. However, this approach is technically quite complicated and analytically not tractable. The controlling shareholder’s optimization problem is one with both incomplete markets consumption-savings problem and a capital accumulation problem with agency costs. We know from the voluminous consumption-savings literature that there is no analytically tractable model with constant relative risk aversion utility.\(^8\) If even solving a subset of such an optimization problem is technically difficult, we naturally anticipate that the joint consumption and production optimization problem for the controlling shareholder is intractable, not to mention finding the equilibrium fixed point.

Here we adopt the alternative approach by directly conjecturing, and then verifying, the equilibrium allocations and prices. Specifically, we conjecture an equilibrium in which the interest rate is constant and there is no trading for the risk-free asset and then show that such an equilibrium satisfies all the optimality and market clearing conditions. We start with the controlling shareholder’s optimization problem.

\(^7\)To completely describe the equilibrium in our economy, we of course also need understand the determinant of the initial level of capital stock \( K_0 \).

3.1 The Controlling Shareholder’s Optimization

We first conjecture that the controlling shareholder holds zero risk-free assets in equilibrium, in that \( B_1(t) = 0 \), for all \( t \geq 0 \). Therefore, his consumption is given by \( C_1(t) = M(t) \), where \( M(t) \) is given in (6).\(^9\) We will then show that under this conjecture, the rate \( r \) that satisfies the controlling shareholder’s optimality condition is equal to the equilibrium interest rate given in (18), presented in Theorem 1. In order to demonstrate that our conjectured interest rate is the equilibrium one, we also need verify that the optimality condition for the minority investors under the conjectured interest rate implies zero demand for the risk-free asset. We verify this later in the section.

Recall that the only tradable asset for the controlling shareholder in this economy is the risk-free asset. Therefore, together with our conjectured equilibrium demand for the risk-free asset by the controlling shareholder, we may equivalently write the controlling shareholder’s optimization problem as the following resource allocation problem:

\[
J_1(K_0) = \max_{D,s} E \left[ \int_0^{\infty} e^{-\rho t} u(M(t)) dt \right],
\]

subject to the firm’s capital accumulation dynamics (1)–(2), the cost of stealing (7), and the transversality condition specified in the appendix.

The controlling shareholder’s optimal payout decision \( D \) and “stealing” decision \( s \) solve the following Hamilton-Jacobi-Bellman equation:\(^{10}\)

\[
0 = \sup_{D,s} \left\{ \frac{1}{1 - \gamma} \left( M^{1-\gamma} - 1 \right) - \rho J_1(K) + (I - \delta K) J_1'(K) + \frac{\epsilon^2}{2} I^2 J_1''(K) \right\}. \tag{19}
\]

The first-order conditions with respect to dividend payout \( D \) and cash diversion \( s \) are

\[
M^{-\gamma} \alpha - \epsilon^2 I J_1''(K) = J_1'(K), \tag{20}
\]
\[
M^{-\gamma} (hK - \eta shK) - \epsilon^2 I J_1''(K) hK = J_1'(K) hK. \tag{21}
\]

In (20) and (21), the controlling shareholder trades off the marginal benefits of receiving higher current cash flows, attainable either via a higher dividend payout as in (20), or more stealing as in (21), against the marginal costs of doing so. Higher current cash flows to the controlling shareholder provide two benefits. One is higher current utility, which is a standard result in any consumption Euler equation. This incremental gain in current utility associated with higher dividends and more stealing is given in the first term on

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\(^9\)An auxiliary condition is that the initial wealth of the controlling shareholder is all in equity.

\(^{10}\)We verify the solution and provide technical regularity conditions in the appendix.
the left-hand side of both (20) and (21). More interestingly, higher current cash flows to
the controlling shareholder also increase his value function by reducing the volatility of
capital stock accumulation, as seen from the second term on the left-hand side of both
(20) and (21). The intuition for this volatility effect comes from (i) the concavity of the
value function due to risk aversion and (ii) that the dividend payout reduces investment,
which in turn lowers the volatility of capital accumulation. The marginal costs of higher
current cash flows are losses of marginal values from investing that amount in the firm.
The right-hand sides of (20) and (21) are the controlling shareholder’s marginal costs
of paying out one more unit of dividends and stealing one additional fraction of gross
output, respectively.

Equations (20) and (21) together imply that the fraction $s$ of “stealing” satisfies a
static trade-off for the controlling shareholder. Solving the simple trade-off gives a con-
stant solution over time, in that $s(t) = \phi \equiv (1 - \alpha)/\eta$. Intuitively, stealing is higher when
investor protection is worse (lower $\eta$) and the conflicts of interests are bigger (smaller $\alpha$).

Conjecture that the controlling shareholder’s value function $J_1(K)$ is given by

$$J_1(K) = \frac{1}{1-\gamma} \left( A_1 K^{1-\gamma} - \frac{1}{\rho} \right),$$

where the coefficient $A_1$ is given in the appendix. We verify this conjecture by solving
the Hamilton-Jacobi-Bellman equation (19) and the associated first-order conditions (20)-
(21) in the appendix. We show that the controlling shareholder’s consumption-capital
ratio $M(t)/K(t)$, the investment-capital ratio $I(t)/K(t)$, and the dividend-capital ratio
$D(t)/K(t)$ are all constant and are given by (13), (14), and (15), respectively.

The next proposition states the main properties of investment.

**Proposition 1** The equilibrium investment-capital ratio $i$ decreases with investor pro-
tection $\eta$ and the controlling shareholder’s cash flow rights $\alpha$, in that $di/d\eta < 0$ and
$di/d\alpha < 0$, respectively.

This result is quite intuitive. Consider the trade-offs faced by the controlling share-
holder in making the firm’s investment decision. The marginal costs to investing result
from postponing consumption and from the increased volatility of consumption through
the volatility of capital accumulation. None of these depend directly on the corporate
control frictions we highlight. The marginal benefits to investing, net of depreciation,
come from the productivity of capital, which is given by $h$, but also from the private
benefits net of the cost of extracting these benefits. Because the marginal net private
benefits are proportional to productivity $h$, the total future discounted benefit to the controlling shareholder from investing one additional unit today can be summarized by $h$ adjusted for by the agency cost frictions (described by $\psi$). Hence, the presence of weak corporate governance increases the flow of net benefits to the controlling shareholder from investment, and thus generates overinvestment relative to the perfect investor protection benchmark.

Overinvestment as a consequence of weak corporate governance is a story that we think fits well the evidence in many emerging market economies but also in developed economies. Jensen’s (1986) free cash flow hypothesis is that managers are empire-builders if left unconstrained. Harford (1999) documents that US cash-rich firms are more likely to attempt acquisitions, but that these acquisitions are value decreasing from either looking at stock return performance or operating performance. Pinkowitz (1999) finds that managers of US publicly traded firms may hold cash to entrench themselves at the expense of outside shareholders (see also the earlier papers Lang, Stulz, and Walking (1991) and Blanchard, Lopez-de-Silanes, and Shleifer (1994)). In a similar vein, Pinkowitz, Stulz, and Williamson (2003) document that, after controlling for the demand for liquidity, one dollar of cash holdings held by firms in countries with poor corporate governance is worth much less to outside shareholders than if in countries with better corporate governance. For emerging market economies the evidence also abounds. Prior to the 1997 East Asian crisis, the countries in East Asia that suffered the crisis were running significant current account deficits, putting the borrowed money into questionable local investments. Burnside et al. (2001) use Thailand and Korea as examples of countries that borrowed significant amounts in foreign currency at low interest rates to lend locally at higher rates benefitting from a fixed exchange rate regime and from a government bailout policy. The volume of non-performing loans was already at 25 percent of GDP for Korea and 30 of GDP for Thailand prior to 1997! China is yet another example of a country with very large amounts of non-performing loans in the banking sector fruit of a government that tirelessly dumps cash in inefficient state owned enterprises. Allen et al. (2002) show that China has had consistent high growth rates since the beginning of the economic reforms in the late 1970s, even though its legal system is not well developed and law enforcement is poor. Our paper argues that the incentives for the controlling shareholders (for example, government officials running the state-owned enterprises on behalf of the state) to overinvest can at least partly account for China’s high economic growth despite weak investor protection.

We now return to our model’s implication on the controlling shareholder’s problem.
We still need verify that the controlling shareholder’s consumption rule (13) and the equilibrium interest rate (18) are consistent with the implication that his optimal risk-free asset holding is indeed zero. This can be done by showing that the interest rate implied by the marginal utility of the controlling shareholder when \( C_1(t) = M(t) \) is the equilibrium one. The controlling shareholder’s marginal utility is given by \( \xi_1(t) = e^{-\rho t} C_1(t)^{-\gamma} \). In equilibrium, \( \xi_1(t) = e^{-\rho t} m^{-\gamma} K(t)^{-\gamma} \). Applying Ito’s lemma gives the following dynamics for \( \xi_1(t) \):

\[
\frac{d\xi_1(t)}{\xi_1(t)} = -\rho dt - \gamma \frac{dK(t)}{K(t)} + \frac{\sigma^2}{2} \gamma (\gamma + 1) dt.
\]

(22)

In order for \( \xi_1 \) to be the equilibrium stochastic discount factor, the drift of \( \xi_1 \) needs to equal \(-r\xi_1\). This equilibrium restriction and (22) together gives the equilibrium interest rate in (18). We refer the reader to Section 5.1 below for a discussion of the properties of the equilibrium interest rate.

Next, we turn to the minority investor’s optimization problem and his equilibrium security valuation.

### 3.2 Minority Investors’ Optimization

Minority investors trade two securities: the stock and the risk-free asset. Each minority investor faces a standard consumption and asset allocation problem. The minority investor accumulates his wealth by either investing in the risky asset (firm asset) or the risk-free asset. His wealth accumulation process is given by

\[
dW_2(t) = [r(t) W_2(t) - C_2(t) + \omega(t) W_2(t) \lambda(t)] dt + \sigma \omega(t) W_2(t) dZ(t),
\]

where \( \lambda(t) = \mu_P(t) + D(t) / P(t) - r(t) \) is the equilibrium risk premium and \( \omega \) is the fraction of wealth invested in the risky asset. Under the conjecture that both equilibrium risk premium and equilibrium interest rate are constant, we conjecture that the minority investor’s value function as follows:

\[
J_2(W) = \frac{1}{1 - \gamma} \left( A_2 W^{1-\gamma} - \frac{1}{\rho} \right),
\]

(23)

where \( A_2 \) is the coefficient to be determined in the appendix. We obtain the following standard consumption function and asset allocation solutions

\[
C_2(t) = \left( \frac{\rho - r(1 - \gamma)}{\gamma} \right) - \frac{\lambda^2(1 - \gamma)}{2\gamma^2\sigma_P^2} W(t),
\]

\[
\omega(t) = -\frac{J_2'(W)}{W J_2''(W) \sigma_P^2} = \frac{\lambda}{\gamma^2 \sigma_P^2}.
\]
In the proposed equilibrium, the minority investor only holds stock ($\omega = 1$) and no risk-free asset. Hence, the equilibrium excess stock return must satisfy

$$\lambda = \gamma \sigma^2_P = \gamma \epsilon^2 i^2.$$  \hfill (24)

This is the usual result (e.g. Lucas (1978) and Breeden (1979)) that the equity premium commanded by investors to hold the stock is the product of the price of risk, given by the investor’s coefficient of relative risk aversion, and the quantity of risk, as given by the infinitesimal variance of the stock return.

In equilibrium, with zero risk-free asset holdings, the minority investor’s consumption is $C_2(t) = (1 - \alpha) D(t)$. We apply Ito’s lemma to the minority investor’s marginal utility, $\xi_2(t) = e^{-\rho t} C_2(t)^{-\gamma} = e^{-\rho t} [(1 - \alpha) dK(t)]^{-\gamma}$, to obtain the following dynamics of $\xi_2(t)$:

$$\frac{d\xi_2(t)}{\xi_2(t)} = -\rho dt - \gamma \frac{dK(t)}{K(t)} + \frac{\epsilon^2 i^2}{2} \gamma (\gamma + 1) dt.$$  \hfill (25)

Because $\xi_2$ is the equilibrium stochastic discount factor, the drift of $\xi_2$ needs to equal $-r\xi_2$, where $r$ is the equilibrium interest rate. This equilibrium restriction and (25) together give the equilibrium interest rate in (18). Importantly, the implied equilibrium interest rate by the controlling shareholder’s $\xi_1$ and the minority investor’s $\xi_2$ are equal. We thus verify that, like the controlling shareholder, minority investors find optimal not to trade the risk-free asset at the equilibrium interest rate (18).

It remains to be shown that the price process (10) appropriately constructed is an equilibrium one for equity trading among minority investors, and generates a constant excess stock return. Using the minority investor’s marginal utility, we may obtain the per share price of the stock by dividing the discounted value of total dividends paid to minority investors by the number of shares $(1 - \alpha)$:

$$P(t) = \frac{1}{1 - \alpha} E_t \left[ \int_t^\infty \frac{\xi_2(s)}{\xi_2(t)} (1 - \alpha) D(s) \, ds \right] = q K(t),$$

where Tobin’s $q$, also known as the firm’s market-to-book value, is given by (17). Tobin’s $q$ is positive for $1 - \epsilon^2 i \gamma > 0$, which holds under Assumption 5. With constant $q$ and dividend-capital ratio $d$, in equilibrium, it is straightforward to show that the drift coefficients for dividend, stock price, and capital stock are all the same, in that $\mu_D = \mu_P = \mu_K = i - \delta$, and the volatility coefficients for dividend, stock price, and capital stock are also the same, in that $\sigma_P = \epsilon i$. Constant risk premium $\lambda$ is an immediate implication of constant $\mu_P$, constant dividend-capital ratio $d$, and constant equilibrium risk-free interest rate.
3.3 Equity Trading Between the Controlling Shareholder and Minority Investors

So far, we have exogenously assumed that the controlling shareholder cannot trade his shares at all. In this section, we allow the minority investors to trade with the controlling shareholder. We show that in equilibrium both the controlling shareholder and outside minority investors rationally decide not to trade with each other. The key in our analysis is to identify a free-rider situation similar to the free-rider problem Grossman and Hart (1980) identified in the corporate takeover context. Lan and Wang (2004) propose such a free rider argument between the risk-neutral controlling shareholder and risk-neutral outside minority investors. Here we apply the free-rider argument to risk averse agents. To the best of our knowledge, this is the first paper that identifies a free-rider problem with risk-averse agents.

The key insight behind our proof for the no-trade result is that the controlling shareholder is unable to enjoy any surplus generated from increasing firm value (via a more concentrated ownership structure.) The crucial assumption is that the controlling shareholder cannot trade anonymously. The inability to trade anonymously is realistic. For example, in almost all countries, the insiders need to file a report before selling or buying their own firm shares.

Suppose the controlling shareholder plans to increase his share ownership infinitesimally from \( \alpha \) to \( \alpha' \), if the price he pays for incremental shares \( \alpha' - \alpha \) is attractive. With a slight abuse of notation, let \( P_{\alpha'} \) and \( P_\alpha \) denote the equilibrium price of shares, when the controlling shareholder’s ownership is \( \alpha' \) and \( \alpha \), respectively. Because a higher ownership concentration gives a better incentive alignment, investors rationally anticipate that \( P_{\alpha'} > P_\alpha \), if \( \alpha' > \alpha \) (see Proposition 3 below). Obviously, the controlling shareholder will not buy any shares at prices above \( P_{\alpha'} \) and in fact, we show in the appendix, that at most he is willing to pay \( P_\alpha \). Consider the decision of a representative minority investor \( j \). If sufficient numbers of shares are tendered by other minority investors at any prices below \( P_{\alpha'} \) to the controlling shareholder, then the deal will go through even if \( j \) does not sell. As a result, investor \( j \) enjoys a price appreciation and obtains a higher valuation by free riding on other investors. Because each minority investor is infinitesimal and not a pivotal decision maker, the free-rider incentive implies no trade in equilibrium. The appendix contains a detailed and formal proof for our free-rider argument.

\[ \text{11} \] The free-rider argument developed here breaks down if the controlling shareholder instead of buying a small number of shares offers to buy the remaining outstanding shares. In this case however suppose that the controlling shareholder finances his acquisition by borrowing in the bond market. In a more
The next two sections analyze the equilibrium implications of our model on both the real and financial sides in detail. Before delving into the details on the relationship between agency and asset returns, we first analyze equilibrium for the benchmark case under which there are no agency costs.

4 Benchmark: Perfect Investor Protection

This section summarizes the main results on both the real and financial sides of an economy under perfect investor protection.

When investor protection is perfect, the cost of diverting resources away from the firm is infinity, even if the controlling shareholder diverts a negligible fraction of the firm’s resources. Therefore, the controlling shareholder rationally decides not to pursue any private benefits and maximizes the present discounted value of cash flows using the unique discount factor in the economy. That is, there are no conflicts of interest between the controlling shareholder and the outside minority investors. Our model is then essentially a neoclassical production based asset pricing model, similar to Cox, Ingersoll, and Ross (CIR) (1985). We will highlight the main differences between our model and CIR later in this section.

The controlling shareholder chooses the first-best investment level

\[ I^*(t) = i^*K^*(t), \]

where investment-capital ratio \( i^* \) is obtained from (14) by letting \( \eta \to \infty \) and is

\[ i^* = \frac{1 + h\epsilon^2}{(\gamma + 1)\epsilon^2} \left[ 1 - \sqrt{1 - \frac{2(\gamma + 1)\epsilon^2(h - \rho - \delta (1 - \gamma))}{\gamma (1 + h\epsilon^2)^2}} \right]. \tag{26} \]

Starred variables (“*”) denote the equilibrium values of the variables under perfect investor protection. From Proposition 1 we know that there is overinvestment under weak investor protection, \( i^* < i \).

Tobin’s \( q \) under this first-best benchmark is given by

\[ q^* = \frac{1}{1 - \epsilon^2 \gamma i^*} \geq 1. \tag{27} \]

Before analyzing the stochastic case (\( \epsilon > 0 \)), we briefly sketch the model’s prediction when capital accumulation is deterministic (\( \epsilon = 0 \)). It is easy to show that without volatility general framework, one that incorporates weak investor protection in the bond market, these bonds would be bought at a premium possibly large enough to offset the gain from buying all the shares from the minority investors, and no trade would occur. We thank John Long for making this point.
in the capital accumulation equation (1), Tobin’s $q$ is equal to unity. This is implied by no arbitrage when capital accumulation is deterministic and incurs no adjustment cost and production function has the constant return to scale property.

The key prediction of our model on the real side under perfect investor protection is that Tobin’s $q$ is larger than unity, when capital accumulation is subject to shocks ($\epsilon > 0$). That is, the value of installed capital is larger than that of to-be-installed capital. The intuition is as follows. In our model, the net change of capital stock over a fixed time interval $\Delta t$ is subject to both a net investment $(I - \delta K)\Delta t$, but also innovations $\epsilon_1 I \Delta Z(t)$ that are proportional to the level of gross investment. That is, new investment introduces uncertainty into the capital accumulation process. This production risk is systematic and thus must be priced in equilibrium. As a result, risk averse investors view it as costly to adjust capital stock in equilibrium. This in turn drives a wedge between the price of uninstalled capital and the price of installed capital. We call this channel the production risk channel.

The main difference between our model and the CIR model is Tobin’s $q$, or equivalently stated, the price of installed capital. In CIR, the production technology is also constant return to scale. However, in their model, the volatility of output does not depend on the level of gross investment. Therefore, the price of capital in CIR is equal to unity. Dow et al. (2004) incorporate the manager’s empire-building incentive into the neoclassical production-based asset pricing framework such as CIR, retaining the feature that the price of capital is equal to unity.

Because the outside minority investor and the controlling shareholder have the same utility functions, and markets are effectively complete in the perfect investor protection case, we naturally expect that both the controlling shareholder and outside minority investors to hold no risk-free asset in equilibrium and invest all of their wealth in the risky asset in equilibrium. The minority investors’ and the controlling shareholder’s consumption plans are equal to their respective entitled dividends, in that $C^*_2(t) = (1 - \alpha)D^*(t)$, and $C^*_1(t) = \alpha D^*(t)$, and $D^*(t) = d^*K^*(t)$, with the first-best dividend-capital ratio given by $d^* = h - i^*$.

The equilibrium interest rate under perfect investor protection, $r^*$, is given by (18), associated with the first-best investment-capital ratio $i^*$. Equation (18) indicates that the interest rate $r^*$ is constant and is determined by the following three components: (i) the investor’s subjective discount rate $\rho$, (ii) the net investment rate $(i - \delta)$, and (iii)
the precautionary saving motive. In a risk-neutral world, the interest rate must equal the subjective discount rate in order to clear the market. This explains the first term. The second term captures the economic growth effect on the interest rate. A higher net investment rate \((i - \delta)\) implies more resources are available for consumption in the future and thus pushes up demand for current consumption relative to future consumption. To clear the market, the interest rate must increase. The second effect is stronger when the agent is less willing to substitute consumption inter-temporally, which corresponds to a lower elasticity of intertemporal substitution \(1/\gamma.\)

The third term captures the precautionary savings effect on interest rate determination. A high net investment rate increases the riskiness of firm’s cash flows, and thus makes agents more willing to save. This preference for precautionary savings reduces current demand for consumption and lowers the interest rate, *ceteris paribus.*

In this benchmark case, the equilibrium stock price \(P^\ast\) is given by a geometric Brownian motion (10) with drift \(\mu^\ast_P = i^\ast - \delta\) and volatility \(\sigma^\ast_P = \iota^\ast \epsilon.\)

Next, we analyze how different degrees of investor protection affect asset prices and returns.

## 5 Equilibrium Asset Returns

We first analyze the equilibrium interest rate and then turn to the stock return.

### 5.1 Risk-Free Rate

The next proposition relates the interest rate under imperfect investor protection to that in the benchmark case.

**Proposition 2** *Worse investor protection or lower share of equity held by the controlling shareholder are associated with a higher risk-free interest rate if and only if \(1 > \epsilon^2 (\gamma + 1) i.\) Specifically, the interest rate in an economy with imperfect investor protection is higher than that under perfect investor protection if and only if \(1 > \epsilon^2 (\gamma + 1) i.\)

Changes in the degree of investor protection produce two opposing effects on the equilibrium interest rate. Both effects result from investment being higher under weaker
investor protection. First, because of the effect of economic growth on the interest rate, higher investment implies larger output in the future and thus intertemporal consumption smoothing motives makes the agent more willing to finance his current consumption by borrowing from the future, leading to a higher current equilibrium interest rate. Second, higher investment makes capital accumulation more volatile and implies a stronger precautionary saving effect, thus pushing down the current equilibrium interest rate, ceteris paribus. The proposition illustrates that the growth effect dominates the precautionary effect if and only if \( 1 > \epsilon^2 (\gamma + 1) i \). Under Assumption 5, \( 1 > \epsilon^2 i \gamma \). This is not sufficient to sign the derivative of \( r \) with respect to agency costs, but for low values of \( i \) it implies that the growth effect dominates and interest rates are higher under weaker investor protection.

We now turn to equilibrium valuation from both the controlling shareholder and the minority investor’s perspective.

### 5.2 Firm Valuation and Returns

**Controlling Shareholder’s Shadow Equity Valuation.** Even though the controlling shareholder cannot trade firm equity with outside minority investors, the controlling shareholder nonetheless has a shadow value for equity. Let \( \hat{P}(t) \) denote this shadow price of equity for the controlling shareholder. We may compute \( \hat{P}(t) \) as follows:

\[
\hat{P}(t) = \frac{1}{\alpha} E_t \left[ \int_t^{\infty} e^{-\rho(s-t)} \frac{M(s)^{1-\gamma}}{M(t)^{-\gamma}} ds \right] = \frac{1}{1 - \epsilon^2 i \gamma} K(t).
\]

The equilibrium shadow market-to-book value of the firm to the controlling shareholder or shadow Tobin’s \( \hat{q} \) is therefore given by

\[
\hat{q} = \frac{1}{1 - \epsilon^2 i \gamma}.
\]

We note that the shadow value \( \hat{q} \) is higher than \( q^* \), Tobin’s \( q \) under perfect investor protection. The intuition is as follows. The controlling shareholder distorts capital accumulation decision in pursuit of his private benefits and thus leads to his shadow value for the firm higher than \( q^* \). Another way to understand this is by the revealed preference argument. The controlling shareholder could set the investment-capital ratio to \( i^* \) and steal nothing \( s = 0 \), which would imply \( \hat{q} = q = q^* \). Therefore, the controlling shareholder’s decision must yield a higher valuation \( \hat{q} \) than \( q^* \). We next turn to firm valuation from the minority investor’s perspective.
Minority Investors’ Valuation. Theorem 1 shows that the equilibrium price for firm equity is proportional to capital stock and is given by $P(t) = qK(t)$, where $q$ measures the market-to-book ratio, also known as Tobin’s $q$. The next proposition characterizes the monotonic relationship between $q$ and investor protection.

Proposition 3 Tobin’s $q$ increases with investor protection, in that $dq/d\eta > 0$, and increases with the controlling shareholder’s cash flow rights, in that $dq/d\alpha > 0$.

Proposition 3 demonstrates that the model is consistent with the evidence offered in La Porta et al. (2002), Black et al. (2003), Gompers et al. (2003), and Doidge (2004) on the relationship between firm value and investor protection. The model also predicts that firm value increases with the controlling shareholder’s ownership $\alpha$. This incentive alignment effect due to higher cash flow rights is consistent with empirical evidence in Claessens et al. (2002) on firm value and cash flow ownership, and with the evidence for Korea in Baek et al. (2004) where it is found that non-chaebol firms experienced a smaller reduction in their share value during the East Asian crisis.

The intuition for the result in Proposition 3 that $q$ is monotonically increasing in investor protection relies on the fact that private benefits of control are extracted from installed capital and not from new capital goods, thus the possibility of extracting private benefits lowers the value of installed capital relative to new capital goods, which is to say the firm’s market-to-book ratio.

Tobin’s $q$ varies with investor protection with the direction of change depending on the relative strength of the production risk channel and the agency channel. Through the agency channel, higher $\eta$ (or higher $\alpha$) leads to a higher payout rate and lower stealing which both contribute to a higher $q$. However, through the production risk channel, the higher $\eta$ (or lower $\alpha$) leads to lower investment rates and lower volatility of installed capital and dividends. This reduces the wedge between installed capital and new capital goods, i.e. $q$. Proposition 3 shows that the agency channel dominates.

La Porta et al. (2002) and Shleifer and Wolfenzon (2002) provide a theoretical explanation for the decline in $q$ resulting from worse investor protection that relies on static partial equilibrium analysis and risk neutrality. Lan and Wang (2004) extend the analysis to a dynamic partial equilibrium analysis with risk neutral entrepreneurs and outside minority investors. Ours is the first paper to explain this empirical evidence while computing firm value in a dynamic stochastic general equilibrium asset pricing model with risk averse agents.
We next turn to the dividend yield. Let \( y \) be dividend yield, in that \( y = D/P = d/q \). We solve for the dividend yield using the following equilibrium excess return relationship:

\[
\lambda + r = \gamma \sigma_D^2 + r = \mu_D + y, \tag{28}
\]

where the first equality uses \( \lambda = \gamma \sigma_D^2 \) in equilibrium and \( \mu_P = \mu_D \). Using the equilibrium interest rate formula (18), we have the following expression for the dividend yield:

\[
y = \rho + \mu_D (\gamma - 1) - \frac{\gamma(\gamma - 1)\sigma_D^2}{2}. \tag{29}
\]

**Proposition 4** The dividend yield increases (decreases) with worse investor protection if and only if \( \gamma > 1 \) (\( \gamma < 1 \)).

We note that both Tobin’s \( q \) and the dividend-capital ratio \( d \) monotonically increase in investor protection \( \eta \). The later is consistent with the evidence in La Porta et al. (2000a). However, the dividend yield \( y \), which is given by the ratio between dividend-capital ratio \( d \) and Tobin’s \( q \), may either increase or decrease in investor protection, depending on \( \gamma \). This reflects the model’s prediction that the percentage change of dividend-capital \( d \), \( d \log d/d\eta \), and the percentage change of Tobin’s \( q \), \( d \log q/d\eta \), in general are different. Whether \( d \log d/d\eta > d \log q/d\eta \) or not depends on whether \( \gamma > 1 \) or not.

The reason the effect on the dividend yield of changing investor protection depends on risk aversion is that better investor protection leads to lower interest rates thus changing the willingness of investors to save. First, we consider the comparison benchmark case in which the dividend yield does not depend on investor protection. If agents have logarithmic utility functions, the dividend yield is equal to the agent’s subjective discount rate \( \rho \), directly implied by (29). This reflects the myopic nature of logarithmic utility agents. When \( \gamma > 1 \), the substitution effect is stronger than the income effect in the inter-temporal allocation of consumption. Whether \( \gamma \) is interpreted as the risk aversion coefficient or the inverse of the elasticity of inter-temporal substitution, empirical estimates of \( \gamma \) are in general larger than unity.\(^{14}\) Therefore, with a plausible \( \gamma > 1 \), the model predicts a higher dividend yield in countries with weaker investor protection and higher interest rates.

The next proposition gives our main results on equilibrium returns. It indicates that the valuation effects of investor protection are not restricted to price level effects as previously studied, or the dividend yield, but have implications for discount rates as well.

Proposition 5  Expected return $\mu_P$, return volatility $\sigma_P$, and risk premium $\lambda$ decrease in investor protection $\eta$ and ownership $\alpha$.

The proposition shows that the rate of excess equity returns is affected in equilibrium by changes in the degree of investor protection. The intuition is quite simple. Weaker investor protection increases the riskiness of the stock to minority investor and thus the risk premium they charge to hold its shares. To see this reasoning in more detail note that the equilibrium equity risk premium is given by

$$\lambda = \gamma \sigma_P^2 = \gamma \epsilon^2 i^2.$$ 

The risk premium reflects the price attached by the minority investors to the uncertainty associated with the economy’s single factor (i.e. $Z(t)$). A positive shock to capital (i.e. $dZ(t) > 0$) leads to a higher stock return and a higher dividend payment. Minority investors’ consumption increases and their marginal utility declines. This negative correlation between stock returns and marginal utility of consumption is larger in absolute terms when investor protection is weaker and the investment rate is higher. This is because the value of this correlation in equilibrium is tied to the volatility of the capital stock, and a higher investment rate makes the existing capital stock more volatile. Thus, the risk premium is larger.\(^{15}\)

There is preliminary evidence in support of proposition 5. Bekaert and Harvey (1997) and Bekaert and Urias (1999) show that emerging markets display higher volatility of returns and larger equity risk premia. Arguably, these countries have weaker corporate governance. Bekaert and Harvey (1997) correlate their estimated conditional stock return volatilities with financial, microstructure, and macroeconomic variables and find some evidence that countries with lower country credit ratings, as measured by Institutional Investor, have higher volatility. Erb et al. (1996) show that expected returns, as well as volatility, are higher when country credit risk is higher.

More direct evidence is provided in Gompers, Ishii, and Metrick (2003) and Daouk, Lee, and Ng (2004). Gompers et al. (2003) construct a governance index that captures differences in shareholder rights in US corporations. They find that firms with stronger shareholder rights outperformed those with weaker shareholder rights. Consistent with our prediction, Table 8 of Gompers et al. (2001) indicates that the portfolio of firms with weaker shareholder rights consistently displayed greater riskiness (i.e. higher correlation

\(^{15}\)As indicated in Proposition 4, not all of the excess returns come necessarily from higher capital accumulation (as a result of overinvestment) and subsequent price appreciation. If $\gamma < 1$, the dividend yield is higher with worse investor protection.
to market returns), specially in the first half of the analysis. Daouk, Lee, and Ng (2004) create an index of capital market governance which captures differences in insider trading laws, short-selling restrictions, and earnings opacity. They model excess equity returns using the international capital asset market model of Bekaert and Harvey (1995) thus allowing for varying degrees of financial integration. Consistent with proposition 5, they show that improvements in their index of capital market governance are associated with lower equity risk premia.

The minority investors in our model behave much like the investors in a traditional consumption capital asset pricing model augmented to include a production sector. However, minority investors are not the ones choosing the investment rate and in fact they will be faced with too much capital accumulation and demand for savings. These predictions differ from production models where the minority investors are the ones choosing the capital accumulation path and can use the investment rate to smooth out business cycle fluctuations. In these other models, as in the benchmark model described above, the volatility of dividends is smaller and the economy’s risk premium is smaller. Hence the model generates a higher risk premium than do traditional neoclassical asset pricing models with production like the CIR (1985) model.

Note that, because equity pricing in the model is done by outside minority investors, the relevant consumption data to feed into the risk premium calculations is that of minority investors and not of aggregate consumption. Our approach is thus similar to Mankiw and Zeldes (1991) and Vissing-Jorgensen (2003) who focus on consumption data of a smaller sample of stockholders. Relative to these papers, our paper has the advantage that we specify the production channels that induce the risk premium and so it does not require dealing with consumer data to compute the risk premium, which generally produces very noisy estimates. Specifically, our model predicts that, for equal risk aversion $\gamma$ and volatility parameter $\epsilon$, the percentage difference in equity premia between any two countries should be of the same order of magnitude as the percentage difference in squared investment-capital ratios.

Naturally, the disagreement in valuation between the controlling shareholder and outside minority investors approaches zero as investor protection increases, in that $q \to \hat{q}$ as $\eta \to \infty$. In the case of perfect investor protection the controlling shareholder is homogeneous to the minority investors and investment and dividend policies chosen by the former coincide with what the later would do.

Despite the disagreement between minority investors and the controlling shareholder on the firm’s market-to-book value under imperfect investor protection, they agree on
expected returns. The instantaneous “shadow” return to the controlling shareholder is

\[
\frac{d\hat{P}(t) + (M(t)/\alpha) dt}{P(t)} = \left( i - \delta + \frac{m}{\alpha q}\right) dt + \epsilon i dZ(t) = (\mu_P + y) dt + \sigma_P dZ(t) .
\]

Therefore, the instantaneous “shadow” return is equal to \( \mu_P + y \), the expected stock return (including the dividend component) for outside minority investors. Intuitively, the economy grows stochastically on a balanced path. Both the controlling shareholder and outside minority investors share the same marginal valuation.

6 Numerical Simulations

Our model is quite parsimonious in that it has only seven parameter values from both the production and investor side of the economy. The choice of parameters is done in one of two ways. Some parameters are obtained by direct measurements conducted in other studies. These include the rate of time preference \( \rho \), risk aversion coefficient \( \gamma \), the depreciation rate \( \delta \), and the equity share of the controlling shareholder \( \alpha \). The remaining three parameters \((\eta, h, \epsilon)\) are picked so that the model matches three relevant moments in the data.

We calibrate the model for Korea. We focus on an emerging capital market, because corporate governance is likely to be the single most important driver of overinvestment in these countries and it is therefore easier to isolate its effect on the various decisions of economic agents.

6.1 Calibration

We start with the first set of parameters. We choose the coefficient of relative risk aversion to be 2, a commonly chosen level of risk aversion. The depreciation rate is set to an annual value of 0.07. From La Porta et al. (2002) we take the share of stock held by the controlling shareholder for Korea to be \( \alpha = 0.18 \).\(^{16}\) Finally, the subjective discount rate is set to \( \rho = 0.01 \) consistent with empirical estimation results such as those reported in Hansen and Singleton (1982, 1983).

Turning now to the second set of parameters, we calibrate the mean productivity \( h \), the volatility parameter \( \epsilon \), and the investor protection parameter \( \eta \) so that the model matches the following three moments of the data; (i) the mean real annual Korean prime

\(^{16}\)We use the number from La Porta et al. (2002) as they consider direct and indirect equity ownership as opposed to Dahlquist et al. (2003).
lending rate $r$ of 3.7 percent over the 1980-2000 period using the GDP deflator; (ii) the annual standard deviation of detrended per capita real output growth of 4.3 percent observed over the period of 1980-2000; (iii) and the Korean mean investment to GDP ratio of 32.4 percent. The data for all these moments was obtained from the World Bank’s (2002) World Development Indicators database. To see how the model can match these moments consider the model equivalent moments:

$$r = \rho + \gamma (i - \delta) - \frac{\sigma_D^2}{2} \gamma (\gamma + 1),$$

$$\sigma_D = \epsilon i,$$

$$\frac{I}{GDP} = 0.32 \frac{i}{h \left(1 - \frac{(1-\alpha)^2}{2\eta}\right)},$$

where the last equation uses $GDP = C_1 + C_2 + I$. The adjustment factor of 0.32 in the last equation is Korea’s output share of capital from Barro and Sala-i-Martin (1995, Table 10.8 Panel B). This adjustment is needed because in our model GDP measures only the payments to capital and excludes any labor payments.

According to the model there is a unique pair of $(i, \epsilon)$ that can match the interest rate and the volatility of output growth in the economy (see the first two equations above). The resulting calibrated investment to capital ratio is 8.6 percent and the volatility parameter $\epsilon = 0.5$. With $\epsilon$ thus fixed we can choose $h$ and $\eta$ to match the required investment-capital ratio $i = 0.086$, and $I/GDP$. Our choices of $h = 0.102$ and $\eta = 20$ deliver the desired investment-capital ratio, but underestimate the investment to GDP ratio, producing a ratio of 27.5 percent. For fixed $h (\eta)$, choosing a lower $\eta (h)$ can help along this dimension, but at a cost of an implausibly high (low) interest rate and output growth volatility.

With these parameters the cost of stealing as a fraction of gross output is $\frac{\Phi(s,hK)}{hK} = \frac{(1-\alpha)^2}{2\eta} = 0.017$.

### 6.2 Results

We report numerical results for Tobin’s $q$ and the risk premium. The figures below contain two panels A and B. Each figure gives the model’s comparative statics for one of the variables, Tobin’s $q$ or the risk premium when investor protection changes (panel A) and when the share of controlling shareholder changes (panel B).

**Tobin’s $q$.** Consider the market valuation of minority investors and the implied market-to-book value. Figure 1 displays the model’s comparative statics. Panel A fixes $\alpha$ to the
calibrated level for Korea whereas panel B fixes \( \eta \) to the calibrated value for Korea. Recall from our discussion above that while better investor protection leads to a higher \( q \) because of less stealing (the agency channel) it also leads to a lower volatility of output and thus a lower \( q \) (production risk channel). Proposition 3 demonstrated that the agency channel dominates. A sufficiently large \( \eta \) or \( \alpha \) takes Tobin’s \( q \) closer to the benchmark case where we know it is larger than unity. The quantitative results from the model suggest a very low level of \( q \) for Korea given our calibration. These numbers (around 0.5) are most likely an underestimate of the true \( q \). However, it is quite striking that even if \( \eta \) were to increase by 10 fold (from 20 to 200) \( q \) would still be only 0.975, or about 7 percent below the benchmark perfect investor case.

![Figure 1: Tobin’s \( q \).](image)

**Risk Premium.** Recall that proposition 5 showed that the benchmark case of perfect investor protection displayed a smaller risk premium than the imperfect investor protection case. Here, we investigate the quantitative significance of our mechanism. Figure 2 plots the ratio of the risk premium for a specific value of \( \eta \) or \( \alpha \) to the benchmark case (note that in panel B the benchmark level of the risk premium also changes with \( \alpha \)). The figure indicates that Korea can lower the equity premium in the stock market by almost 16 percentage points if it were to move to a world of perfect corporate governance. If
investor protection $\eta$ in Korea was 10 fold higher these gains would be on the order of 1.2 percent. The level of the risk premium under our proposed baseline calibration is 0.2 percent, but with CRRA preferences this is to be expected. We view our model as giving an indication that in relative terms, from imperfect investor protection to perfect investor protection the changes in the risk premium can be substantial.

These model estimated gains are quite large. They compare quite reasonably to the empirically estimated gains on lower cost of equity capital from capital market liberalizations (see Stulz (1999) among others). Stulz (1999) suggests that capital market liberalizations can lead to lower domestic cost of capital for both diversification reasons and improvements in corporate governance. However, the liberalization of the domestic capital market as an indirect way of improving investor protection generates only modest gains in the cost of capital relative to a more direct pursuit of changing local legislation and implementation of policies recommended here.

![Figure 2: Risk Premium.](image)

### 7 The Cost of Imperfect Investor Protection

In this section, we explore the implications of imperfect investor protection on utility costs for outside investors. We measure utility costs in terms of equivalent variation by
asking the following trade-off question for the outside minority investor. What fraction of his personal wealth is the outside minority investor willing to give up for a permanent improvement of investor protection from the current level $\eta$ to the benchmark (first-best) level of $\eta = \infty$. Let $(1 - \zeta)$ denote the fraction of his wealth that the minority investor is willing to give up for such a permanent increase in the quality of investor protection. Then, the minority investor is indifferent if and only if the following equality holds:

$$J^*_2(\zeta W_0) = J_2(W_0),$$

where $J_2$ is the minority investor’s value function and $W_0$ is his initial wealth.\(^\text{17}\)

Since the minority investor’s wealth $W$ is proportional to the firm’s capital stock $K$ in equilibrium, $(1 - \zeta)$ is also the fraction of the capital stock that the minority investors own and are willing to give up, in exchange for better investor protection. Using the value function formula given in Section 3, we may calculate the cost of imperfect investor protection in terms of $\zeta$ and obtain:\(^\text{18}\)

$$\zeta = \left(\frac{y}{y^*}\right)^{1/(1 - \gamma)} \frac{d}{d^*}. \quad (30)$$

**Proposition 6** The minority investors’ utility cost is higher under weaker investor protection, in that $d\zeta/d\eta > 0$. Naturally, for any $\eta < \infty$, $0 < \zeta < 1$.

Figure 3 plots $\zeta$ against various levels of investor protection parameter $\eta$, holding ownership fixed at 0.18 in Panel A; and plots $\zeta$ against the controlling shareholder’s ownership $\alpha$, holding investor protection parameter $\eta$ fixed at 20 in Panel B. The results are quite striking. Investors are willing to give up a substantial part of their own wealth for a stronger investor protection. Consider for example the case where investor protection is 10 fold better than the calibrated number for Korea (i.e., $\eta = 200$). Even at this high level of investor protection, Korean investors are willing to give up 6.5 percent of their wealth (or capital stock) to move to a perfect investor protection world. The large increase in the equity premium of having weak investor protection observed above translates into a substantial cost in terms of utility.

\(^\text{17}\)We use $J^*_2$ to denote the corresponding value function for minority investors under perfect investor protection.

\(^\text{18}\)By applying L’Hopital’s rule to (30) around $\gamma = 1$, we may obtain the formula for $\zeta$ for logarithmic utility. With some algebra, it can be verified that

$$\zeta = \frac{d}{d^*} \exp \left[\frac{(\mu_D - \frac{1}{2}\sigma_D^2) - (\mu^*_D - \frac{1}{2}\sigma^*_D)^2}{\rho}\right].$$
Figure 3: Utility cost of imperfect investor protection, $\zeta$ (percent of wealth).

8 A Re-Interpretation of the Production Setup

Let $K$ be the firm’s total productive capital stock. Total capital is the sum of the firm’s tangible and intangible assets. Changes in total capital consist of investment in tangible or physical capital plus produced intangible assets:

$$dK(t) = (I(t) - \delta K(t)) dt + \epsilon I(t) dZ(t),$$

(31)

where $\delta$ is now the depreciation rate on total capital and $I(t)$ represents the firm’s gross investment in productive capital. Investment spending $I$ includes investment in physical structures and equipment, but also research and development. The amount $\epsilon I(t) dZ(t)$ gives the value of intangible assets produced contemporaneously to the investment expenditures $I$. The assumed production function of intangible assets has constant returns to scale on investment with mean productivity of $\epsilon$ and is subject to a random shock described by the Brownian process $Z(t)$. Gross investment in tangible assets is still given by (2) with the reinterpretation that $K$ is total capital.

This new formulation of the dynamics of capital is meant to capture the idea that “[f]irms produce productive capital by combining plant, equipment, new ideas, and organization” (Hall (2001)). Hall (2001) argues that securities markets record in their
valuation of a firm not only the increases in physical capital but also the increases in intangible capital and shows that, in fact, substantial variation in valuations is derived from changes in intangible capital as opposed to changes in tangible capital. This feature of our model makes it different from previous production based asset pricing models that rely exclusively in the connection of firm returns and firm value to physical capital returns (e.g. Cox, Ingersoll, and Ross (1985), Cochrane (1991, 1996), Naik (1994), Kogan (1999), and Singal and Smith (1999)).

This reformulation of our model allows the identification of the dynamics of tangible capital under the assumption that $I$ includes investment in tangible and intangible assets in fixed proportions. Denote by $\bar{K}(t)$ the firm’s tangible capital and suppose that $I$ is only physical capital investment. Then, because tangible and intangible assets are assumed to have the same depreciation rate, we have that

$$d\bar{K}(t) = (I(t) - \delta\bar{K}(t))dt.$$ 

This allows us to compare firm valuations as given by Tobin’s $q$ by taking as the book value of assets either total productive capital $K(t)$ or only the firm’s physical capital $\bar{K}(t)$.

9 Conclusion

Agency conflicts are at the core of modern corporate finance. The large corporate finance literature on investor protection has convincingly documented that corporations in most countries, especially with weak investor protection often have controlling shareholders and that these controlling shareholders derive private benefits at the cost of outside minority shareholders. In fact, several recent papers have shown that firm value is lower in countries with weaker investor protection thus providing support for the hypothesis that weak investor protection allows the distortion of investment and payout decisions by controlling shareholders.

We have provided a model that integrates corporate governance frictions into a production based asset pricing model. A critical part in solving the model is dealing with investor heterogeneity that arises naturally from having a controlling shareholder running the firm and minority investors trading on the stock of the firm conditional on a stochastic dividend stream. In the equilibrium we analyze this is greatly simplified by making use of homogeneity of degree one of our solution.

The model delivers four main results. First, under weak investor protection there is
overinvestment. Second, this investment distortion leads to a lower Tobin’s $q$ in countries with weaker investor protection. Third, overinvestment leads to higher volatility of returns and higher excess expected returns. The text refers to evidence consistent with these predictions. Finally, and fourth, provided risk aversion is higher than 1 the dividend yield is higher in countries with imperfect investor protection. To the best of our this last prediction has not been tested.
Appendices

A Proofs

This appendix contains the proofs for the theorem and the propositions.

A.1 Proof of Theorem 1

The FOC (20) gives
\[ m^{-\gamma} \alpha = A_1 (1 - \epsilon^2 i \gamma), \]  \tag{A.1}
where \( m = M/K \) and \( i = I/K \) are the controlling shareholder’s equilibrium consumption-capital ratio, and the firm’s investment-capital ratio, respectively. Plugging the stealing function into (6) gives
\[ m = \alpha d + \frac{1 - \alpha^2}{2\eta} h = \left( \frac{1 - \phi}{\gamma} \right) h - i + \frac{1 - \alpha^2}{2\alpha \eta} h = \alpha \left( (1 + \psi) h - i \right), \]  \tag{A.2}
where \( d \) is the dividend-capital ratio. Plugging (A.1) and (A.2) into the HJB equation (19) gives
\[ 0 = \frac{1}{1 - \gamma} m^{1-\gamma} - \rho \frac{A_1}{1 - \gamma} + (i - \delta) A_1 - \frac{\epsilon^2}{2} i^2 \gamma A_1 \]
\[ = \frac{A_1}{1 - \gamma} ((1 + \psi) h - i) \left( 1 - \epsilon^2 \gamma i \right) - \rho \frac{A_1}{1 - \gamma} + (i - \delta) A_1 - \frac{\epsilon^2}{2} i^2 \gamma A_1. \]
The above equality implies the following relationship:
\[ ((1 + \psi) h - i) \left( 1 - \epsilon^2 \gamma i \right) = y, \]  \tag{A.3}
where \( y \) is dividend yield and is given by
\[ y = \rho - (1 - \gamma) (i - \delta) + \frac{1}{2} \gamma (1 - \gamma) \epsilon^2 i^2. \]  \tag{A.4}

We note that (A.3) and (A.4) automatically imply the following inequality for investment-capital ratio:
\[ i < (\epsilon^2 \gamma)^{-1}. \]  \tag{A.5}
This above inequality will be used in proving propositions.

We may further simplify (A.3) and give the following quadratic equation for investment-capital ratio \( i \):
\[ \gamma \left( \frac{\gamma + 1}{2} \right) \epsilon^2 i^2 - \gamma [1 + (1 + \psi) \gamma \epsilon^2] i + (1 + \psi) h - (1 - \gamma) \delta - \rho = 0. \]  \tag{A.6}
For $\gamma > 0$, solving the quadratic equation (A.6) gives

$$i = \frac{1}{\gamma(\gamma + 1)e^2} \left[ \gamma \left[ 1 + (1 + \psi) h e^2 \right] \pm \sqrt{\Delta} \right],$$  \hspace{1cm} (A.7)

where

$$\Delta = \gamma^2 \left[ 1 + (1 + \psi) h e^2 \right]^2 \left[ 1 - \frac{2\gamma(\gamma + 1)e^2 ((1 + \psi) h - (1 - \gamma) \delta - \rho)}{\gamma^2 [1 + (1 + \psi) h e^2]^2} \right].$$

In order to ensure that investment rate given in (A.7) is a real number, we require that $\Delta > 0$, which is explicitly stated in Assumption 3. Next, we choose between the two roots for investment-capital ratio given in (A.7). We note that when $\epsilon = 0$, investment-capital ratio is

$$i = \frac{[(1 + \psi) h - (1 - \gamma) \delta - \rho]}{\gamma},$$

as directly implied by (A.6). Therefore, by a continuity argument, for $\epsilon > 0$, the natural solution for the investment-capital ratio is the smaller root in (A.7) and is thus given by

$$i = \frac{1}{\gamma(\gamma + 1)e^2} \left[ \gamma \left[ 1 + (1 + \psi) h e^2 \right] - \sqrt{\Delta} \right].$$

We may also solve for the value function coefficient $A_1$ and obtain

$$A_1 = \frac{m^{-\gamma} \alpha}{1 - e^{2i\gamma}} = \frac{m^{1-\gamma}}{y},$$

where $y$ is the dividend yield and is given by (A.4).

Next, we check the transversality condition for the controlling shareholder:

$$\lim_{T \to \infty} E \left( e^{-\rho T} | J_1(K(T)) \right) = 0.$$  \hspace{1cm} (A.8)

It is equivalent to verify $\lim_{T \to \infty} E \left( e^{-\rho T} K(T)^{1-\gamma} \right) = 0$. We note that

$$E \left( e^{-\rho T} K(T)^{1-\gamma} \right) = E \left[ e^{-\rho T} K_0^{1-\gamma} \exp \left( (1 - \gamma) \left[ \left( i - \delta - \frac{\epsilon^2 t^2}{2} \right) T + eiZ(T) \right] \right) \right],$$

$$= e^{-\rho T} K_0^{1-\gamma} \exp \left[ (1 - \gamma) \left( i - \delta - \frac{\epsilon^2 t^2}{2} + \frac{1 - \gamma}{2} \epsilon^2 t^2 \right) T \right].$$  \hspace{1cm} (A.9)

Therefore, the transversality condition will be satisfied if $\rho > 0$ and dividend yield is positive ($y > 0$), as stated in Assumption 5.

Now, we turn to the optimal consumption and asset allocation decisions for the controlling shareholder. The transversality condition for the minority investor is

$$\lim_{T \to \infty} E \left( e^{-\rho T} | J_2(W(T)) \right) = 0.$$  \hspace{1cm} (A.10)
Recall that in equilibrium, the minority investor’s wealth is all invested in firm equity and thus his initial wealth satisfies $W_0 = (1 - \alpha) q K_0$. Since the minority investor’s wealth dynamics and the firm’s capital accumulation dynamics are both geometric Brownian motions with the same drift and volatility parameters, it is immediate to note that the transversality condition for minority investor is also met if and only if dividend yield $y$ is positive, as stated in Assumption 5. Moreover, we may verify that the minority investor’s value function is given by

$$J_2(W_0) = E \left[ \int_0^\infty e^{-\rho t} \frac{1}{1 - \gamma} \left( [(1 - \alpha) dK(t)]^{1-\gamma} - 1 \right) dt \right] = \frac{1}{1 - \gamma} \left( [(1 - \alpha) dK_0]^{1-\gamma} \frac{1}{y} - \frac{1}{\rho} \right) = \frac{1}{1 - \gamma} \left( W_0^{1-\gamma} \frac{1}{y^\gamma} \frac{1}{\rho} \right),$$

where the second line uses (A.9). Thus, the value function coefficient $A_2$ is given by $A_2 = 1/y^\gamma$. In Section 7, we use the explicit formula for the minority investor’s value function $J_2(W_0)$ to calculate the utility cost of imperfect investor protection.

**A.2 Proof For the Free-Rider Argument in Section 3.3**

We elaborate on the details of how our free-rider argument gives rise to a constant ownership structure over time. We use $J_1(K; \alpha)$ to denote the explicit dependence of the controlling shareholder’s value function on his ownership $\alpha$. Using the envelope theorem, we have

$$\frac{d}{d\alpha} J_1(K; \alpha) = E \left[ \int_t^\infty e^{-\rho(s-t)} M(s)^{-\gamma} D(s) ds \right] = A_1 K(t)^{1-\gamma} \frac{d}{dm}.$$  

(A.12)

The derivative in (A.12) describes the increase in lifetime utility for the controlling shareholder of marginally increasing his firm ownership from $\alpha$ to $\alpha'$. This is not his monetary valuation because we are assuming risk aversion, i.e. $\gamma > 0$. To get his monetary valuation, or willingness to pay, note that we can express the controlling shareholder’s lifetime utility as a function of his wealth

$$J_1(K; \alpha) = J_1 \left( \frac{W}{\alpha \hat{q}}; \alpha \right),$$

where $W$ represents his stock market wealth and $\hat{q}$ is the controlling shareholder’s Tobin’s-$q$ given in section 5. Thus, we have

$$\frac{d}{dK} J_1(K; \alpha) = \alpha \hat{q} \frac{d}{dW} J_1 \left( \frac{W}{\alpha \hat{q}}; \alpha \right).$$
The controlling shareholder’s willingness to pay or the monetary value of the utility value described in (A.12) is

\[
\frac{d}{d\alpha} J_1 (K; \alpha) = AK^{1-\gamma} \frac{d}{d\alpha} \left( \frac{W_{aq}}{\alpha} \right) = \alpha \hat{q} \frac{d}{d\alpha} K (t) = qK (t) = P_\alpha (t),
\]

where the next to last equality follows from the definition of \(q\), and \(P_\alpha (t)\) is the time \(t\) price per share set by minority shareholders. Note that \(\hat{P}_\alpha (t)\) as given in section 5 represents the value of the existing shares for the controlling shareholder and is different from his willingness to pay as given by \(P_\alpha (t)\) when acquiring more shares.

The free rider problem is now apparent. If the equilibrium is for all minority shareholders to sell at \(P_\alpha (t)\) then by deviating from this equilibrium, an infinitesimal investor can gain because trading with other minority investors after the trade with the controlling shareholder has taken place yields a higher valuation \(P_{\alpha'} (t)\). This higher valuation results from a higher \(q\) due to a higher equity share of the controlling shareholder (see Proposition 3). Finally, note that selling by the controlling shareholder is not desirable because he stands to lose the private benefits, but minority investors would also not buy at \(P_\alpha (t)\) because trade that occurs in the next instant will be already at a lower price.

A.3 Proofs for propositions

Proof of Proposition 1. Define

\[
f(x) = \frac{\gamma (\gamma + 1)}{2} e^2 x^2 - [1 + (1 + \psi) \frac{h e^2}{\alpha}] \gamma x + (1 + \psi) h - \rho - \delta (1 - \gamma). \tag{A.13}
\]

Note that \(f(i) = 0\), where \(i\) is the equilibrium investment rate and the smaller of the zeros of \(f\). Also, \(f(x) < 0\) for any value of \(x\) between the two zeros of \(f\) and is greater than or equal to zero elsewhere. Now,

\[
f (\gamma^{-1} \epsilon^{-2}) = \frac{1 - \gamma}{2\epsilon e^2} - \rho - \delta (1 - \gamma).
\]

Therefore, \(f (\gamma^{-1} \epsilon^{-2}) < 0\), if and only if Assumption 5 is met. Hence, under Assumption 5, \(i < \gamma^{-1} \epsilon^{-2}\). Also, under Assumption 1, \(f(0) = (1 + \psi) h - \rho - \delta (1 - \gamma) > 0\) which implies that \(i > 0\).
Abusing notation slightly use (A.13) to define the equilibrium investment rate implicitly \( f(i, \psi) = 0 \). Taking the total differential of \( f \) with respect to \( \psi \), we obtain

\[
\frac{di}{d\psi} = \frac{1}{\gamma} \left( 1 - \gamma^2 i \right) \frac{h (1 - \gamma^2 i)}{1 - \gamma \epsilon^2 i + ((1 + \psi) h - i) \epsilon^2}.
\]

At the smaller zero of \( f \), \( i < \gamma^{-1} \epsilon^{-2} \). Together with \((1 + \psi) h - i > (1 - \phi) h - i = d > 0 \), implies that \( di/d\psi > 0 \).

**Proof of Proposition 2** Differentiate (18) with respect to the agency cost parameter \( \psi \) to get:

\[
\frac{dr}{d\psi} = \gamma \left[ 1 - \epsilon^2 (\gamma + 1) i \right] \frac{di}{d\psi}
\]

and note that \( di/d\psi > 0 \).

**Proof of Proposition 3** We prove the proposition for investor protection. The case for the equity share of the controlling shareholder is then immediate. Use the expression for the dividend yield in (29) to express Tobin’s \( q \) as the ratio between dividend-capital ratio \( d \) and dividend yield \( y \). Differentiating \( \log q \) with respect to investor protection gives

\[
\frac{dq}{d\eta} = \frac{1}{y} \left[ -h \frac{d\phi}{d\eta} - \frac{di}{d\eta} - \left( \frac{d}{y} \frac{dy}{d\eta} \right) \right]
\]

\[
= \frac{1}{y} \left[ -h \frac{d\phi}{d\eta} - \frac{di}{d\eta} - q \left( \frac{d}{d\eta} \left( \frac{d}{d\eta} - \gamma (\gamma - 1) \epsilon^2 i \frac{di}{d\eta} \right) \right) \right]
\]

\[
= \frac{1}{y} \left[ \frac{1 - \alpha h}{\eta^2} - \frac{di}{d\eta} \left( 1 + \frac{1 - \alpha^2}{2 \eta h} \frac{d}{d\eta} h + \gamma \right) \right] > 0,
\]

where the inequality uses \( \gamma > 0 \) and \( di/d\eta < 0 \).

**Proof of Proposition 4** Differentiate the dividend yield with respect to \( \psi \) to get:

\[
\frac{dy}{d\psi} = \frac{di}{d\psi} \left( \gamma - 1 \right) \left( 1 - \gamma \epsilon^2 i \right) \leq 0 \text{ iff } \gamma \leq 1,
\]

and note that the agency cost parameter \( \psi \) decreases with both investor protection and \( \eta \) and ownership \( \alpha \).
Proof of Proposition 5  Weaker investor protection or lower share of equity held by the controlling shareholder both lead to a higher agency cost parameter $\psi$. Proposition 1 shows that a higher $\psi$ leads to more investment hence higher volatility of stock returns $\sigma_p^2 = \epsilon^2 i^2$ and higher expected excess returns $\lambda = \gamma \sigma_p^2$. Finally, we get that the change in total returns when investor protection changes depends on $\gamma$. From (28):

$$\frac{d(\gamma \epsilon^2 i^2 + r)}{d\psi} = \gamma (\epsilon^2 i + 1 - \epsilon^2 i \gamma) \frac{di}{d\psi},$$

which is strictly positive under Assumption 5. Expected returns are higher with weaker investor protection or lower share of equity held by the controlling shareholder.

Proof of Proposition 6  Differentiating $\log \zeta$ with respect to $\eta$ gives

$$\frac{d \log \zeta}{d\eta} = \frac{d \log d}{d\eta} + \frac{1}{1 - \gamma} \frac{d \log y}{d\eta}$$

$$= \frac{d \log d}{d\eta} + \frac{1}{1 - \gamma} \frac{1}{y} \left( (\gamma - 1) \frac{di}{d\eta} - \gamma(\gamma - 1) \frac{\epsilon^2 i}{d\eta} \right)$$

$$= \frac{d \log d}{d\eta} - \frac{di}{d\eta} \frac{1}{y} (1 - \gamma \epsilon^2 i).$$

Using $1 - \gamma \epsilon^2 i > 0$ and $di/d\eta < 0$, the results reported in Proposition 1, and $d \log d/d\eta > 0$, implies $d\zeta/d\eta > 0$. 

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References


