Abstract

After making a loan, a bank finds out if the loan needs contract enforcement ("monitoring"); it also decides whether to lay off credit risk in order to release costly capital. A bank can lay off credit risk by either selling the loan or by buying insurance through a credit default swap (CDS). With a CDS, the originating bank retains the loan’s control rights but no longer has an incentive to monitor; with loan sales, control rights pass to the buyer of the loan, who can then monitor, albeit in a less-informed manner. In a single-period setting, for high levels of base credit risk, only loan sales are used in equilibrium; risk transfer is efficient, but monitoring is excessive. For low levels of credit risk, equilibrium depends on the cost of capital shortfalls. When capital costs are low, only poor quality loans are sold or hedged; risk transfer is inefficient, and monitoring may also be too low. When capital costs are high, CDS and loan sales can coexist, in which case risk transfer is efficient but monitoring is too low. In both cases, if gains to monitoring are sufficiently high, the borrowing firm may choose to borrow more than is needed to finance itself so as to induce monitoring. Restrictions on the bank’s ability to sell the loan expand the range where CDS are used and monitoring does not occur.

In a repeated setting, reputation concerns may support efficient outcomes where CDS are used and the bank still monitors. Because loan defaults trigger a return to inefficient outcomes in the future, total efficiency cannot be sustained indefinitely. Reputational equilibria are most likely for firms that have high base credit quality or for firms where monitoring has a high impact on default probabilities.

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1 Introduction

If contracts are incomplete, then the assignment of control rights is important; further how they are transferred can affect the value of the firm. Consequently, there is a large literature in corporate finance on how these should be allocated between stakeholders, what happens when they change hands, and the effect that such transfers have on firm value. To date, the focus in the literature has been on equity stakes; by contrast, in this paper we specifically focus on debtors and consider how cash flow rights and control or monitoring rights are transferred among agents and how their value changes when agents can unbundle them.

New insights can be gained from looking at loan markets because of the institutional differences between equity and loan markets; namely the public nature of a CDS market and the active nature of bank monitoring. First, the rapid growth of the credit default swap market means that economic actors exposed to credit risk can lay it off cheaply and anonymously over a long time horizon. By contrast, equity swaps are bespoke and therefore expensive to negotiate, while options markets typically do not offer contracts over a long time horizon. In short, if there is a liquid CDS market in a name, any economic actor exposed to credit risk can lay it off at a price which is observable (even if the transaction is not); whereas for equity holdings it is impossible to gauge if such opportunities exist. For equities, it is therefore impossible to verify if the announced holdings correspond to the economic holdings. By contrast, for loans we observe if agents can lay off risk at a low cost.

Second, loan contracts are a more natural instrument with which to evaluate control rights, as banks are held to perform an important monitoring function that debt holders or dispersed equity holders may or may not fulfill. (The recent crisis illustrated the important role that banks play in the real economy.) Indeed, even if a large equity holder seeks to affect firm policy, there are few issues on which she could hold sway compared to the usually more comprehensive loan covenants. In addition, the practice of selling loans is well-established—whereas equity holders exchanging large minority voting blocks less so. We therefore base our model on the institutional features of the the bank debt market and determine when control rights will be exercised optimally, and under what conditions they can they be effectively transferred to another potential monitor.

In our model, a firm has a risky, positive NPV project and seeks funding from a compet-

1Recent contributions to this literature include Admati and Pfleiderer (2009), Edmans and Manso (2009), Edmans (2009), Faure-Grimaud and Gromb (2004).
2A current example of the divergence between announced interest and economic interest is the issue of “empty votes,” in which agents use equity swaps or complex trading strategies to separate cash flow and voting rights. which is an area of concern for the SEC, for example see http://www.sec.gov/spotlight/proxyprocess/proxy-transcript050707.pdf
itive bank. After making the loan, the bank receives private information about the project’s success probability. The bank is also hit by a capital shock that makes it costly to continue to hold the credit risk of the loan.\textsuperscript{3} It can lay off this risk either through a CDS or through a loan sale. The critical difference between the two is that, with a CDS, the originating bank retains ownership and thus control rights over the loan that it made, whereas a loan sale transfers these control rights to the buyer of the loan. Control rights matter because the loan’s owner may enforce these control rights (“monitor the loan”) at a cost.

Overall, our equilibria differ on two possible dimensions. First, either the bank always lays off credit risk (we call these pooling equilibria), or it only lays off credit risk when it knows that the firm is subject to moral hazard and should be monitored (separating equilibria). In the pooling case, risk transfer is efficient, but potential loan buyers do not learn the bank’s private information; in the separating case, risk transfer is inefficient (some banks do not lay off credit risk), but loan buyers do learn the bank’s information.\textsuperscript{4} The reason why this is important is because information about the underlying loans is pertinent for efficient monitoring. Indeed, equilibria also differ by whether the loan buyers monitor the loan or not; they have the ability to monitor but not the same information. There is therefore a tradeoff between efficient risk sharing and efficient monitoring. If loan buyers do monitor, then loan sales dominate CDS (since CDS do not allow monitoring by the CDS seller). If loan buyers do not monitor, then loan sales and CDS have equivalent effects and can coexist, but monitoring is too low. The parameter regions in which this occurs is of particular interest in that control rights are used inefficiently. Of course, the lack of a transparent equity market in “empty votes” means that the situations in which control rights are used inefficiently cannot be easily identified.

Our paper establishes three important relationships. First, we demonstrate that there is a natural tradeoff between bank solvency and bank monitoring. In our model we capture banks’ cost of capital in a reduced form way, however the higher the cost of making a loan to a bank (through a higher cost of capital) the more likely it is to reduce monitoring incentives in equilibrium given that banks can easily go to credit risk transfer markets. Therefore, regulators considering affecting the cost of bank capital should also consider how such changes might change aggregate risk reduction and efficient bank monitoring in the economy. Second, we illustrate that the relationship between success probability and price is not monotonic. Indeed, it depends on the equilibrium amount of monitoring. We establish that even though the uncertainty in our model has a very simple structure, a simple linear

\textsuperscript{3}Specifically, the bank is faced with a correlated and profitable lending opportunity, which requires that it either raise more costly capital or forgo the opportunity.

\textsuperscript{4}This result is consistent with Acharya and Johnson’s (2007) finding that trading in CDS does reveal banks’ inside information about impending credit problems of borrowers.
regression would not allow an econometrician to estimate true default probabilities or the value of the underlying control rights. Conditioning on the existence of the CDS market and on banks’ cost of capital should improve estimates of default risk. Finally, by explicitly modelling how control rights are used, we can analyze some common contractual forms and consider how they affect ex post monitoring. For example, if the loan agreement contains restrictions on the bank’s ability to sell loans, then legally the bank cannot transfer material aspects of the business relationship: in other words, the possibility of monitoring by a loan buyer is reduced. Such anti-assignment clauses increase the bank’s incentive to use CDS, reducing its incentive to monitor. Thus, loan sales restrictions may be counterproductive.

We also consider the robustness of our arguments: our basic analysis posits that the originating bank is the only seller of credit risk. If others sell the firm’s credit risk for portfolio reasons, then the CDS market may be active even when loan sales should dominate. Thus, the social benefit of allowing investors to trade in credit derivatives should be balanced against their effects on the real economy. If there is always portfolio trade in CDS, then the originating bank will prefer to lay off risk through CDS rather than loan sales, and efficient monitoring will not occur.

The possible use of reputation to enforce efficient use of control rights has not been explored in the equity literature. Nevertheless, it is possible that, in a repeated setting, reputation concerns may allow the bank to commit to monitoring even while it makes use of CDS; this would be optimal, since the bank would use its information to monitor efficiently while also engaging in efficient risk transfer.

We show that, in an infinitely-repeated version of our model, limited reputation effects are possible if market participants use loan defaults as a noisy signal that the bank has not monitored, and loan defaults are followed by repetition of the single-period equilibrium as a “punishment” for the bank. Because monitored loans have some chance of default, there is always some chance that, even when the bank honors its commitment to monitor, defaults will occur and inefficient behavior will follow. This reputational equilibrium is generally more likely to be feasible the higher the base credit quality of the bank’s loan and the higher the impact of monitoring on default probability. Thus, our single-period result that CDS are most likely to be used when a firm’s base credit risk is relatively low remains true in the case of multiple periods, with the added proviso that, for these firms, CDS can actually add value compared to loan sales.Ł

The rest of our paper proceeds as follows. Section 2 sets out our single-period model.

\footnote{Ashcraft and Santos (2007) find that the introduction of CDS increases the borrowing rates of firms that are most likely to need monitoring but slightly improves the borrowing rates of firms that are safe and transparent; this is consistent with our result that the introduction of CDS is most likely to add value for firms with low levels of credit risk, whereas it has a negative effect on firms with higher levels of credit risk.}
Section 3 characterizes possible equilibria, taking the face value of the bank’s loan as given; Section 4 then endogenizes this face value and describes which equilibria will in fact prevail. Section 5 analyzes a number of extensions, including the impact of other protection-buyers in the CDS market, the effect of loan sales restrictions, and reputational equilibria in a repeated version of the basic model. In this section, we also discuss the relationship between our results and the existing literature. Finally, Section 6 concludes.

2 Model

Consider the following five-date model of an entrepreneur who raises funds from a bank to undertake a risky project. After the loan is originated, the bank can lay off credit risk either through a credit default swap market or a loan sales market. The owner of the loan can exert costly effort and in some cases decrease the default probability.

At $t = 0$ an entrepreneur raises money from a bank, by making a take-it-or-leave-it offer, in order to fund a project of fixed size 1 which pays off $R$ with some probability and $C$ otherwise. The bank’s contract is characterized by the pair $(R^\ell, C)$, where $R^\ell \leq R$ is the payoff conditional on the project’s success. Here, $C$ is the firm’s collateral or liquidation value, and so $R^\ell - C$ is the risky portion of the loan.

At $t = 1$ the bank gets a private signal about the project’s governance. With probability $\theta$ it learns that the project is “good” and will succeed with probability $p + \Delta$; with probability $1 - \theta$ the project is “risky.” In this case, the entrepreneur can choose either the good project or a riskier one that succeeds with probability $p$ but gives the entrepreneur a private benefit $B$. Thus, with probability $1 - \theta$, the project is prey to moral hazard. We refer to a bank’s private information as its “type.” There are two types of originating banks, $p$ and $p + \Delta$, depending on which project the bank expects the entrepreneur to pick.

At $t = 1$, the bank also receives an opportunity to invest in another project correlated with the existing one. Due to regulatory constraints, (i.e., there is a risk limit and banks have to raise costly external capital), the bank bears a cost of $\beta > 0$ per unit of outstanding risk. In what follows, we refer to the arrival of a correlated opportunity as a capital shock. As all agents in this economy are risk neutral, little is gained by assuming that the capital shock happens stochastically. Assuming that it always occurs, simplifies the analysis without changing the results. We also note in passing that a capital shock plays the same role as the liquidity shocks assumed in the equity literature.

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$^6$For example, suppose that the ex ante variance (beliefs are relative to the public information) of its position is restricted, so that $(R^\ell - C)^2 p (1 - p) \leq \bar{V}$. Another project that is perfectly correlated would violate the bank’s variance constraint: In particular, $4 (R^\ell - C)^2 p (1 - p) > \bar{V}$. Costly external funds prevent a bank from instantaneously raising more capital.
At \( t = 2 \), the bank can offload credit risk from its balance sheet. There are two ways in which a bank can do this. First, it can trade in the CDS market. Second, the bank can sell the loan. If a bank enters into a credit default swap (buys protection), it buys insurance that pays off the face value of the loan if the firm defaults. This is achieved either through physical delivery, in which case the bank delivers the instrument conditional on default (valued at \( C \)) and receives \( R^\ell \), or through cash settlement, in which case the CDS seller pays out \( R^\ell - C \) and the bank retains the existing loan. In both cases, after default the value of the loan plus the CDS is \( R^\ell \). The bank’s aggregate trades in either market are not observable, and so we focus on the case where the bank completely hedges its risk.\(^7\)

Both the loan and CDS market participants are risk neutral and competitive. Therefore, prices are equal to the expected value of the loan or CDS contract to the market. The unconditional probability of project success is \( p + \theta \Delta \). However, as different bank types have different preferences over the credit risk transfer markets, participation in each of the markets may be informative about the project’s underlying success probability. Therefore, the market belief of the default probability is an important endogenous variable that affects the payoffs to each of these actions. Let \( p^{CDS} \) denote the market belief about the probability of success of the loan given that the bank participates in the CDS market, and let \( p^{LS} \) be the market belief if the bank sells the loan. For notational ease we sometimes describe \( p^{CDS} = p + \phi^{CDS} \Delta \), and \( p^{LS} = p + \phi^{LS} \Delta \). Belief about the success probability depends on the equilibrium.

At \( t = 3 \), the owner of the loan can exert costly effort (“monitor”). The cost of monitoring is \( b \). Monitoring represents the use of control levers (e.g., enforcing covenants, threatening to call the loan) to improve the borrower’s financial position. Monitoring is beneficial when the project admits moral hazard. If the loan owner monitors, the entrepreneur is prevented from choosing the riskier project; thus, the project’s probability of success increases from \( p \) to \( p + \Delta \). By contrast, if the project’s probability of success was already \( p + \Delta \), then there is no effect. It follows that information about the firm’s probability of success is valuable for informing the monitoring decision.

Finally, at \( t = 4 \), all claims pay off. Figure 1 summarizes this sequence of events.

To resolve indifference, we assume that when banks are indifferent between laying off risk and retaining it, they retain it on their balance sheets. In addition, if a bank is indifferent between monitoring and not monitoring, we assume that it does not monitor.

We impose parameter restrictions to ensure that there is a moral hazard problem and that monitoring is socially efficient. First, malfeasance on the part of the entrepreneur arises

\(^7\)If the market thought that the bank was hedging less than its total exposure, this would work as a good signal; however, knowing this, the bank would then have incentive to hedge everything at a favorable price.
if any lending rate $R^e$ that lets the bank break even induces the entrepreneur to choose the riskier project. For any $R^e$, an entrepreneur will shirk if

$$B + p(R - R^e) > (p + \Delta)(R - R^e),$$

or, $B > \Delta(R - R^e)$. The lowest possible break-even value of $R^e$ is the one for which the bank assesses the default probability at $p + \Delta$, for a profit of $(p + \Delta)R^e + (1 - p - \Delta)C - 1$. If the bank makes a zero profit, this implies a minimum value of $R^e$. Therefore, a sufficient condition for moral hazard is that:

**Assumption 1** $\Delta \left( R - C - \frac{(1-C)}{p+\Delta} \right) < B$, so given the chance, the entrepreneur prefers to choose the riskier project and consume private benefits.

Second, this behavior is undesirable if a social planner, who knows the entrepreneur can consume private benefits, prefers to monitor the project. Or,

$$C + (p + \Delta)(R - C) - b > C + p(R - C) + B.$$  

Thus, if $\Delta(R - C) > B + b$, aggregate surplus is higher if the bank monitors the entrepreneur when he is subject to moral hazard. Eschewing the risky project is therefore socially efficient.

**Assumption 2** $\Delta(R - C) > B + b$ so that it is socially inefficient for the entrepreneur to choose the riskier project.

In our simple framework, risk transfer is efficient if banks lay off their credit risk and thus bear no capital cost. Monitoring is efficient if projects that the originating bank has identified as being subject to moral hazard and thus having a low success probability of $p$ are monitored. Thus, the ex ante value of a loan if there is efficient monitoring and efficient risk transfer is

$$\Omega^* = C + (p + \Delta)(R - C) - 1 - \theta b.$$
There are two potential sources of inefficiency in the CRT markets. First, if the price of loans is sufficiently low (cost of CDS is sufficiently high), then a bank with a capital shock may not hedge its position, forcing it to raise more costly capital. In this case there is inefficient risk sharing. Second, if there is pooling, market participants cannot update their prior probability that the project is risky and requires monitoring. In this case, there may be inefficient monitoring; monitoring may not take place at all, or monitoring may take place even when the originating bank knows that the loan’s success probability is \( p + \Delta \) so that monitoring is inefficient. We explore these possibilities in the next section.

3 Characterization of Equilibria

If the bank sells the loan, then the loan buyer, although less informed than the originator, updates his beliefs about the type of the loan conditional on the loan being sold. He then handles the loan optimally, given his beliefs. The buyer compares the cost of monitoring to the expected benefit. Suppose that his belief of the success probability on observing a loan sale is \( p_{LS} \), then he monitors if

\[
\frac{C + (p + \Delta)(R^f - C) - b}{C + p_{LS}(R^f - C)} > \frac{C + p_{LS}(R^f - C)}{\text{Value if monitored}} > \frac{C + p_{LS}(R^f - C)}{\text{Value if not monitored}}.
\]

(4)

The action that the buyer takes determines the market price of the loan: This is just a buyer’s willingness to pay for the loan, and so

Lemma 1 An originating bank can lay off credit risk:
(i) Through a loan sale at price \( C + (p + \Delta)(R^f - C) - b \) if the loan buyer plans to monitor;
(ii) Through a loan sale at price \( C + p_{LS}(R^f - C) \) if the loan buyer does not plan to monitor;
(iii) By entering into a CDS for a payment of \( (1 - p^{CDS})(R^f - C) \).

Part (iii) of the Lemma just restates the fact that the price of a CDS is the expected payment to the buyer. It is clear from Lemma 1 that if a bank wants to lay off credit risk, the decision whether to use loan sales or CDS depends on two things: first, the inference that the market draws from the credit risk transfer method and second, the action that the loan purchaser takes (i.e., whether it plans to monitor or not). This fact that the potential credit risk transfer participants could include informed monitors affects both the price of risk, the bank’s incentive to hold it and how effectively the control rights can be transferred.

If the loan buyer does not plan to monitor, then there is no economic difference between CDS and loan sales; control rights are immaterial. Therefore, to simplify matters, in what follows, we assume that if the loan buyer does not plan to monitor, then beliefs are the
same across the two credit risk transfer methods; i.e., no inferences are drawn from the fact
that the bank uses loan sales or CDS. By contrast, if the loan buyer plans to monitor, then
holding beliefs fixed, loan sales may be preferred by some banks to CDS. They are strictly
preferred if the originating bank knows that the loan should be monitored (i.e., the bank’s
type is $p$).

What private information would lead a bank to lay off its credit risk, and how will it do
so? Due to the capital costs, there is always an incentive to lay off risk. However while the
sale price of the loan or the CDS in addition to the capital cost yield the amount that the
bank could get from going to the market, the bank’s valuation of the loan depends on what
it would do, if it kept it on the books. This calculus is identical to that of the uninformed
loan buyer, except that the originating bank has perfect information about the quality of
the loan.

**Lemma 2**  
(i) An originating bank that knows that the project is good will never monitor.
(ii) An originating bank that knows that the project is risky and does not buy CDS monitors
if $R^l - C > \frac{\Delta}{2}$

Notice that, unless there is perfect communication of the originating bank’s private
information, there will be a range of credit exposures for which an originating bank would
monitor if it retained ownership and did not purchase a CDS, but the buyer of the loan will
not. This illustrates that information in this economy is useful because it allows agents to
make optimal monitoring decisions.

The bank that knows that its loan is of high quality, is best-served by laying off its
credit risk with a credit default swap. However, in this case, the market price of risk in the
CDS market will reflect a low default probability. Of course, a bank with a bad credit risk
would have an incentive to also lay its risk off in this market and receive credit protection
at the reduced price. For this reason, any time the bank with good information lays off its
credit risk, a bank with a worse loan will also lay off its risk in the same way. Therefore,
either banks always lay off their credit risk (“pool”) or only banks with bad loans lay off
their credit risk (“separate”). In the latter case, they strictly prefer loan sales if the cost of
monitoring is not too high or else they prefer credit default swaps. Further, given rational
inferences about the underlying quality of the loan given the actions of the bank, the loan
buyer will choose to monitor or not. Therefore, equilibria can only be one of four possible
types; either all banks lay off their credit risk or only some lay off risk.

Let $\Omega^{ij}$ denote the expected social surplus to a loan where $i \in \{p, s\}$ indicates if there is
pooling or separating. If there is pooling, in the CRT market then both banks lay off risk. If there is separation, then only type $p$ lays off risk. In addition, $j \in \{m,n\}$ indicates how the control rights are used in the loan sale market. If $j = m$ then the loan buyer monitors; if $j = n$ the loan buyer does not.

We will exhaustively characterize these equilibria, but for now we observe that each of these possibilities differs in expected social surplus, which can be expressed as deviations from the efficient outcome. Notice, that as credit risk prices are merely transfers between risk neutral agents, these do not affect social surplus.

$$
\begin{align*}
\Omega^{p,n} &= \Omega^* - (1 - \theta)[\Delta(R - C) - (b + B)] \\
\Omega^{p,m} &= \Omega^* - \theta b \\
\Omega^{s,n} &= \Omega^* - (1 - \theta) [\Delta(R - C) - (B + b)] - \theta \beta (R^t - C) \\
\Omega^{s,m} &= \Omega^* - \theta \beta (R^t - C)
\end{align*}
$$

Note that none of the equilibria achieve the maximal social surplus. In each, there is a tradeoff between efficient monitoring and efficient risk sharing. For example, if the equilibrium is pooling, then there is efficient risk sharing. However, in this case a potential loan buyer does not have enough information to correctly assess the value of monitoring and so the equilibrium is inefficient. Consider $\Omega^{p,n}$, in which there is pooling and no monitoring. Social surplus is reduced if the loan is subject to moral hazard (with probability $(1 - \theta)$), by an amount equal to the expected benefit of monitoring, $\Delta(R - C)$. In this case, the monitoring cost $b$ is not incurred and the manager consumes his private benefit $B$. By contrast, if there is monitoring and pooling, $\Omega^{p,m}$, then the loan buyer cannot distinguish between high and low quality loans and so monitors high quality loans, which entails a social cost of $\theta b$. Similar inefficiencies arise if there is separation. In these cases there is always inefficient risk sharing as the bank with the high quality loan retains it on its books and bears a capital cost of $\beta (R^t - C)$, instead of selling it off.

In what follows, we characterize the equilibria that can obtain (i.e., which of the social surpluses is generated) as a function of the parameters for loans of arbitrary credit risk amounts $(R^t - C)$. Effectively, this takes the distribution of loan sizes as exogenous. In the next section, we further endogenize the credit risk of the loan. The cost of bank capital is...
an important determinant of the equilibrium. This is because for any loan size, the larger the cost of bank capital the more likely a bank with a high quality loan is to be willing to shed it, irrespective of the prices that obtain in the CRT market.

**Proposition 1** Suppose that $\Delta \leq \beta$ so that the cost of capital is large relative to the benefit of monitoring then:

i) If $R^\ell - C \leq \frac{b}{(1-\theta)\Delta}$ there is an equilibrium in which only type $p$ sheds credit risk. Trade is possible in both the CDS and loan sales market and there is no monitoring. Social Surplus is $\Omega^{p,n}$.

ii) If $R^\ell - C > \frac{b}{(1-\theta)\Delta}$, then there is an equilibrium in which both types shed credit risk. Trade is only possible in the loan sales market and all loans are monitored. Social Surplus is $\Omega^{p,m}$.

This equilibria are illustrated in Figure 3 below. In this case, the cost of capital is sufficiently high that banks have a strong incentive to lay off their credit risk. In this case, the market cannot use CRT to infer the type of the underlying loan and so rational agents retain their prior of $(1 - \theta)$ that it should be monitored. Therefore, given a fixed monitoring cost of $b$, will only choose to do so if the loan size is sufficiently high. The threshold of $R^\ell - C = \frac{b}{(1-\theta)\Delta}$ is simply the point at which loan buyers are indifferent between monitoring and not given their priors.

![Figure 3: Possible social values of the loan for $\Delta \leq \beta$](image-url)

By contrast, if the opportunity cost of capital is low, then a bank that is holding a “good” project on its books, might choose to retain it rather than incur the pooling costs of trading in the CRT market.

**Proposition 2** Suppose that $\beta < (1 - \theta)\Delta$, so that the cost of capital is small relative to the uninformed beliefs about the value of the debt, then

i) If $R^\ell - C \leq \frac{b}{\Delta}$, then there is an equilibrium in which only the $p$ type lays off credit risk and there is no monitoring. Both the loan sales and CDS market are active. The Social Surplus is $\Omega^{s,n}$.
(ii) If $\frac{b}{\Delta} < R^\ell - C \leq \frac{b}{\beta}$, then there is an equilibrium in which only the $p$ type lays off credit risk, there is monitoring and only the loan sales market is active. The Social Surplus is $\Omega_{s,m}$.

(iii) If $R^\ell - C > \frac{b}{\Delta}$, then there is an equilibrium in which all banks lay off credit risk in the loan sales market and all loans are monitored. The Social Surplus is $\Omega_{p,m}$.

These are illustrated in Figure 4 below. When the amount of credit risk is relatively small, then the high quality bank retains the loan on its books and only the low quality loan is sold. All market participants know the type of the loan and therefore make an efficient monitoring decision. That is, for $R^\ell - C \leq \frac{b}{\Delta}$ they do not monitor, whereas for larger credit amounts they do. At some point, however even though the cost of capital is low; even a bank with a high quality loan will choose to lay it off in which case pooling ensues.

Figure 4: Possible social values of the loan for $\beta < (1 - \theta)\Delta$

If the cost of capital is of an intermediate value, then the relationship is more complex. Indeed, different types of equilibria are possible for the same parameter range and the existence of a CDS market emerges as an important conditioning variable.

**Proposition 3** Suppose that the cost of capital is of an intermediate size so that $(1-\theta)\Delta \leq \beta < \Delta$; then

i) if $R^\ell - C \leq \frac{b}{\Delta}$ then only the $p$ type lays off credit risk, both the CDS market and the loan sales market may be active and no loans are monitored. The Social Surplus is $\Omega_{s,n}$.

(ii) If $\frac{b}{\Delta} < R^\ell - C < \frac{b}{\beta}$ then only the $p$ type lays off credit risk and only the loan sales market is active and all loans are monitored. The Social Surplus is $\Omega_{s,m}$.

(iii) If $R^\ell - C > \frac{b}{(1-\theta)\Delta}$ then both types lay off credit risk, only the loan sales market is active and loans are monitored. The Social Surplus is $\Omega_{p,m}$.

(iv) If $R^\ell - C \leq \frac{b}{(1-\theta)\Delta}$ then both types lay off credit risk, both the loans sales market and the CDS market are active and no loans are monitored. The Social Surplus is $\Omega_{p,n}$. 

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The parameter regions are illustrated in Figure 5. Clearly, for some loan sizes multiple equilibria are possible. In the next section, we consider the loan size that an entrepreneur would pick and this serves as a selection device.

\[
\Omega^{p,n} \quad \Omega^{p,m} \\
\Omega^{s,n} \quad \Omega^{s,m} \\
\frac{b}{\Delta} \quad \frac{b}{\beta} \quad \frac{b}{(1-\theta)\Delta} \quad R^\ell - C
\]

Figure 5: Possible social values of the loan for \((1-\theta)\Delta < \beta < \Delta\)

Intuitively, when monitoring is ruled out \((R^\ell - C \leq \frac{b}{\Delta})\), there are two equilibria: a pooling one in which both banks shed credit risk, and a separating one in which only the worse type of bank sheds credit risk. A bank that knows the project is of high quality compares the benefit of laying off risk (which depends on the capital cost) to the adverse selection discount in the market. In the pooling equilibrium, the type \(p + \Delta\) bank must prefer to sell or hedge the loan at the pooling price rather than hold the loan and face the capital cost. By contrast, in the separating equilibrium, the type \(p + \Delta\) bank prefers to retain the loan and so the price of a sold loan is lower and the premium on CDS is higher.

The upshot is that, when capital costs are sufficiently low \((\beta \leq \Delta)\), a bank with a good project may keep it on its balance sheet and the separating equilibrium exists. When capital costs are sufficiently high \((\beta > (1-\theta)\Delta)\), the good bank may lay off credit risk and so the pooling equilibrium exists. If \((1-\theta)\Delta < \beta \leq \Delta\), both equilibria are possible.

In each of the equilibria, the CDS market is equivalent to the loan sales market when monitoring is not optimal. The result that CDS are never strictly preferred to loan sales is consistent with Minton et al’s (2008) finding that the two tend to be complements. Nevertheless, if CDS involve lower transaction costs than loan sales, they will dominate loan sales in the cases where we find indifference. Moreover, when we introduce reputation concerns, we show that there are cases where CDS dominate even in the absence of differential transaction costs.
Empirical evidence on the quality of sold loans is mixed. In the context of our model, because we establish when a loan will be monitored and its credit quality enhanced, it is important to distinguish between the exogenous base credit quality of the loan and the endogenous default probability that obtains after the monitoring decision is made. Gupta, Singh, and Zebedee (2008) point out that loans sold are typically senior, secured, and sold piecemeal. While Drucker and Puri (2007) find that borrowers whose loans are sold are more than 1.5 times the size of borrowers whose loans are not sold, and have higher leverage and lower distance-to-default than borrowers whose loans are not sold, they report that loans that are sold have more (and more-restrictive) covenants than those that are not sold. Similarly, tighter covenants increase the probability of sale when the initial lender is less reputable (lower market share, or not in the top-ten lead banks). Finally, Berndt and Gupta (2008) find that firms whose loans are traded on the secondary market under perform other borrowers by 8-14% on a risk-adjusted basis during the three years after their loans begin trading, which is consistent with such loans being less well-monitored.

What is clear, however, is that higher capital costs favor pooling equilibria over separating equilibria, because the cost of inefficient risk-sharing increases. Since pooling equilibria have a larger range where monitoring is not supported (the relevant cut-off is \( \frac{b}{(1-\theta)\Delta} \) rather than \( \frac{b}{\Delta} \)), higher capital costs combined with credit risk transfer tend to undermine monitoring. Also, because the pooling equilibrium with no monitoring exists over a larger region than the separating equilibrium with no monitoring, an increase in capital costs makes it more likely the CDS market is active.

The cases in which the cost of capital is either very small or very large admit only one type of equilibrium for each parameter range. However, the structures of these equilibria are different. Given the assumptions of the model, there are four possible market values that can obtain:

<table>
<thead>
<tr>
<th>Social Value</th>
<th>Market Valuation of the Loan</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega^{p,n} )</td>
<td>( C + (p + \theta \Delta)(R^k - C) )</td>
</tr>
<tr>
<td>( \Omega^{p,m} )</td>
<td>( C + (p + \Delta)(R^k - C) - b )</td>
</tr>
<tr>
<td>( \Omega^{s,n} )</td>
<td>( C + p(R^k - C) )</td>
</tr>
<tr>
<td>( \Omega^{s,m} )</td>
<td>( C + (p + \Delta)(R^k - C) - b )</td>
</tr>
</tbody>
</table>

When there is no monitoring, then the credit risk in the CDS market and loan sales market are priced in the same way; as both markets may be active. In both the pooling and separating cases if there is monitoring, then the market valuation of the loan is the same post monitored value — namely at the high success rate \( (p + \Delta) \) and net of the monitoring cost, \( b \). Otherwise, the loan is priced given the best available information: either the knowlege
that it is low quality \( p \) (in the case of \( \Omega^{p,n} \)), or it is assigned the prior probability (in the case of \( \Omega^{p,n} \)).

Even in this simple framework, the fact that banks can lay off risk and monitoring rights affects the relationship between the price of loans in the secondary market or of credit risk in the CDS market and the underlying characteristics of the project can be very different. Consider the case in which bank capital costs are high \( \Delta \leq \beta \) depicted in Figure 3. For loans with a small credit risk (i.e., face value in excess of the collateral value), increases in the size of the loan are priced at the pooling probability \( p + \theta \Delta \), whereas for large amounts, they are priced at the lower default probability (or higher success probability of \( p + \Delta \)). Therefore, even though loans might have identical cash flow characteristics the the market prices may exhibit different sensitivities to the default risk because of actions taken by the potential monitors. By contrast, for bank characteristics depicted in Figure 4, all the loans below \( R^\ell - C = \frac{b}{\Delta} \) are priced at the unmonitored success rate \( p \), while those above are priced at the post monitored value \( p + \Delta \). This suggests that empirical specifications that fail to take into account how control rights are used, will systematically mis-estimate default probabilities. Indeed, in the first case, an a linear estimation of the relationship will underestimate the success probability, while in the latter case, it will overestimate the success probability.

Further, conditioning on the existence (or not) of a CDS market does not by itself capture the differences in the equilibria. This is readily seen in the case depicted in Figure 5. The fact that a bank could lay off credit risk in a CDS market affects the parameters (and hence underlying characteristics) that support different equilibria. This suggests that the ability to lay off positions which should be monitored has a complex effect on equilibrium.

4 Choosing Debt Values and Equilibria

The loan’s face value, \( R^\ell \), plays a critical role in the type of equilibrium that takes place and therefore on the efficiency properties and the value of control rights. In this section, we endogenize the choice. Recall that the entrepreneur makes a take-it-or-leave-it offer to the originating bank, and that the bank’s net opportunity cost of funds is 0. It follows that the entrepreneur gets all surplus, and so she will choose a loan face value that maximizes ex ante welfare subject to feasibility constraints. There are two such constraints: the bank must lend at least one unit so as to fund the project, and the bank must expect to break even given the equilibrium that will occur. (If the bank lends more than one unit, the entrepreneur invests 1 in the project and consumes the rest.)

To calculate the bank’s expected payoff for each equilibrium, let \( \pi^{i,j} \) be the expected
continuation payoff to the bank, where \( i \in \{p, s\} \) and \( j \in \{n, m\} \) denotes the type of equilibrium: pooling or separating, and no monitoring and monitoring, respectively. The bank must earn at least 1 in expectation, and this break-even condition implies the following minimum feasible loan face values.

**Lemma 3** (i) If the equilibrium is type \((i,j)\), where \( i \in \{p, s\} \) and \( j \in \{n, m\} \), then the bank’s break-even condition requires that the loan’s face value \( R^\ell \) must weakly exceed \( R^\ell_{i,j} \), where

\[
R^\ell_{i,j} = \begin{cases} 
\frac{1-C}{p+\theta\Delta} + C & \text{if } i,j = p,n \\
\frac{1-C}{p+\Delta} + C & \text{if } i,j = p,m \\
\frac{1-C}{p+\theta\Delta - \theta\beta} + C & \text{if } i,j = s,n; \\
\frac{1-C}{p+\Delta - \theta\beta} + C & \text{if } i,j = s,m 
\end{cases}
\]  

(ii) \( R^\ell_{i,j} \) is decreasing in the firm’s collateral value \( C \), base probability of success \( p \), and impact of monitoring \( \Delta \). It is weakly decreasing in the probability, \( \theta \), that there is no moral hazard. It is weakly increasing in the cost of monitoring \( b \) and the cost of capital \( \beta \).

It is immediate that \( R^\ell_{i,j} \) is decreasing in \( C, p, \) and \( \Delta \). These parameters govern the firm’s base credit risk; an increase in any of them reduces the bank’s expected credit exposure and thus reduces the face value it needs to break even. Thus, for any given type of equilibrium in the credit risk transfer market, feasibility constraints are less binding as the firm’s base credit risk is lower.

We also show that \( R^\ell_{i,j} \) is weakly decreasing in \( \theta \). If monitoring does not take place, an increase in \( \theta \) increases the expected value of the firm. If monitoring does take place, an increase in \( \theta \) reduces the probability with which monitoring takes place, reducing expected monitoring costs. Either increases the expected value of a loan, reducing the face value the bank needs to break even. Although it is true that, in a separating equilibrium, an increase in \( \theta \) increases the probability that efficient risk transfer does not occur, this effect is dominated by the previous two.

Finally, \( R^\ell_{i,j} \) is weakly increasing in the cost of monitoring \( b \) and the cost of capital \( \beta \). Both factors decrease the bank’s expected profits from making the loan, increasing the bank’s break-even face value.

While minimum credit exposures tend to move with the firm’s base credit risk, it is also apparent that the exact sensitivity depends on the type of equilibrium that results. As a result, a parameter change that results in a new type of equilibrium may result in a jump in the feasibility requirement. Nevertheless, we can establish that there is continuity between
pooling equilibria with and without monitoring, and between separating equilibria with and without monitoring.

Lemma 4  The minimum feasible loan face value, $R^\ell_{i,j}$ exhibits continuity in that
(i) $R^\ell_{p,n} - C > R^\ell_{p,m} - C$ if and only if $R^\ell_{p,n} - C > \frac{b}{(1-\theta)\Delta}$ if and only if $R^\ell_{p,m} - C > \frac{b}{(1-\theta)\Delta}$.
(ii) $R^\ell_{s,n} - C > R^\ell_{s,m} - C$ if and only if $R^\ell_{s,n} - C > \frac{b}{\Delta}$ if and only if $R^\ell_{s,m} - C > \frac{b}{\Delta}$.
(iii) $R^\ell_{s,m} - C > R^\ell_{p,m} - C$ if and only if $R^\ell_{s,m} - C > \frac{b}{\beta}$ if and only if $R^\ell_{p,m} - C > \frac{b}{\beta}$.

Parts (i) and (ii) of the lemma show that there is continuity of loan face values between both pooling (separating) with no monitoring equilibria and pooling (separating) with monitoring equilibria. Part (iii) shows there is continuity of loan face values between pooling with monitoring equilibria and separating with monitoring equilibria.

As the entrepreneur gets all net surplus, she bears the costs of any inefficient monitoring or inefficient risk sharing. By selecting a loan face value $R^\ell$, she can to a large extent select the equilibrium that will result and thus the amount of surplus generated. However, in some cases a given $R^\ell$ can give rise to more than one equilibrium, creating indeterminacy. We identify three cases that differ in the level of the capital cost $\beta$.

Proposition 4  Suppose that capital costs are high ($\beta > \Delta$).
(i) If $R^\ell_{p,n} - C \leq \frac{b}{(1-\theta)\Delta}$, then the entrepreneur chooses the pooling with no monitoring equilibrium $(p,n)$ if $b > (1-\theta)[\Delta(R-C) - B]$, and the pooling with monitoring equilibrium $(p,m)$ otherwise. Loan face values that implement this are $R^\ell = R^\ell_{p,n}$ for $(p,n)$ and $R^\ell = \frac{b}{(1-\theta)\Delta} + \epsilon$ (where $\epsilon$ small) for $(p,m)$.
(ii) If $R^\ell_{p,n} - C > \frac{b}{(1-\theta)\Delta}$, then the entrepreneur chooses the pooling with monitoring equilibrium $(p,m)$. This can be implemented with $R^\ell = R^\ell_{p,m}$. This outcome is more likely if collateral $C$, the firm’s base probability of success $p$, or the cost of monitoring $b$ is low. It is also more likely if the probability that monitoring is needed $(1-\theta)$ is high.

When capital costs are high, banks prefer to pool and get efficient risk-sharing rather than separate. If the firm’s base credit quality is relatively high, the cost of monitoring is relatively high, or it is likely that moral hazard is not a problem, then the pooling with no monitoring equilibrium may be feasible. The entrepreneur prefers this to pooling with monitoring if the costs of excessive monitoring outweigh the social gains from monitoring, which leads to the condition in part (i) of the proposition. If the costs of monitoring are relatively low or moral hazard is likely, the entrepreneur prefers the pooling with monitoring equilibrium. She can implement this by choosing a loan face value that exceeds what is needed to fund the project, leveraging up the firm. Also, if base credit risk is high, pooling with no monitoring may be infeasible, because the minimum face value required on the loan...
is so high that loan buyers prefer to monitor. In this case, the entrepreneur must choose pooling with monitoring.

Next, we turn to the case where capital costs are low.

**Proposition 5** Suppose that capital costs are low \((\beta \leq (1 - \theta)\Delta)\).

(i) If \(R_{s,m}^\ell - C \leq \frac{b}{\Delta}\), then the entrepreneur chooses the separating with no monitoring equilibrium \((s,n)\) if \((1 - \theta + \frac{\beta}{\Delta})b - \theta \beta (R_{s,n}^{\ell} - C) \geq (1 - \theta)\Delta(R - C) - B\), and the separating with monitoring equilibrium \((s,m)\) otherwise. Loan face values that implement this are \(R_{s,n}^{\ell} = R_{s,m}^{\ell}\) for \((s,n)\) and \(R_{s,m}^{\ell} = \frac{b}{\Delta} + \epsilon\) (where \(\epsilon\) small) for \((s,m)\).

(ii) If \(\frac{b}{\Delta} < R_{s,m}^{\ell} - C \leq \frac{b}{\beta}\), then the entrepreneur chooses the separating with monitoring equilibrium \((s,m)\). This can be implemented with \(R_{s,m}^{\ell} = R_{s,m}^{\ell}\).

(iii) If \(R_{s,m}^{\ell} - C > \frac{b}{\beta}\), then the entrepreneur chooses the pooling with monitoring equilibrium \((p,m)\). This can be implemented with \(R_{p,m}^{\ell} = R_{p,m}^{\ell}\).

When capital costs are low, the separating equilibria may be feasible. If the firm’s base credit risk is low, then part (i) of the proposition applies; both of the separating equilibria (with and without monitoring) and the pooling equilibrium with monitoring are feasible. The pooling equilibrium with monitoring has one welfare cost, namely overmonitoring, which hurts welfare by \(-\theta b\). The separating equilibrium with monitoring has inefficient risk transfer, leading to a welfare cost of \(-\theta \beta (R_{s,m}^{\ell} - C)\); by the requirements of this equilibrium, \(R_{s,m}^{\ell} - C \leq \frac{b}{\beta}\), and so this equilibrium dominates the pooling equilibrium with monitoring. Intuitively, capital costs are low, so the losses from inefficient risk transfer are less than those from excessive monitoring.

It follows that the entrepreneur chooses between the two separating equilibria in this case. In both equilibria, welfare is maximized by choosing a loan face value that is as low as is feasible. When the condition in the proposition holds, the separating equilibrium with no monitoring dominates: because it uses a lower face value, it has lower capital costs than the separating equilibrium with monitoring, and these offset the losses from no monitoring if the latter are small enough. Note that this condition is less likely to hold than the similar condition in Proposition 4(i). Also, if the entrepreneur chooses the separating equilibrium with monitoring, she deliberately issues more debt than is needed to fund the project so as to induce monitoring; the possibility of credit risk transfer and the undermining effect of CDS on monitoring leads to higher leverage being chosen.

In part (ii) of the proposition, the firm’s base credit risk is somewhat higher, ruling out the separating equilibrium with no monitoring, but still low enough that the separating equilibrium with monitoring is feasible. As in part (i), this dominates the pooling equilibrium with monitoring, so the entrepreneur chooses the separating equilibrium with
monitoring.

Finally, in part (iii) of the proposition, the firm’s base credit risk is so high that only the pooling equilibrium with monitoring is feasible. In this case, the entrepreneur must choose this equilibrium, leading to efficient risk transfer and excessive monitoring.

The final case to be considered is that where capital costs fall into an intermediate region. As we will see, there is now a possibility of indeterminacy of equilibrium, complicating the entrepreneur’s choice of loan face value.

**Proposition 6** Suppose that capital costs are at intermediate levels \(((1 - \theta)\Delta < \beta \leq \Delta)\).

(i) If \(R_{p,m}^l - C \leq \frac{b}{1 - \theta}\), then any loan face value that would allow a separating equilibrium also allows the pooling equilibrium with no monitoring \((p,n)\), leading to indeterminacy.

(a) If such a loan face value leads to the relevant separating equilibrium, then the entrepreneur chooses between implementing \((s,m)\) with \(R^l - C = \max\{R_{s,m}^l - C, \frac{b}{\Delta} + \epsilon\}\) (where \(\epsilon\) is small) and implementing \((p,n)\) with \(R^l - C \in (\frac{b}{1 - \theta}\Delta, \frac{b}{1 - \theta}\Delta + \epsilon]\). She chooses equilibrium \((p,n)\) if and only if \((1 - \theta)b + \theta\beta \max\{R_{s,m}^l - C, \frac{b}{\Delta}\} \geq (1 - \theta)[\Delta(R - C) - B]\).

(b) If such a loan value leads to \((p,n)\), then the entrepreneur chooses between implementing \((p,n)\) with \(R^l - C \in [R_{p,n}^l - C, \frac{b}{1 - \theta}\Delta]\) and implementing \((p,m)\) with \(R^l - C \in (\frac{b}{1 - \theta}\Delta, R - C]\). She chooses equilibrium \((p,n)\) if and only if \(b > (1 - \theta)[\Delta(R - C) - B]\).

(ii) If \(\frac{b}{1 - \theta} < R_{p,m}^l - C \leq \frac{b}{(1 - \theta)\Delta}\), then the separating equilibria are infeasible. The entrepreneur chooses between implementing \((p,n)\) and \((p,m)\) as in (i,b).

(iii) If \(R_{p,m}^l - C > \frac{b}{(1 - \theta)\Delta}\), then only equilibrium \((p,m)\) is feasible. The entrepreneur can implement this with \(R^l = R_{p,m}^l\).

In part (i) of the proposition, the separating equilibrium with monitoring \((s,m)\) is feasible (this follows from Proposition 3(v)), but any rate that allows that equilibrium or the separating equilibrium with no monitoring \((s,n)\) also allows the pooling equilibrium with no monitoring \((p,n)\). The equilibrium that is realized will depend on the beliefs of market participants, which the entrepreneur cannot control. Also, as noted in the discussion of Proposition 5, \((s,m)\) dominates the pooling equilibrium with monitoring \((p,m)\) whenever \((s,m)\) exists. We can also show that \((p,n)\) dominates \((s,n)\), since both lead to no monitoring but \((p,n)\) leads to efficient risk transfer whereas \((s,n)\) does not.

If market participants’ beliefs favor the separating equilibria, then the entrepreneur effectively chooses between setting a loan face value that leads to separating with monitoring and a higher loan face value that leads to pooling with no monitoring. (For a face value that is high enough, \((s,m)\) is not feasible whereas \((p,n)\) is.) She chooses pooling with no monitoring if the losses from no monitoring are less than the losses from inefficient risk transfer; rearrangement yields the condition in the proposition.
If instead market participants favor the pooling equilibrium with no monitoring, then the entrepreneur must choose between the two pooling equilibria, as in Proposition 4(i), with the same condition for when pooling with no monitoring is preferred to pooling with monitoring. Note that this condition is more likely to be met than that in part (i.a), since \( R_{s,m}^\ell - C \leq \frac{b}{\beta} \), which again implies that welfare under \((s, m)\) exceeds welfare under \((p, m)\).

In part (ii) of the proposition, the firm’s base credit risk is higher, making the separating equilibria infeasible, but still low enough that the pooling equilibrium with no monitoring is feasible. Again, the entrepreneur chooses between the two pooling equilibria. Finally, in part (iii), base credit risk is so high that only the pooling equilibrium with monitoring is feasible.

Looking over the results of Propositions 4, 5, and 6, a few patterns emerge. First, sufficiently high base credit risk (relative to \( \frac{b}{\Delta} \), the per unit cost of reducing default probabilities through monitoring) tends to favor equilibria with monitoring by making other equilibria infeasible. Since equilibria with monitoring have active loan sales markets and inactive CDS markets, this suggests that banks will use CDS less frequently in cases where credit risk is high and monitoring is attractive (\( \frac{b}{\Delta} \) is low).

If gains from monitoring are sufficiently high, or the probability of moral hazard \( 1 - \theta \) is sufficiently high, monitoring equilibria are more attractive. The entrepreneur can implement these by choosing a loan face value that is higher than the amount needed to fund the firm’s investment. Thus, in these cases, leverage will be higher, loan sales will dominate CDS, and loan buyers will monitor.

These patterns are not always clear cut. For example, in the case of intermediate capital costs, increases in base credit risk may shift the equilibrium from separating with monitoring to pooling with no monitoring and then to pooling with monitoring. Once again in each of these equilibria, the control rights are used differently and so the default probabilities for loans of different sizes are different.

5 Extensions

Our analysis so far has focused on a model with a number of restrictive assumptions: for example, monitoring is always cost efficient, the originating bank can always sell its loan without restriction, and there is only one period. In this section, we consider a number of extensions. We show that the use of “covenant light” loans may enhance the development of CDS markets. Conversely, an active CDS market with nonbank liquidity traders helps undermine efficient monitoring. Surprisingly, loan sales restrictions may also increase the viability of CDS markets and undermine monitoring. Finally, we consider the impact of
reputation concerns in an infinite horizon version of our model. We find that reputation concerns may support first-best behavior in the short-run (the originating bank makes use of CDS to lay off credit risk yet still monitors efficiently), and this is more likely for loans of better base credit quality. Even so, eventually defaults will occur and undermine monitoring incentives.

5.1 Monitoring costs and default

In reality, loans vary in the ease with which covenants can be enforced. For example, “covenant light” loans do not include standard protective clauses such as minimum cash flow requirements or restrictions on future debt and therefore do not provide a monitor with levers in the event of financial distress. In the context of our model, this is equivalent to an increase in the cost of monitoring \( b \), possibly making monitoring prohibitively expensive. If monitoring is ruled out there is no value in control rights and, in equilibrium, both the loan sales and CDS markets will be active. Effectively, the barriers to monitoring make the CDS market more viable.

The optimal use of control rights naturally translates into default probabilities. Since the only cases in which both the CDS and loan sales markets are active are those in which there is no monitoring, it immediately follows that:

**Corollary 1** If both the CDS and loan sales market are active then the default probabilities are too high relative to the social optimum.

Thus, because CDS do not bundle control rights with the underlying cash flows, their use undermines optimal control. More generally, credit risk transfer often results in a loss of the originating bank’s private information, making monitoring decisions less efficient.

5.2 The microstructure of the CDS and loan markets

Our model makes stark predictions about when the CDS market will be viable, but given that other investors may trade these instruments for portfolio or speculative reasons, the markets may be active even when banks should not trade. Such “liquidity” or “noise” trading alters the equilibrium, because the presence of “noise” traders allows banks to trade in CDS markets anonymously. This is because both CDS and loans are traded over the counter, and there is no centralized clearing or record of any economic agent’s full position. Further, through the use of intermediaries it would be very easy to disguise a CDS position.

To see how this alters matters, consider any equilibrium in which the loan buyer monitors. In this case, the future success rate of the loan is \( p + \Delta \). However, it cannot be an equilibrium for the price in the CDS market to reflect the true default probability \( p + \Delta \).
If risk were priced this way, then the bank could lay off credit risk through CDS at a price that does not deduct the cost of monitoring, as opposed to selling the loan at a price that does deduct the buyer’s monitoring cost. Of course, if it were to do so and there was an active CDS market which priced the credit risk at the true default probability \( p + \Delta \), the bank would not monitor. Thus, \( p + \Delta \) will not be the success probability of the project.

In sum, if the CDS market is active, then the price in the CDS market must be less than or equal to the price in the loan sales market. This is not consistent with zero profits for the CDS market makers as their contracts will pay off with the post monitoring probability \( p + \Delta \). Therefore, if there is any monitoring of credit risk, prices in the CDS market must generate positive economic profits for the intermediaries. (Effectively, they capture the monitoring cost as profit.) The prices that the intermediaries charge act as a screening device and change both the actions of market participants and those who choose to enter the market.

**Proposition 7** Suppose that a CDS market is always active, then either intermediaries make positive profits or loan buyers do not monitor.

In as much as CDS are synthetic contracts, they should not affect the real economy. However, because transactions in these markets are essentially unobservable, the existence of the CDS market prevents optimal monitoring, as in Morrison (2005).\(^8\)

### 5.3 Restrictions on loan sales and bank monitoring

In our model, banks do not contract on the amount of credit risk that they retain. However, the right to assign a loan is contractible. Therefore, a natural question to ask is if a bank or issuer would choose ex ante to commit to not selling the loan. (This is known as an anti-assignment clause.) In fact, a recent study by Pyles and Mullineaux (2008) finds that 63% of the loans in their sample of syndicated loans have clauses that require borrower approval before sales are allowed.

Such clauses have two effects. First, in the presence of a CDS market, a loan that cannot be sold (a binding anti-assignment clause) may be monitored less than a loan that permits sales. This is because the originating bank has the option of purchasing a CDS if its desire to lay off risk is sufficiently high. In this case, it is no longer exposed to the economic risk of the underlying loan and therefore has no incentive to monitor.

\(^8\)In a more complex model where default was followed by a workout or Chapter 11 procedure, the existence of CDS may be compatible with monitoring by the bank after default has occurred. The CDS pay off on the default, but the bank still holds the loan, so it has incentive to try to increase its post-default value. On the other hand, pre-default monitoring would still tend to be undermined. We thank Doug Diamond for this point.
The second effect is that clauses restricting future sales are frequently not binding. Currently, under Article 9 of the Uniform Commercial Code, a bank may sell participation in a loan even though the underlying loan agreement has an anti-assignment clause. However, in the presence of an anti-assignment clause, the bank is not allowed to transfer collection rights (effectively monitoring rights) to the buyer. In sum, under current law, such clauses do not actually prevent a bank from selling a loan, but do prevent a bank from transferring the business relationship. In the context of our model, this suggests that a bank with a loan that has an anti-assignment clause has effectively committed that the purchaser will not monitor.

Economically, then, an anti-assignment clause followed by loan participation is equivalent to CDS in our model. To the extent anti-assignment clauses either lead a bank to use CDS or are themselves equivalent to CDS because they allow loan participation, their widespread use would have a critical impact on our results. Separating and pooling equilibria would still exist, but any transfer of credit risk would lead to no monitoring, decreasing welfare and increasing loan default rates and thus credit exposures. In fact, Pyles and Mullineaux (2008) find that loans with sales restrictions do pay higher spreads. Although they attribute this to the resulting illiquidity of the loans, increased chance of default is another possible explanation, particularly for larger loans where CDS on the borrower may be available.

5.4 The role of reputation

Central to the inefficiency of credit default swaps is the idea that the originating bank is typically best informed about the benefit to monitoring but has no incentive to monitor. Of course, this affects the price of credit risk in the market. Ex ante, it is efficient for a bank to be able to retain the loan and monitor when necessary yet lay off credit risk through CDS if it has a capital shock; ex post, this cannot be supported in a single-period setting.

It is reasonable to ask whether reputation concerns could lead a bank to monitor when it used CDS. In the appendix, we present an infinite horizon version of our framework in which in each period the (infinitely-lived) bank faces a new borrower and a new set of investors in the market. As we will see, such a setting may support behavior that is closer to the efficient outcome. This is generally more likely as the bank’s discount factor is larger, the borrowing firm’s base chance of default is smaller, and the impact of monitoring on firm default is higher. Nevertheless, there are cases where the repeated setting does not support better behavior. Moreover, the first-best is unlikely to be attainable in any case. Too many defaults signal market participants that the bank probably has not monitored, which is

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9A detailed description of this Article appears in Schwartz(1999).
followed by a reversion to inefficient behavior. Since even a bank that monitors will face some defaults, some “false positives” are inevitable. This drives behavior away from the first-best even when the bank tries to fulfill its role as a monitor. Gopolan, Nanda, and Yerramilli (2007) find empirical evidence of similar behavior in the loan syndication market: after defaults on a lead loan arranger’s loans, the lead arranger loses loan volume and finds it more difficult to offload loans to other syndicate members.

These results are consistent with the findings of Ashcraft and Santos (2007). They find that the introduction of CDS on a borrower reduces loan spreads for high credit quality firms, but worsens loan spreads for low credit quality borrowers. In terms of our model, high credit quality borrowers are most likely to support the reputational equilibrium, where CDS play a positive role, improving borrower surplus. By contrast, low credit quality borrowers are likely to fall into the single-period equilibrium, where CDS either have no effect or (if they are introduced into a setting where loan sales should dominate) actually worsen matters.

More generally, our result that reputation concerns make CDS more attractive suggests that, if larger banks find it easier to project a reputation, they will be the biggest users of CDS. This is consistent with Minton et al’s (2008) finding that CDS use is concentrated among the largest banks in their sample. Briefly, Minton, Stulz, and Williamson (2008) examine banks’ credit risk transfer practices over a sample period of 1999-2003. Of all U.S. bank holding companies with $1 billion or more assets, only 23 use credit derivatives during the sample period however, these are the largest firms, accounting for over 60% of sample assets. To the extent larger banks have better-established reputations, this is consistent with our finding that reputation makes CDS more attractive. Minton et al. also find some evidence that loan sales and credit derivatives are complementary products, which is consistent with our findings that the two markets may coexist.

Regardless of whether the reputational equilibrium penalizes one or more defaults, reputation in this setting will be unable to attain the first-best outcome (monitoring by the bank in every period). The reason is that punishment follows too many defaults, and there is always a chance that defaults are generated through bad luck rather than through negligence; thus, large numbers of defaults will be followed by inefficient behavior during the punishment phase. The intuitive idea of extending the time segment during which signals (defaults) are accumulated to get a clearer sense of the bank’s behavior is misleading; here, the bank chooses each period whether to monitor or not, and extending the time over which signals are accumulated makes it easier for the bank to shirk during this time.¹⁰

¹⁰For further discussion of this point in the context of seller moral hazard, see Dellarocas (2005).
5.5 Related Literature

The set of problems that we study is somewhat different than that that has been examined in the equity literature. First, we consider an environment in which there is no cost to trading either the control rights or the cash flow rights separately or in a bundle and characterize the equilibrium. In focusing on trading shares, the equity literature has typically not considered the possibility of unbundling the two. In addition, because we can specify when the CDS market will not be active, we can provide a characterization of the conditions under which the two will not be sold separately. In addition, we explicitly consider the sale of the control rights and cash flows (i.e., the loans) from one monitor to another equally efficient one and consider whether this can be done socially efficiently.

In the equity literature, there has been debate about the optimal size of the blockholder or whether multiple small blockholders can perform as effectively. For example, Edmans and Manso (2009) argue that many small block holders may lead to more information begin impounded in price which can act as a disciplining device, while Burkhardt, Gromb and Panunzi (1997) show that too large a blockholder may inefficiently interfere in managerial actions. We endogenize the size of the bank’s stake and show what affect this has on efficient monitoring. In addition to the size of a bank’s stake efficient monitoring depends on the effectiveness and cost of monitoring the the benefit to a bank of laying off credit risk.

The informativeness of price plays a distinct role in our analysis. In most equity models, insiders’ trading is a noisy signal of managerial effort and, in as much as managers’ compensation is tied to the underlying value of the stock may affect his incentives to shirk. By contrast, in our framework, monitoring by debt holders is much more direct and information is important to guide this monitoring. Therefore, in contrast to the equity literature, we illustrate that banks’ propensity to sell can in fact obfuscate the underlying nature of the project and lead to less efficient monitoring.

Several theoretical papers examine the effect of credit risk transfer on relationship banking. Pennacchi (1988) and Gorton and Pennacchi (1995) look at optimal loan sale structures which maintain the originating bank’s incentive to monitor. In both models, it is assumed that banks can commit to retain part of the loan’s credit risk so as to have incentive to monitor. Parlour and Plantin (2006) consider how the ability to sell loans to meet liquidity needs affects banks’ incentives to monitor. They provide conditions under which a liquid credit risk transfer market can be socially inefficient.

Turning to credit derivatives, Arping (2004) formalizes CDS as a relaxation of the firm’s limited-liability constraint. By paying out cash in the case of a credit event, protection

\footnote{For a survey of the issues credit risk transfer raises for bank-borrower relationships, see Kiff et al. (2003).}
sellers reinforce the bank’s incentive for efficient early liquidation of the borrower’s project. Critically, the maturity of the credit derivative must be observably shorter than that of the underlying loan, which in turn must be longer than the point at which bank monitoring is efficient; otherwise, monitoring incentives are undermined. Morrison (2005) shows that the unobservability of CDS transactions can undermine bank monitoring, causing corporate borrowers to prefer bond market finance; this disintermediation reduces welfare. Chiesa (2008) examines credit risk transfer when bank monitoring improves returns in downturns. She finds that optimal credit risk transfer takes the form of a securitization of the entire loan portfolio with limited credit enhancement. Proper risk-based capital regulations are needed to implement this; otherwise, the bank provides too much credit enhancement and shirks on monitoring.\textsuperscript{12}

None of these papers examines the distinction between loan sales and CDS which is the key focus of our paper. By contrast, Duffee and Zhou (2001) do consider the co-existence of loan sales and CDS markets. As in Arping (2004), they emphasize the fact that, unlike a loan sale, CDS allow a bank to shed a loan’s risk of early default risk without shedding the risk of default at maturity. This flexibility can undermine the loan sales market, which can lead to both good or bad outcomes. If high quality banks do not use the loan sales market in order to signal their type, introducing credit derivatives can promote pooling, which enhances welfare. However, if loan sales would otherwise lead to risk-sharing by both high- and low-quality banks, credit derivatives may undermine risk-sharing. Critical assumptions are that bank loan maturities are known, loan sales and credit derivative transactions are jointly observable, and asymmetric information is greater later in the life of the loan rather than earlier. Thompson (2006) adapts the model of Duffee and Zhou (2001) to consider the case of imperfect competition in the CDS market. He finds that CDS can lead to efficient outcomes if the insurance provider has market power. We differ from both of these papers in that we explicitly consider the value of control rights in the loan sales market, and model when and how they are exercised.

\section{Conclusion}

We have presented a simple model in which a bank with a loan and private information about the loan’s default probability chooses between laying off risk synthetically through a CDS or by selling the loan. In a single-period setting, when the firm’s credit risk base is high, the only equilibrium has the bank use loan sales to offload credit risk, regardless of its

\textsuperscript{12}Nicolo and Pelizzon (2008) look at banks’ use of credit derivatives to signal their type when required bank capital is costly. They find that the opacity of credit derivatives markets makes it more difficult for high-quality banks to separate from low-quality banks.
information about the borrowing firm; risk transfer is efficient but monitoring is excessive. When the firm’s base credit risk is lower, however, other equilibria may be possible. If the capital cost of retaining credit risk is sufficiently high, then in equilibrium both loan sales and CDS coexist, risk transfer is efficient, and monitoring is too low. If the capital cost of retaining credit risk is low, then in equilibrium only loans in need of monitoring are laid off, inefficiently reducing risk transfer; in some cases, loan sales and CDS coexist and monitoring is too low, while in other cases, only loan sales are used and monitoring is efficient.

We have also analyzed how our results change in a repeated setting. In such a setting, CDS and monitoring may coexist for some period of time, if bank required returns are not too high (δ is high) and the probability of the loan’s default is not too high (p and Δ are high). Nevertheless, the chance of monitored loans defaulting means that it will generally be impossible to support the first-best outcome for an unlimited period of time. A period of high defaults will be followed with a collapse in the bank’s reputation for monitoring and reversion to an inefficient equilibrium outcome.

Our discussion of securitization and the retention of risk suggests some further caveats. To the extent that anti-assignment clauses are common, banks may simply resort to CDS or use loan participations to reduce risk. Since these leave borrower control rights in the hands of the bank, efficient monitoring is totally undercut, unless the bank can maintain a reputation for monitoring. As we have just said, monitoring incentives will be easiest to maintain if the loan’s credit risk is relatively low; for weaker credits, such clauses may prevent useful monitoring by loan buyers in return for no monitoring at all by the originating bank. Similarly, if some of the sellers of credit risk in the markets are not originating banks, their activity makes it easier for the originating bank to hide its trades, especially in the CDS market. This enhances activity in CDS, again undermining monitoring incentives. Overall, our model suggests that activity in CDS markets will be more likely to undermine monitoring than activity in loan sales, especially for weaker credits.

The focus of this paper is on how the use of control rights is affected by the markets in which those rights are traded and the cash flows with which they are bundled. While this is not an investigation of optimal security design, it does suggest that contractual rights can affect the underlying value of cash flows. This has broad implications for valuation. A risky dollar traded in one market is not equivalent to the same dollar of credit risk traded in another market.
7  Supporting efficient monitoring in an infinite horizon setting

Suppose that the bank exists for an infinite number of periods, and it discounts profits with a factor \( \delta \in (0, 1) \). Each period, the bank makes loans to a new borrower that only exists for a single period, and the bank may also engage in credit risk transfer with competitive investors that only live for one period. The question we now explore is whether a Bayesian perfect equilibrium exists in which the bank makes use of CDS yet still monitors when appropriate.

If the bank is to “do the right thing” and monitor despite the presence of CDS, it must earn higher profits through such behavior than through using CDS and not monitoring (“shirking”). This requires that market participants have some means of assessing whether the bank has monitored, and some way of rewarding the bank for monitoring and “punishing” it for shirking.

We assume that a bank’s monitoring is not directly observable, but this is not critical for our results.\(^{13}\) In this setting, only the bank knows whether it has actually monitored when appropriate in the past; investors and the current borrower only know whether the bank’s past borrowers defaulted or not. Nevertheless, because the bank’s loans are more likely to default if the bank does not monitor, market participants can use past defaults as a noisy signal that the bank has not monitored.

This brings us to the question of how the bank is to be punished. To simplify matters, we assume that if market participants decide that the bank is probably shirking, they revert to single-period equilibrium beliefs and behavior, as does the bank; that is, they all follow a “trigger strategy” equilibrium of the sort first proposed by Green and Porter (1984) in the context of collusive oligopoly. For simplicity, we continue to assume that the bank has no market power in normal circumstances. Thus, the bank’s rents when it is punished for apparent shirking are 0 per period. For this to be punishment, it is critical that the bank earn some form of rents as long as it maintains a reputation for monitoring. We will assume that, if the bank maintains a reputation for monitoring, its borrowers will agree to a loan face value \( R^\ell_* \) that gives the bank the additional surplus \( S^\ell* \) it generates vis-a-vis the

\(^{13}\) Even if monitoring could be observed, third parties might not know whether monitoring is useful or not (i.e., whether the borrower can engage in moral hazard or not). In this case, seeing no monitoring followed by a default is not a guarantee of shirking; instead, the bank might have genuinely seen that monitoring was not useful, and yet through bad luck the borrower subsequently defaulted anyway.
single-period equilibrium.\footnote{One way of creating such an outcome would be if banks with reputations were relatively rare, whereas banks that behaved as single-period maximizers were common. In this case, single-period maximizing would only obtain zero rents, but a bank with a good reputation could extract rents.} Note that

\[ S^* = (p + \Delta)R^s_\ell + (1 - p - \Delta)C - (1 - \theta)b - 1 = \Omega^* - \Omega^{i,j}(i \in \{s, p\}, j \in \{m, n\}). \]

Assuming that the bank gets this entire surplus is the most favorable framework for giving the bank incentive to monitor when using CDS.

First, let us consider the case where a single default is used as a signal that the bank has shirked and is followed by reversion to single-period equilibrium behavior forever. Suppose that the bank has not yet suffered any defaults. Define \( V_N \) as the expected present value of the bank’s future stream of profits in this case. Similarly, if the bank’s loan defaults, define \( V_D \) as the expected present value of the bank’s future stream of profits. Since a default is followed by single-period Nash equilibria, and the bank’s rents in this case are assumed to be zero per period, we have \( V_D = 0 \).

Assuming that the bank does not shirk in the current period, we have the following equation for \( V_N \):

\[
V_N = S^* + (p + \Delta)\delta V_N + (1 - p - \Delta)\delta V_D = S^* + (p + \Delta)\delta V_N.
\]

Solving for \( V_N \), we have

\[
V_N = \frac{S^*}{1 - (p + \Delta)\delta}; \tag{6}
\]

the bank discounts the stream of potential rents \( S^* \) at an effective rate of \( (p + \Delta)\delta \). Since the first-best value of rents would be \( \frac{S^*}{\delta} \), there is a loss of efficiency even when this reputation equilibrium can be sustained initially: eventually, default will occur, and the single-period equilibrium will follow.

Of course, this assumes that the bank does not find it worthwhile to shirk. If the bank shirks, it still gets \( S^* \) in the current period (it locks these profits in with a CDS) and saves an additional \((1 - \theta)b\) by not monitoring. The downside to shirking is that the chance of default in the current period is higher, reducing the odds that the bank will collect rents in the future. Letting \( V(\text{shirk}) \) be the value of shirking given that there have not yet been any defaults, we have

\[
V(\text{shirk}) = S^* + (1 - \theta)b + (p + \theta\Delta)\delta V_N. \tag{7}
\]
Note that this calculation assumes that the bank’s future value if the current loan does not default is $V_N$, which has been derived under the assumption that the bank will not shirk.\footnote{If $V_N < V(\text{shirk})$, then the reputation equilibrium fails, so we work under the assumption that $V_N \geq V(\text{shirk})$.} Continuing under this assumption, we have that shirking is not attractive if $V_N \geq V(\text{shirk})$, which is equivalent to

\[(1 - \theta)\Delta\delta V_N \geq (1 - \theta)b,\]  

or

\[\frac{\Delta\delta S^*}{1 - (p + \Delta)\delta} \geq b.\]  

(8)  

(9)

From this expression, a number of implications follow. Holding the surplus level $S^*$ constant, the bank is more likely to monitor rather than shirk if the discount factor $\delta$ is higher, if the borrower’s base chance of success $p$ is higher, and if the increment $\Delta$ added by monitoring is higher. The first result is standard in such reputational models, but the others are novel. Intuitively, the bank is less likely to face defaults in the future if $p$ is higher; this reduces the chance of false signals of shirking, making monitoring more attractive. Higher $\Delta$ means that monitoring reduces the chance of default by more, increasing the likelihood of collecting future rents.

This analysis ignores changes in $S^*$. The next proposition shows what happens when the impact of parameter changes on $S^*$ is taken into account.

**Proposition 8** Consider the reputational equilibrium in the infinitely-repeated game where the first-best outcome is achieved so long as the bank has not suffered any defaults, but a single default leads to repetition of the optimal single-period equilibrium.

(i) An increase in the discount factor $\delta$ makes it more likely that the reputational equilibrium exists.

(ii) If the single-period equilibrium is pooling, an increase in the borrowing firm’s base probability of success $p$ makes it more likely that the reputational equilibrium exists. If the single-period equilibrium is separating with monitoring, then if $R^i - C = \frac{b}{\Delta} + \epsilon$ or $\delta[2(p + \Delta) - \theta\beta] > 1$, an increase in $p$ makes it more likely that the reputational equilibrium exists. If the single-period equilibrium is separating with no monitoring, then if $\delta[2p + (1 + \theta)\Delta - \theta\beta] > 1$, an increase in $p$ makes it more likely that the reputational equilibrium exists.

(iii) If the single-period equilibrium is pooling, an increase in the impact of monitoring $\Delta$ makes it more likely that the reputational equilibrium exists. If the single-period equilibrium is separating with pooling, then if $R^i - C = \frac{b}{\Delta} + \epsilon$ or $(p - \theta\beta)(1 - p\delta) + \Delta^2\delta \geq 0$, an increase in $\Delta$ makes it more likely that the reputational equilibrium exists. If the single period
equilibrium is separating with no monitoring, then so long as $(p - \theta \beta)(1 - \rho \delta) + \theta \Delta^2 \delta \geq 0$, an increase in $\Delta$ makes it more likely that the reputational equilibrium exists.

Generally speaking, even when the endogeneity of $S^*$ is taken into account, the results previously mentioned still hold: the reputational equilibrium is more likely to exist when $\delta$, $p$, or $\Delta$ is higher. The exception occurs when the single-period equilibrium is separating and the lending face value just meets the bank’s break-even constraint (i.e., $R^\ell = R^\ell_{s,m}$ or $R^\ell_{s,n}$), because here $S^*$ increases with the single-period loan face value, which is decreasing in $p$ and $\Delta$. If the credit quality $p + \Delta$ of the borrower is sufficiently low and $\theta \beta$ sufficiently high, this can offset the impact of increasing $p$ and $\Delta$ on the bank’s effective discount rate. However, low $p + \Delta$ or high $\theta \beta$ make it more likely that the equilibrium is pooling with monitoring, in which case these caveats do not apply.

One can argue that punishing the bank forever after a single default is fairly draconian. Other reputational equilibria are in fact possible. For example, the market might punish the bank after a default for some finite number of periods $n$, after which the bank’s reputation is “cleansed” and behavior can revert to the first-best. Since such an equilibrium will attain first-best behavior more often, it will clearly be attractive if it exists. The difficulty is that incentive compatibility is harder to meet for such an equilibrium: by making punishment less severe (loss of rent for $n$ periods instead of forever), the bank’s incentives to shirk are enhanced. This suggests that, if the bank’s relative rents for monitoring ($S^*$) are lower, the punishment period will need to increase so as to maintain incentives.

Another possible objection is that if the borrower’s underlying risk of default is fairly high ($p$ is low), a single default has a high chance of happening even if the bank does monitor when appropriate. We have explored the case where the bank maintains a reputation for monitoring unless it experiences two defaults in a row, after which market behavior reverts to the single-period equilibrium forever. In the interests of space, we do not present the full results here, but instead offer an overview.

Suppose that this new reputational equilibrium exists. It is easy to show that this is a more efficient equilibrium than that where a single default leads the bank to lose its reputation forever. Intuitively, in the new equilibrium, one default through bad luck does not immediately lead to the inefficient single-period equilibrium in the future. Conditions for incentive compatibility are more complex, however. There are separate conditions governing the bank’s incentives depending on whether it has just experienced one or no borrower defaults. Incentive compatibility is more binding in the case where the bank has not yet experienced a default. Intuitively, since a first default does not lead to an immediate loss of reputation, the bank has less to lose by shirking when it has had no defaults; by contrast, when it has just had one default, shirking increases the chance of another default.
and consequent loss of reputation. This suggests that, in a more complete model, incentives to maintain a reputation for monitoring credit exposures may be weakest just when performance seems to be highest.

8 Proofs

Proof of Lemma 1

Results (i) and (ii) follow from the net value of a loan to investors depending on whether or not it is monitored. Result (iii) follows from the fact that a loan that has a CDS outstanding will not be monitored. For part (iii) observe that if a bank buys CDS then it receives \( R^\ell - C \) if the borrower defaults and 0 otherwise.

Proof of Lemma 2

(i) If a bank buys CDS then it receives \( R^\ell - C \) if the borrower defaults and 0 otherwise. Therefore, the bank’s payoff is \( R^\ell \), independent of the default probability. If the bank monitors, it still receives \( R^\ell \) but also incurs a cost \( b > 0 \). Thus, it will not monitor.

(ii) If the bank does not have protection it monitors if and only if

\[
C + (p + \Delta)(R^\ell - C) - b > C + p(R^\ell - C).
\]

The result follows.

Proof of Proposition 1

(i) In this case, loan sales and CDS are equivalent and \( p^{CDS} = p^{LS} = p + \theta \Delta \). The condition that the loan buyers do not monitor is \( R^\ell - C \leq \frac{b}{(1-\theta)\Delta} \). The price of a sold loan is \( C + (p + \theta \Delta)(R^\ell - C) \).

Type \( p + \Delta \) sheds risk if and only if

\[
C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C) < C + p^{LS}(R^\ell - C)
\]

\[
\implies \beta > (1 - \theta)\Delta.
\]

Type \( p \) sells the loan rather than holding and monitoring it if and only if

\[
C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C) - b < C + p^{LS}(R^\ell - C)
\]

\[
\implies \frac{b}{R^\ell - C} + \beta > (1 - \theta)\Delta
\]
which must hold if $\beta > (1 - \theta)\Delta$. Thus, the two necessary and sufficient conditions for this equilibrium are $R^\ell - C \leq \frac{b}{\Delta(1 - \theta)}$ and $\beta > (1 - \theta)\Delta$.

(ii) Both types shed credit risk and loan buyers monitor. Thus, $p^{LS} = p + \theta\Delta$. The condition that buyers monitor becomes $R^\ell - C > \frac{b}{\Delta(1 - \theta)}$. The price of a sold loan is $C + (p + \Delta)(R^\ell - C) - b$.

Type $p + \Delta$ prefers to sell the loan rather than holding it if

$$C + (p + \Delta)(R^\ell - C) - b > C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C)$$

$$\implies R^\ell - C > \frac{b}{\beta}$$

Type $p$ sells the loan if $C + (p + \Delta)(R^\ell - C) - b > C + p(R^\ell - C) - \beta(R^\ell - C)$, which is always true because $\Delta(R^\ell - C) > b$.

Thus we require that $R^\ell - C > \frac{b}{(1 - \theta)\Delta} = \frac{b}{\Delta}$ and $R^\ell - C > \frac{b}{\beta}$.

A further requirement of equilibrium is that $p^{CDS}$ must be such that the banks prefer selling the loan to hedging it with CDS:

$$C + (p + \Delta)(R^\ell - C) - b \geq C + p^{CDS}(R^\ell - C),$$

which is equivalent to $p^{CDS} \leq p + \Delta - \frac{b}{R^\ell - C}$. Since $R^\ell - C > \frac{b}{(1 - \theta)\Delta}$, the bound on $p^{CDS}$ is greater than $p + \theta \Delta$, so setting $p^{CDS} = p + \theta \Delta$ meets this condition.

**Proof of Proposition 2**

(i) In a no pooling equilibrium, only type $p$ sheds credit risk, so $p^{CDS} = p^{LS} = p$. Again, the resale price of the loan follows immediately. For this to be an equilibrium, we need two conditions. First, type $p + \Delta$ holds the loan, so

$$C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C) \geq C + p^{LS}(R^\ell - C)$$

$$\implies \beta \leq \Delta.$$

Second, type $(p, \beta)$ sells the loan, so $C + p(R^\ell - C) - \beta(R^\ell - C) < C + p^{LS}(R^\ell - C)$, which holds because $p^{LS} = p$.

(ii) Suppose that only $p$ sheds credit risk, so $p^{CDS} = p^{LS} = p$. In this case, $\frac{b}{(1 - \theta)\Delta} = \frac{b}{\Delta} < R^\ell - C$, so loan buyers will monitor the loan. Since loan buyers monitor, the resale price of the loan is $C + (p + \Delta)(R^\ell - C) - b$. 

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Type $p + \Delta$ prefers to hold the loan rather than sell it if

$$C + (p + \Delta)(R^\ell - C) - b \leq C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C)$$

$$\implies R^\ell - C \leq \frac{b}{\beta}.$$  

Type $p$ prefers to sell the loan if $C + (p + \Delta)(R^\ell - C) - b > C + p(R^\ell - C) - \beta(R^\ell - C)$; this always holds since $\Delta(R^\ell - C) > b$.

Thus, for $R^\ell - C \leq \frac{b}{\beta}$, only $p$ sheds credit risk.

We need to verify that banks prefer loan sales to CDS in this case. The condition is:

$$p^{CDS} \leq p + \Delta - \frac{b}{R^\ell - C}.$$ Since $R^\ell - C > \frac{b}{\Delta}$, the bound on $p^{CDS}$ is greater than $p$, so setting $p^{CDS} = p$ meets this condition.

(iii) Both types shed credit risk and loan buyers monitor. Thus, $p^{LS} = p + \theta \Delta$. The condition that buyers monitor becomes $R^\ell - C > \frac{b}{(1 - \theta)\Delta}$. The price of a sold loan is $C + (p + \Delta)(R^\ell - C) - b$.

Type $p + \Delta$ prefers to sell the loan rather than holding it if

$$C + (p + \Delta)(R^\ell - C) - b > C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C)$$

$$\implies R^\ell - C > \frac{b}{\beta}.$$  

Type $p$ sells the loan if $C + (p + \Delta)(R^\ell - C) - b > C + p(R^\ell - C) - \beta(R^\ell - C)$, which is always true because $\Delta(R^\ell - C) > b$.

Thus we require that $R^\ell - C > \frac{b}{(1 - \theta)\Delta} = \frac{b}{(1 - \theta)p^{LS}}$ and $R^\ell - C > \frac{b}{\beta}$.

A further requirement of equilibrium is that $p^{CDS}$ must be such that the banks prefer selling the loan to hedging it with CDS:

$$C + (p + \Delta)(R^\ell - C) - b \geq C + p^{CDS}(R^\ell - C),$$

which is equivalent to $p^{CDS} \leq p + \Delta - \frac{b}{R^\ell - C}$. Since $R^\ell - C > \frac{b}{(1 - \theta)\Delta}$, the bound on $p^{CDS}$ is greater than $p + \theta \Delta$, so setting $p^{CDS} = p + \theta \Delta$ meets this condition.

Proof of Proposition 3

(i) In a no pooling equilibrium, only type $p$ sheds credit risk, so $p^{CDS} = p^{LS} = p$. Again, the resale price of the loan follows immediately. For this to be an equilibrium, we need two conditions. First, type $p + \Delta$ holds the loan, so

$$C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C) \geq C + p^{LS}(R^\ell - C)$$

$$\implies \beta \leq \Delta.$$
Second, type \((p, \beta)\) sells the loan, so \(C + p(R^\ell - C) - \beta(R^\ell - C) < C + p^{LS}(R^\ell - C)\), which holds because \(p^{LS} = p\). Buyers do not monitor if \(R^\ell - C \leq \frac{b}{\Delta}\).

(ii) Suppose that only \(p\) sheds credit risk, so \(p^{CDS} = p^{LS} = p\). In this case, \(\frac{b}{(1-p^{LS})\Delta} = \frac{b}{\Delta} < R^\ell - C\), so loan buyers will monitor the loan. Since loan buyers monitor, the resale price of the loan is \(C + (p + \Delta)(R^\ell - C) - b\).

Type \(p + \Delta\) prefers to hold the loan rather than sell it if

\[
C + (p + \Delta)(R^\ell - C) - b \leq C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C)
\]

\[\implies R^\ell - C \leq \frac{b}{\beta};\]

Type \(p\) prefers to sell the loan if \(C + (p + \Delta)(R^\ell - C) - b > C + p(R^\ell - C) - \beta(R^\ell - C)\); this always holds since \(\Delta(R^\ell - C) > b\).

Thus, for \(R^\ell - C \leq \frac{b}{\beta}\), only \(p\) sheds credit risk.

We need to verify that banks prefer loan sales to CDS in this case. The condition is:

\[p^{CDS} \leq p + \Delta - \frac{b}{R^\ell - C}.\]

Since \(R^\ell - C > \frac{b}{\Delta}\), the bound on \(p^{CDS}\) is greater than \(p\), so setting \(p^{CDS} = p\) meets this condition.

(iii) and (iv) follow the same arguments as in Proposition 1.

\[\blacksquare\]

**Proof of Lemma 3**

(i) The proof follows by establishing the participation constraint of the bank in each possible equilibrium. We denote the profit of type \(p\) bank by \(\pi(p)\) with an equivalent expression for type \(p + \Delta\).

(a) If the equilibrium is pooling with no monitoring, then

\[
\pi(p) = C + (R^\ell - C)p
\]

\[
\pi(p + \Delta) = C + (R^\ell - C)(p + \Delta)
\]

The bank is of type \(p\) with probability \(1 - \theta\) and type \(p + \Delta\) with probability \(\theta\). Therefore, we obtain the bank’s expected value, which must exceed 1:

\[C + (p + \theta\Delta)(R^\ell - C) \geq 1\]

The expression for \(R^\ell_{p,n}\) follows easily.

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(b) If the equilibrium is pooling with monitoring, then
\[
\pi(p) = C + (p + \Delta)(R^e - C) - b \\
\pi(p + \Delta) = C + (p + \Delta)(R^e - C) - b,
\]
and weighting by the appropriate probabilities and rearranging yields the expression for \( R_{p,m}^e \).

(c) If the equilibrium is separating with no monitoring, then
\[
\pi(p) = C + (p)(R^e - C) \\
\pi(p + \Delta) = C + (p + \Delta)(R^e - C) - \beta(R^e - C),
\]
which yields the expression for \( R_{s,n}^e \).

(d) If the equilibrium is separating with monitoring, then
\[
\pi(p) = C + (p + \Delta)(R^e - C) - b \\
\pi(p + \Delta) = C + (p + \Delta)(R^e - C) - \beta(R^e - C),
\]
which yields the expression for \( R_{s,m}^e \).

(ii) The results for \( C, p, \) and \( \Delta \) are obvious, as are those for \( b \) and \( \beta \). It is clear that \( R_{p,n}^e \) is decreasing in \( \theta \). So is \( R_{s,n}^e \), since \( \Delta > \beta \) when that equilibrium exists. \( R_{p,m}^e \) is unaffected by \( \theta \). Finally, if the equilibrium is separating with monitoring, we have
\[
\frac{\partial R_{s,m}^e}{\partial \theta} = (p + \Delta - \theta \beta)^{-2} \left[ (p + \Delta - \theta \beta)(-b) - (1 - C + (1 - \theta)b)(-\beta) \right] \\
= (p + \Delta - \theta \beta)^{-2} \left[ (1 - C)\beta - (p + \Delta - \beta)b \right].
\]

But we also have \( R_{s,m}^e - C \leq \frac{b}{\beta} \) if the equilibrium is to exist, so
\[
\frac{1 - C + (1 - \theta)b}{p + \Delta - \theta \beta} \leq \frac{b}{\beta} \Rightarrow (1 - C)\beta \leq (p + \Delta - \beta)b.
\]
It follows that \( R_{s,m}^e \) is decreasing in \( \theta \).
Proof of Lemma 4  
(i) $R_{p,n}^\ell - C > R_{p,m}^\ell - C$ is the same as $\frac{1-C}{p+\theta \Delta} > \frac{1-C+b}{p+\Delta}$. Multiplying both sides by the product of the denominators and rearranging, we have $(1-C)(1-\theta)\Delta > b(p+\theta \Delta)$, which is the same as $\frac{1-C}{p+\theta \Delta} = R_{p,n}^\ell - C > \frac{b}{(1-\theta)\Delta}$. Also, $(1-C)(1-\theta)\Delta > b(p+\theta \Delta)$ is the same as $(1-C)(1-\theta)\Delta + b(1-\theta)\Delta > b(p+\Delta)$, which is the same as $\frac{1-C+b}{p+\Delta} = R_{p,m}^\ell - C > \frac{b}{(1-\theta)\Delta}$.

(ii) $R_{s,n}^\ell - C > R_{s,m}^\ell - C$ is the same as $\frac{1-C}{p+\theta \Delta - \theta \beta} > \frac{1-C+(1-\theta)b}{p+\Delta-\theta \beta}$. This can be rearranged to yield $(1-C)(1-\theta)\Delta > (1-\theta)b(p+\theta \Delta - \theta \beta)$, which then yields $\frac{1-C}{p+\theta \Delta - \theta \beta} = R_{s,n}^\ell - C > \frac{b}{\Delta}$. It also yields $(1-C)\Delta > b(p+\theta \Delta - \theta \beta)$; adding $(1-\theta)\Delta b$ to both sides yields $(1-C)\Delta + (1-\theta)\Delta b > b(p+\Delta - \theta \beta)$, which is the same as $\frac{1-C+(1-\theta)b}{p+\Delta-\theta \beta} = R_{s,m}^\ell - C > \frac{b}{\Delta}$.

(iii) Analysis similar to that in (iv) shows that all of these conditions are equivalent to $\frac{1-C}{p+\Delta-\beta} > \frac{b}{\beta}$.

Proof of Proposition 4  
(i) In this case, it is clear that equilibrium $(p, n)$ is feasible. Any loan face value $R^\ell$ such that $R^\ell - C > \frac{b}{(1-\theta)\Delta}$ implements equilibrium $(p, m)$. Thus, the entrepreneur chooses between these two. By definition, $\Omega^{p,n} > \Omega^{p,m}$ if and only if $-(1-\theta)[\Delta (R-C) - (b+B)] > -\theta b$. Rearranging yields the condition in the text.

(ii) In this case, equilibrium $(p, n)$ is not feasible. The comparative statics results follow from Proposition 3.

Proof of Proposition 5  
Because the $(p, n)$ equilibrium requires $(1-\theta)\Delta < \beta$, it is clearly not feasible in this case.

(i) Proposition 3(iv) implies that $R_{s,n}^\ell - C < \frac{b}{\Delta}$, so $(s, n)$ is feasible and can be implemented by $R^\ell = R_{s,n}^\ell$. Also, a loan face value of $R^\ell - C = \frac{b}{\Delta} + \epsilon$ with $\epsilon$ small feasibly implements $(s, m)$ (note that this rules out $(s, n)$, and $\frac{b}{\Delta} < R^\ell - C < \frac{b}{(1-\theta)\Delta} < \frac{b}{\beta}$, ruling out $(p, m)$). This loan face value also implies $\Omega^{s,m} = \Omega^* - \theta b < \Omega^* - \theta \beta (R^\ell - C) = \Omega^{s,m}$, since $R^\ell - C < \frac{b}{\beta}$. Thus $(p, m)$ will never be chosen in this case.

The entrepreneur chooses between $(s, n)$ and $(s, m)$. Note that, for each equilibrium, social welfare is maximized by choosing a loan face value that is as low as feasible, so the previously specified values are the best choices.\(^\text{16}\) $\Omega^{s,n} > \Omega^{s,m}$ if and only if $-(1-\theta)[\Delta (R-C) - (b+B)] - \theta \beta (R_{s,n}^\ell - C) > -\theta \beta (\frac{b}{\Delta} + \epsilon - C)$. This can be rearranged to yield $(1-\theta + \frac{\theta}{\Delta} b) - \theta \beta (R_{s,n}^\ell - C) + \theta \beta \epsilon > (1-\theta)[(\Delta (R-C) - B)]$; letting $\epsilon$ go to zero yields the inequality in the proposition.

\(^{16}\) Technically, there is no optimal face value for $(s, m)$, since one can always choose a smaller $\epsilon > 0$ that implements this equilibrium. Nevertheless, our analysis shows when some such equilibrium will be chosen and when it won’t.
(ii) In this case, from Proposition 3(iv), \( R_{s,n}^\ell - C > R_{s,m}^\ell - C > \frac{b}{\Delta} \), so \((s,n)\) is not feasible and \( R_{s,m}^\ell \) is the smallest face value implementing \((s,m)\). The same steps as in the proof of (i) above show that \( \Omega^{p,m} < \Omega^{s,m} \), so the entrepreneur chooses \((s,m)\) and maximizes welfare by choosing loan face value \( R_{s,m}^\ell \).

(iii) In this case, \((s,n)\) and \((s,m)\) are not feasible. Proposition 3(v) shows that \( R_{p,m}^\ell - C > \frac{b}{\beta} \), and \( \frac{b}{\beta} > \frac{b}{(1-\theta)\Delta} \), so a loan face value of \( R_{p,m}^\ell \) implements \((p,m)\).

**Proof of Proposition 6**

(i) \( R_{p,m}^\ell - C \leq \frac{b}{\beta} < \frac{b}{(1-\theta)\Delta} \), so by Proposition 3(iii) and (v), \( R_{p,n}^\ell - C \leq \frac{b}{(1-\theta)\Delta} \) and \( R_{s,m}^\ell - C \leq \frac{b}{\beta} \). As shown in the proof of Proposition 3(v), \( R_{s,m}^\ell - C \leq \frac{b}{\beta} \) if and only if \( \frac{b - C}{b + \Delta - \beta} \leq \frac{b}{\beta} \), so we have \( R_{p,n}^\ell - C = \frac{1}{p + \Delta} - \frac{1}{\beta} \geq \frac{b - C}{b + \Delta - \beta} > \frac{1}{p + \Delta} - \frac{1}{\beta} \). Thus, any \( R^\ell - C \in \left[ \frac{b}{\beta}, \frac{b}{(1-\theta)\Delta} \right] \) implements \((p,n)\) uniquely and feasibly.

Any loan face value such that \( R^\ell - C \leq \frac{b}{\beta} \) supports one of the separating equilibria \(((s,n)\) if \( R^\ell - C \leq \frac{b}{\beta} \) and \((s,m)\) otherwise) and also supports \((p,n)\).

(a) Suppose that \( R^\ell - C \leq \frac{b}{\beta} \) leads to the relevant separating equilibrium. It is easy to show that \( \Omega^{p,n} > \Omega^{s,n} \), and \( \Omega^{s,m} > \Omega^{p,m} \), as shown in Proposition 5(i). So the entrepreneur chooses between implementing \((s,m)\) with \( R^\ell \) as small as possible, i.e., \( R^\ell - C = \max\{R_{s,n}^\ell - C, \frac{b}{\Delta} + \epsilon\} \), and implementing \((p,n)\) with any \( R^\ell - C \in \left[ \frac{b}{\beta}, \frac{b}{(1-\theta)\Delta} \right] \). Substituting in, we have \( \Omega^{p,n} > \Omega^{s,m} \) if and only if \( \theta \beta \max\{R_{s,n}^\ell - C, \frac{b}{\Delta} + \epsilon\} > (1-\theta)(\Delta(R-C) - (b+B)) \); rearranging and letting \( \epsilon \) go to 0 yields the condition in the proposition.

(b) If \( R^\ell - C \leq \frac{b}{\beta} \) leads to equilibrium \((p,n)\), the separating equilibria cannot be implemented. The entrepreneur chooses between the two pooling equilibria as in Proposition 4(i), leading to the condition given in this proposition.

(ii) \( \frac{b}{\beta} < R_{p,m}^\ell - C \leq \frac{b}{(1-\theta)\Delta} \) implies that, by Proposition 3(iii)-(v), \( R_{p,n}^\ell - C \leq \frac{b}{(1-\theta)\Delta} \) and \( R_{s,m}^\ell - C > \frac{b}{\beta} > \frac{b}{\Delta} \), so \( R_{s,n}^\ell - C > \frac{b}{\Delta} \) too. Thus, neither \((s,n)\) nor \((s,m)\) can be feasibly implemented, but \((p,n)\) is feasible and can be uniquely implemented through \( R^\ell - C \in \left( \max\{R_{p,n}^\ell - C, \frac{b}{\beta}\}, \frac{b}{(1-\theta)\Delta} \right) \), and \((p,m)\) is also feasible and can be implemented through any \( R^\ell - C > \frac{b}{(1-\theta)\Delta} \). The rest of the analysis is as in the proof of (i.b) given above.

(iii) \( R_{p,m}^\ell - C > \frac{b}{(1-\theta)\Delta} \) implies that \( R_{p,n}^\ell - C > \frac{b}{(1-\theta)\Delta} \) and \((p,m)\) can be implemented by loan face value \( R_{p,m}^\ell \).

**Proof of Proposition 7**

The proof follows from the arguments presented in the text.

Suppose that loan buyers do not monitor. In this case, their valuation of the loan depends on how the banks lay off credit risk. As both markets (CDS and loan sales) are open, then no inferences can be made about the quality of the credit risk, and the posterior
value conditional on observing a loan sale is \( p + \theta \Delta \). Therefore, loan buyers do not monitor if \( (p + \theta \Delta)(R^\ell - C) > (p + \Delta)(R^\ell - C) - b \). Or, \( (R^\ell - C) < \frac{b}{\Delta(1 - \theta)} \).

Suppose that the loan buyers monitor. Then, a \( p + \Delta \) bank sells the loan if \( C + (p + \Delta + \beta)(R^\ell - C) - b > C + (p + \Delta)(R^\ell - C) \), or \( R^\ell - C > \frac{b}{\Delta} \). A \( p \) type bank always sells the loan. Therefore, if loan buyers monitor, the default probability of the underlying is \( p + \Delta \), and a zero profit market maker would insure default using this probability. However, if he cannot distinguish between the originating bank and noise traders, then both banks have a strict incentive to sell the risk in the CDS market, which ensures that it will not be monitored and therefore yields a default probability of \( p \) which generates a loss for the Market Maker.

However a price of credit risk that makes the \( p \) type bank indifferent between the loan sales market and the CDS market is consistent with equilibrium. Specifically, consider a \( p < p^* < p + \Delta \), so that \( (p + \Delta - p^*)(R^\ell - C) \geq b \). This generates a positive profit for the MM, and the \( p \) types trader strictly prefers to sell his loan in which case it is monitored.

Proof of Proposition 8

(i) It is easy to see that, for any single-period equilibrium, the surplus \( S^* = \Omega^* - \Omega^\ell \) is not affected by \( \delta \), so the left-hand side of condition (9) is increasing in \( \delta \).

(ii) For a pooling equilibrium, \( S^* \) does not depend on \( p \), so the left-hand side of condition (9) is increasing in \( p \).

If the single-period equilibrium is separating with monitoring, we have \( S^* = \theta \beta (R^\ell - C) \), where \( R^\ell - C = \max \left( \frac{1 - C + \theta b}{\Delta} \cdot \frac{b}{\Delta} \right) \) is the credit exposure that would be chosen in the single-period equilibrium. It follows that condition (9) becomes

\[
\frac{\delta \Delta}{1 - (p + \Delta)\delta} \theta \beta (R^\ell - C) \geq b. \tag{10}
\]

If \( R^\ell - C = \frac{b}{\Delta} + \epsilon \), the left-hand side of condition (10) is increasing in \( p \). Otherwise, the \( R^\ell = R^\ell_{s,m} \), and the derivative of the left-hand side with respect to \( p \) equals

\[
\frac{\delta \Delta}{1 - (p + \Delta)\delta} \theta \beta (R^\ell_{s,m} - C) \left[ \frac{\delta}{1 - (p + \Delta)\delta} - \frac{1}{p + \Delta - \theta \beta} \right]. \tag{11}
\]

It can then be shown that the sign of the term in brackets equals that of \( \delta(2(p + \Delta) - \theta \beta) - 1 \), so the left-hand side of (10) is increasing in \( p \) if \( \delta(2(p + \Delta) - \theta \beta) > 1 \).

If the single-period equilibrium is separating with no monitoring, \( S^* = (1 - \theta)[\Delta(R - C) - (B + b)] + \theta \beta (R^\ell_{s,n} - C) \). It follows that the derivative of the left-hand side of condition (9) with respect to \( p \) is greater than

\[
\frac{\delta \Delta}{1 - (p + \Delta)\delta} \theta \beta (R^\ell - C) \left[ \frac{\delta}{1 - (p + \Delta)\delta} - \frac{1}{p + \theta \Delta - \theta \beta} \right]. \tag{12}
\]
The sign of the term in brackets equals that of $\delta[2p + (1 + \theta)\Delta - \theta\beta] - 1$, which leads to the condition in the proposition.

(iii) For a pooling equilibrium, $S^*$ is weakly increasing in $\Delta$, so the left-hand side of condition (9) is increasing in $\Delta$.

Suppose the single-period equilibrium is separating with monitoring, so $S^* = \theta\beta(R^t - C)$. If $R^t - C = \frac{b}{\Delta} + \epsilon$, the left-hand side of condition (9) equals $\frac{\delta b}{1 - (p + \Delta)\delta} \theta\beta$, which is increasing in $\Delta$. If $R^t = R^t_{s,m}$, the derivative of the left-hand side of condition (9) with respect to $\Delta$ equals

$$\frac{\delta}{1 - (p + \Delta)\delta} \theta\beta(R^t - C) \left[1 + \frac{\Delta\delta}{1 - (p + \Delta)\delta} - \frac{\Delta}{p + \Delta - \theta\beta}\right].$$

(13)

The sign of the term in brackets equals that of $(p - \theta\beta)(1 - p\delta) + \Delta^2\delta$.

If the single-period equilibrium is no pooling with no monitoring, then, similar to part (ii), the derivative of the left-hand side of condition (9) with respect to $\Delta$ is greater than

$$\frac{\delta}{1 - (p + \Delta)\delta} \theta\beta(R^t - C) \left[1 + \frac{\Delta\delta}{1 - (p + \Delta)\delta} - \frac{\theta\Delta}{p + \theta\Delta - \theta\beta}\right].$$

(14)

The sign of the term in brackets equals that of $(p - \theta\beta)(1 - p\delta) + \theta\Delta^2\delta$. 

\[\square\]
References


