## CHAPTER 5

### THE THEORY OF DEMAND

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During the 1990s and early 2000s the tobacco industry became increasingly embroiled in litigation over the damages caused by cigarette smoking. Many states sued tobacco companies to recover health care costs related to smoking. Several tobacco companies agreed to pay billions of dollars to Minnesota, Florida, Mississippi, Texas, New York, and other states. The tobacco companies then faced a difficult question. How would they pay for these legal settlements? Their response was to raise cigarette prices repeatedly.

Why did cigarette producers believe that they could collect more revenues if they raised cigarette prices? And what information would they need to estimate the size of the increase in their revenues from an increase of, say, five cents per pack? As we saw in Chapter 2, firms can predict the effects of a price increase if they know the shape of the market demand curve. An article in the *The Wall Street Journal* summarizes some of the extensive research on the market demand curve for cigarettes. “The average price for a pack of cigarettes is about $2. Prices vary by state because of taxes. Analysts say that for every 10 percent price increase, sales volumes drop between 3.5 percent and 4.5 percent. They say that small price increases generally don’t cause most consumers to try to give up smoking, but that they smoke fewer cigarettes each day.”

Based on this information, we would conclude that the price elasticity of demand for cigarettes is approximately $-0.35$ to $-0.45$. Thus the demand for cigarettes is relatively price inelastic. As we learned in Chapter 2, when demand is relatively inelastic, a small price increase will lead to an increase in sales revenues. In the cigarette market, if price rises by 10 percent, sales volume will fall by about 4 percent. This means that with a 10 percent price increase, the revenues from cigarette sales would increase by about 6 percent. This explains why cigarette producers believed sales revenues would rise if they increased cigarette prices.

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CHAPTER PREVIEW  In this chapter, you will
• Study how a consumer’s demand for a good depends on the prices of all goods and on income.
• Examine how a change in the price of a good affects a consumer through a substitution effect and an income effect.
• Examine how a change in the price of a good affects a consumer in terms of consumer surplus, compensating variation, and equivalent variation.
• See how to use individual consumers’ demand curves to derive market demand curves.
• Study the effect of network externalities on demand curves.
• Examine how consumers choose to allocate their time between labor and leisure.

5.1 OPTIMAL CHOICE AND DEMAND

Where do demand curves come from? In Chapter 4, we showed how to determine a consumer’s optimal basket. Given the consumer’s preferences and income and the prices of all goods, we could ask how much ice cream a consumer will buy each month if the price of a gallon of ice cream is $5. This will be a point on the consumer’s demand curve for ice cream. We can find more points on her demand curve by repeating the exercise for different prices of ice cream, asking what her monthly consumption of ice cream will be if the price is $4, $3, or $2 per gallon. Let’s see how to do this, using a simplified setting in which our consumer buys only two goods, food and clothing.

THE EFFECTS OF A CHANGE IN PRICE

What happens to the consumer’s choice of food when the price of food changes while the price of clothing and the amount of income remain constant? We have two ways to answer this question, one using the optimal choice diagram in Figure 5.1(a) and the second using the demand curve in Figure 5.1(b).

Looking at an Optimal Choice Diagram

The graph in Figure 5.1(a) shows the quantity of food consumed (x) on the horizontal axis and the quantity of clothing (y) on the vertical axis. It also shows three of the consumer’s indifference curves (U₁, U₂, and U₃). Suppose the consumer’s weekly income is $40 and the price of clothing is $4 per unit.

Consider the consumer’s choices of food and clothing for three different prices of food. First, suppose the price of food is $4. The budget line that the consumer faces when $P_x = $4, $P_y = $4, and I = $40 is labeled BL₁ in the figure. The slope of BL₁ is $−P_x/P_y = −4/4 = −1$. The consumer’s optimal basket is A, indicating that her optimal weekly consumption is 2 units of food and 8 units of clothing.

What happens when the price of food falls to $P_x = $2? The vertical intercept of the budget line is the same because income and the price of clothing are unchanged. However, as we saw in Chapter 4, the horizontal intercept moves to the right (to BL₂). The slope of BL₂ is $−P_x/P_y = −2/4 = −1/2$. Her optimal basket is B, with a weekly consumption of 10 units of food and 5 units of clothing.

Finally, suppose the price of food falls to $P_x = $1. The budget line rotates out to BL₃, which has a slope of $−P_x/P_y = −1/4$. The consumer’s optimal basket is C, with a weekly consumption of 16 units of food and 6 units of clothing.

One way to describe how changes in the price of food affect the consumer’s purchases of both goods is to draw a curve connecting all of the baskets that are
5.1 OPTIMAL CHOICE AND DEMAND

optimal as the price of food changes (holding the price of clothing and income constant). This curve is called the price consumption curve.\(^2\) In Figure 5.1(a), the optimal baskets \(A\), \(B\), and \(C\) lie on the price consumption curve.

\(^2\)In some textbooks the price consumption curve is called the “price expansion path.”
Observe that the consumer is better off as the price of food falls. When the price of food is $4 (and she chooses basket $A$), she reaches the indifference curve $U_1$. When the price of food is $2 (and she chooses basket $B$), her utility rises to $U_2$. If the price of food falls to $1, her utility rises even farther, to $U_3$.

**Changing Price: Moving Along a Demand Curve**

We can use the optimal choice diagram of Figure 5.1(a) to trace out the demand curve for food shown in Figure 5.1(b), where the price of food appears on the vertical axis and the quantity of food on the horizontal axis.

Let’s see how the two graphs are related to each other. When the price of food is $4, the consumer chooses basket $A$ in Figure 5.1(a), containing 2 units of food. This corresponds to point $A'$ on her demand curve for food in Figure 5.1(b). Similarly, at basket $B$ in Figure 5.1(a), the consumer purchases 10 units of food when the price of food is $2, matching point $B'$ on her demand curve in Figure 5.1(b). Finally, as basket $C$ in Figure 5.1(a) indicates, if the price of food falls to $1, the consumer buys 16 units of food, corresponding to point $C'$ in Figure 5.1(b). In sum, a decrease in the price of food leads the consumer to move down and to the right along her demand curve for food.

**The Demand Curve Is Also a “Willingness to Pay” Curve**

As you study economics, you will sometimes find it useful to think of a demand curve as a curve that represents a consumer’s “willingness to pay” for a good. To see why this is true, let’s ask how much the consumer would be willing to pay for another unit of food when she is currently at the optimal basket $A$ (purchasing 2 units of food) in Figure 5.1(a). Her answer is that she would be willing to pay $4 for another unit of food. Why? At basket $A$ her marginal rate of substitution of food for clothing is $MRS_{x,y} = 1$. Thus, at basket $A$ one more unit of food is worth the same amount to her as one more unit of clothing. Since the price of clothing is $4, the value of an additional unit of food will also be $4. This reasoning helps us to understand why point $A'$ on the demand curve in Figure 5.1(b) is located at a price of $4. When the consumer is purchasing 2 units of food, the value of another unit of food to her (i.e., her “willingness to pay” for another unit of food) is $4.

Note that her $MRS_{x,y}$ falls to $1/2$ at basket $B$ and to $1/4$ at basket $C$. The value of an additional unit of food is therefore $2 at $B$ (when she consumes 10 units of food) and only $1 at basket $C$ (when she consumes 16 units of food). In other words, her willingness to pay for an additional unit of food falls as she buys more and more food.

**The Effects of a Change in Income**

What happens to the consumer’s choices of food and clothing as income changes? Let’s look at the optimal choice diagram in Figure 5.2(a), which measures the quantity of food consumed ($x$) on the horizontal axis and the quantity of clothing ($y$) on the vertical axis. Suppose the price of food is $P_x = S2 and the price of clothing is $P_y = S4 per unit, with both prices held constant. The slope of her budget lines is $-P_x/P_y = -1/2$.

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1 At A the indifference curve $U_1$ and the budget line $BL_1$ are tangent to one another, so their slopes are equal. The slope of the budget line is $-P_x/P_y = -1$. Recall that the $MRS_{x,y}$ at A is the negative of the slope of the indifference curve (and the budget line) at that basket. Therefore, $MRS_{x,y} = 1$. 

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5.1 OPTIMAL CHOICE AND DEMAND

In Chapter 4 we saw that an increase in income results in an outward, parallel shift of the budget line. Figure 5.2(a) illustrates the consumer’s budget lines and optimal choices of food and clothing for three different levels of income, as well as three of her indifference curves ($U_1$, $U_2$, and $U_3$). Initially, when the consumer’s weekly income is $I_1 = $40, her budget line is $BL_1$. She chooses basket $A$, consuming 10 units of food and 5 units of clothing per week. As her income rises to $I_2 = $68, the budget line shifts out to $BL_2$. She then chooses basket $B$, with a weekly consumption of 18 units of food and 8 units of clothing. If her income increases to $I_3 = $92, she faces budget line $BL_3$. Her optimal basket is $C$, with 24 units of food and 11 units of clothing.

One way we can describe how changes in income affect the consumer’s purchases is by drawing a curve that connects all the baskets that are optimal as income changes (keeping prices constant). This curve is called the income consumption curve. In Figure 5.2(a), the optimal baskets $A$, $B$, and $C$ lie on the income consumption curve.

Changing Income: Shifting a Demand Curve

In Figure 5.2(a) the consumer purchases more of both goods as her income rises. In other words, an increase in income results in a rightward shift in her demand curve for each good. In Figure 5.2(b) we illustrate this by seeing how a change in income affects her demand curve for food. The price of food (held constant at $2) appears on the

Applications 5.1

What Would People Pay for Cable?

The cable television industry is one of the most important sources of programming for households in the United States. Competitors include traditional broadcast stations, direct broadcast satellites, wireless cable, and video cassettes. However, about two-thirds of all households subscribe to cable television.

Public policy toward the cable television industry has changed repeatedly during the last two decades. In 1984 the industry was deregulated, and cable systems rapidly expanded the services they offered. However, by the early 1990s, Congress had become concerned that local cable operators were charging unacceptably high prices and that many home owners lacked adequate access to alternative programming. In 1992, over President Bush’s veto, Congress passed a sweeping set of regulations for the industry. However, in 1996 Congress again removed regulation from much of the cable television industry, recognizing that competition to provide programming had increased.

Public policy debates on this subject often focus on the nature of the demand for cable television. How much will consumers pay for basic cable television services? How sensitive are consumers to changes in the prices charged? In a study of the demand for cable television with data from 1992, Robert Crandall and Harold Furchtgott-Roth found the price elasticity of demand to be about $-0.8$ for the basic service offered by a typical cable television system. Thus, a 10 percent increase in the price of a basic subscription would lead to a loss of 8 percent of the subscribers. Some of those who drop their subscriptions might opt for other forms of programming, while others might choose no programming at all. As competition from other sources of programming (including the Internet and direct access to television signals from satellites) intensifies over time, the demand for cable television will become more elastic.

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CHAPTER 5 THE THEORY OF DEMAND

FIGURE 5.2 The Effects of Changes in Income on Consumption
The consumer buys food at $P_x = 2$ per unit and clothing at $P_y = 4$ per unit. Both prices are held constant as income varies.

(a) Optimal choice diagram. The budget lines reflect three different levels of income. The slope of all budget lines is $-P_x/P_y = -1/2$. $BL_1$ is the budget line when the weekly income is $40$. $BL_2$ and $BL_3$ are the budget lines when income is $68$ and $92$, respectively. We can draw a curve connecting the baskets that are optimal ($A$, $B$, and $C$) as income changes. This curve is called the income consumption curve.

(b) Demand curves for food. The consumer’s demand curve for food shifts out as income rises.

Engel curve A curve that relates the amount of a commodity purchased to the level of income, holding constant the prices of all goods.

vertical axis, and the quantity of food on the horizontal axis. When the consumer’s weekly income is $40$, she buys 10 units of food each week, corresponding to point $A'$ on demand curve $D_1$ in Figure 5.2(b). If her income rises to $68$, she buys 18 units of food, corresponding to point $B'$ on demand curve $D_2$. Finally, if her income rises to $92$, she buys 24 units of food, corresponding to point $C'$ on demand curve $D_3$.

Using a similar approach, you can also show how the demand curves for clothing shift as income changes (see Problem 5.1 at the end of this chapter).

Engel Curves
Another way of showing how a consumer’s choice of a particular good varies with income is to draw an Engel curve, a graph relating the amount of the good consumed to the level of income. Figure 5.3 shows an Engel curve relating the amount of food
consumed to the consumer’s income. Here the amount of food \((x)\) is on the horizontal axis and the level of income \((I)\) is on the vertical axis. Point \(A’\) on the Engel curve shows that the consumer buys 10 units of food when her weekly income is $40. Point \(B’\) indicates that she buys 18 units of food when her income is $68. When her weekly income rises to $92, she buys 24 units of food (point \(C’\)). Note that we draw the Engel curve holding constant the prices of all goods (the price of food is $2 and the price of clothing is $4). For a different set of prices we would draw a different Engel curve.

The income consumption curve in Figure 5.2(a) shows that the consumer purchases more food when her income rises. When this happens, the good (food) is said to be a normal good. For a normal good the Engel curve will have a positive slope, as in Figure 5.3.

From Figure 5.2(a) you can also see that clothing is a normal good. Therefore, if you were to draw an Engel curve for clothing, with income on the vertical axis and the amount of clothing on the horizontal axis, the slope of the Engel curve would be positive.

As you might suspect, consumers don’t always purchase more of every good as income rises. If a consumer wants to buy less of a good when income rises, that good is termed an inferior good. Consider a consumer with the preferences for hot dogs and a composite good (“other goods”) depicted in Figure 5.4(a). For low levels of income, this consumer views hot dogs as a normal good. For example, as monthly income rises from $200 to $300, the consumer would change his optimal basket from \(A\) to \(B\), buying more hot dogs. However, as income continues to rise, the consumer prefers to buy fewer hot dogs and more of the other goods (such as steak or seafood). The income consumption curve in Figure 5.4(a) illustrates this possibility between baskets \(B\) and \(C\). Over this range of the income consumption curve, hot dogs are an inferior good.

The Engel curve for hot dogs is shown in Figure 5.4(b). Note that the Engel curve has a positive slope over the range of incomes for which hot dogs are a normal good, and a negative slope over the range of incomes for which hot dogs are an inferior good.
FIGURE 5.4 Inferior Good
(a) As income rises from $200 to $300, the consumer’s weekly consumption of hot dogs increases from 13 (basket A) to 18 (basket B). However, as income rises from $300 to $400, the consumer’s weekly consumption of hot dogs decreases from 18 to 16 (basket C).
(b) Hot dogs are a normal good between points A’ and B’ (i.e., over the income range $200 to $300), where the Engel curve has a positive slope. But between points B’ and C’ (i.e., over the income range $300 to $400), hot dogs are an inferior good, and the Engel curve has a negative slope.

LEARNING-BY-DOING EXERCISE 5.1
A Normal Good Has a Positive Income Elasticity of Demand

Problem A consumer likes to attend rock concerts and consume other goods. Suppose x measures the number of rock concerts he attends each year, and I denotes his annual income. Show that the following statement is true: If he views rock concerts as a normal good, then his income elasticity of demand for rock concerts must be positive.

Solution In Chapter 2 we learned that the income elasticity of demand is defined as $\epsilon_{x,I} = (\Delta x / \Delta I)(I/x)$, where all prices are held constant. If rock concerts are a normal good, then x increases as income I rises, so $(\Delta x / \Delta I) > 0$. Since income I and the number of rock concerts attended x are positive, it must also be true that $(I/x) > 0$. Therefore, $\epsilon_{x,I} > 0$.

Similar Problem: 5.3
During the early nineteenth century, Ireland’s population grew rapidly. Nearly half of the Irish people lived on small farms that produced little income. Many others who were unable to afford their own farms leased land from owners of big estates. But these landlords charged such high rents that leased farms also were not profitable.

Because they were poor, many Irish people depended on potatoes as an inexpensive source of nourishment. In *Why Ireland Starved*, noted economic historian Joel Mokyr described the increasing importance of the potato in the Irish diet by the 1840s:

> It is quite unmistakable that the Irish diet was undergoing changes in the first half of the nineteenth century. Eighteenth-century diets, the evergrowing importance of potatoes notwithstanding, seem to have been supplemented by a variety of vegetables, dairy products, and even pork and fish. . . . Although glowing reports of the Irish cuisine in the eighteenth century must be deemed unrepresentative since they pertain to the shrinking class of well-to-do farmers, things were clearly worsening in the nineteenth. There was some across-the-board deterioration of diets, due to the reduction of certain supplies, such as dairy products, fish, and vegetables, but the main reason was the relative decline of the number of people who could afford to purchase decent food. The dependency on the potato, while it cut across all classes, was most absolute among the lower two-thirds of the income distribution. ⑥

Mokyr’s account suggests that the income consumption curve for a typical Irish consumer might have looked like the one in Figure 5.4 (with potatoes on the horizontal axis instead of hot dogs). For people with a low income, potatoes might well have been a normal good. But consumers with higher incomes could afford other types of food, and therefore consumed fewer potatoes.

Given the heavy reliance on potatoes as food and as a source of income, it is not surprising that a crisis occurred between 1845 and 1847, when a plant disease caused the potato crop to fail. During the Irish potato famine, about 750,000 people died of starvation or disease, and hundreds of thousands of others emigrated from Ireland to escape poverty and famine.

This exercise demonstrates a general proposition: If a good is normal, its income elasticity of demand is positive. The converse is also true: If a good’s income elasticity of demand is positive, the good is a normal good.

Using similar reasoning you can demonstrate that the following statements are also true: (1) An inferior good has a negative income elasticity of demand. (2) A good with a negative income elasticity of demand is an inferior good.

**The Effects of a Change in Price or Income: An Algebraic Approach**

So far in this chapter, we have used a graphical approach to show how the amount of a good consumed depends on the levels of prices and income. We have shown how to find the shape of the demand curve when the consumer has a given level of income (as in Figure 5.1), and how the demand curve shifts as the level of income changes (as in Figure 5.2).

We can also describe the demand curve algebraically. In other words, given a utility function and a budget constraint, we can find the equation of the consumer’s demand curve. The next two exercises illustrate this algebraic approach.

LEARNING-BY-DOING EXERCISE 5.2

Finding a Demand Curve (No Corner Points)

A consumer purchases two goods, food and clothing. The utility function is \( U(x, y) = xy \), where \( x \) denotes the amount of food consumed and \( y \) the amount of clothing. The marginal utilities are \( MU_x = y \) and \( MU_y = x \). The price of food is \( P_x \), the price of clothing is \( P_y \), and income is \( I \).

Problem

(a) Show that the equation for the demand curve for food is \( x = I/(2P_x) \).

(b) Is food a normal good? Draw \( D_1 \), the consumer’s demand curve for food when the level of income is \( I = $120 \). Draw \( D_2 \), the demand curve when \( I = $200 \).

Solution

(a) In Learning-By-Doing Exercise 3.3, we learned that the indifference curves for the utility function \( U(x, y) = xy \) are bowed in toward the origin and do not intersect the axes. So any optimal basket must be interior, that is, the consumer buys positive amounts of both food and clothing.

How do we determine the optimal choice of food? We know that an interior optimum must satisfy two conditions:

- An optimal basket will be on the budget line. This means that equation (4.1) must hold: \( P_x x + P_y y = I \).

FIGURE 5.5 Demand Curves for Food at Different Income Levels

The quantity of food demanded, \( x \), depends on the price of food, \( P_x \), and on the level of income, \( I \). The equation representing the demand for food is \( x = I/(2P_x) \). When income is \( $120 \), the demand curve is \( D_1 \) in the graph. Thus, if the price of food is \( $15 \), the consumer buys 4 units of food (point A). If the price of food drops to \( $10 \), she buys 6 units of food (point B). If income rises to \( $200 \), the demand curve shifts to the right, to \( D_2 \). In this case, if the price of food is \( $10 \), the consumer buys 10 units of food (point C).
Since the optimum is interior, the tangency condition, equation (4.3), must also hold: $\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$, or, with the marginal utilities given, $y/x = P_x/P_y$, or $y = (P_x/P_y)x$.

We can now solve for $x$ by substituting $y = (P_x/P_y)x$ into the equation for the budget line $P_x x + P_y y = I$. This gives us:

$$P_x x + P_y \left(\frac{P_x}{P_y}x\right) = I$$

or $x = I/(2P_x)$.

This is the equation of the demand curve for food. Given the consumer’s income and the price of food, we can easily find the quantity of food the consumer will purchase.

(b) If income is $120, the equation of the demand curve for food $D_1$ will be $x = 120/(2P_x) = 60/P_x$. We can plot points on the demand curve, as we have done in Figure 5.5.

An increase in income to $200 shifts the demand curve rightward to $D_2$, with the equation $x = 200/(2P_x) = 100/P_x$. Thus, food is a normal good.

**Similar Problems:** 5.5 and 5.7

The solution to part (a) of this exercise starts out looking very much like the solution to Learning-By-Doing Exercise 4.2, where we were interested in finding the optimal consumption of food and clothing given a specific set of prices and level of income. Learning-By-Doing Exercise 5.2, however, goes farther. By using the exogenous variables ($P_x$, $P_y$, and $I$) instead of actual numbers, we find the equation of the demand curve, which lets us determine the quantity of food demanded for any price and income.

**LEARNING-BY-DOING EXERCISE 5.3**

**Finding a Demand Curve (with a Corner Point Solution)**

A consumer purchases two goods, food and clothing. He has the utility function $U(x, y) = xy + 10x$, where $x$ denotes the amount of food consumed and $y$ the amount of clothing. The marginal utilities are $MU_x = y + 10$ and $MU_y = x$. The consumer’s income is $100, and the price of food is $1. The price of clothing is $P_y$.

**Problem**  Show that the equation for the consumer’s demand curve for clothing is

$$y = \frac{100 - 10P_y}{2P_y}, \text{ when } P_y < 10$$

$$y = 0, \text{ when } P_y \geq 10$$

Use this equation to fill in the following table to show how much clothing he will purchase at each price of clothing (these are points on his demand curve):

<table>
<thead>
<tr>
<th>$P_y$</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solution  In Learning-By-Doing Exercise 4.3, we learned that the indifference curves for the utility function \( U(x, y) = xy + 10x \) are bowed in toward the origin. They also intersect the x axis, since the consumer could have a positive level of utility with purchases of food \((x > 0)\) but no purchases of clothing \((y = 0)\). So he might not buy any clothing (i.e., choose a corner point) if the price of clothing is too high.

How do we determine the consumer’s optimal choice of clothing? If he is at an interior optimum, we know that his optimal basket will be on the budget line. This means that equation (4.1) must hold with the price of \( x \) and income given:

\[ x + P_y y = 100. \]

At an interior optimum, the tangency condition as expressed in equation (4.4) must also hold:

\[ \frac{MU_x}{MU_y} = \frac{P_x}{P_y}, \]

or with the marginal utilities given,

\[ \frac{(y + 10)}{x} = \frac{1}{P_y}, \]

or more simply,

\[ x = P_y y + 10P_y. \]

We can now solve for \( y \) by substituting \( x = P_y y + 10P_y \) into the equation for the budget line \( x + P_y y = 100 \). This gives us

\[ 2P_y y + 10P_y = 100, \]

or

\[ y = \frac{(100 - 10P_y)}{(2P_y)}. \]

Note that the value of this equation for the consumer’s demand curve for clothing is positive when \( P_y < 10 \). But if \( P_y \geq 10 \), then \( 100 - 10P_y \) is zero or negative, and the consumer will demand no clothing (in effect, \( y = 0 \) when \( P_y \geq 10 \), since the consumer can’t demand negative amounts of clothing). In other words, when \( P_y \geq 10 \) the consumer will be at a corner point at which he buys only food.

Using the equation for the demand curve, we can complete the table as follows:

<table>
<thead>
<tr>
<th>( P_y )</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>20</td>
<td>7.5</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Similar Problem:  5.14

In the previous section, we analyzed the overall effect of a change in the price of a good. Here, we refine our analysis by breaking this effect down into two components—a substitution effect and an income effect:

- When the price of a good falls, the good becomes cheaper relative to other goods. Conversely, a rise in price makes the good more expensive relative to other goods. In either case, the consumer experiences the substitution effect—the change in the quantity of the good the consumer would purchase after the price change to achieve the same level of utility. For example, if the price of food falls, the consumer can achieve the same level of utility by substituting food for other goods (i.e., by buying more food and less of other goods); similarly, if the price of food rises, the consumer may substitute other goods for food to achieve the same level of utility.

- When the price of a good falls, the consumer’s purchasing power increases, since the consumer can now buy the same basket of goods as before the price decrease and still have money left over to buy more goods. Conversely, a rise in price decreases the consumer’s purchasing power (i.e., the consumer can no longer afford to buy the same basket of goods). This change in purchasing power is termed the income effect because it affects the consumer in much the same way as a change in income would; that is, the consumer realizes a higher or lower level of utility because of the increase or decrease in purchasing power and therefore purchases a higher or lower amount of the good whose price has changed. The income effect accounts for the part of the total difference in the quantity of the good purchased that isn’t accounted for by the substitution effect.
The substitution effect and the income effect occur at the same time when the price of a good changes, resulting in an overall movement of the consumer from an initial basket (before the price change) to a final basket (after the price change). To better understand this overall effect of a price change, we will show how to break it down (decompose it) into its two components—the substitution effect and the income effect.

In the following sections, we perform this analysis in relation to price decreases. (Learning-By-Doing Exercise 5.5, on page 155, shows a corresponding analysis in relation to a price increase.)

**THE SUBSTITUTION EFFECT**

Suppose that a consumer buys two goods, food and clothing, that both goods have a positive marginal utility, and that the price of food decreases. The substitution effect is the amount of additional food the consumer would buy to achieve the same level of utility. Figure 5.6 shows three optimal choice diagrams that illustrate the steps involved in finding the substitution effect associated with this price change.

**Step 1.** Find the initial basket (the basket the consumer chooses at the initial price \(P_x\)). As shown in Figure 5.6(a), when the price of food is \(P_x\), the consumer faces budget line \(BL_1\) and maximizes utility by choosing basket \(A\) on indifference curve \(U_1\). The quantity of food she purchases is \(x_A\).

**Step 2.** Find the final basket (the basket the consumer chooses after the price falls to \(P_x\)). As shown in Figure 5.6(b), when the price of food falls to \(P_x\), the budget line rotates outward to \(BL_2\), and the consumer maximizes utility by choosing basket \(C\) on indifference curve \(U_2\). The quantity of food she purchases is \(x_C\). Thus, the overall effect of the price change on the quantity of food purchased is \(x_C - x_A\). Predictably, the consumer realizes a higher level of utility as a result of the price decrease, as shown by the fact that the initial basket \(A\) lies inside the new budget line \(BL_2\).

**Step 3.** Find an intermediate decomposition basket that will enable us to identify the portion of the change in quantity due to the substitution effect. We can find this basket by keeping two things in mind. First, the decomposition basket reflects the price decrease, so it must lie on a budget line that is parallel to \(BL_2\). Second, the decomposition basket reflects the assumption that the consumer achieves the initial level of utility after the price decrease, so the basket must be at the point where the budget line is tangent to indifference curve \(U_1\). As shown in Figure 5.6(c), these two conditions are fulfilled by basket \(B\) (the decomposition basket) on budget line \(BL_d\) (the decomposition budget line). At basket \(B\), the consumer purchases the quantity of food \(x_B\). Thus, the substitution effect accounts for the consumer’s movement from basket \(A\) to basket \(B\)—that is, the portion of the overall effect on the quantity of food purchased that can be attributed to the substitution effect is \(x_B - x_A\).

**THE INCOME EFFECT**

Still looking at Figure 5.6, suppose the consumer has income \(I\). When the price of food is \(P_x\), she can buy any basket on \(BL_1\), and when the price of food is \(P_x\), she can buy any basket on \(BL_2\). Note that the decomposition budget line \(BL_d\) lies inside \(BL_2\), which means that the income \(I_d\) that would be needed to buy a basket on \(BL_d\) is less...
Step 1: Find the initial basket $A$.
Slope of $BL_1 = -\frac{P_x}{P_y}$

Step 2: Find the final basket $C$.
Slope of $BL_1 = -\frac{P_x}{P_y}$
Slope of $BL_2 = -\frac{P_x}{P_y}$

Step 5: Find the decomposition basket $B$.
Slope of $BL_1 = -\frac{P_x}{P_y}$
Slope of $BL_2 = -\frac{P_x}{P_y}$
Slope of $BL_d = -\frac{P_x}{P_y}$

FIGURE 5.6 Income and Substitution Effects: Case 1 ($x$ is a Normal Good)
As the price of food drops from $P_x$ to $P_x$, the substitution effect leads to an increase in the amount of food consumed from $x_A$ to $x_B$ (so the substitution effect is $x_B - x_A$). The income effect also leads to an increase in food consumption, from $x_A$ to $x_C$ (so the income effect is $x_C - x_A$). The overall increase in food consumption is $x_C - x_B$. When a good is normal, the income and substitution effects reinforce each other.
than the income $I$ needed to buy a basket on $BL_2$. Also note that basket $A$ (on $BL_1$) and basket $B$ (on $BL_d$) are on the same indifference curve $U_1$ (i.e., the consumer would be equally satisfied by baskets $A$ and $B$), which means that the consumer would be indifferent between the following two situations: (1) having a higher income $I$ when the price of food is higher at $P_1$ (i.e., buying basket $A$) and (2) having a lower income $I_d$ when the price of food is lower at $P_2$ (i.e., buying basket $B$). Another way of saying this is that the consumer would be willing to have her income reduced to $I_d$ if she can buy food at the lower price $P_2$.

With this in mind, let’s find the income effect, the change in the amount of a good consumed as the consumer’s utility changes. In the example illustrated by Figure 5.6, the movement from basket $A$ to basket $B$ (i.e., the movement due to the substitution effect) doesn’t involve any change in utility, and as we have just seen, we can view this movement as the result of a reduction in income from $I$ to $I_d$ as the price falls from $P_1$ to $P_2$. In reality, however, the consumer’s income doesn’t fall when the price of food decreases, so her level of utility increases, and we account for this by “restoring” the “lost” income. When we do this, the budget line shifts from $BL_d$ to $BL_2$, and the consumer’s optimal basket shifts from basket $B$ (on $BL_d$) to basket $C$ (on $BL_2$). Thus, the income effect accounts for the consumer’s movement from the decomposition basket $B$ to the final basket $C$—that is, the portion of the overall effect on the quantity of food purchased that can be attributed to the income effect is $x_C - x_B$.

In sum, when the price of food falls from $P_1$ to $P_2$, the total change on food consumption is $(x_C - x_A)$. This can be decomposed into the substitution effect $(x_B - x_A)$ and the income effect $(x_C - x_B)$. When we add the substitution effect and the income effect, we get the total change in consumption.

**INCOME AND SUBSTITUTION EFFECTS WHEN GOODS ARE NOT NORMAL**

As we noted earlier, the graphs in Figure 5.6 are drawn for the case (we call it Case 1) in which food is a normal good. As the price of food falls, the income effect leads to an increase in food consumption. Also, because the marginal rate of substitution is diminishing, the substitution effect leads to increased food consumption as well. Thus, the income and substitution effects work in the same direction. The demand curve for food will be downward sloping because the quantity of food purchased will increase when the price of food falls. (Similarly, if the price of food were to rise, both effects would be negative. At a higher price of food, the consumer would buy less food.)

However, the income and substitution effects do not always work in the same direction. Consider Case 2, in Figure 5.7 (instead of drawing three graphs like those in Figure 5.6, we have only drawn the final graph [like Figure 5.6(c)] with the initial, final, and decomposition baskets). Note that basket $C$, the final basket, lies directly above basket $B$, the decomposition basket. As the budget line shifts out from $BL_d$ to $BL_2$, the quantity of food consumed does not change. The income effect is therefore zero $(x_C - x_B = 0)$. Here a decrease in the price of food leads to a positive substitution effect on food consumption $(x_B - x_A > 0)$ and a zero income effect. The demand curve for food will still be downward sloping because more food is purchased at the lower price $(x_C - x_A > 0)$.

The income and substitution effects might even work in opposite directions, as in Case 3, in Figure 5.8, where food is an inferior good. When a good is inferior, the indifference curves will show that the income effect is negative (i.e., the final basket $C$
FIGURE 5.7  Income and Substitution  
Effects: Case 2 (x Is Neither a Normal Good  
or an Inferior Good)  
As the price of food drops from $P_x$ to $P_x$,  
the substitution effect leads to an increase in  
the amount of food consumed from $x_A$ to $x_B$  
(so the substitution effect is $x_B - x_A$). The  
income effect on food consumption is zero  
because $x_B$ is the same as $x_C$ (so the income  
effect is $x_C - x_B = 0$). The overall effect on  
food consumption is $x_C - x_A$.

will be to the left of the decomposition basket $B$; as the budget line shifts out from  
$BL_d$ to $BL_2$, the quantity of food consumed decreases ($x_C - x_B < 0$). In contrast, the  
substitution effect is still positive ($x_B - x_A > 0$). In this case, because the substitution  
effect is larger than the income effect, the total change in the quantity of food con-  
sumed is also still positive ($x_C - x_A > 0$), and, therefore, the demand curve for food  
will still be downward sloping.

FIGURE 5.8  Income and Substitution  
Effects: Case 3 (x Is an Inferior Good) with  
a Downward-Sloping Demand Curve  
As the price of food drops from $P_x$ to $P_x$,  
the substitution effect leads to an increase in  
the amount of food consumed from $x_A$ to $x_B$  
(so the substitution effect is $x_B - x_A$). The  
income effect on food consumption is neg-  
ative ($x_C - x_B < 0$). The overall effect on  
food consumption is $x_C - x_A > 0$. When a  
good is inferior, the income and substitution  
effects work in opposite directions.
Case 4, in Figure 5.9, illustrates the case of a so-called Giffen good. In this case, the indifference curves indicate that food is a strongly inferior good, with the final basket C lying not only to the left of the decomposition basket B, but also to the left of the initial basket A. The income effect is so strongly negative that it more than cancels out the positive substitution effect.

What about the demand curve for food in the case illustrated by Figure 5.9? When the price of food drops from $P_{x1}$ to $P_{x2}$, the quantity of food actually decreases from $x_A$ to $x_C$, so the substitution effect is $x_B - x_A$. The income effect on food consumption is negative ($x_C - x_B < 0$). The overall effect on food consumption is $x_C - x_A < 0$.

As we have already noted, some goods are inferior over some price ranges for some consumers. For instance, your consumption of hot dogs may fall if your income rises, if you decide to eat more steaks and fewer hot dogs. But expenditures on inferior goods typically represent only a small part of a consumer’s income. Income effects for individual goods are usually not large, and the largest income effects are usually associated with goods that are normal rather than inferior, such as food and housing. For an inferior good to have an income effect large enough to offset the substitution effect, the income elasticity of demand would have to be negative and the expenditures on the good would need to represent a large part of the consumer’s budget. Thus, while the Giffen good is intriguing, it is not of much practical concern.

While researchers have not yet found data that confirm the existence of a Giffen good for human beings, some economists have suggested that the Irish potato famine (see Application 5.2) came close to creating the right environment. However, as Joel Mokyr observed, “For people with a very low income, potatoes might have well been a normal good. But consumers with higher levels of income could afford other types of food, and therefore consumed fewer potatoes.” Thus, while expenditures on potatoes did constitute a large part of consumer expenditures, potatoes may not have been inferior at low incomes. This may explain why researchers have not shown the potato to have been a Giffen good at that time.
Finding Income and Substitution Effects Algebraically

In Learning-By-Doing Exercises 4.2 and 5.2, we met a consumer who purchases two goods, food and clothing. He has the utility function $U(x, y) = xy$, where $x$ denotes the amount of food consumed and $y$ the amount of clothing. His marginal utilities are $MU_x = y$ and $MU_y = x$. Now suppose that he has an income of $72 per week and that the price of clothing is $P_y = $1 per unit. Suppose that the price of food is initially $P_{x1} = $9 per unit, and that the price subsequently falls to $P_{x2} = $4 per unit.

**Problem** Find the numerical values of the income and substitution effects on food consumption, and graph the results.

---

Solution  To find the income and substitution effects, we follow the procedure explained earlier in this section.

**Step 1. Find the initial consumption basket \( A \) when the price of food is $9.** We know that two conditions must be satisfied at an optimum. First, an optimal basket will be on the budget line. This means that \( P_x x + P_y y = I \), or with the given information, \( 9x + y = 72 \).

Second, since the optimum is interior, the tangency condition must hold. From equation (4.3), we know that at a tangency, \( \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \), which with the given information, simplifies to \( y = 9x \).

When we solve these two equations with two unknowns, we find that \( x = 4 \) and \( y = 36 \). So at basket \( A \) the consumer purchases 4 units of food and 36 units of clothing each week.

**Step 2. Find the final consumption basket \( C \) when the price of food is $4.** We repeat step 1, but now with the price of a unit of food of $4, which again yields two equations with two unknowns:

\[
\begin{align*}
4x + y & = 72 \quad \text{(coming from the budget line)} \\
y & = 4x \quad \text{(coming from the tangency condition)}
\end{align*}
\]

---

**APPLICATION 5.4**

**Mexican Tortillas Are Good, but Are They a Giffen Good?**

Corn tortillas are the main staple of the Mexican diet, with the average citizen consuming about 220 pounds annually. In the late 1990s, the price of tortillas in Mexico skyrocketed following the phase-out of government subsidies and the removal of price controls. For example, between 1996 and 1998, the price of tortillas increased more than 40 percent. This increase was especially hard on Mexico’s poorest households, who are the largest consumers of tortillas, and some lamented that the increased cost of tortillas crowded out opportunities to purchase other foods. Lamented one consumer, “I have no choice but to buy more tortillas and less meat, chicken, and vegetables” (emphasis added).

This quote raises the possibility that corn tortillas in Mexico might be an example of a Giffen good. A study by economist David McKenzie explores this question by examining how Mexican households adjusted their consumption of tortillas in response to changes in tortilla prices and income. McKenzie’s data are drawn from the late 1990s, a period that provides potentially fertile ground for identifying a Giffen good. Not only had tortilla prices increased dramatically in this period, but Mexican incomes also dropped significantly as a result of the peso crisis of 1994–1996. In 1995 alone, Mexico’s real GNP fell over 9 percent. One would imagine that changes in prices and incomes of the magnitude experienced by Mexican consumers would be bound to have an effect on household consumption behavior.

However, McKenzie was unable to find evidence that tortillas were a Giffen good. He did find strong evidence that tortillas were an *inferior* good. In fact, the Engel curve for tortillas estimated by McKenzie resembled the curve shown in Figure 5.4(b), with tortillas appearing to be a normal good at extremely low levels of income but quickly becoming inferior for high levels of income. However, his analysis found that increased tortilla prices had a significantly negative impact on tortilla consumption, after controlling for other factors that might also have influenced demand, including changes in income and demographics. This was true for all households taken together, as well as for households at various income levels.

---


When we solve these two equations, we find that $x = 9$ and $y = 36$. So at basket $C$, the consumer purchases 9 units of food and 36 units of clothing each week.

**Step 3. Find the decomposition basket $B$.** The decomposition basket must satisfy two conditions. First, it must lie on the original indifference curve $U_1$ along with basket $A$. Recall that this consumer’s utility function is $U(x, y) = xy$, so at basket $A$, utility $U_1 = 4(36) = 144$. At basket $B$ the amounts of food and clothing must also satisfy $xy = 144$. Second, the decomposition basket must be at the point where the decomposition budget line is tangent to the indifference curve. Remember that the price of food $P_x$ on the decomposition budget line is the final price of $4$. The tangency occurs when $MU_x / MU_y = P_x / P_y$, that is, when $y/x = 4/1$, or $y = 4x$. When we solve the two equations $xy = 144$ and $y = 4x$, we find that, at the decomposition basket, $x = 6$ units of food and $y = 24$ units of clothing.

Now we can find the income and substitution effects. The substitution effect is the increase in food purchased as the consumer moves along initial indifference curve $U_1$ from basket $A$ (at which he purchases 4 units of food) to basket $B$ (at which he purchases 6 units of food). The substitution effect is therefore $6 - 4 = 2$ units of food.

The income effect is the increase in food purchased as he moves from basket $B$ (at which he purchases 6 units of food) to basket $C$ (at which he purchases 9 units of food). The income effect is therefore $9 - 6 = 3$ units of food.

Figure 5.10 graphs the income and substitution effects. In this exercise food is a normal good. As expected, the income and substitution effects have the same sign. The consumer’s demand curve for food is downward sloping because the quantity of food he purchases increases when the price of food falls.
5.2 CHANGE IN THE PRICE OF A GOOD: SUBSTITUTION EFFECT AND INCOME EFFECT

To this point, all our discussions and examples of the substitution and income effects have been in relation to price decreases. Learning-By-Doing Exercise 5.5 shows how these effects work with a price increase.

LEARNING-BY-DOING EXERCISE 5.5

Income and Substitution Effects with a Price Increase

The indifference curves in Figure 5.11 depict a consumer’s preferences for housing \( x \) and a composite good \( y \). The consumer’s marginal utilities for both goods are positive.

Problem On the graph, show what the income and substitution effects on housing would be if the current price of housing were to increase so that the consumer’s budget line shifted from \( BL_1 \) to \( BL_2 \).

Solution At the initial price of housing, the consumer’s budget line is \( BL_1 \) and the consumer’s optimal basket is \( A \). This enables the consumer to reach indifference curve \( U_1 \). When the price of housing increases, the consumer’s budget line is \( BL_2 \). The consumer purchases basket \( C \) and reaches the indifference curve \( U_2 \).

FIGURE 5.11 Income and Substitution Effects with a Price Increase
At the initial basket \( A \) on budget line \( BL_1 \), the consumer purchases \( x_A \) units of food. At the final basket \( C \) on budget line \( BL_2 \), the consumer purchases \( x_C \) units of food. At the decomposition basket \( B \) on budget line \( BL_d \), the consumer purchases \( x_B \) units of food. The substitution effect is \( x_B - x_A \). The income effect is \( x_C - x_B \).
To draw the decomposition budget line $BL_d$, remember that $BL_d$ is parallel to the final budget line $BL_2$ and that the decomposition basket $B$ is located where $BL_d$ is tangent to the initial indifference curve $U_1$. (Students often err by placing the decomposition basket on the final indifference curve instead of on the initial indifference curve.) As we move from the initial basket $A$ to the decomposition basket $B$, housing consumption decreases from $x_A$ to $x_B$. The substitution effect is therefore $x_B - x_A$. The income effect is measured by the change in housing consumption as the consumer moves from the decomposition basket $B$ to the final basket $C$. The income effect is therefore $x_C - x_B$.

Similar Problem:  5.8

LEARNING-BY-DOING EXERCISE 5.6

Income and Substitution Effects with a Quasi-Linear Utility Function

A college student who loves chocolate has a budget of $10 per day, and out of that income she purchases chocolate $x$ and a composite good $y$. The price of the composite good is $1.

The quasi-linear utility function $U(x, y) = 2\sqrt{x} + y$ represents the student’s preferences. (See Chapter 3 for discussion of this kind of utility function.) For this utility function, $MU_x = 1/\sqrt{x}$ and $MU_y = 1$.

Problem

(a) Suppose the price of chocolate is initially $0.50 per ounce. How many ounces of chocolate and how many units of the composite good are in the student’s optimal consumption basket?

(b) Suppose the price of chocolate drops to $0.20 per ounce. How many ounces of chocolate and how many units of the composite good are in the optimal consumption basket?

(c) What are the substitution and income effects that result from the decline in the price of chocolate? Illustrate these effects on a graph.

Solution

(a) At an interior optimum, $MU_x/MU_y = P_x/P_y$, or $1/\sqrt{x} = P_x$. The student’s demand curve for chocolate is therefore $x = 1/(P_x)^2$. When the price of chocolate is $0.50 per ounce, she buys $1/(0.5)^2 = 4$ ounces of chocolate per day.

We can find the number of units of the composite good from the equation for the budget line, $P_x x + P_y y = I$. With the information given, the budget line equation is $(0.5)(4) + (1)y = 10$, so the student buys $y = 8$ units of the composite good.

(b) We use the consumer’s demand curve for chocolate from part (a) to find her demand for chocolate when the price falls to $0.20 per ounce. She buys $x = 1/(0.2)^2 = 25$ ounces of chocolate at the lower price. Her budget line equation now becomes $(0.2)(25) + (1)y = 10$, so she buys $y = 5$ units of the composite good.

(c) In the first two parts of this problem we found all we need to know about the initial basket $A$ and the final basket $C$. Figure 5.12 shows these baskets.

To find the income and substitution effects, we need to find the decomposition basket $B$. We know two things about basket $B$. First, the consumer’s utility at basket $B$ must be the same as at the initial basket $A$, where $x = 4$, $y = 8$, and, therefore, utility is $U_1 = 2\sqrt{4} + 8 = 12$. Thus, at basket $B$, $2\sqrt{x} + y = 12$. Second, the slope of the decomposition budget line at
5.2 CHANGE IN THE PRICE OF A GOOD: SUBSTITUTION EFFECT AND INCOME EFFECT

FIGURE 5.12 Income and Substitution Effects with a Quasi-Linear Utility Function

At the initial basket $A$ on budget line $BL_1$, the consumer purchases 4 ounces of chocolate at a price of $0.50$ per ounce. At the final basket $C$ on budget line $BL_2$, the consumer purchases 25 ounces of chocolate at a price of $0.20$ per ounce. At the decomposition basket $B$ on budget line $BL_d$, the consumer also purchases 25 ounces of chocolate at a price of $0.20$ per ounce. The substitution effect is $25 - 4 = 21$ ounces. The income effect is $25 - 25 = 0$ ounces.

Basket $B$ must be the same as the slope of the final budget line at basket $C$—that is, $MU_x/MU_y = P_x/P_y$. Given that $MU_x = 1/\sqrt{x}$, that $MU_y = 1$, and that, at basket $C$, $P_x = 0.20$ and $P_y = 1$, this equation simplifies to $1/\sqrt{x} = 0.20$. When we solve these two equations with two unknowns, we find that at basket $B$, $x = 25$ and $y = 2$. Basket $B$ is also shown on Figure 5.12.

The substitution effect is the change in the quantity of chocolate purchased as the consumer moves from the initial basket $A$ (where she consumes 4 ounces of chocolate) to the decomposition basket $B$ (where she consumes 25 ounces of chocolate). The substitution effect on chocolate is therefore $25 - 4 = 21$ ounces. The income effect is the change in the quantity of chocolate purchased as the consumer moves from the decomposition basket $B$ to the final basket $C$. Because she consumes the same amount of chocolate at $B$ and $C$, the income effect is zero.

Similar Problem: 5.8

Learning-By-Doing Exercise 5.6 illustrates one of the properties of a quasi-linear utility function with a constant marginal utility of $y$ and indifference curves that are bowed in toward the origin. When prices are constant, at an interior optimum the
consumer will purchase the same amount of $x$ as income varies. In other words, the income consumption curve will be a vertical line in the graph, and the income effect associated with a price change on $x$ will be zero, as in Figure 5.7.

**Consumer surplus** is the difference between the maximum amount a consumer is willing to pay for a good and the amount he must actually pay to purchase the good in the marketplace. Thus, it measures how much better off the consumer will be when he purchases the good and can, therefore, be a useful tool for representing the impact of a price change on consumer well-being. In this section, we will view this impact from two different perspectives: first, by looking at the demand curve, and second, by looking at the optimal choice diagram.

### UNDERSTANDING CONSUMER SURPLUS FROM THE DEMAND CURVE

In the previous section, we saw how changes in price affect consumer decision making and utility in cases where we know the utility function. If we do not know the utility function, but do know the equation for the demand curve, we can use the concept of consumer surplus to measure the impact of a price change on the consumer.

Let’s begin with an example. Suppose you are considering buying a particular automobile and that you are willing to pay up to $15,000 for it. But you can buy that automobile for $12,000 in the marketplace. Because the amount you are willing to pay exceeds the amount you actually have to pay, you will buy it. When you do, you will have a consumer surplus of $3,000 from that purchase. Your consumer surplus is your net economic benefit from making the purchase, that is, the maximum amount you would be willing to pay ($15,000) less the amount you actually pay ($12,000).

Of course, for many types of commodities you might want to consume more than one unit. You will have a demand curve for such a commodity, which, as we have already pointed out, represents your willingness to pay for the good. For example, suppose you like to play tennis, and that you must rent the tennis court for an hour each time you play. Your demand curve for court time appears in Figure 5.13. It shows that you would be willing to pay up to $25 for the first hour of court time each month, $23 for the second hour, $21 for the third hour, and so on. Your demand curve is downward sloping because you have a diminishing marginal utility for playing tennis.

Suppose you must pay $10 per hour to rent the court. At that price your demand curve indicates that you will play tennis for 8 hours during the month, because you are willing to pay $11 for the eighth hour, but only $9 for the ninth hour, and even less for additional hours.

How much consumer surplus do you get from playing tennis 8 hours each month? To find out, you add the surpluses from each of the units you consume. Your consumer surplus from the first hour is $15 (the $25 you are willing to pay minus the $10 you actually must pay). The consumer surplus from the second hour is $13. The consumer surplus from using the court for the 8 hours during the month is then $64 (the sum of the consumer surpluses for each of the 8 hours, or $15 + $13 + $11 + $9 + $7 + $5 + $3 + $1).

As the example illustrates, the consumer surplus is the area below the demand curve and above the price that the consumer must pay for the good. We represented the demand curve here as a series of “steps” to help us illustrate the consumer surplus.
from each unit purchased. In reality, however, a demand curve will usually be smooth and can be represented as an algebraic equation. The concept of consumer surplus is the same for a smooth demand curve.

As we shall show, the area under a demand curve exactly measures net benefits for a consumer only if the consumer experiences no income effect over the range of price change. This may often be a reasonable assumption, but if it is not satisfied, then the area under the demand curve will not measure the consumer’s net benefits exactly. For the moment, let’s assume that there is no income effect, so we need not worry about this complication.

**LEARNING-BY-DOING EXERCISE 5.7**

**Consumer Surplus: Looking at the Demand Curve**

Suppose the equation \( Q = 40 - 4P \) represents a consumer’s monthly demand curve for milk, where \( Q \) is the number of gallons of milk purchased when the price is \( P \) dollars per gallon.

**Problem**

(a) What is the consumer surplus per month if the price of milk is \$3 per gallon?

(b) What is the increase in consumer surplus if the price falls to \$2 per gallon?

**Solution**

(a) Figure 5.14 shows the demand curve for milk. When the price is \$3, the consumer will buy 28 gallons of milk. The consumer surplus is the area under the demand curve and above the price of \$3—that is, the area of triangle \( G \), or \((1/2)(10 - 3)(28) = \$98\).

(b) If the price drops from \$3 to \$2, the consumer will buy 32 gallons of milk. Consumer surplus will increase by the areas \( H \) (\$28) and \( I \) (\$2), or by \$30. The total consumer surplus will now be \$128 (\( G + H + I \)).
CHAPTER 5 THE THEORY OF DEMAND

When the price of milk is $3 per gallon, consumer surplus = area of triangle G = $98. If the price drops to $2 per gallon, the increase in consumer surplus = sum of areas H ($28) and I ($2) = $30. Total consumer surplus when the price is $2 per gallon = $98 + $30 = $128.

UNDERSTANDING CONSUMER SURPLUS FROM THE OPTIMAL CHOICE DIAGRAM: COMPENSATING VARIATION AND EQUIVALENT VARIATION

We have shown how a price change affects the level of utility for a consumer. However, there is no natural measure for the units of utility. Economists therefore often measure the impact of a price change on a consumer’s well-being in monetary terms. How can we estimate the monetary value that a consumer would assign to a change in the price of a good? In this section, we use optimal choice diagrams to study two equally valid ways of answering this question:

- First, we see how much income the consumer would be willing to give up after a price reduction, or how much additional income the consumer would need after a price increase, to maintain the level of utility she had before the price change. We call this change in income the **compensating variation** (because it is the change in income that would exactly compensate the consumer for the impact of the price change).
- Second, we see how much additional income the consumer would need before a price reduction to be as well off as after the price decrease. We call this change in income the **equivalent variation** (because it is the change in income that would be equivalent to the price change in its impact on the consumer).

The optimal choice diagram shown in Figure 5.15 illustrates a case where the consumer buys two goods, food $x$ and clothing $y$. The price of clothing is $1. The price of food is initially $P_{x1}$ and then decreases to $P_{x2}$. With the consumer’s income remaining...
fixed, the budget line moves from $BL_1$ to $BL_2$ and the consumer’s optimal basket moves from $A$ to $C$.

The compensating variation is the difference between the income necessary to buy basket $A$ at the initial price $P_x$, and the income necessary to buy the decomposition basket $B$ at the new price $P_x$. Basket $B$ lies at the point where a line parallel to the final budget line $BL_2$ is tangent to the initial indifference curve $U_1$.

The equivalent variation is the difference between the income necessary to buy basket $A$ at the initial price $P_x$, and the income necessary to buy basket $E$ at the initial price $P_x$. Basket $E$ lies at the point where a line parallel to the initial budget line $BL_1$ is tangent to the final indifference curve $U_2$.

In graphical terms, the compensating and equivalent variations are simply two different ways of measuring the distance between the initial and final indifference curves. Since the price of clothing $y$ is $1$, the segment $OK$ measures the consumer’s income. The segment $OL$ measures the income needed to buy basket $B$ at the new price of food $P_x$. The difference (the segment $KL$) is the compensating variation. Baskets $B$ and $A$ are on the same indifference curve $U_1$, so the consumer would accept a reduction in income of $KL$ if she could buy food at the lower price.

To find the equivalent variation, note that, as before, the segment $OK$ measures the consumer’s income because $P_y = 1$. The segment $OJ$ measures the income needed to buy basket $E$ at the old price of food $P_x$. The difference (the segment $JK$) is the equivalent variation. Baskets $E$ and $C$ are on the same indifference curve $U_1$, so the consumer would require an increase in income of $JK$ to be equally well off buying food at the initial higher price as at the lower final price.

In general the sizes of the compensating variation (the segment $KL$) and the equivalent variation (the segment $JK$) will not be the same, because the price change would have a non-zero income effect (in Figure 5.15, $C$ lies to the right of $B$, so the income effect is positive). That is why one must be careful when trying to measure the monetary value that a consumer associates with a price change.

However, as illustrated in Figure 5.16, if the utility function is quasi-linear, the compensating and equivalent variations will be the same, because the price change
would have a zero income effect (as we saw in Learning-By-Doing Exercise 5.6). Graphically, this is represented by the fact that the indifference curves associated with a quasi-linear utility function are parallel, which means that the vertical distance between any two curves is the same at all values of \( x \).\(^{10}\) Thus, in Figure 5.16, where basket \( C \) lies directly above basket \( B \), and basket \( E \) lies directly above basket \( A \), the compensating variation (\( KL \)) and equivalent variation (\( JK \)) are equal.

Furthermore, if there is no income effect, not only are the compensating variation and the equivalent variation equal to each other, they are also equal to the change in the consumer surplus (the change in the area under the demand curve as a result of the price change). This important point is illustrated by Learning-By-Doing Exercise 5.8 and the discussion following that exercise.

**LEARNING-BY-DOING EXERCISE 5.8**

**Compensating and Equivalent Variations with No Income Effect**

As in Learning-By-Doing Exercise 5.6, a student consumes chocolate and “other goods” with the quasi-linear utility function \( U(x, y) = 2\sqrt{x} + y \). She has an income of $10 per day, and the price of the composite good \( y \) is $1 per unit. For this

\(^{10}\)Suppose the utility function \( U(x, y) \) is quasi-linear, so that \( U(x, y) = f(x) + ky \), where \( k \) is some positive constant. Since \( U \) always increases by \( k \) units whenever \( y \) increases by 1 unit, we know that \( MU_y = k \). Therefore, the marginal utility of \( y \) is constant. For any given level of \( x \), \( \Delta U = k \Delta y \). So the vertical distance between indifference curves will be \( y_2 - y_1 = (U_2 - U_1)/k \). Note that this vertical distance between indifference curves is the same for all values of \( x \). That is why the indifference curves are parallel.
utility function, \( MU_x = \frac{1}{\sqrt{x}} \) and \( MU_y = 1 \). Suppose the price of chocolate is $0.50 per ounce and that it then falls to $0.20 per ounce.

**Problem**

(a) What is the compensating variation of the reduction in the price of chocolate?

(b) What is the equivalent variation of the reduction in the price of chocolate?

**Solution**

(a) Consider the optimal choice diagram in Figure 5.17. The compensating variation is the difference between her income ($10) and the income she would need to purchase the decomposition basket \( B \) at the new price of chocolate of $0.20. At basket \( B \) she buys 25 units of chocolate and 2 units of the composite good, so she would need \( P_x x + P_y y = (0.20)(25) + (1)(2) = 7 \). She would be willing to have her income reduced from $10 to $7 (a change of $3) if the price of chocolate falls from $0.50 to $0.20 per ounce. Thus, the compensating variation equals $3.

**FIGURE 5.17 Compensating and Equivalent Variations with No Income Effect**

The consumer’s income is $10, and the price of the composite good \( y \) is $1 per unit. When the price of chocolate is $0.50 per ounce, the consumer’s budget line is \( BL_1 \) and she buys basket \( A \), with utility \( U_1 = 12 \). After the price of chocolate falls to $0.20 per ounce, her budget line is \( BL_2 \) and she buys basket \( C \), with utility \( U_2 = 15 \). To reach utility \( U_1 \) after the price decrease, she could buy basket \( B \) for $7, so her compensating variation is $10 – $7 = $3. To reach utility \( U_2 \) before the price decrease, she could buy basket \( E \) for $13, so her equivalent variation is $13 – $10 = $3. When there is no income effect (as here, because the utility function is quasi-linear), the compensating variation and the equivalent variation are equal.
(b) In Figure 5.17, the equivalent variation is the difference between the income she would need to buy basket $E$ at the initial price of $0.50$ per ounce of chocolate and her actual income ($10$). To find the equivalent variation, we need to determine the location of basket $E$. We know that basket $E$ lies on the final indifference curve $U_2$, which has a value of $15$. Therefore, at basket $E$, $2\sqrt{x} + y = 15$. We also know that at basket $E$ the slope of the final indifference curve $U_2 (-MU_x/MU_y)$ must equal the slope of the initial budget line $BL_1 (-P_x/P_y)$, or $(1/\sqrt{x})/1 = 0.5/1$, which reduces to $x = 4$. When we substitute this value of $x$ into the equation $2\sqrt{x} + y = 15$, we find that $y = 11$. Thus, at basket $E$ the consumer purchases 4 units of chocolate and 11 units of the composite good. To purchase basket $E$ at the initial price of $0.50$ per ounce of chocolate, she would need an income of $P_x x + P_y y = 0.50(4) + 1(11) = 13$. The equivalent variation is the difference between this amount ($13$) and her income ($10$), or $3$. Thus, the equivalent variation and the compensating variation are equal.

**Similar Problem:** 5.20

Still considering the consumer in Learning-By-Doing Exercise 5.8, let’s see what happens if we try to measure the change in the consumer surplus by looking at the change in the area under her demand curve for chocolate. In Learning-By-Doing Exercise 5.6, we showed that her demand function for chocolate is $x = 1/(P_x)^2$. Figure 5.18 shows the demand curve for chocolate. As the price of chocolate falls from $0.50$ per ounce to $0.20$ per ounce, her daily consumption of chocolate rises from 4 ounces to 25 ounces. The shaded area in the figure illustrates the increase in consumer surplus as the price of chocolate falls. The size of that shaded area is $3$, exactly the same as both the compensating and equivalent variations. Thus, the change in the area under the demand curve exactly measures the monetary value of a price change when the utility function is quasi-linear (i.e., when there is no income effect).

As we have already noted, if there is an income effect, the compensating variation and equivalent variation will give us different measures of the monetary value that a
5.3 Change in the Price of a Good: The Concept of Consumer Surplus

A consumer would assign to the reduction in price of the good. Moreover, each of these measures will generally be different from the change in the area under the demand curve. However, if the income effect is small, the equivalent and compensating variations may be close to one another, and then the area under the demand curve will be a good approximation (although not an exact measure) of the compensating and equivalent variations.

Learning-by-Doing Exercise 5.9

Compensating and Equivalent Variations with an Income Effect

As in Learning-By-Doing Exercise 5.4, a consumer purchases two goods, food \( x \) and clothing \( y \). He has the utility function \( U(x, y) = xy \). He has an income of $72 per week, and the price of clothing is $1 per unit. His marginal utilities are \( MU_x = y \) and \( MU_y = x \). Suppose the price of food falls from $9 to $4 per unit.

Problem

(a) What is the compensating variation of the reduction in the price of food?

(b) What is the equivalent variation of the reduction in the price of food?

Solution

(a) Consider the optimal choice diagram in Figure 5.19. The compensating variation is the difference between his income ($72) and the income he would need to purchase the

![Figure 5.19](image)
decomposition basket $B$ at the new price of food of $4$. At basket $B$ he buys 6 units of food and 24 units of clothing, so he would need $P_x x + P_y y = 4(6) + 1(24) = 48$. The consumer would be willing to have his income reduced from $72 to $48 (a change of $24) if the price of food falls from $9 to $4. Therefore, the compensating variation associated with the price reduction is $24.

(b) In Figure 5.19, the equivalent variation is the difference between the income he would need to buy basket $E$ at the initial price of $9 per unit of food and his actual income ($72). To find the equivalent variation we need to determine the location of basket $E$. We know that basket $E$ lies on the final indifference curve $U_2$, which has a value of 324. Therefore, at basket $E, xy = 324$. We also know that at basket $E$ the slope of the final indifference curve $U_2$ ($-MU_x/MU_y$) must equal the slope of the initial budget line $BL_1$ ($-P_x/P_y$), or $y/x = 9/1$, which reduces to $y = 9x$. When we solve these two equations with two unknowns, we find that $x = 6$ and $y = 54$. Thus, at basket $E$ the consumer purchases 6 units of food and 54 units of clothing. To purchase basket $E$ at the initial price of $9 per unit of food, he would need income equal to $P_x x + P_y y = 9(6) + 1(54) = 108$. The equivalent variation is the difference between this amount ($108) and his income ($72), or $36. Thus, the equivalent variation ($36) and the compensating variation ($24) are not equal.

Still considering the consumer in Learning-By-Doing Exercise 5.9, let’s see what happens if we measure consumer surplus using the area under the demand curve for food. In Learning-By-Doing Exercise 5.4, we showed that his demand function for food is $x = I/(2P_x)$. Figure 5.20 shows his demand curve when his income is $72. As the price of food falls from $9 to $4 per unit, his consumption rises from 4 units to 9 units. The shaded area in Figure 5.20, which measures the increase in consumer surplus, equals $29.20. Note that this increase in consumer surplus ($29.20) is different from both the compensating variation ($24) and the equivalent variation ($36). Thus, the change in the area under the demand curve will not exactly measure either the compensating variation or the equivalent variation when the income effect is not zero.
In the previous sections of this chapter, we showed how to use consumer theory to derive the demand curve of an individual consumer. But business firms and policy makers are often more concerned with the demand curve for an entire market of consumers. Since markets might consist of thousands, or even millions, of individual consumers, where do market demand curves come from?

In this section, we illustrate an important principle: The market demand curve is the horizontal sum of the demands of the individual consumers. This principle holds whether two consumers, three consumers, or a million consumers are in the market.

Let’s work through an example of how to derive a market demand from individual consumer demands. To keep it simple, suppose only two consumers are in the market for orange juice. The first is “health conscious” and likes orange juice because of its nutritional value and its taste. In Table 5.1, the second column tells us how many liters of orange juice the health conscious consumer will buy at different prices.

### TABLE 5.1 Market Demand for Orange Juice

<table>
<thead>
<tr>
<th>Price ($/liter)</th>
<th>Health Conscious (Liters/Month)</th>
<th>Casual (Liters/Month)</th>
<th>Market Demand (Liters/Month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

The restrictions improved the profits of American producers by nearly $9 billion in 1984. However, the higher prices were not good news for consumers. The study estimated that the compensating variation resulting from the higher prices was about $14 billion. In other words, the amount of additional income domestic household vehicle owners would have required to be as well off at the higher prices was about $14 billion.

Instead of estimating the change in consumer surplus to study the impact of price changes in the market for automobiles, it is probably a good idea to measure compensating or equivalent variations. It is widely believed that the income elasticity of demand for automobiles is positive. Since the purchase of an automobile represents a large expenditure for most people, the income effect of a price change is probably significant. As we showed in Learning-By-Doing Exercise 5.9 and Figure 5.19, consumer surplus may not accurately measure the effect of a price change when the income effect is significant.

In 1981, the American automobile industry faced record financial losses, headlined by the Chrysler Corporation, which stood on the brink of bankruptcy. Facing pressure to help improve the health of the industry, the U.S. government negotiated voluntary export restrictions with the Japanese government, limiting the total annual sales of new Japanese passenger cars that could be sold in the United States.11

Clifford Winston and his colleagues examined the effects of the export restrictions on American producers and consumers. They found that by 1984 the restrictions significantly limited competition in the American market, resulting in prices for Japanese automobiles that were about 20 percent higher and prices for American automobiles that were about 8 percent higher than they would have been absent the restrictions.

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of orange juice he would buy each month at the prices listed in the first column. The second user (a “casual consumer” of orange juice) also likes its taste, but is less concerned about its nutritional value. The third column of Table 5.1 tell us how many liters of orange juice she would buy each month at the prices listed in the first column.

To find the total amount consumed in the market at any price, we simply add the quantities that each consumer would purchase at that price. For example, if the market price is $5 per liter, neither consumer will buy orange juice. If the price is $3 or $4, only the health-conscious consumer will buy it. Thus, if the price is $4 per liter, he will buy 3 liters, and the market demand will also be 3 liters; if the price is $3 per liter, the market demand will be 6 liters. Finally, if the market price is below $3, both consumers will purchase orange juice. Thus, if the price is $2 per liter, the market demand will be 11 liters; if the price is $1 the market demand will be 16 liters.

In Figure 5.21 we show both the demand curve for each consumer (\(D_h\) and \(D_c\)) and the market demand (the thick line, \(D_m\)).

Finally, we can describe the three demand curves algebraically. Let \(Q_h\) be the quantity demanded by the health-conscious consumer, \(Q_c\) the quantity demanded by the casual consumer, and \(Q_m\) the quantity demanded in the whole market (which contains only the two consumers). What are the three demand functions \(Q_h(P)\), \(Q_c(P)\), and \(Q_m(P)\)?

As you can see in Figure 5.21, the demand curve \(D_h\) for the health-conscious consumer is a straight line; he buys orange juice only when the price is below $5 per liter. You can verify that the equation of his demand curve is

\[
Q_h(P) = \begin{cases} 
15 - 3P, & \text{when } P < 5 \\
0, & \text{when } P \geq 5
\end{cases}
\]

The demand curve for the casual consumer is also a straight line; she buys orange juice only when the price is below $3 per liter. The equation of her demand curve \(D_c\) is

\[
Q_c(P) = \begin{cases} 
6 - 2P, & \text{when } P < 3 \\
0, & \text{when } P \geq 3
\end{cases}
\]

As shown in Figure 5.21, when the price is higher than $5, neither consumer buys orange juice; when the price is between $3 and $5, only the health-conscious consumer...
buys it. Therefore, over this range of prices, the market demand curve is the same as the demand curve for the health-conscious consumer. Finally, when the price is below $3, both consumers buy orange juice. (This explains why the market demand curve $D_m$ is kinked at point $A$, which is where the casual consumer’s demand kicks in.) So the market demand $Q_m(P)$ is just the sum of the segment demands $Q_h(P) + Q_c(P) = (15 - 3P) + (6 - 2P) = 21 - 5P$. Therefore, the market demand $Q_m(P)$ is

$$Q_m(P) = \begin{cases} 21 - 5P, & \text{when } P < 3 \\ 15 - 3P, & \text{when } 3 \leq P < 5 \\ 0, & \text{when } P \geq 5 \end{cases}$$

The discussion demonstrates that you must be careful when you add segment demands to get a market demand curve. First, since the construction of a market demand curve involves adding quantities, you must write the demand curves in the normal form (with $Q$ expressed as a function of $P$) before adding them, rather than using the inverse form of the demand (with $P$ written as a function of $Q$).

Second, you must pay attention to how the underlying individual demands vary across the range of prices. In the example above, if you simply add the equations for the individual demands to get the market demand $Q_m = Q_h(P) + Q_c(P) = 21 - 5P$, this expression is not valid for a price above $3$. For example, if the price is $4$, the expression $Q_m = 21 - 5P$ would tell you that the quantity demanded in the market would be 1 liter. Yet, as we can see by Table 5.1, the actual quantity demanded in the market at that price is 3 liters. See if you can figure out why this approach leads to an error. (If you give up, look at the footnote.)

Thus far we have been assuming that each person’s demand for a good is independent of everyone else’s demand. For example, the amount of chocolate a consumer wants to purchase depends on that consumer’s income, the price of chocolate, and possibly other prices, but not on anyone else’s demand for chocolate. This assumption enables us to find the market demand curve for a good by adding up the demand curves of all of the consumers in the market.

For some goods, however, a consumer’s demand does depend on how many other people purchase the good. In that case, we say there are network externalities. If one consumer’s demand for a good increases with the number of other consumers who buy the good, the externality is positive. If the amount a consumer demands increases when fewer other consumers have the good, the externality is negative. Many goods and services have network externalities.

Although we can often find network externalities related to physical networks (as in Application 5.6), we may also see them in other settings (sometimes called virtual networks because there is no physical connection among consumers). For example, the computer software Microsoft Word would have some value in preparing written documents even if that software had only one user. However, the product becomes more valuable to each user when it has many users. The virtual network of users makes it possible for each user to exchange and process documents with many other users.

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12 The error arises because we derived the market demand equation $Q_m = 21 - 5P$ by adding $Q_h(P) = 15 - 3P$ and $Q_c(P) = 6 - 2P$. According to these individual demand equations, when $P = 4$, $Q_h(P) = 3$ and $Q_c(P) = -2$. Sure enough, the sum is 1. But you are assuming that the casual consumer demands a negative quantity of orange juice ($-2$ liters) when the price is $4$, and this is economic nonsense! The expression for the demand of the casual consumer $Q_c(P) = 6 - 2P$ is not valid at a price of $4$. At this price, $Q_c(P) = 0$, not $-2$. 

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5.5 NETWORK EXTERNALITIES
A virtual network may also be present if a good or service requires two complementary components to have value. For example, a computer operating system, such as Microsoft Windows 2000, has value only if software applications exist that can run on the operating system. The operating system becomes more valuable as the number of applications that can run on it increases. A software application also has a higher value if it runs on a widely accepted operating system. Thus, more people using an operating system leads to more software applications, raising the demand for the operating system, and so on.

Finally, positive network externalities can occur if a good or service is a fad. We often see fads for goods and services that affect lifestyles, such as fashions of clothing, children's toys, or beer. Advertisers and marketers often try to highlight the popularity of a product as part of its image.

Figure 5.22 illustrates the effects of a positive network externality. The graph shows a set of market demand curves for connections to the Internet. For this example, let's assume that a connection to the Internet refers to a subscription to a provider of access to the Internet, such as America Online or Microsoft Network. The curve $D_{30}$ represents the demand if consumers believe that 30 million subscribers have access to the Internet. The curve $D_{60}$ represents the demand if consumers believe that 60 million subscribers have access. Suppose that access initially costs $20 per month and that there are 30 million subscribers (point A in the graph).

What happens if the monthly price of access drops to $10? If there were no positive network externality, the quantity demanded would simply change to some other point on $D_{30}$. In this case, the quantity of subscriptions would grow to 38 million (point B in the graph). However, there is a positive network externality; as more people use e-mail, instant messaging, and other Internet features, even more people want to sign up. Therefore, at the lower price, the number of consumers wanting access will be even greater than a movement along $D_{30}$ to point B would indicate. The total number of subscriptions actually demanded at a price of $10 per month will grow to 60 million (point C in the graph). The total effect of the price decrease is an increase...
of 30 million subscribers. The total effect is the pure price effect of 8 million new subscribers (moving from point $A$ to point $B$) plus a bandwagon effect of 22 million new subscribers (moving from point $B$ to point $C$). This bandwagon effect refers to the increased quantity demanded as more consumers are connected to the Internet. Thus, a demand curve with positive network externalities (such as the heavy demand curve in Figure 5.22) is more elastic than a demand curve with no network externalities (such as $D_{30}$).

For some goods, there is a negative network externality—the quantity demanded decreases when more people have the good. Rare items, such as Stradivarius violins, Babe Ruth baseball cards, and expensive automobiles are examples of such goods. These goods enjoy a snob effect, a negative network externality that refers to the decrease in the quantity of a good that is demanded as more consumers buy it. A snob effect may arise because consumers value being one of the few to own a particular type of good. We might also see the snob effect if the value of a good or service diminishes because congestion increases when more people purchase that good or service.

Figure 5.23 shows the effects of a snob effect. The graph illustrates a set of market demand curves for membership in a health and fitness club. The curve $D_{1000}$ represents the demand if consumers believe the club has 1000 members. The curve $D_{1300}$ shows the demand if consumers believe it has 1300 members. Suppose a membership initially costs $1,200 per year, and that the club has 1000 members (point $A$ in the graph).

What happens if the membership price decreases to $900? If consumers believed that the number of members would stay at 1000, 1800 would actually want to join the club (point $B$ in the graph). However, consumers know that the fitness club will become more congested as more members join, and this will shift the demand curve inward. The total number of memberships actually demanded at a price of $900 per month will grow only to 1300 (point $C$ in the graph). The total effect of the price decrease is the pure price effect of 800 new members (moving from point $A$ to point $B$) plus a snob effect of −500 members (moving from point $B$ to point $C$), or an increase of only 300 members. A demand curve with negative network externalities (such as the demand curve connecting points $A$ and $C$ in Figure 5.23) is less elastic than a demand curve without network externalities (such as $D_{1000}$).
As we have already seen, the model of optimal consumer choice has many everyday applications. In this section, we use that model to examine a consumer’s choice of how much to work.

As wages rise, leisure first decreases, then increases

Let’s divide the day into two parts, the hours when an individual works and the hours when he pursues leisure. Why does the consumer work at all? Because he works, he earns an income, and he uses the income to pay for the activities he enjoys in his leisure time. The term leisure includes all nonwork activities, such as eating, sleeping, recreation, and entertainment. We assume that the consumer likes leisure activities.

Suppose the consumer chooses to work $L$ hours per day. Since a day has 24 hours, the time available for leisure will be the time that remains after work, that is, $24 - L$ hours.

The consumer is paid an hourly wage rate $w$. Thus, his total daily income will be $wL$. He uses the income to purchase units of a composite good at a price of $1 per unit.

The consumer’s utility $U$ depends on the amount of leisure time and the number of units of the composite good he can buy. We can represent the consumer’s decision on the optimal choice diagram in Figure 5.24. The horizontal axis represents the number of hours of leisure each day, which can be no greater than 24 hours. The vertical axis represents the number of units of the composite good that he purchases from his income. Since the price of the composite good is $1, the vertical axis also measures the consumer’s income.

To find an optimal choice of leisure and other goods, we need a set of indifference curves and a budget constraint. Figure 5.24 shows a set indifference curves for which the marginal utility of leisure and the composite good are both positive. Thus $U_5 > U_4 > U_3 > U_2 > U_1$.
5.6 THE CHOICE OF LABOR AND LEISURE

The consumer’s budget line for this problem will tell us all the combinations of the composite good and hours of leisure \((24 - L)\) that the consumer can choose. If the consumer does no work, he will have 24 hours of leisure, but no income to spend on the composite good. This corresponds to point \(A\) on the budget line in the graph.

The location of the rest of the budget line depends on the wage rate \(w\). Suppose the wage rate is $5 per hour. This means that for every hour of leisure the consumer gives up to work, he can buy 5 units of the composite good. The budget line thus has a slope of \(-5\). If the consumer were to work 24 hours per day, his income would be $120 and he would be able to buy 120 units of the composite good, corresponding to basket \(B\) on the budget line. The consumer’s optimal choice will then be basket \(E\); thus, when the wage rate is $5, the consumer will work 8 hours.

For any wage rate, the slope of the budget line is \(-w\). The figure shows budget lines for five different wage rates ($5, $10, $15, $20, and $25), along with the optimal choice for each wage rate. As the wage rate rises from $5 to $15, the number of hours of leisure falls. However, as the wage rate continues to rise, the consumer begins to increase his amount of leisure time.

The next section discusses a phenomenon that is directly related to this change in the consumer’s choice of labor versus leisure as wage rates rise.

THE BACKWARD-BENDING SUPPLY OF LABOR

Since a day has only 24 hours, the consumer’s choice about the amount of leisure time is also a choice about the amount of labor he will supply. The optimal choice diagram
in Figure 5.24 contains enough information to enable us to construct a curve showing how much labor the consumer will supply at any wage rate. In other words, we can draw the consumer’s supply of labor curve, as shown in Figure 5.25.

The points $E', F', G', H'$, and $I'$ in Figure 5.25 correspond, respectively, to points $E, F, G, H,$ and $I$ in Figure 5.24. When the wage rate is $5, the consumer supplies 8 hours of labor (points $E'$ and $E$). As the wage rate goes up from $5 to $15, the labor supply rises too—at a wage rate of $15, the labor supply is 11 hours (points $G'$ and $G$). But when the wage rate continues to rise past $15, the labor supply begins to fall, until, finally, at a wage rate of $25, the consumer works only 9 hours (points $I'$ and $I$). For most goods and services, a higher price stimulates supply; in this case, however, a higher wage rate decreases the labor supply. (Remember, the wage rate is the price of labor.) To understand this phenomenon, which is reflected in the backward-bending shape of the supply of labor curve in Figure 5.25, let’s examine the income and substitution effects associated with a change in the wage rate.

Look again at the optimal choice diagram in Figure 5.24. Instead of having a fixed income, our consumer has a fixed amount of time in the day, 24 hours. That is why the horizontal intercept of the budget line stays at 24 hours, regardless of the wage rate. An hour of work always “costs” the consumer an hour of leisure, no matter what the wage rate is.

However, an increase in the wage rate makes a unit of the composite good look less expensive to the consumer. If the wage rate doubles, the consumer needs to work only half as long to buy as much of the composite good as before. That is why the vertical intercept of the budget line moves up as the wage rate rises. The increase in the wage rate therefore leads to an upward rotation of the budget line, as Figure 5.24 shows.

An increase in the wage rate reduces the amount of work required to buy a unit of the composite good, and this leads to both a substitution effect and an income effect. The substitution effect on the labor supply is positive—it induces the consumer to substitute more of the composite good for leisure, leading to less leisure and more labor. In contrast, the income effect on labor supply is negative—it leads to more leisure and less labor, because leisure is a normal good for most people (i.e., the consumer wants more leisure as his income rises).

Now let’s examine the income and substitution effects of a wage increase from $15 to $25. Figure 5.26 shows the initial budget line $BL_1$ (with the wage rate of $15$) and
5.6 THE CHOICE OF LABOR AND LEISURE

FIGURE 5.26 Optimal Choice of Labor and Leisure

At the initial basket G on budget line BL1, the consumer has 13 hours of leisure (and works for 11 hours). At the final basket I on budget line BL2, the consumer has 15 hours of leisure (and works for 9 hours). At the decomposition basket J on budget line BLd, the consumer has 12 hours of leisure (and works for 12 hours). The substitution effect on leisure is $-1$ (the change in leisure between G and J). The income effect on leisure is $+3$ (the change in leisure between J and I). Thus, the total effect on leisure is $+2$, and the corresponding total effect on labor is $-2$.

The substitution effect on leisure is thus $-1$ hour (the change in leisure as we move from G to J). The income effect on leisure is $+3$ hours (the change in leisure as we move from J to I). Since the income effect outweighs the substitution effect, the net effect of the change in the wage rate on the amount of leisure is $+2$ hours. Thus, the net effect of the increase in the wage rate on the amount of labor is $-2$ hours. This accounts for the backward-bending shape of the labor supply curve in Figure 5.25 as the wage rate rises above $15.

In sum, the labor supply curve slopes upward over the region where the substitution effect associated with a wage increase outweighs the income effect, but bends backward over the region where the income effect outweighs the substitution effect.
CHAPTER 5  THE THEORY OF DEMAND

5.7  CONSUMER PRICE INDICES

The Consumer Price Index (CPI) is one of the most important sources of information about trends in consumer prices and inflation in the United States. It is often viewed as a measure of the change in the cost of living and is used extensively for economic analysis in both the private and public sectors. For example, in contracts among individuals and firms, the prices at which goods are exchanged are often adjusted over time to reflect changes in the CPI. In negotiations between labor unions and employers, adjustments in wage rates often reflect past or expected future changes in the CPI.

The CPI also has an important impact on the budget of the federal government. On the expenditure side, the government uses the CPI to adjust payments to Social Security recipients, to retired government workers, and for many entitlement programs such as food stamps and school lunches. As the CPI rises, the government’s payments increase. And changes in the CPI also affect how much money the government collects through taxes. For example, individual income tax brackets are adjusted for inflation using the CPI. As the CPI increases, tax revenues decrease.

Measuring the CPI is not easy. Let’s construct a simple example to see what factors might be desirable in designing a CPI. Suppose we consider a representative consumer, who buys only two goods, food and clothing, as illustrated in Figure 5.27. In year 1, the price of food was $3 and the price of clothing was $8. The consumer had an income of $480 and faced the budget line $BL_1$ with a slope of $-P_F/P_C = -3/8$. He purchased the optimal basket $A$, located on indifference curve $U_1$ and containing 80 units of food and 30 units of clothing.

In year 2 the prices of food and clothing increase to $P_F = $6 and $P_C = $9. How much income will the consumer need in year 2 to be as well off as in year 1, that is, to reach the indifference curve $U_1$? The new budget line $BL_2$ must be tangent to $U_1$ and have a slope reflecting the new prices, $-P_F/P_C = -2/3$. At the new prices, the least

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**APPLICATION 5.7**

The Backward-Bending Supply of Nursing Services

Medical groups and hospitals have long had difficulty attracting enough workers. They have often increased pay to stimulate the supply of medical workers. Yet raising wage rates may not always increase the amount of labor supplied.

In 1991 *The Wall Street Journal* described some of these difficulties in an article titled “Medical Groups Use Pay Boosts, Other Means to Find More Workers.” According to the article, the American Hospital Association concluded, “Pay rises may have worsened the nursing shortage in Massachusetts by enabling nurses to work fewer hours.”

Why might this have happened? As we saw in our discussion related to Figure 5.26, a higher wage may induce a consumer to pursue more leisure, and thus supply less labor. In other words, many nurses may be on the backward-bending region of their supply curve for labor.

Since wage increases alone do not always attract more workers, employers have resorted to other strategies. For example, the article in *The Wall Street Journal* states that the M.D. Anderson Cancer Center at the University of Texas gave employees a $500 bonus if they referred new applicants who took “hard-to-fill” jobs. The Texas Heart Institute in Houston recruited nurses partly by showcasing prospects for promotion. The University of Pittsburgh Medical Center started an “adopt-a-high-school” program to encourage students to enter the health care sector, and reimbursed employees’ tuition fees when they enrolled in programs to increase their skills.

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costly combination of food and clothing on the indifference curve is at basket $B$, with 60 units of food and 40 units of clothing. The total expenditure necessary to buy basket $B$ at the new prices is $P_F^2 F + P_C^2 C = ($6)(60) + ($9)(40) = $720.

In principle, the CPI should measure the percentage increase in expenditures that would be necessary for the consumer to remain as well off in year 2 as he was in year 1. In the example, the necessary expenditures increased from $480 in year 1 to $720 in year 2. The “ideal” CPI would be the ratio of the new expenses to the old expenses, that is $720/$480 = 1.5. In other words, at the higher prices, it would take 50 percent more income in year 2 to make the consumer as well off as he was in year 1. In this sense the “cost of living” in year 2 is 50 percent greater than it was in year 1. In calculating this ideal CPI, we would need to recognize that the consumer would substitute more clothing for food when the price of food rises relative to the price of clothing, moving from the initial basket $A$ to basket $B$.

Note that to determine the ideal CPI, the government would need to collect data on the old prices and the new prices and on changes in the composition of the basket (how much food and clothing are consumed). But, considering the huge number of goods and services in the economy, this is an enormous amount of data to collect! It is hard enough to collect data on the way so many prices change over time, and even more difficult to collect information on the changes in the baskets that consumers actually purchase.

In Figure 5.27, the substitution bias in the Consumer Price Index is illustrated. In year 1, the consumer has an income of $480, the price of food is $3, and the price of clothing is $8. The consumer chooses basket $A$. In year 2, the price of food rises to $6, and the price of clothing rises to $9. The consumer could maintain his initial level of utility $U_1$ at the new prices by purchasing basket $B$, costing $720. An ideal cost of living index would be $1.5$ ($720/480$), telling us that the cost of living has increased by 50 percent. However, the actual CPI assumes the consumer does not substitute clothing for food as relative prices change, but continues to buy basket $A$ at the new prices, for which he would need an income of $750. The CPI ($750/480 = 1.56$) suggests that the consumer’s cost of living has increased by about 56 percent, which overstates the actual increase in the cost of living. In fact, if the consumer’s income in year 2 were $750, he could choose a basket such as $E$ on $BL_3$ and achieve a higher level of utility than $U_1$.\[\text{FIGURE 5.27 Substitution Bias in the Consumer Price Index}\]
In practice, therefore, to simplify the measurement of the CPI, the government has historically calculated the change in expenditures necessary to buy a fixed basket as prices change, where the fixed basket is the amount of food and clothing purchased in year 1. In our example, the fixed basket is \( A \). The income necessary to buy basket \( A \) at the new prices is 

\[
P_F F + P_C C = ($6)(80) + ($9)(30) = $750.
\]

If he were given $750 with the new prices, he would face the budget line \( BL_3 \). If we were to calculate a CPI using the fixed basket \( A \), the ratio of the new expenses to the old expenses is 

\[
\frac{750}{480} = 1.5625.
\]

This index tells us that the consumer’s expenditures would need to increase by 56.25 percent to buy the fixed basket (i.e., the basket purchased in year 1) at the new prices. 

As the example shows, the index based on the fixed basket overcompensates the consumer for the higher prices. Economists refer to the overstatement of the increase in the cost of living as the “substitution bias.” By assuming that the consumer’s basket is fixed at the initial levels of consumption, the index ignores the possible substitution that consumers will make toward goods that are relatively less expensive in a later year. In fact, if the consumer were given an income of $750 instead of $720 in year 2, he could choose a basket such as \( E \) on \( BL_3 \) and make himself better off than he was at \( A \).

### APPLICATION 5.8

**The Substitution Bias in the Consumer Price Index**

While economists have long argued that the Consumer Price Index overstates changes in the cost of living, the bias in the CPI took center stage in the 1990s when Congress tried to balance the budget. In 1995 Alan Greenspan, the Chairman of the Federal Reserve, brought this controversy to the fore when he told Congress that the official CPI may be overstating the true increase in the cost of living by perhaps 0.5 to 1.5 percent. The Senate Finance Committee appointed a panel chaired by economist Michael Boskin to study the magnitude of the bias. The panel concluded that the CPI overstates the cost of living by about 1.1 percent.

While estimates of the impact of the substitution bias are necessarily imprecise, they are potentially very important. Greenspan estimated that if the annual level of inflation adjustments to indexed programs and taxes were reduced by 1 percentage point, the annual level of the deficit would be lowered by as much as $55 billion after five years. The Office of Management and Budget estimated that in fiscal year 1996, a 1 percent increase in the index led to an increase in government expenditures of about $5.7 billion, as well as a decrease in tax revenues of about $2.5 billion.

The government has long been aware of the need to periodically update the “fixed basket” used in the CPI calculation. In fact, the basket has been revised approximately every 10 years, with the most recent revision taking place in 2002. In light of the potential biases of the CPI, the government continues to investigate ways to improve how it is calculated. For example, in January 1999 the government began to use a new formula to calculate many of the component indices that form the CPI. The use of this new formula is intended to counteract the substitution bias and was expected to reduce the annual rate of increase in the CPI by about 0.2 percentage points a year.

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14An index that measures the expenditure necessary to buy the fixed basket at the prices in year 2 divided by the expenditure necessary to purchase the same basket at the prices in year 1 is called a Laspeyers index. Let’s see how to calculate this index with the example in the text. Denote the prices of food in years 1 and 2 as \( P_{F1} \) and \( P_{F2} \), and the prices of clothing in years 1 and 2 \( P_{C1} \) and \( P_{C2} \). The fixed basket is the quantity of food \( F \) and clothing \( C \) consumed in year 1. Then the Laspeyers index \( L \) is

\[
L = \frac{P_{F2} F + P_{C2} C}{P_{F1} F + P_{C1} C}
\]

CHAPTER SUMMARY

• We can derive an individual’s demand curve for a good from her preferences and the budget constraint. A consumer’s demand curve shows how the optimal choice of a commodity changes as the price of the good varies. We can also think of a demand curve as a schedule of the consumer’s “willingness to pay” for a good. (LBD Exercises 5.2, 5.3)

• A good is normal if the consumer purchases more of that good as income rises. A good is inferior if he purchases less of that good as income increases. (LBD Exercise 5.1)

• We can separate the effect of a price change on the quantity of a good demanded into two parts, a substitution effect and an income effect. The substitution effect is the change in the amount of a good that would be consumed as the price of that good changes, holding constant the level of utility. When the indifference curves are bowed in toward the origin (because of diminishing marginal rate of substitution), the substitution effect will move in the opposite direction from the price change. If the price of the good decreases, its substitution effect will be positive. If the price of the good increases, its substitution effect will be negative. (LBD Exercises 5.4, 5.5, 5.6)

• The income effect for a good is the change in the amount of that good that a consumer would buy as her purchasing power changes, holding prices constant. If the good is normal, the income effect will reinforce the substitution effect. If the good is inferior, the income effect will oppose the substitution effect.

• If the good is so strongly inferior that the income effect outweighs the substitution effect, the demand curve will have an upward slope over some range of prices. Such a good is called a Giffen good.

• Consumer surplus is the difference between what a consumer is willing to pay for a good and what he must pay for it. Without income effects, consumer surplus provides a monetary measure of how much better off the consumer will be when he purchases a good. On a graph the consumer surplus will be the area under an ordinary demand curve and above the price of the good. Changes in consumer surplus can measure how much better off or worse off a consumer is if the price changes. (LBD Exercise 5.7)

• Using optimal choice diagrams, we can look at the monetary impact of a price change from two perspectives: compensating variation and equivalent variation. The compensating variation measures how much money the consumer would be willing to give up after a reduction in the price of a good to make her just as well off as she was before the price change.

• The equivalent variation measures how much money we would have to give the consumer before a price reduction to keep her as well off as she would be after the price change.

• If there is an income effect, the compensating variation and equivalent variation will differ, and these measures will also be different from the change in the area under the ordinary demand curve. (LBD Exercise 5.9)

• If the income effect is small, the equivalent and compensating variations may be close to one another, and the change in the area under an ordinary demand curve will be a good approximation (although not an exact measure) of the monetary impact of the price change.

• Without an income effect, the compensating variation and equivalent variation will give us the same measure of the monetary value that a consumer would assign to a change in the price of the good. The change in the area under an ordinary demand curve will be equal to the compensating variation and equivalent variation. (LBD Exercise 5.8)

• The market demand curve for a good is the horizontal sum of the demands of all of the individual consumers in the market (assuming there are no network externalities).

• The bandwagon effect is a positive network externality. With a bandwagon effect, each consumer’s demand for a good increases as more consumers buy it. The snob effect is a negative network externality. With a snob effect each consumer’s demand for a good decreases as more consumers buy it.

REVIEW QUESTIONS

1. What is a price consumption curve for a good?
2. How does a price consumption curve differ from an income consumption curve?
3. What can you say about the income elasticity of demand of a normal good? Of an inferior good?
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4. If indifference curves are bowed in toward the origin and the price of a good drops, can the substitution effect ever lead to less consumption of the good?

5. Suppose a consumer purchases only three goods, food, clothing, and shelter. Could all three goods be normal? Could all three goods be inferior? Explain.

6. Does economic theory require that a demand curve always be downward sloping? If not, under what circumstances might the demand curve have an upward slope over some region of prices?

7. What is consumer surplus?

8. Two different ways of measuring the monetary value that a consumer would assign to the change in price of the good are (1) the compensating variation and (2) the equivalent variation. What is the difference between the two measures, and when would these measures be equal?

9. Consider the following four statements. Which might be an example of a positive network externality? Which might be an example of a negative network externality?
   (i) People eat hot dogs because they like the taste, and hot dogs are filling.
   (ii) As soon as Zack discovered that everybody else was eating hot dogs, he stopped buying them.
   (iii) Sally wouldn’t think of buying hot dogs until she realized that all her friends were eating them.
   (iv) When personal income grew by 10 percent, hot dog sales fell.

10. Why might an individual supply less labor (demand more leisure) as the wage rate rises?

PROBLEMS

5.1. Figure 5.2(a) shows a consumer’s optimal choices of food and clothing for three values of weekly income:
   \( I_1 = $40 \), \( I_2 = $68 \), and \( I_3 = $92 \). Figure 5.2(b) illustrates how the consumer’s demand curve for food shifts as income changes. Draw three demand curves for clothing (one for each level of income) to illustrate how changes in income affect the consumer’s purchases of clothing.

5.2. Use the income consumption curve in Figure 5.2(a) to draw the Engel curve for clothing, assuming the price of food is $2 and the price of clothing is $4.

5.3. Show that the following statements are true:
   a) An inferior good has a negative income elasticity of demand.
   b) A good whose income elasticity of demand is negative will be an inferior good.

5.4. If the demand for a product is perfectly price inelastic, what does the corresponding price consumption curve look like? Draw a graph to show the price consumption curve.

5.5. Suez purchases two goods, food and clothing. She has the utility function \( U(x, y) = xy \), where \( x \) denotes the amount of food consumed and \( y \) the amount of clothing. The marginal utilities for this utility function are \( MU_x = y \) and \( MU_y = x \).
   a) Show that the equation for her demand curve for clothing is \( y = I / (2P_x) \).
   b) Is clothing a normal good? Draw her demand curve for clothing when the level of income is \( I = 200 \). Label this demand curve \( D_1 \). Draw the demand curve when \( I = 300 \) and label this demand curve \( D_2 \).
   c) What can be said about the cross-price elasticity of demand of food with respect to the price of clothing?

5.6. Karl’s preferences over hamburgers (H) and beer (B) are described by the utility function: \( U(H, B) = \min(2H, 3B) \). His monthly income is \( I \) dollars, and he only buys these two goods out of his income. Denote the price of hamburgers by \( P_H \) and of beer by \( P_B \).
   a) Derive Karl’s demand curve for beer as a function of the exogenous variables.
   b) Which affects Karl’s consumption of beer more: a one dollar increase in \( P_H \) or a one dollar increase in \( P_B \)?

5.7. David has a quasi-linear utility function of the form \( U(x, y) = \sqrt{x} + y \), with associated marginal utility functions \( MU_x = 1 / (2 \sqrt{x}) \) and \( MU_y = 1 \).
   a) Derive David’s demand curve for \( x \) as a function of the prices, \( P_x \) and \( P_y \). Verify that the demand for \( x \) is independent of the level of income at an interior optimum.
   b) Derive David’s demand curve for \( y \). Is \( y \) a normal good? What happens to the demand for \( y \) as \( P_x \) increases?

5.8. Rick purchases two goods, food and clothing. He has a diminishing marginal rate of substitution of food for clothing. Let \( x \) denote the amount of food consumed and \( y \) the amount of clothing. Suppose the price
of food increases from \( P_{x1} \) to \( P_{x2} \). On a clearly labeled graph, illustrate the income and substitution effects of the price change on the consumption of food. Do so for each of the following cases:

a) Case 1: Food is a normal good.
b) Case 2: The income elasticity of demand for food is zero.
c) Case 3: Food is an inferior good, but not a Giffen good.
d) Case 4: Food is a Giffen good.

5.9. Reggie consumes only two goods: food and shelter. On a graph with shelter on the horizontal axis and food on the vertical axis, his price consumption curve for shelter is a vertical line. Draw a pair of budget lines and indifference curves that are consistent with this description of his preferences. What must always be true about Reggie’s income and substitution effects as the result of a change in the price of shelter?

5.10. Ginger’s utility function is \( U(x, y) = x^2 y \), with associated marginal utility functions \( MU_x = 2xy \) and \( MU_y = x^2 \). She has income \( I = 240 \) and faces prices \( P_x = $8 \) and \( P_y = $2 \).

a) Determine Ginger’s optimal basket given these prices and her income.
b) If the price of \( y \) increases to \$8 and Ginger’s income is unchanged, what must the price of \( x \) fall to in order for her to be exactly as well off as before the change in \( P_y \)?

5.11. Some texts define a “luxury good” as a good for which the income elasticity of demand is greater than 1. Suppose that a consumer purchases only two goods. Can both goods be luxury goods? Explain.

5.12. Scott consumes only two goods, steak and ale. When the price of steak falls, he buys more steak and more ale. On an optimal choice diagram (with budget lines and indifference curves), illustrate this pattern of consumption.

5.13. Dave consumes only two goods, coffee and doughnuts. When the price of coffee falls, he buys the same amount of coffee and more doughnuts.

a) On an optimal choice diagram (with budget lines and indifference curves), illustrate this pattern of consumption.
b) Is this purchasing behavior consistent with a quasi-linear utility function? Explain.

5.14. (This problem shows that an optimal consumption choice need not be interior, and may be at a corner point.) Suppose that a consumer’s utility function is \( U(x, y) = xy + 10y \). The marginal utilities for this utility function are \( MU_x = y \) and \( MU_y = x + 10 \). The price of \( x \) is \( P_x \) and the price of \( y \) is \( P_y \), with both prices positive. The consumer has income \( I \).

a) Assume first that we are at an interior optimum. Show that the demand schedule for \( x \) can be written as \( x = I/(2P_x) - 5 \).
b) Suppose now that \( I = 100 \). Since \( x \) must never be negative, what is the maximum value of \( P_x \) for which this consumer would ever purchase any \( x \)?
c) Suppose \( P_y = 20 \) and \( P_x = 20 \). On a graph illustrating the optimal consumption bundle of \( x \) and \( y \), show that since \( P_x \) exceeds the value you calculated in part (b), this corresponds to a corner point at which the consumer purchases only \( y \). (In fact, the consumer would purchase \( y = I/P_y = 5 \) units of \( y \) and no units of \( x \).)
d) Compare the marginal rate of substitution of \( x \) for \( y \) with the ratio \((P_x/P_y)\) at the optimum in part (c). Does this verify that the consumer would reduce utility if she purchased a positive amount of \( x \)?
e) Assuming income remains at 100, draw the demand schedule for \( x \) for all values of \( P_x \). Does its location depend on the value of \( P_y \)?

5.15. The figure below illustrates the change in consumer surplus, given by Area \( ABEC \), when the price decreases from \( P_1 \) to \( P_2 \). This area can be divided into the rectangle \( ABDC \) and the triangle \( BDE \). Briefly describe what each area represents, separately, keeping in mind the fact that consumer surplus is a measure of how well off consumers are (therefore the change in consumer surplus represents how much better off consumers are). (Hint: Note that a price decrease also induces an increase in the quantity consumed.)

5.16. Lou’s preferences over pizza (\( x \)) and other goods (\( y \)) are given by \( U(x, y) = xy \), with associated marginal utilities \( MU_x = y \) and \( MU_y = x \). His income is $120.

a) Calculate his optimal basket when \( P_x = 4 \) and \( P_y = 1 \).
b) Calculate his income and substitution effects of a decrease in the price of food to $3.
c) Calculate the compensating variation of the price change.
d) Calculate the equivalent variation of the price change.

5.17. Suppose the market for rental cars has two segments, business travelers and vacation travelers. The demand curve for rental cars by business travelers is \( Q_b = 35 - 0.25P \), where \( Q_b \) is the quantity demanded by business travelers (in thousands of cars) when the rental price is \( P \) dollars per day. No business customers will rent cars if the price exceeds $140 per day.
The demand curve for rental cars by vacation travelers is \( Q_v = 120 - 1.5P \), where \( Q_v \) is the quantity demanded by vacation travelers (in thousands of cars) when the rental price is \( P \) dollars per day. No vacation customers will rent cars if the price exceeds $80 per day.

a) Fill in the table to find the quantities demanded in the market at each price.

<table>
<thead>
<tr>
<th>Price ($/day)</th>
<th>Business (thousands of cars/day)</th>
<th>Vacation (thousands of cars/day)</th>
<th>Market Demand (thousands of cars/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
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<td>50</td>
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</tbody>
</table>

b) Graph the demand curves for each segment, and draw the market demand curve for rental cars.
c) Describe the market demand curve algebraically. In other words, show how the quantity demanded in the market \( Q_m \) depends on \( P \). Make sure that your algebraic equation for the market demand is consistent with your answers to parts (a) and (b).
d) If the price of a rental car is $60, what is the consumer surplus in each market segment?

5.18. One million consumers like to rent movie videos in Pulmonia. Each has an identical demand curve for movies. The price of a rental is \( P \). At a given price, will the market demand be more elastic or less elastic than the demand curve for any individual. (Assume there are no network externalities.)

5.19. Suppose that Bart and Homer are the only people in Springfield who drink 7-UP. Moreover their inverse demand curves for 7-UP are, respectively, \( P = 10 - 4Q_B \) and \( P = 25 - 2Q_H \), and, of course, neither one can consume a negative amount. Write down the market demand curve for 7-UP in Springfield, as a function of all possible prices.

5.20. Joe’s income consumption curve for tea is a vertical line on an optimal choice diagram, with tea on the horizontal axis and other goods on the vertical axis.

a) Show that Joe’s demand curve for tea must be downward sloping.
b) When the price of tea drops from $9 to $8 per pound, the change in Joe’s consumer surplus (i.e., the change in the area under the demand curve) is $30 per month. Would you expect the compensating variation and the equivalent variation resulting from the price decrease to be near $30? Explain.

5.21. Consider the optimal choice of labor and leisure discussed in the text. Suppose a consumer works the first 8 hours of the day at a wage rate of $10 per hour, but receives an overtime wage rate of $20 for additional time worked.

a) On an optimal choice diagram, draw the budget constraint. (Hint: It is not a straight line.)
b) Draw a set of indifference curves that would make it optimal for him to work 4 hours of overtime each day.

5.22. Terry’s utility function over leisure \( (L) \) and other goods \( (Y) \) is \( U(L, Y) = Y + LY \). The associated marginal utilities are \( MU_Y = 1 + L \) and \( MU_L = Y \). He purchases other goods at a price of $1, out of the income he earns from working. Show that, no matter what Terry’s wage rate, the optimal number of hours of leisure that he consumes is always the same. What is the number of hours he would like to have for leisure?

5.23. Consider Noah’s preferences for leisure \( (L) \) and other goods \( (Y) \), \( U(L, Y) = \sqrt{L} + \sqrt{Y} \). The associated marginal utilities are \( MU_L = \frac{1}{2\sqrt{L}} \) and \( MU_Y = \frac{1}{2\sqrt{Y}} \). Suppose that \( P_Y = $1 \). Is Noah’s supply of labor backward bending?