The yield curve and the macro-economy across time and frequencies

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Abstract

We assess the relation between the yield curve and the macroeconomy in the U.S. between 1961 and 2011. We add to the standard parametric macro-finance models, uncovering evidence simultaneously on the time and frequency domains. We model the shape of the yield curve by latent factors corresponding to its level, slope and curvature. The macroeconomic variables measure real activity, inflation and monetary policy. The tools of wavelet analysis, the set of variables and the length of the sample allow for a thorough appraisal of the time-variation in the direction, intensity, synchronization and periodicity of the yield curve-macroeconomy relation.

Keywords: Macro-finance; Yield curve; Kalman filter; Continuous wavelet transform; Wavelet coherency; Phase-difference.

JEL classification: C32; C49; E43; E44

1 Introduction

The last 25 years have witnessed the development of a prolific literature on the relation between the shape of the sovereign debt yield curve and the main macroeconomic variables. Such relation – possibly bidirectional – is relevant for policymakers in a twofold sense: first, the information content of the yield curve may be valuable for the prediction of business cycles, inflation and monetary policy; second, the response of the yield curve may be informative about the transmission of monetary policy and, overall, the dynamic impact of shocks on the macroeconomy.

Early analyses have focused mainly on the slope of the yield curve shape to forecast output or inflation. Typically the authors set *a priori* a number of possible lead horizons for the dynamic relation between the yield curve and the macro variables, and only infrequently allowed for bidirectional relations (e.g. Harvey (1988), Stock and Watson (1989), Estrella and Hardouvelis (1991), Mishkin (1990a, 1990b and 1990c)). Many papers, including recent ones such as Chauvet and Potter (2005), Benati and Goodhart (2008), and Rudebusch and Williams (2009), have used empirical proxies for the slope – in some others, also for the level and curvature – that roughly account for the shape of the yield curve. In such literature, the identification of changes in the relation between the yield curve and the macroeconomy was based on structural break tests (as in, e.g., Estrella, Rodrigues and Schich (2003) and Giacomini and Rossi (2006)).

An alternative literature, following Ang, Piazzesi and Wei (2006) – with an arbitrage-free model – and Diebold, Rudebusch and Aruoba (2006) – with the Nelson and Siegel (1987) decomposition of the yield curve – has specified macro-finance models in which the shape of the yield curve is modelled with a set of latent factors that try to distill the whole information of the curve, at each period of time, into three factors corresponding to the level, slope and curvature. Such macro-finance vector autoregressive (VAR) models allowed for progress in the study of the relation between the yield curve and the main macroeconomic variables along two paths: first, the assessment of bidirectional feedbacks with some flexibility; second, the assessment of time variation, in the continuous framework of time-varying VARs (see e.g. Ang, Boivin, Dong and Loo-Kung 2009, Mumtaz and Surico 2009 and Bianchi, Mumtaz and Surico 2009a and 2009b).

The literature so far has been conducted strictly in the time-domain, thus being essentially uninformative about the frequencies at which the relation between the yield curve components and the macroeconomic variables occur. Yet, the co-movement between the yield curve shape and the main macroeconomic variables surely has been subject to time-variation and structural changes not only as regards its intensity, direction and synchronicity (the lead-lag horizons), but also as regards to its frequency. Hence the contribution of this paper: we adopt a time-frequency framework, that is a natural econometric approach to progress in the study of the relation between the yield curve and the macroeconomy; in particular, we employ wavelet tools, which are one of the most promising time-frequency methods. To be more, precise, we study the relation between the level, slope and curvature of the yield curve and macroeconomic activity,

inflation and the policy interest rate, in the U.S., across time and frequencies, using the wavelet power spectrum, coherency and phase-difference.

To measure the shape of the yield curve, we adopt the Nelson and Siegel (1987) decomposition of the curve into three latent factors – level, slope and curvature –, which has a long tradition in the finance literature, is model-based, accounts for the whole shape of the curve and is implemented by means of formal econometric techniques. Studying the yield curve-macro relation in the time-frequency domain with such a latent factors approach to the yield curve, rather than using empirical proxies, is a further contribution of the paper.

We reach several conclusions. First, the yield curve level has been determined by the fed funds rate (FFR) at low frequencies, specially after the outset of Alan Greenspan's mandate in 1987 at which point the high coherency between these variables moved progressively to cycles of longer period (larger than 12 years). Inflation has led the yield curve level at business cycle frequencies (oscillations of period between 4 and 12 years), but only until 1993, when inflation volatility went down markedly. As expected, the yield curve level has not related closely with real economic activity, unless this is measured by the unemployment rate – less volatile, more persistent, and lagging monthly output growth – which has co-moved with the level with some lag at business cycle frequencies until 2005.

Second, consistent with the monetary policy explanation for the predictive power of the yield curve slope, the FFR and the slope have significantly co-moved in the same direction at all cyclical frequencies across most of the sample period, with the slope either leading or moving contemporaneously. At business cycle frequencies, increases in the slope led increases in inflation and anticipated recessions with a larger lag – consistent with monetary policy reacting to expectations of inflation but impacting only with a lag, first on output and then on inflation. The predictive power of the slope vanished after 1985, when the Great Moderation began, to reappear in 1990 regarding real activity and in 1993 regarding inflation (but here only at cycles of 4$\sim$8 years and with a considerably smaller lag, which is compatible with more effective inflation targeting). Since the early 2000s, at the business cycle frequencies, flatter yield curves became associated with expansions, rather than recessions, which led to the well-known yield curve *conundrum* of 2006.

We have not found evidence of a significant role for the curvature either as a leading or as a coincident indicator of economic activity, nor did we find a clear-cut relation between the curvature and inflation. However, during the *conundrum*, the curvature and the slope were good predictors of the FFR, which indicates that the yield curve may have failed to forecast economic activity but not monetary policy.

The remaining of the paper is organized in five sections. In the second section we describe the related literature, showing how its evolution motivates the use of time-frequency methods. In the third section we present the wavelet analysis tools that are used in the paper. In the fourth section we present the data, with a special focus on the modeling and estimation of the yield

curve latent factors. In the fifth section we present and discuss our empirical results. Section six concludes.

2 Literature overview

In this section we review the literature on the relation between the yield curve and the macroeconomy. We first describe its evolution regarding the set of yield curve components as well as of macroeconomic variables. We then highlight how time-variation or structural breaks in the yield curve-macro relation became dominant in the literature and how it has remained silent about the frequency-domain aspects of the relation, thus establishing the motivations for our paper. We finally refer to the literature that is closer to our paper, clarifying our contributions.

2.1 The yield curve, output and inflation

The ability of the yield curve slope to predict real activity or inflation has been assessed with two classes of regression models. On the one hand, discrete (binary) regression models, in which the dependent variable corresponds to a state of recession or expansion (or to a state of inflation pressure or no pressure); on the other hand, continuous dependent variable models, in which the dependent variable is the growth rate of real output (or changes in the rate of inflation). In some papers, both formulations have, alternatively, been tested and their stability compared – e.g. Estrella, Rodrigues and Schich (2003), Rudebusch and Williams (2009).

Theoretically, only the expectations component of the term spread should help to predict business cycles, as its term premium component reflects the demand for higher yields to compensate for the loss of liquidity and the risk associated to holding longer-term securities. However, Hamilton and Kim (2002) found that both components make statistically significant contributions, similar at short horizons but larger for the expectations component for predicting output more than two years ahead (with interest rate volatility explaining part of the contribution of the term premium component).[1]

Following the seminal paper by Harvey (1988), the term spread (the yield curve slope, typically – but not always – measured as the difference between zero-coupon interest rates of 3-month Treasury bills and 10-year Treasury bonds) has been considered relevant for forecasting business cycles. Stock and Watson (1989) found that interest rate spreads added value to their multivariate index of leading economic indicators. Evidence on the ability of the yield curve slope to predict real economic activity has then been put forth by, e.g., Estrella and Hardouvelis (1991) and Estrella and Mishkin (1998) for the U.S. and Estrella and Mishkin (1997) and Plosser and Rouwenhorst (1994) for several industrialized countries. More recently, it has been shown that

[1] More recently, Estrella and Wu (2008) found that decomposing the spread into expectations and term premium components does not significantly enhance the predictive power of the yield curve. Decomposing the yield slope is beyond the scope of this paper.

the yield slope has a good record in forecasting recessions in real-time (see e.g. Estrella and Trubin, 2006) and has marginal predictive power for U.S. recessions over the Survey of Professional Forecasters (Rudebusch and Williams, 2009). The relevance of the yield slope has survived – and has even been reinforced – in the context of more complex dynamic models and iterative forecasting procedures (see e.g. Kauppi and Saikonen, 2009).[2]

As regards inflation, Mishkin (1990a, 1990b and 1990c) and Jorion and Mishkin (1991) found that the difference between the n-month yield and the m-month yield helped to predict the change in inflation between n and m months ahead.

While most of the earlier literature has focused on the ability of the yield curve to predict real activity or inflation, in theory there could be influences in the opposite direction – see e.g. Estrella (2005) – essentially through the feed-back from the macroeconomy to monetary policy and its impact on the yield curve. Empirical examination of such effects has been made by, e.g., Estrella and Hardouvelis (1991) and Estrella and Mishkin (1997). In the context of VAR models, Ang and Piazzesi (2003) found evidence that, in the U.S., macro variables explain a large part of the variation in yields, which Evans and Marshall (2007) confirmed and attributed mostly to the systematic reaction of monetary policy. In a similar context, Diebold, Rudebusch and Aruoba (2006) found that, in the U.S., the influence from macroeconomic activity to the yield curve is stronger than the opposite way around. Overall, there are theoretical and empirical results that are consistent with a bidirectional relationship.

The literature has recently evolved along two major paths. One has been the enhancement of the yield curve components used to forecast output and, more generally, the build of macro-finance models with a joint modeling of the yield curve components and the main macro variables. The other has been the explicit consideration of time-variation in the relation between the yield curve components and the macro variables. We now discuss these in turn, as both are crucial motivations for this paper.

2.2 The yield curve latent components and the main macroeconomic variables

Following the seminal introduction of macroeconomic variables in the standard affine term structure framework by Ang and Piazzesi (2003), a number of no-arbitrage macro-finance models have been proposed. These have been used in the analysis of several topics, ranging, for example, from the prediction of the yield curve – e.g. Hordahl, Tristani and Vestin (2006) – to the role of inflation expectations in modeling long-term bond yields – e.g. Dewachter and Lyrio (2006)

[2] In spite of the overwhelming evidence, there is still less theoretical agreement about why does the yield curve slope predict real output fluctuations. Traditional explanations rely either on the effects of monetary policy – see e.g. Estrella (2005) – or on movements of the real yield curve and their effect on expectations – see e.g. Harvey (1988). Recently, Adrian, Estrella and Shin (2010) suggested a new causal mechanism deriving from the balance sheet management of financial intermediaries who borrow short and lend long. Disentangling the theoretical linkages between the yield curve and the macro variables is, however, beyond the scope of this paper.

– and to the analysis of the monetary policy regime and the macroeconomic structure – e.g. Rudebusch and Wu (2008).[3] Closer to our purposes, Ang, Piazzesi and Wei (2006) showed that, in such a macro-finance model, including the two first principal components of the curve – corresponding closely to the short interest rate, a proxy for the curve level, and the term spread, a proxy for its slope – enhances the ability of the model to forecast growth.[4]

Parallel to the no-arbitrage literature, another branch has explored the parsimonious modeling of the yield curve suggested by Nelson and Siegel (1987). First, Diebold and Li (2006) showed how to estimate the Nelson-Siegel components as time-varying parameters that distill the entire yield curve shape period-by-period and interpreted them as the level, slope and curvature of the yield curve. Diebold, Rudebusch and Aruoba (2006) augmented the model with time-series of inflation, output and the policy interest rate, suggested a state-space representation for such macro-finance model and estimated it by maximum-likelihood with the Kalman filter.

Following Diebold, Rudebusch and Aruoba (2006), among others, it became relatively consensual to associate the yield curve level to inflation – especially at low frequencies, reflecting a possible link with inflation expectations –, and the slope to the business cycle. The slope-business cycle association is not, however, as consensual as the level-inflation association (Moench, 2010 found that innovations to the slope generate immediate but mild and insignificant responses of real output). As regards the curvature, while Dewachter and Lyrio (2006) suggested it is associated with monetary policy, its relation with the macroeconomy has been harder to establish. Recently, while Modena (2008) has suggested that it could be a coincident indicator for economic activity, Moench (2010) has argued that it is a leading indicator, finding that unexpected increases of the curvature factor (higher concavity) precede a flattening of the yield curve (higher slope) and a significant decline of output more than 1 year ahead.

More recently, Christensen, Diebold and Rudebusch (2009) have specified a generalized no-arbitrage Nelson-Siegel model of the yield curve, bridging the gap between the two above referred branches of the macro-finance literature (see also Rudebusch, 2010). However, as Diebold and Li (2006) and Diebold, Rudebusch and Aruoba (2006) state, it is not clear that arbitrage-free models are necessary or even desirable for forecasting exercises: if the data abides by the no-arbitrage assumption, then the parsimonious but flexible Nelson-Siegel curve should at least approximately capture it; if it's not, imposing it would depress the model's ability to forecast the yield curve and the macro variables. Motivated by these and other additional arguments, as further detailed in section 4, in this paper we follow the parsimonious Nelson-Siegel decomposition of the yield curve.

[3] See Diebold, Piazzesi and Rudebusch (2005) for a review of the recent evolution and challenges facing the macro-finance models. For the inclusion of a yield curve in the new-keynesian dynamic stochastic general equilibrium models that are currently used for monetary policy conduction and assessment see, e.g., De Graeve, Emiris and Wouters (2009).

[4] Wright (2006) confirmed that there is more information in the shape of the yield curve about the probability of recessions than that provided by the term spread, in the context of probit regression models for predicting U.S. recessions.

2.3 Time-variation in the relation between the yield curve and the main macroeconomic variables

The possible time-variation in the yield curve-macro relation has been receiving an increased attention. Initially, within bivariate models of an yield curve factor – generally the slope – and a macro variable, focusing on changes in the intensity and in the time-lags of the relations and using structural break tests; recently, within time-varying parameters models; and, very recently, in the context of macro-finance models.

Stock and Watson (1999) documented econometric instability in the cyclical behavior of a number of U.S. macroeconomic time-series, including the yield curve slope. Haubrich and Dombrosky (1996) found that the predictive ability of the yield spread, although very good, has changed over time. Dotsey (1998) showed that, in contrast to previous periods, the information content of the slope is not statistically significant between the beginning of 1985 and the end of 1997. Estrella, Rodrigues and Schich (2003) tested for structural breaks in models of the slope and real output or inflation, for discrete and continuous dependent variable regressions; overall, they found that models of real output are more stable than models of inflation, and that discrete regression models are more stable than continuous models of the growth rate of output or the inflation rate. Using several alternative measures for the yield slope and multiple structural break tests, Giacomini and Rossi (2006) found a significant breakdown in the forecasting performance of the slope in 1974-76 and in 1979-87. Kucko and Chinn (2009) compared the ability of the yield slope to forecast industrial production growth in samples before and after 1999, finding that overall the predictive ability of the yield slope has decreased after 1998. As Hamilton (2010) refers, recent anecdotal evidence of instability in the yield curve–macro relation is the well-known episode of the summer of 2006 when an inverted yield curve was not followed by a recession, possibly because of the very low level of the curve.

A second line of literature has modeled time-variation with more sophisticated methods, but has overall remained focused on a single component of the yield curve – the slope – and its relation to one macro variable. Using Bayesian time-varying parameters VARs with stochastic volatility, Benati and Goodhart (2008) detected changes in the marginal predictive power of the yield slope for output growth at several forecast horizons in a number of countries, which have not always followed the same pattern for alternative forecast horizons. Time-varying parameter models relating the yield slope with output growth, with ex-post and real-time data, have been used by De Pace (2009) to find a decrease in the marginal predictive power in the recent years in the U.S. and U.K. and a marked instability of the relation in continental European countries. Chauvet and Potter (2005) allowed for time-varying parameters and for auto-correlated errors (to account for the duration of business cycles) in a discrete regression model of the yield slope and output growth in the U.S., finding that once such time-variation is considered, inversions of the yield curve are associated to high probabilities of recessions. In a dynamic bi-factor model that produces an yield curve cycle and a business cycle, each following its own two-state Markov

switching process, Chauvet and Senyuz (2009) found evidence of time-variation and breaks in the forecast-horizon at which yields predict output growth.

Very recently, some papers have allowed for time-varying dynamic relations within macro-finance models, rather than bivariate models. And while some have done so imposing no-arbitrage restrictions – e.g. Ang, Boivin, Dong and Loo-Kung (2009) – others have pursued versions of the Nelson and Siegel (1987) parsimonious yield curve model – e.g. Mumtaz and Surico (2009) and Bianchi, Mumtaz and Surico (2009a, 2009b).

2.4 Time and frequency variation in the relation between the yield curve and the macroeconomy: the Wavelet approach

Having devoted most of the effort to tackling time-variation issues, the literature has remained silent on the frequencies (cyclical periodicity) at which the relation between the yield curve components and the macroeconomic variables occurs. Yet, given the changes in the structure of the economy and in the monetary policy regimes, there surely may have been frequency variations in the yield curve–macro relation.

Overall, progress in the study of the yield curve-macro relation may be pursued with a framework that (i) considers the whole yield curve shape (level, slope, curvature), (ii) allows for time-varying sensitivity and lead/lags, and (iii) allows for time-varying frequencies. The continuous time-frequency framework thus emerges as an approach with unique advantages to study this topic.

Against this background, we use wavelet analysis tools – previously employed by Aguiar-Conraria, Soares and Azevedo (2008) and Aguiar-Conraria and Soares (2011a) – to disentangle the time-frequency relations between the 3 Nelson-Siegel latent factors of the yield curve (level, slope and curvature) and 4 macroeconomic variables (unemployment, an index of macroeconomic activity, inflation and the monetary policy interest rate) in the U.S. Following analyses using the wavelet power spectrum, we then compute the cross-wavelet transform and coherence as well as the phase difference. For each pair formed by a yield curve factor and a macroeconomic variable, these tools give us quantified indications of, respectively, the similarity of power between each time series and a measure of the lead-lags between their oscillations, at each time and frequency. These wavelet tools provide a thorough vision of the inter-relation between the yield curve components and the macro variables that is almost impossible to obtain with purely time-domain or frequency-domain analysis.

3 Wavelets Analysis

This section presents a (necessarily very brief) introduction to wavelet analysis. If interested in a detailed technical overview the reader can check Aguiar-Conraria and Soares (2011b). For

a thorough intuitive discussion on these concepts, the reader is referred to Aguiar-Conraria, Magalhães and Soares (2012).

3.1 The Wavelet

A function ψ qualifies for being a mother wavelet only if ψ is a square integrable function and also if it fulfills a technical condition, usually referred to as the *admissibility condition*. For most of the applications, the wavelet ψ must be a well localized function, both in the time domain and in the frequency domain, in which case the admissibility condition reduces to requiring that ψ has zero mean, i.e. $\int_{-\infty}^{\infty} \psi(t)\, dt = 0$. This means that the function ψ has to wiggle up and down the $t-$axis, i.e. it must behave like a wave; this, together with the assumed decaying property justifies the choice of the term wavelet (small wavelet) to designate ψ.

3.1.1 The Continuous Wavelet Transform

Starting with a mother wavelet ψ, a family $\psi_{\tau,s}$ of "wavelet daughters" can be obtained by simply scaling and translating ψ:

$$\psi_{\tau,s}(t) := \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right), \quad s, \tau \in \mathbb{R}, s \neq 0, \tag{1}$$

where s is a scaling or dilation factor that controls the width of the wavelet and τ is a translation parameter controlling the location of the wavelet. Scaling a wavelet simply means stretching it (if $|s| > 1$) or compressing it (if $|s| < 1$), while translating it simply means shifting its position in time. Given a time series $x(t)$, its continuous wavelet transform with respect to the wavelet ψ is a function of two variables, $W_x(\tau, s)$:

$$W_x(\tau, s) = \int x(t) \frac{1}{\sqrt{|s|}} \overline{\psi}\left(\frac{t-\tau}{s}\right) dt, \tag{2}$$

where the bar denotes complex conjugation.

3.1.2 The Choice of the Mother Wavelet

There are several types of wavelet functions available with different characteristics, such as, Morlet, Mexican hat, Haar, Daubechies, etc. Since the wavelet coefficients $W_x(s, \tau)$ contain combined information on both $x(t)$ and $\psi(t)$, the choice of the wavelet is an important aspect to be taken into account, which will depend on the particular application one has in mind.

If quantitative information about phase interactions between two time-series is required, continuous, rather than discrete, and complex wavelets provide the best choice. When the wavelet $\psi(t)$ is chosen as a complex-valued function, the wavelet transform $W_x(\tau, s)$ is also complex-valued. In this case, the transform can be separated into its real part, $\Re(W_x)$, and imaginary

part, $\Im(W_x)$, or in its amplitude, $|W_x(\tau,s)|$, and phase, $\phi_x(\tau,s) : W_x(\tau,s) = |W_x(\tau,s)|\, e^{i\phi_x(\tau,s)}$. The phase-angle $\phi_x(\tau,s)$ of the complex number $W_x(\tau,s)$ can be obtained from the formula: $\tan(\phi_x(\tau,s)) = \frac{\Im(W_x(\tau,s))}{\Re(W_x(\tau,s))}$, using the information on the signs of $\Re(W_x)$ and $\Im(W_x)$ to determine to which quadrant the angle belongs to.

Analytic wavelets[5] are ideal for the analysis of oscillatory signals, since the continuous analytic wavelet transform provides an estimate of the instantaneous amplitude and instantaneous phase of the signal in the vicinity of each time/scale location (τ,s).

Therefore, for our applications it is essential to choose a complex analytic wavelet, as it yields a complex transform, with information on both the amplitude and phase, crucial to study the cycles synchronism. Examples of popular analytic wavelets are the Paul, Gaussian, Morlet, and Shannon mother wavelets. The Morlet wavelet has one major property: it has optimal joint time-frequency concentration.[6]

For all these reasons we will use the Morlet wavelet, first introduced in Goupillaud et al. (1984):

$$\psi_{\omega_0}(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}. \tag{3}$$

All our numerical results are obtained with the particular choice $\omega_0 = 6$. For this parametrization of the Morlet wavelet, there is an inverse relation between wavelet scales and frequencies, $f \approx \frac{1}{s}$, greatly simplifying the interpretation of the empirical results. Thanks to this very simple one-to-one relation between scale and frequency we can use both terms interchangeably.

3.2 Wavelet Tools

In analogy with the terminology used in the Fourier case, the (local) *wavelet power spectrum* (sometimes called scalogram or wavelet periodogram) is defined as

$$(WPS)_x(\tau,s) = |W_x(\tau,s)|^2. \tag{4}$$

This gives us a measure of the variance distribution of the time-series in the time-scale/frequency plane.[7]

The concepts of cross wavelet power, wavelet coherency and phase-difference are natural generalizations of the basic wavelet analysis tools that enable us to deal with the time-frequency dependencies between two time-series. The cross-wavelet transform of two time-series, $x(t)$ and

[5] A wavelet $\psi(t)$ is analytic if its Fourier transform is such that $\hat{\psi}(f) = 0$, for $f < 0$.

[6] Theoretically, the time-frequency resolution of the continuous wavelet transform is bounded by the Heisenberg box, which describes the trade-off relationship between time and frequency. The area of the Heisenberg box is minimized with the choice of the Morlet wavelet.

[7] Sometimes the wavelet power spectrum is averaged over time for comparison with classical spectral methods. When the average is taken over all times, we obtain the global wavelet power spectrum, $(GWPS)_x(s,\tau) = \int_{-\infty}^{\infty} |W_x(\tau,s)|^2\, d\tau$.

$y(t)$, is defined as

$$W_{xy}(\tau, s) = W_x(\tau, s)\,\overline{W_y}(\tau, s), \tag{5}$$

where W_x and W_y are the wavelet transforms of x and y, respectively. We define the *cross wavelet power*, as $|W_{xy}(\tau, s)|$. The cross-wavelet power of two time-series depicts the local covariance between two time-series at each time and frequency. Therefore, the cross-wavelet power gives us a quantified indication of the similarity of power between two time-series. When compared with the cross wavelet power, the *wavelet coherency* has the advantage of being normalized by the power spectrum of the two time-series. In analogy with the concept of coherency used in Fourier analysis, given two time-series $x(t)$ and $y(t)$ one defines their wavelet coherency:

$$R_{xy}(\tau, s) = \frac{|S(W_{xy}(\tau, s))|}{\sqrt{S(|W_{xx}(\tau, s)|)\,S(|W_{yy}(\tau, s)|)}},$$

where S denotes a smoothing operator in both time and scale.

Although there is some work done on the theoretical distribution of the wavelet power (Ge, 2007) and on the distribution of cross wavelets (Ge, 2008), the available tests imply null hypotheses that are too restrictive to deal with economic data. Therefore, we will rely on Monte Carlo simulations for statistical inference.

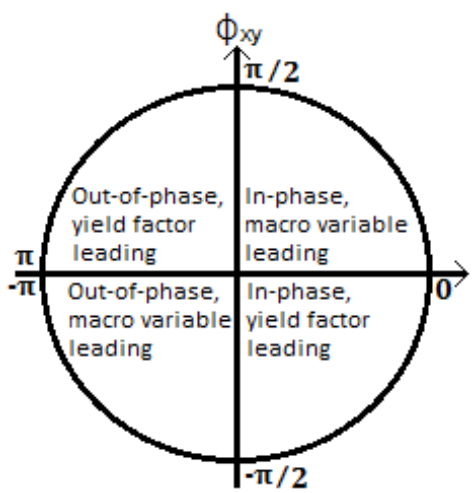

Figure 1: Phase-difference relations

As we have discussed, one of the major advantages of using a complex-valued wavelet is that we can compute the phase of the wavelet transform of each series and thus obtain information about the possible delays of the oscillations of the two series as a function of time and scale/frequency, by computing the phase difference. The *phase difference* can be computed from the cross wavelet transform, by using the formula

$$\phi_{x,y}(s, \tau) = \tan^{-1}\left(\frac{\Im\left(W_{xy}(s, \tau)\right)}{\Re\left(W_{xy}(s, \tau)\right)}\right), \tag{6}$$

and information on the signs of each part to completely determine the value of $\phi_{xy} \in [-\pi, \pi]$. A phase-difference of zero indicates that the time series move together at the specified frequency; if $\phi_{xy} \in (0, \frac{\pi}{2})$, then the series move in phase, but the time-series x leads y; if $\phi_{xy} \in (-\frac{\pi}{2}, 0)$, then it is y that is leading; a phase-difference of π (or $-\pi$) indicates an anti-phase relation; if $\phi_{xy} \in (\frac{\pi}{2}, \pi)$, then y is leading; time-series x is leading if $\phi_{xy} \in (-\pi, -\frac{\pi}{2})$. In the course of this paper the interpretation of the phase-difference is the one suggested in Figure 1.

With the phase difference one can calculate the instantaneous time lag between the two time-series:

$$\Delta T(s, \tau) = \frac{\phi_{x,y}(s, \tau)}{2\pi f(\tau)}, \tag{7}$$

where $f(\tau)$ is the frequency that corresponds to the scale τ.

As with other types of transforms, the continuous wavelet transform applied to a finite length time-series inevitably suffers from border distortions; this is due to the fact that the values of the transform at the beginning and the end of the time-series are always incorrectly computed, in the sense that they involve missing values of the series which are then artificially prescribed. When using a discretized version of formula (2), a periodization of the data is assumed. However, we pad the series with zeros, to avoid wrapping. These edge-effects are larger at lower frequencies. The region in which the transform suffers from these edge effects is called the cone of influence. In this area of the time-frequency plane the results are subject to border distortions and have to be interpreted carefully.

4 Data and Estimation

In this section we present the data used in our wavelet analyses. In a first subsection we describe the source of the zero-coupon yield data and then the modeling choices made to estimate the latent factors that define the shape of the yield curve at each moment. In a second subsection we present the macroeconomic data. For each of our seven time-series, we provide and analyze their wavelet power spectrum, which is a useful preliminary information.

4.1 Yield data and the yield curve latent factors

At each point in time, the yield curve is the set of yields of zero-coupon Treasury securities for each residual maturity. As, in practice, the Treasury issues a limited number of securities with different maturities and coupons, obtaining the yield curve at each moment requires estimation, i.e. inferring what the zero-coupon yields would be across the whole maturity spectrum. Yield curve estimation requires the assumption of some model for the shape of the yield curve, so that the gaps may be filled in by analogy with the yields seen in the observed maturities. Once a model is selected, estimates of its coefficients are chosen so that the weighted sum of the squared deviations between the actual prices of Treasury securities and their predicted prices is

minimized. Once the values for the coefficients are estimated, they may be straightforwardly used to obtain the notional zero-coupon yields for the residual maturities absent from the raw data.[8]

In this paper we use U.S. yield curve data for 1961:6-2011:12 publicly made available by Gurkaynak, Sack and Wright (2007b). These data are estimated with an approach that follows the extension by Svensson (1994) of the functional form originally suggested by Nelson and Siegel (1987). The online database Gurkaynak, Sack and Wright (2007b) provides regularly updated daily zero-coupon yields for all yearly maturities from 1 to 30 years and has been increasingly used in recent research (see e.g. De Graeve, Emiris and Wouters, 2009, and Chauvet and Senyuz, 2009).

Given our purpose of relating the yield curve with macro variables, we are not interested in daily but rather in monthly yield curve data, therefore we use average monthly yield data. Following the literature, we are not interested in the very-long end of the yield curve (maturities above 10 years), while, in contrast, we are interested in a richer set of yield curve points for short and medium term residual maturities than those present in Gurkaynak, Sack and Wright (2007b). Accordingly, we use the appropriate formulae and parameters in Gurkaynak, Sack and Wright (2007a, 2007b) and compute the implied zero-coupon yields for a set of additional relevant intra-year maturities. We end up with monthly-average time-series of zero-coupon yields for the 17 maturities considered in Diebold, Rudebusch and Aruoba (2006): 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months.

We then use these yield curve data to estimate the yield curve latent factors – level, slope and curvature –, following the parsimonious Nelson and Siegel (1987) approach to the modeling of the yield curve used by e.g. Diebold and Li (2006) and Diebold, Rudebusch and Aruoba (2006). Our choice of not following an arbitrage-free approach is motivated by two main arguments. First, as Diebold and Li (2006, pp. 361-362) and Diebold, Rudebusch and Aruoba (2006, pp. 333) argued, for empirical exercises, the actual data are more relevant than data made consistent with theoretical models that preclude behaviors possibly present in the real world: if the market behavior by agents generates arbitrage-free data, then the flexible Nelson-Siegel curve should capture that feature of the data reasonably well; if agents do not completely exploit arbitrage opportunities, it is reasonable to expect that the resulting non-arbitrage-free data are the data that should correlate with the macroeconomic variables actually observed in the economy. It is true that Diebold and Li (2006) and Diebold, Rudebusch and Aruoba (2006) have put forth the argument for forecasting exercises, however our empirical analyses may be seen as a preliminary step of forecasting exercises, given that we focus on detecting co-movements between the yield curve and the macroeconomic variables with possible leads and lags. Second, the zero-coupon yield data that is the basis for this research has been computed with a variant of the Nelson and Siegel (1987) model; for example, its formulae and coefficients are used to interpolate the

[8] Typically, yield curve estimation further requires filtering out some issues that have insignificant liquidity, due to small outstanding amounts or residual life. See Gurkaynak, Sack and Wright (2007a) for additional details.

time-series of yields when, for any residual maturities, no quoted prices have been formed or the market prices resulted from very illiquid markets; while not compulsory, we find it valuable that the estimation of the latent factors of the yield curve is based on a model that is consistent with the one used in the computation of the original time-series of zero-coupon yields.

The yield curve is modeled with the three-component exponential approximation to the cross-section of yields at any moment in time proposed by Nelson and Siegel (1987),

$$y(\tau) = \beta_1 + \beta_2 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_3 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) \tag{8}$$

where $y(\tau)$ denotes the set of (zero-coupon) yields and τ denotes the corresponding maturity.

Following Diebold and Li (2006) and Diebold, Rudebusch and Aruoba (2006), the Nelson-Siegel representation is interpreted as a dynamic latent factor model where β_1, β_2 and β_3 are time-varying parameters that capture the level (L), slope (S) and curvature (C) of the yield curve at each period t, while the terms that multiply the factors are the respective factor loadings:

$$y(\tau) = L_t + S_t \left(\frac{1 - e^{-\lambda\tau}}{\lambda_\tau} \right) + C_t \left(\frac{1 - e^{-\lambda\tau}}{\lambda_\tau} - e^{-\lambda\tau} \right). \tag{9}$$

L_t may be interpreted as the overall level of the yield curve, as its loading is equal for all maturities; S_t has a maximum loading (equal to 1) at the shortest maturity, which then monotonically decays through zero as maturities increase; C_t has a loading that is null at the shortest maturity, increases until an intermediate maturity and then falls back to zero as maturities increase. Hence, S_t and C_t may be interpreted as the short-end and medium-term latent components of the yield curve, with the coefficient λ ruling the rate of decay of the loading towards the short-term factor and the maturity where the medium-term factor has maximum loading. Some authors use, instead, empirical proxies to these factors. We prefer the latent factors approach, not only because it is based on a formal model, facilitating the economic interpretation, but also because it uses the information across the whole yield curve maturities.[9]

As in Diebold, Rudebusch and Aruoba (2006) we assume that L_t, S_t and C_t follow a vector autoregressive process of first order, which allows for casting the yield curve latent factor model in state-space form and using the Kalman filter to obtain maximum-likelihood estimates of the hyper-parameters and the implied estimates of the time-varying parameters L_t, S_t and C_t.

The state-space form of the model comprises the transition system

$$\begin{bmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{bmatrix} + \begin{bmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{bmatrix}, \tag{10}$$

[9] Note that the empirical measures only use information on three different maturities: $Level_t = (y_t(3) + y_t(24) + y_t(120))/3$, $Slope_t = y_t(3) - y_t(120)$, $Curvature_t = -(y_t(3) + y_t(120)) + 2y_t(24)$, where $y_t(m)$ refers to the zero coupon yield of maturity m.

where $t = 1, \ldots .T$ is the sample period, μ_L, μ_S and μ_C are estimates of the mean values of the three latent factors, and $\eta_t(L)$, $\eta_t(S)$ and $\eta_t(C)$ are innovations to the autoregressive processes of the latent factors.

The state-space form further comprises the measurement system, relating a set of N observed zero-coupon yields of different maturities to the three latent factors by

$$\begin{bmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{bmatrix} = \begin{bmatrix} 1 & \left(\frac{1-e^{-\lambda\tau_1}}{\lambda_{\tau_1}}\right) & \left(\frac{1-e^{-\lambda\tau_1}}{\lambda_{\tau_1}} - e^{-\lambda\tau_1}\right) \\ 1 & \left(\frac{1-e^{-\lambda\tau_2}}{\lambda_{\tau_2}}\right) & \left(\frac{1-e^{-\lambda\tau_2}}{\lambda_{\tau_2}} - e^{-\lambda\tau_2}\right) \\ \vdots & \vdots & \vdots \\ 1 & \left(\frac{1-e^{-\lambda\tau_3}}{\lambda_{\tau_3}}\right) & \left(\frac{1-e^{-\lambda\tau_N}}{\lambda_{\tau_N}} - e^{-\lambda\tau_N}\right) \end{bmatrix} \begin{bmatrix} L_t \\ S_t \\ C_t \end{bmatrix} + \begin{bmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{bmatrix}, \tag{11}$$

where $t = 1, \ldots, T$, and $\varepsilon_t(\tau_1), \varepsilon_t(\tau_2), \ldots \varepsilon_t(\tau N)$ are measurement errors, i.e. deviations of the observed yields at each period t and for each maturity τ from the implied yields defined by the shape of the fitted yield curve. In matrix notation, the state-space form of the model may be written, using the transition and measurement matrices A and Λ, as

$$f_t - \mu = A(f_{t-1} - \mu) + \eta_t \tag{12}$$

$$y_t = \Lambda f_t + \varepsilon_t \tag{13}$$

For the Kalman filter to be the optimal linear filter, it is assumed that the initial conditions set for the state vector are uncorrelated with the innovations of both systems: $E\left(f_t \eta_t^T\right) = 0$ and $E\left(f_t \varepsilon_t^T\right) = 0$.

Following Diebold, Rudebusch and Aruoba (2006) we assume that the innovations of the measurement and of the transition systems are white noise and mutually uncorrelated

$$\begin{bmatrix} \eta_t \\ \varepsilon_t \end{bmatrix} \sim WN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & H \end{bmatrix}\right), \tag{14}$$

where the matrix of variance-covariance of the innovations to the transition system Q is unrestricted, while the matrix of variance-covariance of the innovations to the measurement system H is assumed to be diagonal. The latter assumption means that the deviations of the observed yields from those implied by the fitted yield curve are uncorrelated accross maturities and time. Given the large number of observed yields used, this is necessary for computational tractability (Diebold, Rudebusch and Aruoba 2006). Moreover, it is also a quite standard assumption, as, for example, i.i.d. errors are typically added to observed yields in estimating no-arbitrage term structure models.

Given a set of starting values for the parameters (the three latent factors) and for the hyperparameters (the coefficients that define the statistical properties of the model, such as, e.g., the variances of the innovations), the Kalman filter may be run from $t = 2$ through $t = T$ and

the one-step-ahead prediction errors and the variance of the prediction errors may be used to compute the log-likelihood function. The function is then iterated on the hyper-parameters with standard numerical methods and at its maximum yields the maximum-likelihood estimates of the hyper-parameters and the implied estimates of the time-series of the time-varying parameters L_t, S_t and C_t. The latent factors are then recomputed with the Kalman smoother, which uses the whole dataset information to obtain the factors at each period from $t = T$ through $t = 2$ (see Harvey, 1989, for details on the Kalman filter and the fixed-interval Kalman smoother).

The resulting time-series, which are depicted in the left-hand-side panel of Figure 2, are those subject to the cross wavelet analyses of the next section, jointly with the macroeconomic data described in the next sub-section. [10]

The figure further includes (right-hand-side panel) the power spectrum of each of the latent factors.[11] It indicates, for each moment of time the intensity of the variance of the time-series for each frequency of cyclical oscillations. While it is computed exclusively with the information contained in the time-series (left-hand-side panel), it maps it into the time-frequency domain. Before moving to the study of bivariate relations, the wavelet spectra provide a first assessment of the individual behavior of the data in this time and frequency varying framework.

Until 1990, the yield level records a very high variance for long cycles, in frequency bands from 10 to 16 years, which confirms its high persistence and apparent low volatility. Overall, the power spectrum is statistically significant for frequency bands below 8 years throughout the whole sample period, thus indicating a significant variance for such long cycles. In the 1980s, there is a region of significant power at a frequency band between 3 and slightly more than 4 years, which matches the intense short-run cyclical oscillations of the level apparent in the time-series plot during that period, probably associated with the active management of interest rates to disinflate the economy while managing the business cycle initiated after the monetarist experiment of 1979-82, which followed the oil crises, and subsequent inflationary pressures, of the 1970s.[12]

[10] Recall that, by construction, negative values of the slope correspond to the typical upward sloping yield curve (positive values of the slope correspond to inverted yield curves) and thus most of the times the slope factor is negative. In turn, positive, null and even (in most cases) small negative values of the curvature correspond to the typical concave yield curve; below some negative value (which is a function of the value of the slope) the yield curve turns into a convex curve. In short, the lower the value of the slope, the steeper is the yield curve; the lower the value of the curvature, the less concave is the yield curve.

[11] Our wavelet figures throughout the paper depict the power at each time-frequency region associating colder colors (in the extreme, blue) with low power and hotter colors (in the extreme, red) with higher power. The dark lines represent regions of statistically significant powers at 5 percent, while grey lines delimit regions significant at 10 percent. Significance levels have been obtained by bootstraping with 5000 replications. The white stripes show the maxima of the undulations of the wavelet power spectrum. The cone of influence is shown with a black line. This indicates the region affected by edge effects and where caution should be applied when interpreting the evidence.

[12] Given that, at this point, we are just performing univariate analysis, we cannot statistically infer the causes of these regions of high volatility. We do, however, interpret these regions based on previous results from the literature. Of course, other economists may have alternative interpretations. This remark is valid for all our interpretations of the pictures of subsections 4.1 and 4.2.

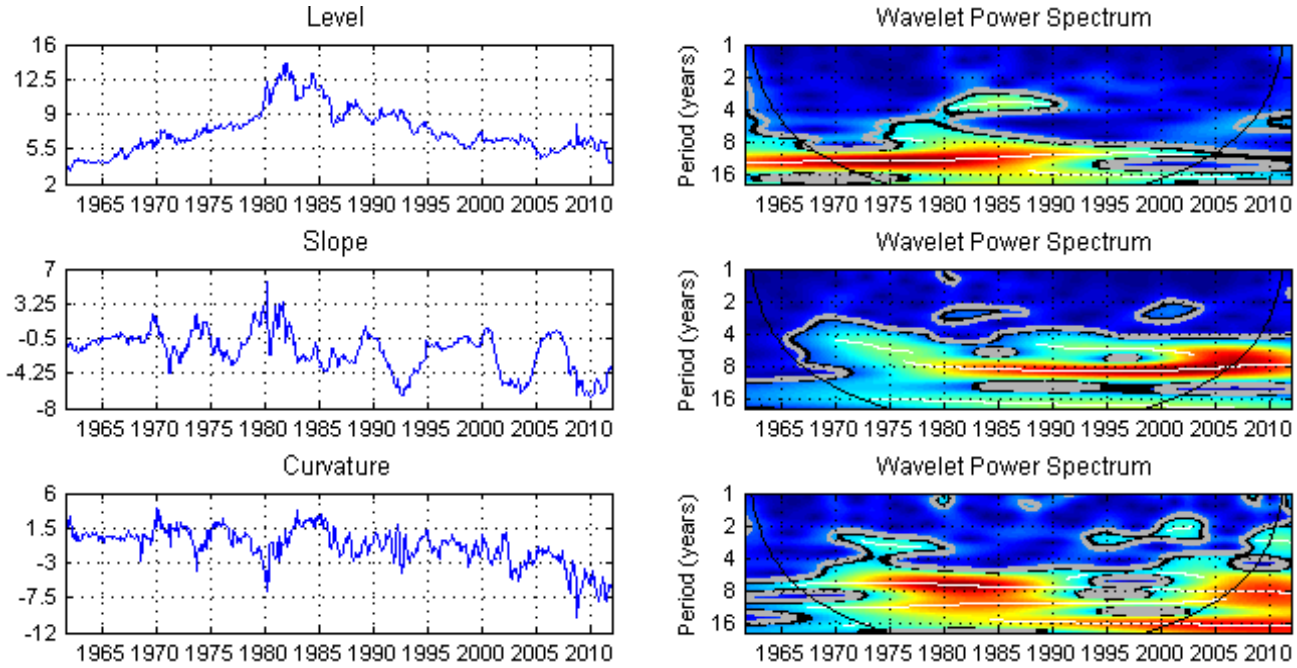

Figure 2: Level, Slope and Curvature of Yield Curve, U.S. 1961:7-2011:12

The power spectrum of the slope is statistically significant at 5 percent throughout the whole sample for frequency bands that correspond to cycles of period above 4 years, indicating that the variance of the slope is significant at business cycles frequencies, as expected under the monetary policy explanation of the relation between the slope and the macroeconomy. The time-frequency regions with higher values for the power spectrum have occurred since the mid-1970s at frequencies around 8 years, becoming stronger after 1980.

The power spectrum of the curvature displays a pattern somewhat similar to the slope, as the variance of the series is statistically significant for cycles of period above 4 years throughout all the sample. The peaks of the power spectrum are less concentrated, in the case of the curvature. In fact, a very high power is detected during the 1970s and 1980s at a frequency band of between 6 and 8 years, but also, since mid-1970s, at a band centered on 12 years – which then gradually moves to a band that approaches the 16 year frequency and is visible until the end of the sample.

4.2 Macroeconomic data

Our macroeconomic data include two variables meant to proxy for real output (the civilian unemployment rate, and an index of economic activity), a measure of price inflation (yearly per cent changes of the Consumer Price Index) and a measure of the monetary policy interest rate (the Effective Federal Funds Rate). With the exception of the economic activity index, the data have been downloaded from the St. Louis Fed website (FRED database).

Given that real GDP is not observed monthly, we must proxy it with alternative monthly indicators. One obvious candidate is the unemployment rate, but it is well known that it

lags behind real GDP growth and that it features less volatility and higher persistence than quarterly real output growth. Given the relevance of timing issues in our empirical analyses, we decide not to rely only on the unemployment rate as indicator of real economic activity, but rather use it as complementary to a coincident index of real economic activity. A possible monthly indicator of economic activity would be the industrial production index. However this indicator is rather limited in its coverage of the overall economic activity – especially as the U.S. increasingly developed non-industrial activities since the 1980s. Another possible indicator would be the Chicago Fed National Activity Index (CFNAI). Unfortunately, there is no data for this indicator before 1967, so using it would imply missing a significant amount of data. All considered, we chose to use the Aruoba Diebold Scotti Index (ADS index, see Aruoba, Diebold and Scotti, 2009), available from the Philadelphia Fed website. This is a coincident index estimated in real-time with six macroeconomic monthly indicators that is available for all our sample period and has been increasingly used in recent research (see the index website for details on its construction). The contemporaneous correlation between the CFNAI and the ADS is above 94% (lagged correlations are smaller), indicating that they are close substitutes. The (negative) correlation between the ADS and the unemployment rate is maximized for a lag of about a year, which is in line with the lag between GDP growth and unemployment, and is a feature of the data that must be taken into consideration for a proper interpretation of our empirical results obtained in the next section.

Figure 3 presents the time-series of our macroeconomic variables, as well as their wavelet power spectrum, showing their behavior in the time-frequency space.

The wavelet power spectrum of the unemployment rate is, overall, statistically significant during most of the sample period for cycles of periodicity higher than 4 years. Around the first oil shock, the power is significant also at cycles of smaller periodicity, a pattern that is also evident in the first half of the 1980s, during the bulk of the disinflation. As regards the time-frequency regions with higher power, one can identify a peak in the 8∼12 frequency band, from 1970 until 2000. Around the 6 year frequency, we observe two peaks, one in the middle and the second half of the 1970s, coinciding with the oil crises, and another in the second half of the first decade of the new millennium, coinciding with the financial and economic crisis.

At business cycle frequencies, the wavelet power spectrum of the index of economic activity ADS was not high in the decade of 1960. After that, during the decade of 1970 and in early 1980s, the variance at the business cycle frequency was quite high again, probably as a result of the severe oil crisis that hit the world economy in 1973 and 1979 and lasted until the early 1980s. These results for the macroeconomic volatility in the United States are compatible with the results of Gallegati and Gallegati (2007), who studied this issue for the G-7 economies using discrete wavelet analysis. After that, volatility decreases. These results from wavelet analysis help to qualify some of the results present in the literature. The literature has identified 1984 as the year that marks the beginning of the Great Moderation (Kim and Nelson 1999;

McConnell and Pérez-Quirós 2000). In fact, in the 1960s the volatility was very low, after the large fluctuations that characterized the previous decades. It then was revived, due to the oil shocks, at the business cycle frequency in the 1970s, however this increase was temporary. These results are in line with Blanchard and Simon (2001) who have argued that the large shocks in the 1970s and the deep contraction in early 1980s hide from view the longer term volatility decline that began a few decades before. As one would expect, given the recent financial and economic crises, after 2005 there is again evidence that volatility is increasing, suggesting that the 'Great Moderation' is not so great anymore. We see this because the wavelet power spectrum becomes statistically significant in the late 2000s at 2 to 8 years frequencies. Although part of this region may be affected by edge effects (because it is under the effect of the cone of influence) it is also true that a part of it is not affected by those edge effects.[13]

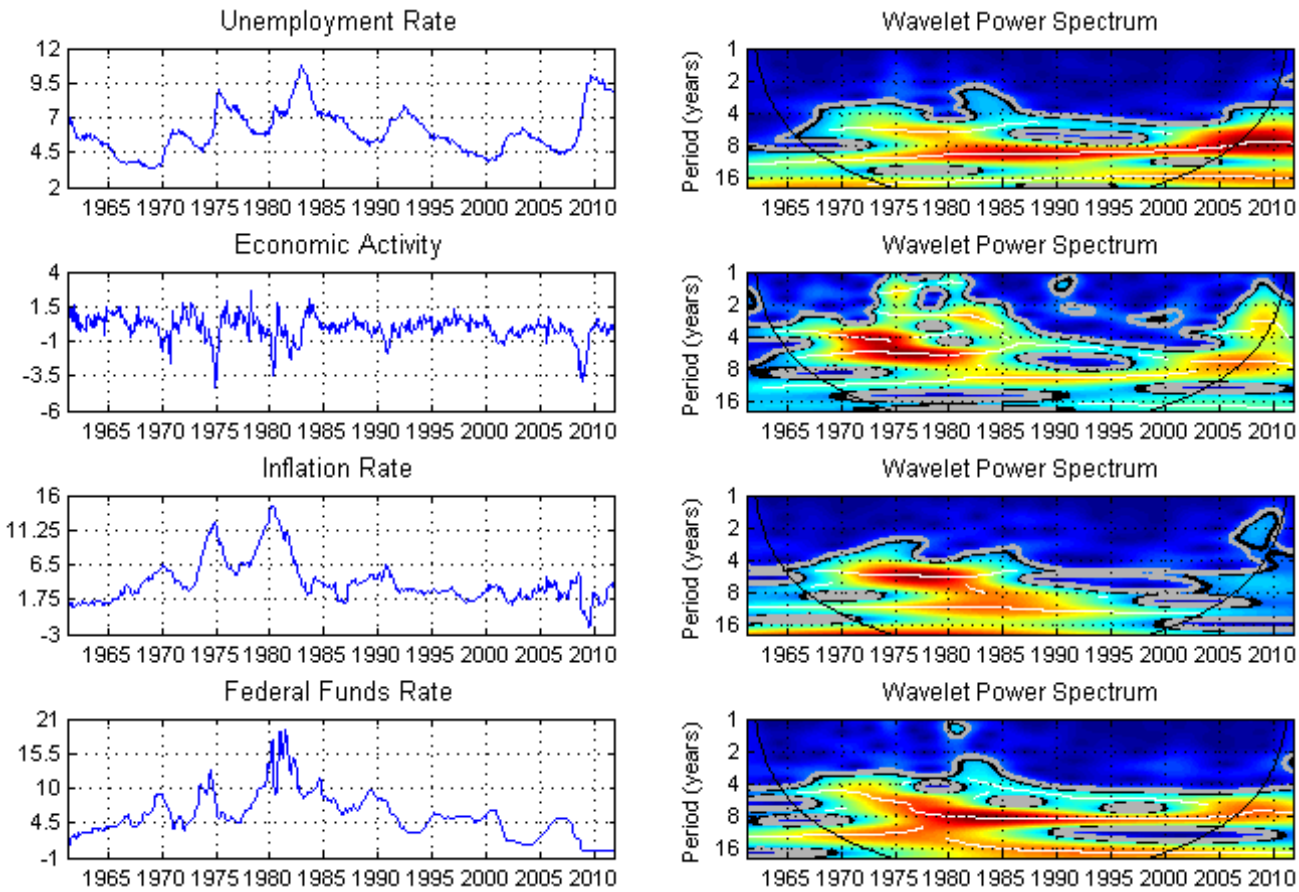

Figure 3: Macroeconomic Variables, U.S. 1961:6-2011:12

The power spectrum of inflation clearly shows the buildup of inflation since the 1960s, with an expansion of the regions with significant power, and the inflationary effect of the oil shocks – with visible peaks in the 4∼8 frequency-band during the 1970s and early 1980s. It then shows

[13] One should also keep in mind that, because of the zero padding (after demeaning the series), this influence will tend to underestimate, not overestimate, the power spectrum, which, actually, reinforces our results.

the gradual control of inflation after the mid-1980s, with a decrease in the regions of significance and a gradual shift of the volatility peaks to cycles of longer period – frequency-bands of more than 8 years – until the disappearance of peaks since the early 1990s, consistent with the recent control of the level and the volatility of inflation. There is evidence that this control includes long-run inflation – and thus inflation expectations – as the peak in frequencies corresponding to more than 16 years that existed since the 1960s also disappears around 1990. Therefore, we also observe a great moderation on the volatility of inflation, which started in the 1950s and was temporarily revived during the oil crises in the 1973 and 1979. These results show that the "Great Moderation" is not just a real phenomenon but also a nominal phenomenon.

In the second half of the 1960s, as the level of interest rates accommodated increasing inflation, the power spectrum of the FFR starts showing peaks at frequency bands around the 12 years. These peaks then extend through cycles of higher frequency and join a region of peaks that appeared during the early 1970s at the 4∼8 frequency-band, to form a region of very strong power in frequencies between 6 and 16 years in the second half of the 1970s. In 1980, at 8 years frequency, starts the region painted with the darkest red, showing that the beginning of the strongest peak in the interest rate variance coincides with the 1979-82 monetarist experiment. The whole disinflationary policy is apparent in the peaks that occur in a broad range of frequencies. The power spectrum then evolves to a concentration of energy around two poles, the 8 and the 16 years frequencies. These eventually fade out during the 1990s, which suggests that monetary policy has been far less active thereafter, given the above mentioned control of inflation. At the end of the sample there are signs of a resurrection of monetary policy, especially at business cycles frequencies (cycles between 6 and 8 years) clearly in response to the financial and economic crisis.

5 Empirical Results

In this section we present the results of the wavelet analyses of our time-series – the coherence and the phase difference between each pair of yield curve latent factor (level, slope, curvature) and macro variable (unemployment, economic activity, inflation and fed funds rate). Our tools give quantified indications of the similarity of power between each time series and a measure of the lead-lags of their oscillations at each frequency and each point in time.[14] We also include significance values generated by bootstrapping. It could be argued that the bootstrapping-based significance levels understate the uncertainty involved in our exercise, as we treat the yield curve latent factors similarly to the macroeconomic data ignoring that there is filter as well as parameter uncertainty associated to their estimates. For robustness, all the calculations done in this section were replicated using the empirical proxies referred above. The main conclusions

[14] Overall, we will not mention often results about the coherency and phase differences at the higher frequencies – cycles of 1 month∼1 year – as they are tipically noisy and, as such, rather uninformative.

we reach are similar (in particular, the results related to the slope of the yield curve are almost identical).

The interpretation of our econometric results proceeds as follows. First, we check the time-frequency regions in which the coherency between the variables is statistically significant, meaning that, in those episodes, we may confidently say that there has been a significant co-movement of the variables for cycles of the indicated period. Then, for the statistically significant time-frequency locations, we analyze the phase differences, to detect whether the co-movement has been positive or negative, and which variables were leading and lagging.

5.1 Real activity and the yield curve

In this sub-section we study the time-frequency relation between our two proxies of real economic activity and the yield curve latent factors. We expect different results for these two macro variables, as the timing of their correlation to real GDP growth is quite different (the unemployment rate lags the ADS index – which is a coincident index of GDP growth – by about one year).

5.1.1 A coincident indicator for economic activity: the ADS index

We firstly take the ADS index as the indicator of the overall economic activity. Given the literature, we expect the slope – and possibly the curvature – to relate significantly with the index, at least during some sub-sample periods. The results are presented in Figure 4, which includes the coherency (lef-hand-side panel) and the phases and phase differences (right-hand-side panel) for the three pairs formed by each yield curve factor and the ADS index.

There is a large region of significant coherency between the level of the yield curve and the ADS index between 1975 and 2011, for cycles in the frequency bands of 4∼8. There is also a significant coherency for cycles of 8∼12 years, but this turns non significant in the late 1990s. The phase differences confirm that the level of the yield curve has not a reasonable predictive power for real economic activity. On the contrary, at the 4∼8 years band most of the time an increase in the level anticipates an increase in the ADS index and at the 8∼12 years band an increase in the ADS index anticipates a fall in the yield curve level.

There is no significant coherency at any frequency band between the yield curve slope and the ADS index in 1985-1990, which is consistent with the structural break in continuous regressions of the yield slope on output growth (with a forecast breakdown) detected, with alternative methods and/or data, by, e.g., De Pace (2009), Chauvet and Potter (2002, 2005), Haubrich and Dombrosky (1996), and Dotsey (1998). Our results confirm the typical association between this forecasting breakdown of the yield slope and the beginning of the Great Moderation as well as the change in monetary policy regime after the Volcker monetarist episode; and they contrast with previous results presented in the literature, as in, for example, Giacomini and Rossi (2006) – who have detected forecast breakdowns of the yield slope in the mid-1970s and in 1979-1987, *i.e.*

during the Burns-Mitchell and Volcker regimes, rather than at the beginning of the Greenspan regime (actually stating, in page 794, that "during the early part of the Greenspan era the yield curve emerged as more reliable model to predict future changes in economic activity").

The phase differences in fact support the above interpretation of a structural change with a forecast breakdown. At the 4∼8 years band, in 1965-1985, and at the 8∼12 years band, in 1972-1985 (the regions of significant coherencies), the phase differences are between $\pi/2$ and π, meaning that an increase in the slope – a flattening of the curve – anticipated (as expected) falls in the ADS index. For cycles of period 4∼8 years, during 1990-2002 the coherency is again statistically significant and the phase differences continue to indicate a leading role for the slope, with flatter yield curves predicting a contraction in real economic activity, meaning that the forecast breakdown has been reverted after the early 1990s, but only for this range of periodicities. In contrast, for cycles of period 8∼12, after 1997, and for cycles of period 4∼8, after 2003, the coherencies are significant and the phase differences are between 0 and $\pi/2$, meaning that increases in the ADS index anticipated a flattening of the yield curve – i.e. the slope fails to forecast economic activity.

These results, and most specially those for the 4∼8 frequency band (2003-2011) for which the phase differences are well within the 0–$\pi/2$ interval, are consistent with the so-called *conundrum* of the Summer of 2006, when the yield curve became inverted and yet no recession emerged – see e.g. Kucko and Chinn (2009) and Hamilton (2010). In fact, the significant coherency and the phase differences at that time-frequency region suggests that the slope and the ADS relate positively with each other, with the ADS index slightly leading the slope of the yield curve – a flatter yield curve was associated to an economic expansion, failing to predict a recession as would be expected in normal times.

There is a large time-frequency region of statistically significant coherency between the curvature of the yield curve and the ADS index, at the 4∼8 years frequency band (in 1965-1985) and at the 8∼12 years band (in 1970-1988). At this region, the phase differences are consistently located within $-\pi/2$ and 0, indicating that increases in the curvature (higher concavity) anticipate increases in economic activity.

There are significant coherencies between the curvature and the ADS index for cycles of 16 or more years, since 1993, and for cycles of period 4∼8 years since 2003. The phase differences consistently located in the region between 0 and $\pi/2$ indicate that in those episodes and frequencies increases in the ADS led to increases in the curvature. In particular for the 4∼8 years cycles, these results are consistent with those obtained for the ADS-slope relation, which we found related to the 2006 *conundrum*. To see that, recall that higher curvatures (higher concavities) are typically associated with higher slopes (flatter curves): in short, for 4-8 years cycles, since 2003, increases in the ADS index have been associated (slightly leading) with increases in the concavity and flattenings of the yield curve.

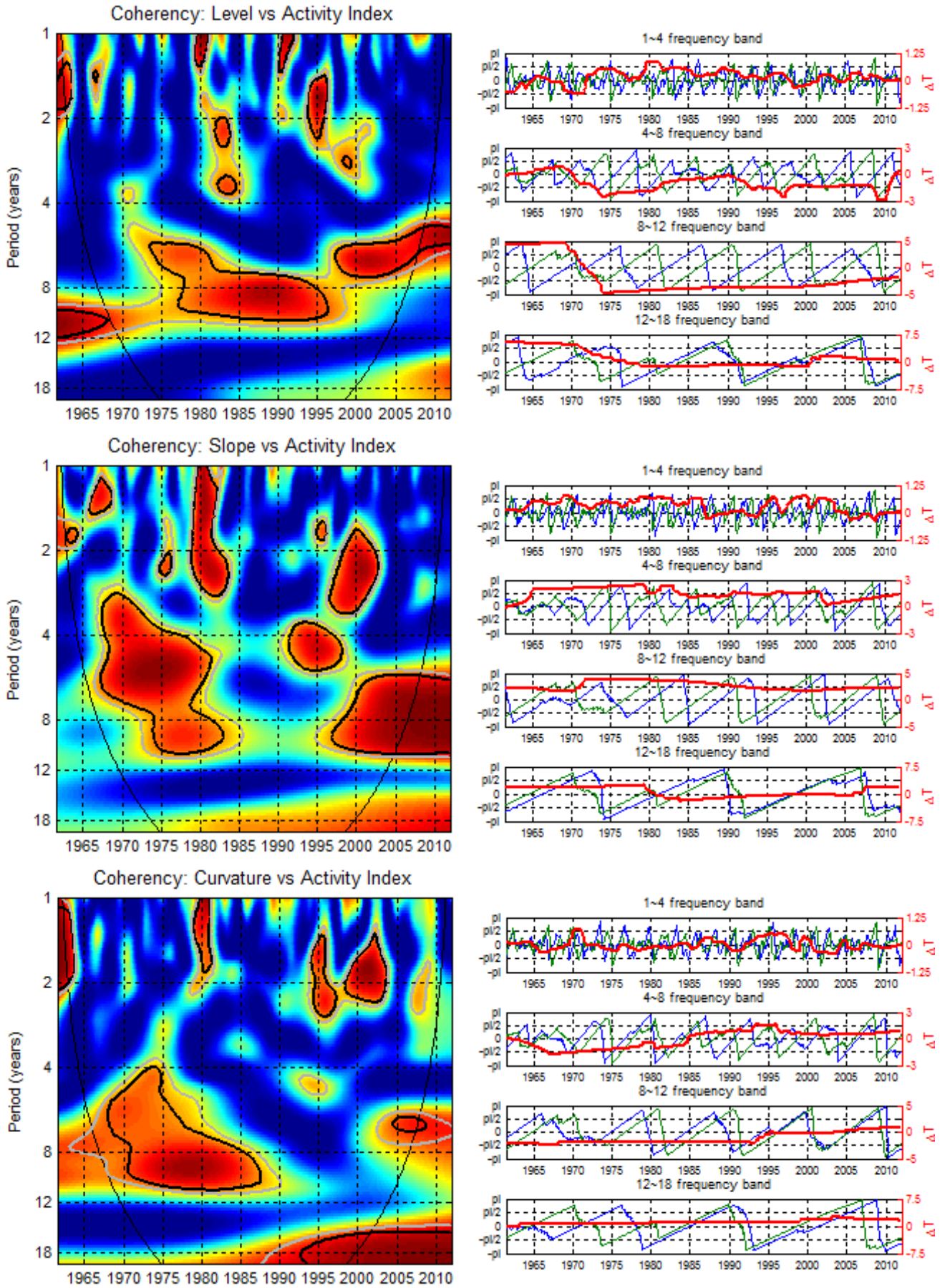

Figure 4: Economic Activity and the Yield Curve, U.S. 1961:6-2011:12

5.1.2 A lagged indicator for economic activity: the unemployment rate

We now take on the unemployment rate as an indicator of macroeconomic activity and assess its relation to the latent factors of the yield curve. By analogy with our previous results, we would expect the slope – and possibly the curvature – to relate significantly with unemployment, at least during some important sub-sample periods. The results are presented in Figure 5.

Most regions of high coherency between the yield curve level and the unemployment rate occur between 1970 and 2005 in cycles of periodicity in the 4∼8 years frequency band and between 1975 and 1997 in the 8∼12 years period cycles. It is visible a gradual shift from shorter-run frequencies (with period cycle closer to 4 years) to longer-run frequencies, with period closer to 12 years, which is reverted after 1995. Independently of the shift in coherency across frequencies, we observe a rather stable phase relationship in all the frequency bands involved (1∼4, 4∼8 and 8∼12 years): for most of the time, the phase difference is between $-\pi/2$ and 0, indicating that the yield curve level leads the unemployment rate and that an increase in the level of yields is associated with an increase in unemployment.

There are important regions of statistically significant coherency between the yield curve slope and the unemployment rate between 1965 and 1985 and, again, between 1990 and the end of the 2000s. The lack of coherency in 1985-1990 at all frequency bands (detected in the previous sub-section also for the ADS index) is consistent with the structural break in continuous regressions of the yield slope on output growth (with a forecast breakdown) well documented in the literature. However, and in contrast to what has been detected in the case of the ADS index; in most of the periods with significant coherency, an increase in unemployment anticipates a decrease in the slope, i.e. a steepening of the yield curve. We interpret this evidence as capturing the lead of higher unemployment rates to a monetary policy ease (which would result in steeper yield curves).

At frequencies with period cycle between 4 and 8 years, from the early 2000s onward, the phase differences indicate that the yield curve slope leads unemployment by around 1.5 to 2 years, with an increase in the slope (flattened curves) anticipating lower unemployment. This is consistent with the *conundrum* of the summer of 2006, as well as with the results seen previously for the ADS index. As Hamilton (2010) points out, it seems that the very low overall levels of interest rates recorded at the time has mitigated the recessionary signal given by the yield slope; our results confirm such conjecture and show that it relates to the ability of the yield curve to predict cycles of the unemployment rate with periodicity in the 4∼8 years frequency band. This mimics the results obtained for the coherency and phase differences between the ADS index and the slope of the yield curve, since the early 2000s, for these cycles as well as for those of 8∼12 years.

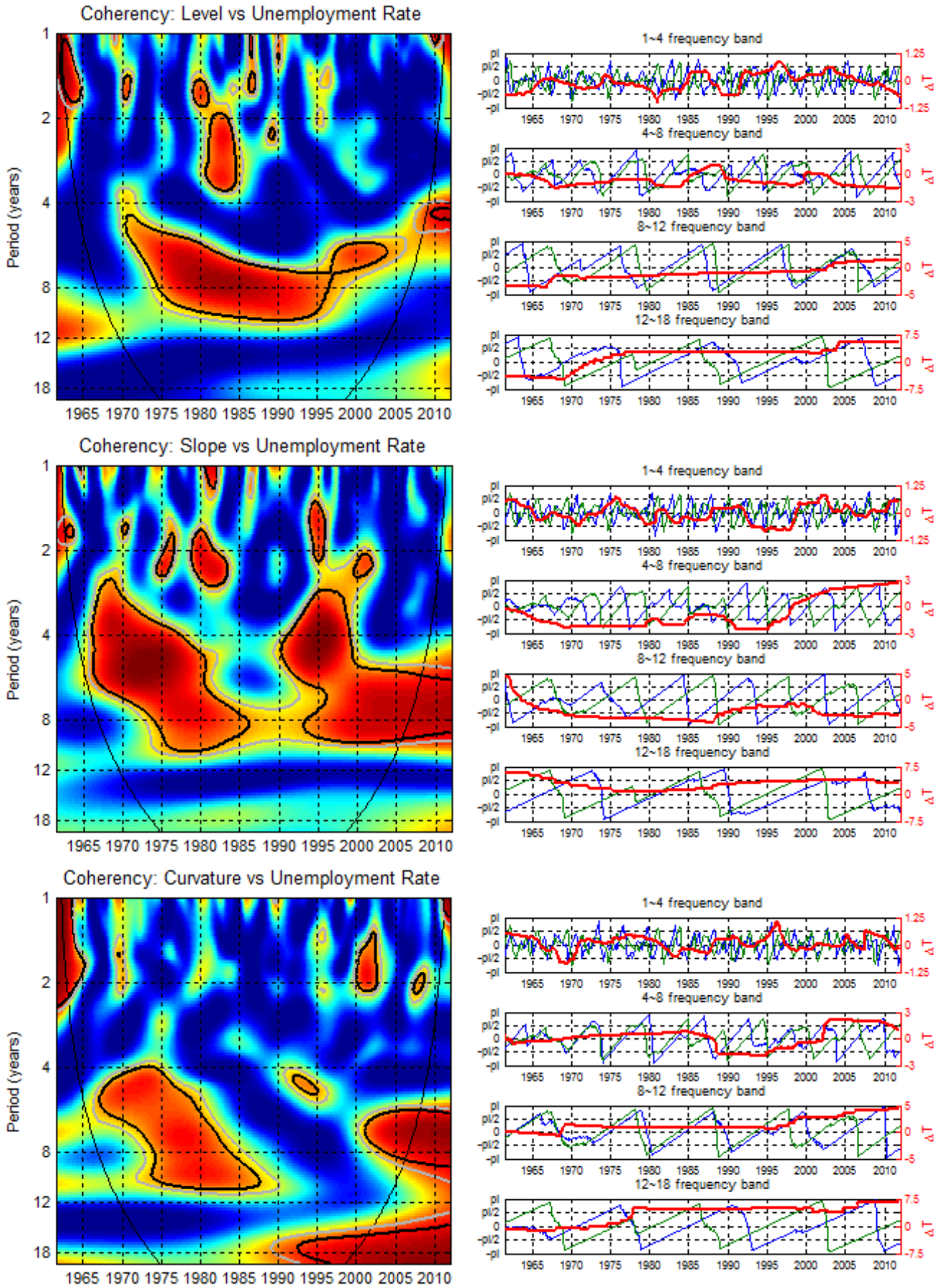

Figure 5: Unemployment and the Yield Curve, U.S. 1961:6-2011.12

Regarding the time-frequency relations between unemployment and the curvature, there is a large region of statistically significant coherency in the 4∼8 frequency band (between the late 1960s and the mid-1980s) and in the 8∼12 years band (between the mid-1970s and the late 1980s). The phase differences indicate, overall, that in these episodes the leading variable has been the unemployment rate, rather than the yield curve curvature. The coherency between the curvature and the unemployment rate is again statistically significant since 1990 for cycles of around 16 years and since 2000 for cycles of period 4∼8 years. For both, the phase-differences between $\pi/2$ and π indicate that the curvature has led the unemployment rate, with increases in the curvature associated with lagged decreases in the unemployment rate. As the curvature is positively correlated with the slope – increases in concavity (curvature) are typically associated with flatter yield curves (increases in the slope) – the evidence here uncovered for cycles of 4∼8 year cycles after 2000 is consistent with the 2006 *conundrum* and the related evidence regarding the slope. It is, furthermore, consistent with the findings above for co-movement between the curvature and the ADS index for these cycles since 2003, once the lags of unemployment to the ADS are considered.

5.1.3 Real activity and the yield curve: summary of results

First, in line with the literature, we find no clear-cut relation between the level of the yield curve and the ADS index. However the level anticipates increases in the unemployment rate for most frequency bands and most of the sample period. Given its persistence and nominal determinants, the yield curve level is not well suited to track monthly real economic growth, but helps tracking the unemployment rate – a novel result that we explain with the fact that the unemployment rate is more persistent and lags real output growth.

Second, the coherency between the slope of the yield curve and real economic activity has not been statistically significant during 1985-1990. In fact, from 1965 (for cycles of period 4∼8 years) and from the early 1970s (for cycles of period 8∼12 years), until 1985, flatter yield curves have led decreases in the ADS, while increases in the unemployment rate have led to steeper yield curves; and this pattern is only resumed after 1990. These results, which may at first sight appear to be inconsistent, should be interpreted having in mind that the unemployment rate lags the ADS index: our phase difference analyses for cycles of period 4∼8 and 8∼12 years capture the lead of the slope over the ADS associated with short-term rates changes determined by monetary policy, while in the case of unemployment they capture the lead from changes in the unemployment rate to monetary policy reactions (which seem to be closer in time than the impact of monetary policy on the unemployment rate).

Our results thus confirm that the breakdown of the slope to predict real activity has been associated with the Great Moderation, as argued in some literature, but further inform that the forecasting ability has reappeared in the 1990s, and establish the frequency of the cyclical

oscillations involved in that structural change (between 4 and 12 years).

Third, our results shed light on the episode known as the yield curve 2006 *conundrum*, and, more in general, on the failure of the yield slope to predict real activity in recent times. In fact, since the early 2000s, for cycles of period 4∼8 and 8∼12 years, the phase differences are consistent with flatter yield curves, such as the one observed in the summer of 2006, being associated to economic expansions, rather than recessions as would be expected in normal times.

Fourth, we do not uncover any significant role for the curvature either as a leading or as a coincident indicator of economic activity, in contrast with evidence obtained elsewhere in the literature with different data and methods.[15] Until 1988 (for 4∼12 year cycles), increases in the curvature lead to increases in the ADS and lag increases in the unemployment rate, which is the symmetric of what would be expected. After 2003, for cycles of period 4∼8 years, the evidence is more in line with the one obtained for the slope and is consistent with the 2006 *conundrum*: increases in the ADS led to increases in the curvature which, in turn, led to decreases in unemployment. Again, this evidence should be interpreted bearing in mind that higher curvatures are mostly associated with higher slopes and that the unemployment rate lags behind the ADS index by about a year.

5.2 Inflation and the yield curve

In the literature, there are two main associations of inflation to the yield curve. On the one hand, its level is seen as reflecting the path of the nominal anchor of the economy (measured by inflation, as in, e.g., Diebold, Rudebusch and Aruoba, 2006, or by inflation expectations, as in, e.g., Mumtaz and Surico, 2009). On the other hand, its slope or changes between slopes computed at different horizons are seen as predictors of changes in inflation at such horizons (see, e.g., Mishkin, 1990a, 1990b, 1990c, and Estrella, Rodrigues and Schich, 2003). In this sub-section we assess the relation of the level, slope and curvature with inflation, in the time-frequency domain.[16] Figure 6 shows our results.

The larger regions of high coherency between inflation and the yield curve level are situated in cycles in the frequency bands of 4∼12 years and occur between the early 1970s and the early 1990s. Across these periods and frequencies, the phase difference is between 0 and $\pi/2$,

[15] As pointed out by a referee, it should be stressed that our framework differs markedly from those of Moench (2010) and Modena (2008) – who have detected a leading and a coincident role of the curvature, respectively –, and so our results are not directly comparable. In fact, they assess the predictive content of shocks to the yield curve curvature while we merely assess the role of the level of the curvature, as our non-parametric approach does not impose restrictions allowing for the identification of shocks to the latent factors of the yield curve.

[16] It should be noted that our assessment of the slope-inflation relation is not directly comparable to the others in the literature. First, most studies use empirical proxies for the yield spread, rather than a model-based one such as ours (and in many cases the proxy differs substantially from the empirical properties of ours). Second, the literature typically looks at regressions of the difference between inflation in period m and inflation in period n, on the difference between the yield for maturity m and the yield for maturity n (e.g. Estrella, Rodrigues and Schich, 2003); our analyses focus on the co-movement (and the time lags therein) between the level of inflation and the level of the slope, for all time periods and cyclical frequencies.

indicating that the yield curve level reacts, in the same direction, to changes in inflation.

A first lesson drawn from our analysis is that until the early 1990s the yield curve level has indeed mirrored the path of inflation, with some delay, in fluctuations of period within the standard concept of business cycles. This is consistent with the view that inflation determines the whole yield curve and is thus supportive of the above mentioned association between inflation (or expectations) and the yield level.

A second lesson is that the coherency between the yield curve level and inflation has vanished since around 1993 (to reappear since around 2003, but only for cycles of period 4∼8 years (and significant merely at the 10 percent level). This breakdown of the inflation-yield level relation coincides with the consolidation of the low inflation regime typically associated with the FED chairmanship of Alan Greenspan – notice, in the third graphic of Figure 3, the apparent fall in the volatility of inflation since 1993, which upheld until the recent financial and economic crisis – as well as with the intensification of the U.S. external imbalance in a context of a global savings glut and a persistently accommodative domestic monetary policy – factors that have somehow detached the overall level of interest rates from macroeconomic conditions in the U.S.

A first idea that emerges from our time-frequency analysis of inflation and the yield slope is that between 1985 and around 1993 there is no significant coherency at any frequency band. Such result is consistent and complements the evidence of a structural change also found in the previous sub-sections regarding the relation between the slope and economic activity; and may explain the difficulties in estimating stable regressions reported in a large part of the literature (see, e.g., Mishkin, 1990a, 1990b, 1990c, and Estrella, Rodrigues and Schich, 2003).

Apart from the mentioned period, most of the coherency appears at the 4∼8 years frequency band, which clearly indicates that the relation between the slope and inflation relates to business cycles. At that frequency band, until the early 1980s the phase difference is located within the $-\pi/2$ and 0 interval, implying that an increase in the slope anticipates (by around 2 to 3 quarters) an increase in inflation. At the 8∼12 years frequency band the relation changes gradually from an in-phase relation in the 1970s – one of perfect synchronization of the slope and inflation – to a similar lead of inflation by some quarters in the 1980s. After the period of absence of coherency (1985-1993) the coherency resumes only within the 4∼8 years frequency band, and the phase difference fluctuates around 0, indicating positive co-movements that in some periods are contemporaneous, while in others there is a slight lead or lag (1 to 2 quarters) of the slope.

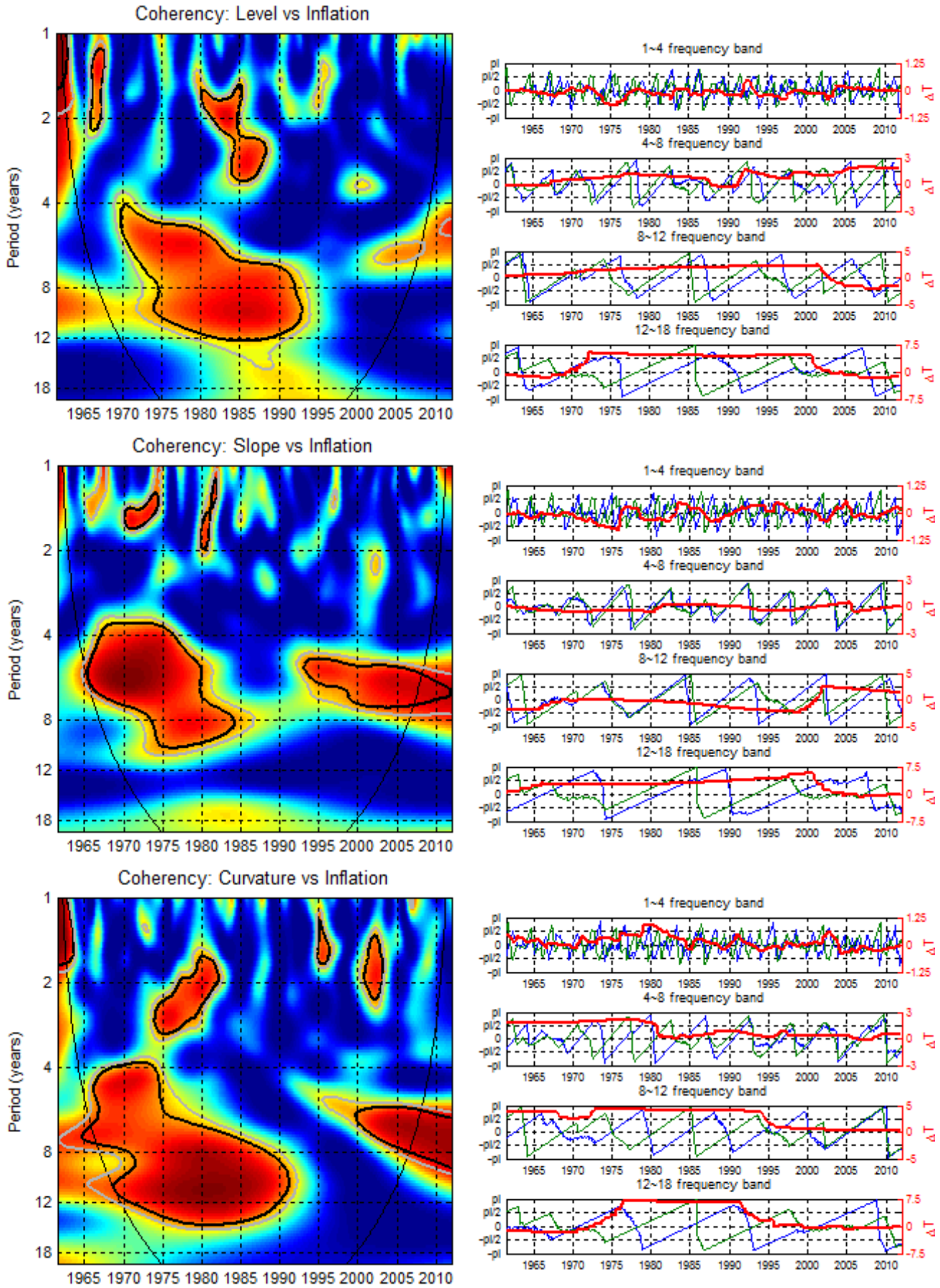

Figure 6: Inflation and the Yield Curve, U.S. 1961:6-2011:12

Such results may seem hard to reconcile with the ability of increases in the slope to predict recessions, as these are typically associated to reductions in inflation. However, once the lags suggested by the phase difference diagrams, the lags in the transmission of monetary policy, and the fact that policy is conducted in reaction to expectations of inflation are considered, a consistent story emerges: our evidence is consistent with a monetary policy explanation for the leading role of the slope. When policy-makers forecast inflationary pressures and implement a tighter monetary policy, thus flattening the yield curve, inflation is not brought to control immediately (and hence the flattening anticipates a rise in inflation) but only with a lag when a recession arises, as a side-effect of such policy (later, as monetary policy reacts to the recession and the ensuing control of inflation, the unemployment rate anticipates the subsequent monetary actions, as seen above). This interpretation is corroborated by the reappearance of the leading role of the slope, for cycles of period 4∼8, at around the same time for the ADS (1990-2002) and for inflation (1993-2002). Hence, the forecast breakdown of the slope since 1985 may be seen as an indicator of success of monetary policy, in that its reaction to inflationary pressures is less related to recessions thereafter and, additionally, it has been more effective in controlling inflationary pressures duly forecasted.

Overall, in the earlier part of the sample, the regions of significant coherency between the curvature of the yield curve and inflation correspond to phase differences within the interval between $\pi/2$ and π, indicating that an increase in the curvature (stronger concavity) anticipates reductions in inflation. This happens in the 4∼8 years frequency band until 1980, and in the 8∼12 and 12∼18 years bands until 1992. We find results hard to reconcile with those on the relation between the slope and inflation, given that curvature and slope co-move positively and suggest that the curvature does not relate closely to inflation in most cyclical frequencies and sub-sample periods. In the early 1980s a change occurs at the 4∼8 years frequency band, and increases in inflation start anticipating increases in the curvature, as the phase differences turn into the interval between 0 and $\pi/2$. After about 10 years with non significant coherency, from 2000 onwards the coherency at the 4∼8 years cycle is again significant, with the phase-differences steady between 0 and $\pi/2$ indicating that increases in inflation led increases in the curvature.[17]

5.3 Monetary policy and the yield curve

The relation between the level of the yield curve and the monetary policy interest rate is surely positive, as increases in the FFR should lead increases in the yield levels. As regards the shape of the yield curve, one could expect a twofold relation between its slope and the monetary policy interest rate. On the one hand, in the course of its transmission mechanism, monetary policy

[17] This positive co-movement is consistent with the one detected between the slope and inflation, but may appear puzzling as the curvature lags inflation, while the slope leads; however, the apparent puzzle is solved once one considers that, in our data, the curvature lags the slope after 1980.

actions should lead to changes in the yield curve slope as changes in the money market interest rates impact first in the short-end of the curve and only then in its longer maturities. On the other hand, financial markets often anticipate the moves of monetary policy-makers and so the yield curve slope may lead the FFR. The relation with the yield curve curvature is less clear-cut, but should broadly mimic the one with the slope, once the lags between the time series are considered. Figure 7 shows that these relations are indeed strong.

There is a high and statistically significant coherency between the yield curve level and the FFR at low frequencies during the whole sample period. Possibly reflecting the changes in the monetary policy regime (with the Greenspan chairmanship since the end of the 1980s) this high coherency moves to cycles of longer period after around 1987. In 2000 this coherency was concentrated in the 16 years period cycle and then it loses significance, at the 5 percent level although remaining important (and significant at 10 percent) until the end of the sample. The phase-differences at those frequencies (8$\sim$12 and 12$\sim$18 years) are overall located between 0 and $\pi/2$ but close to 0, indicating that increases in the FFR slightly anticipate increases in the level of the yield curve.

At the frequency band of 4$\sim$8 years, the largest period of high and statistically significant coherency runs from 1970 to 1987, with the phase differences indicating that the FFR leads the level of the yield curve; such pattern reappears in the period 2000-2005.

The relation between the FFR and the yield slope is even stronger, as there are regions of very high and statistically significant coherency at most of the time-frequency locations. Indeed, since 1965 there are statistically significant coherencies at the 1$\sim$4, 4$\sim$8 and 8$\sim$12 frequency bands, in spite of some change in the specific periodicity of the involved cycles. At the 12$\sim$18 years frequency band the coherencies are significant after 1980. Overall, these significant coherencies are associated with phase differences between $-\pi/2$ and $\pi/2$, meaning that the slope and the FFR co-move in the same direction for all cyclical frequencies throughout all the period, i.e. increases in the slope (flattening of the yield curve) are associated with increases in the FFR – which is consistent with a monetary policy explanation of the changes in the yield curve shape.

At the 4$\sim$8 years cycles, the phase differences indicate a contemporaneous coherency between the slope and the FFR. At longer cycles, for most of the time, the slope leads the FFR.

Overall, we draw two main conclusions. First, tighter monetary policies have been associated with flatter yield curves throughout the whole sample period and across all the frequency bands. This means that monetary policy has impacted differently on the short-end of the yield curve than it has impacted on its long-end, irrespectively of the periodicity of the FFR and slope movements. Second, the yield curve slope has often been a good predictor of monetary policy for cycles of period above the standard business cycle definition (8$\sim$12 and 12$\sim$18 years) but less so for cycles of smaller period.

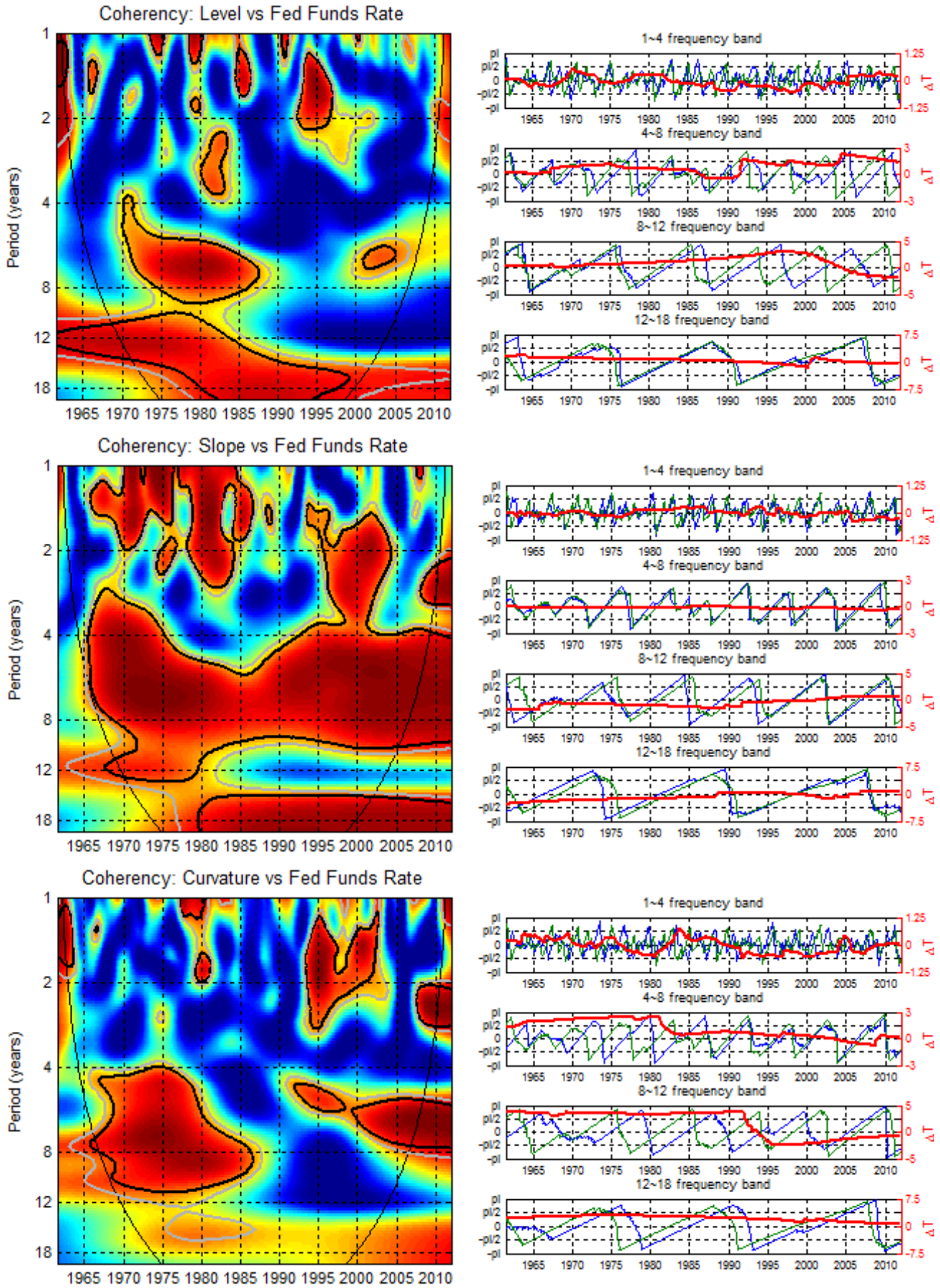

Figure 7: Fed Funds Rate and the Yield Curve, U.S. 1961:6-2011:12

Finally, we assess the results on the relation between the curvature of the yield curve and the FFR. There is a first region of high and significant coherency at the 4∼12 years frequency bands between 1965 and the mid 1980s. For these time-frequency areas, the phase difference is between $\pi/2$ and π, indicating a negative relation with the yield curvature leading: higher degrees of concavity in the yield curve anticipate lower FFRs. This result is puzzling, given that more concave yield curves are typically associated with flatter yield curves, which in turn should be associated with higher interest rates at the short-end of the yield curve and, consistently, with higher monetary policy interest rates. Differently, the curvature-FFR relation is in line with expectations for cycles of period 1∼4, 12∼18 and above, in which strong and sometimes significant coherencies combine with phase differences between $-\pi/2$ and $\pi/2$ that consistently point to a positive co-movement.

After 1992, the coherency between the curvature and the FFR became significant again, for 4∼8 years cycles, and during most of this period, the phase differences have been between 0 and $\pi/2$, indicating that increases in the FFR led to increases in the curvature (associated with flatter curves, as expected). Between 2006 and 2008 the phase differences have been located between 0 and $-\pi/2$, meaning that increases in the curvature anticipated increases in the FFR. Taken together with the results for the slope-FFR relation above, these results suggest that during the 2006 *conundrum* the yield curve may have failed to forecast economic activity but has correctly forecast monetary policy.

6 Conclusion

In this paper, we assessed the relation between the yield curve shape and the U.S. macroeconomy between 1961:6 and 2011:12 across time and frequencies, using wavelet tools. The shape of the yield curve was modeled with three time-varying latent factors corresponding to its level, slope and curvature. The macroeconomic variables are an index of overall economic activity, unemployment, inflation and the federal funds rate (FFR). The time-frequency fills a gap in the literature, which has been conducted so far exclusively in the time-domain. We provide a new description of the relation between the yield curve and the macroeconomy in the US, which, besides their immediate relevance, should prove useful for future research on this area. Among the numerous results, a core set of findings may be summarized as follows.

The level of the yield curve has essentially been determined by nominal variables – FFR and inflation. However, we uncovered a gradual change in the FFR-level relation since 1987 and a structural change in the level–inflation relation around 1993. While the movements in the FFR anticipated movements in the level of the yield curve for most frequencies, after the outset of Alan Greenspan's mandate in 1987 the high coherency between these variables moved progressively to cycles of longer period (larger than 12 years). In turn, inflation has led the yield curve level

only until 1993, when the volatility of inflation fell markedly. We interpret these results jointly, as symptoms of the gradual success of the new monetary regime in anchoring expectations of inflation, as well as of the new macro environment of a global savings glut feeding the US external imbalances with an overall detachment of yields from their macro determinants.

The yield curve level has not related significantly in a meaningful way with an index of economic activity that closely tracks monthly real output growth (the ADS index). Yet, we found that changes in the level anticipate changes in unemployment in the same direction for several periods and frequencies.

Changes in the slope of the yield curve were significantly associated with changes in the FFR in the same direction across almost all the sample period, either contemporaneously or with a small lead by the slope. Such evidence is consistent with the monetary policy explanation for the predictive power of the yield curve slope.

At business cycle frequencies (and lower), increases in the slope led by a few quarters increases in inflation and anticipated recessions with a larger lead. We interpret these results as evidence of a reaction of monetary policy to expectations of inflation, and of the typical outside lag of monetary policy – policy controls inflation only with a lag and after real economic activity responds. Most importantly, we uncovered clear time-frequency evidence of the structural change with a forecast breakdown in the relation between the slope and the main macro variables: the predictive power of the slope at business cycle frequencies vanished after 1985, when the Great Moderation began, to reappear in 1990 regarding real activity and in 1993 regarding inflation (but here only at cycles of 4$\sim$8 years and with a considerably smaller lag, compatible with more effective inflation targeting).

After 2003, flatter yield curves became associated with expansions, rather than recessions, at the business cycle frequencies. Our evidence thus clarifies that the well-known yield curve *conundrum* of 2006 has its roots in a previous change in the relation between the slope and output (and inflation) in 4$\sim$12 year frequency band. Given that the FFR fell considerably from 2001 to 2003 and the yield level fell considerably from 2003 to 2005, our evidence is consistent with the hypothesis that the fall in the level of yields in the last decade has damaged the ability of the slope to predict business cycles and the related changes in inflation.

Finally, we did not find statistical evidence of a consistent role for the curvature either as a leading or as a coincident indicator of economic activity, nor did we find a clear-cut relation between the curvature and inflation, either in the pre-1985 or in the post-1993 period. After 2003, we found evidence indicating that the 2006 *conundrum* has also been present in the curvature. During the yield curve *conundrum* of 2006, the curvature was a good predictor of the FFR, which, together with the evidence regarding the slope-FFR relation above mentioned, indicates that the yield curve may have failed to forecast economic activity but not monetary policy.

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