Scaling Exponents in Heart Rate Variability

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Abstract Long recordings of Heart Rate Variability (HRV) display non-stationary characteristics and exhibit long and short-range correlations. The nonparametric methodology Detrended Fluctuation Analysis (DFA) has become a widely-used technique for the detection of long-range correlations in non-stationary HRV data. Recently, we have proposed an alternative approach based on Fractional Integrated Autoregressive Moving Average (ARFIMA) modelling. These models are an extension of the AR models usual in HRV analysis and have special interest for applications because of their ability for modelling both short- and long-term behaviour of a time series. In this work, DFA is used to assess also short-range scales, further characterizing the data. The methods are applied to 24 hours HRV recordings from the Noltisalis database, collected from healthy subjects, patients suffering from congestive heart failure and heart transplanted patients. The analysis of hort-range scales lead to a better discrimination between the different groups.

1 Introduction

The characterization of the dynamics of a system has become an important and interdisciplinary problem, namely in biomedical applications. Cardiovascular variables such as heart rate, arterial blood pressure and the shape of the QRS complexes in

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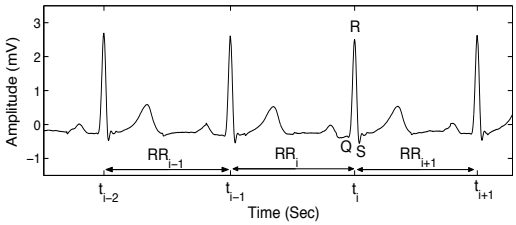

Fig. 1 Schematic representation of electrocardiogram signal and relevant information in each cardiac beat: QRS complexes and RR intervals ($RR_i = t_i - t_{i-1}$).

the electrocardiogram, Fig. 1, show variability on a beat to beat basis. This variability reflects the interaction between perturbations to the cardiovascular variables and the corresponding response of the cardiovascular regulatory systems. Therefore, the analysis of such variability can provide a quantitative and non-invasive method to assess the integrity of the cardiovascular system. The discrete series of successive RR intervals, the tachogram, Fig. 2 (a), is the simplest signal that can be used to characterize Heart Rate Variability (HRV) and has been applied in various clinical situations. The analysis of ambulatory long term HRV series has become important for clinical diagnosis and risk assessment [1, 13]. These series correspond typically to 24 hour recordings and exhibit non-stationary characteristics.

It is well-known in the literature, that HRV series exhibit not only short but also long-range correlations which were firstly studied with DFA [9]. An alternative parametric approach to describe long-range correlation in HRV data, has been proposed by the authors [7] using Fractional Integrated Autoregressive Moving Average (ARFIMA) models which are an extension of AR models. The parametric approach has the advantage of allowing the removal of the long-memory compo-

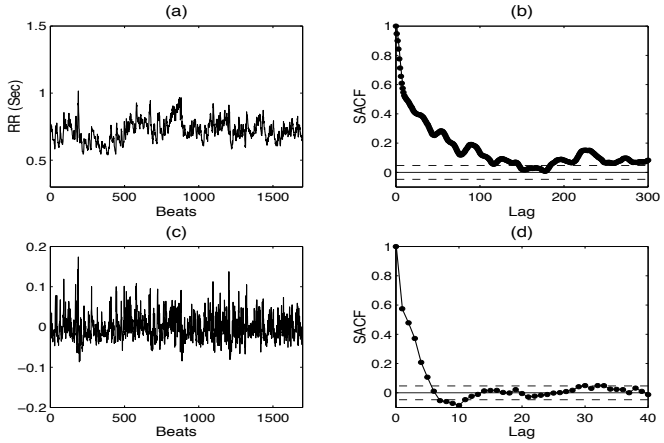

Fig. 2 (a) Tachogram of a normal subject; (c) same tachogram after removing the long-range correlations with an ARFIMA(0,0.47,0) filter; (b) and (d) corresponding SACFs.

nent by applying the adequate fractional differentiating filter. The remaining short memory component may then give further insights of the data, as illustrated in Fig. 2.

In this work, ARFIMA models combined with selective adaptive segmentation [7] and DFA scaling exponents are used in the description of long- and short-range correlations in 24 hour HRV recordings of 30 subjects from the Noltisalis database [12].

2 Scaling exponents

DFA [9] has become an important non parametric tool to assess the correlation properties in non-stationary processes. This methodology was first developed to quantify long-range correlations in non-stationary time series as opposed to ARFIMA models, which require stationarity. Let $x(1),\ldots,x(N)$ be a time series of length N. To compute a scaling exponent with DFA, $x(t)$ is first integrated to give

$$y(i) = \sum_{t=1}^{i} [x(t) - \bar{x}], i = 1, ..., N, \tag{1}$$

where $\bar{x}$ denotes the mean of the series. The integrated time series $y(i)$ is then divided into segments of equal length k and in each segment, the local trend $y_k(i)$ is calculated by a least squares line fit. Next, the integrated time series $y(i)$ is detrended by subtracting the local trend $y_k(i)$ in each segment. The root-mean-square fluctuation of each integrated and detrended time series is given by

$$F(k) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} [y(i) - y_k(i)]^2}. \tag{2}$$

The above computation is repeated for several segments of length k (different time scales). The scaling exponent α at time scale k is obtained by fitting a linear model to the log-log relationship $F(k) \sim k^{\alpha}$. Values of $\alpha > 0.5$ for large scales k indicate long-range correlations in the data. In particular, for 24 hour HRV recordings (approximately 100000 beats) k varies in the interval $[128, 4096]$ [4]. This methodology has the disadvantage of requiring large sample sizes for an unbiased estimation of the long memory, [9]. Further studies with smaller samples indicated that different ranges of values for k lead to the estimation of other scaling exponents, α_1 and α_2, which may be used to characterize the correlation of the series on small and large time scales [9, 10].

Thus, in this work DFA is applied to HRV recordings to assess both long and short range correlations. The value of α_1 corresponds to the slope of $\log F(k)$ as function of $\log k$ in the range $4 \leq k \leq 11$ beats and characterizes the correlation behavior on short time scales. In contrast, we defined the exponent α_2 in the range $64 \leq k \leq 1024$ beats, where the long-range correlation behavior can be observed,

in records with 4096 beats [8]. For short-range correlated data α_1 is larger than 0.5. Values of $\alpha_2 > 0.5$ indicate long-range correlations in the data.

An alternative approachs to describe correlations in HRV data is to use ARFIMA$(p,d,0)$ models, [7]. A stochastic process $x(t)_{t\in Z}$ is an ARFIMA$(p,d,0)$, $p \in N \cup \{0\}$ and $d \in R$, if it satisfies the equation

$$\phi(B)\nabla^d x(t) = \varepsilon(t), \tag{3}$$

where $\varepsilon(t)_{t\in Z}$ is a Gaussian white noise WN(0,σ^2), $\phi(z) = 1 - \phi_1 z - ... - \phi_p z^p$ is a polynomial such that $\phi(z) \neq 0$ for $|z| \leq 1$, B is the backward-shift operator, $Bx(t) = x(t-1)$ and $\nabla^d = (1-B)^d$ is the fractional difference operator [5]. The parameter d determines the long-range correlations, whereas p and the corresponding parameters in $\phi(B)$ allow the modelling of short-range correlations. For $0 < d < 0.5$, the ARFIMA$(p,d,0)$ is stationary and has long-memory. The estimation of d can be obtained using the semi-parametric local Whittle estimator [3]. This estimator is consistent for $-0.5 < d < 1$ [11, 14]. For stationary data with long-range correlations, d is related to α_2 by $d = \alpha_2 - 0.5$.

The ARFIMA models have been found adequate to capture and remove long-range correlations in HRV recordings [7]. This suggests studying short-range scaling exponents of the data obtained after removing long-memory. Sucha na exponent is hereafter denoted by $\alpha_{1,SM}$. In fact, the application of DFA in short-range scales to the data represented in Fig. 2 (a) and (c) (a tachogram before and after filtering by an ARFIMA$(0,0.47,0)$) is represented in Fig. 3 and suggests that $\alpha_{1,SM}$ may provide different information about the data.

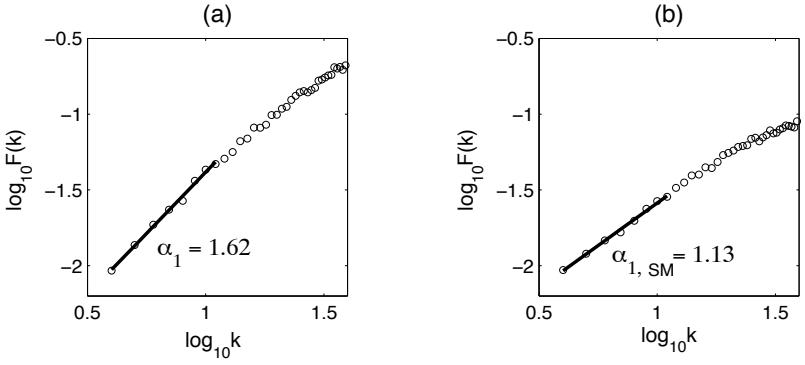

Fig. 3 Results of short-range scaling exponents α_1 and $\alpha_{1,SM}$ with DFA for the data represented in Fig. 2 (a) and (c), respectively.

3 Results and discussion

The methodology presented above is applied to 24 hour HRV recordings of 30 subjects from the Noltisalis database [12]: 10 healthy subjects (N, 34-56 years), 10 patients suffering from congestive heart failure (C, 36-68 years) and 10 heart transplanted patients (T, 18-60 years).

To describe short-range and long-range correlations in the long-term HRV series (24 hours, approximately 100 000 beats), ARFIMA modelling combined with selective adaptive segmentation is used [7]: the long record is decomposed into short records of variable length and the break points, which mark the end of consecutive short records, are determined using the AIC criterion for ARFIMA models. The short records thus obtained have a minimum length 512 and are subsequently modelled using ARFIMA models, to estimate long-range scaling exponent d, and analysed by DFA, to calculate the short-range scaling exponents α_1 and $\alpha_{1,SM}$.

Fig. 4 illustrates the results for an healthy subject-N6 (a), a patient affected by congestive heart failure-C10 (d) and an heart transplanted patient-T3 (g). The long-range scaling exponent $\hat{d}$ in (b), (e) and (h), change over time and the recordings present multifractality characteristics [2, 6, 7, 8]. Moreover, these estimates present a circadian variation, with lowest values during the night periods. The short-range scaling exponents α_1 and $\alpha_{1,SM}$ for healthy subjects, (c), decrease during the night period, for heart transplanted patient, (i), increase during this period and for patients affected by congestive heart failure, (f), is stable during the 24 hours.

The results for the three groups of patients during the 24 hours (P1), 6 hours of night (P2) and 6 hours of day (P3) periods are summarised in Fig. 5 and Table 1. The results indicate that the long-range scaling exponent increases for sick subjects, both during night and day periods, with the highest values for transplanted group. Similar results were obtained using ARFIMA-GARCH approach [6], using long-range scaling exponent α_2 in short records of constant length (4096 beats) [8] and using the value of global scaling exponent α in 24 hours [4]. The short-range scaling exponent decreases for sick subjects, both during night and day periods, with the lowest values for transplanted group. Moreover, Fig. 5 and Table 1 indicate that the short-range scaling exponent $\alpha_{1,SM}$ obtained after removing the long-range correlations in the data, leads to a better discrimination between the different groups, more clearly during the day periods. Furthermore, the results of this study also suggest that short-range scaling exponents α_1 and $\alpha_{1,SM}$ decreases with age, as Fig. 5 shows for the record N3 which belongs to the oldest subject of the healthy group.

4 Conclusion

It is well-know that HRV recordings exhibit long-range correlations. In this work, the long memory is removed by fractional differences filtering, which allows an improved description of short-range correlations. This approach leads to enhanced

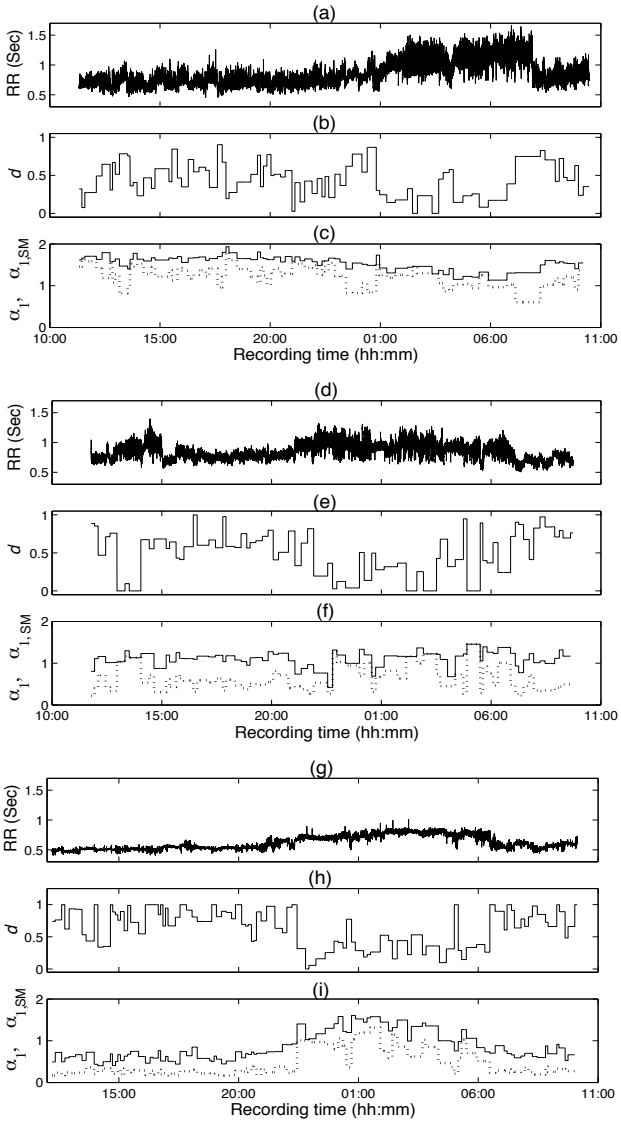

Fig. 4 Tachograms of three subjects, 24 hours Holter recordings: (a) healthy subject-N6, (d) patient affected by congestive heart failure-C10 and (g) heart transplanted patient-T3. Evolution over 24 hours of $\hat{d}$ in (b), (e) and (h) and α_1 (–) and $\alpha_{1,SM}$ (- -) in (c), (f) and (i). $\hat{d}$ is estimated using ARFIMA models combined with selective adaptive segmentation and α_1 and $\alpha_{1,SM}$ using DFA.

short-range scaling exponents and a corresponding better discrimination between the different groups of the Noltisalis database.

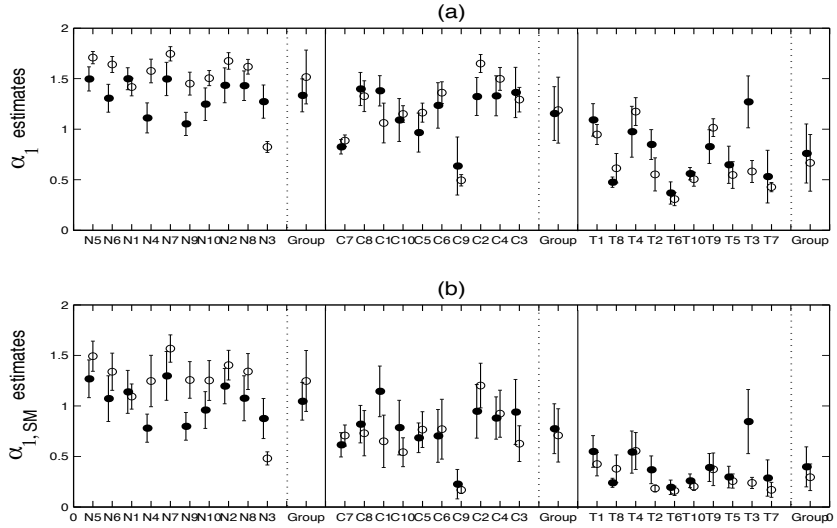

Fig. 5 Average estimates and standard deviations of (a) α_1 and (b) $\alpha_{1,SM}$ for each Holter recording during (•) 6 hours of night and (○) 6 hours of day periods. The estimates are obtained using DFA combined with segmentation in the records before and after filtering by an ARFIMA$(0,d,0)$, respectively. The subjects N-healthy, C-congestive heart failure and T-transplanted are ordered by age (ascending) and group estimates are presented on the right of each panel.

Table 1 Long-range scaling exponent $\hat{d}$ and short-range scaling exponents α_1 and $\alpha_{1,SM}$ values for the three groups of patients: healthy, subjects affected by congestive heart failure (CHF) and transplanted, during 24 hours (P1), 6 hours of night (P2) and 6 hours of day (P3) periods. For each case the average estimates $\pm$ standard deviations are presented.

Scaling exponentes	Periods	Healthy	CHF	Transplanted
$\hat{d}$	P1	0.44 ± 0.06	0.52 ± 0.14	0.76 ± 0.10
	P2	0.34 ± 0.07	0.38 ± 0.16	0.67 ± 0.17
	P3	0.46 ± 0.09	0.59 ± 0.16	0.78 ± 0.12
α_1	P1	1.46 ± 0.18	1.18 ± 0.28	0.70 ± 0.25
	P2	1.33 ± 0.16	1.15 ± 0.26	0.76 ± 0.29
	P3	1.52 ± 0.27	1.19 ± 0.32	0.67 ± 0.28
$\alpha_{1,SM}$	P1	1.16 ± 0.22	0.72 ± 0.24	0.32 ± 0.12
	P2	1.05 ± 0.19	0.77 ± 0.25	0.40 ± 0.20
	P3	1.25 ± 0.30	0.71 ± 0.26	0.29 ± 0.13

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