Irrelevance of private information in two-period economies with more goods than states of nature

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Abstract. We introduce a two-period economy with asymmetric information about the state of nature that occurs in the second period. Each agent is endowed with an information structure that describes her (incomplete) ability to prove whether or not a state has occurred. We show that if the number of states of nature is not greater than the number of goods, then, generically, the equilibria of the associated full information economy are also equilibria of the asymmetric information economy. The information structures of the agents are, in that sense, irrelevant.

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1 Introduction

The first attempt to incorporate information asymmetries in general equilibrium theory was made by Radner (1968).[1] In his model, each agent makes contracts that specify deliveries which must be constant across states of nature that she cannot distinguish. This seems to be too restrictive, since the other party may find it to be in his interest to honor the contract, even if a violation of the contract could be concealed.[2]

In previous works (Correia-da-Silva and Hervés-Beloso, 2008, 2009, 2012), we have introduced the model of an *economy with uncertain delivery*, where this informational restriction is relaxed.[3] Agents are allowed to make contracts that specify deliveries which differ across states of nature that they do not distinguish. However, in that case, they may end up receiving a bundle that was supposed to be delivered in a state of nature that they cannot distinguish from the one that actually occurred.[4]

In this paper, we modify our model of general equilibrium with uncertain delivery (Correia-da-Silva and Hervés-Beloso, 2012) by opening spot markets in the second period, i.e., by allowing agents to retrade after receiving the deliveries associated with the contracts that they had made in the first period.[5]

We consider a two-period economy with spot markets in both periods, present and future, and contingent markets (in the first period) for delivery in the second period. In the first period, when there is uncertainty about the future state of nature, agents

[1]See also Yannelis (1991) and the book edited by Glycopantis and Yannelis (2005).

[2]Such contracts are said to be incentive compatible (Hurwicz, 1972). Allowing agents to make incentive compatible contracts, Prescott and Townsend (1984a, 1984b) showed the existence of optimal allocations and sought to decentralize them through a price system. However, to induce agents to select incentive compatible contracts, such decentralization may require non-linear prices (Jerez, 2005; Rustichini and Siconolfi, 2008).

[3]A related contribution was made by de Castro, Pesce and Yannelis (2011).

[4]In our framework, the term *distinguish* refers to the ability to provide evidence that is relevant for the enforcement of a contract.

[5]A general equilibrium model of trade with asymmetric information which also features trade *ex ante* and *ex post* was developed by Bisin and Gottardi (1999). Their contribution is worth mentioning, as their "Hidden Information Economy" addresses related issues, although using a different approach. In their model, there is a publicly observable aggregate shock and a privately observed idiosyncratic shock. Agents trade securities that are (only) payable in a numeraire good, and are able to influence the payoffs of securities through their announcements. Bisin and Gottardi (1999) concluded that existence of equilibrium requires a minimal form of non-linearity of prices (a bid-ask spread).

trade in spot markets and in contingent markets. Trade in spot markets determines present consumption (goods are assumed to be perishable, which means that no storage is possible). Trade in contingent markets determines the bundle that agents should receive in each of the possible future states of nature. In the second-period, agents exchange the bundle that is delivered to them, together with their second-period endowments, for their second-period consumption bundle.

The difference with respect to the classical model of general equilibrium under uncertainty (Debreu, 1959, chapter 7) is that agents are assumed to have incomplete and differential information about the state of nature that occurs in the second period. Each agent is endowed with a private information structure, described by a partition of the set of possible states of nature. In the second period, with the objective of enforcing the delivery contracted in the first period, all that an agent can verify is that the state of nature belongs to a certain element of her information partition.[6]

Trade in contingent markets is mediated by competitive insurance firms.[7] In the first period, each agent makes a contract with an insurance firm, stipulating a net trade for each of the possible future states of nature. In the second period, given the agent's incomplete ability to verify the state of nature that has occurred, the firm may have the opportunity to deliver a less valuable net trade. It may choose among the net trades contracted for each of the states of nature that the agent cannot distinguish from the one that actually occurred.[8] As a result, the agent receives, in each state of nature, the less valuable of these net trades (according to the spot prices in that state of nature).

In our model, agents cannot use the information contained in the second-period spot prices to prove to a third party that a certain state has occurred. This contrasts with the frameworks of Radner (1979) and Allen (1981). We rule out the use of information revealed by prices to enforce contracts because it is well known that this would eliminate the informational asymmetries, rendering the model useless to explain their economic effects. We assume that even if prices allow an agent to infer the true state of nature,

[6]While Townsend (1979) studied the effects of costly state-verification, we assume that verification is free but incomplete. In addition, this incompleteness varies across agents.

[7]Firms also play the role of intermediaries in the works of Prescott and Townsend (1984a, 1984b), Jerez (2005), Bisin and Gottardi (2006) and Rustichini and Siconolfi (2008).

[8]We implicitly assume that, in case of litigation between agent and firm, it is the agent that bears the burden of proof.

these inferences are ineffective as a means of enforcing contracts.[9]

In our framework, a key point is that there are contracts which cannot be enforced because agents have incomplete information. In this sense, markets are incomplete (in a different way that in the model introduced by Radner (1972) and developed by Magill and Quinzii (1996)). Here, agents face endogenous and differential trade possibilities.

We focus on the case in which the number of states of nature is not greater than the number of goods.[10] In this case, we obtain a strong characterization result. Generically (i.e., in almost all economies) equilibria of the corresponding full information economy are also equilibria of our private state-verification economy.[11] Surprisingly, the incompleteness of private information is irrelevant. Unless the second-period spot prices in the different states are linearly dependent (which almost never occurs), agents are able to overcome their incomplete abilities to verify the occurrence of events by selecting an appropriate bridge portfolio, which guarantees truthful delivery of the desired wealth transfers across states of nature.

We provide two numerical examples to shed light on the limits of our results. The first is an example of nonexistence of equilibrium of an asymmetric information economy, which shows that the above result is only generic. In this example, the single equilibrium of the corresponding full information economy is not an equilibrium of the asymmetric information economy. The origin of nonexistence is the following coincidence: if wealth transfers are allowed, state-contingent prices are parallel (therefore, wealth transfers are not informationally feasible); if wealth transfers are not allowed, state-contingent prices are not parallel (therefore, wealth transfers are informationally feasible).

The second example is an equilibrium of an asymmetric information economy which is not an equilibrium of the corresponding full information economy. In this example, there exists an equilibrium of the full information economy that is also an equilibrium

[9]In spite of ruling out the use of information provided by prices to enforce contracts, we will conclude that (generically) an equilibrium allocation in the full information model is also an equilibrium allocation in our asymmetric information model. This means that agents do not even need to use the information revealed by prices. Our result holds if the number of states is not greater than the number of goods.

[10]More precisely, our conclusions apply to the case in which the maximum number of states in an agent's information set is not greater than the number of goods.

[11]As most genericity results, this characterization requires a differentiability assumption. We assume that agents' preferences are *well-behaved* in the sense of Debreu (1972), which implies that their demand functions are continously differentiable.

of the asymmetric information economy. But the asymmetric information economy has an additional equilibrium, in which state-contingent prices are parallel. The coincidence behind this second example is similar to the one that underlies the first example. In this case: if wealth transfers are allowed, state-contingent prices are not parallel (meaning that wealth transfers are informationally feasible); if wealth transfers are not allowed, state-contingent prices are parallel (meaning that wealth transfers are not informationally feasible).

The paper is organized as follows. In Section 2, we present the model of a two-period economy with private state-verification. In Section 3, we show that, generically, there exists an equilibrium allocation that is optimal in the sense of Pareto. In Section 4, we present the two numerical examples. In Section 5, we conclude the paper with some remarks.

2 The model

In an economy that extends over two time periods, $\tau = 1$ and $\tau = 2$, a finite number of agents, $\mathcal{I} = \{1, ..., I\}$, trade contracts for the delivery of a finite number of commodities, $\mathcal{L} = \{1, ..., L\}$.

In the first period, agents trade in the presence of uncertainty (but no private information) about which of a finite set of possible states of the environment, $\mathcal{S} = \{1, ..., S\}$, will occur in the second period.

In the second period, each agent receives private information that is described by a partition of $\mathcal{S}$. If state $s \in \mathcal{S}$ occurs, agent $i \in \mathcal{I}$ is only able to prove that the state of nature belongs to the element of her information partition that contains s, which is denoted by $P^i(s)$.

The initial endowments of each agent $i \in \mathcal{I}$ are $e^i = (e^i_1, e^i_2) \in \mathbb{R}^N_+ \equiv \mathbb{R}^L_+ \times \mathbb{R}^{SL}_+$.[12]

The preferences of each agent $i \in \mathcal{I}$ about consumption in both periods, $x^i = (x^i_1, x^i_2)$,

[12]The second-period endowments are allowed to differ across states of nature that belong to the same set of an agent's information partition, but it is assumed that the agent cannot use the information provided by its initial endowments to enforce contracts. We will conclude that, in equilibrium, the agent is not informationally constrained, which means that this information would not be useful.

are described by a utility function, $U^i : \mathbb{R}_+^N \to \mathbb{R}$.

There are spot markets at $\tau = 1$ and at $\tau = 2$, and contingent markets at $\tau = 1$ for delivery at $\tau = 2$. The deliveries contracted (at $\tau = 1$) in contingent markets are conditional on the state of nature that occurs (at $\tau = 2$), thus each agent i chooses a plan of net deliveries, specifying what she should receive in each state of nature, $y^i = (y^i(1), ..., y^i(s), ..., y^i(S)) \in \mathbb{R}^{SL}$.

Prices in spot markets at $\tau = 1$ and $\tau = 2$ are denoted by p_1 and p_2, respectively, and prices in contingent markets are denoted by q.

At $\tau = 1$, agent i trades her endowments, e_1^i, for a consumption bundle, $x_1^i \in \mathbb{R}_+^L$, and a plan of future net deliveries, $y^i \in \mathbb{R}^{SL}$. The corresponding budget restriction is:

$$\left(x_1^i, y^i\right) \in B_1^i\left(p_1, q\right) = \left\{(z_1, w) \in \mathbb{R}_+^L \times \mathbb{R}^{SL} : \ p_1 \cdot z_1 + q \cdot w \leq p_1 \cdot e_1^i\right\}.$$

Trade in contingent markets is mediated by profit-maximizing insurance firms, who are also price-takers. The relationship between agents and insurers is asymmetric, as it is the agent that bears the burden of proof. At $\tau = 2$, if state s occurs, agent i can only prove that the state of nature belongs to $P^i(s)$, therefore, the insurers decide which of the alternatives among $\{y^i(t)\}_{t \in P^i(s)}$ is delivered to her. Profit-maximization by the insurers implies that only the cheapest alternatives, according to $p_2(s)$, may be delivered.

Hence, agents receive, in each state s, one of the cheapest bundles among those that, according to the information provided by $P^i(s)$, may correspond to the truthful delivery.[13] Accordingly, we can restrict (without loss of generality) the choice of agent i to satisfy the following restrictions, which induce truthful delivery:[14]

$$y^i \in D^i(p_2) = \left\{z \in \mathbb{R}^{SL} : \ p_2(s) \cdot z(s) \leq p_2(s) \cdot z(t), \ \forall t \in P^i(s), \forall s \in \mathcal{S}\right\}.$$

At $\tau = 2$, in state s, agent i receives $y^i(s)$ (truthful delivery), which she trades, together with her endowments, $e_2^i(s)$, for a consumption bundle, $x_2^i(s) \in \mathbb{R}_+^L$. The corresponding

[13]See also Correia-da-Silva and Hervés-Beloso (2008, 2009, 2012) and Correia-da-Silva (2012).

[14]As we shall verify, since p_2 is parallel to q, the choice of $y^i \notin D^i(p_2)$ would never be optimal, as it would lead to the delivery of some $z^i \in D^i(p_2)$, cheaper than y^i. The agent would be better off by choosing z^i instead of y^i.

budget restriction is:

$$x_2^i(s) \in B_{2s}^i(p_2(s), y^i(s)) = \left\{ z \in \mathbb{R}_+^L : \ p_2(s) \cdot z \leq p_2(s) \cdot \left[y^i(s) + e_2^i(s) \right] \right\}.$$

We write $x_2^i \in B_2^i(p_2, y^i)$ if and only if $x_2^i(s) \in B_{2s}^i(p_2(s), y^i(s)), \forall s \in \mathcal{S}$.

Since there is a budget restriction at $\tau = 1$ and a budget restriction for each state of nature at $\tau = 2$, we normalize prices by imposing that $(p_1, q) \in \Delta^N$ and that $\|p_2(s)\| = \|q(s)\|$, for each $s \in \mathcal{S}$.

The insurance firms behave as a single price-taking agent that has full information, $P^f(s) = \{s\}$, $\forall s \in \mathcal{S}$, but no endowments, $e^f = 0$. The choice of the insurer is a triple, (x_1^f, y^f, x_2^f), with $(x_1^f, y^f) \in B_1^f(p_1, q)$ and $x_2^f \in B_2^f(p_2, y^f)$.[15] The objective function of the insurer is strictly increasing:[16]

$$\max_{(x_0^f, y^f, x_1^f)} \ U^f(x_1^f, x_2^f) \quad \text{s.t.} \quad (x_1^f, y^f) \in B_1^f(p_1, q) \ \wedge \ x_2^f \in B_2^f(p_2, y^f).$$

The demand of the insurer would become unbounded if, for some state s, the relative prices in spot markets at $\tau = 2$ were different from the relative prices in markets for contingent delivery in this state. That is, there are arbitrage opportunities unless we have $q(s) = p_2(s)$, for all $s \in \mathcal{S}$. If, for every state of nature, prices in the contingent markets and prices in the spot markets coincide, the insurer cannot obtain any positive consumption plan and is, therefore, indifferent among any alternative in its choice set.

Hence, we will restrict our search for equilibrium price systems to those that are such that $q(s) = p_2(s)$, $\forall s \in \mathcal{S}$. With this *no arbitrage* property, referring to q is redundant. A price system is completely described by $p = (p_1, p_2) \in \Delta^N$.

In the absence of arbitrage possibilities, we suppose that the insurer clears the contingent markets by choosing:

$$x^f = \left(0, y^f, 0\right), \quad \text{with} \ \ y^f = -\sum_{i \in \mathcal{I}} y^i,$$

[15]The budget sets, B_1^f and B_{2s}^f, are defined as B_1^i and B_{2s}^i, but with null endowments.

[16]Our results do not depend on the actual specification of the objective function. With incomplete markets, the difficulties in defining an appropriate objective function for a firm are well known. See, for example, Drèze (1985).

which is a trivially optimal choice.

Let $x^i = (x_1^i, y^i, x_2^i)$. We write $x^i \in B^i(p)$ if and only if $(x_1^i, y^i) \in B_1^i(p_1, p_2)$ and $x_2^i \in B_2^i(p_2, y^i)$. The choice set of agent i is:

$$C^i(p) = \left\{x^i = \left(x_1^i, y^i, x_2^i\right) \in \mathbb{R}_+^L \times \mathbb{R}^{SL} \times \mathbb{R}_+^{SL} : \; x^i \in B^i(p) \wedge y^i \in D^i(p_2)\right\}.$$

In sum, the problem of agent i can be written as:

$$\begin{array}{ll} \max\limits_{\left(x_1^i, y^i, x_2^i\right)} & U^i(x_1^i, x_2^i) \\ \text{s.t.} & p_1 \cdot x_1^i + p_2 \cdot y^i \leq p_1 \cdot e_1^i, \\ & p_2(s) \cdot x_2^i(s) \leq p_2(s) \cdot \left[y^i(s) + e_2^i(s)\right], \; \forall s \in \mathcal{S}, \\ & p_2(s) \cdot y^i(s) \leq p_2(s) \cdot y^i(t), \; \forall t \in P^i(s), \; \forall s \in \mathcal{S}. \end{array}$$

Or, more compactly, as:

$$\max_{x^i} \quad U^i(x_1^i, x_2^i) \;\; \text{s.t.} \;\; x^i \in C^i(p).$$

If agents make optimal choices and markets clear, the economy is in equilibrium.

Definition 1 (Equilibrium)**.**

An equilibrium of the asymmetric information economy, $E = \{e^i, U^i, P^i\}_{i \in \mathcal{I}}$, is a pair, (x^*, p^*), where $x^* = \{x^{i*}\}_{i \in \mathcal{I}}$ are individual choices and $p^* \in \Delta^N$ is a price system, that satisfies:

(i) $x^{i*} \in \underset{z \in C^i(p^*)}{\operatorname{argmax}} \; U^i(z_1, z_2), \; \forall i \in \mathcal{I}$ *[individual optimality];*

(ii) $\sum_{i \in \mathcal{I}} \left(x_1^{i*}, x_2^{i*}\right) = \sum_{i \in \mathcal{I}} \left(e_1^i, e_2^i\right)$ *[feasibility].*

If the correspondences from prices to choice sets, $C_i(p)$, were continuous, it would be straightforward to establish existence of equilibrium (Debreu, 1952). However, the choice correspondences are not lower hemicontinuous. This property may fail when prices in two indistinguished states are collinear ($\exists s \in \mathcal{S}, t \in P_i(s), k \in \mathbb{R}_{++} : \; p_2(s) = kp_2(t)$).

In spite of this discontinuity, we will be able to establish generic existence of equilibrium in economies where the number of states of nature is not greater than the number of goods. To show that it is not possible to go beyond the generic existence result, we will provide an example of an economy in which equilibrium does not exist.

In this kind of differential information economies, the welfare theorems do not necessarily hold. Imperfect information may generate an inefficient allocation of risk-bearing, because agents may be unable to make the desired wealth transfers across time and states of nature.

Interestingly, the existence of markets for the future delivery of various goods (as opposed to contingent claims that are only payable in the numeraire good) generates additional possibilities for the transference of wealth across states and time. Our main conclusion will be that, if the number of states of nature is not greater than the number of goods, then, generically, a complete set of contingent markets allows agents to arrive at the optimal allocation of risk-bearing, while assets that are only payable in a numeraire good are not sufficient. This conclusion contrasts with the equivalence result obtained by Arrow (1953) for the case of public state-verification.

3 Generic existence of an efficient equilibrium

If there were no deliverability restrictions (as in the case of complete state-verification), the equilibrium allocation would be efficient, since our model would coincide with the classical general equilibrium model as presented by Debreu (1959, chapter 7).

On the other hand, in the case in which there is a single good (in each state of nature), our model essentially coincides with the model of Radner (1968).[17] In that case, the incompleteness of state-verification abilities implies efficiency losses. There is only constrained efficiency.[18]

[17] With a single commodity, if prices are strictly positive: $p(s) \cdot y(s) \leq p(s) \cdot y(t) \Leftrightarrow y(s) \leq y(t)$ and $p(t) \cdot y(t) \leq p(t) \cdot y(s) \Leftrightarrow y(t) \leq y(s)$. Therefore, our deliverability restrictions are equivalent to Radner's measurability restriction: $y(s) = y(t)$. Observe also that, since there is a single commodity, there cannot be trade in the second period. This implies that $x_2(s) = x_2(t)$.

[18] Here, constrained efficiency means that equilibrium allocations are Pareto-optimal among those that satisfy Radner's informational restriction (agents must consume the same in states of nature that belong to the same set of their information partitions).

In this section, we show that if there are at least as many goods as states of nature, then, generically, there exists an efficient equilibrium.[19] In order to prove this, we proceed in two steps. First, we prove that if the state-contingent spot price systems are linearly independent, then the agents are able to attain any consumption plan that is in their budget set (i.e., the deliverability constraints are not relevant) by choosing an appropriate "bridge portfolio". Then, we show that in almost all full information economies, the state-contingent equilibrium price systems are linearly independent. This allows us to conclude that, generically, the equilibria of a full information economy are also equilibria of the corresponding economies with asymmetric information.

Lemma 1 (Wealth transfers)**.**

Consider a vector of state-contingent spot price systems that are linearly independent, $p_2 = (p_2(1), ..., p_2(S)) \in \mathbb{R}^{SL}_+$*, and a vector of wealth transfers,* $w = (w(1), ..., w(S)) \in \mathbb{R}^S$*.*

If $L \geq S$*, there exists a portfolio,* $y = (y(1), ..., y(S)) \in \mathbb{R}^{SL}$*, that implements the wealth transfers,* $p_2(s) \cdot y(s) = w(s),\ \forall s \in \mathcal{S}$*, and satisfies the possible deliverability constraints,* $p_2(s) \cdot y(s) \leq p_2(s) \cdot y(t),\ \forall s, t \in \mathcal{S}$*.*

Proof. Let $\bar{w} = \max_{s \in \mathcal{S}} w(s)$. Observe that there exists a $y(s)$ such that:

$$\begin{bmatrix} p_2(1,1) & ... & p_2(1,L) \\ ... & ... & ... \\ p_2(s,1) & ... & p_2(s,L) \\ ... & ... & ... \\ p_2(S,1) & ... & p_2(S,L) \end{bmatrix} \begin{bmatrix} y(s,1) \\ ... \\ y(s,L) \end{bmatrix} = \begin{bmatrix} \bar{w} \\ ... \\ w(s) \\ ... \\ \bar{w} \end{bmatrix},$$

because the number of equations is not greater than the number of variables, $S \leq L$, and the equations are not inconsistent (they could be if the rows of the price matrix were linearly dependent).

This means that $p_2(s) \cdot y(s) = w(s)$ while $p_2(t) \cdot y(s) = \bar{w} \geq w(t),\ \forall t.$ $\square$

The fact that agents are not informationally restricted in their choice of wealth transfers across time and states of nature implies that their consumption choice set does not depend

[19] Our results apply if the maximum number of states in an agent's information set is not greater than the number of goods, i.e., if $\max_{i,s} \#P^i(s) \leq L$. This condition is always verified if $S \leq L$.

on their information, as we show below (notice that Lemma 2 holds independently of the agents' information partitions).

Lemma 2 (Irrelevance of information structures).

Let $B^i_{AD}(p) \equiv \left\{(z_1, z_2) \in \mathbb{R}^N_+ : \; p_1 \cdot z_1 + p_2 \cdot z_2 \leq p_1 \cdot e^i_1 + p_2 \cdot e^i_2\right\}$.

If $L \geq S$ *and the spot price systems* $\{p_2(s)\}_{s \in \mathcal{S}}$ *are linearly independent, then:*

$$(x_1, x_2) \in B^i_{AD}(p) \;\Leftrightarrow\; \exists y : \; (x_1, y, x_2) \in C^i(p).$$

Proof. Let $(x_1, x_2) \in B^i_{AD}(p)$. For each $s \in \mathcal{S}$, calculate the wealth transfers, $w(s) = p_2(s) \cdot [x_2(s) - e_2(s)]$, and obtain $y(s)$ as shown in Lemma 1. By contruction, $y \in D^i(p_2)$. Observe that, since $p_2(s) \cdot y(s) = w(s) = p_2(s) \cdot [x_2(s) - e_2(s)]$, it is guaranteed that $(x_1, y) \in B^i_1(p_1, p_2)$. It is also clear that each $x_2(s) \in B^i_{2s}(p_2(s), y(s))$.

Let $(x_1, y, x_2) \in C^i(p)$. Since, $\forall s \in \mathcal{S}$, $p_2(s) \cdot x_2(s) \leq p_2(s) \cdot [y(s) + e^i_2(s)]$, we conclude that $p_2 \cdot x_2 \leq p_2 \cdot (y + e^i_2)$. Together with $p_1 \cdot x_1 + p_2 \cdot y \leq p_1 \cdot e^i_1$, this implies that $(x_1, x_2) \in B^i_{AD}(p)$. $\square$

The asymmetric information economy is, therefore (conditionally on the linear independence of second-period spot prices), equivalent to a standard Arrow-Debreu economy.

To study the generic properties of equilibria, we impose further restrictions on the preferences of the agents by assuming that they are *well-behaved* in the sense of Debreu (1972, p. 613).

Assumption 1 (Preferences).

The utility function of each agent $i \in \mathcal{I}$ *satisfies the following properties:*

(i) $U^i : \mathbb{R}^N_+ \to \mathbb{R}$ *is continuous in* $\mathbb{R}^N_+$ *and* $\mathcal{C}^2$ *in* $\mathbb{R}^N_{++}$*;*

(ii) $R^i(x) \equiv \left\{z \in \mathbb{R}^N_+ : \; U^i(z) \geq U^i(x)\right\} \subset \mathbb{R}^N_{++}, \; \forall x \in \mathbb{R}^N_{++}$*;*

(iii) $\forall x \in \mathbb{R}^N_{++}, \; \left(\frac{\partial U^i(x)}{\partial x_1}, ..., \frac{\partial U^i(x)}{\partial x_N}\right) \gg 0$*;*

(iv) $\forall x \in \mathbb{R}^N_{++}, \; \sum_{j=i}^N \sum_{k=1}^N h_j h_k \frac{\partial^2 U^i(x)}{\partial x_j x_k} < 0$*, for all* $h \in \mathbb{R}^N \setminus \{0\}$ *s.t.* $\sum_{j=1}^N h_j \frac{\partial U^i(x)}{\partial x_j} = 0$.

The above assumption was introduced by Debreu (1972) and is widely used (Malinvaud, 1972; Allen, 1981; Mas-Colell, 1985; Magill and Quinzii, 1996; Balasko, 2009).[20] It implies that the preferences of the agents can be described by demand functions that are continuously differentiable (Debreu, 1972). Obviously, the resulting aggregate excess demand function is also continuously differentiable.

It is worth commenting on each of the assumed properties. Point (i) is the differentiability assumption. Point (ii) means that the indifference hypersurfaces are contained in $\mathbb{R}^N_{++}$, implying that agents choose consumption plans that belong to the interior of the consumption set. Point (iii) is the assumption of strong monotonicity. Point (iv) imposes, besides strict quasi-concavity, that the indifference hypersurfaces have everywhere a non-zero curvature (which is crucial for the differentiability of the demand function).

We consider a space of Arrow-Debreu economies, $\mathcal{E} = \mathbb{R}^{IN}_{+}$, in which preferences are kept fixed and satisfy Assumption 1. In this space, the vector of initial endowments, $e \in \mathbb{R}^{IN}_{+}$, completely characterizes an economy. It is well-known that equilibrium exists as long as the aggregate endowment is strictly positive ($e^T \equiv \sum_{i\in\mathcal{I}} e^i \gg 0$).

We want to show that, generically (i.e., in an open and dense subset of $\mathcal{E}$), equilibrium prices for state-contingent delivery, $p_2(s)$ for each $s \in \mathcal{S}$, are linearly independent. In this case, we say that $p = (p_1, p_2) \in \mathcal{P}$.

Lemma 3 (Prices)**.**

There exists an open and dense set of economies, $\mathcal{E}^* \subset \mathcal{E}$, for which all equilibrium price systems belong to $p \in \mathcal{P}$.

Proof. Let $\mathcal{P}' \subset \mathbb{R}^{N-1}$ be the open set that is obtained by removing the last coordinate from the interior of Δ^N.

The aggregate excess demand function, $Z : \mathcal{P}' \times \mathcal{E} \to \mathbb{R}^{N-1}$, is defined as the difference between the sum of the individual demands and the aggregate endowment. Once again, we omit the last coordinate as it can be obtained from the others using Walras' Law.

[20]Debreu (1972) designated the preference relations that satisfy an analogue of Assumption 1 as *well-behaved* preferences. The assumption is not too strong, in the sense that any monotone, convex, continuous and complete preference relation can be approximated by a sequence of preference relations that are *well-behaved* (Mas-Colell, 1974).

It is known that Z has no critical point. Therefore, by the regular value theorem, the set $M \equiv Z^{-1}(0)$ is a differentiable manifold of dimension IN (the equilibrium price manifold).[21]

Let $pr : M \to \mathcal{E}$ be the projection of the equilibrium price manifold to the parameter space. An economy is regular, $e \in \mathcal{R}$, if and only if it is a regular value of this projection. Otherwise, it is a critical economy. It is well known that the set of critical economies, $\mathcal{C} = \mathcal{E} \setminus \mathcal{R}$, is null (Debreu, 1970).

A price system is a regular equilibrium price system of the economy $e \in \mathcal{E}$ if and only if $Z(p, e) = 0$ and $\partial_p Z(p, e)$ has full rank. If an economy is regular, all its equilibrium price systems are regular (Dierker, 1982).

Therefore, for $e \in \mathcal{R}$, we can apply the implicit function theorem to obtain the following result. Given a point of the equilibrium manifold, $(p, e) \in M$, there are open sets, $\mathcal{P}'' \subset \mathcal{P}'$ and $\mathcal{E}' \subset \mathcal{E}$, and a C^1 function $g : \mathcal{E}' \to \mathcal{P}''$ such that $g(e) = p$ and, for $(p', e') \in \mathcal{P}'' \times \mathcal{E}'$, $Z(p', e') = 0$ if and only if $g(e') = p'$.

Moreover, $\partial g(e) = -\left[\partial_p Z(p, e)\right]^{-1} \partial_e Z(p, e)$, which implies that $\partial g(e)$ has full rank.

This means that we can move the equilibrium price in any direction by perturbing the initial endowments. Any neighborhood of $(p, e) \cap M$ contains, therefore, equilibrium price systems for which the prices in different states are linearly independent.

On the other hand, since $\partial g(e)$ is continuous, if $p \in \mathcal{P}$, then there is a neighborhood of $(p, e) \cap M$ in which prices in different states are also linearly independent. $\square$

The main result of this section is a straightforward consequence of Lemmas 2 and 3.

Theorem 1 (Optimality)**.**

If $L \geq S$, there exists an open and dense set of economies, $\mathcal{E}^* \subset \mathcal{E}$, where, $\forall e \in \mathcal{E}^*$, the equilibria of the Arrow-Debreu economy are also equilibria of any economy with the same endowments and preferences but with differential information.

This surprising result establishes that, if $L \geq S$, the information partitions of the agents

[21] See, for example, Balasko (2009, p. 28).

are irrelevant. The equilibrium allocation is independent of the information structure of the economy.

4 Two numerical examples

4.1 An example of nonexistence of equilibrium

Consider an economy with two agents (A and B), two commodities (1 and 2) and two states of nature (s and t).[22]

Agent A can verify the state of nature that occurs while agent B cannot.

$$P^A = \{\{s\}\,;\{t\}\} \quad \text{and} \quad P^B = \{\{s,t\}\}\,.$$

The endowments and preferences of the agents are given by:

$$\begin{aligned} e^A &= \left[\left(e^A_{10}, e^A_{20}\right); \left(e^A_{1s}, e^A_{2s}\right); \left(e^A_{1t}, e^A_{2t}\right)\right] = \left[\left(\tfrac{1}{2}, \tfrac{1}{2}\right); (1,1)\,; (2,2)\right], \\ e^B &= \left[\left(e^B_{10}, e^B_{20}\right); \left(e^B_{1s}, e^B_{2s}\right); \left(e^B_{1t}, e^B_{2t}\right)\right] = \left[\left(\tfrac{3}{2}, \tfrac{3}{2}\right); (1,1)\,; (1,1)\right], \end{aligned}$$

$$\begin{aligned} U^A(x^A) &= 2{x^A_{10}}^{\frac{1}{2}} + 2{x^A_{20}}^{\frac{1}{2}} + {x^A_{1s}}^{\frac{1}{2}} + {x^A_{2s}}^{\frac{1}{2}} + {x^A_{1t}}^{\frac{1}{2}} + \tfrac{3}{2}{x^A_{2t}}^{\frac{1}{3}}, \\ U^B(x^B) &= 2{x^B_{10}}^{\frac{1}{2}} + 2{x^B_{20}}^{\frac{1}{2}} + {x^B_{1s}}^{\frac{1}{2}} + {x^B_{2s}}^{\frac{1}{2}} + \sqrt{2}{x^B_{1t}}^{\frac{1}{2}} + \sqrt{2}{x^B_{2t}}^{\frac{1}{2}}. \end{aligned}$$

Computing the standard Arrow-Debreu equilibrium (i.e., ignoring the informational constraints), we obtain:

$$\begin{aligned} p^* &= \left[(p^*_{10}, p^*_{20})\,; (p^*_{1s}, p^*_{2s})\,; (p^*_{1t}, p^*_{2t})\right] = \left[(1,1)\,; \left(\tfrac{1}{2}, \tfrac{1}{2}\right); \left(\tfrac{1}{2}, \tfrac{1}{2}\right)\right], \\ x^A &= \left[\left(x^A_{10}, x^A_{20}\right); \left(x^A_{1s}, x^A_{2s}\right); \left(x^A_{1t}, x^A_{2t}\right)\right] = \left[(1,1)\,; (1,1)\,; (1,1)\right], \\ x^B &= \left[\left(x^B_{10}, x^B_{20}\right); \left(x^B_{1s}, x^B_{2s}\right); \left(x^B_{1t}, x^B_{2t}\right)\right] = \left[(1,1)\,; (1,1)\,; (2,2)\right]. \end{aligned}$$

Notice that the wealth transfers required by the Arrow-Debreu equilibrium are not com-

[22] To simplify notation, the first period is denoted as if it was state 0.

patible with the deliverability constraints. Agent B needs to transfer wealth from the present to the future state t (precisely: $p_t^* \cdot y_t^B = 1$) but not to the future state s (precisely: $p_s^* \cdot y_s^B = 0$). However, the collinearity between prices in states s and t implies that agent B is restricted to transfer the same amount of wealth from the present to the future state s and to the future state t:

$$\begin{cases} p_s^* \cdot y_s^B \leq p_s^* \cdot y_t^B \\ p_t^* \cdot y_t^B \leq p_t^* \cdot y_s^B \end{cases} \Rightarrow \; p_s^* \cdot y_s^B = p_t^* \cdot y_t^B.$$

An equilibrium with non-collinear prices in states s and t would have to be an Arrow-Debreu equilibrium, because the non-collinearity implies that the agents are able to circumvent the deliverability constraints. Since the single Arrow-Debreu equilibrium (computed above) cannot be an equilibrium of the economy with private state-verification, we conclude that, if an equilibrium exists, it must be such that prices for delivery in states s and t are collinear.

The spot prices in state s must be the same for the two goods ($p_{1s}^* = p_{2s}^*$), otherwise, both agents would demand a higher quantity of the cheapest good while the supply is the same for the two goods. The same is true for present prices ($p_{10}^* = p_{20}^*$). Collinearity between p_s^* and p_t^* implies that candidate equilibrium prices have to be of the form:

$$p^* = \left[(p_{10}^*, p_{20}^*) \, ; (p_{1s}^*, p_{2s}^*) \, ; (p_{1t}^*, p_{2t}^*)\right] = \left[(1,1) \, ; (a,a) \, ; (b,b)\right].$$

With these prices in state t, agent B will demand the same quantity of the two goods: $x_{1t}^B = x_{2t}^B$. Feasibility, then, implies that $x_{1t}^A = x_{2t}^A$. For these to be optimal choices for agent A, it is necessary that the marginal rate of substitution between goods $1t$ and $2t$ is equal to 1. This implies that the allocation in state t is the same as in the Arrow-Debreu equilibrium:

$$\begin{cases} \dfrac{\frac{1}{2} {x_{1t}^A}^{-\frac{1}{2}}}{\frac{1}{2} {x_{2t}^A}^{-\frac{2}{3}}} = 1 \\ x_{1t}^A = x_{2t}^A \end{cases} \Rightarrow \; \left(x_{1t}^A, x_{2t}^A\right) = (1,1) \; \Rightarrow \; \left(x_{1t}^B, x_{2t}^B\right) = (2,2).$$

Observe that $p_t^* \cdot y_t^B = p_t^* \cdot x_t^B - p_t^* \cdot e_t^B = 2b$. Therefore, by the deliverability restrictions, $p_s^* \cdot y_s^B = 2b$, which implies that $\left(x_{1s}^B, x_{2s}^B\right) = \left(1 + \frac{b}{a}, 1 + \frac{b}{a}\right)$ and $\left(x_{1s}^A, x_{2s}^A\right) = \left(1 - \frac{b}{a}, 1 - \frac{b}{a}\right)$.

Spending in the present is the difference between the value of the present endowments and the value of the acquired portfolio, $p_0^* \cdot x_0^B = 3-4b$. Therefore: $\left(x_{10}^B, x_{20}^B\right) = \left(\frac{3}{2} - 2b, \frac{3}{2} - 2b\right)$ and $\left(x_{10}^A, x_{20}^A\right) = \left(\frac{1}{2} + 2b, \frac{1}{2} + 2b\right)$.

Since $\left(x_{10}^A, x_{20}^A\right) = \left(\frac{1}{2} + 2b, \frac{1}{2} + 2b\right)$ and $\left(x_{1t}^A, x_{2t}^A\right) = (1,1)$, for the marginal rate of substitution between goods 10 (or 20) and $1t$ (or $2t$) to equal the ratio between the corresponding prices, we must have $b = \frac{\sqrt{3}+1}{4}$ and $\left(x_{10}^A, x_{20}^A\right) = \left(1 + \frac{\sqrt{3}}{2}, 1 + \frac{\sqrt{3}}{2}\right)$.

For the same reason, we must have:

$$\begin{cases} \dfrac{\frac{1}{2} {x_{1s}^A}^{-\frac{1}{2}}}{\frac{1}{2} {x_{1t}^A}^{-\frac{1}{2}}} = \frac{a}{b} \\ x_{1t}^A = 1 \end{cases} \Rightarrow \left(x_{1s}^A, x_{2s}^A\right) = \left(\frac{b^2}{a^2}, \frac{b^2}{a^2}\right) \Rightarrow \left(x_{1s}^B, x_{2s}^B\right) = \left(2 - \frac{b^2}{a^2}, 2 - \frac{b^2}{a^2}\right).$$

Since $\left(x_{1s}^A, x_{2s}^A\right) = \left(1 - \frac{b}{a}, 1 - \frac{b}{a}\right) = \left(\frac{b^2}{a^2}, \frac{b^2}{a^2}\right)$, we obtain $\frac{b}{a} = \frac{\sqrt{5}-1}{2}$ and $a = \frac{1+\sqrt{3}}{2\left(\sqrt{5}-1\right)}$.

Summing up, we have:

$$\begin{aligned} p^* &= \left[\left(p_{10}^*, p_{20}^*\right); \left(p_{1s}^*, p_{2s}^*\right); \left(p_{1t}^*, p_{2t}^*\right)\right] = \left[(1,1); \left(\frac{1+\sqrt{3}}{2\left(\sqrt{5}-1\right)}, \frac{1+\sqrt{3}}{2\left(\sqrt{5}-1\right)}\right); \left(\frac{\sqrt{3}+1}{4}, \frac{\sqrt{3}+1}{4}\right)\right], \\ x^A &= \left[\left(x_{10}^A, x_{20}^A\right); \left(x_{1s}^A, x_{2s}^A\right); \left(x_{1t}^A, x_{2t}^A\right)\right] = \left[\left(1 + \frac{\sqrt{3}}{2}, 1 + \frac{\sqrt{3}}{2}\right); \left(\frac{3-\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right); (1,1)\right], \\ x^B &= \left[\left(x_{10}^B, x_{20}^B\right); \left(x_{1s}^B, x_{2s}^B\right); \left(x_{1t}^B, x_{2t}^B\right)\right] = \left[\left(1 - \frac{\sqrt{3}}{2}, 1 - \frac{\sqrt{3}}{2}\right); \left(\frac{\sqrt{5}+1}{2}, \frac{\sqrt{5}+1}{2}\right); (2,2)\right]. \end{aligned}$$

In order to see that this single equilibrium candidate is not an equilibrium, notice that agent B is transferring too much wealth from the present to the future. Agent B cannot transfer wealth between states s and t, but could attain a higher utility level by spending more in present consumption and less in its portfolio (transferring wealth from the present to both future states s and t):

$$\begin{aligned} \frac{\partial U^B / \partial x_{10}^B}{p_{10}^*} &= {x_{10}^B}^{-\frac{1}{2}} = \left(\frac{2}{2-\sqrt{3}}\right)^{\frac{1}{2}} \approx 2.73, \\ \frac{\partial U^B / \partial x_{1s}^B}{p_{1s}^*} &= \frac{{x_{1s}^B}^{-\frac{1}{2}}}{2p_{1s}^*} = \frac{\sqrt{5}-1}{\sqrt{3}+1} \left(\frac{2}{\sqrt{5}+1}\right)^{\frac{1}{2}} \approx 0.36, \\ \frac{\partial U^B / \partial x_{1t}^B}{p_{1t}^*} &= \frac{{x_{1t}^B}^{-\frac{1}{2}}}{\sqrt{2}p_{1t}^*} = \frac{2}{\sqrt{3}+1} \approx 0.73. \end{aligned}$$

4.2 An example of an inefficient equilibrium

Modify the above economy by changing the endowments of the agents as follows:

$$\begin{aligned} e^A &= \left[\left(e^A_{10}, e^A_{20}\right); \left(e^A_{1s}, e^A_{2s}\right); \left(e^A_{1t}, e^A_{2t}\right)\right] = \left[(1,1)\,;(1,1)\,;(1,1)\right], \\ e^B &= \left[\left(e^B_{10}, e^B_{20}\right); \left(e^B_{1s}, e^B_{2s}\right); \left(e^B_{1t}, e^B_{2t}\right)\right] = \left[\left(\tfrac{8}{3+2\sqrt{2}}, \tfrac{8}{3+2\sqrt{2}}\right); (2,2)\,; (2,2)\right]. \end{aligned}$$

The following price system and allocation is an equilibrium of the asymmetric information economy:

$$\begin{aligned} p^* &= \left[\left(p^*_{10}, p^*_{20}\right); \left(p^*_{1s}, p^*_{2s}\right); \left(p^*_{1t}, p^*_{2t}\right)\right] = \left[(1,1)\,; \left(\tfrac{1}{2}, \tfrac{1}{2}\right); \left(\tfrac{1}{2}, \tfrac{1}{2}\right)\right], \\ x^A &= \left[\left(x^A_{10}, x^A_{20}\right); \left(x^A_{1s}, x^A_{2s}\right); \left(x^A_{1t}, x^A_{2t}\right)\right] = \left[(1,1)\,;(1,1)\,;(1,1)\right], \\ x^B &= \left[\left(x^B_{10}, x^B_{20}\right); \left(x^B_{1s}, x^B_{2s}\right); \left(x^B_{1t}, x^B_{2t}\right)\right] = \left[\left(\tfrac{8}{3+2\sqrt{2}}, \tfrac{8}{3+2\sqrt{2}}\right); (2,2)\,; (2,2)\right]. \end{aligned}$$

It is straightforward to check that the allocation is feasible (agents are in autarchy) and that it is individually optimal for the informed agent.[23]

To check that it is individually optimal for the uninformed agent is slightly more complicated. Start by calculating the vector of marginal utilities:

$$\left[\frac{\partial U^B}{\partial x^B_{10}}, \frac{\partial U^B}{\partial x^B_{20}}, \frac{\partial U^B}{\partial x^B_{1s}}, \frac{\partial U^B}{\partial x^B_{2s}}, \frac{\partial U^B}{\partial x^B_{1t}}, \frac{\partial U^B}{\partial x^B_{2t}}\right] = \left[\tfrac{1}{2\sqrt{2}} + \tfrac{1}{2}, \tfrac{1}{2\sqrt{2}} + \tfrac{1}{2}, \tfrac{1}{2\sqrt{2}}, \tfrac{1}{2\sqrt{2}}, \tfrac{1}{2}, \tfrac{1}{2}\right].$$

Agent B would clearly like to transfer wealth from state s to state t. However, she is informationally restricted by the conditions $p_s \cdot y^B_s \leq p_s \cdot y^B_t$ and $p_t \cdot y^B_t \leq p_t \cdot y^B_t$. Since $p_s = p_t$, the two conditions imply that $p_s \cdot y^B_s = p_t \cdot y^B_t$. This means that agent B must transfer the same wealth from the present to state s and to state t.

Given this informational restriction, for simplicity of exposition and without loss of generality, we can restrict agent B to select a portfolio of the form:

$$\left[\left(y^B_{1s}, y^B_{2s}\right); \left(y^B_{1t}, y^B_{2t}\right)\right] = \left[(Y,0)\,;(Y,0)\right].$$

[23] The vector of marginal utilities of agent A is $\left[\frac{\partial U^A}{\partial x^A_{10}}, \frac{\partial U^A}{\partial x^A_{20}}, \frac{\partial U^A}{\partial x^A_{1s}}, \frac{\partial U^A}{\partial x^A_{2s}}, \frac{\partial U^A}{\partial x^A_{1t}}, \frac{\partial U^A}{\partial x^A_{2t}}\right] = \left[1, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right]$, which coincides with the price vector.

Such a portfolio implies an exchange of Y units of wealth in the present for $\frac{Y}{2}$ units of wealth in state s and $\frac{Y}{2}$ units of wealth in state t. Since the marginal utility of wealth in the present is $\frac{1}{2\sqrt{2}} + \frac{1}{2}$ and the marginal utility of wealth in states s and t is, respectively, $\frac{1}{\sqrt{2}}$ and 1, it is optimal to choose $Y = 0$. We conclude that the above allocation is individually optimal for agent B.

We have found an example of an equilibrium of the asymmetric information economy in which state-contingent prices are parallel, thus preventing the uninformed agent from transferring wealth between the states. The equilibrium allocation is inefficient.

For this economy, the single Arrow-Debreu equilibrium is the following:

$$\begin{aligned}
p^{AD} &= \left[\left(p_{10}^{AD}, p_{20}^{AD}\right); \left(p_{1s}^{AD}, p_{2s}^{AD}\right); \left(p_{1t}^{AD}, p_{2t}^{AD}\right)\right] \approx \left[(1,1); (0.447, 0.447); (0.6758, 0.6658)\right], \\
x^{A} &= \left[\left(x_{10}^{A}, x_{20}^{A}\right); \left(x_{1s}^{A}, x_{2s}^{A}\right); \left(x_{1t}^{A}, x_{2t}^{A}\right)\right] \approx \left[(0.811, 0.811); (1.0256, 1.0256); (1.2906, 1.2384)\right], \\
x^{B} &= \left[\left(x_{10}^{B}, x_{20}^{B}\right); \left(x_{1s}^{B}, x_{2s}^{B}\right); \left(x_{1t}^{B}, x_{2t}^{B}\right)\right] \approx \left[(1.5615, 1.5615); (1.9745, 1.9745); (1.7094, 1.7615)\right].
\end{aligned}$$

Since state-contingent prices are not parallel, this is also an equilibrium of the asymmetric information economy.

5 Conclusion

We have introduced an extension of the classical general equilibrium model of trade under uncertainty (Debreu, 1959, chapter 7) to the case in which agents have incomplete and differential abilities to verify the occurence of events. In contrast with our previous work, we allowed agents to trade *ex post*. This setup is more realistic because, with private state-verification, the second-period spot markets are not redundant. In fact, the combination of contingent markets (that open in the first period) with second-period spot markets expands the possibilities for wealth transfers across states and time.

In this model, market incompleteness arises endogenously as a consequence of incomplete state-verification. We have shown that, generically, markets are only incomplete if the number of states of nature is greater than the number of goods. Otherwise, markets are actually complete (in spite of incomplete state-verification). Agents are able to induce truthful delivery of the wealth transfers that they desire by choosing, for delivery in each state, goods that are relatively cheap in this state but relatively expensive in other states.

This strong and surprising result suggests that the information of the agents (with information being an exogenously given ability to verify the occurrence of events) is irrelevant. There is an equilibrium allocation, which is optimal and independent of the information structure of the economy.

We remark that, despite the fact that the relative prices for future delivery in a given state coincide with the relative prices in the future spot markets in the same state, agents do not buy, in contingent markets, the bundles that they wish to consume in the future (this would render the future spot markets irrelevant). Instead, they select a "bridge portfolio", which is not intended for consumption, but to induce the desired wealth transfers in the absence of complete state-verification.

The optimal allocation cannot, however, be achieved by a system of contingent markets for the delivery of a numeraire good and spot commodity markets (Arrow, 1953).[24] It may be the case that a complete set of contingent markets allows agents to arrive at an optimal allocation of risk-bearing, while a system of contingent markets payable in a numeraire good and commodity markets does not. If agents have incomplete abilities to verify the occurrence of relevant events, what was a redundancy in the ways of transferring wealth across states becomes useful as a means of enforcing truthful deliveries.

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[24] In a seminal work, Arrow (1953) showed that an optimal allocation of risk-bearing can be achieved by such a system of markets. Relatively to a system of complete contingent markets, the former permits economizing on markets. Only $L + S + L$ markets (where S is the number of states of nature, and L is the number of commodities) are needed, instead of a complete set of markets for contingent claims on commodities, which totals a number of $L + SL$ markets.

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