Optimal priority pricing by a durable goods monopolist

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April 24th, 2020

Abstract. A durable goods monopolist proposes selling mechanisms in two periods, being unable to commit in the first period on the mechanism to propose in the second. The monopolist can preserve buyer anonymity and prevent resale. Although buyers have a continuum of possible valuations, the optimal first-period mechanism is a menu with at most two possibilities: a high price guaranteeing delivery and a low price subject to rationing. This characterization is robust to the arrival of additional buyers in the second period. The optimal mechanism is fully characterized for linear demand, with priority pricing being optimal if agents are sufficiently patient.

Keywords: Priority pricing, Coase conjecture, Durable goods monopoly.

JEL Classification Numbers: D42, D82, L12.
1 Introduction

The market power of a monopolist who sells durable goods over time is mitigated by the inability to commit to future prices (Coase, 1972). At each moment in time, the buyers that are most willing to trade are those with the highest valuations for the durable good. As a result, in the following period, the seller will face buyers with lower valuations and thus will have incentives to set a lower price. Anticipating this, buyers become unwilling to pay a price much higher than the price they expect to be charged in the future. In some sense, the monopolist faces competition from future selves.

Research on revenue management by a durable goods monopolist who cannot commit on future behavior has mostly focused on the case in which the monopolist posts prices. Skreta (2006) showed that this restriction is inconsequential in a class of environments with a finite number of periods. If the seller is able to discriminate buyers based on their past behavior, the optimal selling mechanism under sequential rationality constraints is actually a sequence of posted prices, which means that the seller does not benefit from rationing or from offering a menu of combinations of payment and probability of trade. Rationing and priority pricing schemes do not bring additional revenue.

By contrast, in an environment with two periods where the seller is not able to discriminate buyers in the second period based on their first-period choices, and where resale is not possible, Denicolò and Garella (1999) showed that rationing is profitable if buyers are sufficiently patient. By keeping some buyers with high valuations unserved, rationing alters the composition of future demand in a way that increases the sequentially optimal second-period price, thereby alleviating the problem of limited commitment.

One can designate as transparent/identified a trade environment where each buyer’s decisions are observed by the seller and recalled in the future – allowing behavior-based discrimination –, and as opaque/anonymous one where the seller cannot track each buyer’s individual decisions – precluding behavior-based discrimination. With posted prices, this distinction is irrelevant because past history is the same for all buyers in the market.

\footnote{Or, equivalently, chooses output which is then sold at the market-clearing price. See Bulow (1982), Besanko and Winston (1990) and Fudenberg and Tirole (1998), among others.}
(assuming all buyers arrive in the first period and served buyers leave the market forever). With rationing, it becomes relevant. If trade is transparent, rationed buyers suffer a ratchet effect because their relatively high valuations for the durable good are revealed. If trade is opaque, buyers are more willing to risk being rationed because rationed buyers remain pooled with buyers with lower valuations who did not wish to trade. The results of Skreta (2006) and Denicolò and Garella (1999) imply that transparency is detrimental to the seller. The seller may increase profit through rationing only if trade is opaque.

This paper investigates the optimal selling mechanism in a two-period opaque trade environment. It is shown that the optimal selling mechanism in the first period may be either a relatively high posted price, a relatively low price with rationing, or a priority pricing menu composed by a relatively high price guaranteeing delivery and a relatively low price subject to rationing. More complicated mechanisms do not increase the seller’s revenue. With linear demand (uniformly distributed valuations), for a given time discount factor of the seller: a posted price is optimal if the buyer is sufficiently impatient; a single price with rationing is optimal if the buyer is sufficiently patient; priority pricing is optimal in intermediate cases. If buyer and seller have the same rate of time discount, a single price with rationing is never optimal. If they are relatively impatient (i.e., if the common discount factor is below a critical level), it is optimal for the seller to post a price; otherwise, priority pricing is optimal.

This characterization is robust to the arrival of additional buyers in the second period who are indistinguishable from those that arrived in the first period. The optimal first-period selling mechanism continues to be either a relatively high posted price, a relatively low price with rationing, or a menu with the two. In the case of linear demand, as the fraction of buyers arriving in the second period increases, the critical discount factor above which priority pricing is profitable initially decreases. However, if the fraction of buyers arriving in the second period becomes sufficiently large and the discount factor is intermediate, the optimal sequence of posted prices becomes increasing (instead of decreasing) over time. Instead of price skimming, the seller prefers to make an introductory

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2If they were distinguishable, they would constitute a separate market and would thus be irrelevant for the design of the first-period selling mechanism.
offer: post a relatively low price in the first period, and then charge the monopoly price in the second period. In this case, limited commitment lowers the first-period price (instead of raising the first-period price and lowering the second-period price) to incentivize monopoly pricing in the second period. The rationale for the profitability of priority pricing through alleviation of limited commitment remains valid: by keeping unserved some buyers with relatively high valuations, the seller reduces her future incentives to set a lower the price and thus becomes able to charge a higher price in the first period.

The analysis applies equivalently to the case in which there is a single buyer with a privately known valuation instead of a continuum of buyers. On the other hand, extending these results to an environment with three or more periods is challenging because regularity (monotonicity of virtual utility) of the distribution of valuations across buyers is lost after the first round of trade if priority pricing is used.

Plausible applications include revenue management by airlines, hotels, theaters, amusement parks, and distributors of content (such as songs, movies, and apps for electronic devices). In these industries: goods are durable (buyers that obtain the good or enjoy the service no longer demand additional units), buyers strategize over the timing of purchase, and resale can be forbidden. For priority pricing to be profitable, it is crucial that rationing is random, i.e., that the probability of delivery is independent of the buyer’s valuation, and that resale can be prevented.

In the remainder of the paper: we review the existing literature (Section 2); introduce the model (Section 3); establish the optimality of menus with at most two alternatives besides the null contract (Section 4); characterize the optimal menu in the case of linear demand (Section 5); and conclude with a few remarks (Section 6).

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3To enact an opaque environment, the seller can employ a mediation device (human or machine).

4If resale was possible, buyers with greater valuations among those who had been rationed would acquire the good from buyers with lower valuations who had received the good. Rationing would thus tend to become efficient (rationed buyers are those with lowest valuations) rather than random.
2 Literature review

This paper focuses on the role of rationing and priority pricing as an instrument for a durable goods monopolist to relax competition from future selves. It is well known that competition from future selves can be eliminated by renting rather than selling the durable good, thereby effectively transforming it into a non-durable (Bulow, 1982). However, renting may not be feasible: if the durable good is an intermediate good that the buyer transforms (such as steel or raw diamonds); if transaction costs are high, possibly due to adverse selection (as in the case of used cars); or if the durable good is consumed only once (as in the case of movie or flight tickets). In these and other environments where renting is not economically feasible, rationing and priority pricing may be useful for the monopolist to relax competition from future selves.

The role of rationing as a means of countering Coasian price-skimming was explained by Denicolò and Garella (1999). They showed that rationing can be profitable because keeping buyers with high valuations unserved reduces the incentives for decreasing prices in the future. Their conclusion hinges on the assumption of random rationing (the probability of being rationed does not depend on the buyer’s valuation), which is realistic when modeling environments without a resale market and without queues. Otherwise, efficient rationing (rationed buyers are those with the lowest valuations) is more realistic.

This paper characterizes the optimal selling mechanism in a canonical model where a monopolist without capacity constraints faces a continuum of consumers with privately known valuations arriving over two periods. In a related contribution, Deb and Said (2015) studied a two-period model where buyers arrive over time, and those who arrive in

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5 Other aspects of the economic environment may grant market power to a durable goods monopolist, such as: capacity costs, even if very small (McAfee and Wiseman, 2008); and buyer outside payoffs, even if only slightly positive (Board and Pycia, 2014).

6 In the latter case, the good is durable in the sense that additional units will be much less valued after consumption of the first unit. See Bulow (1982) for a thorough discussion of these examples.

7 Courty and Nasiry (2016) reached similar conclusions in a model where the rationing rule is irrelevant because consumers only learn their valuations after the first period and are thus homogeneous at the moment of rationing. Other contributions where rationing and priority pricing play a related role are those by Desai et al. (2007) and Celis et al. (2014). Beccuti and Möller (2018) compared sale and rental in a model with binary valuations and a finite number of periods. They found rationing in the optimal rental contract but not, in contrast with the results presented below, in the optimal sale contract.
the first period receive additional information about their valuations in the second period. Restricting to deterministic mechanisms (no rationing), they showed that the optimal first-period contract is such that buyers with intermediate valuations delay contracting. Dilme and Li (2019) addressed the case where a monopolist has a fixed number of units to sell to buyers with homogeneous valuations arriving over a finite time interval, and showed that the monopolist should periodically offer fire sales (of a single unit) to reduce inventory and thus mitigate her incentives to set lower prices in the future.

Rationing and priority pricing have other advantages besides mitigating the inability to commit to future prices. If buyers need to support a sunk cost to be able to buy, rationing fosters entry by buyers with low valuations (Gilbert and Klemperer 2000); if buyers have affiliated values, rationing alleviates the winner’s curse (Bulow and Klemperer 2002); if buyers face decreasing prices, rationing in the second period induces early purchases at a higher price (Liu and Van Ryzin 2008). Under uncertainty about demand or supply, priority pricing can be profitable even in static models or in dynamic models with full commitment regarding future prices and output (Harris and Raviv 1981; Chao and Wilson 1987; Wilson 1989; Gale and Holmes 1992; Dana 1998; Nocke and Peitz 2007; Su 2007; Su and Zhang 2008; Su 2010; Möller and Watanabe 2010).

8 Board and Skrzypacz (2016) studied a similar problem, but assuming full commitment.
9 Harris and Raviv (1981) showed that, if supply is scarce, priority pricing is the optimal selling mechanism for a monopolist facing uncertain demand (finite number of buyers with privately known valuations). Profit-maximization implies full separation of buyers in priority classes, with buyers with higher valuations paying a higher price and being served with higher probability. Chao and Wilson (1987) and Wilson (1989) developed the study of priority pricing in environments with uncertain supply, with application to the electric power industry. Nocke and Peitz (2007) considered a two-period model with full commitment and showed that it may be optimal to guarantee delivery at a high price in the first period and ration buyers at a low price in the second period (clearance sale). Su (2007) studied a model where buyers with heterogeneous valuations and discount factors arrive over a finite time interval. In the model of Su and Zhang (2008), there is an uncertain mass of buyers with homogeneous valuations. The seller chooses the first-period price (observed) and total output (unobserved). The remaining inventory is sold in the second period at a low exogenous price. In such an environment, the seller benefits from committing to a given output (i.e., from being able to credibly disclose the output level). Su (2010) considered a model where a monopolist sells a finite inventory in two periods, comparing the case in which the price is set once for all at the beginning with the case in which the monopolist can change the price in the second period. The focus is on the impact of speculators, who act as buyers in the first period and as sellers (who respond to the monopolist) in the second period. Celis et al. (2014) studied a particular selling mechanism which allows buyers to buy a good immediately or take-a-chance in a second-price auction (one period, one unit).
3 Model

A seller of an indivisible durable good produced at zero marginal cost faces buyers in two periods. In the first period there is a unit mass of buyers with valuations in \([0, 1]\) distributed according to a cumulative distribution function, \(F\), with continuous and strictly positive density, \(f\), and such that \(v - \frac{1-F(v)}{f(v)}\) is strictly increasing.\(^{10}\) In the second period arrives an additional mass of buyers, \(G(1) \geq 0\), with valuations in \([0, 1]\) distributed according to a cumulative function \(G\), with continuous and strictly positive density \(g\).\(^{11}\)

In each period, the seller proposes a menu composed by pairs \((\hat{r}, \hat{t})\), where \(\hat{r} \in [0, 1]\) is a probability of trade (in that period) and \(\hat{t} \in \mathbb{R}\) is a transfer (from buyer to seller). Each buyer chooses a pair from the menu. The pair \((0, 0)\) is always in the menu to reflect the fact that buyers can reject to trade. Buyers who obtain the good in the first period exit the market while those unserved remain in the market and may trade in period 2.\(^{12}\)

The seller cannot make commitments in period 1 regarding the selling mechanism to propose in period 2. Moreover, the seller cannot observe each buyer’s choice from the menu. The seller only observes whether or not trade takes place. Therefore, the seller cannot discriminate buyers in period 2 based on their behavior in period 1.\(^{13}\) The seller also cannot discriminate buyers based on their time of arrival to the market. Otherwise, buyers arriving in period 2 could be treated as a separate market and would thus be irrelevant for the choice of the selling mechanism to be proposed in period 1.

Buyers and seller are risk-neutral and have possibly different discount factors, \(\delta_b \in \mathbb{R}^+\).\(^{14}\)

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\(^{10}\)This assumption, known as regularity or monotonicity of virtual surplus, is standard and weaker than weakly-increasing hazard rate, \(\frac{f(v)}{1-F(v)}\).

\(^{11}\)The corresponding cumulative distribution function is \(\frac{G(v)}{G(1)}\), with density \(\frac{g(v)}{G(1)}\). Setting \(G(1) = 0\) captures the case in which all buyers arrive in the first period.

\(^{12}\)Similar models where the seller is restricted to post prices include those by Fudenberg and Tirole (1983) and Sobel and Takahashi (1983). The model by Fudenberg and Villas-Boas (2006) also differs in that goods are non-durable: served buyers do not leave the market.

\(^{13}\)Skreta (2006) studied the case in which the seller is able to discriminate buyers based on their past behaviour, and showed that the optimal mechanism is a decreasing sequence of posted prices. That mechanism does not require observing buyers’ choices, therefore, it is not detrimental for the seller to be unable to observe buyers’ choices. We will conclude that not observing buyers’ choices is actually beneficial if agents are sufficiently patient. By committing not to use the information revealed by past choices, the seller shuts down the ratchet effect (Freixas et al., 1985; Fudenberg and Tirole, 1998).
[0, 1) and \( \delta_s \in [0, 1] \). The seller wishes to maximize the expected discounted value of the transfer, while buyers wish to maximize the expected discounted value of the difference between the value derived from the good and the transfer made to the seller\(^{14}\)

In the second period, the optimal strategy for the seller is to post a price:\(^{15}\)

\[
p_2 \in \arg\max_p \{ p [1 - F_2(p)] \},
\]

where \( F_2(v) \) is the posterior belief of the seller about the distribution of buyers’ valuations after conditioning on sales made in period 1.

### 4 Analysis

The seller chooses a menu in period 1, denoted \( m_1 \), and a price in period 2, denoted \( p_2 \). Her objective is to maximize \( \pi(m_1, p_2) \), but she cannot precommit to some price \( p_2 \). Given this sequential rationality constraint, her problem can be written as:

\[
\max_{(m_1, p_2)} \pi(m_1, p_2) \text{ s.t. } p_2 \in \arg\max_z \pi(m_1, z).
\]

This problem is usually tackled by backward induction. For each \( m_1 \), find a sequentially optimal \( p_2 \) to construct a function \( p_2^*(m_1) \). Then, find the optimal \( m_1 \) by solving \( \max_{m_1} \pi(m_1, p_2^*(m_1)) \). An alternative approach is to write the seller’s problem as:

\[
\max_{p_2} \left\{ \max_{m_1} \pi(m_1, p_2) \text{ s.t. } p_2 \in \arg\max_z \pi(m_1, z) \right\}.
\]  \( \text{(1)} \)

Under this alternative approach, we start by taking \( p_2 \) as given and solve the one-shot mechanism design problem of choosing \( m_1 \) under the restriction that the given \( p_2 \) is

\(^{14}\)The case in which there is a different seller in the second period can be captured by setting \( \delta_s = 0 \). More generally, the case in which the period 1 seller gets a fraction \( \tau \in [0, 1] \) of period 2 revenues, and \( \delta \in [0, 1] \) is her time-discount factor, can be captured by setting \( \delta_s = \tau \delta \).

\(^{15}\)See Riley and Zeckhauser (1983) or Börgers (2015, Section 2.2).
optimal in period 2. Then, find the optimal value of $p_2$. The revelation principle holds because $\pi(m_1, z)$ only depends on $m_1$ through the resulting allocation. Another way to verify that the revelation principle holds is to view the inner problem in (1) as a static mechanism design problem with an exogenous restriction on the period 1 allocation (allowed allocations are those for which $p_2$ is sequentially optimal).

Restricting to incentive compatible direct mechanisms, let $r : [0, 1] \rightarrow [0, 1]$ and $t : [0, 1] \rightarrow \mathbb{R}$ be integrable functions specifying probability of trade in period 1 and transfer (from buyer to seller) in period 1 as a function of the valuation announced by the buyer.

The problem of the seller can be written as:

$$
\max_{p_2} \left\{ \max_{r, t} \left\{ \int_0^1 t(v) f(v) \, dv + \delta_s p_2 \int_{p_2}^1 [1 - r(v)] f(v) \, dv + \delta_s p_2 \int_{p_2}^1 g(v) \, dv \right\} \right. \\
\left. \quad v \in \arg\max_{v'} \{ r(v') v - t(v') + 1_{v' \geq p_2} \delta_b (v - p_2) [1 - r(v')] \} \right. \\
\left. \quad p_2 \in \arg\max_p \left\{ p \int_p^1 [1 - r(v)] f(v) \, dv + p \int_p^1 g(v) \, dv \right\} \right\} 
$$

Using standard techniques, we incorporate the first-order truth-telling constraint in the objective function and replace the global truth-telling constraint by the second-order truth-telling constraint, which is weak monotonicity of the probability of trade.

**Lemma 1.** The inner problem in (2) is equivalent to:

$$
\max_{r} \left\{ \int_0^{p_2} r(v) \left[ v - \frac{1 - F(v)}{f(v)} \right] f(v) \, dv + (1 - \delta_b) \int_{p_2}^1 r(v) \left[ v - \frac{1 - F(v)}{f(v)} + \frac{\delta_b - \delta_s}{1 - \delta_s} p_2 \right] f(v) \, dv \\
\quad + \delta_s p_2 [1 - F(p_2) + G(1) - G(p_2)] \right\} \\
\text{s.t.} \left\{ \begin{array}{l}
\quad r \text{ is weakly-increasing} \\
\quad p_2 \in \arg\max_p \left\{ p \int_p^1 [1 - r(v)] f(v) \, dv + p [G(1) - G(p)] \right\} .
\end{array} \right.
$$

**Proof.** See Appendix A.1.

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16Whenever the maximization is over an empty set, consider a default value of zero or $-\infty$.  

The following result establishes that the optimal period 1 menu includes at most two alternatives besides the null contract: one is to pay a relatively high price to obtain the good with 100% probability; the other is to pay a relatively low price to obtain the good with a probability that is lower than 100%.

**Theorem 1.** The solution of problem (3) satisfies:

\[
    r(v) = \begin{cases} 
    1, & \text{if } v \in (\bar{v}, 1) \\
    \hat{r}, & \text{if } v \in (\underline{v}, \bar{v}) \\
    0, & \text{if } v \in (0, \underline{v}),
    \end{cases}
\]

where \(\underline{v} \in [0, p_2]\), \(\bar{v} \in [p_2, 1]\), and \(\hat{r} \in [0, 1]\).

**Proof.** See Appendix A.1.

Note that Theorem 1 does not rule out the optimality of a degenerate menu in which there is a single alternative besides the null contract. In particular, if \(\hat{r} = 0\) or \(\hat{r} = 1\), it is a posted price greater or smaller than \(p_2\). If \(\bar{v} = 1\), all buyers are rationed.

If priority pricing (a menu with both alternatives besides the null contract) is optimal: buyers with high valuations \((v > \bar{v})\) pay a high price and guarantee delivery; buyers with intermediate valuations \((\underline{v} < v < \bar{v})\) pay a low price and risk being rationed; buyers with low valuations \((v < \underline{v})\) do not trade. Among buyers who are rationed in period 1, those with relatively high valuations \((p_2 < v < \bar{v})\) buy the good in period 2. Buyers who do not trade in period 1 also do not trade in period 2 \((v < \underline{v} \Rightarrow v < p_2)\).

The idea of the proof of Theorem 1 is to start with an allocation, \(\tilde{r}\), that satisfies the restrictions stated in Lemma 1 (i.e., is weakly-increasing and such that a given \(p_2\) is sequentially optimal), but does not satisfy the characterization in Theorem 1 and then construct an allocation \(r\) that improves on \(\tilde{r}\) in terms of the objective function while satisfying the restrictions in Lemma 1 and the characterization in Theorem 1.

\[^{17}\text{Note that Theorem 1 allows one or even two of the three branches to have empty domain.}\]
The construction of $r$ is illustrated in Figure 1: (i) set $r(p_2) = \hat{r}(p_2)$; (ii) set $\bar{v}$ such that the mass of buyers with valuations below $p_2$ that do not obtain the good in period 1 is the same under $r$ as under $\hat{r}$; and (iii) set $\bar{v}$ such that the mass of buyers with valuations above $p_2$ that do not obtain the good in period 1 is the same under $r$ as under $\hat{r}$. Inspection of the objective function in Lemma 1 allows to conclude that, under the regularity assumption, $r$ improves on $\hat{r}$. To verify that $p_2$ remains the sequentially optimal second-period price, notice that: the profit that results from setting $p_2$ remains the same; the profit that results from setting a price below $p_2$ decreases; and the profit that results from setting a price above $p_2$ also decreases.

The intuition for Theorem 1 is partly related to the optimality of a posted price in the single-period model (Myerson 1981; Riley and Zeckhauser 1983). With a single period, a posted price maximizes the probability of trade of buyers with positive virtual surplus and minimizes the probability of trade of buyers with negative virtual surplus. With two periods, for a given $p_2$, there are two groups of buyers in the first period: those that will be willing to trade in the second period ($v > p_2$) and those that will not ($v < p_2$).
For each of these groups, separately, monotonicity of virtual surplus continues to hold, implying that the seller prefers first-period trade to occur with higher probability for higher valuations and lower probability for lower valuations. This “bang-bang” solution is bounded by the value of \( r \) at \( p_2 \). Since \( r \) is weakly increasing, we must have \( r(v) \leq r(p_2) \) for \( v < p_2 \), and \( r(v) \geq r(p_2) \) for \( v > p_2 \).

Therefore, if we ignore the sequential rationality restriction and keep fixed \( p_2, r(p_2) \), the mass of buyers with valuations below \( p_2 \) served in period 1, and the mass of buyers with valuations above \( p_2 \) served in period 1, then the step function \( r \) in Figure 1 is the one that maximizes total virtual surplus. But is there a tension between maximizing total virtual surplus and sequential rationality of \( p_2 \)? The answer is no. The step function \( r \) is also the one that relaxes sequential rationality the most. While second-period demand at \( p_2 \) is kept fixed, \( r \) is the function that minimizes second-period demand at all other prices (keep in mind that we are keeping fixed \( p_2, r(p_2) \), and the masses of buyers with valuations below and above \( p_2 \) that do not get the good in the first period). Hence, \( r \) not only maximizes revenue but also minimizes the incentives to deviate from \( p_2 \) to a different price level in the second period.

Theorem 1 greatly simplifies the reduced problem (3): we only need to find the optimal values of \( \underline{v} \in [0, p_2] \), \( \overline{v} \in [p_2, 1] \), and \( \hat{r} \in [0, 1] \). Equivalently, the period 1 mechanism can be described by a triple \((p_1, p_r, \hat{r})\), where \( p_1 \) is the price paid to get the object with certainty, and \( p_r \) is the conditional price paid to get the object with probability \( \hat{r} \). Truth-telling implies that \( p_r = \underline{v} \), and \( p_1 = (1 - \delta_b)(1 - \hat{r})\overline{v} + \hat{r}\underline{v} + (1 - \hat{r})\delta_b p_2 \).

To solve the complete problem (2), besides \((\underline{v}, \overline{v}, \hat{r})\) or \((p_1, p_r, \hat{r})\), we also need to find the optimal value of \( p_2 \), which is taken as given in the reduced problem (3).

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18 From risk-neutrality, an unconditional price \( p_r \hat{r} \) is equivalent to a price \( p_r \) conditional on delivery.

19 This expression for \( p_1 \) follows from indifference of the buyer with valuation \( \overline{v} \) between paying \( p_1 \) to guarantee delivery and paying \( p_r \) and risk being rationed: \( \overline{v} - p_1 = \hat{r}(\overline{v} - p_r) + (1 - \hat{r})\delta_b(\overline{v} - p_2) \).
5 Linear demand

5.1 All buyers arrive in the first period

We now study the optimal selling mechanism, \((v, \hat{v}, \hat{r}, p_2)\), assuming that all buyers arrive in the first period and have uniformly distributed valuations: \(F(v) = v, \forall v \in [0,1]^{20}\). We say that the optimal first-period selling mechanism is: (i) a \textit{posted price} whenever \(\hat{r} \in \{0, 1\} \text{ or } v = \bar{v}\); (ii) a \textit{priority pricing} menu, i.e., a menu consisting of a higher price without rationing and a lower price with rationing, whenever \(\hat{r} \in (0, 1) \text{ and } v < \bar{v} < 1\); (iii) a \textit{single price with rationing} whenever \(\hat{r} \in (0, 1) \text{ and } v < \bar{v} = 1\).

**Proposition 1.** The optimal first-period selling mechanism is:

(i) a posted price if and only if: \(\delta_s \geq -\frac{4+8\delta_b+\delta_b^2-2\delta_b^3}{2\delta_b+\delta_b^2}\); 

(ii) a priority pricing menu if and only if: \(\frac{-\delta_b^2+6\delta_b-4-4(1-\delta_b)^{\frac{3}{2}}}{\delta_b} < \delta_s < \frac{-4+8\delta_b+\delta_b^2-2\delta_b^3}{2\delta_b+\delta_b^2}\); 

(iii) a single price with rationing if and only if: \(\delta_s \leq \frac{-\delta_b^2+6\delta_b-4-4(1-\delta_b)^{\frac{3}{2}}}{\delta_b}\).

**Proof.** See Appendix A.2.

As illustrated in Figure 2 for a given seller’s discount factor: a single price guaranteeing delivery is optimal if the buyer is sufficiently impatient; a single price with rationing is optimal if the buyer is sufficiently patient; a menu with the two is optimal in intermediate cases. If buyer and seller have a common discount factor: a single price with rationing is never optimal. If they are relatively impatient, it is optimal for the seller to post a single price guaranteeing delivery; otherwise, it is optimal to offer a menu with a relatively high price guaranteeing delivery and a relatively low price subject to rationing.

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20 The case of linear demand and all buyers arriving in the first period corresponds to the model of Besanko and Winston (1990) with two periods, but allowing rationing and priority pricing instead of restricting the monopolist to post a price and satisfy demand, and to the model of Denicolò and Garella (1999) but allowing priority pricing instead of restricting to a single price with or without rationing.
The more patient buyers are, the stronger is the competition the first-period seller faces from the second-period seller (when $\delta_b \to 1$, they sell homogeneous goods), and the more buyers are willing to risk being rationed (when $\delta_b \to 1$, there is no loss from postponing consumption). Regarding the impact of the seller’s discount factor, note that $\delta_s$ can be interpreted as the share of the second-period seller owned by the first-period seller. Hence, the more patient is the seller, the weaker is the competition the first-period seller faces from the second-period seller. This explains why buyers’ patience and seller’s impatience contribute to the profitability of rationing and priority pricing.

The optimal selling mechanism with a common discount factor is represented in Figure 3. In the extreme cases where $\delta = 0$ and $\delta = 1$, the seller is able to attain the same profit as with full commitment.\textsuperscript{21} If $\delta = 0$, second-period profit is irrelevant and thus the seller posts the monopoly price in the first period. If $\delta = 1$, the seller posts a price that no buyer is willing to pay in the first period, and then sets the monopoly price in the second period. It is when $\delta$ is intermediate that the inability to commit to a second-period price reduces the profit of the seller. If $\delta \leq \frac{2}{3}$, posting a price is optimal. But if $\delta > \frac{2}{3}$, the seller gains from offering a priority pricing menu, as rationing buyers with $v \in [v, \bar{v}]$ has a positive impact on second-period price (countervails the problem of limited commitment).

\textsuperscript{21}With full commitment, the seller would post $p_1 = \frac{1}{2}$ and $p_2 \geq \frac{1}{2}$, and obtain profit $\pi = \frac{1}{4}$. 

Figure 2: Optimal kind of selling mechanism with linear demand.
As represented in Figure 4, priority pricing increases profit up to 0.93% (attained at \( \delta \approx 0.885 \)). However, it decreases consumer surplus (up to 2.12%, attained at \( \delta \approx 0.889 \)) and total welfare (up to 0.28%, attained at \( \delta \approx 0.896 \)).

**Figure 3:** Optimal selling mechanism with linear demand and a common discount factor (recall that \( p_r = v \)).

**Figure 4:** Impact of priority pricing on profit (solid line), consumer surplus (dashed line) and welfare (dotted line).
5.2 Additional buyers arriving in the second period

We now study the optimal selling mechanism in the case where additional buyers arrive in period 2, assuming that seller and buyers are equally patient, $\delta_b = \delta_s = \delta \in (0, 1)$, and that both cohorts of buyers have uniformly distributed valuations: $F(v) = v$ and $G(v) = \beta v$, $\forall v \in [0, 1]$, with $\beta \in [0, 1]$.

Restricting the seller to post prices, we find that it can be optimal to engage in price skimming ($p_2 < \frac{1}{2} < p_1$) or to make an introductory offer exactly as aggressive as necessary to make the monopoly price optimal in the second period ($p_1 < \frac{1}{2} = p_2$). An introductory offer is preferable if and only if the proportion of buyers arriving in the second period is sufficiently large and the discount factor is intermediate (see Figure 5).

**Proposition 2.** If the mass of buyers arriving in the second period, $\beta$, is sufficiently large, and the common discount factor, $\delta$, is intermediate, the optimal sequence of posted prices is increasing ($p_1 < \frac{1}{2} = p_2$). Otherwise, it is decreasing ($p_2 < \frac{1}{2} < p_1$).

**Proof.** See Appendix A.2

We investigate whether priority pricing increases profit by perturbing the optimal price sequence. To improve on price skimming, we introduce rationing with a probability of delivery close to zero; to improve on an introductory offer, we do the same with a probability close to one. The parameter region where priority pricing is shown to improve on the optimal sequence of posted prices is represented in Figure 6.

**Proposition 3.** Priority pricing is profitable if the common discount factor is sufficiently high. It improves on decreasing sequences of posted prices if $\delta > \frac{4 + 4\beta - 2\sqrt{1 + 4\beta^2}}{3 + 8\beta}$; and improves on increasing sequences of posted prices if $\delta > \frac{2\sqrt{\beta(1 + \beta) - 1 - \beta}}{\beta}$.

**Proof.** See Appendix A.2
6 Concluding remarks

Priority pricing is a relevant tool for a durable good monopolist to relax competition from future selves. By leaving buyers with relatively high valuations unserved, the monopolist can profitably increase future prices and, therefore, current demand. In a canonical model with two rounds of trade, the optimal selling mechanism is either a single price (with or without rationing) or the simplest form of priority pricing: the seller posts two prices, one guaranteeing delivery and another entailing a probability of rationing. More complex selling mechanisms can be ignored without loss.

The relevance of rationing and priority pricing hinges on several ingredients of the economic environment: the monopolist is unable to commit to future prices, able to preserve buyer anonymity, and able to prevent resale. Still, there are many plausible applications of the insights provided in this paper, including revenue management by airlines, hotels, amusement parks, and distributors of content (such as songs, movies, and apps for electronic devices). In these industries: goods are durable (buyers that obtain the good or service no longer demand additional units), buyers strategize over the time of purchase, and resale can be prevented.
A Appendix

A.1 Proofs of results in Section 4

Proof of Lemma 1

Let $U(v) \equiv \max_{v' \in [0,1]} \{ r(v')v - t(v') + \mathbb{1}_{v' \geq p_2} \delta_b(v - p_2) [1 - r(v')] \}$. The truth-telling condition in problem 2 immediately implies that $U(v) = r(v)v - t(v) + \mathbb{1}_{v \geq p_2} \delta_b(v - p_2) [1 - r(v)]$, which is used to determine $t$ as a function of $r$ and $U$.

Applying Theorem 2 in Milgrom and Segal (2002), we obtain $U$ as a function of $r$:

$$U(v) - U(0) = \begin{cases} \int_0^{p_2} r(\tilde{v}) \, d\tilde{v} + (1 - \delta_b) \int_{p_2}^v r(\tilde{v}) \, d\tilde{v} + \delta_b(v - p_2), & \text{if } v \geq p_2 \\ \int_0^v r(\tilde{v}) \, d\tilde{v}, & \text{if } v < p_2. \end{cases}$$

From Lemma 2 (see Appendix A.3), it is necessary that $r$ is weakly-increasing. We now check that this condition is also sufficient for incentive compatibility.

(i) Supposing that $v' < v \leq p_2$:

$$U(v) \geq r(v')v - t(v') \iff U(v) - U(v') \geq r(v')(v - v') \iff \int_{v'}^{v} r(\tilde{v}) \, d\tilde{v} \geq \int_{v'}^{v} r(v') \, d\tilde{v}.$$

(ii) Supposing that $p_2 \leq v' < v$:

$$U(v) \geq r(v')v - t(v') + \delta_b(v - p_2) [1 - r(v')] \iff U(v) - U(v') \geq r(v')(v - v') + \delta_b(v - v') [1 - r(v')] \iff \int_{v'}^{v} (1 - \delta_b) r(\tilde{v}) + \delta_b d\tilde{v} \geq \int_{v'}^{v} (1 - \delta_b) r(v') + \delta_b d\tilde{v} \iff \int_{v'}^{v} r(\tilde{v}) \, d\tilde{v} \geq \int_{v'}^{v} r(v') \, d\tilde{v}.$$

(iii) Finally, supposing that $v' < p_2 < v$: 


\[ U(v) \geq r(v')v - t(v') + \delta_b(v - p_2) \left[ 1 - r(v') \right] \]

\[ \Leftrightarrow U(v) - U(v') \geq r(v')(v - v') + \delta_b(v - p_2) \left[ 1 - r(v') \right] \]

\[ \Leftrightarrow \int_{v'}^{P_2} r(\tilde{v}) d\tilde{v} + \int_{v'}^{P_2} (1 - \delta_b) r(\tilde{v}) + \delta_b d\tilde{v} \geq \int_{v'}^{P_2} r(v') d\tilde{v} + \int_{v}^{v'} (1 - \delta_b) r(v') + \delta_b d\tilde{v} \]

\[ \Leftrightarrow \int_{v'}^{P_2} r(\tilde{v}) d\tilde{v} + (1 - \delta_b) \int_{v}^{v'} r(\tilde{v}) d\tilde{v} \geq \int_{v'}^{P_2} r(v') d\tilde{v} + (1 - \delta_b) \int_{v}^{v'} r(v') d\tilde{v}. \]

In all cases (i)-(iii), the conditions hold if \( r \) is weakly-increasing. Hence, using the fact that \( t(v) = r(v)v + 1_{v \geq p_2} \delta_b(v - p_2) \left[ 1 - r(v) \right] - U(v) \), the inner problem in (2) becomes:

\[
\max_{r, U} \left\{ \int_{0}^{1} \{ r(v)v + 1_{v \geq p_2} \delta_b(v - p_2) \left[ 1 - r(v) \right] - U(v) \} f(v) \, dv + \delta_p p_2 \int_{p_2}^{1} [1 - r(v)] f(v) + g(v) \, dv \right\} \\
\text{s.t.} \quad \left\{ \begin{array} {l}
\text{r is weakly-increasing} \\
p_2 \in \argmax_{p} \left\{ p \int_{p}^{1} [1 - r(v)] f(v) + g(v) \, dv \right\} .
\end{array} \right.
\]

Manipulating the objective function, and setting \( U(0) = 0 \), we obtain:

\[
\int_{0}^{P_2} \left\{ r(v)v - \int_{0}^{P_2} r(\tilde{v}) d\tilde{v} \right\} f(v) \, dv + \int_{P_2}^{1} \left\{ r(v)v + \delta_b(v - p_2) \left[ 1 - r(v) \right] \right\} f(v) \, dv \\
- \int_{p_2}^{1} \left\{ \int_{0}^{P_2} r(\tilde{v}) d\tilde{v} + (1 - \delta_b) \int_{P_2}^{v} r(\tilde{v}) d\tilde{v} + \delta_b(v - p_2) \right\} f(v) \, dv + \delta_p p_2 \int_{p_2}^{1} [1 - r(v)] f(v) + g(v) \, dv \\
= \int_{0}^{P_2} r(v)v f(v) \, dv - \int_{0}^{P_2} r(\tilde{v}) f(\tilde{v}) \, dv + (1 - \delta_b) \int_{P_2}^{1} r(v)v f(v) \, dv \\
- [1 - F(p_2)] \int_{0}^{P_2} r(v) f(v) \, dv - (1 - \delta_b) \int_{P_2}^{1} r(\tilde{v}) f(\tilde{v}) \, dv \\
+ (\delta_b - \delta_s) p_2 \int_{p_2}^{1} r(v) f(v) \, dv + \delta_p p_2 [1 - F(p_2) + G(1) - G(p_2)].
\]

Using the fact that:

\[
\int_{0}^{P_2} \int_{0}^{v} r(\tilde{v}) d\tilde{v} f(v) \, dv = \int_{0}^{P_2} \int_{v}^{P_2} r(\tilde{v}) f(v) \, dv d\tilde{v} = \int_{0}^{P_2} [F(p_2) - F(\tilde{v})] r(\tilde{v}) d\tilde{v},
\]

and that

\[
\int_{P_2}^{1} \int_{P_2}^{v} r(\tilde{v}) d\tilde{v} f(v) \, dv = \int_{P_2}^{1} \int_{v}^{1} r(\tilde{v}) f(v) \, dv d\tilde{v} = \int_{P_2}^{1} [1 - F(\tilde{v})] r(\tilde{v}) d\tilde{v},
\]

the objective function becomes:
\[
\int_0^{p_2} r(v)v f(v) \, dv - \int_0^{p_2} [F(p_2) - F(v)] r(v) \, dv + (1 - \delta_b) \int_{p_2}^1 r(v)v f(v) \, dv \\
- \int_0^{p_2} [1 - F(p_2)] r(v) \, dv - (1 - \delta_b) \int_{p_2}^1 [1 - F(v)] r(v) \, dv \\
+ (\delta_b - \delta_s) p_2 \int_{p_2}^1 r(v) f(v) \, dv + \delta_s p_2 [1 - F(p_2) + G(1) - G(p_2)]
\]

which is the objective function in the statement of this Lemma. \qed

**Proof of Theorem [1]**

Let us start by establishing that a solution exists. The seller must choose a function \( r : [0, 1] \to [0, 1] \) that is non-decreasing. All such functions are Lebesgue measurable and integrable thus there is no loss in restricting to the space of bounded and integrable functions from \([0, 1]\) to \(\mathbb{R}\) endowed with the \(L^1\)-norm.\(^{22}\) The subset consisting of those functions that are non-decreasing and have values in \([0, 1]\), denoted \(\mathcal{M}\), is compact and convex.\(^{23}\) The seller is also restricted by the sequential optimality condition, which is satisfied in a compact and convex subset of \(\mathcal{M}\), denoted \(\mathcal{S}\). This subset is closed (and thus compact) and convex because it is defined by inequalities of the form \(h(r) \geq s\), where \(h(r)\) is a linear functional of \(r\) and \(s\) is a constant\(^{24}\).

Since the seller maximizes a continuous functional on a compact set, a maximum (i.e., a solution) exists by the Bolzano-Weierstrass theorem.

Now consider an arbitrary non-decreasing function \(\tilde{r}\) such that \(p_2\) is sequentially-optimal (i.e., an arbitrary \(\tilde{r} \in \mathcal{S}\)) which does not satisfy the characterization established by the Theorem. We proceed by constructing \(r \in \mathcal{S}\) that has strictly greater payoff than \(\tilde{r}\) and satisfies the characterization established by the Theorem. This will finish the proof.

Let \(D_2(\tilde{r}) \equiv \int_{p_2}^1 [1 - \tilde{r}(v)] f(v) \, dv\) and \(D_2'(\tilde{r}) \equiv \int_0^{p_2} [1 - \tilde{r}(v)] f(v) \, dv\). Note that \(D_2(\tilde{r})\) and \(D_2'(\tilde{r})\) are the masses of buyers with valuations above \(p_2\) and below \(p_2\), respectively, that are

\(^{22}\)Note also that two functions in the same equivalence class in \(L^1\) yield the same payoff in \([3]\).

\(^{23}\)Convexity is trivial. For compactness, see Börgers [2015, Lemma 2.6], who refers to Helly’s selection theorem [Rudin 1976, Exercise 13, p. 167] and the bounded convergence theorem of Lebesgue integration [Rudin 1976, Corollary, p. 322).

\(^{24}\) Precisely, \(h(r) \equiv -p_2 \int_{p_2}^1 r(v)f(v)dv + p \int_p^1 r(v)f(v)dv\) and \(s \equiv p_2 [1 - F(p_2) + G(1) - G(p)] - p [1 - F(p) + G(1) - G(p)]\).
not served in period 1 under \( \tilde{r} \).

Construct \( r \) by letting:

\[
r(v) = \begin{cases} 
1, & \text{if } v \in (\underline{v}, 1] \\
\tilde{r}(p_2), & \text{if } v \in [\underline{v}, \overline{v}] \\
0, & \text{if } v \in [0, \underline{v}),
\end{cases}
\]

where \( \underline{v} \in [0, p_2] \) is such that \( D_2^x(r) = D_2^x(\tilde{r}) \), and \( \overline{v} \in [p_2, 1] \) is such that \( D_2(r) = D_2(\tilde{r}) \). This means that the thresholds \( \underline{v} \) and \( \overline{v} \) are such that the masses of buyers with valuations above \( p_2 \) and below \( p_2 \) that are not served in period 1 remain constant when \( \tilde{r} \) is replaced by \( r \). This construction is possible by Bolzano’s theorem. Observe that \( D_2^x(r) \equiv \int_0^{p_2} [1 - r(v)] f(v) dv = F(p_2) - [F(p_2) - F(\underline{v})] \tilde{r}(p_2) \) is continuously increasing in \( \underline{v} \) ranging from \( F(p_2) [1 - \tilde{r}(p_2)] \leq D_2^x(\tilde{r}) \) to \( F(p_2) \geq D_2^x(\tilde{r}) \). Similarly, \( D_2(r) \equiv \int_{p_2}^{1} [1 - r(v)] f(v) dv = [F(\overline{v}) - F(p_2)] [1 - \tilde{r}(p_2)] \) is continuously increasing in \( \overline{v} \) ranging from \( 0 \leq D_2(\tilde{r}) \) to \( [1 - F(p_2)] [1 - \tilde{r}(p_2)] \geq D_2(\tilde{r}) \).

The next step of the proof is to show that \( r \) has strictly greater payoff than \( \tilde{r} \). In the objective function in (3), the two integrals represent total virtual surplus of buyers served (the first integral for buyers with \( v \geq p_2 \), the second integral for buyers with \( v \geq p_2 \)). By construction of \( r \), the masses of buyers served with \( v \geq p_2 \) and with \( v \geq p_2 \) are the same under \( r \) and \( \tilde{r} \). Since virtual surplus strictly increases in \( v \), showing that \( r \) places strictly more weight on higher virtual surpluses (and less on lower virtual surpluses) than \( \tilde{r} \), we will conclude that \( r \) has strictly greater payoff than \( \tilde{r} \).

Let us see this formally in two steps. Firstly, let us show that the distribution induced by \( r \) first-order stochastically dominates that induced by \( \tilde{r} \), both for buyers with \( v \in [0, p_2] \) and for buyers with \( v \in [p_2, 1] \). That is: \( \int_0^x r(v)f(v)dv \leq \int_0^x \tilde{r}(v)f(v)dv \), for all \( x \in [0, p_2] \), and \( \int_{p_2}^x r(v)f(v)dv \leq \int_{p_2}^x \tilde{r}(v)f(v)dv \), for all \( x \in [p_2, 1] \). Note that \( \int_0^{p_2} r(v)f(v)dv = \int_0^{p_2} \tilde{r}(v)f(v)dv = F(p_2) - D_2^x(r) \) and \( \int_{p_2}^{1} r(v)f(v)dv = \int_{p_2}^{1} \tilde{r}(v)f(v)dv = 1 - F(p_2) - D_2(r) \). For \( x \in [0, \underline{v}] \), the inequality is trivial because \( \int_0^x \tilde{r}(v)f(v)dv = 0 \). For \( x \in [\underline{v}, p_2] \), the inequality follows from \( \int_0^x r(v)f(v)dv = F(p_2) - D_2^x(r) - \int_x^{p_2} \tilde{r}(p_2)f(v)dv \leq F(p_2) - D_2^x(r) - \int_x^{p_2} \tilde{r}(v)f(v)dv = \int_0^x \tilde{r}(v)f(v)dv \). For \( x \in [p_2, \overline{v}] \), the inequality follows from \( \int_{p_2}^x r(v)f(v)dv = \int_{p_2}^x \tilde{r}(p_2)f(v)dv \leq \int_{p_2}^x \tilde{r}(v)f(v)dv \). For \( x \in [\overline{v}, 1] \), the inequality follows from \( \int_{p_2}^{1} r(v)f(v)dv = 1 - F(p_2) - D_2(r) - \int_1^{p_2} f(v)dv \leq 1 - F(p_2) - D_2(r) - \int_1^{x} \tilde{r}(v)f(v)dv = \int_{p_2}^{1} \tilde{r}(v)f(v)dv \). In addition, observe that \( \int_0^x r(v)f(v)dv = \int_0^x \tilde{r}(v)f(v)dv \), for all \( x \in [0, 1] \) would imply that \( r \) and \( \tilde{r} \) are equal almost
everywhere and therefore \( \tilde{r} \) would satisfy the characterization of the Theorem (contradiction). Note that \( r \) is defined in a full measure open set, \((0, v) \cup (v, p_2) \cup (p_2, \overline{v}) \cup (\overline{v}, 1)\) and that monotonicity implies that if \( \tilde{r} \) differs from \( r \) in a point \( z \) in that open set then it also differs in an interval \([z, z + \epsilon)\) or \((z - \epsilon, z]\). We thus also conclude that the distribution induced by \( r \) strictly first-order stochastically dominates that induced by \( \hat{r} \) for buyers with \( v \in [0, p_2] \) or for buyers with \( v \in [p_2, 1] \).

Secondly, the integrals in the objective function, \( \int_{p_2}^{p_2} [v - \frac{1 - F(v)}{f(v)}] r(v) f(v) \, dv \) as well as \( \int_{0}^{p_2} [v - \frac{1 - F(v)}{f(v)} + \frac{\delta_b - \delta_s}{1 - \delta_b} p_2] \, r(v) f(v) \, dv \), are expected values (modulo a scaling constant) of strictly increasing utility functions, \( v - \frac{1 - F(v)}{f(v)} \) and \( v - \frac{1 - F(v)}{f(v)} + \frac{\delta_b - \delta_s}{1 - \delta_b} p_2 \). It is well known that the expected value of a strictly increasing utility function increases as a result of a first-order stochastic dominance shift, and that the increase is strict if the shift is strict. We thus conclude that \( r \) has strictly greater payoff than \( \tilde{r} \).

To finish the proof, we need to verify that \( r \) satisfies the sequential optimality condition of problem (3). Setting \( p_2 \) yields the same demand (and thus profit) in period 2 under \( r \) and under \( \tilde{r} \). By construction: \( \int_{p_2}^{p_2} [1 - \tilde{r}(v)] f(v) \, dv = \int_{p_2}^{p_2} [1 - r(v)] f(v) \, dv = D_2(r) \). We now show that setting \( p \neq p_2 \), demand (and thus profit) in period 2 cannot be higher under \( r \) than under \( \tilde{r} \). As a result, if under \( \tilde{r} \) there is no gain from deviating from \( p_2 \) to \( p \), then under \( r \) there is also no gain from deviating from \( p_2 \) to \( p \). We can ignore buyers who arrive in period 2 because, for any given \( p \), their demand is the same under \( r \) and under \( \tilde{r} \). We only need to establish that: \( \int_{p}^{p} [1 - r(v)] f(v) \, dv \leq \int_{p}^{p} [1 - \tilde{r}(v)] f(v) \, dv \). This is a straightforward consequence of \( \int_{0}^{p} [1 - r(v)] f(v) \, dv = \int_{0}^{p} [1 - \tilde{r}(v)] f(v) \, dv \) (both equal to \( D_2(r) + D_2^c(r) \)) and \( \int_{0}^{p} [1 - r(v)] f(v) \, dv \geq \int_{0}^{p} [1 - \tilde{r}(v)] f(v) \, dv \) (the first-order stochastic dominance shift which was shown above).}

### A.2 Proofs of results in Section 5

#### Proof of Proposition 1

The single-period monopoly price is \( p^m = \frac{1}{2} \), thus \( p_2 \leq \frac{1}{2} \). From Lemma 3 (see Appendix A.3), \( \overline{v} = 2p_2 \). Incentive compatibility implies that \( p_r = \overline{v} \), and that:
\[ \overline{v} - p_1 = r(\overline{v} - p_r) + (1 - r)\delta_b(\overline{v} - p_2) \iff p_1 = \left(1 - \frac{\delta_b}{2}\right)(1 - r)\overline{v} + r\overline{v}. \]

The resulting profit of the seller can be written as:

\[
p_1(1 - \overline{v}) + rp_r(\overline{v} - \overline{u}) + \delta_s p_2(1 - r)(\overline{v} - p_2)
\]
\[
= \left[\left(1 - \frac{\delta_b}{2}\right)(1 - r)\overline{v} + r\overline{u}\right](1 - \overline{v}) + r\overline{u}(\overline{v} - \overline{u}) + \delta_s(1 - r)\overline{v}^2
\]
\[
= (1 - r)\left[\left(1 - \frac{\delta_b}{2}\right)\overline{v} - \left(1 - \frac{\delta_b}{2} - \frac{\delta_s}{4}\right)\overline{v}^2\right] + r(\overline{v} - \overline{u}^2).
\]

To check optimality of \( p_2 = \frac{\overline{v}}{2} \), it is sufficient to check that it is not profitable to deviate to \( p_d = \frac{1}{2}[(1 - r)\overline{v} + r\overline{u}] \), which is the single alternative satisfying the first-order condition:

\[
(1 - r)(\overline{v} - p_2)p_2 \geq (1 - r)(\overline{v} - \overline{u})p_d + (\overline{u} - p_d)p_d
\]
\[
\iff \frac{1}{4}(1 - r)\overline{v}^2 \geq \frac{1}{4}[(1 - r)\overline{v} + r\overline{u}]^2 \iff r \leq \frac{(\overline{v} - 2\overline{u})\overline{v}}{(\overline{v} - \overline{u})^2}.
\]

Our problem is thus reduced to:

\[
\max_{\overline{v}, \overline{u}, r} \left\{ (1 - r)\left[\left(1 - \frac{\delta_b}{2}\right)\overline{v} - \left(1 - \frac{\delta_b}{2} - \frac{\delta_s}{4}\right)\overline{v}^2\right] + r(\overline{v} - \overline{u}^2) \right\} \quad \text{s.t.} \quad r \leq \frac{(\overline{v} - 2\overline{u})\overline{v}}{(\overline{v} - \overline{u})^2}.
\]

Since the objective function is an affine function of \( r \), the optimum is either attained with \( r = 0 \) or \( r = \frac{(\overline{v} - 2\overline{u})\overline{v}}{(\overline{v} - \overline{u})^2} \). With \( r = 0 \), that is, with a posted price in period 1, the maximum profit is:

\[
\pi_0 = \max_{\overline{v}, \overline{u}} \left\{ \left(1 - \frac{\delta_b}{2}\right)\overline{v} - \left(1 - \frac{\delta_b}{2} - \frac{\delta_s}{4}\right)\overline{v}^2 \right\} = \frac{(2 - \delta_b)^2}{4(1 - 2\delta_b - \delta_s)}, \quad (4)
\]

which is attained with \( \overline{u} = \frac{2 - \delta_b}{4 - 2\delta_b - \delta_s} \). Since \( \overline{u} \) is irrelevant, we can pick \( \overline{v} = \frac{\overline{v}}{2} = p_2 \), implying \( r = 0 = \frac{(\overline{v} - 2\overline{u})\overline{v}}{(\overline{v} - \overline{u})^2} \). Hence, the optimum is always attained with \( r = \frac{(\overline{v} - 2\overline{u})\overline{v}}{(\overline{v} - \overline{u})^2} \).

Let \( x \in [0, \frac{1}{2}] \) be the ratio between \( \overline{v} \) and \( \overline{u} \). With \( r = \frac{(\overline{v} - 2\overline{u})\overline{v}}{(\overline{v} - \overline{u})^2} = \frac{1 - 2x}{(1 - x)^2} \), profit is given by:

\[
\pi(x, \overline{v}) = \frac{1 - \frac{2\overline{u}}{(1 - x)^2}}{1 - \frac{2\overline{u}}{(1 - x)^2}} \left[(1 - \frac{\delta_b}{2})\overline{v} - \left(1 - \frac{\delta_b}{2} - \frac{\delta_s}{4}\right)\overline{v}^2\right] + \frac{1 - 2x}{(1 - x)^2} (x\overline{v} - x^2\overline{v}^2)
\]
\[
= \frac{x\overline{v}}{(1 - x)^2} \left[1 - \frac{2\overline{u}}{(1 - x)^2} - \left(1 - \frac{\delta_b}{2} - \frac{\delta_s}{4}\right)\overline{v}\right] + \frac{x(1 - 2x)}{(1 - x)^2} (1 - x\overline{v})
\]
\[
= \frac{x\overline{v}}{4(1 - x)^2} \left[4 - 4x - 2\delta_bx - (8 - 8x - 2\delta_b - \delta_s)x\overline{v}\right].
\]

Setting \( x = \frac{1}{2} \), we obtain \( r = 0 \) and the expression for profit in (4). This is the profit that results from an optimally chosen posted price. Setting \( x = 0 \) is never optimal because the resulting
profit is zero.

For \( x \in (0, \frac{1}{2}) \), profit \( \pi(x, \cdot) \) is strictly concave as a function of \( \pi \). The first-order condition with respect to \( \pi \) yields:

\[
\begin{align*}
[4 - 4x - 2\delta_b x - (8 - 8x - 2\delta_b - \delta_s)x\pi_1] - x\pi_1 (8 - 8x - 2\delta_b - \delta_s) &= 0 \\
\Leftrightarrow \pi_1 &= \frac{2-2x-\delta_s}{x(8-8x-2\delta_b-\delta_s)}. 
\end{align*}
\]

It is straightforward to check that \( \pi_1 \) is a strictly positive, continuous, and decreasing function of \( x \). Since \( \pi_1 > 1 \) for \( x \to 0 \) and \( \pi_1 < 1 \) for \( x \to \frac{1}{2} \), there is a threshold value of \( x, x_1 = \frac{10-\delta_b-\delta_s-\sqrt{(10-\delta_b-\delta_s)^2-64}}{16} \in (0, \frac{1}{2}) \), below which \( \pi_1 > 1 \), and above which \( \pi_1 < 1 \).

For \( x \in (0, x_1] \), the first-order condition yields \( \pi_1 \geq 1 \), thus we have a corner solution, with \( \pi = 1 \) and profit given by: \( \pi(x, 1) = \frac{x(4-12x+8x^2+\delta_s x)}{4(1-x)^2}. \)

For \( x \in (x_1, \frac{1}{2}) \), the first-order condition yields \( \pi_1 < 1 \), thus we have an interior solution, with \( \pi = \frac{2-2x-\delta_s x}{x(8-8x-2\delta_b-\delta_s)} \) and profit given by: \( \pi^*(x) = \max_{\pi} \pi(x, \pi) = \frac{(2-2x-\delta_s x)^2}{4(1-x)^2(8-8x-2\delta_b-\delta_s)}. \) Attainable profit can thus be written as a function of \( x \), which is continuously differentiable (even at \( x_1 \)):

\[
\pi(x) = \begin{cases} 
\frac{x(4-12x+8x^2+\delta_s x)}{4(1-x)^2}, & \text{if } x \in (0, x_1] \\
\frac{(2-2x-\delta_s x)^2}{4(1-x)^2(8-8x-2\delta_b-\delta_s)}, & \text{if } x \in (x_1, \frac{1}{2}).
\end{cases}
\]

The function in the second branch of (5), \( \pi^*(x) \), is an upper bound to the expression in the first branch of (5), \( \pi(x, 1) \). Therefore, if the maximum of \( \pi^*(x) \) is attained with \( x \in (x_1, \frac{1}{2}) \), it is the global solution.

Maximizing \( \pi^*(x) \), the first-order condition yields three solutions: \( x_a = \frac{2}{2+\delta_s} \), which is outside the domain; \( x_b = \frac{4-\delta_b+\sqrt{5\delta_b^2-2\delta_b+\delta_b^2-\delta_s^2\delta_s}}{2(2+\delta_b)} \), which is also outside the domain because \( \frac{4-\delta_b}{2(2+\delta_b)} \geq \frac{1}{2} \); and \( x_c = \frac{4-\delta_b-\sqrt{5\delta_b^2-2\delta_b+\delta_b^2-\delta_s^2\delta_s}}{2(2+\delta_b)} \), the lowest of the three, which may be inside the domain. Since \( \pi^*(x) \) is increasing at \( x = 0 \), \( x_c \) is a local maximizer (whenever it is well-defined).

If \( x_c \in \left( \frac{1}{2}, +\infty \right) \) or if \( x_c \) is not well-defined, the maximizer is \( x = \frac{1}{2} \), which means that the optimal mechanism is a posted price. If \( x_c \in (x_1, \frac{1}{2}) \), then it maximizes \( \pi(x) \). If \( x_c \in (0, x_1) \), the maximizer is in the first branch of (5).
The third candidate (whenever it exists) belongs to $[\frac{1}{2}, +\infty)$ if and only if:

$$4 - \delta_b - \sqrt{\frac{5\delta_b^2 - 2\delta_b - 2\delta_s - \delta_s^2}{2(2 + \delta_b)}} \geq \frac{1}{2} \iff 2 - 2\delta_b \geq \sqrt{5\delta_b^2 - 2\delta_b - 2\delta_s - \delta_s^2}$$
$$\implies 4 - 8\delta_b - \delta_s^2 + 2\delta_b^3 + 2\delta_s\delta_b + \delta_s^2\delta_b \geq 0$$
$$\implies \delta_s \geq \frac{-4 + 8\delta_b + \delta_s^2 - 2\delta_b^3}{2\delta_b + \delta_s^2}, \quad (6)$$

which is surely the case if $\delta_b \leq \frac{1}{2}$, because expression (6) is strictly positive if and only if $\delta_b > \frac{1}{2}$.

Observe that if $\delta_s < \frac{-4 + 8\delta_b + \delta_s^2 - 2\delta_b^3}{2\delta_b + \delta_s^2}$, then $x_c$ is well-defined and is lower than $\frac{1}{2}$, which means that $x_c - \epsilon$, for small $\epsilon > 0$ improves on $x_c$.

It can be verified that $x_c \in (0, x_1)$ if and only if: $\delta_s < \frac{-4 + 8\delta_b + \delta_s^2 - 2\delta_b^3}{2\delta_b + \delta_s^2}$.

Hence, $x_c \in (x_1, \frac{1}{2})$ if and only if:

$$\frac{-\delta_s^2 + 6\delta_b - 4 - 4(1 - \delta_b)^2}{\delta_b} < \delta_s < \frac{-4 + 8\delta_b + \delta_s^2 - 2\delta_b^3}{2\delta_b + \delta_s^2}.$$

When $x_c \in (0, x_1)$, the maximum is in the first branch of $[\delta]$. The expression of $\pi(x, 1)$ does not depend on $\delta_b$. As a function of $x$, $\pi(\cdot, 1)$, is concave in the relevant domain and increasing at $x = 0$. We are sure that the maximum is interior because: $\left. \frac{\partial \pi(x, 1)}{\partial x} \right|_{x = x_1} = \left. \frac{d \pi^*(x)}{dx} \right|_{x = x_1} < 0$.

$$\left. \frac{\partial \pi(x, 1)}{\partial x} \right|_{x = x_1} = \left. \frac{d \pi^*(x)}{dx} \right|_{x = x_1} < 0. \quad \Box$$

**Proof of Proposition 2**

We start by finding the optimal price sequence (subject to sequential rationality) under the restriction $p_2 < p_1$, and under the restriction $p_2 \geq p_1$. The solution will be the best of these two candidates.

**Optimal decreasing price sequence ($p_2 < p_1$)**

Suppose w.l.o.g. that buyers with $v \geq \overline{v}$, where $\overline{v} \in [0, 1]$, pay price $p_1 \in [0, 1]$ to obtain the good in period 1. For $p_2 \leq \overline{v}$, we obtain $\pi_2 = p_2(\overline{v} - p_2) + \beta p_2(1 - p_2)$. The second-period profit-maximizing price is $p_2^* = \frac{\overline{v} + \beta}{\beta(1 + \beta)} \leq \frac{1}{2}$, and the corresponding profit is $\pi_2^* = \frac{(\overline{v} + \beta)^2}{\beta(1 + \beta)}$. The single candidate deviation is to $p_2^D = \frac{1}{2}$. If $\overline{v} < \frac{1}{2}$, deviating yields profit $\pi_2^D = \frac{\beta}{4}$; if $\overline{v} \geq \frac{1}{2}$, deviating is surely unprofitable.
Comparing $\pi^*_2$ and $\pi^D_2$, it is straightforward to check that it is profitable to deviate if and only if $v < \sqrt{\beta + \beta^2} - \beta$. We must thus maximize under the restriction $v \geq \sqrt{\beta + \beta^2} - \beta$.

It can be checked that $v \geq \sqrt{\beta + \beta^2} - \beta$ implies $p_2^* \leq v$. It follows that $v - p_1 = \delta(v - p_2^*) \Leftrightarrow p_1 = (1 - \delta)v + \delta p_2^*$. Total profit is $\pi = p_1(1 - v) + \delta p_2^*[v - p_2^* + \beta(1 - p_2^*)]$.

Replacing the expressions for $p_1$ and $p_2^*$, we obtain $\pi$ as a concave function of $v$. Maximizing, we find $v^* = \frac{1}{2} + \frac{\delta}{2[1+4\beta(1-\delta)-3\delta]} \geq \frac{1}{2} > \sqrt{\beta + \beta^2} - \beta$. The corresponding $p_2^*$ is thus sequentially optimal.

The corresponding profit is $\pi^* = \frac{(1 + \beta)[(2 - \delta)^2 + 4\beta\delta(1 - \delta)]}{4[4 + 4\beta(1 - \delta) - 3\delta]}$.

Optimal increasing price sequence ($p_2 \geq p_1$)

Suppose w.l.o.g. that buyers with $v \geq \bar{v}$, where $\bar{v} \in [0, 1]$, pay price $p_1 \in [0, 1]$ to obtain the good in period 1. Since $p_2 \geq p_1$, we have $v = p_1$, $p_2^{**} = \frac{\beta}{4}$ and $\pi = \bar{v}(1 - v) + \delta \beta \frac{\beta}{4}$.

In period 2, the monopolist may be tempted to deviate to $p_2^d = \frac{v + \beta}{4(1 + \beta)}$, which yields profit $\pi^d_2 = \frac{(v + \beta)^2}{4(1 + \beta)}$. The deviation is unprofitable if and only if $v \leq \sqrt{\beta + \beta^2} - \beta < \frac{1}{2}$. This restriction is binding because $\pi$ is increasing in $v$ for $v < \frac{1}{2}$. Therefore, $v^{**} = \sqrt{\beta + \beta^2} - \beta$ and $\pi^{**} = (1 + 2\beta)\sqrt{\beta(1 + \beta)} - 2\beta(1 + \beta) + \frac{\delta}{4}\beta$.

Comparison between the two candidates

Comparing $\pi^*$ with $\pi^{**}$, we find that increasing prices are optimal with high $\beta$ and intermediate $\delta$. The analytical expression for the interval of $\delta$ where increasing prices are more profitable (for given $\beta$) can be obtained but is intricate. It is represented in Figure 5.

Proof of Proposition 3

We start by characterizing the optimal priority pricing mechanism. After obtaining the derivative of the objective function with respect to $\hat{r}$, we study the sign of this expression when $\hat{r} \to 0$ (perturbation of a decreasing price sequence) and when $\hat{r} \to 1$ (perturbation of an increasing price sequence).

Partial characterization of optimal priority pricing

Setting $p_2 \in [\bar{v}, \pi]$ yields profit $\pi_2 = (1 - \hat{r})p_2(\bar{v} - p_2) + \beta p_2(1 - p_2)$. In this range, $p_2 = \frac{(1 - \hat{r})(\pi + \beta)}{2(1 - \hat{r} + \beta)}$ yields the maximum profit, which is equal to $\pi_2 = \frac{(1 - \hat{r})(\pi + \beta)^2}{4(1 - \hat{r} + \beta)}$.  

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Since \( p_1 = \pi - \hat{r}(\pi - v) - \delta(1 - \hat{r})(\pi - p_2) \) and total profit is \( \pi = p_1(1 - \pi) + r\pi(\pi - v) + \delta(1 - \hat{r})p_2(\pi - p_2) + \delta\beta p_2(1 - p_2) \), replacing \( p_2 = \frac{(1 - \hat{r})\pi + \beta}{2(1 - \hat{r} + \beta)} \) yields:

\[
\pi = (1 - \hat{r})\pi(1 - \pi) + \hat{r}\pi(1 - \pi) + \delta(1 - \hat{r})^2\pi(3\pi - 2) + 2\beta(1 - \hat{r})[1 - 2\pi(1 - \pi)] + \beta^2 \overline{v}^2,
\]

which increases with \( \overline{v} \) if \( \overline{v} < \frac{1}{2} \), and decreases with \( \overline{v} \) if \( \overline{v} > \frac{1}{2} \) (strictly if \( \hat{r} > 0 \)).

If \( \hat{r} > 0 \), the seller may prefer to charge \( p_{2d} < \overline{v} \) to earn profit \( \pi_{2d} = p_{2d}(\overline{v} - p_{2d}) + p_{2d}(1 - \hat{r})(\pi - v) + \beta p_{2d}(1 - p_{2d}) \). In that range, the optimal price is \( p_{2d} = \frac{v - r\pi + r\pi + \beta}{2(1 + \beta)} \) and yields profit \( \pi_{2d} = \frac{[(1 - \hat{r})\pi + \hat{r}\pi + \beta]^2}{4(1 + \beta)} \), which increases with \( \overline{v} \). For a deviation to be unprofitable, it is necessary that \( \overline{v} \leq \frac{1}{2} \).

We find that \( \pi_2 \geq \pi_{2d} \) if \( \overline{v} \leq \frac{-(1 - r)(1 - r + \beta)\pi - \beta(1 - r - \beta + (\pi - \pi + \beta)\sqrt{(1 + \beta)(1 + r + \beta)})}{r(1 - r + \beta)} \). Since the threshold is never above \( \frac{1}{2} \), this restriction is binding and yields the optimal \( \overline{v} \).

Replacing the optimal \( \overline{v} \) in the objective function, we obtain:

\[
\pi = \frac{1}{4r^2\pi^2} \left\{ 2(1 - r)\pi \left[ -8\beta^2 + 2r^2 \sqrt{1 + \beta} - \delta r(1 - r) \right] + (1 - r)\pi^2 \left[ 8(1 - r)z \sqrt{1 + \beta} - 4\beta(2 - r - r\delta) - (1 - r)(8 - 3r\delta) \right] + \beta \left( 2r^2(2 - \delta) - 8\beta \left( 1 + \beta - z \sqrt{1 + \beta} \right) - r \left( 4 - 4z \sqrt{1 + \beta} - 2\delta + \beta \delta \right) \right] \right\},
\]

where \( z \equiv \sqrt{1 - r + \beta} \).

The derivative with respect to \( \hat{r} \) yields:

\[
\frac{1}{4r^2\pi^2 w^4} \left\{ -8(1 + \beta)^2(\pi + \beta)^2(1 + \beta - w) + 4r(1 + \beta)(\pi + \beta)^2(5 + 5\beta - 4w) \right. \\
+ w^4 \left[ -(1 + \beta) + 4\pi(1 + \beta) + 2w\beta - 3w\overline{v} \right] \\
+ 2r^3(1 + \beta) \left\{ -\beta + \overline{v} [2 + 7\beta - 2w\delta - \overline{v} (2 + 6\beta - 3w\delta)] \right. \\
- r^2\beta [-2 + \beta (8 + 10\beta - 4w + w\delta)] \\
- 2r^2\pi [1 + 6\beta^3 - w\delta + \beta^2 (25 - 4w - 2w\delta) - 2\beta (-10 + 4w + w\delta)] \\
- r^2\pi^2 \left\{ 12 - 8\beta^3 - 8w + 3w\delta - 4\beta^2 [4 - (1 + \delta)w] + \beta (4 + 6w\delta) \right\} \right\},
\]

where \( w \equiv \sqrt{(1 + \beta)(1 - r + \beta)} \).

Marginal improvement over the optimal decreasing price sequence
From (7), the derivative of $\pi$ with respect to $\hat{r}$ is indeterminate at $\hat{r} = 0$. After removing the indeterminacy (applying L'Hôpital's rule twice), we find that the limit when $\hat{r} \to 0$ is:

$$\frac{1}{4(1 + \beta)^2} \left( 2\beta + \beta^2(1 - \delta) + \tau^2 \left[ 3(1 - \delta) + 8\beta + 4\beta^2(1 - \delta) - 6\beta \delta \right] - 2\tau \left[ 1 + 4\beta + 2\beta^2 - \delta - \beta(2\delta + 2\beta \delta) \right] \right).$$

Replacing $\tau$ by its optimal value when $\hat{r} = 0$, that is, by $\tau = \frac{1}{2} + \frac{\delta}{2[4 + 3\beta(1 - \delta) - 3\delta]} \in \left[ \frac{1}{2}, 1 \right]$, the derivative of $\pi$ with respect to $\hat{r}$ at $\hat{r} = 0$ becomes:

$$\frac{(1 - \delta) \left\{ -4 + \delta [8 + 8\beta(1 - \delta) - 3\delta] \right\}}{4 [4 + 4\beta(1 - \delta) - 3\delta]^2},$$

which is strictly positive if and only if: $\delta > \frac{4 + 4\beta - 2\sqrt{1 + 4\beta^2}}{3 + 8\beta}$. This is thus a sufficient condition for priority pricing to improve on increasing price sequences.

Marginal improvement over the optimal increasing price sequence

Using (7), we find that the derivative of $\pi$ with respect to $\hat{r}$ when $\hat{r} = 1$ is:

$$2\beta^2 + 4\beta\tau - 1 - \frac{\delta}{4} + \tau \left[ 2 + \delta + \tau(1 - \delta) \right] + \left( \frac{1 - 2\tau}{2\beta} + 1 - 2\beta - 4\tau \right) \sqrt{\beta(1 + \beta)}.$$

The above expression is quadratic in $\tau$. Maximizing with respect to $\tau$, we find:

$$\tau^\ast = \frac{(1 + 4\beta) \sqrt{\beta(1 + \beta)} - 2\beta - 4\beta^2 - \beta \delta}{2\beta(1 - \delta)}$$

and, replacing this expression for $\tau$ in $\pi$,

$$\frac{\partial \pi}{\partial r} = -\frac{1 - 6y + \beta \left[ 17 + \delta - 28y - 4\delta y + 8\beta^2(3 + \delta) + 8\beta (5 + \delta - 3y - \delta y) \right]}{4\beta(1 - \delta)},$$

where $y \equiv \sqrt{\beta(1 + \beta)}$.

It is strictly negative if and only if $\delta > \frac{2\sqrt{\beta(1+\beta) - 1 - \beta}}{3 + 8\beta}$. In the parameter region where $\tau^\ast > 1$, setting $\tau = 1$ yields strictly negative $\frac{\partial \pi}{\partial r}$. Hence, $\delta > \frac{2\sqrt{\beta(1+\beta) - 1 - \beta}}{3 + 8\beta}$ is a sufficient condition for priority pricing to improve on increasing price sequences. □
A.3 Auxiliary results

Lemma 2. Truthful revelation implies that \( r \) is weakly-increasing.

Proof. To verify that \( r \) must be weakly-increasing, consider simultaneously two incentive constraints: a buyer with valuation \( v \) does not want to announce valuation \( v' \) and vice-versa:

\[
\left\{ \begin{array}{l}
 r(v)v - t(v) + \mathbb{1}_{v \geq p_2} \delta_b(v - p_2) [1 - r(v)] \geq r(v')v - t(v') + \mathbb{1}_{v \geq p_2} \delta_b(v - p_2) [1 - r(v')] \\
 r(v')v' - t(v') + \mathbb{1}_{v' \geq p_2} \delta_b(v' - p_2) [1 - r(v')] \geq r(v)v' - t(v) + \mathbb{1}_{v' \geq p_2} \delta_b(v' - p_2) [1 - r(v)].
\end{array} \right.
\]

Adding the two constraints and simplifying, we obtain:

\[
[r(v) - r(v')] [v - v' + \mathbb{1}_{v' \geq p_2} \delta_b(v' - p_2) - \mathbb{1}_{v \geq p_2} \delta_b(v - p_2)] \geq 0.
\]

If \( v > v' \), the expression inside the second parenthesis is strictly positive: (i) if \( p_2 \geq v \), it is \( v - v' \); (ii) if \( p_2 \leq v' \), it is \((1 - \delta_b)(v - v')\); (iii) if \( v' < p_2 < v \), it is \((1 - \delta_b)v + \delta_b p_2 - v'\). Hence, \( r(v) - r(v') \) cannot be strictly negative. \( \square \)

Lemma 3. Let \( \frac{v f(v)}{1 - F(v)} \) be strictly increasing and \( G(v) = \beta F(v), \forall v \).\(^{25}\) If \((\overline{v}, \underline{v}, \hat{r}, p_2)\) is a solution of problem (2), then: \( p_2 \leq p^m \), where \( p^m = \argmax_p \{p [1 - F(p)]\} \). Moreover: \( (1 - \hat{r}) F(\overline{v}) = (1 - \hat{r} + \beta) [F(p_2) + f(p_2)p_2] + \beta \).

Proof. From Theorem 1, we know that \( p_2 \in [\underline{v}, \overline{v}] \). If a different price \( p \in [\underline{v}, \overline{v}] \) is posted in period 2, the resulting profit is \( p \int_{\overline{v}}^{\underline{v}} (1-\hat{r}) f(v) \, dv + \int_{\overline{v}}^{\underline{v}} \beta f(v) \, dv = (1 - \hat{r} + \beta) p [F(\overline{v}) - F(p)] + \beta p [1 - F(\overline{v})] \). The first-order condition for sequential optimality is:

\[
(1 - \hat{r}) F(\overline{v}) - (1 - \hat{r} + \beta) [F(p) + p f(p)] + \beta = 0.
\]

The expression is strictly decreasing in \( p \), thus there is a single \( p_2 \) that satisfies this condition, which is a strictly increasing function of \( \overline{v} \). If \( \overline{v} = 1 \), we obtain \( p_2 = p^m \). Thus, \( p_2 \leq p^m \). \( \square \)

\(^{25}\)Assuming increasing proportional hazard rate guarantees that the single-period profit function, \( p[1 - F(p)] \), is single-peaked.
References


