The profit-sharing rule that maximizes sustainability of cartel agreements

João Correia-da-Silva* and Joana Pinho†

CEF.UP and Faculdade de Economia. Universidade do Porto.

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Abstract. We study the profit-sharing rule that maximizes the sustainability of cartel agreements assuming that firms can make side-payments. This rule is such that the critical discount factor is the same for all firms (“balanced temptation”). If a cartel applies this rule, contrarily to the typical finding in the literature, asymmetries among firms may not hinder the sustainability of the cartel. In the simplest case of a Cournot duopoly in which firms differ in their stocks of capital, the cartel is the least sustainable when one of the firms is approximately two times bigger than the other.

Keywords: Collusion sustainability, Side-payments, Balanced temptation.

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*E-mail: joao@fep.up.pt
†E-mail: jpinho@fep.up.pt
1 Introduction

The literature on the sustainability of cartels typically studies an infinitely repeated game where each firm may either: act in accordance with a collusive agreement that specifies each firm’s output (or the price charged to consumers); or break the agreement by increasing output (or decreasing price), taking advantage of the fact that the other firms are restricting output (or charging the cartel price). If one of the firms deviates from the agreement, the industry becomes non-cooperative and thus profits in the subsequent periods become lower.

The choice of a firm between these two options depends on the relative importance that it gives to present and future profits. If the discount factor is sufficiently close to unity (meaning that the same importance is given to present and future profits), the firm will prefer to abide by the cartel rules; if it is sufficiently close to zero, then the firm will prefer to break the cartel to enjoy a higher profit in the present, in spite of suffering the consequent decrease of future profits. The critical value of the discount factor, above which no firm finds it profitable to break the agreement, is an indicator of the sustainability of the cartel.

When firms are heterogeneous, sharing the profit of the cartel is not a trivial matter. If they avoid side-payments in order to decrease the probability of detection, a natural profit-sharing rule is to allocate production efficiently and let each firm keep its individual profit.1 Side-payments provide an additional degree of freedom: the cartel can allocate production efficiently and, independently, choose how to share profits. In this context, a natural profit-sharing rule is based on the solution of the Nash bargaining game: firms get the profits that they would obtain under competition plus an equal share of the increase of the industry profit generated by the cartel agreement.2

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1 This profit-sharing rule is perhaps the most commonly used in the literature. See the discussion by Bos and Harrington (2010).
2 This alternative has been proposed by Osborne and Pitchik (1993).
Depending on the source of firm heterogeneity and on the profit-sharing rule adopted by the cartel, some firms will be more prone to break the cartel than others. Since the sustainability of the cartel only depends on the behavior of the firm that has the strongest propensity to deviate, it seems to be straightforward that firm heterogeneity decreases the sustainability of cartels. This conclusion is supported by the theoretical contributions of Bae (1987), Harrington (1989), Compte et al. (2002), Kühn (2004), Vasconcelos (2005), Miklós-Thal (2011) and Brandão et al. (2014).

To address the issue of sustaining collusion in the presence of asymmetries among firms when side-payments are possible, we study the profit-sharing rule that maximizes the sustainability of the cartel. With this rule, all the firms end up having the same critical discount factor. It satisfies, therefore, the balanced temptation property proposed by Friedman (1971). If one of the firms had a higher critical discount factor (i.e., a stronger propensity to break the cartel), the sustainability of the cartel agreement could be increased by marginally increasing this firm’s share of the cartel profits.

Applying this profit-sharing rule to a simple example of a Cournot duopoly in which firms differ in their stocks of capital (and, as a result, in their marginal costs of production), we find that the relationship between the asymmetry between firms and the sustainability of collusion is U-shaped. The cartel is the least sustainable when one of the firms is approximately two times bigger than the other. It is slightly more sustainable when firms are symmetric, and much more sustainable when one of the firms has almost all the industry capital.

Previous contributions have also considered the balanced temptation solution to study the effect of cost asymmetry on the sustainability of a cartel, but in environments without side-payments.\footnote{This solution concept was criticized by Harrington (1991), who advocated the application of a bargaining solution to select one among the set of sustainable collusive agreements. Verboven (1997) showed that when perfect collusion is not sustainable, the outcome of bargaining is likely to satisfy the balanced temptation property.} In a price-setting supergame where firms have constant unit costs, Bae
(1987) concluded that cost asymmetry hinders the sustainability of collusion. Closer to our contribution is the study of Collie (2004), who provided numerical evidence suggesting that cost asymmetry enhances the sustainability of collusion in a quantity-setting supergame with linear demand and quadratic costs.

2 Model

Consider an industry composed by heterogeneous firms in a stationary infinite-horizon setting. The objective of each firm is to maximize the discounted value of its flow of profits: \[
\sum_{t=0}^{\infty} \delta^t \Pi_{it},
\]
where \( \Pi_{it} \) denotes the profit of firm \( i \) in period \( t \). All firms have the same discount factor, \( \delta \in (0, 1) \).

Before period \( t = 0 \), firms \( i \in \{1, \ldots, N\} \) establish a collusive agreement. This agreement lasts forever, as long as there are no defections. If any of the firms violates the agreement, firms engage in competition in all the subsequent periods (firms use grim trigger strategies). There is no renegotiation.

Let \( \Pi^s_i \) denote the single-period profit of firm \( i \) in each of the possible competitive scenarios: \( s = c \) if firms compete; \( s = m \) if firms collude; and \( s = d \) if firm \( i \) unilaterally deviates from the collusive agreement.

Firms prefer to abide by the collusive agreement if the discounted value of the flow of collusive profits exceeds the sum of the deviation profit with the discounted value of the flow of the subsequent competitive profits. More precisely, collusion is sustainable if and only if the following incentive compatibility constraint is satisfied for all firms

\[4\text{ We take the collusive agreement as a given. It may include or not all the firms in the industry, may maximize or not the profits of the cartel, and may stipulate the output or price set by each firm, a geographical sharing of the market, or any other type of collusive behavior.} \]
\(i \in \{1, \ldots, N\}:
\sum_{t=0}^{\infty} \delta^t \Pi_i^m \geq \Pi_i^d + \sum_{t=1}^{\infty} \delta^t \Pi_i^c \iff \delta \geq \frac{\Pi_i^d - \Pi_i^m}{\Pi_i^d - \Pi_i^c} \equiv \delta_i^*.
\)

Therefore, the critical discount factor for collusion sustainability is: \(\delta^* = \max_{i \in \{1, \ldots, N\}} \delta_i^*\).

The magnitude of each firm’s critical discount factor depends on how the profit of the cartel is shared among firms. Denote by \(\lambda_i \in (0, 1)\) the profit-share of firm \(i\): \(\Pi_i^m = \lambda_i M\), where \(M\) denotes the industry profit under collusion.

We consider the profit-sharing rule that maximizes sustainability of collusion. Firms are assumed to choose the allocation, \(\lambda \equiv (\lambda_1, \ldots, \lambda_N)\), that minimizes \(\delta^*\).

**Proposition 1.** The allocation of the industry profit, \(\lambda^*\), that maximizes sustainability of collusion is such that:

\[\delta^* = \frac{D - M}{D - C} \quad \text{and} \quad \lambda_i^* = \frac{M - C}{M(D - C)} \Pi_i^d + \frac{D - M}{M(D - C)} \Pi_i^c,\]

where \(C = \sum_{i=1}^{N} \Pi_i^c\) is the industry profit under competition and \(D = \sum_{i=1}^{N} \Pi_i^d\) is the sum of the individual deviation profits.

**Proof.** It is rather trivial to show that \(\delta_i^* = \delta^*\), \(\forall i, j\). Let \(A = \{i : \delta_i^* = \delta^*\}\) and \(B = \{i : \delta_i^* < \delta^*\}\). If \(B\) were not empty, then a marginal increase of the profit shares of the firms in \(A\) at the cost of the profit shares of the firms in \(B\) would decrease \(\delta^*\).

Thus, the \(N\) incentive compatibility constraints (1) are satisfied with the equality sign. We have, therefore, a linear system of \(N + 1\) equations with \(N + 1\) unknowns:

\[\begin{cases}
\delta^* = \frac{\Pi_i^d - \lambda_i^* M}{\Pi_i^d - \Pi_i^c}, \forall i \\
\sum_{i=1}^{N} \lambda_i^* = 1,
\end{cases}\]

whose solution is (2). \(\square\)
Observe that the resulting critical discount factor is always lower than 1, which means that collusion is sustainable if firms are sufficiently patient.

3 Cournot duopoly with asymmetric production costs

Consider the case of an infinitely-repeated Cournot duopoly with homogeneous goods but asymmetric production costs. In each period $t \in \{0, 1, \ldots\}$, firms 1 and 2 simultaneously choose their output levels, $q_{1t}$ and $q_{2t}$. The price charged to consumers is the same for both firms and given by:

$$P_t = 1 - (q_{1t} + q_{2t}).$$

Firms need capital to produce. A firm with more capital can produce the same units of output at a lower cost. The cost function of firm $i$ is given by:

$$C_i(q_i) = \frac{q_i^2}{2k_i},$$

where $k_i$ is the capital stock of firm $i$. For simplicity, let $k_1 = k$ and $k_2 = 1 - k$.

Under competition, each firm produces the quantity that maximizes its individual profit:

$$\max_{q_i} \left\{ (1 - q_i - q_j) q_i - \frac{q_i^2}{2k_i} \right\}.$$

The competitive output and profit of each firm $i \in \{1, 2\}$ is given by:

$$q_i^c = \frac{k_i}{(1 + k_i)(1 + S)} \quad \text{and} \quad \Pi_i^c = \frac{k_i(1 + 2k_i)}{2(1 + k_i)^2(1 + S)^2},$$

where $S = \sum_{i=1}^{2} \frac{k_i}{1 + k_i}$.

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5 This is a simplified version of the cost function proposed by Perry and Porter (1985).
We follow most theoretical contributions that study collusion sustainability in assuming that, along the collusive path, firms maximize their joint profit:

$$\max_{(q_1, q_2)} \left\{ (1 - q_1 - q_2) (q_1 + q_2) - \frac{q_1^2}{2k} - \frac{q_2^2}{2(1 - k)} \right\}.$$  

Under perfect collusion, the output of each firm is proportional to its capital stock, $q^m_i = \frac{k_i}{3}$, and the cartel profit is $M = \frac{1}{6}$.

If firm $i$ disrupts the collusive agreement, it will produce the quantity that maximizes its individual profit, assuming that the other firm produces the collusive output:

$$\max_{q_i} \left\{ \left( 1 - q_i - \frac{k_j}{3} \right) q_i - \frac{q_i^2}{2k_i} \right\}.$$  

The deviation output and the corresponding single-period profit are:

$$q^d_i = \frac{k_i (2 + k_i)}{3(1 + 2k_i)} \quad \text{and} \quad \Pi^d_i = \frac{k_i (2 + k_i)^2}{18(1 + 2k_i)}.$$  

Using (2), we obtain the critical discount factor that is associated with the profit-sharing rule that maximizes collusion sustainability (illustrated in Figure 1):

$$\delta^* = \frac{(3 - 4k + 4k^2) (1 + k - k^2)^2}{6 + 3k - 12k^2 + 14k^3 + 3k^4 - 12k^5 + 4k^6}. \quad (3)$$  

Collusion is easiest to sustain if one firm owns almost the whole industry capital (i.e., when $k \to 0$ or $k \to 1$). On the contrary, collusion is less likely when one firm owns approximately 33.2% of the industry capital. The most surprising result is that symmetry between firms does not necessarily facilitates collusion. This is in contrast with the existing literature, with the exception of Collie (2004), which defends symmetry across firms as facilitating collusion (Motta, 2004).
Alternatively, if firms divide their joint profit according to their relative bargaining power, the share of firm $i$ is the solution of the associated Nash bargaining game:

$$\max_{\lambda_i^N} \left\{ (\lambda_i^N M - \Pi_i^C) \left[ (1 - \lambda_i^N) M - \Pi_j^C \right] \right\}.$$

After some algebra, we obtain the critical discount factors for firms to abide by the collusive agreement (illustrated in Figure 2):

$$\delta_1^N = \frac{-4 - 8k + 45k^2 + 62k^3 - 68k^4 + 6k^5 + 51k^6 - 57k^7 + 6k^8 + 6k^9}{3k^2 (-1 - k + k^2) (3 - 2k + 2k^2) (-4 - 6k + 3k^2 + k^3)}$$

and

$$\delta_2^N = \frac{-39 + 35k + 165k^2 - 285k^3 + 92k^4 + 207k^5 - 324k^6 + 207k^7 - 60k^8 + 6k^9}{3(-1 + k)^2 (-1 - k + k^2) (3 - 2k + 2k^2) (6 + 3k - 6k^2 + k^3)}.$$

We conclude that, in this case, the smaller firm is the most tempted to deviate.

If side-payments between firms are not feasible (assuming that firms continue to max-
imize cartel profits and allocate production efficiently), the critical discount factors are:

\[
\delta_1^P = \frac{(1 - k) (1 + k - k^2)^2}{k^2 (4 + 6k - 3k^2 - k^3)} \quad \text{and} \quad \delta_2^P = \frac{k (1 + k - k^2)^2}{(1 - k)^2 (6 + 3k - 6k^2 + k^3)}.
\]

Figure 2: Critical discount factors of firm 1 (solid line) and firm 2 (dashed line) if profits are shared according to the Nash bargaining rule.

Figure 3: Critical discount factors for firm 1 (solid line) and firm 2 (dashed line), if there are no side-payments.
In this case, it is the biggest firm that has the strongest incentives to deviate (see Figure 3). If firms are very asymmetric, i.e., if \( k \in (0, 0.39) \cup (0.61, 1) \), perfect collusion is not sustainable without side-payments.

Figure 4 shows that the profit-sharing rule that maximizes the sustainability of collusion by balancing temptation significantly decreases the critical discount factor.

4 Conclusions

When asymmetries between firms are very pronounced, perfect collusion without side-payments may not be possible. However, if firms can make side-payments in order to share their collusive profits in an appropriate way, they become able to sustain perfect collusion (as long as they are sufficiently patient). With the profit-sharing rule that maximizes collusion sustainability, which is the one that satisfies the “balanced temptation” requirement of Friedman (1971), asymmetries between firms may increase the sustainabil-
ity of collusion. This is in line with the results of Collie (2004), but contrasts with the typical message in the literature.

References


