Spatial competition between shopping centers

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Abstract. We study competition between two shopping centers that sell the same set of goods and are located at the extremes of a linear city, without restricting consumers to make all their purchases at a single place. In the case of competition between a shopping mall (set of independent single-product shops) and a department store (single multiproduct shop), we find that: if the number of goods is low, all consumers shop at a single place; if it is moderately high, some consumers travel to both shopping centers to buy each good where it is cheaper (a single good is cheaper at the shopping mall). The shops at the mall, taken together, obtain a lower profit than the department store. Nevertheless, two shopping malls should be expected to appear endogenously.

Keywords: Multiproduct firms, Spatial competition, Two-stop shopping.

JEL Classification Numbers: D43, L13, R32.

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1 Introduction

Shopping centers have existed for many centuries as galleries, market squares, bazaars or seaport districts. Today, they are mainly organized in two alternative formats: shopping malls and department stores. Both are spaces where consumers can buy a huge variety of goods. But, while a department store can be seen as a multiproduct firm, a shopping mall is constituted by independent shops.

Competition between shopping centers exists in most large cities, with physical distance between them playing a relevant role.[1] Even when they offer similar product lines, the fact that they are spatially differentiated provides them with some market power that they can exploit when setting prices.[2] This market power is limited by the fact that some consumers may find it worthwhile to visit more than one shopping center in order to purchase goods where they are cheaper.

To study competition between shopping centers, one should take into account the demand for multiple goods and also the cost of traveling to one or more shopping centers. Most of the existing spatial competition models fail to do so, because they either restrict the analysis to markets with a single good or assume that consumers make all their purchases at the same place (Bliss, 1988; Beggs, 1994; Smith and Hay, 2005; Innes, 2006; Lahmandi-Ayed, 2010). This "one-stop shopping" assumption is very convenient because it allows treating multiple goods as a single bundled good.

We provide a study of competition between shopping centers by extending the standard model of spatial competition (Hotelling, 1929; d'Aspremont, Gabszewicz and Thisse, 1979) to the case of multiple goods, without assuming one-stop shopping. We consider the existence of two shopping centers located at the extremes of a linear city, selling the same set of goods. Consumers are uniformly spread across the city and buy exactly one unit of each good.[3] They may travel to a shopping center and buy all goods there, or travel to both

[1] Our hometown, Porto, having a metropolitan area with 1.3 million residents, is served by seven shopping centers with a commercial area above 39,000 m^2: six shopping malls (ArrábidaShopping, Dolce Vita, GaiaShopping, MAR Shopping, NorteShopping and Parque Nascente), and one department store (El Corte Inglés). They are more or less evenly distributed around the city, so that a car trip between two adjacent shopping centers can take around 10 minutes.

[2] In his empirical study on store choice in spatially differentiated markets, which uses data on store sales of packaged goods in Pittsfield (Massachusetts, U.S.A.), Figurelli (2013) reports that 99% of expenditure is on items for sale in more than one store and 96% in more than one chain of stores.

[3] Total demand is inelastic, as in the standard single-product model. It would undoubtedly be of interest to relax this simplifying assumption. A possible approach could be to maintain indivisibility, but introduce heterogeneous valuations for each good as in the multiproduct monopoly model of Rhodes (2012).

shopping centers and buy each good where it is cheaper.[4]

A shopping center may be either a shopping mall (where each good is sold by an independent firm) or a department store (where a single firm sells all the goods).[5] We solve for equilibrium prices, market shares and profits in three scenarios: (*i*) competition between a department store and a shopping mall; (*ii*) competition between two department stores; (*iii*) competition between two shopping malls.

In the case of competition between a department store and a shopping mall, we find that there may be consumers visiting the two extremes of the city or not, depending on the number of goods that are sold by the shopping centers. If the number of goods is low, all consumers make their purchases at a single place (one-stop shopping). If the number of goods is moderately high, some consumers are willing to travel to both extremes of the city to buy each good where it is cheaper (two-stop shopping). In this case, there is only one good that is cheaper at the shopping mall than at the department store. However, its price is low enough for some consumers to travel there just to buy this good. If the number of goods is moderately low or very high, there is no price equilibrium in pure strategies.

Regardless of the number of goods, the equilibrium price of the bundle is lower at the department store than at the shopping mall. This occurs because unrelated goods become complements when they are sold at the same location (and substitutes when they are sold at opposite extremes of the city).[6] When a shop at the mall considers the possibility of decreasing its price, it only cares about the increase of its own demand and not about the increase of the demand of the other shops at the mall. In contrast, the department store internalizes this effect, and takes into account that a decrease in the price of one good also increases the demand for its other goods.[7] In spite of charging a lower price for the bundle, the department store obtains a higher profit than the shops at the mall taken together.

The scenario in which prices are lowest is that of competition between two department stores. In this case, the price charged for the bundle of goods is equal to the price charged

[4]Consumers are assumed to be fully informed about the prices charged in each extreme of the city. Multiproduct pricing in the presence of search costs has been recently studied by Rhodes (2012). One of his main conclusions is that a firm that sells more products attracts consumers that are less price-elastic and, therefore, has lower incentives to surprise customers with higher prices once they have visited the store.

[5]We rule out bundling strategies, i.e., we restrict the price of a bundle of goods to be equal to the sum of the prices of the individual goods. For an analysis of the bundle pricing problem in a related context, see Armstrong and Vickers (2010). See also Hanson and Martin (1990).

[6]See Stahl (1987).

[7]Gould, Parshigian and Prendergast (2005) showed that rental contracts in shopping malls typically include incentives for an individual shop to act in a way that is beneficial for the other shops at the mall. Therefore, one should not expect a shopping mall to behave exactly as a set of independent shops.

in the single-good model (independently of the number of goods). The two department stores obviously capture equal shares of the market and obtain equal profits. These are, unsurprisingly, lower than the profits obtained when competing against a shopping mall.

Finally, in the scenario of competition between two shopping malls, we find that each good is sold at the same price as in the single-good model. The shops behave as if consumers only bought their good. This is the competitive scenario in which prices are highest. The explanation is the same as before: the shops at the mall set the same price as in the single-good model because they do not internalize the positive effect of a price decrease on the other shops at the same mall.

After finding the equilibrium prices and profits in each of the three competitive scenarios, it is straightforward to analyze whether it is more profitable to have a department store or several independent shops at a mall.[8] We answer this question by considering a two-stage game in which the shopping centers start by acquiring land and then compete in prices. We find that, if the number of goods is low, shopping malls are willing to bid higher for the land. Therefore, the competitive scenario that appears in equilibrium is that of competition between two shopping malls. However, if the number of goods is moderately high, there is another self-fulfilling equilibrium, which is Pareto-inferior: competition between two department stores.

As explained previously, a department store has stronger incentives to charge lower prices than the independent shops at a mall. If the prices of the rival retailers remained the same, the greater aggressiveness of the department store would be profitable. However, setting lower prices induces the rivals to lower their prices as well. If the number of goods is low, this effect dominates, leading to lower profits for everyone. The reason why both sides would win if a department store separated itself into several independent shops was explained by Innes (2006): "*a multi-product retailer can effectively pre-commit to higher prices by organizing itself as a mall of independent outlets*". If the number of goods is moderately high, it becomes more profitable to compete against a department store by behaving as a department store. But it is still better to compete against a shopping mall by behaving as a shopping mall. This is why there are two scenarios that may emerge endogenously: competition between two department stores or competition between two shopping malls.

We also compare the consumer surplus and the total surplus in the different competitive scenarios. Since all consumers are assumed to buy exactly one unit of each good, a change

[8]Since otherwise unrelated goods become complements when they are sold at the same shopping center, this question is related to the literature on mergers between firms that sell complementary goods. See, for example, Economides and Salop (1992), Matutes and Regibeau (1992) or Bart (2009).

in prices simply transfers surplus between consumers and stores. Therefore, total surplus is maximized when consumers shop at the closest shopping center (transportation costs are minimized).[9] This occurs when there are either two department stores or two shopping malls. Unsurprisingly, consumer surplus is maximal in the case of competition between two department stores. Competition between two shopping malls is actually the worst scenario for consumers. In spite of bearing a higher total transportation cost, consumers are better off when there is competition between a department store and a shopping mall than when there is competition between two shopping malls.

One possible policy implication of our work concerns the debate regarding the regulation of big-box retail.[10] According to Griffith, Harrison, Haskel and Sako (2003), the "wholesale and retail" sector is responsible for 20% of the productivity gap between the U.K. and the U.S.A.. This may partly be due to the stricter regulatory environment in the U.K., which is restricting the development of large out-of-town retail stores.[11] While the welfare-losses associated with regulatory restrictions have been mainly focused on productivity losses due to reduced store-size, the recent contribution of Schiraldi, Seiler and Smith (2011) focuses on competition between large retailers and small stores with the objective of understanding whether out-of-town big-box retail may have a predatory effect on small city centre stores and, as a result, be detrimental to the vitality of the city centre.[12] Our contribution seems to be relevant for that discussion, as it provides a theoretical model of spatial competition between a large retailer and a set of small independent stores. Despite the very stylized nature of our model (in particular, the inelasticity of total demand and the absence of substitution or complementarity between goods), which warns against taking our results literally, we believe that our conclusions should not be readily dismissed.[13]

The remainder of the article is organized as follows. In Section 2, we briefly review the

[9] Under the assumption that total demand is inelastic, statements about total surplus should be taken with a grain of salt. In more realistic settings, high prices entail a deadweight loss.

[10] We thank a referee for describing to us how our work could be meaningful to this policy issue.

[11] See Griffith and Harmgart (2008), Sadun (2011), Schiraldi, Seiler and Smith (2011), and Cheshire, Hilber and Kaplanis (2012). See also Bertrand and Kramarz (2002) and Schivardi and Viviano (2011) on the effects of retail sector regulation in France and Italy, respectively.

[12] Using a model of consumer choice about which stores to visit and how much to spend in each store, Schiraldi, Seiler and Smith (2011) address the conflict that seems to exist between: environmental policy authorities, according to whom out-of-town big-box stores lead to the shutdown of small downtown stores; and competition policy authorities, which view big-box stores as mainly competing among themselves. The settlement of this issue crucially depends on the extent to which consumers view the two types of stores as substitutes. According to Schiraldi, Seiler and Smith (2011), two-stop shoppers are an important consumer segment for such an analysis.

[13] The recent contribution of Sloev, Thisse and Ushchev (2013) also seems very relevant for that discussion.

existing literature. In Section 3, we setup the model and obtain the demand and profit functions. In Section 4, we find the equilibrium prices, demand and profits in each of the three different competitive scenarios. We obtain endogenous modes of retail in Section 5, and dedicate Section 6 to a welfare analysis. Section 7 concludes the article with some remarks. Most of the proofs are collected in the Appendix.

2 Literature review

Our model extends the standard duopoly model of spatial competition to analyze multiproduct competition between department stores and shopping malls. To the best of our knowledge, Lal and Matutes (1989) were the first to present a multiproduct version of the linear city model of Hotelling (1929). Their objective was to study price discrimination between two types of consumers.[14] Building on this contribution, Lal and Matutes (1994) and Lal and Rao (1997) introduced imperfect information about prices and the possibility of firms announcing the prices that they charge for one or more goods (with advertising acting as a commitment device).[15] In all these works, the analysis is restricted to the case of competition between two department stores that sell two goods. We allow a finite number of goods and an alternative mode of retail: the shopping mall.[16]

Other authors have analyzed multiproduct price competition, but did not use the spatial competition model. Moreover, most of them based the analysis on the assumption that consumers make all their purchases at the same shopping center (Bliss, 1988; Beggs, 1994; Smith and Hay, 2005; Innes, 2006). The empirical evidence regarding this assumption is mixed. On the one hand, consumers concentrate most of their expenditure in a single store. Rhee and Bell (2002) estimated that consumers make 94% of their weekly groceries expenditures at the same supermarket, while Schiraldi, Seiler and Smith (2011) estimated this fraction to be 86%. On the other hand, a large fraction of consumers engages in multi-

[14]In the model of Lal and Matutes (1989), there are two types of consumers: *poor* and *rich*. The *poor* have low reservation prices and do not bear transportation costs. Therefore, they buy each good where it is cheaper (one-stop shopping is not assumed). In contrast, the *rich* have high reservation prices and bear transportation costs. In equilibrium, these consumers are not interested in shopping around.

[15]This line of research has been recently developed by Rhodes (2012). His model is quite general in that agents have heterogeneous valuations for a finite number of indivisible goods. But, contrarily to us, he does not build upon the linear city model. Instead, he assumes that agents have homogeneous search costs.

[16]There are other extensions of the spatial competition model that allow for multiproduct firms, but in which consumers only buy one of the goods that are available (Giraud-Héraud, Hammoudi and Mokrane, 2003; Laussel, 2006). In such settings, goods available in a shopping center are substitutes instead of complements. This is also the case in the model of Gehrig (1998).

store shopping. Schiraldi, Seiler and Smith (2011) found that, in an average week, 39% of shoppers visit more than one store. Stassen, Mittelstaedt and Mittelstaedt (1999) estimated a much higher fraction, 75%. Figurelli (2013) estimated that 95% of consumers visit three or more retail stores over a year.

A quite general multiproduct duopoly model in which consumers decide whether to buy goods from a single seller or to bear an additional cost to buy goods from two sellers was proposed by Klemperer (1992). This is a notable exception to the one-stop shopping assumption. He analysed two cases: one in which the two department stores offer differentiated product lines; and another in which they offer the same product line. In the first scenario, some consumers make two-stop shopping to benefit from a greater variety of goods.[17] When the product lines are identical, the motive for two-stop shopping disappears. Consumers never make two-stop shopping to take advantage of price differences (as they do in our model).[18]

There are other studies of multiproduct competition with spatial differentiation in which two-stop shopping emerges because firms offer different sets of goods. In the model of Thill (1992), a firm that sells two goods competes against a firm that sells only one of the goods. He concluded that some of the consumers that need to buy both goods choose to make two-stop shopping.[19] In a recent contribution, Chen and Rey (2012a) also considered competition between a firm that sells two goods and a firm (or a fringe of firms) that offers only one of the goods. They show that heterogeneity in consumers' shopping costs makes it profitable for the multiproduct firm to price below cost the good in which it faces competition. This loss-leading strategy allows it to discriminate two-stop shoppers from one-stop shoppers. Interestingly, they conclude that the multiproduct firm can actually obtain a higher profit in the presence of competition than if it were a monopolist in the sale of the two goods.

Another multiproduct duopoly model with spatial differentiation and two-stop shopping was also recently developed by Chen and Rey (2012b). Two department stores sell the same pair of goods, but each of them has a competitive advantage in the supply of one of the goods. Consumers are heterogeneous in the additional cost of buying goods from both sellers instead of a single seller. As a result, some consumers buy both goods at a single store, while

[17]In the contributions of Kim and Serfes (2006) and Anderson, Foros and Kind (2012), where each firm offers a single product, consumers may also engage in two-stop shopping in order to consume both goods instead of a single good. This kind of setup is related to the literature on two-sided markets where multi-homing is permitted. Our contribution diverges from this strand of literature because our focus is on the interaction between multiproduct competition and multi-stop shopping.

[18]The main point of the contribution of Klemperer (1992) is that the two firms may choose to offer the same product line to decrease the level of competition.

[19]For a similar reason, some consumers make two-stop shopping in the model of Zhu, Singh and Dukes (2011). There, two firms sell goods 1 and 2, while a third firm sells goods 2 and 3.

the others buy each good where the difference between utility and price is greater. Chen and Rey (2012b) concluded that fierce competition for one-stop shoppers dissipates profits in this segment, but firms still earn positive profits from two-stop shoppers. This contrasts with the results of our model, where the one-stop shoppers are the high-value segment.[20]

Armstrong and Vickers (2010) also studied competition between two department stores that sell two horizontally differentiated goods, but with consumers located in a square according to their preferences for the different varieties of the two types of goods. In their model, some consumers make two-stop shopping because they prefer the variety of good A that is sold by one firm and the variety of good B that is sold by the other firm.[21]

Regarding the comparison between the behavior of department stores and shopping malls, the first result in the literature was presented by Edgeworth (1925), who found that it is better, for consumers, to have a single monopolist selling two complementary goods than to have two separate monopolists. Salant, Switzer and Reynolds (1983) arrived at a similar conclusion, but in a model of Cournot competition. Using a framework that is closer to ours, Bertrand competition with linear demand, Beggs (1994) concluded that it may be desirable for a department store to separate into several shops or not, depending on the degree of substitutability between goods. As a result, either two department stores or two shopping malls emerge in equilibrium. Innes (2006) studied the effect of entry and concluded that only department stores survive in equilibrium because they compete more aggressively and, therefore, are more effective in deterring entry. Shopping malls would be driven out of the market by department stores because when there is competition between department stores and shopping malls, the former have higher profits.[22]

[20]In the work of Shelegia (2012), there are also two firms offering the same pair of goods, but consumers do not necessarily buy the two goods (which may be complements or substitutes). In his model, there is no horizontal differentiation between firms. Instead, some consumers only have access to a single firm (*captives*), while others have access to both firms without incurring in an additional cost (*shoppers*).

[21]The contribution of Armstrong and Vickers (2010) has also the notable feature of addressing the bundle pricing problem (they allow the price of the bundle to be lower than the sum of the prices of the individual goods), in contrast to the existing literature on spatial multiproduct competition.

[22]Smith and Hay (2005) have also studied price competition under alternative modes of retail organization (shopping streets, shopping malls and department stores), but did not consider two-stop shopping nor competition between different modes of retail.

3 The model

3.1 Basic setup

We consider a multiproduct version of the model of Hotelling (1929). There is a continuum of consumers uniformly distributed across a linear city, $[0,1]$. Each consumer buys one unit of each of the products, $i \in \mathcal{I} \equiv \{1, ..., n\}$, which are sold at both extremes of the city ($x = 0$ and $x = 1$).[23] The price of good i at the left extreme (L) is denoted by p_{i_L} and the price of good i at the right extreme (R) is denoted by p_{i_R}.

The reservation price for each good is assumed to be high enough for the market to be fully covered. Thus, total demand is perfectly inelastic and the only decision of consumers is where to buy each product. Each consumer chooses among three possibilities:

(L) to buy all the goods at the left extreme;

(R) to buy all the goods at the right extreme;

(LR) to travel to both extremes and buy each good where it is cheaper.

We denote by P_L and by P_R the price that a consumer pays for all the goods at L and at R, respectively ($P_L = \sum_{i=1}^{n} p_{i_L}$ and $P_R = \sum_{i=1}^{n} p_{i_R}$). By P_{LR}, we denote the price that a consumer pays for all the goods if she buys each good where it is cheaper ($P_{LR} = \sum_{i=1}^{n} \min\{p_{i_L}, p_{i_R}\}$).

To make their decision, consumers take into account not only the prices of the goods, but also the transportation costs that they must bear to acquire them. We assume that transportation costs are linear in distance.[24] Let $u_L(x)$, $u_R(x)$ and $u_{LR}(x)$ denote the utility attained by an agent located at $x \in [0,1]$ who chooses to purchase, respectively: all the

[23] As mentioned by Lal and Rao (1997), to assume that the same goods are sold at both extremes has the advantage of focusing the analysis on price competition. A possible interpretation is that the goods are supplied by manufacturers that are common to both stores.

[24] Using data from the motion picture exhibition market in the U.S.A., Davis (2006) concluded that the marginal transportation cost is decreasing in distance, ranging from 31 cents for the first mile to 19 cents for the 15^{th} mile (which he refers as being the maximum distance that consumers are willing to travel). Figurelli (2013) arrived at similar conclusions for the supermarket industry, having found that the average distance of a trip to a supermarket is 3.1 miles, and that the shopping cost ranges from 12 cents for the first mile to 10 cents for the 7^{th} mile. In spite of this empirical evidence, we consider linear transportation costs for reasons of tractability.

goods at L; all the goods at R; each good where it is cheaper. Then:

$$\begin{aligned} u_L(x) &= V - P_L - tx, \\ u_R(x) &= V - P_R - t(1-x), \\ u_{LR}(x) &= V - P_{LR} - t, \end{aligned}$$

where V denotes the reservation price of the bundle of goods and $t > 0$ denotes the transportation cost per unit of distance.

It is important to keep in mind that if a consumer travels to both extremes, she bears higher transportation costs than if she purchases all the goods at the same location. For this reason, the demand for each product at a certain location is related to the demand for all the other products at the same and at the other location. Products sold at the same location are complementary goods, while products sold at different locations are substitutes.

3.2 Demand and profit functions

The consumers that are most likely to purchase a good that is sold at one extreme are those who are located closer to that extreme. When all the goods have strictly positive demand at both locations, the consumers near the left extreme are surely buying all the goods at L, while those near the right extreme are surely buying all the goods at R.

Depending on the prices charged for each good at each location, some consumers may find it worthwhile to travel to both extremes of the city, to buy each good where it is cheaper. This occurs if some goods are sufficiently cheaper at L while other goods are sufficiently cheaper at R. On the contrary, if the price differences across locations are relatively small, then all the consumers make their purchases at a single location (at L or at R). These possible demand scenarios (one-stop shopping and two-stop shopping) are illustrated in Figure 1.

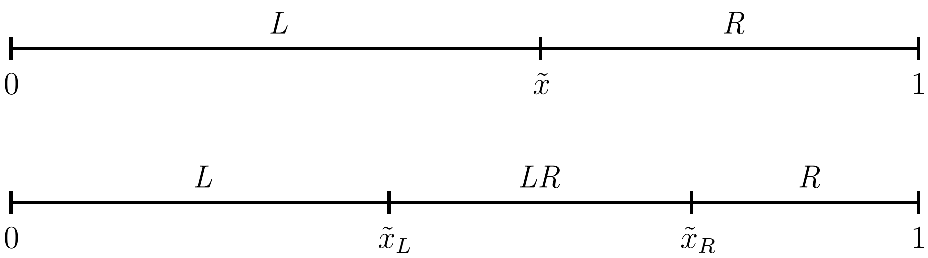

Figure 1: Possible demand scenarios.

To obtain the demand for each good at each location, it is useful to find the location of the consumer that is indifferent between each pair of choices (among L, R and LR). Accordingly, we use some additional notation.

By $\tilde{x}_L$, we denote the location of the consumer that is indifferent between L and LR:

$$u_L(\tilde{x}_L) = u_{LR}(\tilde{x}_L) \;\Leftrightarrow\; \tilde{x}_L = 1 - \frac{P_L - P_{LR}}{t}.$$

We denote by $\tilde{x}_R$ the consumer that is indifferent between R and LR:

$$u_R(\tilde{x}_R) = u_{LR}(\tilde{x}_R) \;\Leftrightarrow\; \tilde{x}_R = \frac{P_R - P_{LR}}{t}.$$

Finally, we denote by $\tilde{x}$ the consumer that is indifferent between L and R. It is clear from the expression below that $\tilde{x} = \frac{\tilde{x}_L + \tilde{x}_R}{2}$:

$$u_L(\tilde{x}) = u_R(\tilde{x}) \;\Leftrightarrow\; \tilde{x} = \frac{1}{2} + \frac{P_R - P_L}{2t}. \tag{1}$$

Consumers located in $x \in (\tilde{x}_L, \tilde{x}_R)$ choose LR. Hence, there are consumers traveling to both extremes of the city if $\tilde{x}_L < \tilde{x}_R$, which is equivalent to $\sum_{i\in\mathcal{I}} |p_{i_L} - p_{i_R}| > t$. Otherwise, all consumers make their purchases at a single place. It is easy to verify that $\sum_{i\in\mathcal{I}} |p_{i_L} - p_{i_R}| \leq t$ implies that $0 \leq \tilde{x} \leq 1$.[25] Therefore, in this case, the demand for each good sold at L is $\tilde{x}$ and the demand for each good sold at R is $1 - \tilde{x}$.

It is convenient to denote the vector of prices of all the goods at both locations by $\boldsymbol{p} \in \mathbb{R}^{2n}_{+}$ and to consider the following sets:

$$\begin{aligned}\mathcal{P}_1 &\equiv \left\{\boldsymbol{p} \in \mathbb{R}^{2n}_{+} :\; \textstyle\sum_{i\in\mathcal{I}} |p_{i_L} - p_{i_R}| \leq t\right\};\\ \mathcal{P}_2 &\equiv \left\{\boldsymbol{p} \in \mathbb{R}^{2n}_{+} :\; \textstyle\sum_{i\in\mathcal{I}} |p_{i_L} - p_{i_R}| > t\right\}.\end{aligned}$$

If there are consumers that travel to both extremes, the demand for a good depends on whether this good is cheaper at L or at R. Denoting by $\mathcal{I}_L$ and $\mathcal{I}_R$ the sets of goods that are strictly cheaper at L and R, respectively, we can write the expressions for the indifferent consumers as follows:[26]

$$\tilde{x}_L = 1 - \frac{1}{t}\sum_{i\in\mathcal{I}_R} (p_{i_L} - p_{i_R}) \tag{2}$$

[25]There are prices for which the location of the consumer that is indifferent between L and R is outside the interval $[0, 1]$. Precisely, this occurs when $|P_L - P_R| > t$. But $\sum_{i\in\mathcal{I}} |p_{i_L} - p_{i_R}| \leq t$ implies that $|P_L - P_R| \leq t$, which guarantees that $\tilde{x} \in [0, 1]$.

[26]Formally, $\mathcal{I}_L \equiv \{i \in \mathcal{I} :\; p_{i_L} < p_{i_R}\}$ and $\mathcal{I}_R \equiv \{i \in \mathcal{I} :\; p_{i_R} < p_{i_L}\}$.

and

$$\tilde{x}_R = \frac{1}{t} \sum_{i \in \mathcal{I}_L} (p_{i_R} - p_{i_L}) \,. \tag{3}$$

If $p_{i_L} = p_{i_R}$, consumers that travel to both extremes may either buy good i at L or at R. We can assume, for example, that half of the consumers buys good i at L and the other half buys it at R. Any other tie-breaking assumption would lead to the same results.

The demand for good i at L is:

$$q_{i_L} = \begin{cases} \tilde{x} & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_1 \\ \min\{\tilde{x}_R, 1\} & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_2 \;\wedge\; p_{i_L} < p_{i_R} \\ \frac{1}{2}\left(\min\{\tilde{x}_R, 1\} + \max\{0, \tilde{x}_L\}\right) & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_2 \;\wedge\; p_{i_L} = p_{i_R} \\ \max\{0, \tilde{x}_L\} & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_2 \;\wedge\; p_{i_L} > p_{i_R} \end{cases},$$

while the demand for the same good at $x = 1$ is $q_{i_R} = 1 - q_{i_L}$.

The marginal cost of producing one unit of each of the goods is assumed to be zero.[27] Therefore, profits coincide with sales revenues.

The profit that results from selling good i at L is:

$$\Pi_{i_L} = \begin{cases} p_{i_L}\left(\frac{1}{2} + \frac{P_R - P_L}{2t}\right) & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_1 \\ p_{i_L} \min\left\{\frac{P_R - P_{LR}}{t}, 1\right\} & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_2 \;\wedge\; p_{i_L} < p_{i_R} \\ \frac{p_{i_L}}{2}\left(\min\left\{\frac{P_R - P_{LR}}{t}, 1\right\} + \max\left\{0, 1 - \frac{P_L - P_{LR}}{t}\right\}\right) & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_2 \;\wedge\; p_{i_L} = p_{i_R} \\ p_{i_L} \max\left\{0, 1 - \frac{P_L - P_{LR}}{t}\right\} & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_2 \;\wedge\; p_{i_L} > p_{i_R} \end{cases}.$$

By symmetry, the profit that results from selling good i at the right extreme of the city is:

$$\Pi_{i_R} = \begin{cases} p_{i_R}\left(\frac{1}{2} + \frac{P_L - P_R}{2t}\right) & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_1 \\ p_{i_R} \min\left\{\frac{P_L - P_{LR}}{t}, 1\right\} & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_2 \;\wedge\; p_{i_R} < p_{i_L} \\ \frac{p_{i_R}}{2}\left(\min\left\{\frac{P_L - P_{LR}}{t}, 1\right\} + \max\left\{0, 1 - \frac{P_R - P_{LR}}{t}\right\}\right) & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_2 \;\wedge\; p_{i_R} = p_{i_L} \\ p_{i_R} \max\left\{0, 1 - \frac{P_R - P_{LR}}{t}\right\} & \text{if} \quad \boldsymbol{p} \in \mathcal{P}_2 \;\wedge\; p_{i_R} > p_{i_L} \end{cases}.$$

Before moving on to study the profit-maximizing behavior of firms, it is worth making a remark on the characteristics of the demand and profit functions.

Observe that, under two-stop shopping, the slope of the functions that give the indifferent consumers ($\tilde{x}_L$ and $\tilde{x}_R$) as a function of prices is $\frac{1}{t}$, whereas the slope for the indifferent

[27] The introduction of strictly positive marginal costs would add an unnecessary confounding factor.

consumer under one-stop shopping ($\tilde{x}$) is $\frac{1}{2t}$. Demand is, therefore, more sensitive to price under two-stop shopping than under one-stop shopping. Technically, this generates kinks in the demand functions, which translate into profit functions with kinks and that may not be quasi-concave.

To understand the origin of the different slopes, consider an indifferent consumer, $\tilde{x}$, who obtains a utility of $\tilde{u}$ from either L or R. Then, a consumer located at $\tilde{x}+\epsilon$ obtains a utility $\tilde{u}-t\epsilon$ from L and a utility $\tilde{u}+t\epsilon$ from R. It is necessary to decrease P_L by $2t\epsilon$ for a consumer at $\tilde{x}+\epsilon$ to become indifferent between L and R. This explains the slope $\frac{1}{2t}$ when there is one-stop shopping. Now consider a consumer, $\tilde{x}_R$, that is indifferent between LR and R, obtaining a utility of $\tilde{u}$ from either choice. A consumer at $\tilde{x}+\epsilon$ obtains the same utility, $\tilde{u}$, from LR, but a greater utility, $\tilde{u}+t\epsilon$, from R. Therefore, it is sufficient to decrease P_{LR} by $\frac{1}{t}$ for a consumer at $\tilde{x}+\epsilon$ to become indifferent between LR and R.

3.3 Modes of retail and price-setting behavior

On the supply side, we consider two different modes of retail: department store and shopping mall. A department store is a multiproduct firm that sells the n goods at the same location. For example, a department store at L sells goods $\{i_L\}_{i\in\mathcal{I}}$, seeking to maximize its profit, $\Pi_L=\sum_{i=1}^{n}\Pi_{i_L}$. A shopping mall is a group of single-product firms that sell each of the n goods at the same location. For example, a shopping mall at R is composed by n firms, each firm selling one good, i_R, with the objective of maximizing its individual profit, Π_{i_R}. We exclude the possibility of coordinated behavior among shops at a mall. Each shop chooses how much to charge for the product it sells, taking the remaining prices as given.[28]

3.3.1 Profit maximization by a department store

With prices that induce one-stop shopping, $\boldsymbol{p}\in\mathcal{P}_1$, the profit of a department store is:

$$\Pi_L = P_L\left(\frac{1}{2}+\frac{P_R-P_L}{2t}\right).$$

[28] In reality, shops located at a mall exhibit some coordination (Gould, Parshigian and Prendergast, 2005). Assuming the extreme cases of no coordination in price setting at a mall and perfect coordination in price setting at a department store results in a more clearcut comparison between the two modes of retail.

The bundle price, P_L, that maximizes profits in this domain is:

$$P_L = \frac{P_R}{2} + \frac{t}{2}. \tag{4}$$

It is always possible for the department store to set prices that add up to this P_L and induce one-stop shopping. For example, by setting $p_{i_L} = \frac{P_L}{P_R} p_{i_R}, \forall i \in \mathcal{I}$.

With prices that induce two-stop shopping, $\boldsymbol{p} \in \mathcal{P}_2$, the profit of the department store is given by:

$$\Pi_L = \frac{1}{t} \sum_{i \in \mathcal{I}_L} p_{i_L} \sum_{j \in \mathcal{I}_L} (p_{j_R} - p_{j_L}) + \sum_{i \in \mathcal{I}_R} p_{i_L} \left[1 - \frac{1}{t} \sum_{j \in \mathcal{I}_R} (p_{j_L} - p_{j_R}) \right]. \tag{5}$$

For the choice of any p_{i_L} such that $i \in \mathcal{I}_L$, the first-order condition is:

$$\frac{\partial \Pi_L}{\partial p_{i_L}} = 0 \;\Leftrightarrow\; \sum_{i \in \mathcal{I}_L} p_{i_L} = \frac{1}{2} \sum_{i \in \mathcal{I}_L} p_{i_R}. \tag{6}$$

While, for the choice of any p_{i_L} such that $i \in \mathcal{I}_R$, the first-order condition is:

$$\frac{\partial \Pi_L}{\partial p_{i_L}} = 0 \;\Leftrightarrow\; \sum_{i \in \mathcal{I}_R} p_{i_L} = \frac{1}{2} \sum_{i \in \mathcal{I}_R} p_{i_R} + \frac{t}{2}. \tag{7}$$

We need to check that these candidate profit-maximizing prices are inside the domain, i.e., that they induce two-stop shopping. Otherwise, there are no profit-maximizing prices in the interior of $\mathcal{P}_2$, which, in turn, means that the profit-maximizing prices are in $\mathcal{P}_1$.

The prices that satisfy the first-order conditions (6) and (7) are inside the domain of the objective function (5) if and only if:

$$\sum_{i \in \mathcal{I}_L} p_{i_R} - \sum_{i \in \mathcal{I}_R} p_{i_R} > t. \tag{8}$$

This is actually a necessary and sufficient condition for the department store to be interested in inducing two-stop shopping.

Lemma 1. *Let* $P_R - \min_{i \in \mathcal{I}} \{p_{i_R}\} \leq 2t$. *A department store located at L prefers to induce two-stop shopping with a given partition instead of one-stop shopping if and only if:* $\sum_{i \in \mathcal{I}_L} p_{i_R} - \sum_{i \in \mathcal{I}_R} p_{i_R} > t$.

Proof. See the Appendix. □

Adding (6) and (7), we obtain (4). This means that the department store charges the same price for the bundle of n goods regardless of whether it induces two-stop shopping or not.

Underlying this result is the fact that, while under one-stop shopping the demand of a department store located at L is linear with reservation price equal to P_R+t, under two-stop shopping the department store faces two linear demand functions whose reservation prices add up to P_R+t.[29] The bundle composed by the goods that are more expensive at L than at R, which is only demanded by the one-stop shoppers at L, has a reservation price given by $\sum_{i\in\mathcal{I}_R} p_{i_R}+t$. The bundle composed by the goods that are cheaper at L than at R is also demanded by the two-stop shoppers, with a reservation price given by $\sum_{i\in\mathcal{I}_L} p_{i_R}$.

Lemma 2. *Let $P_R-\min_{i\in\mathcal{I}}\{p_{i_R}\}\leq 2t$. A department store located at L sets the price of the bundle of n goods to: $P_L=\frac{P_R}{2}+\frac{t}{2}$.*

A department store that induces two-stop shopping always finds it optimal to set prices such that a single good is more expensive there than at the other shopping center (all the other goods are cheaper).

Lemma 3. *Let $P_R-\min_{i\in\mathcal{I}}\{p_{i_R}\}\leq 2t$. When a department store located at L induces two-stop shopping, it sets prices $\boldsymbol{p}_L$ so that the set $\mathcal{I}_R$ contains a single element, $j\in \text{argmin}_{i\in\mathcal{I}}\{p_{i_R}\}$. The only exception is when there is more than one good i with $p_{i_R}=0$. In this case, $\mathcal{I}_R$ can be any non-empty subset of those goods.*

Proof. See the Appendix. □

Given that the two-stop shoppers pay $\frac{1}{2}\sum_{i\in\mathcal{I}_L} p_{i_R}$, while the one-stop shoppers pay $\frac{1}{2}(P_R+t)$, it is not surprising that a department store that induces two-stop shopping always finds it optimal to choose the partition of goods that maximizes $\sum_{i\in\mathcal{I}_L} p_{i_R}$. In fact, besides being the partition that maximizes the price paid by the two-stop shoppers, it is also the partition that maximizes the mass of this segment. Having more two-stop shoppers is an additional advantage because an increase of this segment is associated with equal decreases of the masses of one-stop shoppers at L and at R (because we are keeping P_L fixed), and a two-stop shopper is more than half as profitable than a one-stop shopper (otherwise, two-stop shopping would not be preferred to one-stop shopping).

Our result is related to the well-known *loss leader* marketing strategy, in which a single product is sold at a low price to attract customers to the store. However, what we find is

[29] When a firm's demand is linear and costs are null, the profit-maximizing price is half of the reservation price. Therefore, in both cases, the profit-maximizing price for the bundle is $P_L=\frac{P_R}{2}+\frac{t}{2}$.

that the department store should follow a pricing strategy in which all products except one are *leaders*. It should set low prices for all goods except one, to attract customers, and a very high price for a single good, to cream-skim the consumers that live nearby (who are less willing to travel to the opposite extreme of the city).

The profit-maximizing behavior of a department store is summarized in Proposition 1.

Proposition 1. *Let $P_R \leq 2t$ and $j \in argmin_{i \in \mathcal{I}} \{p_{i_R}\}$. If $\sum_{i \neq j} p_{i_R} - p_{j_R} \leq t$, a department store located at L induces one-stop shopping, setting prices that are such that: $\sum_i |p_{i_L} - p_{i_R}| \leq t$ and $P_L = \frac{P_R}{2} + \frac{t}{2}$. If $\sum_{i \neq j} p_{i_R} - p_{j_R} > t$, the department store induces two-stop shopping, setting $\sum_{i \neq j} p_{i_L} = \frac{1}{2} \sum_{i \neq j} p_{i_R}$ (with $p_{i_L} < p_{i_R}, \forall i \neq j$) and $p_{j_L} = \frac{1}{2} p_{j_R} + \frac{t}{2}$.*

3.3.2 Profit maximization by the shops at the mall

In this subsection, we consider the profit-maximization problem of an individual shop located at the right extreme of the city.

To study the behavior of Π_{i_R} as a function of p_{i_R}, it is convenient to define a partition of the domain of p_{i_R} and consider separately the cases in which: (D_1) all consumers buy good i at R; (D_2) there is two-stop shopping with $i \in \mathcal{I}_R$; (D_3) there is one-stop shopping; (D_4) there is two-stop shopping with $i \in \mathcal{I}_L$; (D_5) no consumer buys good i at R:

$$\begin{cases} D_1 &= [0 \ , \ -t + p_{i_L} + s_{Ri}] \\ D_2 &= (-t + p_{i_L} + s_{Ri} \ , \ -t + p_{i_L} + s_{Li} + s_{Ri}) \\ D_3 &= [-t + p_{i_L} + s_{Li} + s_{Ri} \ , \ t + p_{i_L} - s_{Li} - s_{Ri}] \\ D_4 &= (t + p_{i_L} - s_{Li} - s_{Ri} \ , \ t + p_{i_L} - s_{Li}) \\ D_5 &= [t + p_{i_L} - s_{Li} \ , \ +\infty) \, , \end{cases}$$

where $s_{Li} = \sum_{j \in \mathcal{I}_L \setminus \{i\}} (p_{j_R} - p_{j_L})$ and $s_{Ri} = \sum_{j \in \mathcal{I}_R \setminus \{i\}} (p_{j_L} - p_{j_R})$. The above partition is valid as long as $s_{Li} + s_{Ri} \leq t$. Otherwise, D_3 becomes empty and the transition between D_2 and D_4 occurs at $p_{i_R} = p_{i_L}$.

Accordingly, the demand for good i_R, as a function of p_{i_R}, is:[30]

$$q_{i_R} = \begin{cases} 1, & p_{i_R} \in D_1 \\ \frac{1}{t}\sum_{j\in\mathcal{I}_R}\left(p_{j_L} - p_{j_R}\right), & p_{i_R} \in D_2 \\ \frac{1}{2} + \frac{1}{2t}P_L - \frac{1}{2t}P_R, & p_{i_R} \in D_3 \\ 1 - \frac{1}{t}\sum_{j\in\mathcal{I}_L}\left(p_{j_R} - p_{j_L}\right), & p_{i_R} \in D_4 \\ 0, & p_{i_R} \in D_5 \end{cases}. \tag{9}$$

If D_3 is not empty ($s_{Li} + s_{Ri} \leq t$), demand and profit are globally continuous functions of p_{i_R}. Otherwise ($s_{Li} + s_{Ri} > t$), they exhibit a downwards jump at the transition between D_2 and D_4 (i.e., at $p_{i_R} = p_{i_L}$), which is the point at which the shop loses the two-stop shoppers.

In this subsection, we will not provide a complete characterization of the price-setting behavior of the shops at a mall. We only report, for later use, the profit function in the relevant branches (D_2, D_3 and D_4) and the corresponding first-order conditions.

In D_2, the profit of the shop is:

$$\Pi_{i_R} = \frac{1}{t}p_{i_R}\sum_{j\in\mathcal{I}_R}\left(p_{j_L} - p_{j_R}\right),$$

which leads to the following first-order condition:

$$p_{i_R} = \sum_{j\in\mathcal{I}_R}\left(p_{j_L} - p_{j_R}\right) \;\Leftrightarrow\; p_{i_R} = \frac{1}{2}\sum_{j\in\mathcal{I}_R}p_{j_L} - \frac{1}{2}\sum_{j\in\mathcal{I}_R\setminus\{i\}}p_{j_R}. \tag{10}$$

In D_3, the profit of the shop is:

$$\Pi_{i_R} = p_{i_R}\left(\frac{1}{2} + \frac{P_L - P_R}{2t}\right).$$

The corresponding first-order condition is:

$$p_{i_R} = P_L - P_R + t \;\Leftrightarrow\; p_{i_R} = \frac{P_L}{2} - \frac{1}{2}\sum_{j\in\mathcal{I}\setminus\{i\}}p_{j_R} + \frac{t}{2}. \tag{11}$$

[30] The demand function is continuous and piecewise linear. Its derivative is initially zero (in D_1), then it is $-\frac{1}{t}$ (in D_2), changes to $-\frac{1}{2t}$ (in D_3), becomes $-\frac{1}{t}$ again (in D_4) and, finally, vanishes (in D_5). Accordingly, the profit function is concave in each branch. It starts at zero (for $p_{i_R} = 0$) and ends at zero (for $p_{i_R} \in D_5$).

Finally, in D_4, the profit of the shop is:

$$\Pi_{i_R} = \frac{p_{i_R}}{t}\left[t - \sum_{j\in\mathcal{I}_L}(p_{j_R} - p_{j_L})\right],$$

and the first-order condition is given by:

$$p_{i_R} = t - \sum_{j\in\mathcal{I}_L}(p_{j_R} - p_{j_L}) \;\Leftrightarrow\; p_{i_R} = \frac{t}{2} + \frac{1}{2}\sum_{j\in\mathcal{I}_L} p_{j_L} - \frac{1}{2}\sum_{j\in\mathcal{I}_L\setminus\{i\}} p_{j_R}. \tag{12}$$

4 Competitive scenarios

In the previous section, we have studied the profit-maximizing behavior of the department store and of an individual shop at a mall. In this section, we will characterize the equilibrium prices, demand and profits in three possible competitive scenarios:

– a department store at L and a shopping mall at R;
– two department stores, one at L and another at R;
– two shopping malls, one at L and another at R.

We will show that, when a department store competes against a shopping mall, existence and characteristics of pure strategy equilibrium crucially depend on the number of goods. If $n \leq 4$, there is a unique equilibrium in which all consumers make one-stop shopping. If $n > 4$, a one-stop shopping equilibrium does not exist because the department store would deviate and induce two-stop shopping by setting low prices for $n-1$ goods and a high price for a single good. Such a strategy allows the department store to profit the most from two-stop shoppers, who demand the cheap goods, while maintaining the surplus extracted from one-stop shoppers, who also buy the expensive good. If $7 \leq n \leq 11$, there is a unique equilibrium in which some consumers make two-stop shopping to buy $n-1$ goods at the department store and the other good at the shopping mall. If $n \leq 6$ or $n \geq 12$, a two-stop shopping equilibrium cannot exist. If $n \leq 6$, there are very few two-stop shoppers in the candidate equilibrium. Thus, the single shop at the mall that is setting a low price and capturing the two-stop shoppers prefers to set a higher price to profit more from one-stop shoppers, and this deviation from the candidate equilibrium eliminates the reason for two-stop shopping. If $n \geq 12$, the opposite occurs. There are so many two-stop shoppers that the shops at the mall which are setting high prices and selling only to one-stop shoppers prefer to undercut the price set by the department store, in order to capture the two-stop shoppers.

The analysis of the symmetric scenarios is more straightforward. Under competition between two department stores, the equilibrium is identical to that of the single-product model. The n goods are treated as a single bundled good. In the case of competition between two shopping malls, the equilibrium corresponds to a n times replicated equilibrium of the single-product model. Each pair of shops that sell the same good in opposite locations behaves as in the single-product duopoly model.

4.1 Competition between a department store and a shopping mall

We start by considering the case in which there is a department store located at L and a shopping mall located at R. The department store chooses the prices of the n goods with the objective of maximizing its total profit ($\sum_{i=1}^{n} \Pi_{i_L}$), while each shop at the mall seeks to maximize its individual profit (Π_{i_R}).

4.1.1 Equilibria with one-stop shopping

In an equilibrium with one-stop shopping, the first-order conditions for profit-maximization by the shops at the mall (11) imply that:

$$p_{i_R} = P_L - P_R + t \;\Rightarrow\; P_R = np_{i_R} = nP_L - nP_R + nt \;\Rightarrow\; P_R = \frac{n}{n+1}\left(P_L + t\right). \tag{13}$$

Combining this condition with the first-order condition for profit-maximization by the department store (4), we obtain the candidate equilibrium prices (see Figure 2):

$$\begin{cases} P_L = \frac{P_R}{2} + \frac{t}{2} \\ P_R = \frac{n}{n+1}\left(P_L + t\right) \end{cases} \Rightarrow \begin{cases} P_L = \frac{2n+1}{n+2}t \\ P_R = \frac{3n}{n+2}t \quad \text{with} \quad p_{i_R} = \frac{3}{n+2}t, \ \forall i \in \mathcal{I} \end{cases}. \tag{14}$$

Figure 2: Candidate equilibrium with one-stop shopping.

By Lemmata 1 and 3, we know that the department store prefers to deviate from this candidate equilibrium and set prices that induce two-stop shopping if and only if:

$$\sum_{i\in\mathcal{I}_L} p_{i_R} - \sum_{i\in\mathcal{I}_R} p_{i_R} > t \;\Leftrightarrow\; \frac{3(n-2)}{n+2}t > t \;\Leftrightarrow\; n > 4.$$

This means that the candidate (14) can only be an equilibrium for $n \leq 4$. As the number of goods increases, individual prices at R decrease while the total price at R increases. For $n > 4$, it becomes possible to form two bundles whose prices are sufficiently different at R for the department store to prefer two-stop shopping. More precisely, as the number of goods increases, good j becomes cheaper ($p_{j_R} = \frac{3}{n+2}t$) while the bundle composed by the remaining goods becomes more expensive ($\sum_{i\neq j} p_{i_R} = \frac{3n-3}{n+2}t$). At $n = 5$, the threshold in Proposition 1 is attained, and it becomes profitable for the department store to induce two-stop shopping in order to exploit the greater market power it enjoys in the segment of one-stop shoppers (consumers that are nearer) relatively to two-stop shoppers (consumers that are more distant).

To better understand why there is no equilibrium with one-stop shopping if $n \geq 5$, see the comparison (for $n = 5$) between the equilibrium candidate and the profit-maximizing deviation by the department store in Figure 3.

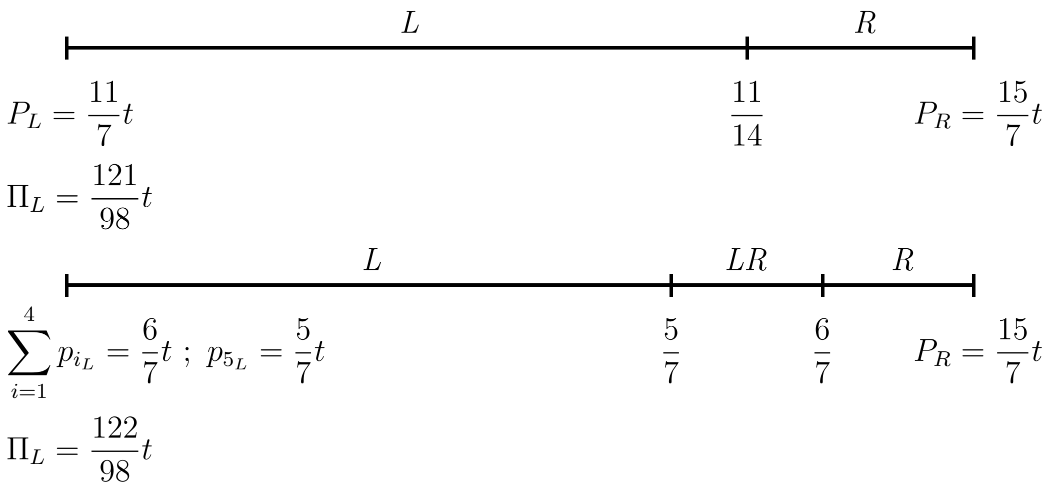

Figure 3: Comparison between the candidate one-stop shopping equilibrium (top) and the optimal deviation by the department store (bottom) when $n = 5$.

The shops at the mall do not deviate to a situation with two-stop shopping as long as the prices at the department store satisfy the following condition:[31]

$$\sum_{i=1}^{n} \left| p_{i_L} - \frac{3}{n+2}t \right| \leq \frac{n + 6\sqrt{2} - 7}{n+2}t < t.$$

Proposition 2. *In the case of competition between a department store and a shopping mall, there is an equilibrium with one-stop shopping if and only if* $n \leq 4$*. It is such that:*

(1) It is cheaper to buy the n *goods at the department store than at the shopping mall:*

$$\begin{cases} P_L = \sum_{i=1}^{n} p_{i_L} = \frac{2n+1}{n+2}t, \quad \text{with} \quad \sum_{i=1}^{n} \left| p_{i_L} - \frac{3}{n+2}t \right| \leq \frac{n+6\sqrt{2}-7}{n+2}t < t \\ P_R = \sum_{i=1}^{n} p_{i_R} = \frac{3n}{n+2}t, \quad \text{with} \quad p_{i_R} = \frac{3}{n+2}t, \ \forall i \in \mathcal{I} \end{cases}.$$

(2) The demand is greater at the department store:

$$\begin{cases} q_{i_L} = \frac{2n+1}{2n+4} \\ q_{i_R} = \frac{3}{2n+4} \end{cases}.$$

(3) The department store earns more profits than the shops at the mall taken together:

$$\begin{cases} \Pi_L = \sum_{i=1}^{n} \Pi_{i_L} = \frac{(2n+1)^2}{2(n+2)^2}t \\ \Pi_R = \sum_{i=1}^{n} \Pi_{i_R} = \frac{9n}{2(n+2)^2}t \end{cases}.$$

Proof. See the Appendix. □

The department store does not care about how much to charge for each individual good because all of its customers buy the entire bundle of goods. What matters for the department store is the price of the bundle.

The department store charges a lower price for the bundle of n goods, because, when compared with the shops at the mall, it has an additional incentive to set low prices. By decreasing the price of one good (for example, the price of books), the department store increases the demand for all the goods that are sold there (books, groceries, etc.). At the shopping mall, the bookshop, when choosing the price to set for books, only takes into account the effect on its own demand, ignoring the effect of the price of books on the demand for groceries and the remaining goods.

[31] This is shown in the proof of Proposition 2. See the Appendix.

As a result of setting lower prices, the department store captures more than half of the market. It does not capture the whole market because the customers that are closer to the shopping mall weigh the price advantage of the department store against the proximity advantage of the shopping mall. In equilibrium, the shopping mall retains the consumers that are sufficiently close.

Comparing the joint profit at each extreme of the city, we find that the department store earns more than the shops at the mall taken together.

4.1.2 Equilibria with two-stop shopping

In an equilibrium with two-stop shopping, by Lemma 3, the department store must be selling $n-1$ goods at a lower price than the shopping mall and a single good at a higher price.

Adding the first-order conditions for profit-maximization by the $n-1$ shops at the mall that sell goods in $\mathcal{I}_L$, given by (12), we obtain the following best-response function:

$$\sum_{i\in\mathcal{I}_L} p_{i_R} = (n-1)t - (n-1)\sum_{i\in\mathcal{I}_L}(p_{i_R} - p_{i_L}) \Leftrightarrow \sum_{i\in\mathcal{I}_L} p_{i_R} = \frac{n-1}{n}\left(t + \sum_{i\in\mathcal{I}_L} p_{i_L}\right).$$

Combining it with the first-order condition for profit-maximization by the department store (6), we obtain:

$$\begin{cases} \sum_{i\in\mathcal{I}_L} p_{i_R} = \frac{2n-2}{n+1}t, \quad \text{with} \quad p_{i_R} = \frac{2}{n+1}t, \forall i \in \mathcal{I}_L \\ \sum_{i\in\mathcal{I}_L} p_{i_L} = \frac{n-1}{n+1}t \end{cases}.$$

For the single shop at the mall that sells a good $i \in \mathcal{I}_R$, the first-order condition for profit-maximization is (10):

$$p_{i_R} = \frac{p_{i_L}}{2}.$$

While the corresponding first-order condition for profit-maximization by the department store is (7):

$$p_{i_L} = \frac{p_{i_R}}{2} + \frac{t}{2}.$$

Combining the two conditions, we obtain:

$$p_{i_R} = \frac{t}{3} \quad \text{and} \quad p_{i_L} = \frac{2t}{3}.$$

In spite of charging a higher price, the department store has a greater demand because many of its customers do not find it profitable to make two-stop shopping:

$$q_{i_R} = \frac{1}{3} \quad \text{and} \quad q_{i_L} = \frac{2}{3}.$$

The candidate equilibrium with two-stop shopping is represented in Figure 4.

Figure 4: Candidate equilibrium with two-stop shopping.

The department store is only interested in setting prices that induce two-stop shopping if condition (8) is satisfied. Substituting the expressions for the candidate equilibrium prices, this condition becomes:

$$\frac{2n-2}{n+1}t - \frac{t}{3} > t \;\Leftrightarrow\; n > 5.$$

For $n \leq 5$, the candidate equilibrium is upset because the department store prefers to set prices that induce one-stop shopping.[32]

We must also verify that the shop at the mall that is setting the low price does not deviate to a price that induces one-stop shopping. From, the first-order condition (11), we find the following candidate deviation (which induces one-stop shopping for $n \leq 8$):

$$p_{i_R} = \frac{P_L}{2} - \frac{1}{2} \sum_{j \in \mathcal{I} \setminus \{i\}} p_{j_R} + \frac{t}{2} = \frac{n-1}{2(n+1)}t + \frac{t}{3} - \frac{n-1}{n+1}t + \frac{t}{2} = \frac{n+4}{3(n+1)}t.$$

The corresponding profit is:

$$\Pi_{i_R} = p_{i_R}(1 - \tilde{x}) = \frac{(n+4)^2}{18(n+1)^2}t.$$

[32]Recall that in the candidate equilibrium with one-stop shopping, the department store preferred to deviate and induce two-stop shopping for $n \geq 5$. It is not contradictory that, for $n = 5$: the department store prefers to induce two-stop shopping when facing the one-stop shopping candidate equilibrium; and prefers to induce one-stop shopping when facing the two-stop shopping equilibrium. This behavior prevents, however, the existence of an equilibrium when $n = 5$.

It is higher than the candidate equilibrium profit when:

$$\frac{(n+4)^2}{18(n+1)^2}t > \frac{1}{9}t \;\Leftrightarrow\; (n+4)^2 > 2(n+1)^2 \;\Leftrightarrow\; n < 7.$$

This means that, in the candidate equilibrium: if $n \geq 7$, the shop at the mall that is setting the low price (and capturing the two-stop shoppers) is setting a globally optimal price; if $n \leq 6$, the shop would prefer to raise its price in a way that induces one-stop shopping.

In the candidate equilibrium with two-stop shopping, as the number of goods increases, the segment of two-stop shoppers becomes increasingly important while the segment of one-stop shoppers loses importance (see Figure 4). There is a threshold (for the number of goods) below which it is not profitable for the shop at the mall to be setting such a low price. For $n \leq 6$, it is preferable for the shop to increase its price substantially in order to extract more surplus from the one-stop shoppers, in spite of losing all the two-stop shoppers (and even some of the one-stop shoppers).

The profit function of this shop (with all the other prices fixed at their candidate equilibrium levels), for $n = 6$ and $n = 7$, is shown in Figure 5. It has a kink at the price below which there are consumers making two-stop shopping just to buy this good at the mall ($p_{i_R} \in D_2$) and above which there are no two-stop shoppers ($p_{i_R} \in D_3$).[33] If $n = 7$, the global maximum of the profit of the shop is in the interior of D_2, as in the candidate equilibrium. However, if $n = 6$, the local maximum in D_2 is not the global maximum, which means that the shop prefers to deviate from the candidate equilibrium and set a substantially higher price.

Finally, we need to verify that the shops at the mall that are setting higher prices than the department store do not deviate to a price that slightly undercuts the price set by the department store in order to capture the two-stop shoppers. In the Appendix, we show that they deviate from the candidate equilibrium when $n \geq 12$.

As illustrated in Figure 6, there is a discontinuity in the profit function of a shop at the mall at the price $p_{iR} = p_{iL}$, below which the shop at the mall captures the two-stop shoppers ($p_{iR} \in D_2$) and above which it loses all the two-stop shoppers ($p_{iR} \in D_4$). For $n = 10$, it does not compensate to deviate in order to capture the two-stop shoppers, while for $n = 12$ such a unilateral deviation from the candidate equilibrium would be profitable.

An equilibrium with two-stop shopping requires a balance between the profits of the shops

[33]Recall that: D_2 is the set of values of p_{i_R} for which the two-stop shoppers buy good i at R; D_3 is the set for which there are no two-stop shoppers; D_4 is the set for which the two-stop shoppers buy good i at L; and D_5 is the set for which no consumer buys good i at R.

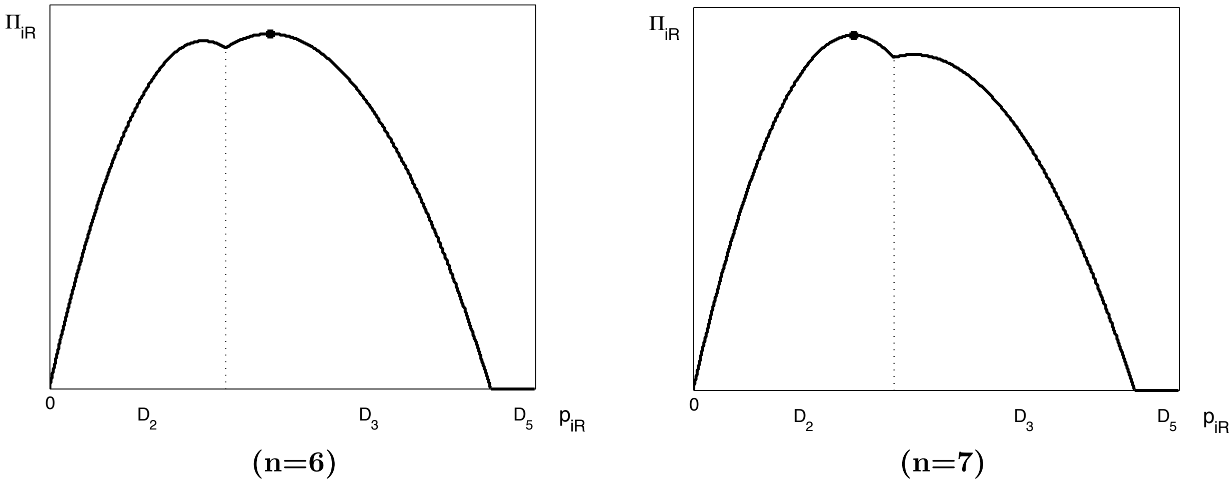

Figure 5: Profit function of the shop that sells the cheap good at the mall, given that the other stores are charging the prices of the candidate equilibrium with two-stop shopping.

at the mall that are setting high prices and the profits of the shop that is setting a low price, so that neither wishes to change its pricing strategy. When the number of goods is low ($n \leq 6$), the two-stop shopping segment is small, therefore, the shop that is setting a low price deviates. When the number of goods is high ($n \geq 12$), it is the one-stop shopping segment that is small. As a result, a shop that is setting a high price prefers to deviate and lower its price substantially, in order to capture all the two-stop shoppers. It is only when the number of goods is intermediate ($7 \leq n \leq 11$) that none of the shops deviates from the two-stop shopping equilibrium.

Proposition 3. *In the case of competition between a department store and a shopping mall, there is an equilibrium with two-stop shopping if and only if* $7 \leq n \leq 11$. *It is such that:*

(1) It is cheaper to buy $n-1$ *of the* n *goods at the department store than at the shopping mall:* $\#\mathcal{I}_L = n-1$ *and* $\#\mathcal{I}_R = 1$.

(2) The prices of the goods that are cheaper at the department store, $i \in \mathcal{I}_L$, *are such that:*

$$\begin{cases} \sum_{i \in \mathcal{I}_L} p_{i_L} = \frac{n-1}{n+1} t, \quad \text{with} \quad p_{i_L} \leq \frac{12}{(n+1)^2} t, \forall i \in \mathcal{I}_L \\ \sum_{i \in \mathcal{I}_L} p_{i_R} = \frac{2(n-1)}{n+1} t, \quad \text{with} \quad p_{i_R} = \frac{2}{n+1} t, \forall i \in \mathcal{I}_L \end{cases},$$

and the corresponding demands are:

$$q_{i_L} = \frac{n-1}{n+1} \quad \text{and} \quad q_{i_R} = \frac{2}{n+1}.$$

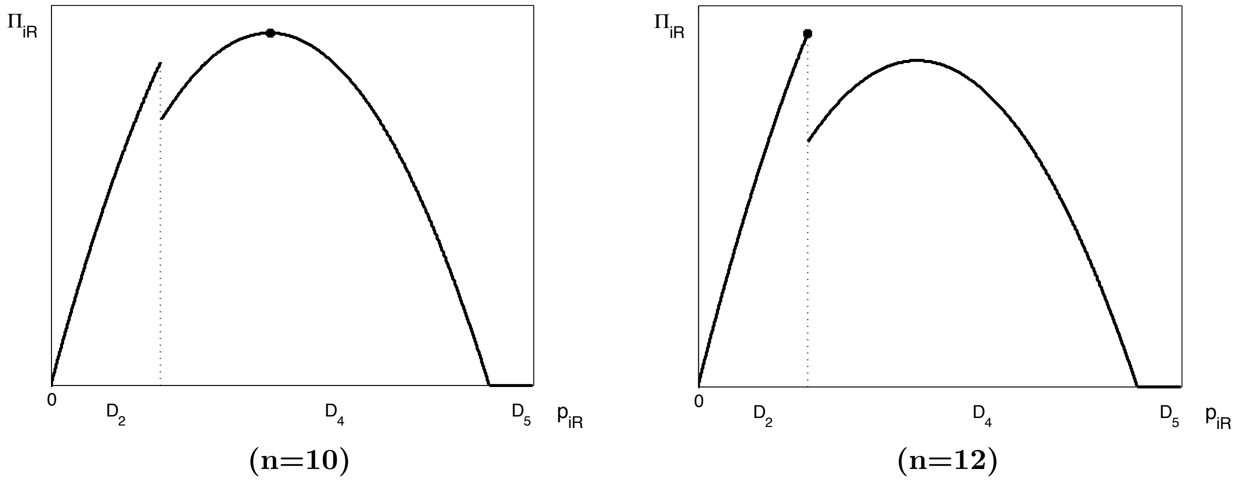

Figure 6: Profit function of a shop that sells an expensive good at the mall, given that the other stores are charging the prices of the candidate equilibrium with two-stop shopping.

(3) The prices of the only good that is cheaper at the shopping mall, $i \in \mathcal{I}_R$, are:

$$p_{i_L} = \frac{2}{3}t \quad \textit{and} \quad p_{i_R} = \frac{1}{3}t,$$

and the demands are:

$$q_{i_L} = \frac{2}{3} \quad \textit{and} \quad q_{i_R} = \frac{1}{3}.$$

(4) The department store earns more profits than the shops at the mall taken together:

$$\begin{cases} \Pi_L = \frac{(n-1)^2}{(n+1)^2}t + \frac{4t}{9} \\ \Pi_R = \frac{4(n-1)}{(n+1)^2}t + \frac{t}{9} \end{cases}.$$

Curiously, independently of the number of goods, exactly two thirds of the consumers buy all the products at the department store. For the remaining, which are located at $x \in \left(\frac{2}{3}, 1\right]$, the cost of visiting the shopping mall is smaller than $\frac{t}{3}$. However, the difference in the price of good $i \in \mathcal{I}_R$ is: $p_{i_L} - p_{i_R} = \frac{t}{3}$. Thus, for all these consumers, it is worthwhile to buy the good $i \in \mathcal{I}_R$ at the shopping mall. As the number of goods increases, there are more consumers willing to make their purchases at both extremes of the city.

When the number of goods is low ($n \leq 6$), the shop that sells the cheap good at the mall is not capturing (in the candidate equilibrium) much more consumers than the shops that sell the expensive goods (see Figure 4). It prefers to deviate and set a higher price, which induces one-stop shopping. As the number of goods increases, to sell a cheap good becomes more profitable, because the two-stop shopping segment, $x \in \left(\frac{2}{3}, \frac{n-1}{n+1}\right)$, increases.

As a result, when the number of goods is high ($n \geq 12$), the shops that sell the expensive goods at the mall prefer to deviate and set a lower price, to capture the customers that make two-stop shopping.

There only exists equilibrium with one-stop shopping when $n \leq 4$ and with two-stop shopping when $7 \leq n \leq 11$. Therefore, there is no equilibrium (in pure strategies) when the number of goods is $5 \leq n \leq 6$ or $n \geq 12$.

4.2 Competition between two department stores

Now, we consider the case in which there are two department stores, one at each extreme of the city. Each department store chooses the price to charge for each of the n goods, with the objective of maximizing its profit, taking as given the prices set by its competitor.

Proposition 4. *In the case of competition between two department stores:*

(1) The price of the bundle is equal to the transportation cost parameter:

$$P_L = P_R = t, \quad \textit{with} \quad \sum_{i=1}^{n} |p_{i_L} - p_{i_R}| \leq t.$$

(2) Consumers make all their purchases at the closest department store:

$$q_{i_L} = q_{i_R} = \frac{1}{2}.$$

(3) The resulting profits are also independent of the number of goods:

$$\Pi_L = \Pi_R = \frac{t}{2}.$$

Proof. See the Appendix. □

In equilibrium, the department stores charge the same price for the bundle of n goods (there is, once more, some indeterminacy regarding the split of the bill between the goods). No consumer is willing to travel to both extremes of the city. All consumers buy the n goods at the closest department store.

What may be surprising is that the margin (difference between price and marginal cost) with n goods is the same as in the standard Hotelling model, in which a single good is sold. The reason why the margin is not greater with n goods is related to the fact that the reservation utility of the customer is not relevant for the pricing decisions of the firms (as long as it is high enough, as it is typically assumed). With one-stop shopping, the n goods are equivalent to a single bundled good. Therefore, even if customers attribute a higher utility to the n goods than to a single good, the margin remains constant and equal to the transportation cost parameter.[34]

4.3 Competition between two shopping malls

In the case of competition between two shopping malls (one at each extreme of the city), the shops that sell the same good at different locations are direct competitors. However, their demand also depends on the prices of the other goods. This interdependence across shops selling different goods exists because, when deciding where to buy each good, consumers take into account not only the price but also the transportation costs that they have to bear. A shop benefits from having low prices for the goods sold at the same location (since this attracts customers to its location); and high prices for the goods sold at the other location (since this repels customers from the other location). But since this externality has no influence on the pricing decisions of the shops, the equilibrium of the model replicates that of the single-product model.

Proposition 5. *In the case of competition between two shopping malls:*

(1) The price of each good is equal to the transportation cost parameter:

$$p_{i_L} = p_{i_R} = t, \ \forall i \in \mathcal{I}.$$

(2) Consumers make all their purchases at the closest shopping mall:

$$q_{i_L} = q_{i_R} = \frac{1}{2}, \ \forall i \in \mathcal{I}.$$

(3) The profit of each firm is also independent of the number of goods:

$$\Pi_{i_L} = \Pi_{i_R} = \frac{t}{2}, \ \forall i \in \mathcal{I}.$$

Proof. See the Appendix. □

[34] The same occurs in the case of Bertrand competition with homogeneous products. Independently of the number of products that firms sell, their equilibrium margin is always null.

Notice that the joint profit of the n shops located at a shopping mall is greater than the profits obtained in any of the alternative scenarios that we have considered.

5 Endogenous modes of retail

Until now, we have assumed that the organization of each shopping center was exogenous. In this section, we analyze whether one should expect department stores or shopping malls to appear endogenously.

More precisely, we consider a two-stage game in which two land owners (one at L and another at R) start by simultaneously auctioning their land for the construction of department stores or shopping malls.[35] Price competition as studied in the previous section takes place afterwards.

We assume that land is allocated to the form of retail that is more profitable, because managers of more profitable projects are willing to bid higher. Therefore, what we need is to compare the profit of a department store with the joint profit of the shops that constitute a shopping mall.[36]

For $n \leq 4$, the profits of the shopping centers in each competitive scenario are shown in Table 1.

	Department Store	**Shopping Mall**
Department Store	$\left(\frac{1}{2}t \ , \ \frac{1}{2}t\right)$	$\left(\frac{(2n+1)^2}{2(n+2)^2}t \ , \ \frac{9n}{2(n+2)^2}t\right)$
Shopping Mall	$\left(\frac{9n}{2(n+2)^2}t \ , \ \frac{(2n+1)^2}{2(n+2)^2}t\right)$	$\left(\frac{n}{2}t \ , \ \frac{n}{2}t\right)$

Table 1: Profits of the competing shopping centers if $n \leq 4$.

Proposition 6. *If $n \leq 4$, a shopping mall is more profitable than a department store, regardless of whether its competitor is a department store or a shopping mall.*

[35]What we designate by shopping mall is simply a co-located group of independent shops. In reality, such a group may constitute a shopping street instead of a mall. In that case, there would be many independent shops bidding for fractions of land.

[36]As in the previous section, we do not consider the possible combination of a smaller department store (selling a subset of the existing goods, $\mathcal{I}^{ds} \subset \mathcal{I}$) with several independent shops (each selling one of the remaining goods, $i \in \mathcal{I} \backslash \mathcal{I}^{ds}$). This could change the outcome of the game, as suggested by the contributions of Salant, Switzer and Reynolds (1983) and Kamien and Zang (1990).

We expect, therefore, that the land is allocated to the development of shopping malls or shopping streets and not to the development of department stores.

For $7 \leq n \leq 11$, the payoff matrix is given in Table 2.

	Department Store	**Shopping Mall**
Department Store	$\left(\frac{1}{2}t \ , \ \frac{1}{2}t\right)$	$\left(\frac{(n-1)^2}{(n+1)^2}t + \frac{4}{9}t \ , \ \frac{4(n-1)}{(n+1)^2}t + \frac{1}{9}t\right)$
Shopping Mall	$\left(\frac{4(n-1)}{(n+1)^2}t + \frac{1}{9}t \ , \ \frac{(n-1)^2}{(n+1)^2}t + \frac{4}{9}t\right)$	$\left(\frac{n}{2}t \ , \ \frac{n}{2}t\right)$

Table 2: Profits of the competing shopping centers if $7 \leq n \leq 11$.

Proposition 7. *If $7 \leq n \leq 11$, a shopping mall is more profitable than a department store when competing against a shopping mall, while a department store is more profitable than a shopping mall when competing against a department store.*

If the land at L and at R is auctioned simultaneously, there are two equilibrium outcomes. If department stores are expected to form, department stores will bid higher than shopping malls; if the opening of shopping malls is expected, shopping malls will bid higher.

Of course, the retailers will prefer to coordinate on the equilibrium with two shopping malls. Therefore, we should expect to have shopping malls at both extremes of the city. This would also be the case if the allocation of land at L and R were sequential instead of simultaneous.

Our results are robust to a variation of the first-stage game in which there is a single owner of land or, equivalently, there is collusion between the two owners. It is clear that two shopping malls emerge in equilibrium.

We have implicitly assumed that the regulatory environment does not allow a retailer to own both shopping centers. If that was possible, a single retailer would be the highest bidder for the two pieces of land - as it would, then, enjoy monopoly profits.[37]

[37] In this model, since total demand is inelastic, a monopolist would be able to raise prices up to the reservation price of the consumer located in the middle of the city.

6 Welfare analysis

6.1 Total surplus

In this model, the total demand is assumed to be perfectly inelastic (each consumer buys one unit of each good that is available in the market, independently of the prices of the goods). Therefore, a change in prices only leads to a transfer of surplus between consumers and firms. The total surplus remains constant. In this context, the maximization of total surplus is equivalent to the minimization of total transportation costs incurred by consumers.

Total transportation costs are minimized when each consumer shops at the closest shopping center. This occurs in the case of competition between two department stores and in the case of competition between two shopping malls. When there is a department store competing with a shopping mall, the indifferent consumer is no longer located at the middle of the city. If $n \leq 4$, there are more consumers shopping at the department store than at the shopping mall ($\tilde{x} > \frac{1}{2}$). If $7 \leq n \leq 11$, there are consumers who shop at both extremes of the city. Total transportation costs are even higher in such a situation. Thus, the existence of different modes of retail diminishes total surplus.

The assumption of inelastic demand seems to be crucial for the conclusion that total surplus is higher when there are two shopping malls than when there is one shopping mall and one department store. With elastic demand, since production costs are null, the social optimum would consist in having null prices (with inelastic demand, symmetric prices suffice). It seems, therefore, that in the presence of a significant level of demand elasticity, the deadweight loss due to high prices and low demand would render the scenario with two shopping malls the least desirable from a social point of view.

Given the limitations of our model, it seems preferable to focus the attention on consumer surplus rather than total surplus as a guide for policy design.

6.2 Consumer surplus

Let CS_{DD}, CS_{MM} and CS_{DM} denote the consumer surplus in each of the three scenarios: (DD) competition between two department stores; (MM) competition between two shopping malls; and (DM) competition between a department store and a shopping mall.

If the shopping centers are organized in the same way, they charge the same price for the bundle of goods. As a result, the indifferent consumer is located at the middle of the city and the total transportation cost is minimized. We have:

$$CS_{DD} = \tilde{x}\,(V-t) + \tilde{x}\,(V-t) - t\int_0^{\tilde{x}} x\ dx - t\int_{\tilde{x}}^1 (1-x)\ dx = V - \frac{5}{4}t;$$
$$CS_{MM} = \tilde{x}\,(V-nt) + \tilde{x}\,(V-nt) - t\int_0^{\tilde{x}} x\ dx - t\int_{\tilde{x}}^1 (1-x)\ dx = V - \frac{4n+1}{4}t.$$

When a department store competes with a shopping mall and $n \leq 4$ (equilibrium with one-stop shopping), consumer surplus is:

$$CS_{DM} = \int_0^{\tilde{x}} (V - P_L - tx)\ dx + \int_{\tilde{x}}^1 [V - P_R - t(1-x)]\ dx = V - \frac{10n^2+28n+7}{4(n+2)^2}t.$$

When $7 \leq n \leq 11$ (equilibrium with two-stop shopping), it is given by:

$$\begin{aligned} CS_{DM} &= \int_0^{\tilde{x}_L} (V - P_L - tx)\ dx + \int_{\tilde{x}_L}^{\tilde{x}_R} (V - P_{LR} - t)\ dx + \int_{\tilde{x}_R}^1 [V - P_R - t(1-x)]\ dx \\ &= V - \frac{19n^2+20n-17}{9(n+1)^2}t. \end{aligned}$$

For consumers, competition between department stores is the most favorable scenario. Prices are lower than in the other scenarios, and transportation costs are minimized. It is not so straightforward to compare the case of competition between two shopping malls (lower transportation costs) with the case of competition between a shopping mall and a department store (lower prices). We find that the price effect dominates. Consumers prefer competition between a shopping mall and a department store rather than competition between two shopping malls.

Proposition 8. *Comparing the consumer' surplus in the three competitive scenarios, for* $2 \leq n \leq 4$ *and for* $7 \leq n \leq 11$*, we obtain:*

$$CS_{DD} > CS_{DM} > CS_{MM}.$$

Proof. Straightforward from the above expressions for CS_{DD}, CS_{DM} and CS_{MM}. □

It is somewhat surprising that the lower is the number of independent stores in the market, the higher is the consumer surplus. This result contradicts the typical intuition, according to which as the number of firms in the market increases, competition becomes stronger, leading to lower prices. In our model, this is not the case, since the price for the bundle of goods is cheaper when there are only two department stores.

7 Conclusions

We have developed a multiproduct version of the model of Hotelling (1929) to study competition between shopping centers that can be organized as department stores or as shopping malls. In particular, we analyzed how the modes of retail affect prices, market shares and profits, and which retail structure is more likely to emerge endogenously.

Relatively to the existing theoretical literature, our focus was on relaxing the one-stop shopping assumption.[38] For the model to remain tractable, we have maintained the hypotheses of the model of Hotelling (1929): inelasticity of total demand, linearity of transportation costs and uniform distribution of consumers. In addition, we have assumed that the two competing shopping centers are exogenously located at opposite extremes of the city and that they offer the same exogenous product line. Our stylized model has, therefore, limitations that advise against taking our results literally.

A department store competes more aggressively than a shopping mall, because it takes into account that a decrease of the price of one good increases the demand for all its goods. In contrast, a shop at a mall only takes into account its individual demand when setting the price of its good.

As a result, when a department store competes with a shopping mall, the bundle of goods is cheaper at the department store than at the shopping mall. The greater demand more than compensates the lower price, hence, the department store obtains higher profits than the shops at the mall taken together. In spite of having higher profits, the department store has incentives to separate itself into a shopping mall. In fact, the competitive scenario that is expected to emerge endogenously is competition between two shopping malls.

When the mode of retail is the same in the two shopping centers, our results are identical to those of the single-product model. In the case of competition between department stores, multiple goods are treated as a single bundled good. In the case of competition between shopping malls, multiple goods originate replicas of the single-product model. In these cases, no consumer finds it worthwhile to shop at both shopping centers.

It is when a department store competes with a shopping mall that our results are more interesting. If the number of goods is low, there is one-stop shopping in equilibrium, but, if the number of goods is moderately high, some consumers travel to both extremes of the city to buy each product where it is cheaper. In this case, the department store sells all

[38] The empirical contributions of Stassen, Mittelstaedt and Mittelstaedt (1999), Schiraldi, Seiler and Smith (2011) and Figurelli (2013) suggest that assuming one-stop shopping would be too restrictive.

goods except one at a low price, to attract and profit the most from two-stop shoppers, and a single good at a high price, to cream-skim the customers who live nearer to the store.

It would be of interest to investigate whether this pricing strategy remains optimal without the assumption that total demand is inelastic. With elastic demand, concentrating the upward distortion on a single good and distributing the downward distortions among all the remaining goods may not be the most profitable way to induce two-stop shopping. The demand for the single expensive good could become too reduced.

If the number of goods is moderately low or very high, there is no equilibrium in pure strategies for the case of competition between a department store and a shopping mall. Therefore, a natural extension of this work would be to allow mixed strategies. The shops would set prices simultaneously and irreversibly, and then consumers would observe the prices and decide their purchases. An alternative would be to consider that consumers would have to travel to a shopping center to observe the prices that were actually set, and then decide whether to travel to the other shopping center or not. We leave this extension for future work.

The dependence of results on the number of goods is one of the most peculiar aspects of our model. In order to have some intuition for the impact of the number of goods, we may view the shopping mall as becoming less coordinated when the number of goods increases. This, given the discontinuities and failures of quasi-concavity of the profit functions of the department store and of the shops at the mall, originates the qualitative changes on the existence of one-stop shopping or two-stop shopping equilibria.

In reality, the number of goods is an endogenous variable. Unfortunately, our model is not appropriate to account for this endogeneity. In the case of competition between shopping malls, profits are proportional to the number of goods. Therefore, the shopping mall would like the number of goods to be as high as possible. However, increasing the number of goods indefinitely strains the assumption that consumers are willing to buy one unit of each good.

We believe that our conclusions show that reducing the research on multiproduct competition to the case of two goods may be restrictive. In the specific case of our stylized model, more than two goods are necessary to generate two-stop shopping. Moreover, in a model with only two goods, the strategy of setting a high price for a single good and low prices for the remaining goods would be undistinguishable from a strategy of selling a single good at a low price and the remaining at a high price.

8 Appendix

Proof of Lemma 1.

($\Rightarrow$) Let us start by showing that if the department store prefers to induce two-stop shopping, then $\sum_{i\in\mathcal{I}_L} p_{i_R} - \sum_{i\in\mathcal{I}_R} p_{i_R} > t$.

Suppose, by way of contradiction, that $\sum_{i\in\mathcal{I}_L} p_{i_R} - \sum_{i\in\mathcal{I}_R} p_{i_R} \leq t$. In this case, to induce two-stop shopping, the department store has to sell the bundle of goods $\mathcal{I}_L$ at a lower price than the bundle $\mathcal{I}_R$:

$$\begin{cases} \sum_{i\in\mathcal{I}_L} p_{i_R} - \sum_{i\in\mathcal{I}_R} p_{i_R} \leq t \\ \sum_{i\in\mathcal{I}_L} (p_{i_R} - p_{i_L}) + \sum_{i\in\mathcal{I}_R} (p_{i_L} - p_{i_R}) > t \end{cases} \Rightarrow \sum_{i\in\mathcal{I}_L} p_{i_L} < \sum_{i\in\mathcal{I}_R} p_{i_L}.$$

From the expressions that yield the indifferent consumers, (1)-(3), for any given P_L (the same with one-stop shopping and two stop-shopping), the indifferent consumer with one-stop shopping is at the midpoint between the indifferent consumers with two-stop shopping: $\tilde{x}_R - \tilde{x} = \tilde{x} - \tilde{x}_L$.

This implies that, for any given P_L, the one-stop shopping scenario is more profitable than the two-stop shopping scenario:

$$\begin{aligned} \sum_{i\in\mathcal{I}_L} p_{i_L} < \sum_{i\in\mathcal{I}_R} p_{i_L} \quad &\Leftrightarrow \quad (\tilde{x}_R - \tilde{x}) \sum_{i\in\mathcal{I}_L} p_{i_L} < (\tilde{x} - \tilde{x}_L) \sum_{i\in\mathcal{I}_R} p_{i_L} \\ &\Leftrightarrow \quad \tilde{x}_R \sum_{i\in\mathcal{I}_L} p_{i_L} + \tilde{x}_L \sum_{i\in\mathcal{I}_R} p_{i_L} < \tilde{x} P_L. \end{aligned} \tag{15}$$

This contradicts our hypothesis. Therefore, $\sum_{i\in\mathcal{I}_L} p_{i_R} - \sum_{i\in\mathcal{I}_R} p_{i_R} > t$ is a necessary condition for the department store to prefer two-stop shopping.

($\Leftarrow$) Let us now show that if $\sum_{i\in\mathcal{I}_L} p_{i_R} - \sum_{i\in\mathcal{I}_R} p_{i_R} > t$, then the department store prefers to induce two-stop shopping.

We start by showing that, under the current hypotheses and with the p_{i_L} given by the first-order conditions (6) and (7), we have: (i) $\tilde{x}_R \in [0,1]$; (ii) $\tilde{x}_L \in [0,1]$; (iii) $\boldsymbol{p} \in \mathcal{P}_2$; (iv) $\sum_{i\in\mathcal{I}_L} p_{i_L} < \sum_{i\in\mathcal{I}_L} p_{i_R}$; and (v) $\sum_{i\in\mathcal{I}_R} p_{i_L} > \sum_{i\in\mathcal{I}_R} p_{i_R}$.

Conditions (i) and (ii) ensure that the indifferent consumers are inside the interval $[0,1]$, and condition (iii) ensures that $\tilde{x}_L < \tilde{x}_R$. This implies that the demand for goods in $\mathcal{I}_L$ is $\tilde{x}_R$ and the demand for goods in $\mathcal{I}_R$ is $\tilde{x}_L$. From conditions (iv) and (v), the individual prices are not incompatible with the partitions. More precisely, there exist prices $\boldsymbol{p}_L$ that induce the given partitions and satisfy the first-order conditions (6) and (7).

Using (6) and (7) to replace the p_{i_L} by expressions that involve only the p_{i_R}, we obtain:

(i) $\tilde{x}_R \in [0,1] \;\Leftrightarrow\; \sum_{i\in\mathcal{I}_L} p_{i_R} \leq 2t$.

(ii) $\tilde{x}_L \in [0,1] \;\Leftrightarrow\; \sum_{i\in\mathcal{I}_R} p_{i_R} \leq t$.

(iii) $\sum_{i\in\mathcal{I}_L} (p_{i_R} - p_{i_L}) + \sum_{i\in\mathcal{I}_R} (p_{i_L} - p_{i_R}) > t \;\Leftrightarrow\; \sum_{i\in\mathcal{I}_L} p_{i_R} - \sum_{i\in\mathcal{I}_R} p_{i_R} > t$.

(iv) $\sum_{i\in\mathcal{I}_L} p_{i_L} < \sum_{i\in\mathcal{I}_L} p_{i_R} \;\Leftrightarrow\; \sum_{i\in\mathcal{I}_L} p_{i_R} > 0$.

(v) $\sum_{i\in\mathcal{I}_R} p_{i_L} > \sum_{i\in\mathcal{I}_R} p_{i_R} \;\Leftrightarrow\; \sum_{i\in\mathcal{I}_R} p_{i_R} < t$.

Observe that: (i) and (iii) are true by hypothesis; (iv) is a consequence of (iii); (ii) and (v) are a consequence of (i) and (iii).

Since all these conditions are satisfied, the first-order conditions (6) and (7) yield the maximum profit of the department store with two-stop shopping:

$$\Pi_L^{2ss} = \tilde{x}_R \sum_{i\in\mathcal{I}_L} p_{i_L} + \tilde{x}_L \sum_{i\in\mathcal{I}_R} p_{i_L} = \tilde{x}_R \left(\frac{1}{2} \sum_{i\in\mathcal{I}_L} p_{i_R} \right) + \tilde{x}_L \left(\frac{1}{2} \sum_{i\in\mathcal{I}_R} p_{i_R} + \frac{t}{2} \right).$$

On the other hand, applying the first-order condition (4), we obtain the maximum attainable profit with one-stop shopping:

$$\Pi_L^{1ss} = \tilde{x} P_L = \tilde{x} \left(\frac{1}{2} \sum_{i\in\mathcal{I}_L} p_{i_R} + \frac{1}{2} \sum_{i\in\mathcal{I}_R} p_{i_R} + \frac{t}{2} \right).$$

Subtracting the maximum profit with one-stop shopping from the maximum profit with two-stop shopping, we obtain:

$$\Pi_L^{2ss} - \Pi_L^{1ss} = (\tilde{x}_R - \tilde{x}) \left(\frac{1}{2} \sum_{i\in\mathcal{I}_L} p_{i_R} \right) - (\tilde{x} - \tilde{x}_L) \left(\frac{1}{2} \sum_{i\in\mathcal{I}_R} p_{i_R} + \frac{t}{2} \right).$$

From conditions (4), (6) and (7), the optimal price of the bundle is $P_L = \frac{1}{2}(P_R + t)$, regardless of whether the department store induces one-stop shopping or two-stop shopping. Therefore (from the expressions that give the indifferent consumers): $\tilde{x}_R - \tilde{x} = \tilde{x} - \tilde{x}_L$.

This allows us to conclude that:

$$\Pi_L^{2ss} - \Pi_L^{1ss} = (\tilde{x}_R - \tilde{x}) \left(\frac{1}{2} \sum_{i\in\mathcal{I}_L} p_{i_R} - \frac{1}{2} \sum_{i\in\mathcal{I}_R} p_{i_R} - \frac{t}{2} \right).$$

Since $\tilde{x}_R - \tilde{x} > 0$:

$$\Pi_L^{2ss} > \Pi_L^{1ss} \;\Leftrightarrow\; \sum_{i \in \mathcal{I}_L} p_{i_R} - \sum_{i \in \mathcal{I}_R} p_{i_R} - t > 0. \qquad \square$$

Proof of Lemma 3.

When there is two-stop shopping, the profit of the department store is given by:

$$\begin{aligned}\Pi_L &= \tilde{x}_R \sum_{i \in \mathcal{I}_L} p_{i_L} + \tilde{x}_L \sum_{i \in \mathcal{I}_R} p_{i_L} \\ &= \frac{1}{t}\left(\sum_{i \in \mathcal{I}_L} p_{i_R} - \sum_{i \in \mathcal{I}_L} p_{i_L}\right) \sum_{i \in \mathcal{I}_L} p_{i_L} + \left[1 - \frac{1}{t}\left(\sum_{i \in \mathcal{I}_R} p_{i_L} - \sum_{i \in \mathcal{I}_R} p_{i_R}\right)\right] \sum_{i \in \mathcal{I}_R} p_{i_L}.\end{aligned}$$

For simplicity of exposition, denote: $z_L = \sum_{i \in \mathcal{I}_L} p_{i_L}$, $z_R = \sum_{i \in \mathcal{I}_R} p_{i_L}$, $y_L = \sum_{i \in \mathcal{I}_L} p_{i_R}$ and $y_R = \sum_{i \in \mathcal{I}_R} p_{i_R}$. Using this notation, we have:

$$\Pi_L = \frac{1}{t}(y_L - z_L)z_L + \left[1 - \frac{1}{t}(z_R - y_R)\right] z_R.$$

Observe, from the expression above, that the choice of prices $\boldsymbol{p}_L$ by the department store impacts its profit through: (i) the partition, $\{\mathcal{I}_L, \mathcal{I}_R\}$, which determines how P_R is split between y_L and y_R; (ii) the price charged for the bundle $\mathcal{I}_L$, denoted z_L; and (iii) the price charged for the bundle $\mathcal{I}_R$, denoted z_R.

We will now show that the department store should induce the partition that minimizes the value of y_R.

Consider a given partition, $\{\mathcal{I}_L, \mathcal{I}_R\}$. From Lemma 1: if the partition implies that $y_L \leq t + y_R$, one-stop shopping is preferred to two-stop shopping (with this partition); if the partition implies that $y_L > t + y_R$, two-stop shopping (with this partition) is preferred to one-stop shopping.

We are working under the hypothesis that the department store prefers to induce two-stop shopping, therefore, we can reduce our search for an optimal partition among those that imply that $y_L > t + y_R$. Equivalently, $P_R - y_R > t + y_R$.

Since, by hypothesis, $P_R \leq 2t$ and $y_L > t + y_R$, conditions (i)-(v) in the proof of Lemma 1 hold. Therefore, the maximum profit under two stop-shopping for a given partition results from the application of the first-order conditions, (6) and (7), which yield $z_L = \frac{1}{2}y_L$ and

$z_R = \frac{1}{2}(t + y_R)$:

$$\begin{aligned}\Pi_L &= \frac{1}{4t}y_L^2 + \left[1 - \frac{1}{2t}(t - y_R)\right]\frac{1}{2}(t + y_R)\\ &= \frac{1}{4t}(P_R - y_R)^2 + \frac{1}{4t}(t + y_R)^2.\end{aligned}$$

The value of P_R is not affected by the partition, but y_R is. Thus, the resulting partition impacts the attainable profit through the value of y_R that results. When choosing the price vector $\boldsymbol{p}_L$, besides satisfying the first-order conditions, the department store wishes to induce a profit-maximizing value of y_R:

$$\max_{y_R} \left\{\frac{1}{4t}(P_R - y_R)^2 + \frac{1}{4t}(t + y_R)^2\right\}.$$

Under the restriction $P_R - y_R > t + y_R$, the objective function is decreasing in y_R. Therefore, the maximum occurs when y_R is minimum.

We conclude, then, that it is in the interest of the department store to choose a price vector $\boldsymbol{p}_L$ such that $z_L = \frac{1}{2}y_L$, $z_R = \frac{1}{2}(t + y_R)$ and y_R is minimized. To minimize y_R, the department store should induce $\mathcal{I}_R$ to contain only the good j that is the cheapest at R.[39] The department will set $p_{j_L} = \frac{1}{2}(t + p_{j_R})$ and, for example, set $p_{i_L} = \frac{1}{2}p_{i_R}$, $\forall i \neq j$. □

Proof of Proposition 2.

To finish the proof, we must verify that $p_{i_R} = \frac{3}{n+2}t$ maximizes the profit of the shop at the mall that sells good i_R. So far, we only know that it maximizes profit in D_3, as long as $\sum_{i=1}^{n} |p_{i_L} - \frac{3}{n+2}t| \leq t$ (which implies that $p_{i_R} \in D_3$ and, thus, $\boldsymbol{p} \in \mathcal{P}_1$).

With $P_L = \frac{2n+1}{n+2}t$ and $p_{j_R} = \frac{3}{n+2}t$, $\forall j \neq i$, the profit function has a local maximum in D_3, attained at $p_{i_R}^* = \frac{3}{n+2}t$. The derivative of the demand with respect to price is $-\frac{1}{2t}$ in D_3 and $-\frac{1}{t}$ in D_4. Therefore, if a marginal price increase is not profitable, it is also not profitable to make a greater price increase to D_4, where the demand is more price-sensitive. The maximum is either $p_{i_R}^* = \frac{3}{n+2}t$ or attained in D_2.

In D_2, the demand for good i_R is $q_{i_R} = \frac{1}{t}\left(s_{Ri} + p_{i_L} - p_{i_R}\right)$ and the corresponding profit is $\Pi_{i_R} = \frac{1}{t}\left(s_{Ri}p_{i_R} + p_{i_L}p_{i_R} - p_{i_R}^2\right)$. The first-order condition is satisfied at $p_{i_R}^{**} = \frac{s_{Ri}}{2} + \frac{p_{i_L}}{2}$.

[39] Unless there is more than one good with null price at R. In that case, it is optimal for the department store to induce any set $\mathcal{I}_R \subseteq \{i \in \mathcal{I} : \ p_{i_R} = 0\}$.

If $p_{i_R}^{**} \in D_2$:

$$\Pi_{i_R}\left(p_{i_R}^{**}\right) = \frac{\left(s_{Ri} + p_{i_L}\right)^2}{4t}.$$

Then, $p_{i_R}^{*} = \frac{3}{n+2}t$ is surely a global maximizer if:

$$\Pi_{i_R}\left(p_{i_R}^{**}\right) \leq \Pi_{i_R}\left(p_{i_R}^{*}\right) \;\Leftrightarrow\; \frac{\left(s_{Ri} + p_{i_L}\right)^2}{4t} \leq \frac{9}{2(n+2)^2}t \;\Leftrightarrow\; s_{Ri} + p_{i_L} \leq \frac{\sqrt{18}}{n+2}t.$$

Otherwise, the alternative maximizer, $p_{i_R}^{**} = \frac{s_{Ri}}{2} + \frac{p_{i_L}}{2}$, upsets our equilibrium if it belongs to the domain, D_2. This is occurs if:

$$\begin{cases} \frac{s_{Ri}}{2} + \frac{p_{i_L}}{2} \leq -t + p_{i_L} + s_{Li} + s_{Ri} \\ \frac{s_{Ri}}{2} + \frac{p_{i_L}}{2} \geq -t + p_{i_L} + s_{Ri} \end{cases}. \tag{16}$$

In (the candidate) equilibrium, we have $P_R - P_L = \frac{n-1}{n+2}t$. As a result:

$$\sum_{j \in I \setminus \{i\}} \left(p_{j_R} - p_{j_L}\right) + \frac{3}{n+2}t - p_{i_L} = \frac{n-1}{n+2}t \;\Leftrightarrow\; s_{Li} = \frac{n-4}{n+2}t + s_{Ri} + p_{i_L}. \tag{17}$$

Substituting (17) in (16), we find that $p_{i_R}^{**}$ is in the domain D_2 when:

$$s_{Ri} + p_{i_L} \in \left[\frac{4}{n+2}t, 2t\right].$$

We have $s_{Ri} + p_{i_L} \leq 2t$, because:

$$s_{Li} \leq t \;\Leftrightarrow\; \frac{n-4}{n+2}t + s_{Ri} + p_{i_L} \leq t \;\Leftrightarrow\; s_{Ri} + p_{i_L} \leq \frac{6}{n+2}t.$$

Then, if $s_{Ri} + p_{i_L} > \frac{\sqrt{18}}{n+2}t$, we have $p_{i_R}^{**}$ in D_2 and this upsets our candidate equilibrium, $p_{i_R}^{*}$. As a result, our candidate is an equilibrium if and only if, $\forall i \in \mathcal{I}$:

$$s_{Ri} + p_{i_L} \leq \frac{\sqrt{18}}{n+2}t \;\Leftrightarrow\; s_{Li} \leq \frac{n + \sqrt{18} - 4}{n+2}t.$$

If $\mathcal{I}_R$ is non-empty, the conditions that bind are those for $i \in \mathcal{I}_R$ (because s_{Li} is maximal), which can be written as:

$$\sum_{i\in\mathcal{I}_L} |p_{i_R} - p_{i_L}| \leq \frac{n+\sqrt{18}-4}{n+2}t.$$

If $\mathcal{I}_R$ is empty, then all the above conditions are satisfied because, in this case:

$$\sum_{i\in\mathcal{I}_L} |p_{i_R} - p_{i_L}| = \sum_{i\in\mathcal{I}} (p_{i_R} - p_{i_L}) = P_R - P_L = \frac{n-1}{n+2}t < \frac{n+\sqrt{18}-4}{n+2}t.$$

To obtain a more elegant condition, notice that (in the candidate equilibrium):

$$P_R - P_{LR} - P_L + P_{LR} = \frac{n-1}{n+2}t \iff \sum_{i\in\mathcal{I}_R} |p_{i_L} - p_{i_R}| = \sum_{i\in\mathcal{I}_L} |p_{i_R} - p_{i_L}| - \frac{n-1}{n+2}t \iff$$
$$\iff \sum_{i\in\mathcal{I}} |p_{i_L} - p_{i_R}| = 2\sum_{i\in\mathcal{I}_L} |p_{i_R} - p_{i_L}| - \frac{n-1}{n+2}t.$$

Therefore, the equilibrium condition can be written as:

$$\sum_{i\in\mathcal{I}} |p_{i_L} - p_{i_R}| \leq \frac{2\left(n+\sqrt{18}-4\right)}{n+2}t - \frac{n-1}{n+2}t = \frac{n+6\sqrt{2}-7}{n+2}t. \qquad \square$$

Proof of Proposition 3.

To finish the proof, we need to check that the shops at the mall that are selling goods at a higher price than the department store do not wish to deviate as long as $7 \leq n \leq 11$. The candidate equilibrium maximizes their profits in the domain D_4, but we must guarantee that the shops do not prefer to choose prices in D_3 (if it exists) or in D_2.

It is convenient to start with some preliminary calculations. In the candidate equilibrium, for the shop that sells good $i \in \mathcal{I}_L$, we have:

$$s_{Li} = \sum_{j\in\mathcal{I}_L\backslash\{i\}} (p_{j_R} - p_{j_L}) = \frac{n-3}{n+1}t + p_{i_L};$$
$$s_{Ri} = \sum_{j\in\mathcal{I}_R\backslash\{i\}} (p_{j_L} - p_{j_R}) = \frac{t}{3}.$$

We know that D_3 is empty if and only if:

$$s_{Li} + s_{Ri} > t \iff \frac{n-3}{n+1}t + p_{i_L} + \frac{t}{3} > t \iff p_{i_L} > \frac{11-n}{3(n+1)}t.$$

For $n \geq 12$, the domain D_3 is empty (for $n = 11$ it is either empty or a singleton).

Let us start by considering the case in which D_3 is empty and study whether a deviation to D_2 is profitable. From (10), the interior maximizer in D_2 is:

$$p_{i_R}^{**} = \frac{p_{i_L}}{2} + \frac{t}{6}. \tag{18}$$

This alternative only belongs to D_2 if $p_{i_R}^{**} < p_{i_L}$. Equivalently, if:

$$\frac{p_{i_L}}{2} + \frac{t}{6} < p_{i_L} \;\Leftrightarrow\; p_{i_L} > \frac{t}{3}. \tag{19}$$

But since $p_{i_L} < \frac{2}{n+1}t$, condition (19) cannot hold for $n \geq 7$. This means that the optimal choice in D_2 is at the frontier ($p_{i_R} \to p_{i_L}$). The resulting profit is $\Pi_{i_R} = \frac{p_{i_L}}{3}$. We conclude that the shop gains with this deviation if:

$$\frac{p_{i_L}}{3} > \frac{4}{(n+1)^2}t \;\Leftrightarrow\; p_{i_L} > \frac{12}{(n+1)^2}t.$$

It is straightforward to confirm that $p_{i_L} > \frac{12}{(n+1)^2}t$ implies that $p_{i_L} > \frac{11-n}{3(n+1)}t$, which was a prerequisite (for D_3 to be empty).

Notice that if $p_{i_L} \leq \frac{12}{(n+1)^2}t, \forall i \in \mathcal{I}_L$, then:

$$\sum_{i \in \mathcal{I}_L} p_{i_L} \leq \frac{12(n-1)}{(n+1)^2}t \;\Leftrightarrow\; \frac{n-1}{n+1}t \leq \frac{12(n-1)}{(n+1)^2}t \;\Leftrightarrow\; n \leq 11.$$

For $n \geq 12$, the candidate is not an equilibrium because at least one of the shops deviates.

Now suppose that D_3 is not empty. We must have $p_{i_L} \leq \frac{11-n}{3(n+1)}t$ and $p_{i_L} < \frac{2}{n+1}t$. The first condition can only hold for $n \leq 11$, while the second is implied by the first. In this case, we must consider deviations to D_3 and also to D_2.

The interior maximum in D_2, given by (18), is again outside the domain because:

$$p_{i_L} \leq \frac{11-n}{3(n+1)}t \;\Rightarrow\; \frac{p_{i_L}}{2} + \frac{t}{6} > -t + p_{i_L} + s_{Li} + s_{Ri}.$$

The candidate deviation is, therefore, at the frontier. But if it is profitable to increase the price until the frontier of D_2 it is surely profitable to keep increasing the price after entering D_3 because the price-sensitivity of demand is lower in D_3. It is enough to consider deviations in D_3.

From (11), the candidate deviation is:

$$p_{i_R} = \frac{P_L}{2} - \frac{1}{2}\sum_{j\in\mathcal{I}\setminus\{i\}} p_{j_R} + \frac{t}{2} = \frac{n+13}{6(n+1)}t.$$

This deviation is outside D_3, because:

$$\frac{n+13}{6(n+1)}t \leq t + p_{i_L} - s_{Li} - s_{Ri} \;\Leftrightarrow\; \frac{n+13}{6(n+1)}t \leq \frac{11-n}{3(n+1)}t \;\Leftrightarrow\; n \leq 3,$$

which is false.

Therefore, the maximum in D_3 is at the frontier with D_4. Global continuity of the profit function (satisfied as long as D_3 is not empty) and concavity in D_4 imply that the local maximum at D_4 is actually the global maximum. □

Proof of Proposition 4.

We start by showing that an equilibrium with two-stop shopping is not possible.

With $n = 2$, one of the goods must be cheaper at $x = 0$ and the other must be cheaper at $x = 1$. Combining the first-order conditions, (6) and (7), we obtain, for $i \in \mathcal{I}_L$:

$$\begin{cases} p_{i_L} = \frac{p_{i_R}}{2} \\ p_{i_R} = \frac{p_{i_L}}{2} + \frac{t}{2} \end{cases} \Leftrightarrow \begin{cases} p_{i_L} = \frac{1}{3}t \\ p_{i_R} = \frac{2}{3}t. \end{cases}.$$

For $j \in \mathcal{I}_R$, we obtain $p_{j_L} = \frac{2}{3}t$ and $p_{j_R} = \frac{1}{3}t$. This is an equilibrium, but no consumer gains by shopping at both extremes (only the consumer in the middle, $x = \frac{1}{2}$, is indifferent between one-stop shopping and two-stop shopping).

For $n > 2$, both department stores wish to have a single good that is more expensive than at the other extreme. From Proposition 1, the profit-maximizing behavior of the department store located at L implies that there is a single good j for which $p_{j_L} > p_{j_R}$. Simultaneously, the profit-maximizing behavior of the department store located at R implies that there is a single good j' for which $p_{j'_R} > p_{j'_L}$. For these behaviors to be compatible, we must have, $\forall i \notin \{j, j'\}$, $p_{i_L} = p_{i_R}$. Actually, we must have $p_{i_L} = p_{i_R} = 0, \forall i \notin \{j, j'\}$, otherwise the department stores would gain by undercutting each other's prices (so capture the demand of all the two-stop shoppers). Since, from Proposition 1, j' is one of the cheapest goods at L, we also have $p_{j'_L} = 0$. It follows that the department store at L (resp. R) is setting a strictly positive price for good j (resp. j'), $p_{j_R} = \frac{t}{2}$ (resp. $p_{j'_R} = \frac{t}{2}$) and offering all the remaining goods for free. This is neither optimal nor induces two-stop shopping.

Therefore, in equilibrium, we must have one-stop shopping. With one-stop shopping, the first-order conditions for profit-maximization by the department stores imply that:

$$P_L = \frac{P_R}{2} + \frac{t}{2} \quad \text{and} \quad P_R = \frac{P_L}{2} + \frac{t}{2},$$

yielding:

$$P_L = P_R = t.$$

It is straightforward to verify that they do not have incentives to deviate (8).

The equilibrium demand and profits follow immediately. □

Lemma 4. *When there is competition between two shopping malls, no consumer shops at both extremes of the city (in equilibrium).*

Proof of Lemma 4.

By way of contradiction, suppose that the vector of prices that maximize the profits of the shops is such that $\boldsymbol{p} \in \mathcal{P}_2$. More precisely, that $0 < \tilde{x}_L < \tilde{x} < \tilde{x}_R < 1$.

(**i**) There cannot be any $i \in \mathcal{I}$ for which $p_{i_L} = p_{i_R} > 0$. If that was the case, the shop selling good i_L could infinitesimally reduce its price and conquer all consumers at $x \in [\tilde{x}_L, \tilde{x}_R]$. The shop selling good i_R would have the same incentive to decrease its price.

It cannot also be the case that $p_{i_L} = p_{i_R} = 0$ for some $i \in \mathcal{I}$. In such a situation, both shops would obtain a null profit. However, the shop selling good i at $x = 0$, could choose $p_{i_L} > 0$ and profit $\Pi_{i_L} = p_{i_L}\tilde{x}_L > 0$. The same argument applies to the shop selling good i_R.

(**ii**) Since $p_{i_L} \neq p_{i_R}$, $\forall i \in \mathcal{I}$, we have $\mathcal{I}_L \cup \mathcal{I}_R = \mathcal{I}$. Thus, if the cardinality of $\mathcal{I}_L$ is k, the cardinality of $\mathcal{I}_R$ is $n - k$.

The profit function of the shop that sells the good i_L is:

$$\Pi_{i_L} = \begin{cases} p_{i_L}\ \tilde{x}_R, & i \in \mathcal{I}_L \\ p_{i_L}\ \tilde{x}_L, & i \in \mathcal{I}_R \end{cases},$$

while the profit function of the shop selling the good i_R is:

$$\Pi_{i_R} = \begin{cases} p_{i_R}\left(1 - \ \tilde{x}_R\right), & i \in \mathcal{I}_L \\ p_{i_R}\left(1 - \ \tilde{x}_L\right), & i \in \mathcal{I}_R \end{cases}.$$

If $i \in \mathcal{I}_L$, the first-order conditions are:

$$\begin{cases} \frac{\partial \Pi_{i_L}}{\partial p_{i_L}} = 0 \\ \frac{\partial \Pi_{i_R}}{\partial p_{i_R}} = 0 \end{cases} \Leftrightarrow \begin{cases} p_{i_L} = P_R - P_{LR} \\ p_{i_R} = t - P_R + P_{LR} \end{cases} \Rightarrow p_{i_R} = t - p_{i_L}.$$

The expressions above imply that $\forall i, j \in \mathcal{I}_L$: $p_{i_L} = p_{j_L}$ and $p_{i_R} = p_{j_R}$. Moreover:

$$p_{i_L} = \sum_{j \in \mathcal{I}_L} (p_{j_R} - p_{j_L}) = k\,(t - 2p_{i_L}) \;\Leftrightarrow\; p_{i_L} = \frac{k}{2k+1}t$$

and

$$p_{i_R} = \frac{k+1}{2k+1}t.$$

Analogously, if $i \in \mathcal{I}_R$, then:

$$p_{i_L} = \frac{n-k+1}{2n-2k+1}t \quad \text{and} \quad p_{i_R} = \frac{n-k}{2n-2k+1}t.$$

The expressions for the marginal consumers, $\tilde{x}_L$ and $\tilde{x}_R$, follow immediately:

$$\tilde{x}_L = \frac{n-k+1}{2n-2k+1} \quad \text{and} \quad \tilde{x}_R = \frac{k}{2k+1}.$$

It is straightforward to see that $\tilde{x}_L > \tilde{x}_R$. Contradiction. $\square$

Proof of Proposition 5.

By Lemma 4 (in this Appendix), there is no equilibrium with prices in $\mathcal{P}_2$. Therefore, we must seek prices satisfying the condition $\sum_{i=1}^{n} |p_{i_L} - p_{i_R}| \leq t$.

As obtained in (13), the first-order conditions of the n shops at the malls imply that:

$$P_L = \frac{n}{n+1}(t + P_R) \quad \text{and} \quad P_R = \frac{n}{n+1}(t + P_L).$$

Therefore:

$$P_L = P_R = nt.$$

Using (11), we obtain the individual prices:

$$p_{i_L} = p_{i_R} = t, \forall i \in \mathcal{I}.$$

To complete the proof, we must verify that these local maxima are global maxima. We need to check if each shop chooses the price t, when the remainders charge t for their products. Without loss of generality, we consider the shop that sells good i_R.

Substituting $p_{j_L} = p_{j_R} = t, \forall j \neq i$ and $p_{i_L} = t$ in the demand for good i_R, given in (9):

$$q_{i_R} = \begin{cases} 1 - \frac{p_{i_R}}{2t}, & p_{i_R} \in [0, 2t] \\ 0, & p_{i_R} \in (2t, +\infty) \end{cases}$$

and

$$\Pi_{i_R} = \begin{cases} p_{i_R}\left(1 - \frac{p_{i_R}}{2t}\right), & p_{i_R} \in [0, 2t] \\ 0, & p_{i_R} \in (2t, +\infty) \end{cases} .$$

The profit function is globally concave and continuous. Therefore, the local maximum is also the global maximum. The equilibrium demand and profits follow immediately. □

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