Generic non-existence of general equilibrium with EUU preferences under extreme ambiguity

João Correia-da-Silva∗†

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Abstract. In the presence of three or more realisations of the aggregate endowment that are extremely ambiguous, in the sense that all relative probabilities are admissible, if agents have preferences that are representable by expected uncertain utility functions (Gul and Pesendorfer, 2014), general equilibrium does not generically exist in finite economies. It always exists, however, in continuum economies.

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∗Toulouse School of Economics. 21 Allée de Brienne, 31000 Toulouse, France. E-mail: joao.correia@tse-fr.eu.
†CEF.UP and Faculdade de Economia, Universidade do Porto. Rua Dr. Roberto Frias, 4200-464 Porto, Portugal. E-mail: joao@fep.up.pt.
1 Introduction

In this paper, it is shown that general equilibrium does not generically exist in economies under extreme ambiguity about the aggregate endowment when agents have preferences that are representable by expected uncertain utility (EUU) functions (Gul and Pesendorfer, 2014).

Gul and Pesendorfer (2014) have proposed, as a normative description of choice under ambiguity, a kind of preferences according to which an agent maximises the expected value of an interval utility function, which attributes a utility index to a set of possible outcomes whose relative probabilities the agent cannot quantify. This utility index only depends on the worst and the best possible outcomes.

They have generalised the notion of maxmin expected utility (MEU), axiomatised by Gilboa and Schmeidler (1989), according to which the utility of an ambiguous prospect only depends on the worst possible outcome, by allowing utility to depend also on the best possible outcome. Since this dependence is not necessarily separable, they have also generalised the notion of $\alpha$-maxmin expected utility ($\alpha$-MEU) due to Arrow and Hurwicz (1972).

EUU preferences are able to rationalise various types of deviations from expected utility maximisation (Allais, 1953; Ellsberg, 1961). However, under extreme ambiguity about the aggregate endowment, the choice behaviour that they prescribe is, to some extent, incompatible with general equilibrium theory (Arrow and Debreu, 1954) because it implies that equilibrium does not exist in economies with a finite number of agents.

More precisely, non-existence of equilibrium ensues when there are three or more different realisations of the aggregate endowment whose relative probabilities are completely unknown. In the language of Gul and Pesendorfer (2014): when these realisations are contained and diffuse in an event with known probability.\footnote{In EUU theory, an event $E$ is ideal if the agent can perfectly quantify its probability of occurrence and a subset $D$ is diffuse in $E$ if no strict subset of $E$ that is ideal contains $D$ or its complement.} In the language of $\alpha$-MEU
theory: when, conditionally on an event with known probability, these realisations have a probability that can be as low as zero and as high as one.

It is not surprising that existence of equilibrium is problematic, since EUU preferences are not convex (except for the particular case in which they coincide with MEU preferences). What may be surprising is that, under this form of extreme ambiguity about the aggregate endowment, non-existence of equilibrium is generic.

The failure of existence of equilibrium has its origin in the combination of the lack of convexity associated with EUU preferences and the lack of strict monotonicity associated with extreme ambiguity. Concretely, it originates from the fact that the marginal utility of consumption in one of a set of completely ambiguous states is strictly positive for consumption levels below the minimum or above the maximum consumption level over the remaining states, but is null between these two thresholds.

For example, suppose that there are three possible states of nature and consider the consumption plan \((x_1, x_2, x_3) \in \mathbb{R}_+^3\), with \(x_1 < x_2 \leq x_3\). Under extreme ambiguity, an EUU maximiser is indifferent between \((x_1, x_2, x_3)\) and \((x_1, x_1, x_3)\), but the second consumption plan has the advantage of being cheaper. Therefore, given a set of completely ambiguous states, an agent with EUU preferences will choose the same consumption level across all states except one. For the state in which the price is the lowest, the agent may choose a higher level of consumption.

Hence, all agents will choose the same consumption level across completely ambiguous states, with the possible exception of a single state in which the price is the lowest. Suppose that there is a state of nature for which the price is the lowest, i.e., suppose that \(p = (p_1, p_2, p_3) \in \mathbb{R}_+^3\) is such that \(p_1 \geq p_2 > p_3 > 0\). In this case, all agents will choose the same (low) level of consumption for the first two states and a possibly different (high) level of consumption for the third state.

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2It is well known that existence of equilibrium is guaranteed if agents have MEU preferences because these are convex. See, for example, the contributions of Correia-da-Silva and Hervés-Beloso (2009) and de Castro, Pesce and Yannelis (2011), which are based on the model of Radner (1968).
However, if the aggregate endowment is different across the first two states, equality of aggregate demand implies that exact feasibility fails. This is incompatible with existence of equilibrium with strictly positive prices because preferences are non-satiated (which implies that agents choose a consumption plan in the frontier of their budget sets).

An equilibrium with some null price is also impossible, because the demand for the corresponding contingent good would become unbounded. Therefore, there exists no equilibrium (neither with strictly positive prices nor with at least one null price).

The above reasoning was conditional on the absence of ties for the lowest price across completely ambiguous states. In the example described above, if $p_2 = p_3$, some agents could demand higher consumption for state 2 while others demand higher consumption for state 3. As a result, it would not be possible to conclude that aggregate demand is the same in states 1 and 2.

To establish generic non-existence of equilibrium, it is, therefore, necessary to confirm that the probability of such ties for the lowest price being compatible with market-clearing is null. This is accomplished with recourse to an auxiliary economy in which the highest and lowest levels of consumption are treated as actual commodities.

In economies with a continuum of agents, existence of equilibrium is guaranteed. This is a direct consequence of existence results that are well known (Hildenbrand, 1970). Perhaps more interestingly, equilibrium is typically characterised by some particular features. Among a set of completely ambiguous states: price is highest in one state and lowest in all the others; and individual consumption is also highest in one state and lowest in all the others.

The remainder of the paper is organised as follows: the model is presented in Section 2; the core of the analysis is carried out in Section 3; generic non-existence is established in Section 4; economies with a continuum of agents are studied in Section 5; an example with symmetric agents is presented in Section 6; concluding remarks are made and possible extensions are briefly discussed in Section 7.
2 Model

Consider an economy with a finite number of agents, \( I = \{1, \ldots, I\} \), a single consumption good, and a finite number of states of nature, \( S = \{1, \ldots, S\} \). Each agent \( i \in I \) possesses state-contingent endowments, denoted by \( e^i \in \mathbb{R}^S_+ \), which she trades under uncertainty (risk and ambiguity).

The structure of risk and ambiguity faced by the agents is described by a partition, \( P \), of the set of states of nature. Agents know that the probability of occurrence of any event \( E \) that belongs to the \( \sigma \)-algebra generated by \( P \) is given by \( \sum_{s \in E} \mu_s \), where \( \mu = (\mu_1, \ldots, \mu_S) \), with \( \mu_s > 0 \), \( \forall s \in S \), and \( \sum_{s \in S} \mu_s = 1 \).

The preferences of agent \( i \in I \) over consumption plans, \( x^i \in \mathbb{R}^S_+ \), that are measurable with respect to the \( \sigma \)-algebra generated by \( P \) can be described through an expected utility function, \( U^i : \mathbb{R}^S_+ \rightarrow \mathbb{R} \), of the form: \( U^i(x^i) = \sum_{s \in S} \mu_s v^i(x^i_s) \), where \( v^i : \mathbb{R}_+ \rightarrow \mathbb{R} \) is a standard von Neumann-Morgenstern utility index.

It is assumed that agents face complete ambiguity about the probabilities of occurrence of states of nature that belong to the same set of the partition \( P \). The preferences of agent \( i \in I \) over consumption plans that are not measurable with respect to the \( \sigma \)-algebra generated by \( P \) are described by expected uncertain utility (EUU) functions (Gul and Pesendorfer, 2014).

To evaluate the expected uncertain utility of a consumption plan, \( x^i \), that is not measurable with respect to the \( \sigma \)-algebra generated by \( P \), agent \( i \) considers a lower bound, \( x^i_L \), and an upper bound, \( x^i_H \), defined as follows (for each \( s \in S \)):

\[
x^i_L(s) \equiv \min_{t \in P(s)} x^i_t, \\
x^i_H(s) \equiv \max_{t \in P(s)} x^i_t.
\]

The interval utility function of agent \( i \), \( u^i : \mathbb{R}^2_+ \rightarrow \mathbb{R} \), attributes a utility level,
$u^i(x^i_{L(s)}, x^i_{H(s)} - x^i_{L(s)})$, to an ambiguous prospect with consumption levels between $x^i_{L(s)}$ and $x^i_{H(s)}$. The objective function of agent $i \in I$ is, thus:

$$U^i(x^i) = \sum_{s \in S} \mu_s u^i(x^i_{L(s)}, x^i_{H(s)} - x^i_{L(s)}).$$

Consistency of the proposed extension of preferences from risky prospects (consumption plans that are measurable with respect to the $\sigma$-algebra generated by $P$) to ambiguous prospects (consumption plans that are not) requires that $u^i(z, 0) = v^i(z)$, $\forall z \in \mathbb{R}_+$. The following assumption excludes the particular case in which agents are maxmin expected utility (MEU) maximisers. If agents had MEU preferences (with a concave utility index function), preferences would be convex. Therefore, in that case, existence of equilibrium would be guaranteed.

**Assumption 1.** The interval utility functions of the agents are strictly increasing in the first and in the second variable.

It is only when there are elements of $P$ containing three or more states of nature that the interval utility functions impose some structure on preferences and demand. It is easy to verify that, in that case, the preference relations of the agents are not convex and are not strictly increasing.

**Remark 1.** If there are three or more completely ambiguous states of nature, EUU preferences violate convexity and strict monotonicity.

**Proof.** Suppose that $P(s) = \{1, 2, 3\}$, $\forall s \in \{1, 2, 3\}$. Consider the consumption plans $x^i = (3, 1, 1, ..., 1)$, $y^i = (1, 3, 1, ..., 1)$ and $z^i = (2, 2, 1, ..., 1)$. All plans yield the same utility outside $\{1, 2, 3\}$. In $\{1, 2, 3\}$, the utility of $x^i$ and $y^i$ is the same: $u^i(1, 2)$. Under Assumption 1, the utility of $z^i$, which is a convex combination, is strictly lower: $u^i(1, 1)$. Thus, preferences are not convex. Observe also that the plan $w^i = (3, 2, 1, ..., 1)$ has the same utility as $x^i$. Thus, preferences are not strictly increasing.
In the economy under scrutiny, agents trade contingent claims under uncertainty (Arrow, 1953; Debreu, 1959). Each agent $i \in I$ trades her state-contingent endowments, $e^i \in \mathbb{R}^S_+$, for a consumption plan, $x^i \in \mathbb{R}^S_+$, that maximizes her expected uncertain utility, subject to the following budget restriction:

$$ x^i \in B^i(p) \iff p \cdot x^i \leq p \cdot e^i, $$

where $p = (p_1, ..., p_S)$ is a price system. Each coordinate, $p_s$, represents the price paid ex ante for the delivery of one unit of consumption if state $s$ occurs. Price systems are normalized to belong to the simplex, $\Delta^S \equiv \{ q \in \mathbb{R}^S_+ : \sum_{s \in S} q_s = 1 \}$.

A general economic equilibrium (Arrow and Debreu, 1954) consists of a price system, $p^* \in \Delta^S$, and an allocation, $x^* = (x^i, ..., x^I) \in \mathbb{R}^I_+$, that are compatible with individual optimization and market clearing:

- $x^i_s^* \in \arg \max_{x^i \in B^i(p^*)} U^i(x^i), \forall i \in I$,
- $\sum_{i \in I} x^i_s^* \leq \sum_{i \in I} e^i_s, \forall s \in S$.

### 3 Analysis

Strict monotonicity of the interval utility functions with respect to the second variable implies that equilibrium prices (if they exist) must be strictly positive. If there existed a null price, the demand for consumption in the corresponding state would be unbounded.

**Remark 2.** Equilibrium prices (if they exist) must be strictly positive.

**Proof.** Suppose that $(p^*, x^*)$ is an equilibrium and that there exists a state $s \in S$ such that $p_s = 0$. Let $x^i_s = x^i_{H(s)} + \epsilon$, where $\epsilon > 0$, and $x^i_t = x^i_t^*$, $\forall t \neq s$. From strict monotonicity of the interval utility function, $U^i(x^i) > U^i(x^i^*)$. Since $p^* \cdot x^i = p^* \cdot x^i^*$, the equilibrium consumption plan of agent $i$ is not individually optimal. Contradiction. $\square$
The fact that preferences satisfy non-satiation implies that agents choose a consumption plan in the frontier of their budget sets. In an equilibrium with strictly positive prices, this implies that feasibility is exact.

Remark 3. An equilibrium allocation (if it exists) must satisfy exact feasibility.

Proof. Under Assumption 1, the preferences of the agents satisfy non-satiation, i.e., \( y^i \gg x^i \Rightarrow U^i(y^i) > U^i(x^i), \forall i \in \mathcal{I}. \) Therefore, agents choose in the frontier of their budget sets. Thus: \( \sum_{i \in \mathcal{I}} p \cdot x^i = \sum_{i \in \mathcal{I}} p \cdot e^i. \) Since \( \sum_{i \in \mathcal{I}} x^i \leq \sum_{i \in \mathcal{I}} e^i \) and prices are strictly positive (Remark 2), feasibility must be exact: \( \sum_{i \in \mathcal{I}} x^i = \sum_{i \in \mathcal{I}} e^i. \)

Null marginal utility of consumption in the interior of the interval \( (x^i_{L(s)}, x^i_{H(s)}) \) implies that agents are not willing to pay a strictly positive price to increase their consumption level in state \( s \) from \( x^i_{L(s)} \) to any level not higher than \( x^i_{H(s)} \). This leads to the following characterisation of individual demand.

Remark 4. If prices are strictly positive, EUU maximisation implies that, in any given element of \( P \), there is at most one state in which the agent does not consume the minimum over states that belong to that element of \( P \).

Proof. Suppose that there exists \( i \in \mathcal{I}, s \in \mathcal{S} \) and \( s' \in P(s) \) such that \( x^i_{L(s)} < x^i_s \leq x^i_{s'}. \) If that is the case, \( x^i \) does not maximise the utility of agent \( i \) in \( B^i(p) \). Agent \( i \in \mathcal{I} \) is better-off by slightly decreasing consumption in state \( s \) and slightly increasing consumption in the remaining states. For example, let \( y^i_s = x^i_s - \epsilon/p_s + \epsilon \) and \( y^i_t = x^i_t + \epsilon, \forall t \neq s. \) This modification is budget-neutral and increases utility as long as \( 0 < \epsilon < p_s(x^i_s - x^i_{L(s)}). \)

Henceforth, with the objective of ruling out existence of equilibrium, we work under the restrictions established above: prices are strictly positive, feasibility is exact, and demand satisfies the characterisation in Remark 4.

\[ ^3 \text{Formally, this property can be written as: } \# \left\{ t \in P(s) : x^i_t > x^i_{L(s)} \right\} \leq 1, \forall (i, s) \in \mathcal{I} \times \mathcal{S}. \]
For each state \( s \in \mathcal{S} \), let \( N_s \) denote the cardinality of the cell of \( P \) that contains \( s \), i.e., let \( N_s \equiv \#P(s) \). If this cardinality is sufficiently high relatively to the number of agents, there must exist states in \( P(s) \) with equal consumption profiles.

**Remark 5.** Suppose that \( N_s \geq I + 2 \). If prices are strictly positive, there exists a set \( A \subset P(s) \) with cardinality \( N_s - I \) such that all agents choose constant consumption across states in \( A \): \( x^i_t = x^i_{L(s)} \), \( \forall i \in \mathcal{I}, \forall t \in A \).

**Proof.** Consider the \( N_s \) states that constitute the cell \( P(s) \). From Remark 4, each agent chooses a constant consumption level across all \( N_s \) states with the possible exception of a single state. Therefore, there must exist \( N_s - I \) states across which the consumption profile is the same. \( \square \)

Let \( N \) denote the maximum cardinality of the cells of \( P \), i.e., let \( N \equiv \max_{s \in \mathcal{S}} \#P(s) \).

The above remark implies that, if \( N \geq I + 2 \) and aggregate endowments differ across states, markets cannot clear. There always exist at least two states with the same aggregate demand, \( \sum_{i \in \mathcal{I}} x^i_s \), but different aggregate supply, \( \sum_{i \in \mathcal{I}} e^i_s \). Therefore, feasibility would require free disposal, which, in turn, would require null prices (see Remark 3). However, with null prices, demand would be unbounded (see Remark 2).

**Proposition 1.** Suppose that \( N \geq I + 2 \) and that \( \sum_i e^i_s \) differs across all states \( s \in \mathcal{S} \). Under Assumption 1, there does not exist an equilibrium.

**Proof.** Consider a cell of \( P \) with \( N \) states. From Remark 5, there exist at least two states in that cell, \( s \) and \( t \), across which agents demand the same. Therefore, \( \sum_i x^i_s = \sum_i x^i_t \). If \( \sum_i e^i_s < \sum_i e^i_t \), exact feasibility is violated in state \( t \). Otherwise, it is violated in state \( s \). From Remark 2, equilibrium prices must be strictly positive. Thus, \( \sum_i \sum_s p_s x^i_s < \sum_i \sum_s p_s e^i_s \). However, since optimising agents choose consumption plans in the frontier of their budget sets, \( \sum_i \sum_s p_s x^i_s = \sum_i \sum_s p_s e^i_s \). Contradiction. \( \square \)
Observe that the aggregate endowment differs across any pair of states of nature in an open, dense and full Lebesgue measure subset of the space of possible initial endowment profiles, which is $\mathbb{R}_+^{I_S}$.

4 Generic non-existence of equilibrium

Let $\mathcal{E} \equiv \mathbb{R}_+^{I_S}$ denote a space of economies in which the profile of preferences is kept fixed, as well as the structure of uncertainty and information. In this space of economies, the initial endowments completely describe an economy.

Let $\mathcal{E}_0 \subset \mathcal{E}$ denote the full measure subset composed by the economies in which the aggregate endowment differs across any pair of states of nature. We say that a property holds generically in some set whenever it holds in a full measure subset.

Corollary 1. If $N \geq I + 2$, non-existence of equilibrium always holds in $\mathcal{E}_0$ and, thus, holds generically in $\mathcal{E}$.

Proof. This is a direct consequence of Proposition 1 together with the fact that $\mathcal{E}_0$ is a full measure subset of $\mathcal{E}$. 

The purpose of this Section is to establish that non-existence of equilibrium still holds generically in $\mathcal{E}$ under a much weaker condition on the cardinality of sets of completely ambiguous states ($N \geq 3$ instead of $N \geq I + 2$).

Consider a cell of $P$ with three or more states. If there is a single state in which the price is the lowest, all agents demand the highest level of consumption for that state and the lowest level of consumption for the other states. This implies that the consumption profile is the same across these other states, violating exact feasibility if the aggregate endowment is different across states. Hence, conditionally on the absence of ties for the lowest price, $N \geq 3$ is a sufficient condition for non-existence of equilibrium of the economies in $\mathcal{E}_0$.
Considering an economy in $\mathcal{E}^0$, denote by $a(s) \in P(s)$ the state with the lowest level of total endowment across states in $P(s)$, for each $s \in S$. Exact feasibility implies that if there is a state in $P(s)$ for which all agents demand the lowest consumption level, $x^s_{L(s)}$, it is state $a(s)$. Exact feasibility also implies that, in any other state $t \in P(s)$, there must be some agent choosing the highest consumption level, $x^t = x^s_{H(s)}$. Therefore, if an equilibrium exists, there must be ties for the lowest price across all states in $P(s) \setminus \{a(s)\}$.

**Remark 6.** Suppose that $\sum_i e^i_s$ differs across all states $s \in S$. If an equilibrium exists, prices must be such that: $p^*_t = \min_{z \in P(s)} p^*_z$, $\forall t \in P(s) \setminus \{a(s)\}$.

**Proof.** If $p^*_t > \min_{z \in P(s)} p^*_z$, with $t \in P(s) \setminus \{a(s)\}$, no agent demands the highest consumption level for state $t$. With all agents choosing the lowest consumption level for state $t$, i.e., with $x^t_i = x^i_{L(t)}$, $\forall i \in I$, we have $\sum_i x^t_i \leq \sum_i x^s_{a(s)}$. Since $\sum_i e^t_i > \sum_i e^s_{a(s)}$, exact feasibility fails. Thus, $x^*$ is not an equilibrium allocation. $\square$

To show that these price ties imply generic non-existence of equilibrium, a difficulty that needs to be surmounted is that EUU preferences (which are not strictly monotone) do not satisfy the assumptions that are necessary for the application of differential topology techniques. Fortunately, as we will show, it is sufficient to impose those assumptions on the interval utility functions.

We proceed, therefore, by assuming that the interval utility functions satisfy the following conditions, formalised below: (i) twice differentiability; (ii) differentiable strict monotonicity; and (iii) differentiable strict concavity.\(^4\)

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\(^4\)These assumptions are less restrictive than those of Debreu (1972), as the indifference curves may intersect the boundary of the consumption set. Imposing interior indifference curves would mean that an agent would prefer $(x_L, x_H - x_L) = (1 - \epsilon, \epsilon)$ to $(x_L, x_H - x_L) = (1, 0)$, which would be nonsensical. Under these assumptions on utility, demand is locally Lipschitz continuous (Mas-Colell, 1985, Proposition 2.7.3). This implies that demand is continuously differentiable almost everywhere and that it maps null sets into null sets (Rader, 1973, Lemma 3), which are sufficient conditions to establish that local uniqueness of equilibrium is a generic property (Rader, 1973, Corollary 14). See also Shannon (1994, Theorems 17 and 18). Páscoa and Werlang (1999) established generic local uniqueness of equilibrium under the even weaker assumption that agents have continuous, monotone and strictly concave utility functions with non-singular bordered hessian outside a null set.
Assumption 2.

The interval utility function of each agent $i \in I$ satisfies the following properties:

(i) $u^i : \mathbb{R}_+^2 \to \mathbb{R}$ is continuous in $\mathbb{R}_+^2$ and $C^2$ in $\mathbb{R}_+^{2+}$;

(ii) $\forall x \in \mathbb{R}_+^{2+}$, $\left( \frac{\partial u^i(x)}{\partial x_1}, \frac{\partial u^i(x)}{\partial x_2} \right) \gg 0$;

(iii) $\forall x \in \mathbb{R}_+^{2+}$, $\sum_{j=1}^2 \sum_{k=1}^2 h_j h_k \frac{\partial^2 u^i(x)}{\partial x_j \partial x_k} < 0$, for all $h \in \mathbb{R}_+^2 \{0\}$ s.t. $\sum_{j=1}^2 h_j \frac{\partial u^i(x)}{\partial x_j} = 0$.

The idea of the proof of generic non-existence of equilibrium is the following. For simplicity, consider an economy with $N$ completely ambiguous states. Restrict prices so that there are ties for the lowest price across $N - 1$ states. Then, construct an auxiliary economy in which the $N$ contingent goods are replaced by only two goods: $L$ and $H$. Preferences are the same as in the original economy (but now the consumption levels of $L$ and $H$ enter directly in the interval utility functions), while the initial endowments (of goods $L$ and $H$) are constructed in such a way that, with the assumed ties for the lowest price in the original economy (which is a requisite for existence of equilibrium), the budget constraints in the auxiliary economy are equivalent to those in the original economy while the feasibility constraints are weaker ($N - 2$ conditions are omitted).

As a result of this construction: existence of an equilibrium of the original economy implies the existence of an equilibrium of the auxiliary economy which is such that the omitted feasibility constraints are also satisfied. Under Assumption 2, the auxiliary economy has, generically, a finite number of equilibria. This implies that the demand side of the omitted feasibility constraints has values in a finite set. Since, without modifying the auxiliary economy, the supply side (aggregate endowment) can have values in a set of dimension $N - 2$, we conclude that non-existence of equilibrium is generic.

**Proposition 2.** If $N \geq 3$, under Assumption 2, non-existence of equilibrium holds generically in $E_0$ and, thus, also holds generically in $E$.

**Proof.** For simplicity of exposition, let us restrict the analysis to economies with $N$ completely ambiguous states ordered by increasing total endowment (which differs across
states) and normalise prices so that the price for contingent delivery in the state with lower total endowment is unity: \( p_1 = 1 \).

From Remark 6, equilibrium prices must be of the form \((p_1, p_2, ..., p_N) = (1, q, ..., q)\), with \( q \leq 1 \). Start by supposing that \( q < 1 \). In this case, each agent \( i \in \mathcal{I} \) chooses the highest level of consumption, denoted \( x^i_H \), for a state in \( \{2, ..., N\} \) and the lowest level of consumption, denoted \( x^i_L \), for the remaining \( N - 1 \) states. Denote by \( \mathcal{I}_s \) the set of agents that plan the highest level of consumption for state \( s \).

The exact feasibility constraints can be written as:

\[
\sum_{i \in \mathcal{I}_s} x^i_H + \sum_{i \in \mathcal{I} \setminus \mathcal{I}_s} x^i_L = \sum_{i \in \mathcal{I}} e^i_s, \quad \forall s \in S.
\]

Consider the feasibility constraint for \( s = 1 \) and the feasibility constraint that results from adding all the feasibility constraints for \( s \in \{2, ..., N\} \) and subtracting \( N - 2 \) times the feasibility constraint for \( s = 1 \):

\[
\sum_{i \in \mathcal{I}} x^i_L = \sum_{i \in \mathcal{I}} e^i_1, \\
\sum_{i \in \mathcal{I}} x^i_H = \sum_{i \in \mathcal{I}} \left[ \sum_{s=2}^{N} e^i_s - (N - 2)e^i_1 \right].
\]

With prices of the form \((p_1, p_2, ..., p_N) = (1, q, ..., q)\), the budget restrictions of the agents can be manipulated in a similar manner:

\[
x^i_1 + q \sum_{s=2}^{N} x^i_s = e^i_1 + q \sum_{s=2}^{N} e^i_s \Leftrightarrow \\
[1 + (N - 2)q] x^i_L + q x^i_H = [1 + (N - 2)q] e^i_1 + q \left[ \sum_{s=2}^{N} e^i_s - (N - 2)e^i_1 \right].
\]

We can, therefore, conceive an auxiliary economy with only two goods, \( L \) and \( H \), preferences given by the interval utility functions, and initial endowments given by \( e^i_L = \).
\(e^i_1\) and \(e^i_H = \sum_{s=2}^N e^i_s - (N - 2)e^i_1\), for all \(i \in I\). Under Assumption 2, in a full measure subset of the space of auxiliary economies, \(\mathcal{E}_0^A \subset \mathcal{E}^A \equiv \mathbb{R}_{++}^{2J}\), the auxiliary economy has a finite number of equilibria. This was shown by Mas-Colell (1985, Proposition 8.7.3) and Shannon (1994, Theorem 18).\(^5\)

An original economy that belongs to a full measure subset of the space of original economies, \(\mathcal{E}_1 \subset \mathcal{E}_0\), originating an auxiliary economy in \(\mathcal{E}_0^A\) (which has a finite number of equilibria). Since the equilibrium conditions of the original economy include those of the auxiliary economy, there is only a finite number of candidate equilibria of the original economy.

Recall that \(N - 2\) feasibility conditions of the original economy were omitted in the auxiliary economy. For example, the exact feasibility condition for state \(s = 2\) is:

\[
\sum_{i \in I} x^i_H + \sum_{i \in I \setminus I_2} x^i_L = \sum_{i \in I} e^i_2.
\]

Observe that any perturbation that increases \(e^i_2\) and decreases \(e^i_3\) by the same (small) amount has no impact on the auxiliary economy because the endowments \(e^i_1\) and \(\sum_{s=2}^N e^i_s\) remain constant. However, such a perturbation will typically imply that omitted feasibility conditions are violated. We conclude, therefore, that an equilibrium with prices of the form \((p_1, p_2, ..., p_N) = (1, q, ..., q)\), with \(q < 1\), cannot exist in a full measure subset of \(\mathcal{E}_1\) (which is also a full measure subset of \(\mathcal{E}_0\) and of \(\mathcal{E}\)).

To conclude the proof, suppose that \(q = 1\). This means that prices are constant across all states: \((p_1, ..., p_N) = (1, ..., 1)\). Under Assumption 2, there is a single pair \((x^i_{H,*}, x^i_{L,*})\) that maximises the EUU of agent \(i \in I\). The aggregate demand in state 1 is, then:

\[
\sum_{i \in I_1} x^i_{H,*} + \sum_{i \in I \setminus I_1} x^i_{L,*}.
\]

\(^5\)Páscoa and Werlang (1999) have established that local uniqueness of equilibrium is generic under even weaker conditions.
Observe that there is a finite number of possible sets \( I_1 \subseteq I \). Therefore, aggregate demand in state 1 can only take a finite number of possible values. This implies that the feasibility constraint will not be satisfied (this price vector does not clear the markets) in a full measure subset of \( \mathcal{E} \).

\[ \square \]

## 5 Economies with a continuum of agents

The non-existence results presented above do not extend to economies with a continuum of agents. In fact, requiring that the interval utility functions are continuous and strictly increasing is sufficient to guarantee existence of equilibrium (Hildenbrand, 1970).

We have in mind an economy with a continuum of agents, \( E \), in which: the set of agents is the unit interval, \( I = [0, 1] \), endowed with the Lebesgue measure; the initial endowments are described by an integrable mapping, \( e : I \to \mathbb{R}^3_{++} \), such that \( \int_I e^i \, di \gg 0 \); and the preference profile is described by a measurable mapping, \( u : I \to \mathcal{U} \), where \( \mathcal{U} \) is a space of preference relations that can be represented by interval utility functions that are continuous and strictly increasing in both variables.\(^6\)

**Proposition 3.** There exists an equilibrium in \( E \).

**Proof.** Theorem 1 of Hildenbrand (1970) establishes existence of a quasi-equilibrium. A quasi-equilibrium is composed by a price system, \( p^* \in \Delta^3 \), and a feasible allocation, \( x : I \to \mathbb{R}^2_{++} \), that maximizes the utility of each agent in her budget set, except for agents whose wealth is zero. Since \( \int_I e^i \, di \gg 0 \), there is a non-null set of agents whose wealth is strictly positive. For these agents to be maximizing utility in their budget sets, it is necessary that prices are strictly positive (otherwise, from Remark 2, their demand

\[ \text{More precisely, an element of } \mathcal{U} \text{ is denoted } \preceq_u \text{ and is a subset of } \mathbb{R}^4 \text{ that corresponds to an interval utility function } u \text{ through the following equivalence:} \]

\[ \preceq_u \equiv \{ (x_L, x_H - x_L, x'_L, x'_H - x'_L) \in \mathbb{R}^4_+ \mid u(x_L, x_H - x_L) \geq u(x'_L, x'_H - x'_L) \} . \]
would be infinite). Then, since prices are strictly positive, all agents have strictly positive wealth, which means that the quasi-equilibrium is an equilibrium.

Since equilibrium exists (in economies with a continuum of agents), we are interested in characterising it. From Remarks 2 and 3, prices are strictly positive and feasibility is exact. But the strongest characterisation results are implied by Remarks 4 and 6. Given a set of completely ambiguous states: individual consumption is constant across all states with the possible exception of a single state (among those in which the price is lowest), in which consumption may be higher. Moreover, prices are constant across all states with the possible exception of the states with lowest aggregate endowment, in which prices may be higher.

For the characterisation of equilibrium and the market-clearing mechanism to become clear, it is instructive to focus on an economy with a simplified structure, \( \tilde{E} \), in which there are \( N \) completely ambiguous states of nature with different aggregate endowments. Supposing, w.l.o.g., that states are ordered according to increasing aggregate endowment, from Remark 6, equilibrium prices are of the form \( (p_1, p_2, \ldots, p_N) = (1, q, \ldots, q) \), with \( 0 < q \leq 1 \). If \( q < 1 \), the market-clearing mechanism is composed by the 1-dimensional adjustment of prices (i.e., of \( q \)) and the \( (N - 2) \)-dimensional adjustment of the distribution of agents in their choice of the state of highest consumption (they are indifferent among states \( \{2, \ldots, N\} \)). If \( q = 1 \), markets clear through the \( (N - 1) \)-dimensional adjustment of the distribution of agents in their choice of state of highest consumption (they are indifferent among all states).

To go a bit deeper and investigate under which conditions there exists an equilibrium with prices that are constant across states \( (q = 1) \), define a simplified demand correspondence, \( \tilde{\gamma}^i : \mathbb{R}_+ \to \mathbb{R}^2_+ \), as follows:

\[
\tilde{\gamma}^i(q) \equiv \arg \max \left\{ u^i(x^i_L, x^i_H - x^i_L) \mid \frac{1}{1 + q(N - 2)} x^i_L + q x^i_H \leq e^i_H + q \sum_{s=2}^{N} e^i_s \right\},
\]
let $L^i$ be the lowest $x^i_L$ in the demand set of agent $i \in \mathcal{I}$ when $q = 1$:

$$L^i \equiv \min \{ \text{proj}_1 (x_L, x_H - x_L) \mid (x_L, x_H - x_L) \in \tilde{\gamma}^i(1) \},$$

and define its aggregate, $L \equiv \int_{\mathcal{I}} L^i$.

**Remark 7.** The simplified economy, $\tilde{E}$, has an equilibrium in which prices are constant across states, $p^* = (1, \ldots, 1)$, if and only if $L \leq \int_{\mathcal{I}} e_1^i \, di$.

**Proof.** ($\Rightarrow$) If $q = 1$, each agent $i \in \mathcal{I}$ maximizes $u^i(x^i_L, x^i_H - x^i_L)$ subject to the budget restriction $(N - 1)x^i_L + x^i_H \leq \sum_{s=1}^{N} e^i_s$. For $p^* = (1, \ldots, 1)$ to be an equilibrium price system, there must exist optimal choices such that $\int_{\mathcal{I}} x^i_L \, di \leq \int_{\mathcal{I}} e_1^i \, di$ (feasibility in state 1). If this condition is not satisfied when we pick the lowest $x^i_L$ (and, consequently, the highest $x^i_H$) from the demand set of each agent $i \in \mathcal{I}$, i.e., if $L > \int_{\mathcal{I}} e_1^i \, di$, there cannot exist an equilibrium with $q = 1$.

($\Leftarrow$) Let us construct an equilibrium with $q = 1$ and $x^i_L = L^i$ (which implies individual maximization). Since $L \leq \int_{\mathcal{I}} e_1^i \, di$, then $(N - 1) \int_{\mathcal{I}} x^i_L \, di < \sum_{s=1}^{N-1} \int_{\mathcal{I}} e^i_s \, di$ (the inequality is strict because $\int_{\mathcal{I}} e^i_1 \, di < \int_{\mathcal{I}} e^i_s \, di, \forall s > 1$). Since agents choose from the frontier of their budget sets, aggregating, we obtain $(N - 1) \int_{\mathcal{I}} x^i_L \, di + \int_{\mathcal{I}} x^i_H \, di = \sum_{s=1}^{N} \int_{\mathcal{I}} e^i_s \, di$. Hence, $\int_{\mathcal{I}} x^i_L \, di \leq \int_{\mathcal{I}} e^i_1 \, di < \int_{\mathcal{I}} e^i_N \, di \leq \int_{\mathcal{I}} x^i_H \, di$. Therefore, there exists a subset $\mathcal{I}_1 \subset \mathcal{I}$ (the set of agents that choose a highest consumption level for state 1), such that $\int_{\mathcal{I} \setminus \mathcal{I}_1} x^i_L \, di + \int_{\mathcal{I}_1} x^i_H \, di = \int_{\mathcal{I}} e^i_1 \, di$ (which is the exact feasibility condition for state 1). There are many ways of constructing the set $\mathcal{I}_1$. For example, from the intermediate value theorem, $\mathcal{I}_1$ can be of the form $[0, a_1]$.\(^7\) Subtracting the feasibility condition in state 1 from the aggregate budget restriction satisfied in equality, we obtain $(N - 2) \int_{\mathcal{I}} x^i_L \, di + \int_{\mathcal{I}_1} x^i_L \, di + \int_{\mathcal{I} \setminus \mathcal{I}_1} x^i_H \, di = \sum_{s=2}^{N} \int_{\mathcal{I}} e^i_s \, di$. Therefore, since $(N - 2) \int_{\mathcal{I}} x^i_L \, di < \sum_{s=3}^{N} \int_{\mathcal{I}} e^i_s \, di$, we have $\int_{\mathcal{I}_1} x^i_L \, di + \int_{\mathcal{I} \setminus \mathcal{I}_1} x^i_H \, di > \int_{\mathcal{I}} e^i_2 \, di$. Hence, there exists $\mathcal{I}_2 \subset \mathcal{I} \setminus \mathcal{I}_1$ such that $\int_{\mathcal{I} \setminus \mathcal{I}_2} x^i_L \, di + \int_{\mathcal{I}_2} x^i_H \, di = \int_{[0, a_1]} x^i_H \, di$ is a continuous and increasing function of $a_1$, being equal to $\int_{\mathcal{I}} x^i_H \, di$ for $a_1 = 0$ and equal to $\int_{\mathcal{I}} x^i_L \, di$ for $a_1 = 1$; while $\int_{\mathcal{I}} e^i_1 \, di$ does not depend on $a_1$ and lies between $\int_{\mathcal{I}} x^i_L \, di$ and $\int_{\mathcal{I}} x^i_H \, di$.\(^7\)

\(^7\)Because $\int_{[0, a_1]} x^i_L \, di + \int_{[0, a_1]} x^i_H \, di$ is a continuous and increasing function of $a_1$, being equal to $\int_{\mathcal{I}} x^i_H \, di$ for $a_1 = 0$ and equal to $\int_{\mathcal{I}} x^i_L \, di$ for $a_1 = 1$; while $\int_{\mathcal{I}} e^i_1 \, di$ does not depend on $a_1$ and lies between $\int_{\mathcal{I}} x^i_L \, di$ and $\int_{\mathcal{I}} x^i_H \, di$.\(^7\)
\( \int_I e^i_H di \) (which is the exact feasibility condition for state 2). Again, there are many ways of constructing \( \mathcal{I}_2 \). For example, if \( \mathcal{I}_1 = [0, a_1] \), from the intermediate value theorem, \( \mathcal{I}_2 \) can be of the form \((a_1, a_2]\).\(^8\) Similarly, for each state \( s \in \{3, ..., N - 1\} \), there exists \( \mathcal{I}_s \subset \mathcal{I}_s \cup [s-1] \mathcal{I}_2 \) such that \( \int_{\mathcal{I}_s} x^i_H di + \int_{\mathcal{I}\setminus\mathcal{I}_s} x^i_L di = \int_I e^i_s di \). Finally, let \( \mathcal{I}_N \equiv \mathcal{I}_1 \cup [N-1] \mathcal{I}_2 \). Adding the exact feasibility conditions obtained by construction for \( s \in \{1, ..., N - 1\} \), we obtain: 
\[
(N - 1) \int_I x^i_L di + \int_{\mathcal{I}\setminus\mathcal{I}_N} x^i_H di - \int_{\mathcal{I}\setminus\mathcal{I}_N} x^i_L di = \sum_{s=1}^{N-1} \int_I e^i_s di.
\]
Subtracting this condition from the aggregate budget restriction, we obtain: 
\[
\int_{\mathcal{I}\setminus\mathcal{I}_N} x^i_L di + \int_{\mathcal{I}_N} x^i_H di = \int_I e^i_N di,
\]
which is exactly the feasibility condition for \( s = N \) (that was the only equilibrium condition yet to be obtained).

\[\square\]

In the case in which \( L > \int_I e^i_1 di \), all equilibria of \( \tilde{E} \) have prices of the form \( p = (1, q, ..., q) \), with \( q < 1 \). To understand that such an equilibrium exists, observe that each agent \( i \in \mathcal{I} \) maximizes \( u_i(x^i_L, x^i_H-x^i_L) \) subject to the budget restriction \( [1 + (N - 2)q] x^i_L + q x^i_H \leq e^i_1 + q \sum_{s=1}^{N-1} e^i_s \). When \( q \to 1 \), optimal choices are such that \( \int_I x^i_L di > \int_I e^i_1 di \), which, from the aggregation of the agents’ budget restrictions satisfied in equality, implies that \( \int_I x^i_H di < \sum_{s=2}^{N} \int_I e^i_s di - (N - 2) \int_I e_1 di \). On the other hand, when \( q \to 0 \), from strict monotonicity, optimal choices are such that \( \lim_{q \to 0} \int_I x^i_H di = +\infty \). Therefore, from continuity of aggregate demand in continuum economies, there exists \( q \) for which optimal choices are such that \( \int_I x^i_H di = \sum_{s=2}^{N} \int_I e^i_s di - (N - 2) \int_I e_1 di \). This, in turn, implies that \( \int_I x^i_L di = \int_I e^i_1 di \). Having obtained the equilibrium prices and individual choices \((x^i_L, x^i_H-x^i_L)\), the remainder of the construction of the equilibrium allocation, i.e., finding \((\mathcal{I}_2, ..., \mathcal{I}_N)\), is analogous to that of the equilibrium made for constant prices.

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\(^8\)Because \( \int_{\{0,a_1\} \cup \{a_2,a_3\}} x^i_L di + \int_{\{a_1,a_3\}} x^i_H di \) is a continuous and increasing function of \( a_2 \), being equal to \( \int_I x^i_L di \) for \( a_2 = a_1 \) and equal to \( \int_{\mathcal{I}\setminus\mathcal{I}_1} x^i_H di \) for \( a_2 = a_1 \); while \( \int_I e^i_2 di \) does not depend on \( a_2 \) and lies between \( \int_I x^i_L di \) and \( \int_{\mathcal{I}\setminus\mathcal{I}_1} x^i_H di \).
6 Example with a continuum of symmetric agents

Consider an economy with three states of nature, and suppose that all agents have the same initial endowment, \((e_1, e_2, e_3) \in \mathbb{R}^+\), with \(e_1 < e_2 < e_3\), and the same interval utility function, \(u(x_L, x_H - x_L)\), which is assumed to be continuous, strictly increasing and strictly quasi-concave. In this scenario, all agents have the same simplified demand function, which, for prices of the form \(p = (1, q, q)\), with \(0 < q \leq 1\), gives a single individually optimal choice of \((x_L, x_H - x_L)\) as a continuous function of \(q\).

As explained in the previous Section, the equilibrium characteristics depend on whether the demanded value of \(x_L\) when \(p = (1, 1, 1)\), denoted \(L\), is smaller or greater than \(e_1\).

**Result 1.** If \(L \leq e_1\), then \(p^* = (1, 1, 1)\) is an equilibrium price vector and the following is a corresponding equilibrium allocation:

\[
x^{i*} = \begin{cases}
(e_1 + e_2 + e_3 - 2L, L, L), & \text{if } i \in [0, a_1) \\
(L, e_1 + e_2 + e_3 - 2L, L), & \text{if } i \in [a_1, a_2] \\
(L, L, e_1 + e_2 + e_3 - 2L), & \text{if } i \in (a_2, 1],
\end{cases}
\]

where \(a_1 = \frac{e_1 - L}{e_1 + e_2 + e_3 - 3L}\) and \(a_2 - a_1 = \frac{e_2 - L}{e_1 + e_2 + e_3 - 3L}\).

**Proof.** The allocation is individually optimal by construction. The exact feasibility conditions for states 1 and 2 allow us to pin down \(a_1\) and \(a_2\):

\[
\begin{align*}
(a_1 x_H + (1 - a_1) x_L &= e_1 \iff a_1 = \frac{e_1 - x_L}{x_H - x_L} \\
(a_2 - a_1) x_H + [a_1 + (1 - a_2)] x_L &= e_2 \iff a_2 - a_1 = \frac{e_2 - x_L}{x_H - x_L}.
\end{align*}
\]

Exact feasibility in states 1 and 2 implies exact feasibility in state 3. \(\Box\)

\(\)
If individual demand for \( x_L \) when \( q = 1 \), denoted \( L \), is greater than \( e_1 \), there cannot exist an equilibrium with symmetric prices (feasibility in state 1 is impossible). We must have \( q < 1 \), which implies that no agent chooses the highest consumption level for state 1. The equilibrium value of \( q \) will be one which implies that individual demand is such that \( x_L = e_1 \), which is the exact feasibility condition for state 1.

**Result 2.** If \( L > e_1 \), then \( p^* = (1, q^*, q^*) \), with \( q^* < 1 \), is an equilibrium price vector and the following is a corresponding equilibrium allocation:

\[
x^{i*} = \begin{cases} 
(e_1, e_2 + e_3 - e_1, e_1), & \text{if } i \in [0, a_2] \\
(e_1, e_1, e_2 + e_3 - e_1), & \text{if } i \in (a_2, 1],
\end{cases}
\]

where \( a_2 = \frac{e_2 - e_1}{e_2 + e_3 - 2e_1} \).

**Proof.** There exists an equilibrium value, \( q^* < 1 \), which implies that the demand for \( x_L \) is equal to \( e_1 \). Existence follows from the intermediate value theorem, because \( x_L > e_1 \) when \( q = 1 \) while \( x_L < e_1 \) when \( q \to 0 \) (since \( x_H \) tends to infinity as it becomes free). The fact that \( x_L < e_1 \) for sufficiently low \( q \) can be verified by writing the budget restriction of the agents in terms of \( x_L \) and \( x_H \):

\[
(1 + 2q)x_L + q(x_H - x_L) = (1 + 2q)e_1 + q(e_2 - e_1) + q(e_3 - e_1)
\]

\[
\iff (1 + 2q)x_L + q(x_H - x_L) = (1 + 2q)e_L + q(e_H - e_L),
\]

where \( e_L \equiv e_1 \) and \( e_H \equiv e_2 + e_3 - e_1 \). As \( x_H \) tends to infinity, we have \( x_H - x_L > e_H - e_L \), which implies that \( x_L < e_L \), i.e., \( x_L < e_1 \).

The value of \( a_2 \) (the measure of agents choosing \( x_H \) for state 2) can be obtained from the feasibility condition for state 2:

\[
a_2(e_2 + e_3 - e_1) + (1 - a_2)e_1 = e_2 \iff a_2 = \frac{e_2 - e_1}{e_2 + e_3 - 2e_1}.
\]

For concreteness, let us now consider a functional form for the utility function.
Result 3. Let \( u(x_L, x_H - x_L) = \ln(x_L) + \alpha \ln(x_H - x_L) \), with \( \alpha > 0 \). In this case:

\[
L = \frac{e_1 + e_2 + e_3}{3(1 + \alpha)} \quad \text{and, if} \quad L > e_1, \quad q^* = \frac{\alpha e_1}{e_2 + e_3 - 2(1 + \alpha)e_1}.
\]

Proof. For given prices of the form \( p = (1, q, q) \), with \( 0 < q \leq 1 \), the simplified problem of the agents is to maximize \( \{\ln(x_L) + \alpha \ln(x_H - x_L)\} \) subject to \( (1 + q)x_L + qx_H = e_1 + q(e_2 + e_3) \). This yields the following demand:

\[
(x_L, x_H - x_L) = \left( \frac{e_1 + q(e_2 + e_3)}{(1 + \alpha)(1 + 2q)}, \frac{\alpha [e_1 + q(e_2 + e_3)]}{(1 + \alpha)q} \right).
\]

Setting \( q = 1 \), we obtain the demand \( x_L = \frac{e_1 + e_2 + e_3}{3(1 + \alpha)} \). Equating the demanded value of \( x_L \) to \( e_1 \), we obtain \( q^* = \frac{\alpha e_1}{e_2 + e_3 - 2(1 + \alpha)e_1} \).

7 Concluding remarks

It has been shown that if agents are expected uncertain utility (EUU) maximisers and there are three or more states of nature regarding which agents face complete ambiguity, an Arrow-Debreu equilibrium rarely exists in economies with a finite number of agents. It does, however, exist in economies with a continuum of agents.

Non-existence of equilibrium results from the clash between the structure of demand that results from EUU maximisation (in a set of completely ambiguous states of nature, demand is constant across all states except one) and the exact feasibility restrictions. The demand-convexifying effect of the continuum of agents is able to resolve this conflict.

The analysis was carried out in a simple framework in which the mechanism that originates non-existence of equilibrium operates, but it extends to more general frameworks.

Agent-specific structures of risk and ambiguity
Agents could have different partitions of the state space, $P^i$, describing their individual perceptions of risk and ambiguity. To address such a scenario, since equilibrium prices may reveal the probabilities of otherwise ambiguous events, the equilibrium concept requires more elaboration. In an equilibrium with full revelation, agents would learn the probabilities of any event in the join (coarsest common refinement) of the partitions of all the agents, $P \equiv \bigvee_{i \in I} P^i$. In an equilibrium with partial revelation, each agent would end up with a partition that may be coarser but not finer than $P$. Therefore, if $P$ contains a cell with three or more states, all the results apply.

General state spaces

Instead of a finite state space, it would be possible to consider a general state space, $(S, \mathcal{P}, \mu)$, where $\mathcal{P}$ is the $\sigma$-algebra of events with known probability and $\mu$ is a prior probability measure on $\mathcal{P}$. Denote by $\mathcal{P}_+$ the set of events $E \in \mathcal{P}$ such that $\mu(E) > 0$. The results extend as long as there are three or more different realisations of the aggregate endowment that are diffuse subsets of an event in $\mathcal{P}_+$.\footnote{This does not mean that trade takes place under asymmetric information. Trade is still assumed to take place ex ante, like in the model of Rigotti, Shannon and Strzalecki (2008) and unlike in the framework of Kajii and Ui (2009).}

CEU and $\alpha$-MEU preferences

Although the exposition has focused on EUU preferences, the results extend to the case in which agents behave as $\alpha$-maxmin expected utility ($\alpha$-MEU) or Choquet expected utility (CEU) maximisers. What is crucial is extreme ambiguity, in the sense that agents cannot probabilistically distinguish between non-empty strict subsets of an element of $P$.

In $\alpha$-MEU theory, extreme ambiguity corresponds to the consideration of all priors that agree with the known probability measure $\mu$ in the $\sigma$-algebra generated by $P$.\footnote{The probability of an event $D$ that is a diffuse subset of an event $E \in \mathcal{P}_+$ can be as low as zero ($D$ does not contain any event in $\mathcal{P}_+$) and as high as $\mu(E)$ (the complement of $D$ in $E$ does not contain any event in $\mathcal{P}_+$).\footnote{If $P$ is the trivial partition, this is the full set of priors.}}
In CEU theory, extreme ambiguity corresponds to the capacity only depending on the larger subset and on the smaller superset that belong to the $\sigma$-algebra generated by $P$, while agreeing with $\mu$ in the $\sigma$-algebra generated by $P$.\textsuperscript{13}

For each event $A$, let $A^-$ denote the larger subset that belongs to the $\sigma$-algebra generated by $P$ and let $A^+$ denote the complement of $A^-$. Accordingly, let $\pi_*(A) = \mu(A^-)$ and $\pi^*(A) = \mu(A^+)$, which are interpreted as the lower and upper bounds on the probability of $A$.\textsuperscript{14} Consider the particular case in which the interval utility function can be written as a convex combination of the utilities of the minimum and the maximum consumption level, i.e., $u'(x'_L, x'_H - x'_L) = \alpha v^i(x'_L) + (1 - \alpha)v^i(x'_H)$. In this case, EUU preferences under extreme ambiguity correspond to CEU preferences with utility index $v^i$ and capacity $\alpha \pi_* + (1 - \alpha)\pi^*$ and to $\alpha$-MEU preferences with utility index $v^i$ and set of priors equal to the core of $\pi_*$. 

\textsuperscript{13}If $P$ is the trivial partition, this is a Hurwicz capacity (Chateauneuf et al., 2007).

\textsuperscript{14}The capacities $\pi_*$ and $\pi^*$ correspond to the Dempster-Shafer belief function and plausibility function, respectively (Dempster, 1967; Shafer, 1976).
References


