Competitive Effects of Partial Control in an Input Supplier

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**Abstract.** Motivated by recent competition policy cases, we study an industry where downstream firms partially own a supplier. If ownership corresponds to control, then consumer surplus is higher and possibly non-monotonic with respect to the ownership share. We provide conditions such that consumers are better off when ownership of the upstream firm is shared by the downstream firms; and when ownership is partial (i.e., less than 100%). These results are based on two effects of partial ownership: first, a vertical-control effect, which effectively reduces the extent of double marginalization; and second, a tunneling effect, whereby the downstream firms use the wholesale price as a means to transfer value from independent upstream shareholders.
1. Introduction

SportTV is Portugal’s main supplier of football (soccer) TV broadcasts. It is currently owned by one of the cable companies (Zon) and a third party (Sportinveste), each with a 50% share. In 2014, an operation was proposed whereby half of Zon’s shares in SportTV would be sold to PT, one of Zon’s rivals in the cable market.

Portugal is by no means the only instance of a cable operator’s upstream acquisition (Waterman and Weiss, 1997): for example, in 2006 BSkyB, a UK leading TV broadcaster, acquired 17.9% stake in ITV, the country’s largest TV content producer. Nor is the cable industry unique when it comes to vertical ownership deals. For example, Scandinavian banking group Nordea owns a partial stake in Bankgirot, a provider of payment system services to banks (Greenlee and Raskovich, 2006).

Vertical ownership by one or more downstream firms raises various concerns, including collusion and the possibility of upstream or downstream foreclosure. In this paper, we examine the unilateral effects of partial ownership of a supplier, with a particular focus on ownership that induces some degree of control (of the upstream firm by the downstream firms).

Our analysis of active shareholdings uncovers two main effects. The first one, which we denote by vertical-control effect, generalizes well-known ideas from the vertical integration literature: in a world of linear wholesale pricing (i.e., absent two-part tariffs or other forms of nonlinear pricing), separation of upstream and downstream control creates a double-marginalization effect. As such, any movement in the direction of vertical integration tends to reduce the double-marginalization effect, to the benefit of consumers. In our context this has the implications that, as far as the vertical-control effect goes, consumer surplus is increasing in the share of the upstream firm owned by downstream firms; and in the concentration of that share among downstream firms.

However, this is not the end of the story: there is a second effect, which we denote by tunneling effect (LaPorta et al., 2000). A downstream firm with partial control of the upstream firm will use its power to transfer value from independent shareholders of the upstream firm. In our model, the only instrument available to the downstream firm is wholesale price: a lower wholesale price effectively transfers value from the upstream independent shareholders, for it decreases the upstream firm’s profits and increases the downstream firm’s profits — and benefits consumers too.

In general, both the vertical-control and the tunneling effects suggest that partial ownership is beneficial to the final consumer. There are, however, two important qualifications. First, to the extent that the downstream firms own different shares of the upstream firm, we have a tunneling externality: a lowering of the wholesale price benefits all downstream firms, not just the firm that attempts to tunnel value from the upstream firm. This implies that, contrary to the vertical-control effect, equal sharing of upstream ownership (as in the SportTV case) may be better for consumers as it internalizes the tunneling externality and increases the extent of this effect.

The second surprising effect of the tunneling effect is that consumer surplus is not necessarily monotonic with respect to the share owned by the downstream firms. In fact, as that share approaches 100% the tunneling effect converges to zero. Although the vertical-control effect is increasing (culminating with vertical integration at 100%), we provide conditions such that the tunneling effect dominantes. When that is the case, consumers are
better off when the downstream firm owns a share of the upstream firm that is greater than 0 but strictly less than 100%.

**Related literature.** We are hardly the first to consider the competitive effects of partial vertical ownership. Considering the variety of results obtained in the literature, it is fair to say that, unlike horizontal acquisitions, the effect of vertical ownership is highly controversial.

Flath (1989) and Greenlee and Raskovich (2006) analyze vertical structures similar to ours: forward integration in the case of Flath (1989) and backward integration in the case Greenlee and Raskovich (2006). However, their analysis is limited to the case of passive ownership. We too begin our analysis by considering the case of passive ownership. This analysis restates previous results and proves some new ones. More important, it establishes a benchmark with which to compare the core of our paper, namely the analysis of ownership with control.

Chen and Ross (2003) and Rossini and Vergari (2011) analyze the related case of input production joint ventures (JV) set up by duopolists that equally own the joint venture. They consider two types of JV governance: (i) the input price is set to maximize the downstream (parent) firm’s aggregate profit; and (ii) the input price is set to maximize the joint venture’s own operating profit. Our analysis differs from theirs in that we focus on partial shareholdings, as well as on the way these shareholdings are distributed among the downstream competitors.

Höffler and Kranz (2011a) and Höffler and Kranz (2011b) find that passive ownership of the upstream bottleneck may be optimal in terms of downstream prices, upstream investment incentives and prevention of foreclosure. However, they assume that wholesale price is exogenously given, whereas the effects of ownership on wholesale price is a key element of our analysis.

Hunold and Stahl (2016) focus on the case when there is (strong) upstream competition. They show that passive backward integration induces an effect which is equivalent to horizontal coordination; it exacerbates double marginalization and increases downstream prices. As acknowledged by the authors, and as we show later in our paper, the assumption of upstream competition is quite important.

All of the above papers consider ownership shares that do not accrue any measure of control. By contrast, we are interested primarily in the situation when ownership is accompanied by control. In this sense, our paper is related to Baumol and Ordover (1994). They show that when an upstream firm controls but partially owns a downstream firm, the efficient use of differently efficient downstream firms may not be guaranteed. In this sense, there is an important difference between full ownership or no ownership, on the one hand; and partial ownership, on the other hand. Our paper is similar in showing that partial ownership may lead to outcomes that are quite different from the opposite extremes of no ownership or full ownership. However, our focus is on final consumer price, rather than the efficient use of downstream firms (our downstream firms are equally efficient).

More recently, Spiegel (2013) examines a model in which (partial) vertical integration affects the incentives of the downstream firms to invest in product quality. The paper shows that, relative to full integration, partial vertical integration may either alleviate or exacerbate the concern for input foreclosure; and examines the resulting implications for consumers.
Levy et al. (2016) show that the effect of partial vertical integration on foreclosure depends on the initial ownership structure of the target firm. Their focus is on the acquisition game itself: determining when a downstream firm has an incentive to acquire an input supplier at a particular price. They assume that control is obtained by acquiring a share greater than some threshold $\alpha$. They show that partial backward integration, which leads to input foreclosure, is particularly profitable when the upstream supplier is initially held by dispersed shareholders. Our focus and our approach are different. First, we take ownership shares as given and look at their impact of firm prices. Second, instead of assuming a minimum threshold for full control we assume that control is proportional to shareholding. In this sense, we follow the assumption in O’Brien and Salop (2000). However, whereas O’Brien and Salop (2000) apply their framework to the study of horizontal cross-ownership, we are primarily interested in vertical relations.

**Roadmap.** Section 2 lays down the basic model, the timing of events, and our assumption regarding ownership and control. Section 3 looks at the case of passive shareholding, a case that links our analysis to previous research and serves as a benchmark to the central part of the paper, Section 4, where we look at shareholdings that allow for partial control. Section 5 considers various extensions: $n > 2$ downstream firms, non-linear demand, wholesale price discrimination, downstream price competition, and nonlinear ownership-to-control mappings. Section 6 concludes the paper.

### 2. Model

There is an upstream monopolist, which we denote by Firm 0; two downstream retailers, Firms 1 and 2, each owning a share $s_i$ in Firm 0, $(i = 1, 2)$; and an independent shareholder, Firm 3, owning $s_3$ of Firm 0.

Each firm $i$ $(i = 1, 2)$ maximizes its own value:

$$v_i = \pi_i + s_i \pi_0$$

where $\pi_i$ is Firm $i$’s profit, $i = 0, 1, 2$. The independent shareholder, in turn, maximizes its shareholding value:

$$v_3 = s_3 \pi_0$$

#### Passive and active control.

We consider two possible cases regarding the nature of Firm 0’s ownership. First, we consider passive (or silent) ownership. In this case, Firm 0 looks after its shareholders’ interests in a narrow sense:

$$v_0 = \pi_0$$

Alternatively, we also consider the possibility of active ownership (or control). There are many ways in which this can be modeled. Following O’Brien and Salop (2000), we consider the specific case of proportional control, whereby the weight given by Firm 0 to a given shareholder is proportional to that shareholder’s share:

1. In Brito et al. (2014), we extend O’Brien and Salop (2000)’s analysis of horizontal cross-holdings. In Section 5, we consider the extension to a more general ownership-to-control mapping.
2. The extension to the $n$ case is fairly straightforward; see Section 5.
3. For notational simplicity, some expressions include $\pi_3$; we simply assume $\pi_3 = 0$. 

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Proportional control refers to a (...) scenario in which the Board and managers of the acquiring firm take into account their shareholders’ interests in other firms. However, rather than trying to maximize joint profits, they take the shareholders’ interests into account in proportion to their financial interests in the acquired firm. For example, if the acquiring firm is the only competitor with a financial interest in the acquired firm and it has a 25 percent stake, then the acquired firm’s managers will make pricing and output decisions as if the acquired firm has a 25 percent financial interest in the acquiring firm. (O’Brien and Salop, 2000)

Formally, this corresponds to the assumption that the upstream firm maximizes

\[ v_0 = \sum_{i=1}^{3} s_i v_i \]

The proportional control assumption is not entirely innocuous and deserves further discussion. An alternative assumption is that the acquirer has full control of the target if and only if its share is greater than 50%. However, there are cases when a lower share arguably allows for full control. For example, News Corp. acquired a de facto control in Hughes Electronics Corporation in 2004 by acquiring a 34% stake; and the European Commission concluded that BSkyB’s 2006 acquisition of a 17.9% stake in ITV in 2006 gave the acquirer effective control. On the other hand, the case can be made that minority shareholders may exert some power, to the extent that even a majority shareholder does not have full control.

We believe that our assumption of proportional control provides a reasonable benchmark. In Section 5, we consider the extension to a nonlinear (logistic) ownership-to-control mapping (of which the 50%-threshold rule is a limit case), concluding that the main qualitative results remain valid.

**Demand and costs.** We assume downstream competition takes place à la Cournot: firms simultaneously set output levels of a homogeneous product. Moreover, we assume that final demand is linear. With no further loss of generality, we assume that the final inverse demand takes the form

\[ p = 1 - Q \]

where \( Q \) is total output; whereas retailer cost, other than the wholesale price, is zero.

**Timing.** The timing of the game is as follows. First, Firm 0 sets wholesale price \( c \) to maximize \( v_0 \). We assume that Firm 0 cannot discriminate between downstream firms. Then, given a wholesale price, Firms 1, 2 simultaneously set output \( q_i \) to maximize \( v_i \).

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5. See Paragraph 2 in Competition Commission, 2007, “Acquisition by British Sky Broadcasting Group plc of 17.9 per cent of the Shares in ITV plc.”
6. In Section 5, we consider the extension to non-linear demand and to downstream price competition. We conclude that the main qualitative results remain valid.
7. In Section 5, we consider the extension to the case when Firm 0 sets downstream-specific wholesale prices \( c_i \). We conclude that the main qualitative results remain valid.
3. Passive ownership and consumer welfare

In this section we consider the case of passive ownership. This is the case when Firm 0 simply maximizes its own profit with no regard to its investors’ interests beyond the share they own in Firm 0. We first show (in line with previous results in the literature) that consumer surplus is invariant with respect to Firm 0’s ownership structure, except for the possibility of Firm 2’s foreclosure.\footnote{In a related result, Bergstrom and Varian (1985), Salant and Shaffer (1999) and Van Long and Soubeyran (2001) show that a mean-preserving spread of marginal costs has no effect on total output or consumer surplus.} Recall that \( s \) denotes the ownership share of downstream Firm \( i \) \((i = 1, 2)\) in the upstream Firm 0.

**Proposition 1.** Under passive ownership, consumer surplus is invariant with respect to the level of \( s \) or its distribution among downstream firms so long as both are active. However, if \( s_1 > \frac{2}{5} + \frac{3}{5} s_2 \) then \( q_2 = 0 \) and \( Q \) is lower than if \( s_1 < \frac{2}{5} + \frac{3}{5} s_2 \).

In order to understand the first part of Proposition 1, which restates Proposition 2 in Greenlee and Raskovich (2006), notice that the upstream firm’s profit is simply given by wholesale price, \( c \), times downstream demand, \( Q \). The value of \( Q \) in turn depends on \( c \) and \( s_i \). Specifically, an increase in \( s_i \) decreases firm \( i \)’s effective marginal cost (part of the payment to Firm 0 is recouped in the form of dividends). We now come to an important feature of Cournot competition: equilibrium output is a function of the sum of costs, not of each firm’s individual cost. In fact, as we show in Section 5, this is a property of any aggregative game, of which Cournot competition is a particular case.

Given that final demand is linear, it follows that Firm 0’s profit has the form

\[
\pi_0 = c \left(1 - c \left(1 - s/2\right)\right)
\]

This problem is isomorphic to the problem of monopoly profit maximization with variable market size, where \( c \) is price and \( s \) a parameter affecting market size. And under linear demand optimal monopoly price is independent of market size.\footnote{By market size we mean the value of \( \varsigma \) in \( Q = \varsigma D(p) \).}

Intuitively, as \( s \) increases the effective marginal cost faced by downstream firms is lower, as they anticipate recouping some of the input costs in the form of dividends. This implies that, for a give value of \( c \), a higher \( s \) leads to higher input demand. And anticipating this increase in derived demand, the upstream firm finds it optimal to increase the wholesale price.

In sum, we have two steps leading to the irrelevance result stated in the first part of Proposition 1. First, the wholesaler’s optimal solution is independent on the share distribution among downstream firms (a property that does not require the assumption of demand linearity; see Section 5). Second, the wholesaler’s optimal output is independent on the value of the total share held by downstream firms (a property that is special to the linear demand case).

When \( s_1 > s_2 \), the wholesaler is conflicted in its choice of \( c \): it would like to set a high \( c \) for Firm 1, since the latter is less wholesale-price sensitive. Since the wholesaler is unable to price discriminate (a possibility we consider in Section 5), the wholesaler sets a value of \( c \) that strikes a balance between optimal \( c_1 \) and optimal \( c_2 \). However, if \( s_1 \) is substantially higher than \( s_2 \), then the wholesaler’s optimal \( c \) effectively squeezes Firm 2.
out of the market. Given that Firm 2 is no longer part of the equation, we observe a discontinuous increase in $c$ (it is now the optimal $c$ set to Firm 1 only). This in turn leads to a discontinuous decrease in $Q$, as stated in the second part of Proposition 1.

Figure 1 summarizes the results of Proposition 1 for the particular case when $s_2 = 0$ and $s_1 = s$. The left axis measures output levels, both firm level and total; whereas the right axis measures wholesale price. As $s_1 = s$ increases, Firm 0, anticipating that Firm 1’s effective costs is lower than $c$, increases the value of $c$, though at a lower rate than the increase in $s$. As a result, Firm 1 increases its output. Firm 2, facing a higher wholesale price, as well as a higher output by Firm 1, reduces its output. Firm 2’s output decrease is exactly compensated by Firm 1’s increase, so that total output $Q$ remains constant.

If $s$ is greater than $\frac{2}{5}$, then Firm 0 finds it optimal to increase wholesale price so that Firm 2 is effectively excluded from the market. The assumption that Firm 0 is unable to price discriminate between Firms 1 and 2 plays an important role here (as we will see in greater detail in Section 5). Firm 0 would like to increase Firm 1’s cost by a greater amount, for since Firm 1 owns a share in Firm 0, Firm 1 is less sensitive to increases in wholesale price. We thus have a discontinuity in Firm 0’s optimal strategy: once it decides to set a $c$ that forecloses Firm 2, it increases $c$ to the Firm 1 optimal level.

If the assumptions in this section were true, then we would expect Sport TV and Zon to squeeze PT out of the cable market by setting a high wholesale price. (Recall that Zon’s share $s$ is greater than $\frac{2}{5}$, the critical threshold in Proposition 1.) We know that PT (and a much smaller rival) have not been squeezed out of the Portuguese cable market. The difference between model and reality may be reconciled in several ways. First, the threshold value $s = \frac{2}{5}$ depends on the particular functional form we considered, namely linear demand. Second, price competition may be a better description of the retail competition mode (in Section 5 we consider the extension to price competition at the retail level). Finally, and more important, the assumption of passive ownership may be unwarranted; and as we show

10. Greenlee and Raskovich (2006) assume that changes in ownership do not induce entry or exit (cf their Assumption 3). As such, they effectively exclude the possibility considered in the second part of Proposition 1.
in the next section, foreclosure fails to take place when Firm 1 has (partial) control over Firm 0.

4. Active ownership and consumer welfare

We now consider the possibility of active ownership. Specifically, we consider the case when a firm’s control is proportional to its shareholding of Firm 0. In other words, following O’Brien and Salop (2000) we assume that Firm 0 maximizes a value function that consists of the weighted sum of its shareholders’ value functions, with weights given by the shareholders’ shares:

\[ v_0 = \sum_{i=1}^{3} s_i v_i \]

where

\[ v_i = \pi_i + s_i \pi_0 \]

We first show that, contrary to the case of passive holdings, consumer welfare is increasing in the share held by downstream firms.

**Proposition 2.** (a) consumer surplus is greater when the downstream firms’ share corresponds to active control; (b) there exists \( s' \in (0, 1) \) such that, if \( s < s' \), then, under active control, consumer surplus is increasing in \( s \).

Both parts of Proposition 2 result from two effects of vertical ownership. First there is the well-known problem of double marginalization in vertical structures. Since we assume linear wholesale pricing, separate pricing by upstream and downstream firms leads to inefficiently high retail prices. Upstream firm control by the downstream firms helps alleviate this double-marginalization problem. This explains that consumer surplus is higher with active ownership as well as that it may be increasing in the share of the upstream firm owned by the downstream firms.

A second effect of (partial) vertical ownership is tunneling, by which we mean the strategy of transferring value from Firm 0 to its shareholders. We do not necessarily refer to the legality or morality of the practice, rather to the fact that, given our assumption of partial control, the value of \( c \) (wholesale price) takes into account the interests of the payers, namely Firms 1 and 2. A lower value of \( c \) is effectively a means of transferring value from Firm 3 (the independent shareholder) to Firms 1 and 2. To the extent that the downstream firms have control over the decision of setting \( c \), we would expect lower values to be set, which ultimately benefits consumers.

We note that the restriction \( s < s' \) is necessary. In particular, as we will show later, the tunneling effect is not monotonic in \( s \) and in fact may counter the double marginalization effect to the point that consumer welfare is decreasing in \( s \).

Recall that the motivating example of Sport TV and Zon, to which we alluded in the introduction, focused on the sale of a share owned by a downstream firm to another downstream firm. Proposition 2 is about the total holdings by downstream firms, not their

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11. In Section 5 we consider a more general ownership-to-control mapping. The qualitative results remain valid, which suggests that the main difference is between passive and active ownership, not the particular mapping of ownership to control.
distribution among downstream firms. The next result deals with the issue of the distribution of shares.

**Proposition 3.** Ownership concentration increases consumer welfare if and only if \( s < \frac{1}{2} \).

Concentration of shareholdings has two effects of opposite sign. First, given our assumption of proportional control, total control by downstream firms is increasing in the concentration of their holdings of the upstream firm, just as the market power commanded by two firms is greater the greater the concentration of their market shares. In fact, the proof of Proposition 3 shows that the index

\[
h \equiv \sum_{i=1}^{2} \left( \frac{s_i}{s} \right)^2
\]

(akin to the Herfindahl-Hirschman index of market concentration) is a sufficient statistic of the distribution of ownership shares for the purpose of consumer welfare. A higher value of \( h \) corresponds to a greater concentration of ownership shares and, ultimately, a more effective control of the upstream firm by the downstream firms. And, as suggested by Proposition 2, for low values of \( s \) this greater control by the downstream firms leads to greater consumer welfare, essentially by alleviating the double-marginalization problem.

But this is not the end of the story. Concentration also implies a divergence of interests between the downstream firms. Specifically, to the extent that Firm 0 cannot discriminate between downstream firms, we are in the presence of a *tunneling externality:* as Firm 1 exerts its power to bring down the value of \( c \), it effectively benefits its rival as well; and it harms itself, for a transfer from Firm 0 to Firm 2 amounts to a transfer from Firm 1’s shareholders to Firm 2’s shareholders. For this reason, balanced shareholdings, by internalizing the tunneling externality, may lead to a lower value of \( c \). Proposition 3 shows that, for high values of \( s \), the effect of internalizing the tunneling externality outweighs the effect of greater control of the upstream firm.

Since the share owned by Zon was precisely \( s = \frac{1}{2} \), Proposition 3 would predict a neutral effect in terms of consumer surplus. Naturally, this result depends critically on the value of \( s \) as well as on our functional form assumptions. More on this in Section 5. However, to the extent that all of our expressions are continuous functions, we would expect that the unilateral effect of Zon’s sale would be relatively small.

Figure 2 illustrates Proposition 3. It plots total output as a function of \( s \) in two extreme cases: \( (s_1 = s, s_2 = 0) \); and \( (s_1 = s_2 = s/2) \). As can be seen, the two curves cross at \( \frac{1}{2} \), with the curve corresponding to \( (s_1 = s_2 = s/2) \) reaching higher values of \( s > \frac{1}{2} \).

Figure 2 also depicts an intriguing feature of total output under the shared-ownership case (dashed line), namely that it is non-monotonic with respect to changes in \( s \). Our last result in this section formalizes this point:

**Proposition 4.** Under shared control by downstream firms, that is, \( s_1 = s_2 = s/2 \), consumer surplus is highest when \( s = s^* \in (0, 1) \).

Proposition 4 shows that Proposition 2 is “tight,” that is, the restriction that \( s < s' \), required for part (b), is necessary. The intuition for Proposition 4 is closely related to the tunneling effect referred to above. When \( s_1 = s_2 = s/2 \), there is no tunneling externality: both downstream firms agree that a lower \( c \) is good insofar as it transfers value from Firm 3
to Firms 1 and 2. However, to the extent that upstream and downstream pricing decisions are independent, a lower \( c \) induces a more competitive behavior by Firms 1 and 2. In other words, rather than the tunneling externality referred to above, we now have a tension between the goal of transferring value from Firm 3 and the goal of not transferring value to consumers. The equilibrium value of \( c \) strikes a balance between these two goals. As the value of \( s \) converges to 1, the goal of transferring value from Firm 3 becomes relatively less important, for the simple reason that Firm 3’s profits are very small. Thus, the downward pressure on \( c \) is alleviated, which in turn results in a lower value of \( Q \).

Another way to understand this intuition — and also a way to show how general it is — is to consider values of \( s \) in the neighborhood of 1. At \( s = 1 \) we have vertical integration and the value of \( Q \) corresponds to that of a vertically integrated monopoly.\(^{12}\) If \( s \) is close to, but less than, 1, then a small change in output level implies a second-order change in the downstream firms’ profits (by the envelope theorem). However, the tunneling effect is of first order. This implies that, as \( s \) drops from 1 to a value slightly lower than 1, the equilibrium value of \( c \) decreases and the equilibrium value of \( Q \) increases.

5. Extensions

In this section, we consider various extensions to our basic framework. We consider the case of more than two downstream firms, non-linear final demand, discriminatory wholesale prices, downstream price competition, and finally a non-linear ownership-to-control mapping.

- \( n > 2 \) retailers. In all of our analysis we assumed there are only two retailers. However, our results can be extended to the \( n \)-retailer case. In fact, several of our results (e.g., Proposition 3) can be stated in terms of total ownership \( s = \sum s_i \) and the Herfindahl-type concentration coefficient of ownership shares by downstream firms, \( h = \sum s_i^2 \), both

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\(^{12}\) As Chen and Ross (2003) show, the downstream competitors effectively use joint input ownership as a means to achieve the monopoly solution.
of which are independent of the number of retailers. The general n-retailer case generates more complicated algebra but does not produce any additional intuitions with respect to the two-retailer case.

**Non-linear consumer demand.** Considering that Cournot competition is an aggregative game, we can extend our analysis to the case when consumer demand is non linear. For example, for a given (common) wholesale price, it can be shown that total output is only a function of the aggregate ownership share \( s = \sum s_i \). Beyond that, generalizing our various results to the case of non-linear demand requires a series of conditions regarding the curvature of demand. It can also be shown that all results are continuous with respect to the primitive functional forms. Therefore, it is not surprising that, if the demand curve is neither too convex nor too convex, then the main qualitative results remain valid.

**Discriminatory wholesale pricing.** In the US, EU and other jurisdictions, several limitations are imposed on upstream firms’ ability to discriminate among downstream firms. For example, the US Robinson–Patman Act prohibits price discrimination where competition may substantially be harmed; and Articles 101.1(d) and 102(c) of the Treaty of the European Union prohibit sellers from applying dissimilar conditions to equivalent transactions. Consistently with this pattern, in the previous sections we assumed that Firm 0 sets the same wholesale price to Firms 1 and 2. In practice, however, there may be ways of discriminating between downstream firms (at least partly). Moreover, the comparison between discrimination and no-discrimination is interesting as a basis for policy making. For these reasons, we consider the extension to the case when wholesale prices are retailer specific: \( c_i \).

By symmetry, if \( s_1 = s_2 = \frac{1}{2}s \) then Firm 0’s optimal wholesale prices are \( c_1 = c_2 = c \); and discrimination becomes a moot point. Our discussion of discrimination thus focuses on the (opposite extreme) case when \( s_2 = 0 \) and \( s_1 = s \). Our main result is that discrimination is good for consumers if and only if the share of the upstream firm owned by the downstream firms is sufficiently high.

**Proposition 5.** For each form of ownership (active or passive), there exists \( s' \in (0, 1) \) such that discriminating wholesale prices lead to weakly higher consumer surplus if and only if \( s > s' \).

Figure 3 illustrates Proposition 5. It plots the value of total output as a function of \( s \). Five different cases are considered. Four cases correspond to the combinations passive ownership (P) and partial control (A); discrimination (D) or no discrimination (ND). Finally, for reference, the case when the two downstream firms hold equal (active) shares in the upstream firm is also considered (case A/50-50, already plotted in Figure 2). (Note that, in this case, it makes no difference whether there is or there isn’t price discrimination: even if Firm 0 could discriminate, its optimal solution is not to.)

The more salient feature of Figure 3 is that, for high enough \( s \), consumers are better off when price discrimination is allowed. Consider first the case of passive ownership. Since Firm 0 is aware that Firm 1’s effective marginal cost is given by \( c(1 - s) \), Firm 0 would like to set a high value of \( c \). However, absent discrimination such cost must also be offered

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13. For a related analysis of price discrimination in input markets, see Inderst and Valletti (2009).
to Firm 2. One way to “solve” this trade-off is to set a value of $c$ that is high enough to foreclose Firm 2. This is bad for consumers, for a downstream duopoly turns into a downstream monopoly. By contrast, once wholesale price discrimination is allowed there is no need to foreclose Firm 2 from the market. (The downside is that, if $s$ is low enough that Firm 2 would not be excluded under no discrimination, allowing for discrimination allows Firm 0 to partially squeeze Firm 2 out of the market, to the detriment of consumers.)

Consider now the case of partial control of Firm 0 by Firm 1. If $s$ is high enough, Firm 0 now faces an additional trade-off. Since its interests are partially Firm 1’s interests, Firm 0 would like to lower the wholesale price it offers Firm 1 as a way of reducing double marginalization and transferring profit from independent shareholders to Firm 1. If no discrimination is allowed, such tendency is checked by the fact that a lower wholesale price to Firm 1 is also a lower wholesale price to Firm 2. If discrimination is allowed, then Firms 0 and 1 are able to better solve the double marginalization problem, which implies a gain to Firms 0 and 1 but also to consumers.

Finally, Figure 3 helps understand that our assumption of no-discrimination is not very important for the results presented in the previous section. First, Proposition 2, stating that under active ownership consumer surplus is increasing for small $s$ (and higher than under passive ownership) is also true in the case when Firm 0 can price discriminate. Second, Proposition 3, while not true literally under price discrimination, is still qualitatively true under price discrimination. In particular, as Figure 3 shows, for high enough values of $s$, consumer welfare is higher when the downstream firms’ ownership shares are equal rather than concentrated on Firm 1. Finally, Proposition 4, stating that consumer welfare is non-monotonic with respect to $s$, is proven by considering the case when $s_1 = s_2$. As mentioned earlier, in this case it does not matter whether discrimination is or is not allowed: even if it is, Firm 0 finds it optimal not to.

**Price competition.** In the previous sections we assumed downstream quantity competition; we now consider the extension to the case of price competition. We consider a
Hotelling model with “hinterlands” as in Armstrong and Wright (2009). Firms 1 and 2 are located at the extremes of a line segment of length 1. Consumers are distributed uniformly from $-1$ to $2$, have a reservation value $v$, and incur a transportation cost $t$. Consumers to the left of 0 or to the right of 1 correspond to each firm’s “hinterland.”

Let $x$ denote the address of the indifferent consumer. Consumers located in $[0,1]$ purchase from Firm 1 if and only if

$$p_1 + tx < p_2 + t(1-x)$$

or simply

$$x < \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

Consumers in $[-1,0]$ purchase from Firm 1 if and only if

$$v - p_1 - tx > 0$$

or simply

$$x < \frac{v - p_1}{t}$$

It follows that Firm 1’s demand function is given by

$$q_1 = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{v - p_1}{t}$$

A similar expression applies for Firm 2.

We then solve the game in a similar way to the downstream quantity competition case. Figure 4 depicts consumer surplus under active control when shares are concentrated in Firm 1 or equally split between Firms 1 and 2. Comparing to Figure 2, we observe a similar qualitative pattern: for high values of $s$, consumers are better off under shared ownership; and consumer surplus under shared ownership is a non-monotonic function of $s$.

There are however two differences between downstream price and output competition. First, unlike Figure 2, Figure 4 shows that, when $s = 1$, consumer welfare is different.
under shared and concentrated ownership. Under price competition, the \((s_1 = s, s_2 = 0)\) case corresponds to a vertically integrated firm, whereas the \((s_1 = s/2, s_2 = s/2)\) case corresponds to an Input Production Joint Venture (IPJV). Under quantity competition with homogeneous product both result in the same consumer surplus. Not so under product differentiation: the IPJV corresponds to the full merger outcome (Chen and Ross, 2003), which differs from vertical integration by one firm when another independent retailer is active. From the consumers’ perspective, the effect of internalizing the vertical externality dominates the effect of internalizing the horizontal externality.

A second difference is that, unlike Cournot downstream competition, Hotelling competition breaks the neutrality result under passive ownership: As shown by Greenlee and Raskovich (2006), neutrality only takes place when all downstream firms hold equal shares in the upstream firm.

**Non-linear ownership-to-control mappings.** Our analysis of ownership and control is based on O’Brien and Salop’s (2000) linearity assumption: the weight that Firm 0 assigns to Firm \(i\)’s profit is proportional to the latter’s ownership of the former. We believe this is a reasonable benchmark, but it certainly is not the only reasonable assumption. One might argue that a shareholder has control of a firm if and only if the shareholder owns more than 50% of the shares. Proportional control and the 50% rule provide two extreme assumptions regarding the mapping from ownership to control.

In order to assess the importance of the ownership-to-control mapping, we consider a general logistic functional form. Specifically, we assume that the weight given to shareholder \(i\) is given by \(w_i/\sum w_j\), where

\[
w_i = \frac{s_i}{1 + \exp(-\alpha (s_i - 1/2))}
\]

Under this formulation, the O’Brien and Salop (2000) proportional-control case corresponds to \(\alpha = 0\), whereas the 50% rule corresponds to the limit as \(\alpha \to \infty\). For strictly positive,
finite values of $\alpha$ we get a family of logistic functions symmetric with respect to $(.5,.5)$ that are steeper the greater the value of $\alpha$.

The analysis of ownership and control proceeds very much in the same way as before, with the difference that the value function maximized by Firm 0 now includes the general weights (1) rather than simply $s_i$. Figure 5 summarizes the analysis. As before, we measure output under active ownership and downstream Cournot competition. The light lines, taken from Figure 2, correspond to proportional control (linear ownership-to-control mapping); the dark lines, by contrast, correspond to $\alpha = 10$. As in Figure 2, the solid lines correspond to the case of concentrated ownership ($s_1 = s, s_2 = 0$), whereas the dashed lines correspond to the case of shared ownership ($s_1 = s_2 = \frac{1}{2}s$).

Figure 5 suggests that the effects of a non-linear ownership-to-control mapping are to make the relation between ownership and welfare more "drastic," that is, more dependent on the degree of ownership. However, we also observe that the main qualitative features — the relation between concentrated and shared ownership, the non-monotonicity of welfare with respect to $s$ — remain unchanged.

The various curves are continuous in $\alpha$, and similar patterns are observed for different values of $\alpha$. All in all, we believe our analysis is fairly robust with respect to specific assumptions regarding the relation between ownership and control, so long as it satisfies continuity at the extremes (that is, no ownership and complete ownership).

6. Concluding remarks

In July 2014, the Portuguese Competition Authority blocked the sale by Zon of a 25% share in SportTV. In terms of our paper’s notation, this would imply a shift from $s_1 = \frac{1}{2}, s_2 = 0$ shareholder structure to $s_1 = s_2 = \frac{1}{4}$. The Portuguese Competition Authority justified its decision with arguments related to collusion and market foreclosure. While these arguments are valid — and arguably of primary importance in the present case — we show that, in general, they are incomplete: one must also consider the unilateral effects of the operation.

When unilateral effects are taken into consideration, we show that comparative statics with respect to consumer surplus can be quite complex. In particular, we show that shared ownership may lead to higher consumer welfare; and that consumer welfare may be non-monotonic with respect to the downstream ownership of an upstream firm.

Our extensions show that the results are relatively robust with respect to assumptions such as the ability of the upstream firm to price discriminate or the nature of downstream competition. There is one assumption, however, that likely plays a central role: linear contracts. To our defense, there is evidence that in some industries, such as the US and Portuguese cable TV industries, linear contracts are the norm (Crawford and Yurukoglu, 2012). There are also industries where the opposite is true, that is, two-part tariffs and other forms of non-linear pricing play an important role. This suggests that, as far as modeling vertical relations is concerned, one size does not fit all.
Appendix

Proof of Proposition 1: Suppose first that both firms are active, that is, \( q_i > 0 \). Firm \( i \) \((i = 1, 2)\) maximizes
\[
v_i = (1 - c - Q) q_i + s_i c Q
\]
The first-order condition for value maximization is given by
\[
q_i^* = \frac{1}{2} (1 - c (1 - s_i)) - \frac{1}{2} q_j
\] (2)
Adding up and simplifying we get
\[
\hat{Q} = \frac{2}{3} \left( 1 - c \left( 1 - \frac{1}{2} s \right) \right)
\] (3)
where \( s = s_1 + s_2 \). If follows that Firm 0’s profit (and Firm 0’s value) only depends on \( s_i \) through the sum \( s = s_1 + s_2 \). This proves the first part of the proposition.

Continuing to assume that both firms are active, Firm 0’s optimum is simply determined by maximizing
\[
\pi_0 = c Q = \frac{2}{3} c \left( 1 - c \left( 1 - \frac{1}{2} s \right) \right)
\]
which implies
\[
\hat{c} = \frac{1}{2 - s}
\] (4)
\[
\hat{Q} = \frac{1}{3}
\]
\[
\hat{v}_0 = \frac{1}{3 (2 - s)}
\] (5)
For this solution to be valid, it must be that \( q_2 > 0 \), as assumed. Substituting the optimal \( c \) into Firm 2’s equilibrium output level, we get
\[
s_1 < \frac{1 + s_2}{2}
\]
Suppose now that \( c \) is high enough that \( q_2 = 0 \). Then
\[
\hat{q}_1 = \frac{1}{2} (1 - c (1 - s))
\]
Upstream optimization then implies
\[
\hat{c} = \frac{1}{2 (1 - s)}
\]
which in turn implies
\[
\hat{Q} = q_1 = \frac{1}{4}
\]
and
\[
\hat{v}_0 = \frac{1}{8 (1 - s)}
\] (6)
Comparing (5) and (6), we conclude that foreclosure is Firm 0’s optimal choice if and only if
\[
s_1 > \frac{2}{5} + \frac{3}{5} s_2
\] (7)
Finally, one can check that, if (7) does not hold and \( c \) is given by (4), then equilibrium \( q_2 \) is positive as assumed. ■

**Proof of Proposition 2:** Firm 0’s value is given by

\[
v_0 = \sum_{i=1}^{3} s_i v_i = \sum_{i=1}^{3} s_i (s_i \pi_0 + \pi_i) = \sum_{i=1}^{3} s_i^2 c (q_1 + q_2) + (1 - q_1 - q_2 - c) \sum_{i=1}^{2} s_i q_i \tag{8}
\]

Solving (2), we get

\[
\hat{q}_i = \frac{1}{3} (1 - c - s_j + 2 s_i) = \frac{1}{3} (1 - c - s_i) + s_i
\]

\[
\hat{Q} = \frac{1}{3} (2 - 2 c + s) \tag{9}
\]

Substituting for \( q_i \) in (8) and simplifying, we get

\[
v_0 = \frac{1}{3} c (1 - s)^2 (cs + 2 (1 - c)) + \frac{1}{3} s (1 - c (1 + s)) (1 - c (1 - 2 s)) + \frac{1}{3} h s^2 c (2 - c (2 - s)) - \frac{1}{3} (1 - h) s^2 c (1 - c (1 + s)) \tag{10}
\]

where

\[
s \equiv \sum_{i=1}^{2} s_i
\]

\[
h \equiv \sum_{i=1}^{2} \left( \frac{s_i}{s} \right)^2
\]

Solving the first-order condition for maximizing \( v_0 \) with respect to \( c \) we get

\[
c^* = \frac{6 - 14 s + (4 + 9 h) s^2}{12 - 32 s + 2 (10 + 9 h) s^2 - 8 s^3} \tag{11}
\]

It follows that

\[
\frac{dc^*}{ds} \bigg|_{s=0} = \frac{1}{6}
\]

From (9), it follows that

\[
\frac{d\hat{Q}}{ds} \bigg|_{s=0} = \left( \frac{\partial \hat{Q}}{\partial s} + \frac{\partial \hat{Q}}{\partial c} \frac{dc^*}{ds} \right) \bigg|_{s=0} = \frac{1}{3} \left( 1 - \frac{2}{6} \right) > 0
\]

At \( s = 0 \), \( Q \) is the same with passive or active control. Moreover, Proposition 1 implies that \( Q \) is invariant with respect to \( s \). Finally, it is straightforward to check that all relevant functions are continuous at \( s = 0 \). ■

**Proof of Proposition 3:** Substituting (11) for \( c \) in (9) we get the equilibrium value

\[
Q^* \equiv Q^*_N / Q^*_D,
\]

where

\[
Q^*_N = 4 - 10 s + 6 s^2 (1 + h) - s^3 (4 - 3 h)
\]

\[
Q^*_D = 12 - 32 s + 2 s^2 (10 + 9 h) - 8 s^3 \tag{12}
\]
From the differentiation rule for fractions we know that

\[
\text{sgn}\left( \frac{dQ^*}{dh} \right) = \text{sgn}\left( \frac{dQ^*_N}{dh} Q^*_D - \frac{dQ^*_D}{dh} Q^*_N \right) = \text{sgn}\left( 24 s^3 (1 - s) (2 - s) (\frac{1}{2} - s) \right)
\]

which is positive if and only if \( s < \frac{1}{2} \). □

**Proof of Proposition 4:** Substituting \( \frac{1}{2} \) for \( h \) in (12), we conclude that \( Q^* \equiv Q^*_N/Q^*_D \), where

\[
\begin{align*}
Q^*_N &= 4 - 10 s + 9 s^2 - \frac{1}{2} s^3 \\
Q^*_D &= 12 - 32 s + 29 s^2 - 8 s^3
\end{align*}
\]

From the differentiation rule for fractions we know that

\[
\text{sgn}\left( \frac{dQ^*}{ds} \right) \bigg|_{s = 1} = \text{sgn}\left( \frac{dQ^*_N}{ds} Q^*_D - \frac{dQ^*_D}{ds} Q^*_N \right) \bigg|_{s = 1} = \text{sgn}(6.5 - 4 \times 2.5)
\]

which is negative. □

**Proof of Proposition 5:** Consider first the case of passive ownership. With input price discrimination, retailers’ value functions are equal to

\[
v_i = (1 - q_i - q_j - c_i) q_i + s_i (c_i q_i + c_j q_j)
\]

From the first-order conditions we obtain

\[
q_i = \frac{1}{3} + \frac{1}{3} c_j (1 - s_j) - \frac{2}{3} c_i (1 - s_i)
\]

Under passive ownership, the upstream firm maximizes

\[
v_0 = \pi_0 = \sum c_i q_i
\]

where \( q_i \) is given by (13). The corresponding first-order conditions result in

\[
c_i^* = \frac{6 - s_i - 5 s_j}{12 (1 - s_j) (1 - s_i) - (s_i - s_j)^2}
\]

(14)

From (13), total output is given by

\[
Q = \frac{1}{3} \left( 2 - c_1 (1 - s_1) - c_2 (1 - s_2) \right)
\]

Substituting (14) for \( c_i \), setting \( s_2 = 0, s_1 = s \), and simplifying, we get

\[
Q = \frac{4 (1 - s) - s^2}{12 (1 - s) - s^2}
\]

Taking the derivative with respect to \( s \),

\[
\frac{\partial Q}{\partial s} = -\frac{8 s (2 - s)}{(12 (1 - s) - s^2)^2} < 0
\]
If $s$ is high enough (specifically, if $s > \frac{2}{3}$), then we hit the constraint $q_2 \geq 0$ and obtain the same solution as under no discrimination (that is, Firm 2 is effectively not a player, and so it does not matter whether Firm 0 can or cannot discriminate). If $s = \frac{2}{5}$, the value at which $Q$ drops from $\frac{1}{3}$ to $\frac{1}{4}$ under no-discrimination, we have $q_2 > 0$ under discrimination. It follows that $Q$ is greater under discrimination if and only if $s > \frac{2}{5}$ (see Figure 3).

Consider now the case of partial control and suppose that both retailers are active in equilibrium. Their output levels are then given by (13). Firm 0 now maximizes

$$v_0 = (1 - s)^2 \pi_0 + s v_1$$

The first-order conditions for value maximization are given by

$$c_1^* = \frac{(6 - s) (1 - 2 s) (1 - 2 s + 2 s^2)}{(1 - s) (12 - 44 s + 63 s^2 - 36 s^3 - 4 s^4)}$$

$$c_2^* = \frac{6 - 21 s + 29 s^2 - 18 s^3}{12 - 44 s + 63 s^2 - 36 s^3 - 4 s^4}$$

Total output in turn is given by

$$Q = \frac{4 - 14 s + 19 s^2 - 8 s^3 - 4 s^4}{12 - 44 s + 63 s^2 - 36 s^3 - 4 s^4} \quad (15)$$

From (12), under no-discrimination total output is given by

$$Q = \frac{4 - 10 s + 12 s^2 - s^3}{12 - 32 s + 38 s^2 - 8 s^3} \quad (16)$$

Computation establishes that the right-hand side of (15) is greater than the right-hand side of (16) if and only if $s > .34455$.

Finally, if the constraint $q_2 \geq 0$ is binding (i.e., if $s > \frac{1}{2}$), then

$$c_1^* = \frac{1 - 2 s}{(2 - 3 s) (1 - s)}$$

whereas total output is given by $Q = \frac{1}{2}$, which is greater than the value given by (16).
References


