Is Exclusionary Pricing Anticompetitive in Two-Sided Markets?*

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Abstract

This paper studies the competitive effects of exclusionary pricing in two-sided markets. Whilst formally showing that below-cost pricing on one market side can allow an incumbent firm to exclude a potential rival which does not have a customer base yet, the proposed model does not necessarily imply that below-cost pricing in such markets should be taken as anticompetitive conduct. Instead, I find that in sufficiently asymmetric two-sided markets, exclusion is always beneficial and if anything, there is too little of it in the sense that there are cases in which there is inefficient entry. Further, prohibiting below marginal cost pricing may destroy some socially efficient exclusion and worsen the problem of excessive (or inefficient) entry.

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1 Introduction

In recent years, there has been a large literature dealing with two-sided markets, that is, markets where a platform sells to two distinct groups of users which may affect each other’s utility. Common examples of such markets are credit cards (card-holders and merchants), operating systems (computer users and software developers), malls (shoppers and shops), media (viewers or readers and advertisers).\(^1\)

One important feature of such markets is that prices optimally take into account the externalities between the two sides of the market. Similarly to what happens for firms selling complementary products, it may be optimal to sell below cost to - or even subsidize - one group (the group whose demand is more price-sensitive) to increase demand on this side of the market, with the objective of increasing demand on the other side of the market.\(^2\)

This has led several commentators to state that below-cost pricing in two-sided markets should not worry antitrust agencies, since - far from implying an exclusionary objective - they would reflect normal competitive behavior in industries where there exist externalities between different sides of the market. For instance, Evans and Schmalensee (2007) claim that:

“Price equals marginal cost (or average variable cost) on a particular side is not a relevant economic benchmark for two-sided platforms for evaluating either market power, predatory pricing, or excessive pricing under European Community law ... it is incorrect to conclude, as a matter of economics, that deviations between price and marginal cost on one side provide any indication of pricing to exploit market power or to drive out competition.” (p. 27)\(^3\)

Undoubtedly, in most cases pricing below cost on one side of the market does not represent a threat to competition, and in some cases it may be the only way to get ‘both sides on board’, and to ensure that a product is viable. Nevertheless, in this paper, I propose a model in which there exist indirect cross-group network effects and show that, under certain circumstances, pricing below cost on one side of the market may allow a dominant firm with an established and captive customer base to exclude a potential rival from both sides of the market.

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\(^2\)As Caillaud and Jullien (2003) point out, “due to indirect network effects, the key pricing strategies are of a ‘divide-and-conquer’ nature, subsidizing the participation of one side (divide) and recovering the loss on the other side (conquer).” (p. 310)

\(^3\)See also Wright (2004) for a related discussion.
Intuitively, sacrificing profits on one side so as to deter entry allows the dominant incumbent to enjoy monopoly profits on the other side. Obviously, the rival knows that getting consumers on each side is crucial for its overall existence, and this typically results in a price war on one side of the market (in my case, the side which is less affected by demand externalities).\(^4\) There are two effects which determine which firm will win consumers on this market side. On the one hand, the rival is assumed to have lower production costs, and this allows it to make more aggressive price offers. On the other hand, if the incumbent excludes the rival from one side of the market, it will be monopolist on the other, whereas the rival would always have to compete with the incumbent which has already an installed base of customers on both sides. In other words, other things being equal, the incumbent will set prices more aggressively on one side because it anticipates that, if it secures it, it will obtain monopoly (rather than duopoly) profits on the other market side. Only if the rival has a sufficiently strong cost advantage would it manage to overcome this latter effect.

Now, while the proposed model provides a rationale for exclusionary pricing in two-sided markets, it does not imply that below-cost pricing in such markets should necessarily be taken as anti-competitive conduct. Indeed, I find that if a two-sided market is sufficiently asymmetric, i.e. if consumers on one market side care sufficiently less about cross-group network externalities than consumers on the other side of the market, then below-cost pricing does not generate excessive exclusion: when exclusion occurs, it is socially optimal. Instead, some socially optimal exclusion may not occur. So, if anything, there is too little exclusion in equilibrium.

Another important finding is that the model always yields inefficient entry: there always exist cases in which entry occurs in equilibrium but it is socially inefficient (exclusion would be socially preferred).

The paper also studies a scenario of prohibition on below marginal cost pricing and the effects of such a rule on consumers’ surplus and on social welfare. By so doing, it is shown that a policy prohibiting below marginal cost pricing may be counterproductive for two main reasons. First, this policy may destroy socially desirable exclusion by replacing an exclusionary equilibrium (that would occur in a context of unconstrained pricing) with an entry equilibrium which is inferior from a social welfare and also from a consumers’ welfare perspective. Second, adding the policy constraining prices not to be lower than marginal costs may also worsen the problem of excessive

\(^4\)Along similar lines, and considering a situation where two groups (1 and 2) interact via one or more platforms, Armstrong (2007) highlights that “[i]f a member of group 1 exerts a large positive externality on each member of group 2, then it is natural to expect that group 1 will be targeted aggressively (i.e. offered a low price relative to the cost of supply) by platforms. In broad terms, especially in competitive markets, it is group 1’s benefit to the other group that determines group 1’s price, not how much group 1 benefits from the presence of group 2.”
(or inefficient) entry occurring in equilibrium.

The remainder of the paper is structured as follows. In Section 2, I discuss related literature. In Section 3, the basic model is presented, which is chosen as the simplest possible setting where the elements I am interested in could emerge. Section 4 analyses the scenario in which there is competition between an incumbent and an entrant when prices are not constrained and provides a welfare analysis of this case. Section 5 considers the case of prohibition on below marginal cost pricing and studies the welfare effects of such a policy. Section 6 investigates what are the main implications of relaxing the baseline model assumption that cross-group network externalities are unidirectional. Finally, Section 7 concludes the paper.

2 Related literature

An important feature of this paper is that it combines two strands of the recent economic literature. The first strand of the literature is the one on two-sided markets, whose main references have already been mentioned. In terms of modelling assumptions, the closest paper to mine within this strand of the literature is that of Armstrong (2006). In particular, like me, he assumes that the fixed benefit a consumer enjoys from using a platform depends only on which side of the market the agent is on, platform charges are levied as a lump-sum fee and costs are incurred when agents join a platform.\(^5\) There exist, however, a few differences between Armstrong’s framework and mine. In particular, in the proposed model the market is composed of a discrete number of buyers with inelastic demands, implying that buyers are strategic players in my setting: each of them is making a purchasing decision and is, thus, playing a game recognizing the existence of strategic interdependence. Further, I focus on a market that already exists at the moment the game starts, with an asymmetric position between an incumbent and an entrant.

The second strand of the literature deals with exclusionary conduct. In this paper, the incumbent firm exploits demand externalities across buyers to exclude a rival, a mechanism that is in the spirit of anticompetitive exclusion in presence of contracting externalities, as stressed by Bernheim and Whinston (1998). Apart from the literature on exclusive dealing (see e.g. Rasmusen et al. (1991),

\(^5\) There exist a number of important differences in the modelling assumptions between Armstrong (2006) and the pioneering article by Rochet and Tirole (2003) that concern the characterisation of agent’s gross utility, the structure of platforms’ fees and the structure of platforms’ costs (see Section 2 in Armstrong (2006) for a discussion). As Armstrong (2006, p. 671) highlights, “[w]hich assumptions concerning tariffs and costs best reflect reality depends on the context. Rochet and Tirole’s model is well suited to the credit card context, for instance, whereas the assumptions here are intended to apply to markets such as nightclubs, shopping malls, and newspapers.”
Segal and Whinston (2000)), a similar mechanism can be found in models of exclusionary pricing such as Karlinger and Motta (2012) and Fumagalli and Motta (2013). Karlinger and Motta (2012) consider an industry exhibiting network effects and show that an incumbent with an established customer base might charge a price below cost to some crucial group of consumers, thereby depriving the entrant from the scale it needs to operate profitably in the market. Even though in their setting (like in this paper) the incumbent and the entrant choose prices simultaneously, exclusion takes place because of miscoordination among buyers,\(^6\) whereas the mechanism in my benchmark model does not rely on miscoordination. Fumagalli and Motta (2013), on the other hand, propose a theory of predatory pricing in a setting where in addition to an incumbency advantage, which also exists in this paper,\(^7\) the exclusionary effect depends on price discrimination over time (below cost pricing to early buyers so as to deprive the entrant of reaching the efficient scale needed to operate successfully and subsequent extraction of rents from late buyers). In contrast, in the proposed model, and as already mentioned, the rationale for exclusion is different as it relies on the distributional impact of the simultaneous pricing policy (sacrifice profits on one market side to exclude the entrant and, at the same time, recoup monopoly rents on the other market side). In addition, in the very simplified extension dealing with two-sided markets in Fumagalli and Motta (2013), the incumbent and the rival have similar costs, whereas in the present paper it is shown that exclusion may occur even if the entrant benefits from a cost advantage.

\section{The setup}

In this Section, I present a model of two-sided markets which contains some new features: in particular, I model buyers as discrete.

Suppose there are two groups of agents, labelled \(i = 1, 2\), which interact with each other via intermediaries or “platforms”. At the moment the game starts, there exist two competing “platforms”. Platform \(I\) is the dominant incumbent, and has already an installed base of \(\beta^I > 0\) consumers on each market side, and platform \(E\) is a rival new firm which does not have any customer base, \(\beta^E = 0\). The incumbent’s installed base of consumers can be interpreted in different ways. For instance, the product at issue may be a durable good and “old” consumers are locked-in to the platform they have

\(^6\)Karlinger and Motta’s (2012) exclusionary equilibria crucially depend on some fragmentation of buyers: if buyers could coordinate their choices, exclusion of the more efficient entrant would no longer take place.

\(^7\)It should be highlighted, however, that while in Fumagalli and Motta (2013) the incumbency advantage results from scale/scope economies on the supply side, in the present paper the advantage of the incumbent comes from scale economies on the demand side.
bought from in the past and would not consider buying again; or the product may be non-durable but “old” consumers have arbitrarily large switching costs which make them captive to the platform they chose in the past. Throughout the paper, I choose the former interpretation since it allows to disregard “old” consumers’ purchasing decisions.

Clearly, I am focusing here on a situation similar to the one that antitrust agencies and courts are facing if there was a monopolization (Section 2 of the US Sherman Act) or abuse of dominance (article 102 of the European Union Treaty) investigation. The market already exists and a firm enjoys an incumbency advantage.

On each side of the market, there exists a second group of (“new”) consumers, of size $N$, that is on the market when the game starts. So, the two platforms compete for “new” consumers and all the “new” consumers on a given side are homogeneous. A consumer is assumed to either choose to deal with one platform or to deal with no platform (single-homing). In addition, a consumer in one group is assumed to care about the number of (“old” and “new”) consumers of the other group who use the same platform. In particular, suppose the utility of an agent belonging to group $i$ $(i = 1, 2)$ if she joins platform $k$ $(k = I, E)$ is given by:

$$U^k_i = r_i + z_i v(\beta_k + N^k) - p^k_i, \quad (1)$$

where $r_i \geq 0$ is the fixed benefit the agent obtains from using a platform on market side $i$.\footnote{I assume that firms’ platforms are of the same quality: the fixed benefit only depends on the side of the market the agent is on.} $v(\cdot)$ is a function that represents the benefit the consumer derives from interacting with agents on the other side of the platform, $N^k$ denotes the number of “new” consumers on the other market side that join platform $k$, $z_i$ is a parameter which measures the intensity of the ‘cross-group externality’, and $p^k_i$ is the price charged by platform $k$ to consumers on side $i$.\footnote{Following Armstrong (2006), I assume that platforms charge for their services on a “lump-sum” basis. This approach is different from the one in the paper by Rochet and Tirole (2003), who focus on the case where charges are levied on a “per-transaction” basis. As Armstrong (2007) points out, “[t]he crucial difference between the two forms of tariff is that cross-group externalities are less important with per-transaction charges, since a fraction of the benefit of interacting with an extra agent on the other side is eroded by the extra charge incurred.”} In what follows, for simplicity, I assume that $v(\cdot)$ is a linear function. In addition, throughout the paper, I assume that the two platforms are incompatible.

The most important assumption I make in the baseline model is that $z_1 > z_2 = 0$, i.e. I consider an asymmetric two-sided market: side-1 consumers care about the externality whereas
side-2 consumers’ utility is unaffected by the number of people on the other side of the market. Moreover, suppose \( r_1 = 0 < r_2 = r \) (if \( r = 0 \), side-2 consumers would never be willing to pay for the product). Note, therefore, that - for equal number of users - platforms are perceived by buyers as homogenous. I also assume that \( N > \beta^I \). (As will be clearer later on, assuming \( N \leq \beta^I \) would reinforce the exclusionary results.)

Turning to the cost side, assume that costs are incurred when agents join a platform. Each platform \( k \) has a cost \( c_k, k = I, E \), where \( c_I > c_E \geq 0 \). Note that, for simplicity, platform \( k \)'s costs of serving side 1 or side 2 are assumed to be identical. In addition, the new platform \( E \) is assumed to face zero entry costs, to highlight that entry barriers come only from indirect network effects.

I also impose the following market viability condition.

**Assumption 1** Assume that:

\[
\min \left\{ z_I \beta^I, r \right\} > c_I.
\]

In particular, note that \( z_I \beta^I > c_I \) means that the market was viable when only the “old” cohort of buyers existed.

I consider the following two-stage game:

1. Active platforms set prices simultaneously on each side of the market.
2. “New” buyers (on each side) decide whether to join a platform, and which one if they have the choice, and pay the corresponding price. (Recall that “old” buyers have already purchased the product and do not buy any longer.)

In what follows, I characterize the equilibria of this game under two alternative pricing regimes: (i) a regime where prices are not constrained; and (ii) a regime where there is prohibition on below marginal cost pricing.

### 4 Equilibrium solutions with unconstrained prices

In this section, I assume that platforms can charge different (and possibly negative) prices across the two sides of the market, but cannot charge different prices to subgroups of consumers belonging to the same market side (there is no price discrimination on a given market side).
Given that similar games with uncoordinated buyers and scale economies are typically characterized by multiple equilibria (see for instance Segal and Whinston (2000) and Karlinger and Motta (2012)), in what follows I follow that literature and investigate the existence of two types of pure-strategy equilibria in the proposed game: “exclusionary” equilibrium where all consumers buy from the incumbent, and “entry” equilibrium where all “new” consumers on both sides of the market join the entrant’s platform.

I first look for the equilibrium in which the incumbent serves all buyers.

**Proposition 1** If both platforms can set different (and possibly negative) prices across the two sides of the market, then there exists an equilibrium in which all “new” buyers buy from the incumbent if and only if:

\[ 2N (c_I - c_E) \leq z_1 N \beta I. \]  

(2)

In the equilibrium, \( p_I^1 = z_1 (N + \beta I) \) and \( p_I^2 = 2c_E - z_1 N < c_I \), and the minimum prices that the entrant is willing to charge on each side of the market are respectively given by \( p_E^1 = c_E \) and \( p_E^2 = 2c_E - z_1 N \).

**Proof.** See Appendix. ■

Notice that there is a very simple way to understand the condition under which this type of equilibrium exists. Imagine that the “new” buyers from both market sides form a grand coalition such that this coalition chooses one of the two available platforms. Then, the incumbent platform will win the coalition if and only if inequality (2) is satisfied, which implies that, when all consumers are served by the incumbent platform (rather than by the entrant), the gain in terms of cross-group network externalities benefiting consumers on market side 1 more than compensates for the cost disadvantage that the incumbent platform faces on both sides of the market. Hence, under condition (2), if this grand coalition of “new”-generation buyers from both platform sides were allowed to form, these buyers could not agree to (collectively) switch from the incumbent’s to the entrant’s platform since switching would not increase their coalition payoff.\(^ {11} \) Hence, everything is as though the two firms competed to maximize buyer surplus and platform \( I \) will win the market whenever it generates more total surplus than platform \( E \). However, the distribution of this buyer surplus reflects the strategic position of each buyer type: side-1 buyers get zero surplus, whereas side-2 buyers, being pivotal,\(^ {12} \) get the surplus offered by the “losing” platform \( E \): \( r - p_E^2 = r + z_1 N - 2c_E \).

\(^ {11} \) Notice that \( r + z_1 (\beta I + N) - 2c_I \) is the (average) total surplus generated by platform \( I \), and \( r + z_1 N - 2c_E \) is the (average) total surplus generated by platform \( E \).

\(^ {12} \) The fact that side-2 buyers do not value cross-group network externalities, while side-1 buyers do, implies that
Two other important remarks should be made at this point. First, I have assumed that \( N > \beta I \), that is, the number of "new" consumers is larger than the number of "old" ones: the market is growing (at least doubling in size). It is straightforward to see that, other things being equal, as \( \beta I \) increases - for given \( N \) - the exclusionary equilibria become more likely. Hence, the assumption that the market is growing was relaxed, it would be more likely for exclusionary equilibria to exist. Second, under this type of equilibrium, fierce competition on market side 2 will always induce the incumbent platform to sell at a below marginal cost price to side-2 consumers, who do not care about indirect network externalities, and recovers losses on the other side of the market by charging the monopoly price to side-1 consumers.\(^{13}\) Intuitively, there cannot exist an exclusionary equilibrium where firm \( I \) charges above \( c_I \) on market side 2: firm \( E \), being more cost-efficient, would slightly undercut firm \( I \) thereby getting side-2 consumers and attracting side-1 consumers as well.

Hence, exclusion takes place because firm \( I \) can sacrifice profits on side-2 consumers to keep firm \( E \) out of the market and getting in this way the full monopoly profits, \( z_1(\beta I + N) \), on side-1 consumers. In the proposed model, therefore, there is a distributional impact of the pricing policy: profits on market side 2 are sacrificed (consumption on this side is subsidized) so as to keep the entrant out and reap monopoly profits on market side 1.

I now turn to the analysis of the entry equilibrium. Given that platforms can set different prices across the two sides of the market, if platform \( I \) manages to set a price \( p_I^2 < p_E^2 \), it will induce every consumer on side 2 to strictly prefer platform \( I \), in turn inducing consumers on side 1 to join platform \( I \) as well, since they would not enjoy any indirect network externality from joining platform \( E \).\(^ {14}\) Of course, rival firm \( E \) will also try to lower prices on side 2 (if it lost side-2 consumers it would not sell to side-1 consumers). Aggressive price competition on side-2 will follow, and the result will depend among other things on the cost difference between the firms. More specifically, and as shown in the following proposition, an ‘entry’ equilibrium will only exist if the incumbent’s cost disadvantage is sufficiently large (the lower \( c_E \) relative to \( c_I \) the more likely that platform \( E \) will win side-1 buyers).

**Proposition 2** If both platforms can set different (and possibly negative) prices across the two sides of the market, then there exists an equilibrium in which all “new” buyers buy from the entrant if

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\( ^{13}\) In addition, if the entrant’s platform is sufficiently efficient, i.e. if \( c_E < z_1N/2 \), it turns out that the incumbent will actually have to charge a negative price to side-2 consumers (subsidizing their consumption).

\( ^{14}\) As will be clear, the incumbent platform will have no incentive to embark on a “reverse” deviation wherein it would charge a price below \( c_I \) to side-1 consumers. This is because this deviation would not induce side-2 consumers to join its platform (as they do not care about cross-group indirect network externalities).
and only if:

\[ 2N (c_I - c_E) \geq z_1 N \beta^I + N (z_1 \beta^I - c_I). \quad (3) \]

In the equilibrium, \( p_1^E = c_I + z_1 (N - \beta^I), \) \( p_2^E = 2c_I - z_1 (\beta^I + N), \) and the minimum prices that the incumbent is willing to charge on each side of the market are respectively given by \( p_1^I = c_I \) and \( p_2^I = 2c_I - z_1 (\beta^I + N). \)

**Proof.** See Appendix. ■

One can also note that the larger the incumbent’s customer base the lower the price that the entrant would have to charge for an entry equilibrium to exist \( (\partial p^E_i / \partial \beta^I \leq 0). \) In turn, this implies that the larger the incumbent’s base - for given \( N \) - the less likely that the entry equilibrium exists (for given “new” consumers \( N \), the higher the established base of the incumbent \( \beta^I \) the less likely that platform \( E \) will win side-1 consumers).

Condition (3) can be obtained from (2) by adding \( N (z_1 \beta^I - c_I) \) to the r.h.s. and by changing the sign of the inequality. Now, \( N (z_1 \beta^I - c_I) \) is positive by Assumption 1 and can be called the “incumbency advantage”: compared to the imaginary case of the grand coalition of all “new” buyers discussed above, it makes the existence of the entry equilibrium less likely by this term.

### 4.1 Welfare analysis

This section investigates when is exclusion socially desirable. Note that since I assume inelastic demands, when computing total welfare, prices can be ignored as they reduce consumers’ surplus by the same amount as they increase profits. Hence, under an exclusionary equilibrium, total welfare will be:

\[ W_{\text{exclusion}} = (N + \beta^I) [z_1 (\beta^I + N) + r] - 2Nc_I. \quad (4) \]

Under entry, total welfare will be:

\[ W_{\text{entry}} = N [z_1 N + r] + \beta^I [z_1 \beta^I + r] - 2Nc_E. \quad (5) \]

Therefore, exclusion will be socially optimal if:

\[ (N + \beta^I) [z_1 (\beta^I + N) + r] - 2Nc_I > N [z_1 N + r] + \beta^I [z_1 \beta^I + r] - 2Nc_E \quad (6) \]
or, equivalently, if:

\[ 2N (c_I - c_E) < z_1 N \beta I + z_1 N \beta I, \]

(7)

where the second term on the r.h.s. of the previous condition represents the positive externalities (resulting from exclusion) on the “old” side-I locked-in consumers.\(^{15}\)

So, as far as total welfare is concerned, there are two opposing forces at work. On the one hand, indirect network externalities benefitting side-I consumers imply that society is better-off when consumers use the same platform rather than divide themselves across platforms. More specifically, \((N + \beta I) z_1(\beta I + N) > N z_1 N + \beta I z_1 \beta I, ceteris paribus,\) making welfare higher when both “old” and “new” buyers are served by the incumbent platform. On the other hand, when it is the incumbent which serves all buyers, then there is a productive inefficiency, which is reflected in the l.h.s. of inequality (7).

Now, by contrasting conditions (2) and (7), one may conclude that, in the benchmark setting, when exclusion occurs, it is always socially optimal. Consider again the imaginary case of the grand coalition involving all “new” buyers. Since this grand coalition internalizes the payoffs of all “new” consumers, condition (2) captures perfectly the social welfare related to these “new” consumers. What is missing is the change in the payoffs of the “old” consumers, which, as explained above, amounts to \(z_1 N \beta I > 0\) (the externality effect). Therefore, if condition (2) holds, condition (7) is also satisfied.

Hence, in the benchmark setting, below-cost pricing does not generate excessive exclusion: when exclusion occurs, it is socially optimal. Instead, some socially optimal exclusion may not occur because the incumbent platform does not internalize the positive externalities that exclusion would inflict on the “old” locked-in consumers. So, if anything, there is too little exclusion.\(^{16}\) In addition, since the externality effect \(z_1 N \beta I\) is always larger than the “incumbency advantage” \(N (z_1 \beta I - c_I),\) there are cases in which conditions (3) and (7) both hold and, therefore, entry occurs in equilibrium, but it is socially inefficient (exclusion would be socially preferred in this region of parameter values).\(^{17}\)

\(^{15}\)In terms of indirect network externalities, side-I “old” consumers obtain \(\beta I [z_1(\beta I + N)]\) under exclusion and \(\beta I (z_1 \beta I)\) under entry. Hence, the difference in their payoff resulting from exclusion equals \(z_1 N \beta I.\)

\(^{16}\)When condition (2) fails to hold whereas condition (7) is satisfied, then the exclusionary equilibrium does not exist even though it would be socially desirable.

\(^{17}\)If instead (7) does not hold, entry occurs in equilibrium (condition (3) is satisfied) and it is socially efficient.
5 Prohibition on below marginal cost pricing

Suppose now that there is a policy which prohibits the incumbent (or both firms; since the entrant is more efficient this would be equivalent) from setting prices below cost on any side of the market. In what follows, I study the effects of such a policy on pricing, on consumers’ surplus and on welfare.

In this case, it is easy to see that there is only the entry equilibrium. Intuitively, the incumbent could not set price below-cost on any side. So, the entrant, which benefits from a cost advantage, would just need to set a price a shade below the cost of the incumbent to win the orders from both consumer groups.

Proposition 3 If below-cost pricing is prohibited, then: (i) there always exists an entry equilibrium where \( p_I^1 = p_I^2 = c_I, \) \( p_{E1}^E = c_I + z_1 (N - \beta I), \) and all “new” buyers on both sides buy from the entrant; (ii) there exists no exclusionary equilibrium where all “new” buyers join the incumbent’s platform.

Proof. See Appendix. ■

Now, two different cases should be distinguished depending on the type of equilibrium which would prevail in the absence of this policy (i.e. with unconstrained pricing), which are discussed in what follows.

Consider first the case in which \( 2N (c_I - c_E) < z_1 N \beta I \) (i.e. condition (2) holds). Then, without the constraint on pricing, one has the exclusionary equilibrium, whereas by adding the constraint one obtains only the entry equilibrium. What are then the welfare effects of such a policy? Table 1 presents the consumers’ surplus regarding each consumer group and considering both types of
equilibria.\footnote{In my model, side-1 consumers’ utility increases continuously with the number of (“old” and “new”) consumers on the other market side. Hence, the “old” generation of consumers on market side 1 cannot be ignored when studying welfare effects: even if they do not buy again, side-2 “new” consumers’ decisions have (also) an impact on side-1 “old” consumers’ utility.}

<table>
<thead>
<tr>
<th>Consumer Group</th>
<th>$CS^{entry}$</th>
<th>$CS^{exclusion}$</th>
<th>$\Delta CS = CS^{entry} - CS^{exclusion}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>“new” side-1 consumers</td>
<td>$N(z_1 \beta^I - c_I)$</td>
<td>0</td>
<td>$N(z_1 \beta^I - c_I) &gt; 0$</td>
</tr>
<tr>
<td>“new” side-2 consumers</td>
<td>$N(r - c_I)$</td>
<td>$N(r - 2c_E + z_1 N)$</td>
<td>$-N(c_I + z_1 N - 2c_E) &lt; 0$</td>
</tr>
<tr>
<td>“old” side-1 consumers</td>
<td>$\beta^I z_1 \beta^I$</td>
<td>$\beta^I z_1 (N + \beta^I)$</td>
<td>$-\beta^I z_1 N &lt; 0$</td>
</tr>
<tr>
<td>“old” side-2 consumers</td>
<td>$\beta^I r$</td>
<td>$\beta^I r$</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td>$-N(2(c_I - c_E) + z_1 N) &lt; 0$</td>
</tr>
</tbody>
</table>

**Table 1:** Consumers’ surplus variation per consumer group

Hence, as this Table demonstrates, only the “new” generation of consumers on market side 1 would gain with a policy prohibiting below marginal cost pricing. This is because, as shown in Proposition 1, in the context of exclusionary pricing, the incumbent platform will behave as a monopolist on market side 1, extracting all possible surplus from “new” consumers on this market side, and thereby recouping losses made to win all “new” customers on market side 2. As for the other groups of consumers, it turns out that: (i) “new” consumers on market side 2 loose surplus with the introduction of this policy since they no longer benefit from the subsidization of their consumption by the incumbent platform (that, under exclusion, sells at a price below cost on this market side); (ii) the “old” generation of consumers on market side 1 also faces a welfare loss with the policy introduction since under an entry equilibrium there are no “new” consumers joining the incumbent on market side 2 and this has a negative impact on side-1 “old” consumers’ utility; and (iii) the “old” generation of consumers on market side 2 is not affected by the policy introduction (as they do not care about indirect externalities and do not have to buy again the product).

Overall, however, consumers’ surplus decreases with the policy introduction. In addition, and as shown in the previous section, if condition (2) holds, condition (7) also holds and, therefore, when exclusion occurs (in a context of unconstrained pricing), it is socially optimal. This in turn implies that in the region of parameter values under analysis, adding the policy constraining prices...
not to be set below marginal costs is *counterproductive* as it destroys socially desirable exclusion by inducing the firms to play an entry equilibrium which is inferior from a social welfare (and also from a consumers’ welfare) perspective than the exclusionary equilibrium.

Consider now the case in which instead $2N(c_I - c_E) > z_1N\beta^I$ (i.e. condition (2) fails to hold and thus an exclusionary equilibrium does not exist in a context of unconstrained pricing). When this is the case, we already know that, under unconstrained pricing, an entry equilibrium will exist if condition (3) is verified. In order to see whether the entry is socially excessive or suboptimal in a context where prices are not constrained, we should compare the “incumbency advantage” $N(z_1\beta^I - c_I)$ (see condition (3)) with the positive externality on locked-in consumers, $z_1N\beta^I$ (which is present in condition (7)). Since, as already mentioned, the latter is always larger than the former, there exists excessive entry in my setting. Now, adding the policy constraining prices not to be lower than marginal costs will worsen this problem of excessive entry since the region of parameter values in which excessive (or inefficient) entry occurs in equilibrium is given by $z_1N\beta^I + N(z_1\beta^I - c_I) < 2N(c_I - c_E) < z_1N\beta^I + z_1N\beta^I$ without the constraint and becomes $z_1N\beta^I < 2N(c_I - c_E) < z_1N\beta^I + z_1N\beta^I$ when the policy introduces the constraint on pricing.19

### 6 Extension: two-sided network externalities

So far, it has been assumed that network externalities are unidirectional: consumers on market side 2 were assumed to be indifferent as to the number of consumers on market side 1. In this section, I investigate how the main results change when one considers instead bidirectional network externalities. In particular, in what follows I consider the case in which $z_1 > z_2 > 0$, implying that side-2 consumers care less about cross-group demand externalities than side-1 consumers, but they are not indifferent as to the number of consumers on the other market side. This being the case, the market viability condition is now:

**Assumption 1’** Assume that:

$$\min\{z_1\beta^I, r + z_2\beta^I\} > c_I.$$ 

As before, this condition ensures that the market was viable (on both sides) when only the “old” cohort of buyers existed. I also impose in what follows that $z_2$ is positive but close to zero (small enough).

19 If instead $2N(c_I - c_E) > z_1N\beta^I + z_1N\beta^I$, entry occurs in equilibrium both with and without the constraint on pricing and it is socially efficient.
The next proposition investigates whether an exclusionary equilibrium exists in this more general scenario.

**Proposition 4** If both platforms can set different (and possibly negative) prices across the two sides of the market and $z_1 > z_2 > 0$, then there exists an equilibrium in which all “new” buyers buy from the incumbent if and only if:

$$2N (c_I - c_E) \leq z_1 N \beta^I + z_2 N (\beta^I + N).$$

In the equilibrium, $p^I_1 = z_1 (\beta^I + N)$ and $p^I_2 = 2c_E - z_1 N + z_2 (\beta^I + N)$, and the minimum prices that the entrant is willing to charge on each side of the market are respectively given by $p^E_1 = c_E$ and $p^E_2 = 2c_E - z_1 N$.

**Proof.** See Appendix. ■

Hence, similarly to what happened with regards to the baseline model, also in a more general context wherein there are two-way network externalities, the adoption of a biased pricing structure favouring the group of consumers that exert higher externalities on each consumer belonging to the other market side can still be aimed at excluding a potential entrant from the market. However, contrary to what happened in the baseline model, this biased pricing structure does not necessarily involve charging a price below cost to those consumers who are targeted more aggressively: prices will only be set below $c_I$ for consumers on market side 2 if $z_2$ is sufficiently low.

Now, a natural question one can raise at this point is whether the analogy of some hypothetical buyer coalition pitting the two platform suppliers against each other is still valid in this more general framework to understand the existence condition (8). In what follows, I explain that this analogy fails to go through when one considers two-sided externalities. If both groups of buyers experience some kind of cross-group network benefits from joining the platform, then the group of side-2 buyers is no longer pivotal (nor is the group of side-1 buyers). The exclusionary equilibrium must then be immune to both possible deviations from the entrant: (i) stealing side-2 buyers from the incumbent to win side-1 buyers (as in the benchmark model); and (ii) stealing side-1 buyers to win side-2 buyers (which did not make sense in the benchmark model, as explained in Proposition 1).

In an exclusionary equilibrium, deviation (i) defines the upper bound on the price that platform I can charge side-2 buyers, and deviation (ii) defines the upper bound on the price that platform I can charge side-1 buyers. The fact that both groups now enjoy network benefits distorts competition for the buyers in favor of platform I. In particular, to keep side-2 buyers (i.e. to be immune to deviation
(i), platform $I$ will only have to match the surplus offered by platform $E$ to side-2 buyers if side-1 buyers did not switch simultaneously: $CS_I^f (\beta^I + N, p_2^I) = r + z_2 (\beta^I + N) - p_2^I \geq CS_E^S (0, p_2^E) = r - p_2^E$. Thus, it’s as though $E$’s platform was much more inferior to $I$’s than it actually is. The “fair” comparison between the two platforms would of course be that under a “coordinated” switch from $E$ to $I$ (i.e. if both groups of buyers switched together), so that side-2 buyers compare $r + z_2 (\beta^I + N) - p_2^I$ to $r + z_2 N - p_2^E$ (and side-1 buyers compare $z_1 (\beta^I + N) - p_1^I$ to $z_1 N - p_2^E$). But at the exclusionary equilibrium, the buyers never get to make this “fair” comparison. Instead, buyers from side 2 will stick with platform $I$ whenever $CS_I^f (\beta^I + N, p_2^I) \geq CS_E^S (0, p_2^E)$, which allows platform $I$ to charge a comfortably high $p_2^I = 2c_E - z_1 N + z_2 (\beta^I + N)$ compared to the tight $p_2^I = 2c_E - z_1 N$ when side-2 buyers do not enjoy any network benefits (the case of Proposition 1). Even more striking, the analogous condition implied by deviation (ii), $CS_I^f (\beta^I + N, p_1^I) \geq CS_E^S (0, p_1^E)$, needed for side-1 buyers to buy from $I$, shows that platform $I$ can even extract full surplus from side-1 consumers, $p_1^I = z_1 v(\beta^I + N)$ (which is the same as under one-sided externalities). In other words, there is virtually no competitive restraint exerted by $E$ as far as $I$’s pricing on side 1 is concerned.

But then, one gets that the incumbent can break even under the equilibrium prices $p_1^I = z_1 (\beta^I + N)$ and $p_2^I = 2c_E - z_1 N + z_2 (\beta^I + N)$ whenever the existence condition (8) holds. In essence, the $z_1 N$ contained in $p_1^I$ cancels out because competitive pressure on prices for side 2 implies a $-z_1 N$ in $p_2^I$. But the $z_2 N$ in $p_2^I$ does not cancel with any discount offered on side 1, so that the final condition (8) is much less demanding on platform $I$ than the “fair” comparison made by the grand coalition. More specifically, the grand buyer coalition would stay with platform $I$ if and only if $CS_I^f (\beta^I + N, p_1^I) + CS_2^S (\beta^I + N, p_2^I) > CS_E^S (N, p_1^E) + CS_2^E (N, p_2^E)$ and, at marginal cost pricing, this is equivalent to $2(N (c_I - c_E) < (z_1 + z_2) N \beta^I$.\footnote{The exclusionary equilibrium can only be broken if it is a dominant strategy for side-2 buyers to buy from $E$, even if side-1 buyers stay with $I$, i.e. even absent any network benefits enjoyed on $E$’s platform.}

Now, as for entry equilibria, considering two-way network externalities implies that there are two types of deviations by the incumbent platform that should be investigated. In particular, if the price set by the incumbent platform on market side $i$, $i = 1, 2$, is sufficiently low to induce every side-$i$ consumer to strictly prefer to join the incumbent platform, then consumers on market side $j$, where $j \neq i$, can be easily induced to join the incumbent platform as well since they would not enjoy any indirect network externality from joining the entrant’s platform instead. As a result, as the next proposition shows, an entry equilibrium will only exist if the prices charged by the entrant’s

\footnote{Platform $I$ would then win the grand coalition of “new” buyers if $E$’s cost advantage was not sufficient to compensate buyers for the loss in network benefits generated by the installed base.}
platform to consumers on each side of the market are so low that the incumbent platform cannot profitably set a sufficiently low price on one side of the market while extracting more surplus from consumers on the other side of the market.

**Proposition 5** If both platforms can set different (and possibly negative) prices across the two sides of the market and \( z_1 > z_2 > 0 \), then there exists an equilibrium in which all “new” buyers buy from the entrant if and only if:

\[
2N(c_I - c_E) \geq z_1N\beta^I + N(z_1\beta^I - c_I) - z_2N(N - \beta^I).
\]  

In the equilibrium, \( p_1^E = z_1(N - \beta^I) + c_I \), \( p_2^E = 2c_I - z_1(\beta^I + N) + z_2(N - \beta^I) \), and the minimum prices that the incumbent is willing to charge on each side of the market are respectively given by \( p_1^I = c_I \) and \( p_2^I = 2c_I - z_1(\beta^I + N) \).

**Proof.** See Appendix. ■

Hence, the fact that the entrant’s platform must simultaneously offer to “new” buyers on both sides of the market at least as much as the incumbent platform’s best offer implies that, for an entry equilibrium to exist, the cost difference between the incumbent and the entrant should be sufficiently large, similarly to what happened in the baseline model.

Now, contrasting the existence conditions in Propositions and 4 and 5 with those in Propositions 1 and 2, one may conclude that the presence of a positive \( z_2 \) makes both the exclusionary and the entry equilibria more likely. This is very intuitive. If \( z_2 > 0 \), consumers on each market side care about what happens on the other side of the market, which generates equilibrium multiplicity.

In concluding this section, two important questions should be raised regarding the welfare properties of the two equilibria just characterized. In particular: (i) is exclusion still socially desirable when one considers two-sided network externalities?; and (ii) are there still cases of inefficient entry in this more general setting?

Under an exclusionary equilibrium total welfare will be:

\[
\tilde{W}^{\text{exclusion}} = (N + \beta^I) \left[ z_1(\beta^I + N) + r + z_2(\beta^I + N) \right] - 2Nc_I.
\]  

Under entry, total welfare will be:

\[
\tilde{W}^{\text{entry}} = N \left[ z_1N + r + z_2N \right] + \beta^I \left[ z_1\beta^I + r + z_2\beta^I \right] - 2Nc_E.
\]
Therefore, exclusion will be socially optimal if:

$$2N (c_I - c_E) < 2N \beta^I (z_1 + z_2). \quad (12)$$

By contrasting conditions (8) and (12), one may conclude that if $z_2 < z_1 \beta^I / (N - \beta^I)$, then, like in the baseline model, there is suboptimal exclusion: when exclusion occurs, exclusion is socially optimal and, at the same time, some socially optimal exclusion does not occur.\textsuperscript{22}

Now, in order to understand whether entry is socially excessive or not, one must compare conditions (9) and (12). Notice that the r.h.s. of inequality (9) can be rewritten as $2N \beta^I (z_1 + z_2) - N (c_I + z_2 (N + \beta^I))$. Hence, clearly, there always exist cases where conditions (9) and (12) both hold in this more general setting. Put another way, independently of the level of asymmetries regarding indirect cross-group network effects, the model always yields inefficient entry.

In sum, in a more general context with bidirectional network externalities, the welfare implications are richer than in the benchmark model where cross-group network externalities are unidirectional. Nevertheless, the main qualitative results of the benchmark model still hold good in this more general context as long as the two-sided market is sufficiently asymmetric, i.e. as long as $z_2$ is sufficiently small.

\section{Conclusion}

This paper proposes a model which studies the competitive effects of exclusionary pricing in two-sided markets. The model is one where indirect cross-group network effects exist, and demand-side scale economies imply that unless a sufficient number of customers buy from the entrant on one side of the market, consumers on the other side would not buy from the new entrant either. Therefore, an incumbent may want to sell below costs to the crucial side of the market so as to make sure the rival does not win it, thereby deterring (also) buyers on the other side of the market - whose demand is characterized by a strong(er) positive externality with that of the former side - from buying from the entrant. As a result, they will be obliged to buy from the incumbent, which can behave like a monopolist on this side of the market, recouping any losses made to win the other side.

While model provides a rationale for exclusionary pricing in two-sided markets, it does not nec-

\textsuperscript{22}More formally, when $z_2 < z_1 \beta^I / (N - \beta^I)$, then whenever condition (8) holds, condition (12) also holds. In addition, there are cases in which condition (8) fails to hold whereas condition (12) is satisfied.
essarily imply that below-cost pricing in such markets should be taken as anti-competitive conduct. Indeed, it is shown that if the two-sided market is sufficiently asymmetric, i.e. if consumers on one market side care sufficiently less about cross-group network externalities than consumers on the other side, then there is suboptimal exclusion: when exclusion occurs, exclusion is socially optimal but some socially optimal exclusion may not occur.

Another important finding is that the proposed model always predicts the existence of inefficient entry: there always exist cases in which entry occurs in equilibrium but it is inferior from a social welfare perspective than the exclusionary equilibrium.

The paper also investigates the induced welfare effects resulting from a rule constraining prices not to be lower than marginal costs. It is then shown that such a policy may be counterproductive since: (i) it may induce the replacement of an exclusionary equilibrium (occurring in a context of unconstrained pricing) with an entry equilibrium which is inferior from a social welfare and also from a consumers’ welfare perspective; and (ii) it may also worsen the problem of inefficient entry occurring in equilibrium.

References


Appendix

This Appendix contains the proofs of the Propositions stated in the text.

Proof. (Proposition 1)

In what follows, I look at firms’ decisions and look for deviations which may disrupt an (exclusionary) equilibrium according to which the incumbent sets prices \((p_1^I, p_2^I)\) and all buyers buy from \(I\). There are two possible deviations by the entrant (losing) platform: (i) stealing side-2 buyers from the incumbent to win side-1 buyers; and (ii) stealing side-1 buyers to win side-2 buyers. As will be clear, however, there is no room for deviation (ii) in this benchmark model.

Consider first deviation (i) and let us investigate what is the entrant’s minimum (threat) price for consumers on market side 2. If the entrant attracts all \(N\) “new” consumers on market side 2, then how much surplus will it be able to extract from each “new” consumer on the other side of the market? Optimally, the entrant platform will choose a price \(p_1^E\) such that

\[
CS^E_1 (N, p_1^E) = z_1N - p_1^E \geq CS^I_1 (\beta^I, p_1^{I*}) = z_1\beta^I - p_1^{I*},
\]  

(13)
where \( p^{I*}_1 \) denotes the incumbent’s equilibrium price on market side 1. Further, it must be that
\[
CS_1^E (N, p^E_1) = z_1 N - p^E_1 \geq 0.
\]
Put it another way, if all side-2 buyers buy from \( E \), side-1 buyers will also buy from \( E \) if \( p^E_1 \leq \min \{ p^{I*}_1 + z_1 (N - \beta^I), z_1 N \} \). Given that \( p^{I*}_1 > z_1 \beta^I \) (at the candidate equilibrium, the incumbent will at least extract, on side 1, all surplus generated by the existence of the “old” generation of consumers on the other market side), the previous condition boils down to \( p^E_1 \leq z_1 N \). Hence, \( p^E_1 = z_1 N \) and the entrant’s minimum threat price on market side 2 is given by \( c_E - \max \{ 0, z_1 N - c_E \} \). Now, Assumption 1 ensures that \( z_1 N > c_I > c_E \) and, as a consequence, the entrant’s threat price on market side 2 is given by \( p^{E*}_2 = 2c_E - z_1 N < c_E \). The entrant is then willing to subsidize side-2 buyers, stealing them from the incumbent, to win side-1 buyers as well.

Consider now deviation (ii) described above. In particular, let us now investigate what is the entrant’s minimum (threat) price on market side 1. Suppose that the entrant attracts all \( N \) “new” consumers on market side 1. How much surplus can it then extract on market side 2? Note that the entrant platform will choose \( p^E_2 \) according to

\[
CS_2^E (N, p^E_2) = r - p^E_2 = CS_2^I (\beta^I, p^{I*}_2) = r - p^{I*}_2,
\]

where \( p^{I*}_2 \) denotes the incumbent’s equilibrium price on market side 2. But, in order to be immune to the previously described deviation (i), wherein the losing platform \( E \) adopts its lowest side-2 (threat) price \( p^{E*}_2 = 2c_E - z_1 N, p^{I*}_2 \) should satisfy:

\[
CS_2^I (\beta^I + N, p^{I*}_2) = r - p^{I*}_2 = \max \{ 0, r - p^{E*}_2 \},
\]

where \( CS_2^E (0, p^{E*}_2) = r - p^{E*}_2 = r - 2c_E + z_1 N > 0 \) (under Assumption 1) and, therefore, condition (15) boils down to \( p^{I*}_2 = p^{E*}_2 \). Hence, making use of eq. (14), one may conclude that \( p^E_2 = p^{I*}_2 = p^{E*}_2 \). But since \( p^{E*}_2 = 2c_E - z_1 N \), one finally concludes that \( p^E_2 = 2c_E - z_1 N < c_E \) and \( p^E_2 - c_E = c_E - z_1 N < 0 \) (under Assumption 1). So, clearly, the entrant’s minimum (threat) price on market side 1 is given by \( p^{E*}_1 = c_E - \max \{ 0, p^{E*}_2 - c_E \} = c_E \). Put it another way, the entrant will not be interested in embarking on deviation (ii), wherein it steals side-1 buyers from the incumbent (subsidizing their consumption) so as to win side-2 buyers.

In summary, the entrant’s minimum threat price on market side 1 is \( c_E \) whereas its minimum threat price on market side 2 equals \( 2c_E - z_1 N \). The equilibrium prices of the incumbent platform are then simply a best response to these threat prices of the entrant platform. Hence, as already
mentioned, making use of eq. (15), one may conclude that \( p_1^* = p_2^E = 2c_E - z_1N \) whereas \( p_1^I \) is obtained from

\[
CS_1^I (N + \beta^I, p_1^I) = z_1 (N + \beta^I) - p_1^I = \max \left\{ 0, -p_1^E \right\}.
\]

This then implies that the incumbent is free to set \( p_1^I = z_1 (N + \beta^I) \) on market side 1.

Hence, in order for an exclusionary equilibrium to exist, one must have that \( \pi^I(p_1^I, p_2^I) \geq 0 \), i.e.,

\[
z_1 (N + \beta^I) + 2c_E - z_1N - 2c_I \geq 0
\]
or, equivalently,

\[
2N (c_I - c_E) \leq z_1N \beta^I.
\]

In concluding, notice that \( p_2^I = 2c_E - z_1N = c_E - (z_1N - c_E) < c_I \). This is because \( z_1N > c_E \) is implied by Assumption 1 and by the fact that \( N > \beta^I \), on the one hand, and \( c_I > c_E \), on the other. This completes the proof.

**Proof. (Proposition 2)** Consider an equilibrium where all “new” consumers buy from \( E \). Now, the question is what is the lowest price that the losing (incumbent) platform is willing to charge on each side of the market?

Consider first the incumbent’s minimum threat price regarding market side 2. If the incumbent attracts \( N \) consumers on market side 2, it can extract \( z_1 (\beta^I + N) \) from each “new” consumer on market side 1. Moreover, since side 1 consumers only benefit from the externalities from side 2, if all side 2 consumers move to the incumbent platform, the entrant platform does not exercise any competitive pressure on side 1 (recall here that the fixed benefit a consumer obtains from using a platform on market side 1 is null). Therefore, the incumbent’s minimum price on market side 2 is \( p_2^I = c_I - \max \left\{ 0, z_1 (\beta^I + N) - c_I \right\} \). Now, Assumption 1 ensures that \( z_1 (\beta^I + N) > c_I \) and, therefore, the incumbent’s minimum price on market side 2 equals \( p_2^I = 2c_I - z_1 (\beta^I + N) \).

Let us now investigate what is the incumbent’s minimum threat price on market side 1. Suppose that the incumbent attracts \( N \) consumers on market side 1. How much surplus can it then extract on market side 2? Recall that side-2 buyers do not care about cross-group externalities. Hence, the incumbent platform will choose \( p_2^I \) such that

\[
CS_2^I (\beta^I + N, p_2^I) = r - p_2^I = \max \left\{ r - p_2^E, 0 \right\},
\]

21
where $p_2^{E*}$ denotes the equilibrium price of the entrant platform on market side 2. But, in order to be immune to the previously described deviation, wherein the losing platform I adopts its lowest side-2 (threat) price $p_I^{I*} = 2c_I - z_1 (\beta^I + N)$, $p_2^{E*}$ should satisfy

$$CS_2^I (\beta^I, p_2^{I*}) = r - p_2^{I*} = CS_2^E (N, p_2^{E*}) = r - p_2^{E*},$$

(18)

where $r - p_2^{I*} = r - 2c_I + z_1 (\beta^I + N) > 0$ (under Assumption 1). Therefore, $r - p_2^{E*}$ is positive. Hence, making use of eqs. (17) and (18), one may conclude that $p_2^{I*} = p_2^{E*} = p_2^{I*} = 2c_I - z_1 (\beta^I + N)$. Now, the incumbent’s minimum price on market side 1 is given by $p_1^{I*} = c_I - \max \{0, p_2^{I*} - c_I\} = c_I$ since Assumption 1 implies that $z_1 (\beta^I + N) > c_I$.

In summary, the incumbent’s minimum threat price on market side 1 is $c_I$ whereas its minimum threat price on market side 2 equals $2c_I - z_1 (\beta^I + N)$. The equilibrium prices of the entrant platform are then simply a best response to these threat prices of the incumbent platform. Hence, as already explained, making use of eq. (18), one may conclude that $p_2^{E*} = p_2^{I*} = 2c_I - z_1 (\beta^I + N)$ whereas $p_1^{I*}$ is obtained from

$$CS_1^I (\beta^I, c_I) = z_1 \beta^I - c_I = CS_1^E (N, p_1^{E*}) = z_1 N - p_1^{E*}.$$

(19)

This then implies that $p_1^{E*} = z_1 (N - \beta^I) + c_I$.

Hence, in order for an entry equilibrium to exist, one must have that $\pi^E (p_1^{E*}, p_2^{E*}) \geq 0$, i.e.,

$$z_1 (N - \beta^I) + c_I + 2c_I - z_1 (\beta^I + N) - 2c_E \geq 0,$$

or

$$2(c_I - c_E) \geq 2z_1 \beta^I - c_I,$$

or, equivalently,

$$2N (c_I - c_E) \geq z_1 N \beta^I + N (z_1 \beta^I - c_I),$$

This completes the proof. $\blacksquare$

**Proof. (Proposition 3) (i)** At the candidate equilibrium buyers have no incentive to deviate
since buying from the incumbent would not improve their payoff. Notice that, on the one hand,

$$CS^E_1 (N, c_I + z_1 (N - \beta^I)) = CS^I_1 (\beta^I, c_I) = z_1 \beta^I - c_I > 0$$

and $CS^E_2 (N, c_I) = CS^I_2 (\beta^I, c_I) = r - c_I > 0$ (see Assumption 1), on the other. The incumbent cannot decrease its price by the policy assumption; increasing its price would not win any order on either side. The entrant could decrease its price because $c_E < c_I$, but it has no incentive to do so because it would decrease its profits (which, at the candidate equilibrium, equal $N \left[ z_1 (N - \beta^I) + 2 (c_I - c_E) \right] > 0$). If instead it increased prices on either side, it would lose customers to the incumbent.

(ii) Suppose there is a candidate exclusionary equilibrium at which $p^I_1 = c_I$ and $p^I_2 = c_I$ and both groups of “new” buyers buy from $I$. The entrant could slightly undercut the incumbent on side 2, and set $p^E_1 = c_I + z_1 (N - \beta^I)$, thereby making positive profits on both sides. Therefore, no such equilibrium would exist.

Proof. (Proposition 4) Consider an equilibrium in which all buyers buy from $I$. Now, the question is what is the lowest price that the losing (entrant) platform is willing to charge on each side of the market?

Let us first investigate what is the entrant’s minimum (threat) price for consumers on market side 2. If the entrant attracts all $N$ “new” consumers on market side 2, then how much surplus will it be able to extract from each “new” consumer on the other market side? Optimally, the entrant platform will choose a price $p^E_1$ such that

$$CS^E_1 (N, p^E_1) = z_1 N - p^E_1 \geq CS^I_1 (\beta^I, p^I_* \{p^I_* + z_1 (N - \beta^I), z_1 N\}) = z_1 \beta^I - p^I_*$$

(20)

where $p^I_*$ denotes the incumbent’s equilibrium price on market side 1. Further, it must be that $CS^E_1 (N, p^E_1) = z_1 N - p^E_1 \geq 0$. This then implies that $p^E_1 \leq \min \{p^I_* + z_1 (N - \beta^I), z_1 N\}$. Given that $p^I_* > z_1 \beta^I$ (at the candidate equilibrium, the incumbent will at least extract, on side 1, all surplus generated by the existence of the “old” generation of consumers on the other market side), the previous condition boils down to $p^E_1 \leq z_1 N$. Hence, $p^E_1 = z_1 N$ and, therefore, the minimum price that the entrant is willing to charge on market side 2 is given by $c_E - \max \{0, z_1 N - c_E\}$. Now, Assumption 1’ ensures that $z_1 N > c_I > c_E$ and, as a consequence, the entrant’s minimum

\[\text{To be more precise, the entrant will set prices a shade below } c_I + z_1 (N - \beta^I) \text{ and } c_I \text{ to market sides 1 and 2, respectively.}\]
Let us now investigate what is the entrant’s minimum (threat) price on market side 1. Suppose that the entrant attracts all $N$ “new” consumers on market side 1. How much surplus can it then extract on market side 2? The entrant platform will choose $p_2^{E^*}$ according to

$$CS_2^E (N, p_2^{E^*}) = r + z_2 N - p_2^E = CS_2^I (\beta^I, p_2^{I^*}) = r + z_2 \beta^I - p_2^{I^*}$$  

(21)

where $p_2^{I^*}$ denotes the incumbent’s equilibrium price on market side 2. But, in order to be immune to the previously described deviation, wherein the losing platform $E$ adopts its lowest side-2 (threat) price $p_2^{E^*} = 2c_E - z_1 N$, $p_2^{I^*}$ should satisfy:

$$CS_2^I (\beta^I + N, p_2^{I^*}) = r + z_2 (\beta^I + N) - p_2^{I^*} = \max \left\{ 0, r - p_2^{E^*} \right\}$$  

(22)

where $CS_2^E \left( 0, p_2^{E^*} \right) = r - p_2^{E^*} = r - 2c_E + z_1 N > 0 \text{ (under Assumption 1') }$. This then implies that condition (22) boils down to $p_2^{I^*} = p_2^{E^*} + z_2 (\beta^I + N)$. Hence, making use of eq. (21), one may conclude that $p_2^E = p_2^{I^*} + z_2 (N - \beta^I) = p_2^{E^*} + z_2 (\beta^I + N) + z_2 (N - \beta^I) = p_2^{E^*} + 2Nz_2$. But since $p_2^{E^*} = 2c_E - z_1 N$, one finally concludes that $p_2^E = 2c_E - z_1 N + 2Nz_2$ and, thus, $p_2^E - c_E = c_E - z_1 N + 2Nz_2$. Now, for $z_2$ small enough, it turns out that $p_2^E - c_E < 0$ since Assumption 1’ implies that $z_1 N > c_E$. So, the entrant’s minimum price on market side 1 is $p_2^{I^*} = c_E - \max \left\{ 0, p_2^E - c_E \right\} = c_E$.

That is, for $z_2$ small enough, the entrant is not willing embark on a deviation wherein it steals side-1 buyers from the incumbent (subsidizing their consumption) so as to win side-2 buyers.

In summary, the entrant’s minimum threat price on market side 1 is $c_E$ whereas its minimum threat price on market side 2 equals $2c_E - z_1 N$. The equilibrium prices of the incumbent platform are then simply a best response to these threat prices of the entrant platform. Hence, making use of eq. (22), one may conclude that $p_2^{I^*} = p_2^{E^*} + z_2 (\beta^I + N) = 2c_E - z_1 N + z_2 (\beta^I + N)$ whereas $p_1^{I^*}$ is obtained from

$$CS_1^I (N + \beta^I, p_1^{I^*}) = z_1 (N + \beta^I) - p_1^{I^*} = \max \left\{ 0, -p_2^{E^*} \right\}.$$  

(23)

This then implies that $p_1^{I^*} = z_1 (N + \beta^I)$. 

price on market side 2 is given by $p_2^{E^*} = 2c_E - z_1 N$. The entrant is then willing to set a price below $c_E$ on market side 2 in order to steal side-2 buyers from the incumbent so as to win side-1 buyers as well.
Hence, in order for an exclusionary equilibrium to exist, one must have that \( \pi^I(p^I_1, p^I_2) \geq 0 \), i.e.,
\[
z_1 (N + \beta^I) + 2c_E - z_1 N + z_2 (\beta^I + N) - 2c_I \geq 0,
\]
or, equivalently,
\[
2N (c_I - c_E) \leq z_1 N \beta^I + z_2 N (\beta^I + N).
\]
This completes the proof. ■

**Proof. (Proposition 5)** Consider an equilibrium where all “new” consumers buy from \( E \). Now, the question is what is the lowest price that the losing (incumbent) platform is willing to charge on each side of the market?

Consider first the incumbent’s minimum threat price regarding market side 2. If the incumbent attracts \( N \) consumers on market side 2, it can extract \( z_1 (\beta^I + N) \) from each “new” consumer on market side 1. Moreover, since side 1 consumers only benefit from the externalities from side 2, if all side 2 consumers move to the incumbent platform, the entrant platform does not exercise any competitive pressure on side 1 (recall here that the fixed benefit a consumer obtains from using a platform on market side 1 is null). Therefore, the incumbent’s minimum price on market side 2 is \( p^I_2 = c_I - \max \{0, z_1 (\beta^I + N) - c_I\} \). Now, Assumption 1’ ensures that \( z_1 (\beta^I + N) > c_I \) and, therefore, the incumbent’s minimum price on market side 2 equals \( p^I_2 = 2c_I - z_1 (\beta^I + N) \).

Let us now investigate what is the incumbent’s minimum threat price on market side 1. Suppose that the incumbent attracts \( N \) consumers on market side 1. How much surplus can it then extract on market side 2? The incumbent platform will choose \( p^I_2 \) such that
\[
CS^I_2 (\beta^I + N, p^I_2) = r + z_2 (\beta^I + N) - p^I_2 = \max \{r - p^E_2, 0\},
\]
where \( p^E_2 \) denotes the equilibrium price of the entrant platform on market side 2. But, in order to be immune to the previously described deviation, wherein the losing platform \( I \) adopts its lowest side-2 (threat) price \( p^I_2 = 2c_I - z_1 (\beta^I + N) \), \( p^E_2 \) should satisfy
\[
CS^I_2 (\beta^I, p^I_2) = r + z_2 \beta^I - p^I_2 = CS^E_2 (N, p^E_2) = r + z_2 N - p^E_2,
\]
where I assume that \( r + z_2 \beta^I - p^I_2 > 0 \). Therefore, \( r - p^E_2 \) is positive for \( z_2 \) small enough. Hence, making use of eqs. (24) and (25), one may conclude that \( p^I_2 = p^E_2 + z_2 (\beta^I + N) = \)
\( p^I_2 + z_2 (N - \beta^I) + z_2 (\beta^I + N) = p^I_2 + 2Nz_2 \). But, since \( p^I_2 = 2c_I - z_1 (\beta^I + N) \), one finally concludes that \( p^I_2 = 2c_I - z_1 (\beta^I + N) + 2Nz_2 \). Now, for \( z_2 \) small enough, the incumbent’s minimum price on market side 1 is given by \( p^I_1 = c_I - \max \{ 0, p^I_2 - c_I \} = c_I \) since Assumption 1' implies that \( z_1 (\beta^I + N) > c_I \).

In summary, the incumbent’s minimum threat price on market side 1 is \( c_I \) whereas its minimum threat price on market side 2 equals \( 2c_I - z_1 (\beta^I + N) \). The equilibrium prices of the entrant platform are then simply a best response to these threat prices of the incumbent platform. Hence, making use of eq. (25), one may conclude that \( p^E_2 = p^I_2 + z_2 (N - \beta^I) = 2c_I - z_1 (\beta^I + N) + z_2 (N - \beta^I) \) whereas \( p^E_1 \) is obtained from

\[
CS^I_1 (\beta^I, c_I) = z_1 \beta^I - c_I = CS^E_1 (N, p^E_1) = z_1 N - p^E_1. \tag{26}
\]

This then implies that \( p^E_1 = z_1 (N - \beta^I) + c_I \).

Hence, in order for an entry equilibrium to exist, one must have that \( \pi^E (p^E_1, p^E_2) \geq 0 \), i.e.,

\[
z_1 (N - \beta^I) + c_I + 2c_I - z_1 (\beta^I + N) + z_2 (N - \beta^I) - 2c_E \geq 0,
\]

or

\[
2 (c_I - c_E) \geq 2z_1 \beta^I - z_2 (N - \beta^I) - c_I,
\]

or, equivalently,

\[
2N (c_I - c_E) \geq z_1 N \beta^I + N (z_1 \beta^I - c_I) - z_2 N (N - \beta^I).
\]

This completes the proof. ■