Input Price Discrimination in the presence of Downstream Vertical Differentiation*

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Abstract

This paper investigates the competitive effects of input price discrimination (IPD) in a setting in which an upstream monopolist produces an essential input supplied to the downstream market where there is competition between two vertically differentiated retailers. Two different input pricing regimes are investigated: (i) the uniform pricing regime, in which third-degree input price discrimination is prohibited; and (ii) a discriminatory pricing regime, under which the upstream monopolist may charge different prices to the two downstream firms. We find that despite favouring the low-quality firm, IPD is welfare enhancing if and only if the quality gap is sufficiently high.

JEL Classification: L13; L41.

Keywords: Input Price Discrimination, Vertical Differentiation

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1 Introduction

Third-degree price discrimination means charging different prices in markets which are segmented according to an easily identifiable characteristic. A necessary condition for discriminatory pricing to be welfare-enhancing in a final good market is to increase total output (Schmalensee, 1981; Schwartz, 1990; Varian, 1985).

In input markets, however, the welfare effects of third-degree price discrimination are not straightforward. On the one hand, Katz (1987) and DeGraba (1990) formally show that an input monopolist sets lower input prices to less efficient firms. In line with the case of price discrimination in a final good market, this efficiency distortion is socially harmful if total output decreases or remains unchanged. On the other hand, Yoshida (2000) shows that if firms differ in their efficiency in transforming inputs into the final good (i.e., need more or less inputs per unit of the final good), an increase in the aggregate output of the final good is a sufficient condition for input price discrimination (IPD) to deteriorate welfare. The subsequent literature challenged the above mentioned effects of IPD under different upstream and downstream market assumptions (see, for example, Arya and Mittendorf, 2010; Herweg and Müller, 2016, 2014, 2012; Inderst and Valletti, 2009; Kim and Sim, 2015; O’Brien, 2014) and provides mixed results.

We depart from previous literature (in which downstream firms differ in terms of cost efficiency) by examining the welfare effects of third-degree input price discrimination when downstream firms are vertically differentiated, but symmetric in terms of cost efficiency. A case in point is the pay TV industry, where competition concerns have been raised regarding the wholesale supply of premium content (e.g., live coverage of sports events and movies). In many countries there are competing distributors of premium content that make use of different technologies (e.g., cable and FTTx technologies that all differ in terms of quality). This implies that consumers perceive their quality of service as different (i.e., there is vertical differentiation in the downstream market).

To the best of our knowledge, the effects of IPD in the presence of quality differentiation have only been addressed by Chen (2017). Chen (2017) considers an input monopolist supplying two downstream firms that differ both in terms of cost and quality, with the high-quality firm’s costs being higher than those of the low-quality firm. Therefore, in Chen’s setting there is a trade-off

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1 Fiber-to-the-X are configurations of fiber deployment where X denotes the point at which fiber is terminated (Home, Building, Curb, thus FTTH, FTTB, FTTC, respectively). The closer the fiber to the consumer premises is, the higher the broadband speed will be. Hence, each configuration is qualitatively differentiated not only from the other configurations, but also from alternative technologies, such as cable.

2 For a detailed discussion of the incentives regarding wholesale premium content distribution, on the one hand, and a characterization of the pay TV sector in a number of countries, on the other, see Weeds (2016) and the references cited therein.
between cost and quality efficiency, making it: (i) unclear which firm is the "efficient" one; and (ii) impossible to single out which effects of IPD are exclusively attributed to quality differentiation. Additionally, Chen (2017) assumes the cost difference to be such that, in equilibrium, both firms sell a positive output. Further, it can be shown that in Chen’s (2017) model whenever IPD benefits the "inefficient" firm, welfare decreases with IPD (and when IPD benefits the "efficient" firm, welfare can either increase or decrease).³

In the present paper, we investigate the competitive effects of IPD when the two downstream firms differ solely in terms of quality. Moreover, we allow the upstream producer to set discriminatory input price(s) that may leave one firm with no sales.

Within this structure, our main finding is that although an input monopolist sets lower input prices to the inefficient firm, this efficiency distortion is socially beneficial when the quality gap is significantly high, which contrasts with the conclusions of the literature focusing solely on cost differences. In our case, total output increases as well, contrasting with the result of Yoshida (2000).

The model, the equilibria and the conclusions are presented in sections 2, 3 and 4, respectively. All proofs are relegated to an online appendix.

2 Model

We consider a vertical industry in which an upstream monopolist, firm $M$, produces an input that is supplied to a duopolistic downstream sector. Each downstream firm $i \in \{1, 2\}$ requires one unit of the input to produce each unit of the final product. Although the two downstream firms are symmetric in terms of costs, the quality of their final products is different. Denoting the quality of product $i$ by $v_i$, we assume that $v_1 > v_2$. All production costs are normalized to zero except for the input price, $w_i \in [0, v_i]$, paid by firm $i$ to the upstream monopolist. This market structure is assumed to be fixed.⁴

There is a mass of $N = 1$ consumers with unit demands, each of whom values product quality differently. Consumer valuation for quality is measured by $s$, which is uniformly distributed in $[0, 1]$. Net valuation of firm $i$'s product is then $U_i = sv_i - p_i$, where $p_i$ denotes the retail price. Consumers choose between buying one unit from either firm or not purchasing at all, which results in zero utility.

³Which firm is the efficient firm will depend on specific parameter values. A mathematical appendix comparing our results with those of Chen is available from the authors upon request.

⁴In particular, we do not consider quality to be endogenously determined by the firms as in Choi and Shin (1992) and Wauthy (1996) and we do not allow downstream firms to decide whether or not to be active.
Consumers select firm 1 over firm 2 if and only if $s > s_{1,2} = (p_1 - p_2)/(v_1 - v_2)$ and prefer purchasing from firm $i$ to making no purchase if and only if $s > s_{i,0} = p_i/v_i$. The demand faced by each downstream firm is then:

$$
D_1(p_1, p_2) = \begin{cases} 
1 - \frac{p_1}{v_1} & 0 < p_1 < \frac{v_1}{v_2}p_2 \\
1 - \frac{p_1 - p_2}{v_1 - v_2} & \text{if } \frac{v_1}{v_2}p_2 < p_1 < p_2 + v_1 - v_2 \\
0 & p_2 + v_1 - v_2 < p_1 < v_1 \\
1 - \frac{p_2}{v_2} & 0 < p_2 < p_1 - v_1 + v_2 \\
\frac{p_1 - p_2}{v_1 - v_2} - \frac{p_2}{v_2} & \text{if } p_1 - v_1 + v_2 < p_2 < \frac{v_2}{v_1}p_1 \\
0 & \frac{v_2}{v_1}p_1 < p_2 < v_2 
\end{cases}
$$

$$
D_2(p_1, p_2) = \begin{cases} 
1 - \frac{p_1}{v_1} & 0 < p_1 < \frac{v_1}{v_2}p_2 \\
1 - \frac{p_1 - p_2}{v_1 - v_2} & \text{if } \frac{v_1}{v_2}p_2 < p_1 < p_2 + v_1 - v_2 \\
0 & p_2 + v_1 - v_2 < p_1 < v_1 \\
1 - \frac{p_2}{v_2} & 0 < p_2 < p_1 - v_1 + v_2 \\
\frac{p_1 - p_2}{v_1 - v_2} - \frac{p_2}{v_2} & \text{if } p_1 - v_1 + v_2 < p_2 < \frac{v_2}{v_1}p_1 \\
0 & \frac{v_2}{v_1}p_1 < p_2 < v_2 
\end{cases}
$$

Thus, three outcomes are possible: only firm $i \in \{1, 2\}$ is active and serves $(1 - s_{i,0})$ consumers or both firms are active with firm 1 serving $(1 - s_{1,2})$ consumers and firm 2 serving $(s_{1,2} - s_{2,0})$ consumers.

The timing of the game is as follows. First, the upstream monopolist sets the input price(s). When IPD is possible, firm $M$ can set a different per unit input price $w_i$ to each downstream firm, whereas under uniform pricing both downstream firms pay the same input price $w_i$. Afterwards, downstream firms set retail prices simultaneously.

### 3 Equilibrium

We solve the game by backward induction, characterizing first the downstream equilibrium and determining afterwards firm $M$’s optimal input price(s). We then compare the discriminatory and uniform pricing regimes in terms of social welfare.

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5With two-part tariffs, only the high-quality firm would be active. To obtain the profit of a vertically integrated high-quality monopolist, the upstream firm would optimally set a common menu of prices to both retailers wherein: (i) the per unit charge is set at zero ($w_1 = w_2 = 0$); and (ii) the fixed fee equals the high-quality downstream monopolist profit ($F_1 = F_2 = v_1/4$). This menu would only be accepted by firm 1 and IPD would not result in higher profits for firm $M$. With linear tariffs instead, firm $M$ also serves the low-quality firm so as to reduce the double marginalization problem.
3.1 Retail Price Stage

The profit function of firm $i = 1, 2$ is given by $\pi_i = (p_i - w_i)D_i$. Taking the first order condition of $\pi_i$ with respect to $p_i$ yields firm $i$’s best response to $p_j$, with $j = 1, 2$ and $j \neq i$, which is presented in Lemma 1.6

**Lemma 1** (a) Firm 1’s best response function is:

$$p_1^*(p_2) = \begin{cases} 
  p_2 + v_1 - v_2 & p_2 < p_2^L \\
  \frac{1}{2}(w_1 + v_1 - (v_2 - p_2)) & p_2^L < p_2 < p_2^M \\
  \frac{w_1}{v_2}p_2 & p_2^M < p_2 < p_2^H \\
  \frac{1}{2}(w_1 + v_1) & p_2 > p_2^H \end{cases}.$$  \hspace{1cm} (1)

(b) Firm 2’s best response function is:

$$p_2^*(p_1) = \begin{cases} 
  \frac{w_2}{v_1}p_1 & p_1 < p_1^L \\
  \frac{1}{2w_1}(w_2v_1 + p_1v_2) & p_1^L < p_1 < p_1^M \\
  p_1 - v_1 + v_2 & p_1^M < p_1 < p_1^H \\
  \frac{1}{2}(w_2 + v_2) & p_1 > p_1^H \end{cases}.$$  \hspace{1cm} (2)

All firm $j$ price thresholds above are functions of $w_i$ presented in the Appendix. Firm $i$’s best response function is upward sloping, with four distinct branches.7 When the rival sets a price lower than $p_j^L$, the optimal reaction of firm $i$ is to set a price that does not attract any consumer so as to avoid selling with a negative margin. On the other extreme case, when firm $j$ sets a price above $p_j^M$, the optimal reaction of the rival firm is to set a price such that firm $j$ will not sell. If $p_j$ is sufficiently high (above $p_j^H$) firm $i$ can set its monopoly price.8 For price values between $p_j^L$ and $p_j^M$, the best response of each retailer is such that both downstream firms are active in the retail market.

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6 When indifferent between several prices, we assume that the firm in question sets the lowest one.
7 Choi and Shin (1992) present the two firm’s best-response functions in a similar setting and assume that the two firms neither face costs nor cover the market. Their Eq. (3) corresponds to the particular branch of $p_j^*(p_i)$ for $p_j^L < p_i < p_j^M$.
8 The monopoly price is $p_i = \frac{1}{2}(v_i + w_i)$. 

5
Inspection of the best response functions (1) and (2) reveals that, depending on the input prices set by the upstream monopolist, there are different types of equilibria in the retail pricing subgame. Figures 1(a) to 1(e) depict the possible equilibria. In fact, depending on the price thresholds (which, as mentioned, are functions of \(w_1\) and \(w_2\)), the best-response functions can intersect as in Figure 1(c), giving rise to an interior equilibrium, but can also overlap on segments of \(p_2 = \frac{v_2}{v_1} p_1\) (in which case the demand for firm 2 is zero) or on segments of \(p_2 = p_1 - v_1 + v_2\) (in which case the demand for firm 1 is zero). The next Lemma characterizes the downstream price equilibrium, as a function of \(w_1\) and \(w_2\).

**Lemma 2** In the retail price equilibrium:

(a) If \(w_1 < w_1^{LL}\), then \(p_1^* = \frac{1}{2} (w_1 + v_1)\) and \(p_2^* = \frac{v_2 + w_1}{v_1}\). Firm 2 does not sell in equilibrium.

(b) If \(w_1^{LL} < w_1 < w_1^L\), then \(p_1^* = \frac{w_2 v_1}{v_2}\) and \(p_2^* = w_2\). Firm 2 does not sell in equilibrium.

(c) If \(w_1^L < w_1 < w_1^H\), then

\[
p_1^* = v_1 \frac{2w_1 + w_2 + 2(v_1 - v_2)}{4v_1 - v_2}, \quad p_2^* = \frac{w_1 v_2 + 2w_2 v_1 + v_2 (v_1 - v_2)}{4v_1 - v_2}
\]

and both firms have positive sales in equilibrium.

(d) If \(w_1^H < w_1 < w_1^{HH}\), then \(p_2^* = w_1 - v_1 + v_2\) and \(p_1^* = w_1\). Firm 1 does not sell in equilibrium.

(e) \(w_1 > w_1^{HH}\), then \(p_2^* = \frac{1}{2} (v_2 + w_2)\) and \(p_1^* = \frac{1}{2} (v_2 + 2v_1 - v_2)\). Firm 1 does not sell in equilibrium.

The expressions for \(w_1^{LL}, w_1^L, w_1^H\) and \(w_1^{HH}\), all functions of \(w_2\), are presented in the Appendix. The interpretation of this result is straightforward. For low values of \(w_1\), firm 1, who has a quality advantage over its rival, can set its monopoly price in equilibrium. For higher values of \(w_1\) it can still be the only firm selling in equilibrium but with a price below the monopoly price. For even higher values of its input price it will share the market with firm 2.\(^9\) And if \(w_1\) is even higher, it will not sell (with firm 2 pricing below or at its monopoly level).

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\(^9\)In Wauthy (1996) firms have no costs and parameter \(s\) is uniformly distributed between \(s^-\) and \(s^+\). In our setting, when \(w_i = 0\), it is always true that \(w_i \leq w_i^{LL} < w_i^{HH}\): the retail prices are those in Lemma 2(c) which correspond to the prices in Wauthy’s Proposition 1(A). The several equilibrium possibilities in Lemma 2 are also present in Wauthy (1996) and Gabszewicz and Thisse (1979). However, this is due to different parameters in the distribution of consumers tastes (or income) and not to different relative input prices as in our case.
3.2 Wholesale Price Stage

In this stage, firm $M$ sets the input price(s) anticipating the retail market equilibrium characterized in Lemma 2. Two different input pricing regimes are considered: discriminatory and uniform pricing.

3.2.1 Input Price Discrimination

When IPD is allowed, firm $M$ chooses $w_1$ and $w_2$ to maximize its total profit, $\pi_M = w_1D_1 + w_2D_2$.

From Lemma 2, when $w_1 < w^L$, then $p_1 = \frac{v_1}{v_2} p_2$, $D_1 = 1 - \frac{p_1}{v_1}$ and $D_2 = 0$. Firm $M$’s profit is then $w_1(1 - \frac{p_1}{v_1})$, which is decreasing in $p_1$, with $p_1$ assumed to be the min $\left\{ \frac{1}{2} (w_1 + v_1), \frac{w_2v_1}{v_2} \right\}$ as defined in Lemma 2 (a) and (b). Firm $M$ can use $w_2$ to make this price as low as possible, subject to $w_1 < w^L$. The optimal input price is then $w_1 = \frac{v_1}{2}$.

When $w^L < w_1 < w^H$, both downstream firms are active, and their demand is $D_1 = 1 - s_{1,2}$ and $D_2 = s_{1,2} - s_{2,0}$. Then, $\pi_M$ can be written as a function of $w_1$ and $w_2$, and solving the system of first-order conditions yields the input prices: $w_1 = \frac{v_1}{2}$ and $w_2 = \frac{v_2}{2}$.

Finally, when $w_1 > w_H$, then $p_1 = p_2 + v_1 - v_2$. Hence, $D_1 = 0$ and $D_2 = 1 - \frac{p_2}{v_2}$. The monopolist’s profit is then $w_2(1 - \frac{p_2}{v_2})$, which is decreasing in $p_2 = \min \left\{ \frac{1}{2} (v_2 + w_2), w_1 - v_1 + v_2 \right\}$. Again, firm $M$ can use $w_1$ to lower $p_2$, subject to $w_1 > w_H$. The input price that maximizes its profit is $w_2 = \frac{v_2}{2}$.

The comparison of profits in each case shows that firm $M$ earns more profit when both firms are active. The implications are summarized in the next Proposition:

**Proposition 1** Under IPD, in equilibrium:

(a) Input prices are:

$$w_1 = \frac{v_1}{2} \text{ and } w_2 = \frac{v_2}{2}$$

(b) Retail prices are:

$$p_1 = \frac{3}{2} \frac{2v_1 - v_2}{4v_1 - v_2} v_1 \text{ and } p_2 = \frac{15v_1 - 2v_2}{2} \frac{v_1}{4v_1 - v_2} v_2$$

(c) Downstream quantities are:

$$Q_1 = \frac{v_1}{4v_1 - v_2} \text{ and } Q_2 = \frac{1}{2} \frac{v_1}{4v_1 - v_2}$$
(d) Consumer surplus is:
\[
CS = \frac{1}{8} v_1^2 \frac{4v_1 + 5v_2}{(4v_1 - v_2)^2}
\]

(d) Profits are:
\[
\pi_1 = v_1^2 \frac{v_1 - v_2}{(4v_1 - v_2)^2}; \quad \pi_2 = \frac{1}{4} v_1v_2 \frac{v_1 - v_2}{(4v_1 - v_2)^2} \quad \text{and} \quad \pi_M = \frac{1}{4} v_1 \frac{2v_1 + v_2}{4v_1 - v_2}
\]

(e) Welfare is:
\[
W = v_1 \frac{28v_1^2 - 4v_2^2 + 3v_1v_2}{8 (4v_1 - v_2)^2}
\]

The above result is in line with the corresponding result of the related literature concluding that the input monopolist sets lower input prices to less cost-efficient firms. This equilibrium is as described in Figure 1(c).

3.2.2 Uniform Pricing

With \(w_1 = w_2 = w\), Lemma 2 can be considerably simplified. In particular, it is not possible to have an equilibrium in which firm 1 does not sell.

Firm M’s profit is now \(\pi_M = w(D_1 + D_2)\). If \(w > w^L\), then

\[
D_1 = 1 - s_{1,0} = 1 - \frac{\min \left\{ \frac{1}{2} (w + v_1), \frac{w v_1}{v_2} \right\}}{v_1}
\]

and \(D_2 = 0\), whereas if \(w < w^L\), then \(D_1 = 1 - s_{1,2}\) and \(D_2 = s_{1,2} - s_{2,0}\), hence \((D_1 + D_2) = 1 - \frac{p_2}{v_2} = 1 - \frac{w(2v_1+v_2)+v_2(v_1-v_2)}{v_2(4v_1-v_2)}\).\(^{10}\) Taking the first order condition of \(\pi_M\) with respect to \(w\) for each case and analyzing the results, yields the optimal input pricing strategy of the wholesaler.

For sufficiently low values of \(w\), firm 2 can have positive sales in equilibrium. As \(w\) increases, both retail prices increase, with \(p_1\) increasing more than \(p_2\). However, \(s_{1,2} = \frac{w+2v_1-v_2}{4v_1-v_2}\) increases at a lower rate than \(s_{2,0} = \frac{w^2+2v_1+v_2}{4v_1-v_2}\). Therefore, as \(w\) increases, the demand for firm 2 (given by \(s_{1,2} - s_{2,0}\)) decreases until it eventually reaches zero. The following proposition characterizes the equilibrium of the uniform pricing regime and shows that firm M’s privately optimal input price gives rise to the latter case, where the less efficient firm is left with zero sales.

\(^{10}\) The expressions for \(w^L\) is presented in the appendix.
Proposition 2 Under uniform pricing, in equilibrium:

(a) Input price is:

\[ w = \frac{1}{2} v_1 \text{ if } \frac{v_2}{v_1} < \frac{1}{2} \]
\[ w = \frac{1}{2} v_2 \text{ if } \frac{v_2}{v_1} > \frac{1}{2} \]

(b) Retail prices are:

\[ p_1 = \frac{3}{4} v_1 \text{ and } p_2 = \frac{3}{4} v_2 \text{ if } \frac{v_2}{v_1} < \frac{1}{2} \]
\[ p_1 = \frac{1}{2} v_1 \text{ and } p_2 = \frac{1}{2} v_2 \text{ if } \frac{v_2}{v_1} > \frac{1}{2} \]

(c) Downstream quantities are:

\[ Q_1 = \frac{1}{4} \text{ and } Q_2 = 0 \text{ if } \frac{v_2}{v_1} < \frac{1}{2} \]
\[ Q_1 = \frac{1}{2} \text{ and } Q_2 = 0 \text{ if } \frac{v_2}{v_1} > \frac{1}{2} \]

(d) Consumer surplus, profits and welfare are:

\[ CS = \frac{v_1}{32}; \pi_1 = \frac{v_1}{16}; \pi_2 = 0; \pi_M = \frac{v_1}{8} \text{ and } W = \frac{7v_1}{32} \text{ if } \frac{v_2}{v_1} < \frac{1}{2} \]
\[ CS = \frac{v_1}{8}; \pi_1 = \frac{v_1 - v_2}{4}; \pi_2 = 0 \text{ and } \pi_M = \frac{v_2}{4}; W = \frac{3v_1}{8} \text{ if } \frac{v_2}{v_1} > \frac{1}{2} \]

In contrast with Chen (2017), there is no interior solution under uniform pricing. If \( \frac{v_2}{v_1} < \frac{1}{2} \), firm 1’s quality advantage is so high that firm M’s optimal uniform price leads firm 1 to sell at its monopoly price: there will be double marginalization. If \( \frac{v_2}{v_1} > \frac{1}{2} \), firm 1’s quality advantage allows it to be the only firm selling downstream, but the retail price is constrained by the presence of firm 2. This benefits the upstream monopolist as it mitigates double marginalization. Despite the fact that the input price is lower (to allow firm 2 to put some pressure on firm 1), firm M
benefits from substantially larger sales due to the lower retail prices.\footnote{In the uniform price regime equilibrium, the assumption that the market structure is fixed is therefore relevant. Just like in the asymmetric Bertrand price game, the existence of a less efficient firm affects the equilibrium, although this firm does not have a positive market share.}

\section{Discussion}

Figure 2 presents consumer choices in the two pricing regimes, for different values of $v_2/v_1$. When the quality difference is high ($v_2/v_1$ is low), uniform input pricing leads firm 1 to be a monopolist, whereas IPD leads to a downstream duopoly (in which firm 1 is charged the same input price but firm 2 has a lower one). Competition between the two downstream firms then reduces $p_1$, making more consumers purchase the high-quality product. Additionally, some consumers that would not purchase at all under uniform pricing, now purchase the low-quality product. The two effects lead to higher welfare and consumer surplus levels.

When the quality difference is low ($v_2/v_1$ is high), uniform pricing leads firm 1 to be the only firm with positive sales, but charging a price below its monopoly price. IPD increases the input price for firm 1 and maintains the input price for firm 2. This makes some consumers switch from the high-quality product to the low-quality product while others stop purchasing. As a result, welfare and consumer surplus are lower under IPD. In terms of profits, IPD increases the profit of the upstream monopolist and that of the inefficient firm, while lowering the profit of the more efficient firm. This explains Proposition 3.

**Proposition 3** When compared to uniform pricing, IPD increases welfare if and only if $v_2/v_1 < 1/2$.

Thus, the efficiency distortion stemming from favouring the low-quality firm is not always socially harmful: IPD increases total output and welfare if and only if the quality gap is significantly high.

\section{Conclusions}

In this paper, we studied the impact of third-degree input price discrimination (IPD) when the downstream firms are vertically differentiated. Our main result is that, compared to uniform pricing, IPD increases total output and welfare if and only if the quality gap is significantly high. This finding contrasts with the result of the seminal papers that, assuming instead cost-asymmetric downstream firms, have found that IPD may be socially harmful because it benefits the less efficient firms.
References


Figures

Fig. 1(a): Retail equilibrium as in Lemma 2 (a)

Fig. 1(b): Retail equilibrium as in Lemma 2 (b)

Fig. 1(c): Retail equilibrium as in Lemma 2 (c)

Fig. 1(d): Retail equilibrium as in Lemma 2 (d)

Fig. 1(e): Retail equilibrium as in Lemma 2 (e)
Fig. 2: Consumer choices: Indifferent consumers as a function of $v_2/v_1$ for the two pricing regimes.
Appendix

The critical price and input price thresholds presented in Lemmas 1 and 2 and in the text are the following:

\[
\begin{align*}
 p^L_1 & := \frac{w_2v_1}{v_2} & p^L_2 & := w_1 - v_1 + v_2 & w^{LL}_1 & := v_1 \frac{2w_2-v_2}{v_2} \\
 p^M_1 & := \frac{v_1(w_2+2v_1-2v_2)}{2v_1-v_2} & p^M_2 & := \frac{v_2(w_1+v_1-v_2)}{2v_1-v_2} & w^L_1 & := w_2 \frac{2w_1-v_2}{v_2} - (v_1 - v_2) \\
 p^H_1 & := \frac{1}{2}(w_2 + 2v_1 - v_2) & p^H_2 & := \frac{1}{2}(w_1 + v_1) \frac{v_2}{v_1} & w^H_1 & := p^M_1 \\
 - & & - & & w^{HH}_1 & := p^H_1 \\
 - & & - & & w^L & := \frac{v_2}{2}
\end{align*}
\]