Endogenous Mergers and Collusion in Asymmetric Market Structures*

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Abstract
Recent empirical evidence shows that cartels are often asymmetric, while cartel theory suggests that firm symmetry is conducive to collusion. Including an indivisible cost of cartelization, we show that medium asymmetric market structures are more conducive to collusion, since they balance the small firms’ incentives to stay in the cartel against the need to cover the cartel leaders’ indivisible cartelization cost. Using an endogenous merger model, we also show that forbidding mergers leading to symmetric market structures can induce mergers leading to asymmetric market structures with a higher risk of collusion. Current anti-symmetry merger policy can thus be counterproductive.

Keywords: Collusion; Cost Asymmetries; Merger Policy

JEL classification: D43; K21; L41

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1 Introduction

The detection and prosecution of cartels is one of the main priorities of competition authorities in most jurisdictions, including the EU and the USA. The existing theory has stressed that symmetry is a factor conducive to collusion, since the asymmetry between firms creates incentives to deviate both in the collusive phase and the punishment phase.¹

However, the few empirical studies available on cartels and asymmetries seem to indicate that asymmetries are often quite pronounced. Davies and Olczack (2008) studied 41 EU cartel cases (in the period 1990-2006) and concluded that “size asymmetries among members are often considerable”. Grout and Sonderegger (2005) studied 24 EU cartel cases and concluded “We observe considerable heterogeneity in the market shares held by cartel members”. Moreover, in 10 out of 43 EU cartel cases during the period 2002 to 2007, i.e. in approximately 23 percent of the cases, a ring leader was explicitly identified by the Court.²

These observations raise the question of why so many cartels are asymmetric and have ring leaders. We present one possible theoretical explanation for this phenomenon. Our starting point is that the possibility of sharing the costs associated with running a cartel is limited. For instance, the cost of protecting the cartel by buying out potential entrants typically falls on the acquiring firm,³ thus permitting coconspirators to free-ride on the investment. Moreover, it is less costly to only have a few firms administering and monitoring the price behavior of the colluding firms. Along these lines, Harrington (2006, pp. 49-51) provides some evidence that in real world cartels, the tasks of monitoring sales volumes and auditing sales volumes (to solve problems of misreporting) are usually carried out by a single cartel member. In order to focus on this aspect, we assume there to be an indivisible cost associated with collusion.

Using a standard repeated game oligopoly framework, our first main result is to establish an inverse U-shaped relationship between the likelihood of collusion and the level of cost asymmetries. More specifically, we show that there are levels of the indivisible cost of collusion such that: (i) firms do not collude when asymmetries are small; (ii) firms do collude when asymmetries are moderate; and (iii) firms do not collude when asymmetries are large. On the one hand, firms cannot be too asymmetric, since a large asymmetry gives the smallest firm in the collusive agreement a strong incentive to deviate from the collusive conduct (the smallest firm is the one with the highest potential

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¹For an overview of the main trade-offs studied in this literature and the resulting implications for the analysis of horizontal mergers, see Kaplow and Shapiro (2007).
²Authors’ own reading of the 43 EU cartel cases in the period 2002-2007.
³In principle, a cartel could jointly acquire an entrant. However, in practice, joint acquisition by rivals is exceedingly rare and might trigger in-depth investigations by anti-trust authorities.
of stealing the business of its rivals). On the other hand, firms must be sufficiently asymmetric or else the largest firm (the ring leader) would lack the incentive to cover the indivisible cost of collusion. In other words, an indivisible cost of collusion introduces a second constraint that must be met for collusion to be sustainable.

We then proceed to analyze the equilibrium in an endogenous merger formation model. Within this theoretical framework, we are able to show that firms may have an incentive to merge so as to create asymmetric market structures conducive to collusion, i.e. market structures where one (large) firm could cover the indivisible cost associated with being the cartel leader. More specifically, we show that this result holds in a differentiated products Bertrand model (henceforth DPB model) à la Singh and Vives (1984) for a cartel that: (i) maximizes joint profits; or (ii) imposes an equal increase in prices for all products sold by the cartel members.

We then turn to the implications of the obtained results for merger policy. Symmetry of firms in an industry is a factor that has attracted some attention in merger reviews by antitrust authorities on both sides of the Atlantic. This interest stems from the fact that firm and product homogeneity may potentially be conducive to coordinated effects in an oligopolistic market. The European Commission, for instance, notes in its Guidelines on the assessment of horizontal mergers that “[f]irms may find it easier to reach a common understanding on the terms of coordination if they are relatively symmetric”. Similarly, the US Horizontal Merger Guidelines state that “reaching terms of coordination may be facilitated by product or firm homogeneity”. The relevance of this factor is, for instance, illustrated by the Baby-food case, where the Federal Trade Commission argued that the proposed merger would make the two remaining suppliers’ cost structures more similar, allowing these rivals to “arrive at a mutually advantageous detente”. To capture these aspects

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4 We use the model by Horn and Persson (2001a). This model has, for instance, been applied to study how the pattern of domestic and cross-border mergers depends on trade costs by Horn and Persson (2001b), and the incentives for the formation of domestic and cross-border mergers in unionized oligopoly by Lommerud, Sorgard and Straume (2006).

5 The Guidelines make an explicit reference to the Gencor-Lonrho merger (Case/M 619) and the Nestlé-Perrier merger (Comp/M 190). In the latter case, the Commission found that Nestlé/Perrier and BSN would be jointly dominant in the French market for bottled water and, more specifically, that “[a]fter the merger, there would remain two national suppliers on the market which would have similar capacities and similar market shares (symmetric duopoly) ... Given this equally important stake in the market and their high sales volumes, any aggressive competitive action by one would have a direct and significant impact on the activity of the other supplier and most certainly provoke strong reactions with the result that such actions could considerably harm both suppliers in their profitability without improving their sales volumes. Their reciprocal dependency thus creates a strong common interest and incentive to maximize profits by engaging in anti-competitive parallel behavior. This situation of common interests is further reinforced by the fact that Nestlé and BSN are similar in size and nature, are both active in the wider food industry and already cooperate in some sectors of that industry.”


7 FTC /. H.J. Heinz Company, Public Version of Commission Reply Brief, January 17, 2001, Downloaded from
in a very stylized way, in the proposed model we assume that the anti-trust authority can only
use a simple merger policy rule: either it allows mergers or it decides to block mergers leading to
symmetric market structures.

Our second main result, then, shows that a merger policy that blocks mergers leading to a
symmetric industry structure may be counterproductive. The adoption of such a policy can induce
firms to choose mergers leading to asymmetric market structures which are associated with higher
production costs and a lower aggregate producer and consumer surplus. If a merger induces a sym-
metric industry structure, the large (efficient) firm may not find it profitable to bear the indivisible
cost of collusion after the merger, thus ruling out collusion possibilities. But if an “anti-symmetry”
merger policy is being adopted, it may well happen that equilibrium mergers induce an industry
structure where firms are moderately asymmetric, creating the most favorable conditions for collu-
sion to arise. Moreover, for some parameter values, the gains from successful collusion which are
obtained in this asymmetric industry structure are so high that they will more than compensate
for the fact that one of the colluding firms has high production costs. Consequently, by forbidding
a merger leading to a symmetric industry structure, competition authorities may end up with a
merger that does not only create higher production costs, but also facilitates collusion.

Key to the result described above is that the rule determining the collusive outcome must have
the property that the large (efficient) firm benefits sufficiently more from collusion when there is
an increase in industry asymmetries,\(^8\) which is the case for both the joint profit maximizing cartel
rule and the equal price increasing cartel rule. In contrast, we show that if the rule determining the
collusive outcome instead has the property that the large (efficient) firms’ gains from collusion do
not increase in asymmetry, then asymmetries between firms hurt collusion possibilities even more
when an indivisible cost of collusion is present. For collusion to arise in this case, firms cannot be
too asymmetric since the large (efficient) firm will then have no incentives to cover the indivisible
cost.\(^9\) This is shown to hold in the DPB model under a constant market share cartel behavior
rule.\(^10\)

Since the indivisible cost associated with collusion plays an important role in the model, a
relevant question that should be raised is then in which situations do we expect this indivisible cost
of cartelization to be large? In our opinion, there are, at least, two such situations. First, when there

\(^{8}\)Huck et al (2004) show that coordination between plants in a merged entity can improve merger profitability
through a commitment advantage for the merged firm.

\(^{9}\)A small inefficient firm will have an even weaker incentive to cover the indivisible cost.

\(^{10}\)This rule has been used in several cartels in practice (see Harrington, 2006).
is a threat that a potential entrant will enter the market and destabilize the cartel, a cartel member (most probably, a large one) may decide to prevent entry by conducting a buyout acquisition of the potential entrant. Clearly, however, this buyout will be costly for the acquirer and could thus be viewed as an endogenous indivisible cost which must be incurred by some cartel member so as to protect the cartel. In an extension of our benchmark model,\textsuperscript{11} we explore this issue and show that the cartel member undertaking this buyout needs to be compensated with sufficiently high profits in the subsequent cartel interaction. Therefore, the analysis and the mechanisms described above (for the exogenous indivisible fixed cost of cartelization case) also hold for the case with endogenous indivisible costs. An implication of this result is then that the lower the exogenous entry barriers are, the more asymmetric must the market structure be for collusion to be sustained, since entry-deterring take-overs to protect the cartel then become more costly. Consequently, an empirical prediction of the model is that, we should observe relatively more asymmetric cartels in markets with low exogenous entry barriers. Second, cartelized firms face a high a risk of discovery of illegal cartel behavior by the antitrust authorities. Moreover, in most antitrust jurisdictions, being a ring-leader is usually considered to be an aggravating circumstance, implying that the ring-leader is fined substantially more than other cartel members. Further, it should also be noted that ring-leaders are usually not eligible for leniency, which exacerbates the (expected) fine (fixed cost of cartelization) that a ring-leader must incur in case the cartel ends up being uncovered by competition authorities. Consequently, only in sufficiently asymmetric market structures may ring-leaders be compensated for taking on such risks. However, it should be noted that this also implies that the very symmetric market structure can be more stable since firms then divide the cost of expected fines equally.

There are several empirical facts indicating the potential importance of the mechanisms identified in the analysis of this paper. According to our proposed theory, the medium asymmetric market structures should be more conducive to collusion, whereas the previous literature would predict that collusion is more likely in symmetric industries. A very rough check of these predictions would be to compare the concentration indexes in EU court cartel cases with some EU industry average concentration indexes. Using our cartel cases data set, we find that in the 43 European cartel cases between 2002 and 2007, the size of the second largest firm was on average 70% of the size of the largest firm, and the size of the third largest firm was on average 32% of the size of the largest firm (see Table 1).\textsuperscript{12} We then compare these concentration indexes with those reported in Davies and Lyons (1996) for the EU in 1987. Davies and Lyons (1996) found that the size of the

\textsuperscript{11}See Section 3.

\textsuperscript{12}Authors’ calculations based on the 43 cartel cases in the period 2002-2007.
Table 1: The relative size of the three largest members of EU cartels and average relative size of the three largest firms in EU industries (1987)

<table>
<thead>
<tr>
<th>Cartel mean</th>
<th>EU industry average mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of the second largest firm in relation to the largest firm</td>
<td>Size of the third largest firm in relation to the largest firm</td>
</tr>
<tr>
<td>70%</td>
<td>32%</td>
</tr>
<tr>
<td>64%</td>
<td>46%</td>
</tr>
</tbody>
</table>

second largest firm was on average 64% of the largest firm, and the size of the third largest firm is on average 46% of the largest firm. Consequently, the market structures corresponding to the cartel cases do not seem to be substantially more symmetric than the average market structure in the EU, which seems consistent with our proposed theory. Moreover, in their survey article of the cartel literature, Leveinstein and Suslow (2006, p. 47) report that cost-asymmetries have ambiguous effects on collusive possibilities.

Now, we turn to evidence for cartel leadership. As reported above, a ring leader was explicitly identified by the Court in several EU cartel cases. One example is the pre-insulated pipe cartel in Europe in the 1990s, subsequently referred to as the “ABB case”, which can briefly be described as follows. In 1987, just before the merger with ASEA, Brown Boveri Company embarked on a strategic program of acquiring producers of insulated steel pipes for district heating across Europe. According to the European Commission, “[t]he organization of the cartel represented a strategic plan by ABB to control the district heating industry ... It is abundantly clear that ABB systematically used its economic power and resources as a major multinational company to reinforce the effectiveness of the cartel and to ensure that other undertakings complied with its wishes ... The gravity of the infringement is aggravated in ABB’s case by the following factors: ABB’s role as the ringleader and instigator of the cartel and its bringing pressure on other undertakings to persuade them to enter the cartel.” Moreover, ABB took the bulk of the costs of running the cartel. In particular, in the situation where Powerpipe, a firm outside the cartel, tried to expand its activities, ABB used large resources to try to eliminate the maverick from the market. This predatory activity was multi-dimensional and costly as indicated by the following quote in §§91-92 of the EC Decision: “Numerous passages in ABB strategy documents during this period covered by this Decision refer to plans to force Powerpipe into bankruptcy... In 1993 ABB embarked on a systematic campaign of luring away key employees of Powerpipe, including its then managing

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14 §169 and §171 in the Decision.
15 As phrased by the Commission: “Its systematic orchestration of retaliatory measures against Powerpipe, aimed at its elimination.”
director, by offering the salaries and conditions which were apparently exceptional in the industry.”

The existing cartel theory literature has examined how asymmetries between firms affect the benefits of collusion and cheating (see Leveinstein and Suslow (2006) for an overview). This paper is most closely related to the strand of the literature on the effects of firm cost asymmetries on collusion. The conventional wisdom is that cost asymmetries make it more difficult for firms to sustain a collusive outcome. Along these lines, Ivaldi et al. (2003) emphasize that “even if firms agree on a given collusive price, low-cost firms will again be more difficult to discipline, both because they might gain more from undercutting their rivals and because they have less to fear from a possible retaliation from high-cost firms” (p.36). These issues are explored in detail in Vasconcelos (2005) (see also Compte et al. (2002), Kühn (2004) and, more recently, Bos and Harrington (2010)). In addition, Eswaran (1997) shows that by adjusting market shares across the business cycle (and thereby eliminating the possibility of bankruptcy for its inefficient rivals), a low cost (‘swing’) producer could enlarge the set of self-enforcing collusive equilibria.

We contribute to this literature by allowing for an indivisible cost of collusion and introducing an endogenous merger model pre-collusion. This enables us to show that firms may have an incentive to embark on mergers leading to asymmetric market structures since the merged firm could then cover the indivisible cost associated with cartel leadership. We show that in industries where there are important indivisible costs of collusion, the most severe coordinated effects may be due to mergers inducing a medium asymmetric market structure since it balances the incentive to stay in the collusive agreement against the need to cover the cartel leader’s indivisible costs of collusion.

The paper continues as follows. The model is spelled out in Section 2. Section 2.1 studies the collusion pattern when firms are asymmetric and one firm faces an indivisible cost of collusion, while Section 2.2 shows that an anti-symmetry merger policy can be counterproductive. Section 3 provides an extension of the basic model where the indivisible cost of cartelization is endogenized in the form of a buyout of a potential entrant. Section 4 discusses the implications of our results for the analysis of merger policy regarding the coordinated effects of a merger. In Section 5, we discuss the role of Antitrust Authorities in cartel detection and the problem of the cartel allocation rules. Finally, Section 6 concludes the paper.

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16 Product differentiation has also been found to decrease the benefit both to collusion and to cheating. See, for example, Dick (2003) and Thomadsen and Rhee (2007).
2 The Model

We consider a market with three initial firms, denoted 1, 2 and 3. The firms play an infinitely repeated game. In period 0, there is a merger formation game in which a merger between two of the firms is possible.\(^\text{17}\) Then, from period 1 onwards, the firms resulting from the merger formation process play a standard repeated oligopoly game. The game is solved backwards in the usual fashion.

Pre-merger, the constant variable costs of firms 1, 2 and 3 are \(c_1 = c > 0\), \(c_2 = 0\), and \(c_3 = 0\), respectively. Post merger, two firms will be active in the market, where the merged firm’s variable cost \(c_m\) is equal to the lowest variable cost of the participating firms (i.e. \(\min(c_i, c_j) = 0\) for \(i \neq j\)), and the non-merged firm’s variable cost is denoted \(c_n\). Firm \(i\)’s per-period profit is denoted \(\pi_{i,t}(p_{i,t}, p_{-i,t}, c_i, c_{-i})\), where \(p_{i,t}\) is firm \(i\)’s price in period \(t\).

2.1 The Repeated Duopoly Game

We start our analysis with the study of the infinitely repeated game that follows the merger formation stage and identify conditions for sustainable collusion to arise. Time is denoted by \(t = 0, \ldots, \infty\). At the beginning of the last stage, the present value of firm \(i\)’s profit is

\[
\Pi_i = \sum_{t=0}^{\infty} \delta^t \cdot \pi_{i,t}(p_{i,t}, p_{-i,t}, c_i, c_{-i}),
\]

where \(\delta \in (0, 1)\) is the common discount factor.

Firms will try to collude, which may or may not be possible in the repeated game. As argued in the introduction, there is some cost of collusion that is indivisible and not easily shared. To highlight this feature of collusion, we make the following assumption:

**Assumption 1** Firm \(m\) must incur a fixed indivisible cost \(f\) per period of collusion.

Furthermore, we assume that a deviation is detected instantaneously and firms employ standard grim-trigger strategies (Friedman, 1971) to sustain the collusive agreement, i.e., whenever one firm deviates from the collusive norm, the other firm will play non-cooperatively in the next period and forever after.

Collusion is said to be sustainable if, for each firm \(i\) (\(i = m, n\)), the potential short-run gains from cheating are no greater than the present value of expected future losses that are due to

\(^{17}\text{To explain why the merger opportunity arises now and not before is outside the scope of this paper. However, the merger could be triggered by a cost shock, for instance.}\)
the subsequent punishment. This trade-off is captured by the analysis of each firm’s incentive compatibility constraint (ICC). The ICC for firm \( m \) is given by:

\[
\sum_{t=0}^{\infty} \left[ \pi_m^C \left( p_m^C, p_n^C, c_m, c_n \right) - f \right] \cdot \delta^t \geq \pi_m \left( p_m^* \left( p_n^C \right), p_n^C, c_m, c_n \right) + \sum_{t=1}^{\infty} \pi_m \left( p_m^B, p_n^B, c_m, c_n \right) \delta^t
\]  

(2)

and, correspondingly, for firm \( n \) by

\[
\sum_{t=0}^{\infty} \pi_n^C \left( p_n^C, p_m^C, c_m, c_n \right) \delta^t \geq \pi_n \left( p_n^* \left( p_m^C \right), p_m^C, c_m, c_n \right) + \sum_{t=1}^{\infty} \pi_n \left( p_n^B, p_m^B, c_m, c_n \right) \delta^t
\]

(3)

where \( p_i^* (\bullet) \) represents firm \( i \)'s reaction function, \( p_i^C \) denotes firm \( i \)'s collusive price and \( p_i^B \) the Bertrand-Nash equilibrium price.

Rewriting the ICC (2) of firm \( m \) (which bears the indivisible cost), we can solve for the critical indivisible cost \( f \), which is the greatest indivisible cost that the firm can incur and still find it profitable to stay in the cartel:

\[
\tilde{f} (c_n, \delta) = \pi_m^C \left( p_m^C, p_n^C, c_m, c_n \right) - (1 - \delta) \pi_m \left( p_m^* \left( p_n^C \right), p_n^C, c_m, c_n \right) - \delta \pi_m \left( p_m^B, p_n^B, c_m, c_n \right),
\]

(4)

which is a continuous function in the marginal cost of firm \( n \), and the discount factor (\( \delta \)). Recall that the marginal cost of firm \( m \) is zero.

To focus on the effect of the indivisible cost of collusion, we make the following assumption in the symmetric case:

**Assumption 2** Firms are sufficiently patient, i.e. \( \delta \) is sufficiently high, for collusion to be sustainable when both firms have zero production costs, \( c_n = c_m = 0 \), and the indivisible fixed cost of collusion is zero, \( f = 0 \).

For simplicity, we also assume that:

**Assumption 3** There exists a unique \( c_n \) (denoted \( c^* \)) for which the ICC of firm \( n \), i.e. (3), binds with equality.

In the following, we will use a Differentiated Product Bertrand (DPB) model à la Singh and Vives (1984) to derive specific results. Moreover, to simplify the analysis, we assume that, after a
two-firm merger, the merged entity produces only one of the two original varieties, namely the one that can be produced the cheapest. This will not affect our main finding, but mainly makes the market less asymmetric post-merger. This assumption will hold, for instance, if there is a fixed cost associated with carrying a product variety which is sufficiently large.\textsuperscript{18}

Following Singh and Vives (1984), we assume that a representative consumer maximizes the following utility function:

\[
U(q_1, q_2) = a_1q_1 + a_2q_2 - \frac{1}{2} \left( b_1q_1^2 + b_2q_2^2 + 2\theta q_1q_2 \right) + y, \tag{5}
\]

where \(y\) is a numeraire ‘outside’ good. This utility function gives rise to a linear demand structure, where direct demand can be written as:

\[
q_1(p_1, p_2) = \alpha_1 - \beta_1 p_1 + \gamma p_2, \tag{6}
\]

\[
q_2(p_1, p_2) = \alpha_2 - \beta_2 p_2 + \gamma p_1, \tag{7}
\]

where \(d \equiv (b_1b_2 - \theta^2)\), \(\alpha_i \equiv (a_i b_j - a_j \theta)/d\), \(\beta_i \equiv b_j/d\) for \(i \neq j\), \(i = 1, 2\), and \(\gamma \equiv \theta/d\).

The profit of firm \(i\) is \(\Pi_i = \sum_{t=0}^{\infty} \delta^t \cdot \pi_{i,t}(p_{1,t}, p_{2,t})\), where \(p_{i,t}\) is the price charged by firm \(i\) in period \(t\) and firm \(i\)’s individual profit in period \(t\) is

\[
\pi_i(p_{1,t}, p_{2,t}) = q_i(p_{1,t}, p_{2,t}) \cdot [p_{i,t} - c_i]. \tag{8}
\]

Consider a single period game. If the firms play noncooperatively, each firm solves the following optimization problem

\[
\max_{p_i} q_i(p_1, p_2) \cdot (p_i - c_i). \tag{9}
\]

Let \(c_1 = c_m = 0\) and \(c_2 = c_n = c\), where \(c < \alpha_2/\beta_2\). From the first-order conditions (FOCs) of the previous maximization problem, it can be concluded that the reaction functions of firms 1 and 2

\textsuperscript{18}In Web-Appendix I, we study an extended version of the model in which there is no fixed cost associated with carrying a product variety. When this is the case, it is shown that it is more profitable for the merged entity to keep all varieties in its portfolio active. However, the core of the analysis in the paper (namely, Figure 1 and Proposition 1) is shown to be still valid in an extended setting where no variety is dropped after a merger.
are, respectively, given by:

\[ p_1^* (p_2) = \frac{\alpha_1 + \gamma p_2}{2\beta_1} \quad \text{and} \quad p_2^* (p_1) = \frac{\alpha_2 + \beta_2 c + \gamma p_1}{2\beta_2}. \]  \hspace{1cm} (10)

Now, some algebra shows that in the unique Bertrand-Nash equilibrium of the one-shot game, firms’ prices and individual profits are:

\[ p_1^B = \frac{2\alpha_1 \beta_2 + \alpha_2 \gamma + \beta_2 \gamma c}{4\beta_1 \beta_2 - \gamma^2}, \quad p_2^B = \frac{2\alpha_2 \beta_1 + \alpha_1 \gamma + 2\beta_1 \beta_2 c}{4\beta_1 \beta_2 - \gamma^2} \]  \hspace{1cm} (11)

\[ \pi_1 (p_1^B, p_2^B) = \beta_1 (p_1^B)^2 \quad \text{and} \quad \pi_2 (p_1^B, p_2^B) = \beta_2 \left( \frac{2\alpha_2 \beta_1 + \alpha_1 \gamma + c (\gamma^2 - 2\beta_1 \beta_2)}{4\beta_1 \beta_2 - \gamma^2} \right)^2. \]  \hspace{1cm} (12)

If firms instead decide to maximize their joint profit in the one-shot game, their equilibrium prices result from the following optimization problem:

\[ \max_{p_1, p_2} \sum_{i=1}^{2} q_i (p_1, p_2) \cdot (p_i - c_i). \]  \hspace{1cm} (13)

From the FOCs of the previous maximization problem, and after some rearranging, it is concluded that the cooperative prices of firms 1 and 2 are, respectively, given by:\footnote{Since \( c < \alpha_2/\beta_2 \), the corresponding equilibrium quantities \( q_1 (p_1^C, p_2^C) \) and \( q_2 (p_1^C, p_2^C) \) are both positive.}

\[ p_1^C = \frac{\alpha_2 \gamma + \alpha_1 \beta_2}{2 (\beta_1 \beta_2 - \gamma^2)}, \quad p_2^C = \frac{\alpha_2 \beta_1 + \alpha_1 \gamma + c (\beta_1 \beta_2 - \gamma^2)}{2 (\beta_1 \beta_2 - \gamma^2)}. \]  \hspace{1cm} (14)

In addition, firms’ collusive equilibrium profits are:

\[ \pi_1 (p_1^C, p_2^C) = \frac{(\alpha_1 + c\gamma) (\alpha_2 \gamma + \alpha_1 \beta_2)}{4 (\beta_1 \beta_2 - \gamma^2)}, \]  \hspace{1cm} (15)

\[ \pi_2 (p_1^C, p_2^C) = \frac{(\alpha_2 - c\beta_2) (\alpha_2 \beta_1 + \alpha_1 \gamma + c (\gamma^2 - \beta_1 \beta_2))}{4 (\beta_1 \beta_2 - \gamma^2)}. \]  \hspace{1cm} (16)

If a given firm is considering to deviate in period \( t \), when firms are supposed to set prices (14) then, making use of firms’ reaction functions (10), firms’ optimal deviation prices \( p_1^* (p_2^C) \) and \( p_2^* (p_1^C) \) are obtained. The corresponding deviation profits for firms 1 and 2 are, respectively, given
by:

\[ \pi_1 (p_1^*, p_2^C) = \beta_1 \left( \frac{\alpha_1 (2\beta_1 \beta_2 - \gamma^2) + \gamma \alpha_2 \beta_1 - c \gamma (\gamma^2 - \beta_1 \beta_2)}{4\beta_1 (\beta_1 \beta_2 - \gamma^2)} \right)^2, \]  

(17)

\[ \pi_2 (p_1^C, p_2^* (p_1^C)) = \left( \frac{\alpha_2 (2\beta_1 \beta_2 - \gamma^2) + \gamma \alpha_1 \beta_2 + 2\beta_2 c (\gamma^2 - \beta_1 \beta_2)}{16\beta_2 (\gamma^2 - \beta_1 \beta_2)^2} \right)^2. \]  

(18)

The equilibrium collusion pattern is illustrated in Figure 1, where the variable cost of firm \( n \), \( c_n \), is depicted on the \( x \)-axis and the indivisible fixed cost of collusion \( f \) is depicted on the \( y \)-axis (\( \delta \) is fixed).20

**The symmetric market structure.** Let us first address the case where firm \( n \) faces a zero variable production cost (recall that firm \( m \) is assumed to have a zero variable production cost). \( ICC - s \) then depicts the indivisible fixed cost at which firm \( m \)'s ICC (2) binds when firm \( n \) also faces a zero production cost. We refer to this as \( f' \), where \( f' \equiv f (0, \delta) \) (see eq. (4)). This curve is horizontal since, by assumption, both firms have a marginal cost of production which is zero. It follows that collusion is sustainable in the symmetric market if \( f < f' \).

**The asymmetric market structure.** Let us now turn to the case where firm \( n \) has a positive production cost, \( c_n > 0 \). In this case, \( ICC - n \) shows the production cost \( c_n \) at which firm \( n \)'s ICC (3) binds. This curve is vertical since firm \( n \) is assumed not to pay the per-period fixed indivisible cost. In addition, Assumptions 2 and 3 imply that ICC (3) only holds if cost asymmetries are sufficiently low, i.e., if \( c_n < c^* \).

Finally, \( ICC - m \) shows combinations of \( c_n \) and \( f \) for which firm \( m \)'s ICC (2) binds. At \( c_n = 0 \), this curve obviously coincides with the curve \( ICC - s \) (i.e. firm \( m \)'s ICC under symmetry). The slope of the curve \( ICC - m \) is unfortunately ambiguous. To illustrate our first result, we make use of the following assumption, which implies that the \( ICC - m \) curve is upward sloping at \( c_n = 0 \) and never below \( ICC - s \):

**Assumption 4a** \( \bar{f} (c_n, \delta) \) is weakly increasing in \( c_n \) and \( \left. \frac{\partial \bar{f} (c_n, \delta)}{\partial c_n} \right|_{c_n=0} > 0 \).

Assumption 4a holds in the DPB model with a joint profit-maximizing (JPM) cartel (see Appendix A).21 Clearly, there will be collusion in this asymmetric market structure if and only if we

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20 See Appendix A for details regarding how the curves in Figure 1 are generated. There we show that there exist parameter values in the DPB model, under the joint profit-maximizing rule, such that Figure 1 is valid.

21 Assumption 4a also holds in the DPB model under an equal price increase cartel, where each firm increases its price by an equal amount from the non-collusive equilibrium price (see Ganslandt, Persson and Vasconcelos (2008)).
Figure 1: Equilibrium Collusion Pattern: ICC-m upward sloping

are below the $ICC - m$ curve and to the left of the $ICC - n$ curve. So, we can now characterize the equilibrium collusion pattern under Assumptions 1-3 and 4a. This is done in Figure 1 where \( \{C, *\} \) indicates that collusion can be sustained in the symmetric market structure, but not in the asymmetric market structure, \( \{*, C\} \) indicates that collusion cannot be sustained in the symmetric market structure but can be sustained in the asymmetric market structure, etc.

Now, making use of Figure 1, we have the following result:

**Proposition 1** Under Assumptions 1-3 and 4a, and at a fixed indivisible cost so high that no collusion can be sustained in the symmetric case ($f > f'$), there exists a fixed indivisible cost such that (i) firms do not collude for low asymmetries; (ii) firms do collude for intermediate asymmetries; and (iii) firms do not collude for large asymmetries.

**Proof.** Let $f = f' + \epsilon$ (i.e. ICC-s does not hold), where $\epsilon$ is arbitrarily small and positive, and let $c'$ denote the value of $c_n$ for which the $ICC - m$ binds with equality at $f = f' + \epsilon$. Then, there exist (i) $c_i < c'$ such that $ICC - m$ does not hold; (ii) $c_{ii} \in (c', c^*)$ such that $ICC - m$ and $ICC - n$ hold simultaneously; and (iii) $c_{iii} > c^*$ such that $ICC - n$ does not hold.  

This result thus shows that there exist levels for the fixed indivisible cost of collusion such that (i) firms do not collude when asymmetries are small; (ii) firms do collude when asymmetries are moderate; and (iii) firms do not collude when asymmetries are large. The intuition is straightforward. For collusion to arise, firms cannot be too symmetric since the largest (most efficient) firm
will then not have the incentive to cover the fixed indivisible cost of collusion. On the other hand, firms cannot be too asymmetric either; if there are strong asymmetries, the smallest firm in the collusive agreement will have strong incentives to deviate from the collusive conduct and steal the business of its rival.

To illustrate our second result, we now make use of the following assumption, which implies that the ICC - m curve is downward sloping at \( c_n = 0 \) and is never above ICC - s:

**Assumption 4b** \( f(c_n) \) is weakly decreasing in \( c_n \) and \( \frac{\partial f(c_n, \delta)}{\partial c_n} < 0 \) \( \big|_{c_n=0} \).

Assumption 4b holds, for instance, in the DPB model under the constant market shares rule (see Appendix B), where firms increase prices (reduce quantities) keeping market shares constant (i.e., equal to the market shares in the Bertrand-Nash equilibrium). We can now characterize the equilibrium collusion pattern under Assumptions 1-3 and 4b. This is illustrated in Figure 2. We can now derive the following result:

**Proposition 2** Under Assumptions 1-3 and 4b, and at such a low fixed indivisible cost that collusion can be sustained in the symmetric case \( f < f' \), there exist levels for the fixed indivisible cost such that (i) firms collude for small asymmetries; and (ii) firms do not collude for large asymmetries.

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22 Remember that under the joint profit-maximizing rule, this firm benefits less from cartel formation when the industry is more symmetric.

23 Appendix B also demonstrates that there exist parameter values in the DPB model, under the constant market shares rule, for which Figure 2 is valid.
Proof. Let \( f = f' - \varepsilon \), where \( \varepsilon \) is arbitrarily small and positive, and let \( \tilde{c}' \) denote the value of \( c_n \) for which the ICC - \( m \) binds with equality at \( f = f' - \varepsilon \). Then, there exist (i) \( \tilde{c}_i < \tilde{c}' \) such that ICC - \( m \) and ICC - \( n \) hold; and (ii) \( \tilde{c}_{ii} > \tilde{c}' \) such that ICC - \( m \) does not hold. ■

This result thus shows that there exist levels for the fixed indivisible cost of collusion such that firms collude when asymmetries are minimal but do not collude when asymmetries are larger. For collusion to arise, firms cannot be too asymmetric, otherwise the largest (most efficient) firm will not have the incentive to cover the fixed indivisible cost of collusion, since it benefits less from the cartel being formed under the constant market share rule. Moreover, firms cannot be too asymmetric for another reason. If there are strong asymmetries, the smallest firm in the collusive agreement will have strong incentives to deviate from the collusive conduct and steal the business from its rival.\(^{24}\)

2.2 The First Stage: Merger Formation

We next proceed to analyze the incentives for mergers prior to the repeated game played subsequently. The assumed timeline is: (i) merger formation; (ii) the merged entity decides how many varieties to use after the merger takes place; (iii) oligopoly interaction. Thus firms contemplating merging, will take into account, whether or not the merged entity will use both varieties in its product portfolio or only one of them, and the equilibrium outcomes under different market structures in the subsequent repeated game. Depending on which merger takes place in the first stage, collusion may or may not arise in the following supergame, which is potentially a motive for policy intervention.

As demonstrated by the below analysis, however, efficiency-enhancing regulatory intervention could be difficult. Blocking the most desirable merger from the firms’ perspective may result in a market structure that is more, not less, conducive to coordination. A poorly designed merger policy can thus be counterproductive.

To determine the merger pattern, we make use of an endogenous merger model developed by Horn and Persson (2001a) where merger formation is treated as a cooperative game of coalition formation. The merger model has three basic components: (i) a specification of the owners determining whether one ownership structure (denoted \( M^i \)) dominates another; (ii) a criterion for determining when these owners prefer the former structure to the latter; and (iii) a stability (solution) criterion that selects the ownership structures seen as solutions to the merger formation game.

\(^{24}\)Note that it can be shown that non-merged (inefficient) firms will have a lower incentive to cover the fixed cost than the merged (efficient) firm in these examples.
on basis of all pairwise dominance rankings.

An important remark should be made at this point. When investigating the induced effects of all possible mergers between the three firms in the status quo market structure, we assume that single-merger profitability condition is fulfilled. Put differently, we assume that, for any possible two-firm merger, the merged firm’s profit in the induced duopoly market structure is larger than the aggregate profit earned by the two merging firms in the (ex-ante) triopoly market structure. This will be true as long as the fixed or variable cost savings obtained due to the merger are sufficiently high. Here, to simplify presentation, we rely on fixed cost savings. Thus we could simply add a fixed cost of production for each firm and assume that a merged entity only faces one such cost. In Web-Appendix II, however, we provide an example (with no fixed cost savings) where mergers are driven by variable cost synergies, and where the qualitative results derived below are shown to hold good.

We then use the following Lemma (from Horn and Persson (2001a)) regarding the situation where any merger proposal would be approved by the Antitrust Authority (referred to as the laissez-faire policy).

**Lemma 1** Under the laissez-faire policy and assuming that the aggregate profit of the merging firms increases with the merger the equilibrium merger gives rise to the highest aggregate duopoly profit.

**Proof.** This follows from Proposition 2 in Horn and Persson (2001a) and from the fact that, in our model, the profit flows both under collusion and non-collusion regimes are time invariant. ■

Since we have assumed that in the status quo industry structure \( c_1 = c \) and \( c_2 = c_3 = 0 \), the marginal costs associated with the possible merged entities are \( c_{12} = c_{13} = c_{23} = 0 \). Thus, two different market structures can result from our merger formation game: (i) a symmetric market structure \( M^A \), resulting from a merger between firms 1 and 2 or between firms 1 and 3. In this case, after the merger, both the merged entity and the outsider firm have zero variable costs (\( c_m = c_n = 0 \)); and (ii) an asymmetric market structure \( M^B \) resulting from a merger between firms 2 and 3. In this scenario, the merged entity faces zero variable costs (\( c_m = 0 \)) but the outsider firm 1 will face a variable cost \( c_n > 0 \). Now, a merger leading to a symmetric industry structure \( M^A \) will occur in equilibrium, if the following condition holds:

\[
\Pi^{Ah} = \pi^{Ah}_m + \pi^{Ah}_n > \pi^{Bk}_m + \pi^{Bk}_n = \Pi^{Bk},
\]

where \( h, k = \{*, C\} \).
Our main finding in this context is then that there exist fixed costs of collusion such that the equilibrium of the merger formation game is an efficient, non-collusive, symmetric duopoly under the laissez-faire policy, while the relevant alternative, if this merger is blocked by the Antitrust Authority, is an inefficient, collusive, asymmetric duopoly.

To proceed, we then assume that after a two-firm merger the merged entity produces only one of the original varieties, namely the one that can be produced cheapest. In Web-Appendix I, however, we provide an example where the merged entity decides whether to use one or two varieties, and where the qualitative results derived below are shown to hold good. Then applying condition (19) to the DPB model, we can derive the following result:

Proposition 3 Let $c < c^*$. Under Assumptions 1-3 and 4a, we have that (i) at a fixed indivisible cost so low that collusion can be sustained in the symmetric case ($f < f'$), there will be a merger to a symmetric market structure $M_A$, and (ii) at a fixed indivisible cost so high that collusion cannot be sustained in the symmetric case ($f > f'$), there exist parameter values in the DPB model where we will have mergers to an asymmetric market structure.

Proof. See Appendix C.

Figure 3 illustrates this result by indicating the equilibrium market structures for the different regions of parameter values.

Consider first the region of parameter values to the left of $ICC - n$ and below $ICC - s$. In this
region, there is collusion both in market structure $M^A$ and in market structure $M^B$. When this is the case, then, as shown in the Appendix $C$, condition (19) boils down to

$$ \Pi^{AC} = \pi_{m}^{AC} + \pi_{n}^{AC} > \pi_{m}^{BC} + \pi_{n}^{BC} = \Pi^{BC} $$

which turns out to always be true. Hence, $M^A$ dominates $M^B$ in this specific region.

Consider now the region of parameter values for which $ICC_m$ and $ICC_n$ hold, but $ICC_s$ does not. When this is the case, there will be collusion in market structure $M^B$ but not in market structure $M^A$, implying that the merger condition (19) becomes:

$$ \Pi^{As} = \pi_{m}^{As} + \pi_{n}^{As} > \pi_{m}^{BC} + \pi_{n}^{BC} = \Pi^{BC}. $$

Now, the downward sloping dashed line in Figure 3 represents merger condition (21) when it is binding. As illustrated by Figure 3, this dashed line divides the region under consideration into two different subregions. In the right-hand subregion, $M^A$ dominates $M^B$, whereas the opposite holds in the left-hand subregion. It should be noted, however, that the parameter area where $M^B$ dominates $M^A$ is rather small in Figure 3. The reason is that there are assumed to be no synergies in a merger between the two efficient firms. Allowing for such synergies would increase the likelihood of $M^B$ being the equilibrium outcome game.

Let us now explicitly study the effects of an anti-symmetry merger policy in this context. Assuming that the antitrust authority decides to adopt a merger policy that blocks mergers leading to symmetric market structures but permits mergers leading to asymmetric market structures, we have the following result:

**Proposition 4** A merger policy blocking mergers to a symmetric market structure can induce a merger without any cost synergies (i.e. between firm 2 and firm 3), yielding a higher average production cost and a lower aggregate producer and consumer surplus.

**Proof.** See Appendix D. ■

This result can be illustrated with the help of Figure 3. In what follows, let us focus on the region of parameter values where $ICC - m$ and $ICC - n$ hold, but $ICC - s$ does not. As Figure 3 illustrates, there exists a (right) subregion of this parameters’ space where $M^A$ dominates $M^B$. It is worth noting that in the whole region under analysis, firms will not be able to collude in case a merger results in a perfectly symmetric market structure and, hence, the reason why the
symmetric market structure $M^A$ dominates the asymmetric market structure $M^B$ in the right-hand subregion cannot be that a merger to symmetry results in collusion amongst the remaining firms in the market. Instead, what drives this result is that this merger will give rise to very significant savings of production costs.

So, by adopting an anti-symmetry merger policy, antitrust authorities will actually “force” firms to opt for the merger leading to the asymmetric market structure, $M^B$. Now, in the region under analysis, this alternative merger will, however, create the most favorable conditions for collusion to arise, since the asymmetries between the two remaining firms are moderate after the merger (see Proposition 1 and Figure 1). In addition, for some parameter values, the gains from successful collusion which are obtained in this asymmetric industry structure are so high that they will more than compensate for the fact that one of the colluding firms has high production costs. Consequently, by forbidding a merger leading to a symmetric industry structure, competition authorities may end up with a merger that does not only result in less efficient production, but also facilitates collusion.

3 Endogenous Indivisible Cost of Cartelization: Buyouts of Potential Entrants

As already mentioned in the Introduction, one situation where we expect the indivisible cost of cartelization to be large is when there is the threat that a potential entrant will effectively enter the market and entry would lead to a cartel breakdown.\textsuperscript{25} When this is the case, a cartel member may decide to embark on a buyout acquisition of this potential entrant so as to protect the cartel. This buyout acquisition will, obviously, be costly for the acquirer and can be viewed as an endogenous (indivisible) cost of cartelization.\textsuperscript{26}

As already mentioned, a case in point is the \textit{pre-insulated pipe cartel} in Europe. In 1987, Brown Boveri Company (later ABB) embarked on a strategic program of acquiring district heating pipe producers across Europe. However, this program was considered to be unfairly costly by ABB as

\textsuperscript{25}See also Section 4.2 where we mention another possible way of endogenizing the fix cost of cartelization.

\textsuperscript{26}Investigating the possibility of a buyout acquisition of a potential entrant by a cartel member has, in our view, two main justifications. First, as mentioned in the Introduction, it makes it possible to illustrate how an important (exogenous) parameter in the model (parameter $f$) can be endogenized in a meaningful way. Second, it contributes to shed some light on an issue which has largely been neglected by the extant literature since, as highlighted by Leveinstein and Suslow (2006, p.49), “[m]ost theoretical models ignore or assume away the possibility of entry, which is in fact one of the biggest challenges cartels face. When firms do manage to coordinate their conduct on incentive compatible collusive strategies, they create an incentive for outsiders to enter the industry. Coping with and trying to prevent entry can undermine the best-laid collusive plans.”
indicated by the following quote from §28 of the EC Decision: “ABB for its part considers that it was unfairly having to bear all the cost of industry reorganization while other producers obtained a free ride.” Moreover, in reaction to Powerpipe’s aggressive behavior and entry into new geographical markets, ABB decided to use several predatory strategies to protect the cartel.

To capture these aspects of creating and maintaining the cartel, consider the following modified model set-up, where we keep the repeated oligopoly game we had before, but change the first-stage interaction in the following way. Now, firm 3 is outside the market and contemplates entering it, incurring an exogenous entry cost $E \geq 0$. The cartel might prevent this entry by buying out the potential entrant. To this end, we assume that firm 2 makes a take-it-or-leave-it offer to firm 3.

There are two scenarios to consider: one with, and one without, an acquisition taking place.

**Case 1: Entry.** In this case, firm 3 enters the market facing a variable cost $c_3$. It is assumed that if entry occurs, collusion is not sustainable and firms compete à la Bertrand in each period, generating profits $\pi_1^B(c_1,c_2,c_3)$, $\pi_2^B(c_1,c_2,c_3)$, and $\pi_3^B(c_1,c_2,c_3)$, respectively. This assumption of one-shot Bertrand competition could be the result of a cartel not being sustainable with three firms in the market, or by firm 3, upon entering the market, deciding to announce the existence of the collusive agreement to the antitrust authorities.\(^\text{27}\)

**Case 2: Buyout.** To focus on the buyout as an indivisible collusion cost, we assume that a buyout would not be profitable if collusion did not occur post buyout. Then, we can use the above set-up to determine whether there will be collusion, taking the indivisible buyout cost into account. It immediately follows that if a buyout takes place in equilibrium, firm 2 will make an offer equal to firm 3’s reservation price, i.e. its profit if it enters. The ICC for firm 2 is then given by:

$$\sum_{t=0}^{\infty} [\pi_2^C(p_2^C,p_1^C,c_1,c_2) - A] \cdot \delta^t > \pi_2(p_2^*(p_1^C),p_1^C,c_1,c_2) + \sum_{t=1}^{\infty} \pi_2(p_2^B,p_1^B,c_1,c_2) \delta^t,$$  

where the indivisible acquisition cost $A = \pi_3^B(c_1,c_2,c_3) - E$ is the reservation price of firm 3. The ICC for firm 1 is given by

$$\sum_{t=0}^{\infty} \pi_1^C(p_1^C,p_2^C,c_1,c_2) \delta^t > \pi_1(p_1^*(p_2^C),p_2^C,c_1,c_2) + \sum_{t=1}^{\infty} \pi_1(p_1^B,p_2^B,c_1,c_2) \delta^t.$$

To simplify the presentation, we assume the buyout price $A$ to be independent of the level of

\(^{27}\)See Friedman and Thisse (1994, p. 272).
asymmetry, i.e. $A = \pi^B_2(c_1, c_2, c_3) - E = \bar{A}$. Consequently, it is assumed that if $c_1$ decreases, $c_2$ will increase in such a way that the acquisition price will not change. Then, using assumptions 1-3 and 4a, it directly follows that we can make use of the graphical solution in Figure 1 to describe the equilibrium, where we have $c_1$ (the marginal cost of the incumbent not participating in the merger) on the horizontal axis and the vertical axis now represents $A$ (instead of $f$).

What will then happen if we relax the assumption that the acquisition price is independent of the cost asymmetry? If the buyout price (reservation price) decreases in asymmetries, collusion becomes relatively more likely under asymmetry, whereas the opposite is true if the buyout price increases in asymmetries.

We then make use of the DPB model to show the existence of an equilibrium where the buyout of a collusion breaking potential entrant will be undertaken, if and only if the market structure is medium asymmetric. Thus, we can show that:

**Proposition 5** There exist parameter values in the DPB model under the joint profit-maximizing rule such that firm 2 buys out firm 3 if and only if collusion occurs post-buyout, which is the case if and only if the market structure is moderately asymmetric.

**Proof.** See Ganslandt, Persson and Vasconcelos (2008).

Moreover, it immediately follows that if the entry cost $E$ increases, the buy-out price $A$ decreases. This, in turn, implies that a more symmetric market structure could support collusion when the exogenous entry barriers are high. Consequently, we can state the following result:

**Corollary 1** The lower is the exogenous entry barrier, the more asymmetric must the market structure be to sustain collusion, since entry-deterring takeovers to protect the cartel then become more costly.

Consequently, an empirical prediction of the model is that we should observe more asymmetric cartels in markets with lower exogenous entry barriers.\(^{28}\)

### 4 Merger Policy and Coordinated Effects of Mergers

An important element when evaluating the effects of a merger is the analysis of its coordinated effects, that is, understanding whether the structural change implied by a merger creates more

\(^{28}\)Since the existence of cartels will be very low when entry costs are also very low, this prediction is conditional on cartels still being stable.
favorable conditions for collusion to arise between the remaining firms in the industry.

Previous contributions to the literature have stressed that symmetry, absent indivisible costs associated with cartels, is a factor conducive to collusion. In particular, Compte, Jenny and Rey (2002) and Vasconcelos (2005) investigate how the distribution of firms’ capacities affects the likelihood of collusion in price setting and quantity setting infinitely repeated games, respectively.\(^\text{29}\) Even though the mapping between firms’ capacity holdings and their incentives to collude differs in both studies,\(^\text{30}\) the main outcome that emerges from both analyses is that a more asymmetric distribution of firms’ capacities tends to hurt collusion possibilities. Their analysis then suggests that a merger involving the largest firm tends to hurt collusion whereas a merger involving the smallest one enhances collusion possibilities. Put differently, transferring capacity from the largest to the smallest firm in the industry enhances collusion, i.e. more symmetry helps collusion. Recently, the impact of asymmetries on (tacit) collusion possibilities has also been explored by Fonseca and Normann (2008) in Bertrand-Edgeworth laboratory markets. Their findings confirm that, absent indivisible costs associated with collusion, asymmetries appear to prevent collusion.

We contribute to this literature by showing that if there is an indivisible cost associated with collusion to be paid by a cartel member (say, the cartel leader), a medium asymmetric market structure might be more conducive to collusion since it balances the incentive to stay in the cartel against the need to cover the cartel leader’s indivisible costs. This possibility result is related to the recent findings of Bos and Harrington (2010). In the context of an infinitely repeated capacity constrained game endogenizing the composition of a cartel when firms are heterogeneous in their capacities, they find that the most severe coordinated effects might result from mergers involving moderate-sized firms, rather than the largest or the smallest firms.

Hence, when assessing whether a merger between two firms is likely to enhance oligopolistic coordination in the market, antitrust authorities should give special attention to firms’ cost conditions. In particular, if there are fixed costs associated with running a cartel, our analysis suggests that the most severe coordinated effects may be due to mergers inducing medium asymmetric market structures rather than mergers inducing a high degree of post-merger symmetry among the firms in the industry, as would be suggested by the previous literature.

When considering unilateral effects of mergers, antitrust authorities typically look at the increase

\(^{29}\)See also Kühn (2004) and earlier contributions by Verboven (1997) and Rothschild (1999).

\(^{30}\)Compte, Jenny and Rey (2002) show that the minimum discount factor above which a cartel can sustain the joint-profit maximum increases in the capacity holding of the largest cartel member. Vasconcelos (2005), on the other hand, shows that this minimum value for the discount factor also depends on the capacity of the smallest firm in the agreement. In his setting, the smallest firms are the ones which induce the highest competitive pressure in the short run, while the largest firm is the most reluctant to comply with a punishment strategy leading to lower prices.
in the Herfindahl-Hirshman Index (HHI) caused by the merger. It is important to note that the HHI index tends to penalize asymmetry, i.e. it increases more when a merger gives rise to an asymmetric market structure than when a merger induces a more symmetric market structure. This being the case, antitrust authorities tend to block a merger based on the unilateral effects arguments when the merger under analysis is expected to induce a more asymmetric industry structure. Our findings, that medium asymmetric market structures can be conducive to collusion, then suggest that such standards can also be motivated when competition authorities investigate whether a merger between two firms is likely to enhance oligopolistic coordination in the market.

5 Discussion

In this section, we discuss two assumptions of the previous analysis: (i) the role of the antitrust authorities in cartel detection and (ii) the cartel allocation rule.

5.1 The Role of the Antitrust Authorities in Detecting the Cartel

A limitation of the model is that the only role of the antitrust authority is to define its policy with respect to the mergers it has to evaluate, where we have only considered two simple merger policy rules: (i) a laissez-faire policy according to which any proposed merger obtains approval; and (ii) an anti-symmetry policy which consists of blocking mergers leading to symmetric market structures while accepting mergers inducing asymmetric market structures.

Since antitrust authorities are assumed to play an active role in blocking mergers, a natural question that can be raised is then why the model does not also consider the threat that antitrust authorities discover the cartel. It should be noted, however, that this threat could easily be considered in the model by endogenizing the fixed cartelization cost in the following way. Assume that the per-period probability of detection of illegal behavior is given by $\rho$. Assume also that a discovery by the authority results in a fine $F$ for the large player (the ring-leader), while the other cartel member pays no fine since it is eligible for leniency and could always collaborate with the antitrust authority in case the cartel is uncovered.\(^{31,32}\) For simplicity, also assume that collusion

\(^{31}\)In most jurisdictions, being the ring leader is seen as an aggravating circumstance and ring leaders are not eligible for leniency. See Herre and Rasch (2009) for a discussion on how antitrust authorities should deal with ring leaders, as well as for an investigation of how excluding ring leaders from leniency programs affects collusion sustainability.

\(^{32}\)In any case, if the simplifying assumption that the non-ring leader applies for leniency and pays no fine is relaxed, $f$ can be interpreted as the difference in fines between the two firms.
can be resumed right after the detection. Under these assumptions, \( f \) can be endogenized by simply making it correspond to the per-period reduction in the ring-leader profit due to the expected fine in that period. Consequently, only in sufficiently asymmetric market structures will ring-leaders be compensated for taking on such risks. However, it should also be noted that this also implies that the very symmetric market structure can be more stable since firms then divide the cost of expected fines equally.

In conclusion, it should be highlighted that the above argument is based on the assumption that the cartel can be resumed right after the detection. This simplifying assumption is, however, unrealistic in many situations since it is likely that antitrust authorities will monitor the industry closely for some time after detection, thus rendering resumed collusion impossible. For this reason, Motta and Polo (2003) force the cartel to compete for one period after detection, while the firms in Harrington (2008) cannot collude in any future period after detection. Whether either of these cases will significantly affect the model is left to future work.

5.2 Alternative Allocation Rules

Key to our results is the chosen rule for determining the collusive outcome. It should be noted, however, that both allocating rules which were considered despite being simple and reasonable do have some caveats which we discuss in turn.

If the cartel behavior rule has the property of the gains of the large (efficient) firm from collusion increasing with industry asymmetries, our main results are valid. Basically, what is important is that the “advantages” of the large (efficient) firm in the non-cooperative interaction (e.g. its larger market share, lower production costs, larger capital stock, or higher quality products) are used to give leverage in capturing more of the surplus created by collusion. This will be the case for some particular rules of how to determine the cartel outcome. As shown above, joint profit maximization is a rule with this characteristic. But this rule has some problematic features, as discussed in Harrington (2004). In particular, it could allocate the gains from collusion very unevenly.

It should also be noted that despite being one of the most frequently used allocation rules, the other rule considered in the model for allocating cartel supply – the constant market shares rule – also has an odd feature in our proposed setting: the fact that the small (inefficient) firm gains more from collusion than the large (efficient) firm. The reason is that under the constant market share rule, the large (efficient) firm is forced to reduce output much more than the small (inefficient) firm, which implies that the “cost” due to the output reduction to a larger extent falls on this large
(efficient) firm.\textsuperscript{33} This rule is evidently used in practice, but this property suggests that other rules should be used where the large (efficient) firm gains most from collusion. For example, Ganslandt, Persson and Vasconcelos (2008) show that, in the current setting, the large (efficient) firm gains most from collusion under an equal price increase rule.

Now, at a more general level, one might also wonder whether there exist other alternative allocation rules that compensate the ring leader for the fact that it is in charge of running the cartel and, hence, incurs the indivisible cost of cartelization.\textsuperscript{34} Indeed, in the framework of the proposed model, the compensation can be incorporated in how the firms share the cartel supply and profits. One way of modeling this compensation would then be to let the cartel prices be determined through Nash-bargaining. However, endogenizing the cartel allocation rule based on how firms negotiate would seriously affect the tractability of the model without changing much of the nature of the arguments. In addition, as highlighted by Bos and Harrington (2010, pp. 107-108), “If there is a simple acceptable allocation rule that allows for collusion, cartel members may be hesitant to entertain modifications to it out of fear that it will lead to negotiations that may prove difficult to resolve and could ultimately result in the collapse of collusion.”\textsuperscript{35} For these reasons, endogenizing the allocation rule through bargaining is left to further research. Hopefully, however, the above model can be seen as a stepping stone in the direction of a more complete analysis.\textsuperscript{36}

6 Conclusion

Allowing for indivisible costs of collusion, which must be borne by one firm (a ring leader), we show that forbidding mergers leading to symmetric market structures can induce mergers leading

\textsuperscript{33}See Norbäck and Persson (2006) for a formalization of this argument.

\textsuperscript{34}A possible alternative compensation method not involving the cartel’s supply allocation rule would consist of the implementation of side payments. The other members of the cartel could directly compensate the ring leader for the fact that it is in charge of running the cartel and for having to pay the corresponding indivisible cartelization cost. However, antitrust rules typically prohibit direct transfers and such transfers usually create evidence compromising the cartel. Hence, it seems plausible to assume that direct transfers are out of option. Also, in a more symmetric industry, it could reasonably be argued that the two leaders of an industry could take turns in the payment of the fixed cost of collusion (say, take turns at buying out new entrants). However, one can argue against this approach by saying that such a type of cost-sharing agreement where firms take turns might be easier to detect.

\textsuperscript{35}Along these lines, Levenstein and Suslow (2004) state that “[b]argaining problems were much more likely to undermine collusion than was secret cheating. About one quarter of the cartel episodes ended because of bargaining problems. Bargaining issues affected virtually every industry studied.”

\textsuperscript{36}The evidence presented in the Lysine cartel case might also help us gain further understanding of our findings, where the frequent clashes between Ajinimoto and ADM, the two largest cartel members, resulted in Ajinomoto eventually approaching the Commission to denounce the agreement (see the discussion in Grout and Sonderegger (2005)). Similarities in size can thus be detrimental to collusion, as they are more conducive to conflict. This suggests that organization is easier to achieve when there is one clear leader, rather than several contenders for the leadership position.
to asymmetric market structures with higher production costs and a higher risk of collusion.

We also show that the indivisible cost of cartelization can be high when there is a threat that a potential entrant will enter the market and destabilize the cartel. A cartel leader may then decide to prevent entry by conducting a buyout acquisition of the potential entrant. Clearly, these entry-deterring-buyouts will be costly for the acquirer and may, therefore, be seen as an indivisible fixed cost which must be incurred by some cartel member so as to protect the cartel.

The obtained results are important from a policy perspective since the analysis suggests a weakness of the current policy when it comes to the evaluation of the coordinated effects of mergers. Even though the paper is not designed to evaluate all costs and benefits of a symmetry-opposing merger policy, it does suggest that as far as coordinated effects of mergers are concerned, antitrust authorities should give special attention to firms’ cost conditions and the role played by large firms (ring-leaders) in the collusion process. If there turn out to be fixed costs associated with the running of a cartel (e.g. costs associated with a cartel member buying out potential entrants whose entry would probably destabilize the cartel agreement), then our analysis highlights that the most severe coordinated effects may be due to mergers inducing medium asymmetric market structures rather than mergers inducing a high degree of post-merger symmetry among the firms in the industry, as would be suggested by the previous literature.

A Generating Figure 1

In the following, we derive the curves presented in Figure 1 assuming the following parameter values: 

\( \alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1 \) and \( \gamma = 1/3 \). The corresponding values for the parameters in the utility function (5) are: \( a_1 = a_2 = 3/2, b_1 = b_2 = 9/8 \) and \( \theta = 3/8 \).

A.1 Deriving Figure 1

A.1.1 Curve ICC-s

Making use of eqs. (4), (12), (15), (17), some algebra shows that:

\[
J' = J(0, \delta) = \frac{3}{8} - (1 - \delta) \frac{25}{64} - \delta \frac{9}{25}. \tag{24}
\]

For a given (and sufficiently high) value of the discount factor \( \delta \), the previous equation is represented by the horizontal line ICC-s in Figure 1.
A.1.2 Curve ICC-n

Making use of eqs. (3), (12), (16) and (18), and after some rearranging, it may be concluded that condition (3) becomes:

$$
\frac{(1 - c_n)(3 - 2c_n)}{8} \frac{1}{1 - \delta} > \frac{(5 - 4c_n)^2}{64} + \frac{(21 - 17c_n)^2}{1225} \frac{\delta}{1 - \delta}.
$$

(25)

Now, for a given value of $\delta$, the value of $c_n$ for which the previous condition binds is represented by the vertical line ICC-n in Figure 1.

A.1.3 Curve ICC-m

Combining the results in eqs. (4), (12), (15), (17), it may be concluded that:

$$
\bar{f}(c_n, \delta) = \frac{1}{8} (c_n + 3) - (1 - \delta) \frac{(2c_n + 15)^2}{576} - \delta \frac{9 (c_n + 7)^2}{1225}.
$$

(26)

For a given $\delta$, the previous condition is represented by the ICC-m curve in Figure 1.

Thus, this section proves that there exist parameter values in the DPB model such that Figure 1 is valid, as claimed in Proposition 1.

B The Constant Market Shares Rule

In Figure 1 and in Proposition 1, we have assumed that joint profit maximization was used as a criterion for selecting the collusive outcome. This section studies the properties of an alternative criterion to select the collusive outcome: the so-called constant market shares rule.

As in Appendix A, we focus the attention on a specific parametrization of the DPB model where $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1$ and $\gamma = 1/3$. This being the case, from eqs. (11) and (12), it may be concluded that the Bertrand-Nash equilibrium profits are given by:

$$
\pi_1(p^B_1, p^B_2, c) = \frac{9(7 + c)^2}{1225}, \quad \pi_2(p^B_1, p^B_2, c) = \frac{(21 - 17c)^2}{1225}.
$$

(27)

Suppose that along the collusive path, firm $i$’s output is $q^C_i = \alpha q^B_i$ where $q^B_i$ denotes firm $i$’s quantity in the Bertrand-Nash equilibrium, $i, j = 1, 2, i \neq j$, and $\alpha \in (0, 1)$. When this is the case,
each firm’s market share along the collusive path coincides with its market share at the Bertrand Nash equilibrium. However, the problem is to identify the value of $\alpha$ chosen by the cartel. Note that firm $i$’s optimal value of $\alpha$ results from the following maximization problem:\footnote{Making use of eqs. (6)-(7), it is straightforward to conclude that for the specific parameter values we have chosen, firm $i$’s inverse demand function is given by $p_i = \frac{3}{2} - \frac{9}{8} q_i - \frac{3}{8} q_j - c_i$.} 

$$\max_{\alpha} \left( \frac{3}{2} - \frac{9}{8} \alpha q_i^B - \frac{3}{8} \alpha q_j^B - c_i \right) \alpha q_i^B. \tag{28}$$

It can easily be checked that the optimal values of $\alpha$ for firms 1 and 2 are, respectively, given by:

$$\alpha_1^* = \frac{35}{2(21 - 2c)}, \quad \alpha_2^* = \frac{35}{18} - \frac{2}{4} c. \tag{29}$$

Thus, the two firms in the cartel will have to bargain over the value of $\alpha \in [\alpha_2^*, \alpha_1^*]$. Let the chosen level of $\alpha$ be $\alpha_1^*$.\footnote{Since we have assumed the most efficient firm to be the one which must cover indivisible per-period coordination fixed costs, it seems natural to assume that it has all the bargaining power in the choice of $\alpha$.} The corresponding cooperative prices and profits are then given by:

$$\tilde{p}_1^C = \frac{3}{4}, \quad \tilde{p}_2^C = \frac{3}{4} \frac{21 + 8c}{21 - 2c}, \tag{30}$$

$$\tilde{\pi}_1^C = \frac{9}{8} \frac{7 + c}{21 - 2c}, \quad \tilde{\pi}_2^C = \frac{1}{8} \frac{(21 - 17c)(8c^2 - 60c + 63)}{(21 - 2c)^2}. \tag{31}$$

Now, making use of eqs. (10) and (30), it is shown that the optimal deviation prices for firms 1 and 2 are, respectively, given by:

$$p_1^* (\tilde{p}_2^C) = \frac{105}{8(21 - 2c)}, \quad p_2^* (p_1) = \frac{5}{8} + \frac{1}{2} c. \tag{32}$$

The corresponding optimal deviation profits are:

$$\tilde{\pi}_1^D = \frac{11025}{64(21 - 2c)^2}, \quad \tilde{\pi}_2^D = \frac{(5 - 4c)^2}{64}. \tag{33}$$

37Making use of eqs. (6)-(7), it is straightforward to conclude that for the specific parameter values we have chosen, firm $i$’s inverse demand function is given by $p_i = \frac{3}{2} - \frac{9}{8} q_i - \frac{3}{8} q_j$. 38Since we have assumed the most efficient firm to be the one which must cover indivisible per-period coordination fixed costs, it seems natural to assume that it has all the bargaining power in the choice of $\alpha$.}
B.1 Deriving Figure 2

B.1.1 Curve ICC-s

Making use of eqs. (4), (27), (31) and (33), some algebra shows that:

\[ f' = \bar{f}(0, \delta) = \frac{9}{8} \frac{7}{21} - (1 - \delta) \frac{11025}{64} + \frac{925}{1225} \left(1 + \frac{7}{8} \delta \right). \]  
(34)

For a given (and sufficiently high) value of \( \delta \), the previous equation is represented by the horizontal line ICC-s in Figure 2.

B.1.2 Curve ICC-n

Making use of eqs. (27), (31) and (33), it may be concluded that condition (3) can be rewritten as:

\[ \frac{1}{1 - \delta} \frac{21 - 17c_n}{8} \left(\frac{8c_n^2 - 60c_n + 63}{21 - 2c_n}\right) > \frac{(5 - 4c_n)^2}{64} + \frac{(21 - 17c_n)^2}{1225} \frac{\delta}{1 - \delta}. \]  
(35)

For a given value of \( \delta \), the value of \( c_n \) for which the previous constraint is binding is represented by the vertical line ICC-n in Figure 2.

B.1.3 Curve ICC-m

Combining the results in eqs. (4), (27), (31) and (33), it may be concluded that:

\[ \bar{f}(c_n, \delta) = \frac{9}{8} \frac{7 + c_n}{21 - 2c_n} - (1 - \delta) \frac{11025}{64} \frac{925}{1225} \left(1 + \frac{7 + c_n}{8} \right). \]  
(36)

It can easily be checked that for a given sufficiently high value of the discount factor \( \delta \), \( \bar{f}(c_n, \delta) \) decreases in \( c_n \). So, the previous condition is represented by the downward sloping curve ICC-m in Figure 2.

C Proof of Proposition 3

Take the specific parametrization of the DPB model used above, where \( \alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1 \) and \( \gamma = 1/3 \). If, after the merger, the two firms, whose marginal costs are \( c_m = 0 \) and \( c_n \) play
noncooperatively, then, making use of eq. (12), their Bertrand-Nash equilibrium profits are given by:

\[
\pi^B_m(c_n) = \frac{9}{1225} (c_n + 7)^2, \quad \pi^B_n(c_n) = \frac{1}{1225} (21 - 17c)^2.
\]  

(37)

If firms instead decide to maximize their joint profit in the one-shot game, then, from (15) and (16), their equilibrium profits will be as follows:

\[
\pi^C_m(c_n) = \frac{1}{8} (c_n + 3), \quad \pi^C_n(c_n) = \frac{1}{8} (2c_n - 3) (c_n - 1).
\]  

(38)

Finally, from eqs. (17) and (18) it is concluded that deviation profits are given by

\[
\pi^D_m(c_n) = \frac{1}{576} (2c_n + 15)^2, \quad \pi^D_n(c_n) = \frac{1}{64} (4c_n - 5)^2.
\]  

(39)

Two different cases should now be considered, which we discuss in turn.

1. First, consider the case where \( f < f^* \) so that collusion can be sustained if there is a merger inducing a symmetric market structure, \( M^A \). Notice that since \( c_n < c^* \), we also have that in case there is a merger (between firms 2 and 3) leading to an asymmetric market structure \( M^B \), the remaining firms in the industry will be able to collude (both the ICC-m and the ICC-n hold after the merger, as illustrated by Figure 1). So, condition (19) boils down to:

\[
\Pi^{AC} = \pi^AC_m + \pi^AC_n > \pi^BC_m + \pi^BC_n = \Pi^{BC}.
\]  

(40)

Now, making use of eq. (38), simple algebra shows that:

\[
\pi^AC_m = \left( \frac{3}{8} - f \right) \frac{1}{1 - \delta}, \quad \pi^AC_n = \frac{3}{8} \frac{1}{1 - \delta}.
\]  

(41)

\[
\Pi^{AC} = \pi^AC_m + \pi^AC_n = \left( \frac{3}{4} - f \right) \frac{1}{1 - \delta},
\]  

(42)

\[
\pi^BC_m = \left( \frac{1}{8} (c_n + 3) - f \right) \frac{1}{1 - \delta}, \quad \pi^BC_n = \frac{1}{8} (1 - c_n) (3 - 2c_n) \frac{1}{1 - \delta},
\]  

(43)

\[
\Pi^{BC} = \pi^BC_m + \pi^BC_n = \frac{1}{4} \frac{c_n^2 - 2c_n + 3 - 4f}{1 - \delta}.
\]  

(44)
Hence, some algebra shows that condition (40) holds if

\[ \frac{1}{4} \frac{c_n (2 - c_n)}{1 - \delta} > 0, \]  

(45)

2. Consider now the case where \( f > f' \). More specifically, consider the region of Figure 1 where ICC-m and ICC-n hold, but ICC-s does not. In this region, firms will not collude in the symmetric market structure \( M^A \). Hence, making use of eq. (37), it can easily be concluded that in this symmetric market structure, the present discounted value of individual firms’ profits and the industry profit is, respectively, given by:

\[ \Pi^A_m = \Pi^A_n = 18 \frac{1}{25} \frac{1}{1 - \delta}, \]

(46)

\[ \Pi^A = 18 \frac{1}{25} \frac{1}{1 - \delta}. \]

(47)

On the other hand, and restricting the attention to the same region of parameter values, it is shown that firms will be able to collude in the asymmetric market structure \( M^B \). Therefore, and as already shown, in this asymmetric market structure \( M^B \), the present discounted value of individual firms’ profits and the industry profit is, respectively, given by eqs. (43) and (44).

Thus, since in this region, firms will not collude in the symmetric market structure \( M^A \) but will be able to collude in the asymmetric market structure \( M^B \) (see Figure 1), condition (19) boils down to:

\[ \Pi^A_m = \Pi^A_n = \pi^A_n - \pi^A_m > \pi^B_C + \pi^B_C = \Pi^B_C. \]

(48)

Now, making use of eqs. (44) and (47), it may be concluded that the previous merger condition will be satisfied if the fixed indivisible costs associated with collusion are sufficiently high, i.e.:

\[ f > \frac{c_n^2 - 2c_n}{4} + 3 - \frac{18}{25}. \]

(49)

The previous condition, when it is binding, is represented by the downward sloping dashed curve in Figure 3.

As a final remark, note that in Figure 3, we assume that \( \delta = 0.9 \). This being the case and making use of eqs. (24) - (26), it is straightforward to show that the equations representing
the ICC-s, the ICC-n and the ICC-m curves are, respectively, given by:

\[ f' = f(0, 0.9) = 0.011938, \]  
\[ c^* = 0.35742, \]  
\[ f(c_n, 0.9) = \frac{1}{8}(c_n + 3) - 0.1 \frac{(2c_n + 15)^2}{576} - 8.1 \frac{(c_n + 7)^2}{1225}. \]

D Proof of Proposition 4

Let us restrict the attention to the region of parameter values where the ICC-m and the ICC-n hold, but the ICC-s does not. As shown in Proposition 3 (and illustrated in Figure 3), there exists a subregion of parameter values where \( M^A \) dominates \( M^B \). This subregion is the one to the right of the dashed line in Figure 3. So, in this area, if the antitrust authority forbids a merger creating a symmetric market structure \( M^A \), firms will be induced to choose a merger creating an asymmetric industry structure, \( M^B \).

By creating an asymmetric industry structure, this merger between firms 2 and 3 yields a higher average production cost and a lower aggregate producer surplus than that leading to a symmetric market structure \( M^A \) (recall that, as shown in Proposition 3, \( M^A \) dominates \( M^B \) under a laissez-faire policy). To complete the proof, however, we also need to show that a merger leading to \( M^B \) gives rise to a lower consumer surplus than a merger leading to \( M^A \), i.e., we need to check whether the prices charged by the merged entity and the non-merged firm are higher in market structure \( M^B \) than in market structure \( M^A \). Regarding the merged entity (firm \( m \)), making use of eqs. (11) and (14), it is easy to check that it will charge higher prices in case the ex-post merger market structure is \( M^B \), since:

\[ p^{A*}_m = \frac{3}{5} < p^{BC}_m = \frac{3}{4}. \]  

As for the non merged entity (firm \( n \)), making use of eqs. (11) and (14), it is simple to check that it will also set a higher price in market structure \( M^B \) than in market structure \( M^A \), since:

\[ p^{A*}_n = \frac{3}{5} < p^{BC}_n = \frac{1}{4}(2c_n + 3), \]  

which is true for any \( c_n > 0 \). Hence, a merger leading to an asymmetric market structure \( M^B \) will,
in the region under consideration, give rise to a lower consumer surplus than a merger leading to a completely symmetric industry structure, $M^A$. This completes the proof of Proposition 4.

References


