Inter-firm Bundling and Vertical Product Differentiation*

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Abstract

This paper studies the competitive effects of bundled discounts offered by pairs of independent firms. In a setting with vertically differentiated goods where firms decide whether to participate in a discounting scheme before prices are set, it is shown that, in equilibrium, all pairs of firms producing goods of the same quality level offer bundled discounts and, relative to the no-bundling benchmark: (i) all headline prices rise; (ii) all bundle prices, net of the respective discount, decrease; and (iii) only high quality sellers will obtain higher profits. Furthermore, this equilibrium corresponds to the worst scenario in terms of consumers’ welfare and decreases social welfare.

Keywords: Bundled Discounts, Bilateral Bundling, Vertical Differentiation.

JEL Classification: D43; L13; L41.

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I. Introduction

Bundled discounts provide purchasers with the opportunity to pay less for a bundle than the sum of the prices of the bundled products when purchased separately. Consumers are therefore faced with the choice between meeting all their requirements by buying a package at a discounted price, or purchasing items individually à la carte.

Examples of companies offering bundled discounts include fast food restaurants, telephone companies, book stores, grocery stores and gasoline retailers, to name just a few. Despite the fact that bundled discounts are a widespread business practice, the academic literature has devoted little attention to this issue. While the case where a retailer bundles several of its products has received some attention in the literature, the case where independent firms bundle their products has seen very little investigation. In this article, we contribute to addressing this uncovered issue in the previous literature by proposing a model of strategic interaction among producers of different and unrelated products to study the likely competitive effects of bundled discounts.

There are several examples of discounts for bundles where each component good is sold by a different and independent firm. A typical example is the case of supermarket and retail gasoline chains that frequently offer bundled discounts to consumers who purchase from them. These discounts are usually a fixed amount off the headline (or stand-alone) prices that the partner firms continue to set independently. Another example, presented by Armstrong (2012), refers to bundles in the pharmaceuticals industry: “Pharmaceuticals are sometimes used in isolation and sometimes as part of an approved ‘cocktail’ with one or more drugs supplied by other firms. Drugs companies have the ability to set different prices depending on whether the drug is used on a stand-alone basis or in a cocktail”. (p.2) One such ‘cocktail’ is Atripla, a combination of three HIV medicines: Sustiva, Emtriva and Viread. Sustiva is a registered trademark of Bristol-Myers Squibb while Emtriva and Viread are trademarks of Gilead Sciences, Inc., a different firm. Other examples of bundling between independent firms are the cases of hotels bundling with restaurants, airlines/fast trains bundling with car rental companies or with hotels and

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1 As pointed out by Nalebuff (2005, p.364), “[t]he practice of bundle discounts is prevalent, but their effects on competition are not well understood.” Along these lines, Kobayashi (2005) highlights that his “review of the economic literature on commodity bundling generally confirms the US Solicitor General’s view in 3M v. LePage’s regarding the underdeveloped state of the economics literature (…).”

2 The literature focusing on bundling by multiproduct firms has analyzed two important issues. Firstly, it shows that bundled discounts can change the market structure by affecting firms’ incentives to enter or exit the market (see, for example, Peitz (2008), Greenlee et al. (2008) and Nalebuff (2004, 2005), to name a few). Secondly, it shows that by introducing an additional instrument for firms to engage in price discrimination, bundled discounts may have an important impact on both consumer surplus and social welfare (see the initial works along these lines by Carbajo et al. (1990) and Matutes and Regibeau (1992) and, more recently, the studies by Thanassoulis (2007), Armstrong and Vickers (2010) and Goic et al. (2011)).
public transportation bundling with cultural/leisure institutions, to name just a few.\textsuperscript{3}

Bundling by independent and otherwise unrelated firms differs from bundling by a multi-product firm in the following two ways. First, the firms’ objective functions are different in the sense that the multiproduct firm takes into consideration how changing the price of a given product affects the demand for the other products in its portfolio. Second, when the discount is shared by independent firms, there is the need to sign a contract defining the discount magnitude, on the one hand, and determining how the corresponding cost will be divided between the partner firms, on the other. In addition, since these contract decisions must occur before the price setting stage, there are additional strategic considerations to be dealt with when setting the optimal discount.

Gans and King (2006), the most similar previous study to ours, investigated the case where each of the two components in the bundle is produced by two independent single product firms.\textsuperscript{4} In their setting, unilateral bundling, i.e. bundling by a single pair of firms, increases the profits of the partner firms to the detriment of the remaining non-bundling firms. Nevertheless, if both pairs of independent firms offer bundled discounts, i.e. if there is bilateral bundling, then each firm’s profits and output end up being \textit{the same} as in the case where there are no bundled discounts. Moreover, bilateral bundling leads to a social-welfare reduction, as some consumers simply find themselves consuming a sub-optimal branding mix.

One of the assumptions of Gans and King (2006) is that products are horizontally differentiated.\textsuperscript{5} However, there are several examples of bundled discounts in industries where, at least with respect to one of the products in the bundle, differentiation is clearly vertical. This is the case of the grocery-gasoline bundled discounts where there is vertical differentiation, at least, on the groceries’ side: most consumers consider that supermarket chains and discount stores offer products and services of different quality. Another case in point refers to bundled discounts offered to consumers looking for airline tickets and car rental. Low cost airlines and national carriers are also, in most cases, vertically differentiated, as well as local rent-a-car versus multinational rent-a-car agencies. Vertical differentiation, at least as far as consumer perception is concerned, is also present in the pharmaceutical industry, with the distinction

\textsuperscript{3}Some airline and credit card/hotel/car rental partnerships are based on accumulating miles and are, therefore, loyalty reward schemes. Here, we are referring instead to the possibility of obtaining lower prices when renting a car upon choosing to fly with a given airline: in many instances, by presenting the boarding passes and a discount code, travelers will be given a discount.

\textsuperscript{4}Other work that focuses on bundled discounts by independent producers is Maruyama and Minamikawa (2009), who study the incentives for vertical integration. Also, Armstrong (2010) discusses the impact on profits, consumer surplus and welfare induced by inter-firm discounts, focusing on independent, partially substitutable and complementary products.

\textsuperscript{5}This assumption is shared by several articles that studied bundled discounts by multiproduct firms, e. g. Matutes and Regibeau (1992), Thanassoulis (2007), Peitz (2008) and Armstrong and Vickers (2010).
between branded products and generics. Needless to say, hotels and restaurants are also vertically differentiated. To the best of our knowledge, however, vertical differentiation has been neglected by the extant literature on bundled discounts.

In the present paper, we contribute to closing this gap in the literature by developing a two-stage model in which independent firms first decide on whether to offer a bundled discount involving the joint purchase of their products and, subsequently, compete in prices. With respect to the supply side of the market, it is assumed that each available product is produced by two different firms, a high quality producer and a low quality producer. As for the demand structure, we assume that consumers are arrayed on a unit square, where each axis measures consumers’ valuations for the quality of each product.

With respect to the competitive effects of the introduction of bundled discounts in this context, our main results are the following. First, and relative to the no-discounting benchmark case, the headline prices of the bundling firms always rise. Second, bundled discounts may induce a decrease in consumer surplus and always induce a reduction in total welfare. In line with Gans and King (2006), this leads to the conclusion that bundled discounts may raise antitrust concerns.

As for the firms’ decisions regarding their eventual participation in a bundled discount scheme, our results differ from those of Matutes and Regibeau (1992), Gans and King (2006), Thanassoulis (2007) or Maruyama and Minamikawa (2009). In equilibrium, the high quality firms earn higher profits than in the status quo no-discounting situation, and therefore firms do not find themselves in a prisoner’s dilemma situation: allowing for vertical differentiation leads to the elimination of the Bertrand bundling super-trap identified by Gans and King (2006). Nevertheless, the equilibrium scenario is shown to be the one leading to the most adverse consequences in terms of consumer welfare and also leads to a reduction of social welfare, when compared to the no bundled discounts benchmark.

Our results then suggest that competition authorities should scrutinize bundling discounting by independent producers of vertically differentiated goods in detail. To our knowledge, there have been no antitrust cases in Europe where there is bundling of two products which not only are in different relevant markets, but are also produced by different firms. Furthermore, it is arguably difficult to deal with cases of this type under EU competition law. Abuse-of-dominance cases require a dominant position. The problem is that in the types of situations

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6 The prisoner’s dilemma is a recurrent result in the price discrimination literature. As Armstrong (2008) highlights, “(...) an oligopolistic firm is always better off if it can price discriminate compared to when it cannot, for given prices offered by its rivals. However, as in many instances of strategic interaction, once account is taken of what rivals too will do, firms in equilibrium can be worse off when price discrimination is permitted. Firms then find themselves in a classic prisoner’s dilemma.”
we address, it is not clear which firm should have a dominant position. Both of the bundling firms? Just one of these firms? Also, since the bundling scheme involves two independent firms, it requires an agreement between the two firms. So one would think that these cases would fall under article 101 rather than under article 102. However, it is difficult to argue that article 101 would cover agreements between independent and otherwise unrelated firms regarding inter-firm bundling. Hence, this paper deals with cases which appear not to be covered by EU competition law, but are, nevertheless, shown to be welfare detrimental, suggesting that competition authorities should be more careful with regard to this type of promotion strategy.

The remainder of the paper is organized as follows. In Section II, we lay down our general framework and specify the timing of the proposed game. In Section III, we present the equilibrium of the game. Section IV discusses the results and investigates their robustness. Finally, Section V concludes the paper. All proofs and some longer expressions are relegated to the online Appendix.

II. The model

Firms

We consider the case of two distinct products, X and Y, each sold by two firms, a high quality producer and a low quality producer. X and Y also denote the high quality producers of the two products and x and y denote the two low quality producers of the two products. The high and low quality level indices are denoted by S and s respectively and we assume there are no costs associated with the production of either product or quality level. The producer i’s unit price is denoted by $p_i$ and its individual profit by $\pi_i$, with $i = X, Y, x, y$. The exogenous quality difference $S - s$ is denoted by the positive parameter $\sigma$.

Each pair of producers of different products may agree to participate in a bundled discount scheme. Let $\beta_{jk}\sigma$, with $jk = XY, xY, Xy$ or $xy$, denote the discount offered by producer $j$ and $k$, who jointly decide upon the level of $\beta_{jk}$. When $\beta_{jk} > 0$ we say that firms $j$ and $k$ are partner firms in the discounting scheme. For instance, when the two high quality producers, firms $X$ and $Y$, are partners, consumers that purchase the high quality bundle will pay $p_X + p_Y - \beta_{XY}\sigma$. We denote by $\alpha_{jk}$ the percentage of the discount financed by firm $j$.

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7To economize on notation, $X$ simultaneously represents one of the products and also the corresponding high quality producer, and likewise for $Y$. To avoid unnecessary confusion, we always explicitly mention if we are referring to product $X$ or to producer/firm $X$.

8As Armstrong (2010) points out, this additive bundled discount is “probably more easily implemented in practice relative to a system of choosing a rigid bundle price (...) and then negotiating how to share that revenue.”
Like Gans and King (2006), we are interested in the profitability of relatively small discounts. As such, for each scenario, we define a small discount as a discount that does not exceed 50% of the average price of the two products in the bundle, evaluated at the no bundle equilibrium. Such a discount restriction implies that the equilibrium at the pricing stage is such that the four possible combinations of products have a positive market share for all discount-sharing possibilities and for all discount levels offered by the competitors (if any).\(^9\)

Our assumption of small discounts is justified by some management literature that has underlined the ineffectiveness of high discounts. Raghubir (1998) illustrates that consumers use the coupon value (either percentage discount or cents-off coupon) to form their expectations about price. A high discount signals a high (standalone) price, and therefore may make purchase less likely. In addition, Barat and Paswan (2005), using questionnaire data, conclude that, “for low face values of coupon, intention to redeem is positively associated with face value, whereas, for the higher face values of the coupon, the intention remains more or less unchanged. The correlation between intention to redeem the coupon and the perceived sticker price of the product is positive at the lower levels of coupon face value, but becomes negative for higher face values.” Furthermore, Raghubir (2004) analyzes the contexts that trigger negative deductions about the price or quality of a product from coupon discounts among the consumers. One of the implications of the obtained results is that “doubling coupon values may neither profitably, nor reliably increase a product’s sales”. Along different lines, the management literature has also addressed how the perception of price unfairness affects consumer behavior, and ultimately sellers’ profits. In particular, it turns out that the perception of unfairness may arise from price discrimination, as recently analyzed in Wu et al. (2011) whose results “show that post-purchase disclosure of discrimination information elicits higher negative emotions for indirect discrimination involving coupon and purchase quantity, but is rather inconsequential for direct discrimination or indirect discrimination through membership.”

Small discounts are also empirically more relevant. In fact, gasoline discounts usually amount to 5 to 10 euro cents and restaurant discounts for guests at hotels are usually around 10%. As for the HIV medicines mentioned above, online information states that “Atripla is cheaper ($1,465 vs. $1,479 per 30 days’ supply) than if the three components are purchased individually”, which represents a discount of around 1% on the corresponding wholesale prices.\(^10\)

It should be pointed out at this point that imposing restrictions on the discount choice variable is common in the literature on bundled discounts, see e.g. Gans and King (2006),

\(^9\)In the online appendix, we show that the results are robust for other definitions of small discounts, provided that the four combinations of products have a positive demand.

\(^10\)http://www.ncbi.nlm.nih.gov/pmc/articles/PMC2504066/
Aydemir (2009) or Goic et al. (2011). An important argument in favor of introducing such a restriction, in addition to the arguments above, is that it makes the analysis more tractable as it significantly limits the number of cases that one has to address. Moreover, note that the discount constraint we imposed is only binding in one of the five scenarios investigated.

Consumers

The way we model consumers’ preferences for quality follows Gabszewicz and Thisse (1979). Consumers purchase at most one unit of each good and we assume, in line with Thanassoulis (2007), that some consumers are only interested in buying one of the products while others want to buy both products. We also assume that the three groups of consumers are of equal size: the number of consumers who value only one of the two goods is equal to the number of consumers who value only the other good and also equal to the number of consumers who value both goods, and we normalize the measure of consumers in each of these three groups to be 1.

Assuming no discounts, consumer $k$’s net utility when purchasing product $X$ from the high quality producer is given by $V_X + \theta^k_X s - p_X$, whereas consumer $k$’s net utility when purchasing it from the low quality producer is given by $V_X + \theta^k_X s - p_x$. The same applies for product $Y$, with the obvious differences. Therefore, parameter $\theta^k_X$ measures consumer $k$’s valuation for the quality of product $X$. In one of the three groups of consumers, $V_i$, $i = X, Y$, is assumed to be sufficiently high so that every consumer purchases one unit of each good. In the other two consumer groups, however, either $V_X$ and $\theta^k_X$ or $V_Y$ and $\theta^k_Y$ are assumed to be zero. We also assume that the consumers’ valuations of the products they value are sufficiently high so that the market is fully served. That is, all consumers interested in both products will buy one unit of each product, and consumers who are in the market for one product only will buy one unit of that product (only).

Consumers are assumed not to get any extra benefit or any reduction in transaction costs from purchasing a specific pair of products. Hence, in the absence of a discount, the demand for one product is independent of the demand for the other. Furthermore, we also assume that for consumers who value both products, valuations for the quality, $(\theta^k_X, \theta^k_Y)$, are uniformly distributed in $[0,1] \times [0,1]$. In addition, for consumers who value only one of the available products, valuations for this product are uniformly distributed along a line of unit length.

Finally, when a consumer purchases, say, the high quality version of good $X$ and the low

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11 In contrast, both Matutes and Regibeau (1992) and Gans and King (2006) assume that all consumers in the market must purchase both available goods in order to derive any positive utility.

12 We exclude $V_X$ and $V_Y$ from the levels of consumer and social surplus presented below.
quality version of good Y we denote such a consumer purchase as \(X_y\). We denote the quantity of such a pair of products by \(q_{Xy}\). As for the consumers who care for only one product, the quantity demanded of, say, the high quality version of product \(X\) is denoted by \(q_{X}\). Aggregate demand for producer \(i\) is denoted by \(Q_i\) with \(i = X, Y, x, y\).

Deriving the relevant demand for each pair of products relies heavily on the identity of several consumers who are indifferent between purchasing different pairs of products. Assume that there is a discount for consumers who purchase the two high quality products, \(\beta_{XY}\), and consider, for instance, those consumers who have a relatively high valuation for quality in product \(X\). In these circumstances they are most likely to purchase from producer \(X\). With respect to product \(Y\), however, their decision to purchase the high or low quality version of \(Y\) depends on the comparison between the value of additional quality and the corresponding payment of a higher price. As a result, only for the subset of consumers such that \(\frac{p_Y - p_y}{\sigma} + \sigma \beta_{XY} > \frac{p_Y - p_y}{\sigma} \beta_{xy}\) will the option be to purchase from producer \(Y\) rather than from producer \(y\). In this case, \(\vartheta_Y^{XY}\) denotes the consumer who is indifferent between buying \(XY\) and buying \(Xy\), when there is a discount for those who purchase \(XY\). We thus have \(\vartheta_Y^{XY} := \frac{p_Y - p_y - \sigma \beta_{XY}}{\sigma}\).

More generally, \(\vartheta_i^{jk}\) represents the consumers who are indifferent between the pair \(jk\), which entitles them to a discount \(\sigma \beta_{jk}\), and the pair with different quality for product \(i = X, Y\) and the same quality for product \(j \neq i\), which does not include any discount.\(^{13}\)

The following lemma presents the relevant demand functions for different types of discounts.

**Lemma 1:**

(i) Consider the case where discounts are given by firms with the same quality. Assume that \(\left(\vartheta_Y^{XY}, \vartheta_X^{XY}, \vartheta_Y^{xy}, \vartheta_X^{xy}\right) \subset [0, 1]^4\). Then:

\[
q_{Xy} = \left(1 - \frac{p_X - p_x + \sigma \beta_{xy}}{\sigma}\right) \left(\frac{p_Y - p_y - \sigma \beta_{XY}}{\sigma}\right),
\]

\[
q_{xy} = \left(1 - \frac{p_Y - p_y - \sigma \beta_{XY}}{\sigma}\right) \left(1 - \frac{p_X - p_x - \sigma \beta_{xy}}{\sigma}\right) - \frac{\left(\beta_{XY} + \beta_{xy}\right)^2}{2}.
\]

(ii) Consider the case where discounts are given by firms with different quality. Assume that

\[
\vartheta_Y^{XY} := \frac{p_X - p_x - \sigma \beta_{XY}}{\sigma}, \quad \vartheta_Y^{XY} := \frac{p_Y - p_y - \sigma \beta_{XY}}{\sigma}, \quad \vartheta_Y^{xy} := p_Y - p_y - \sigma \beta_{XY}, \quad \vartheta_X^{XY} := \frac{p_Y - p_y - \sigma \beta_{XY}}{\sigma}, \quad \vartheta_X^{xy} := \frac{p_X - p_x - \sigma \beta_{xy}}{\sigma}
\]

13 In the case of discounts by producers with the same quality, let \(\vartheta_Y^{XY} := \frac{p_X - p_x - \sigma \beta_{xy}}{\sigma}, \vartheta_Y^{XY} := \frac{p_Y - p_y - \sigma \beta_{XY}}{\sigma}, \vartheta_Y^{xy} := \frac{p_Y - p_y - \sigma \beta_{xy}}{\sigma}, \vartheta_X^{XY} := \frac{p_Y - p_y - \sigma \beta_{XY}}{\sigma}, \vartheta_X^{xy} := \frac{p_X - p_x - \sigma \beta_{xy}}{\sigma}\). In the case of discounts by producers of different quality, the corresponding values are \(\vartheta_Y^{XY} := \frac{p_Y - p_y - \sigma \beta_{xy}}{\sigma}, \vartheta_X^{XY} := \frac{p_Y - p_y - \sigma \beta_{XY}}{\sigma}, \vartheta_Y^{xy} := \frac{p_Y - p_y - \sigma \beta_{xy}}{\sigma}, \vartheta_X^{xy} := \frac{p_X - p_x - \sigma \beta_{xy}}{\sigma}\).
\( \left( \theta_{XY}^X, \theta_{Xy}^X, \theta_{YX}^Y, \theta_{yX}^Y \right) \in [0, 1]^4 \). Then:

\[
\begin{align*}
q_{XY} &= \left( 1 - \frac{p_X - p_x - \sigma \beta_{XY}}{\sigma} \right) \left( \frac{p_Y - p_y + \sigma \beta_{XY}}{\sigma} \right) - \frac{(\beta_{XY} + \beta_{xY})^2}{2}, \\
q_{xY} &= \left( 1 - \frac{p_Y - p_y - \sigma \beta_{xy}}{\sigma} \right) \left( \frac{p_X - p_x + \sigma \beta_{xy}}{\sigma} \right) - \frac{(\beta_{XY} + \beta_{xY})^2}{2}, \\
q_{XY} &= \left( 1 - \frac{p_Y - p_y + \sigma \beta_{XY}}{\sigma} \right) \left( 1 - \frac{p_X - p_x + \sigma \beta_{xy}}{\sigma} \right), \\
q_{xy} &= \left( \frac{p_Y - p_y - \sigma \beta_{XY}}{\sigma} \right) \left( \frac{p_X - p_x - \sigma \beta_{XY}}{\sigma} \right).
\end{align*}
\]

(iii) Consider the group of consumers who only value one product. The corresponding demand functions for each quality level of each product are given by:

\[
\begin{align*}
q_X &= \left( 1 - \frac{p_X - p_x}{\sigma} \right) \quad \text{and} \quad q_x = \left( \frac{p_X - p_x}{\sigma} \right), \\
q_Y &= \left( 1 - \frac{p_Y - p_y}{\sigma} \right) \quad \text{and} \quad q_y = \left( \frac{p_Y - p_y}{\sigma} \right).
\end{align*}
\]

Making use of Lemma 1 and of the assumption that the number of consumers interested in buying product X only, product Y only or both products is the same, the aggregate demand function faced by each individual firm can be obtained from:

\[
\begin{align*}
Q_X &= q_{XY} + q_{XY} + q_X \quad \text{and} \quad Q_x = q_{xy} + q_{xY} + q_x, \\
Q_Y &= q_{xY} + q_{XY} + q_Y \quad \text{and} \quad Q_y = q_{xy} + q_{XY} + q_y.
\end{align*}
\]

Clearly, all quantities are a function of prices and discounts. For the sake of brevity, however, we omit the arguments of these functions.

**Timing**

The timing of the game played between firms is divided into two stages: (1) a coalition formation stage and (2) a competition stage.

In the first stage, firms play a coalition formation game from where the identities of the partner firms that finance the discount are obtained. At this stage, we assume that nature randomly selects one firm which has the opportunity to select another firm (if any) that it wants to offer a bundled discount with. If no offer being made or in the event of rejection, another firm is randomly selected. Whenever a pair of firms agrees to the match, the remaining
two firms are also paired and the process ends. Otherwise, the process ends after all the firms have been given the opportunity to select a partner. The assumption that the two remaining firms are paired after the other two have been matched reflects the inability of firms to commit not to offer discounts. If it is optimal for them not to offer a discount, we assume they are still able to do so by deciding upon a zero discount level. As a result of this inability to commit not to offer discounts, after this stage ends all four firms are paired.

Afterwards, prices and discounts are set at the competition stage, which can be further divided into two stages: First, the pairs of partner firms agree to their bundled discount, if any, and to the way of sharing it. Second, firms set their headline prices simultaneously.

Partner firms choose both the discount level and the percentage of the discount financed by each producer so as to maximize their joint profits.\(^\text{14}\) Then, each single-product firm sets its headline price (non-cooperatively and simultaneously). Since the discount is set before the headline prices, we are implicitly assuming that it is easier for a firm to change its own price than to change the bundled discount level. This is a natural assumption given that any firm is free to unilaterally change its headline price at short notice, whereas changing the discount would involve renegotiation with the partner firm. Moreover, it is often the case that firms advertise the discount scheme but not the headline prices. This is a way of committing to the agreed upon discount in advance while still allowing for future changes in the headline prices.\(^\text{15}\)

### III. Equilibrium

In this section, we present the equilibrium of the game described above. We solve the game following the usual backward induction procedure, starting with the competition stage and then moving to the coalition formation stage. Before proceeding, however, we first present the benchmark case where there are no discounts, i.e. \(\beta_{jk} = 0\) for all \(j = x, X\) and \(k = y, Y\). In the absence of discounts, the decision to purchase a high or low quality version of one product is independent of the prices of the two different quality variants of the other product. Purchasing choices regarding the group of consumers interested in both products in this no-discounting benchmark are illustrated in Figure 1.

**Lemma 2:** If no pair of firms offers a bundled discount, then, in equilibrium:

(i) the headline prices for the high quality firms will be \(p_X = p_Y = 2\sigma/3\).

(ii) the headline prices for the low quality firms will be \(p_x = p_y = \sigma/3\).

\(^{14}\)This can be thought of as a joint organization setting the discount on behalf of the two firms involved.

\(^{15}\)Goic et al. (2011) also make this assumption despite the fact that discounts are only offered by multi-product firms.
(iii) consumer surplus will be $CS = \frac{2}{9} (11s - 2S)$.

(iv) the individual profit of the high quality firms will be $\pi_X = \pi_Y = 8\sigma / 9$ and the individual profit of the low quality firms will be $\pi_x = \pi_y = 2\sigma / 9$.

(v) social surplus will be $SS = \frac{2}{9} (8S + s)$.

\textit{Competition stage}

There are two possible outcomes of the coalition formation stage. Either the two pairs of partner firms that offer discounts sell products of the same quality or they sell products of different quality. We start with the former.

Consider first the case of discounts offered by firms selling products of the same quality level. Assume that the high quality pair of firms and the low quality pair of firms offer discounts, respectively denoted by $\beta_{XY} \geq 0$ and $\beta_{xy} \geq 0$ and recall that the competition stage involves two sub-stages. First, the two pairs of firms simultaneously make their decisions about the discount scheme. Second, each firm individually sets its price.

So, at the final stage all four firms choose prices simultaneously to maximize their own profit, implying that the equilibrium prices result from the individual maximization of the following objective functions:

\[
\begin{align*}
\pi_X &= p_X Q_X - \alpha_{XY} \sigma_{XY} q_{XY} \\
\pi_Y &= p_Y Q_Y - (1 - \alpha_{XY}) \sigma_{XY} q_{XY} \\
\pi_x &= p_x Q_x - \alpha_{xy} \sigma_{xy} q_{xy} \\
\pi_y &= p_y Q_y - (1 - \alpha_{xy}) \sigma_{xy} q_{xy}
\end{align*}
\]
The equilibrium prices, as a function of \( \alpha_{XY} \), \( \alpha_{xy} \), \( \beta_{XY} \) and \( \beta_{xy} \), are obtained by solving the system made up of four linear first-order conditions. The corresponding expressions are presented in the online Appendix.

Given the equilibrium prices, each pair of firms simultaneously decides upon its discount scheme variables (namely, the discount level and the sharing rule) so as to maximize aggregate profits, i. e. \( \alpha_{XY} \) and \( \beta_{XY} \) are set to maximize \( \pi_X + \pi_Y \), and \( \alpha_{xy} \) and \( \beta_{xy} \) are set to maximize \( \pi_x + \pi_y \).

When deciding upon the optimal discount level, the pair of partner firms will take into consideration the effects that different discount levels will have on the equilibrium prices. These effects can be described as follows. All else being equal, a higher discount offered by any pair of producers increases the demand for their bundle. Also, the introduction of the discount can be interpreted as a unit cost, partially incurred by each partner firm, for the units entitled to the discount. As a result of the higher demand and the higher unit cost, headline prices for the goods in the bundle increase with the discount, although the “net” bundle price decreases with it. For the other pair of firms, a higher discount given by their competitors gives rise to two different effects: a lower demand for their products, on the one hand, but on the other hand “higher costs” for their competitors. The first effect turns out to dominate the second one and, as a result, their headline price decreases in line with their competitors’ discount.

In the online Appendix, we show that for any level of the two discounts, if the rival pair of firms sets \( \alpha_j = 1/2 \), the other pair of firms maximizes its profit by also setting \( \alpha_k = 1/2 \), where \( j, k = XY, xy \). Given that the rival pair of symmetric firms is equally distributing the discount cost between the two partner firms, it is optimal for the other pair to do the same. Now, setting \( \alpha_{XY} = \alpha_{xy} = 1/2 \), it can also be shown that the equilibrium discount levels are given by \( \beta_{XY}^* = 1/3 \) and \( \beta_{xy}^* = 1/6 \), which represents a corner solution.

The following Lemma presents the equilibrium under bilateral bundling by firms selling products of the same quality.  

**Lemma 3:** If both the low and high quality pairs of firms offer a bundled discount, then, in equilibrium and relative to the situation without bundling:

(i) the headline prices for the high quality firms will rise to \( p_X = p_Y = 0.75\sigma \).

(ii) the headline prices for the low quality firms will rise to \( p_x = p_y = 0.35\sigma \).

(iii) the price of the high quality bundle, net of the discount, will decrease to \( p_X + p_Y - \beta_{XY}\sigma = 1.19\sigma \) and the price of the low quality bundle, net of the discount, will decrease to

\[ p_x + p_y - \beta_{xy}\sigma \]

\[ 0.66\sigma \]

---

\[ ^{16} \]We have rounded all the numeric results to two decimal places. The online appendix presents the numeric solutions rounded to five decimals.
\[ p_x + p_y - \beta_{xy}^{*} \sigma = 0.55\sigma. \]

(iv) consumer surplus will fall to \( CS = 2.47s - 0.47S \).

(v) the individual profit of the high quality firms will rise to \( \pi_X = \pi_Y = 0.90\sigma \) and the individual profit of the low quality firms will fall to \( \pi_x = \pi_y = 0.21\sigma \).

(vi) social surplus will fall to \( SS = 1.74S + 0.26s \).

Hence, in this bilateral bundling scenario, only the high quality pair of firms will be better off with the introduction of the two bundled discounts. In addition, and perhaps more importantly, both social and consumer surplus will decrease. Figure 2 illustrates how consumers are divided between the four product combinations when there are discounts offered by firms with the same quality.

We now turn to the case in which the pairs of partner firms have different quality levels. In particular, the high quality producer of \( X \) offers a joint discount with the low quality producer of \( Y \), denoted by \( \beta_{xy} \). In this case, the percentage of the discount financed by the high quality producer is denoted by \( \alpha_{xy} \). As for the remaining pair of firms also offering a bundled discount, the corresponding notation regarding the discount level and the percentage of the discount financed by the high quality producer will be \( \beta_{xY} \) and \( \alpha_{xY} \), respectively.

Equilibrium prices result from the individual maximization of the following objective functions:

\[
\begin{align*}
\pi_X &= p_X Q_X - \alpha_{Xy}^{*}\sigma\beta_{Xy}q_{Xy} \\
\pi_Y &= p_Y Q_Y - \alpha_{xY}^{*}\sigma\beta_{xY}q_{xY} \\
\pi_x &= p_x Q_x - (1 - \alpha_{xy})\sigma\beta_{xY}q_{xY} \\
\pi_y &= p_y Q_y - (1 - \alpha_{XY})\sigma\beta_{Xy}q_{Xy}
\end{align*}
\]

The corresponding analytical expressions regarding the equilibrium prices are also presented in the online Appendix.

In the following analysis, and given that the two pairs of firms are symmetric, we look for a symmetric equilibrium.\(^{17}\) It can be shown that \( \partial (\pi_X + \pi_y) / \partial \alpha_{xy} \) and \( \partial (\pi_x + \pi_Y) / \partial \alpha_{xY} \) are both positive for all admissible symmetric discounts and discount sharing rules, i.e. for all \( \beta_{xy} = \beta_{xY} \) and \( \alpha_{Xy} = \alpha_{xY} \). Hence, for any level of the two discounts, we have \( \alpha_{Xy}^{*} = \alpha_{xY}^{*} = 1 \). Taking this optimal sharing rule into account, one can solve the first-order conditions with respect to the discount level (looking for a symmetric solution), obtaining \( \beta_{XY}^{*} = \beta_{xY}^{*} = 0.23 \).

It should be noted that there is a slight difference between this case and that of section 3.1.1

\(^{17}\) Each pair involves a high quality producer of one good and the low quality producer of the other good.
with respect to the way headline prices of the products outside the bundle change with the
discount. In this case, the price of the low quality product increases with the rivals’ discount,
whereas in the previous case, the headline prices of all products outside a specific bundle would
decrease with the increase in the discount for that bundle (the rivals’ discount). Recall that
an increase in the rivals’ discount has two opposite effects from the perspective of a given firm:
on the one hand, there is a lower demand for its product, but on the other hand, its direct
competitor will face “higher costs” in some of the units it sells. In the current scenario, as the
high quality producer of $X$ is paying for the entirety of the discount, the increase in its costs
is very large. Therefore, from the perspective of the low quality producer of $X$, the second
effect dominates the first one, and as a result its equilibrium price will increase in the rivals’
discount level.

**Lemma 4:** If the two pairs of firms of different quality levels offer a bundled discount, then,
in equilibrium and relative to the situation without bundling:

(i) the headline prices for the high quality firms will rise to $p_X = p_Y = 0.70\sigma$.
(ii) the headline prices for the low quality firms will rise to $p_x = p_y = 0.37\sigma$.
(iii) the price of the bundles, net of the discount, will decrease to $p_X + p_Y - \beta_{xy}\sigma = 0.84\sigma$.
(iv) consumer surplus will rise to $CS = 2.42s - 0.42S$.
(v) the individual profit of the high quality firms will fall to $\pi_X = \pi_Y = 0.79\sigma$ and the
individual profit of the low quality firms will rise to $\pi_x = \pi_y = 0.27\sigma$.
(vi) social surplus will fall to $SS = 1.70S + 0.30s$.  

Hence, the negotiated bundled discount leads partner firms to raise (all) headline prices
relative to the benchmark case in which the products are marketed independently. Additionally,
in this “crossed” bilateral bundling scenario, the discount associated cost will be fully incurred
by the high quality producer, implying that the high quality firms’ profit will decrease whereas
the individual profit of low quality producers will rise with the introduction of the two bundled
discounts. Overall, however, total welfare will decrease relative to the no discount benchmark.
Figure 3 illustrates consumer choices when discounts are offered by firms with different quality.
Coalition formation stage

We now analyze the stage of the game where firms make their offers regarding the formation of discount coalitions, which may be accepted or rejected. From Lemma 3 and Lemma 4, whenever a high quality firm, say firm $X$, receives an offer by the other high quality firm, the payoff of accepting it will be $\pi_X = 0.90\sigma$, whereas if the offer is made by a low quality firm, the payoff will be $\pi_X = 0.79\sigma$. As a result, the high quality firm should always accept an offer by another high quality firm as this results in the highest possible payoff it can obtain. In case of a low quality firm, say firm $x$, acceptance of an offer by a high quality firm results in $\pi_x = 0.27\sigma$ (see Lemma 4) while acceptance of an offer by a low quality firm will yield $\pi_x = 0.21\sigma$ (see Lemma 3).

If a sub-game is reached where three firms have made offers that were previously rejected, the following may occur. If the last firm chosen by nature is a high quality firm, it will make an offer to the other high quality firm and the offer is accepted. If instead this last firm chosen by nature is a low quality firm, any of its offers will be rejected by the target firm. With this in mind, the same reasoning applies to a sub-game that is reached after two (or less) offers have been made and rejected. As a consequence, in equilibrium one will have bilateral bundling by firms with the same quality level. Regardless of the type of coalition formation game, one can establish the following result:

**Proposition 1:** In equilibrium there will be bilateral bundling by firms with the same quality level.
Regardless of the way the coalitions are formed, any scenario involving bundling by partner firms with different quality levels cannot be a Nash equilibrium of this game, if firms are free to unilaterally decline offering discounts. Clearly, if one of the high quality firms is involved in a discounting scheme with a low quality producer, this high quality firm will opt out of the partnership. This is because by so doing its payoff will increase after leaving the partnership, regardless of whether firms outside the partnership are offering a bundled discount together or not. In addition, the inexistence of any pairs of partner firms is not a Strong Nash Equilibrium, because any pair of producers of the same quality can profitably (and cooperatively) deviate and introduce a discount. The same happens when only one pair of firms results from the coalition formation stage: the pair of mismatched firms would profit by cooperatively deviating and offering a bundled discount. Hence, the case of bilateral bundling by firms with the same quality stands out as the unique Strong Nash Equilibrium of the game. In fact, none of the high quality firms can be induced to cooperatively deviate from this equilibrium because this equilibrium results in the highest payoff each of these firms can obtain. As for the low quality firms, any deviation that does not include a high quality firm will also lower their profit.

This outcome is fairly robust to changes in the coalition formation game. In the online Appendix we show that if (i) firms can commit not to offer discounts, (ii) firms simultaneously choose their partner and (iii) a mutual agreement is needed to establish a partnership, then the outcome is the same. We also show that if, instead of modeling the coalition formation stage explicitly, one looks for stable coalitions, e.g. as in Horn and Persson’s (2001) definition of core, then bilateral bundling by firms with the same quality is (again) the only stable outcome in this context.

**IV. Discussion**

In this section, we (i) discuss the different effects that are relevant when setting the discounts; (ii) explore consequences in terms of welfare resulting from the discounts introduction; and (iii) discuss the robustness of our results in terms of welfare.

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18 In the case of unilateral bundling involving producer $X$ and producer $y$, the high quality firm $X$ will increase its payoff from $\pi_X = 0.87\sigma$ to $\pi_X = 0.89\sigma$ by breaking away from the partnership. In the case of bilateral bundling where partner firms offer different quality levels, the high quality firm will increase its payoff from $\pi_X = 0.79\sigma$ to $\pi_X = 0.83\sigma$ by removing itself from the partnership $Xy$. 

Discounts

We start by analyzing the introduction of a bundled discount by a single pair of firms. This (unilateral) bundling scenario can be thought of as a special case of the more general situation in which two pairs of partner firms have been created, but where one of these pairs chooses not to offer a discount (or, equivalently, sets its discount level at zero). The reason we chose to address this case first is simply because it has the advantage of isolating the effects of introducing the discount \textit{per se} from the effects resulting from the competition in discounts involving the two pairs of firms.

Having already discussed the impact of the discounts on the equilibrium prices in section 3.1 we now focus on the impact of discounts on the bundling firms’ profit. Consider, for instance, the case of unilateral bundling by the low quality pair of firms. To analyze the effects that result from the introduction of the discount, we will focus on its impact on the profits of one of the two partner firms, say firm $x$, whose profit is $\pi_x = p_x Q_x - \alpha_{xy} \beta_{xy} q_{xy}$. The impact of a marginal increase in the discount level is given by:\[\frac{\partial \pi_x}{\partial \beta_{xy}} = p_x \frac{\partial q_{xy}}{\partial \beta_{xy}} + p_x \frac{\partial q_x}{\partial \beta_{xy}} + (p_x - \alpha_{xy} \beta_{xy}) \frac{\partial q_{xy}}{\partial \beta_{xy}} - \alpha_{xy} \sigma q_{xy}\] (1)

where the derivatives of quantities with respect to the discount include a \textit{direct effect}, when prices are kept constant, but also \textit{strategic effects}, via the equilibrium prices $p_X$, $p_Y$ and $p_y$. The first two terms in (1) are always negative and reflect the reduction in this firm’s sales that do not involve the bundle. This reduction is due to the decrease in the headline prices of the products sold by firms not involved in the bundling scheme, after the increase (or the introduction) of the discount $\sigma \beta_{xy}$. The last two terms in (1), on the other hand, represent the typical volume versus margin trade-off in the sales of the bundle. Increasing the discount will increase the volume regarding the sales of the bundle (positive third term) but will reduce the profit margin in the infra-marginal units (negative fourth term).

As a result of the interplay between these direct and strategic effects, the equilibrium

\[\text{\textsuperscript{19}}\text{The impact on the profit of firm } y \text{ is symmetric.}\]
\[\text{\textsuperscript{20}}\text{Recall that } Q_x = q_{xy} + q_{xY} + q_x.\]
\[\text{\textsuperscript{21}}\text{The overall effect on } \pi_x \text{ via } p_x \text{ is zero because } p_x \text{ is set at a later stage to maximize firm } x \text{’s profit. If the discount level and the prices were set simultaneously, as in most of the previous literature on bundling by a multi-product firm, the \textit{direct effect} would fully characterize the incentives to offer discounts. However, in our case, there is a pre-commitment to a discount and prices are set at a later stage. This implies that, when setting the discount, firms must also consider the impact this discount will have both on the price set by the non-partner firms, the \textit{external strategic effect}, as well as its impact on the price charged by the partner firms themselves, the \textit{internal strategic effect}. This effect would not exist in the case of bundling by a multiproduct firm. In that case, both prices and the discount are set to maximize the same objective, the profit of the low quality firm. Hence, if there was a merger between the partner firms without any impact on the timing of the game, this effect would disappear.}\]
discount levels and other endogenous variables regarding the possible cases of (unilateral) bundling by a single pair of firms are presented in Table 1.\footnote{Please refer to the online Appendix for the derivation of the results summarized in this Table.}

<table>
<thead>
<tr>
<th>Unilateral bundling by:</th>
<th>firms $x$ and $y$</th>
<th>firms $X$ and $Y$</th>
<th>firms $X$ and $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
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<td>0.28</td>
<td>0.17</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>$\frac{1}{2}$</td>
<td>1.0</td>
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<table>
<thead>
<tr>
<th>Firm</th>
<th>Price</th>
<th>Profit</th>
<th>Price</th>
<th>Profit</th>
<th>Price</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>0.66$\sigma$</td>
<td>0.88$\sigma$</td>
<td>0.74$\sigma$</td>
<td>0.91$\sigma$</td>
<td>0.70$\sigma$</td>
<td>0.87$\sigma$</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.66$\sigma$</td>
<td>0.88$\sigma$</td>
<td>0.74$\sigma$</td>
<td>0.91$\sigma$</td>
<td>0.65$\sigma$</td>
<td>0.83$\sigma$</td>
</tr>
<tr>
<td>$x$</td>
<td>0.35$\sigma$</td>
<td>0.22$\sigma$</td>
<td>0.32$\sigma$</td>
<td>0.20$\sigma$</td>
<td>0.34$\sigma$</td>
<td>0.23$\sigma$</td>
</tr>
<tr>
<td>$y$</td>
<td>0.35$\sigma$</td>
<td>0.22$\sigma$</td>
<td>0.32$\sigma$</td>
<td>0.20$\sigma$</td>
<td>0.35$\sigma$</td>
<td>0.25$\sigma$</td>
</tr>
</tbody>
</table>

$CS$  | $2.44S - 0.44S$ | $2.46S - 0.46S$ | $2.43S - 0.43S$ |
$SS$  | $1.77S + 0.23s$ | $1.76S + 0.24s$ | $1.75S + 0.24s$ |

Table 1: Equilibrium values for the cases of bundling by a single pair of firms

Now consider the case in which the two pairs of firms offer a bundled discount. When this is the case, equilibrium discounts will increase which is a consequence of the fact that discounts are strategic complements in our setting. Clearly, the existence of a discount offered by the pair of high quality firms will affect the magnitude of the effects described above for the low quality producers. First, an increase in the discount offered by the rival pair of firms will, all other things being equal, decrease the headline prices of the low quality producers, i.e. $p_x$ will decrease with the introduction of a discount by the high quality pair of producers. This implies that the revenue loss resulting from an increase in the discount $\beta_{xy}$, represented by the first two terms in eq. (1), will be smaller. This is reinforced by the fact that the loss in the number of consumers associated with the first two terms of eq. (1) is also smaller, because the headline prices of the rival pair of high quality firms will not decrease as much with $\beta_{xy}$ when they also have a discount. With respect to the third and fourth terms in eq. (1), the existence of a rival discount has an ambiguous effect. Considering the fourth term, for instance, it may lead to a decrease in the number of infra-marginal consumers buying the low quality bundle ($q_{xy}$), implying that the direct cost resulting from increasing discount $\beta_{xy}$ may be lower. But, it may also have the opposite effect. As a result of the increase in the high quality headline prices, the high quality discount may actually increase the number of consumers purchasing the low quality bundle. So, the interplay between the effects resulting from a change in the discount level offered by a pair of firms is affected in a non-trivial way by the existence of a discount
offered by the remaining pair of firms, although in our setting the dominant effects result in discounts being strategic complements.

Regarding the way the discount cost is financed by partner firms, it is interesting to note that in the case of unilateral bundling by firms with different quality, the discount cost is fully supported by the high quality producer. To understand the reasoning behind this result, we will focus on the effects of a change in $\alpha_{XY}$. As mentioned above, the discount $\beta_{XY}$, can be interpreted as a unit cost, partially incurred by each partner firm, for the $q_{XY}$ units which are entitled to the discount. This being the case, an increase in $\alpha_{XY}$ induces an increase in the costs of the high quality producer $X$ and, at the same time, a decrease in the costs of the low quality producer $y$, for each unit of the bundle. These effects on the partner firms’ costs will in turn make $p_X$ increase and $p_y$ decrease, and will also affect the prices of the competing products, $p_x$ and $p_Y$, in a similar way but by a smaller magnitude. As a result, the price differences $p_X - p_x$ and $p_Y - p_y$ will increase with $\alpha_{XY}$. It should be noted, however, that the impact that changing $\alpha_{XY}$ has on the profit of the low quality producer $y$ is substantially larger than the impact the same change in $\alpha_{XY}$ has on the profits of the high quality producer $X$. This happens because $q_{XY}$ represents a larger fraction of the sales of the low quality producer $y$ than of the sales of the high quality producer $X$ (the reason being that the high quality producer will have higher aggregate sales). As a result, $p_Y - p_y$ ends up increasing more than $p_X - p_x$, meaning that the $xY$ pair becomes relatively more expensive than the $XY$ bundle.\(^{23}\) Graphically, this implies that the upward sloping line in Figure 3 shifts both upwards and to the right, but the first effect is stronger, implying that the new upward sloping line is above the initial one. In other words, although the increase in $p_X - p_x$ makes some consumers switch away from the high quality producer of $X$, the higher increase in $p_Y - p_y$ more than compensates for this for those consumers who choose the bundle. Hence, as $\alpha_{XY}$ increases, so does the demand for the bundle, resulting in higher profits for the bundling firms and leading to $\alpha_{XY} = 1$.\(^{24}\) The same effect is present in the case of bilateral bundling.

\(^{23}\)This follows from the fact that $\Delta(p_Y - p_y) > \Delta(p_X - p_x)$ is equivalent to $\Delta p_Y + \Delta p_x > \Delta p_y + \Delta p_X$ and from the fact that the discount level does not change.

\(^{24}\)This contrasts with the case of symmetric partner firms. In this case, bundling firms differ only if they do not share the discount cost equally. Without loss of generality, assume bundling by the low quality pair of firms and consider that $\alpha_{xy} < 1/2$. Then, the low quality producer $x$ incurs lower costs associated with the production of $q_{xy}$ than the low quality producer $y$. This means that it will set lower unit prices and will, as a result, sell an overall higher number of units. An increase in $\alpha_{xy}$ will thus have exactly the same effect as the one described above. However, when $\alpha_{xy}$ exceeds $1/2$ this effect is reversed because firm $x$ is now the firm with higher costs.
## Welfare

In this section we briefly discuss the implications of our results on consumer surplus and on social surplus.

Let us start by addressing the implications for consumer surplus of the equilibrium scenario of bilateral bundling by firms offering products of similar quality levels, i.e. where the two bundles available are \(xy\) and \(XY\). From Lemmas 2 and 3, consumer surplus falls by

\[
\frac{2}{9} (11s - 2S) - (2.47s - 0.47S) \simeq 2.61 \times 10^{-2} \sigma,
\]

when compared to the no discounting benchmark. Table 2 refers to this bilateral bundling scenario and presents the ex-ante and ex-post consumer choices, the normalized changes in price faced by each consumer group, and the effect on quality and on the aggregate consumer surplus for each group of consumers.

<table>
<thead>
<tr>
<th>ex-ante</th>
<th>ex-post</th>
<th>% cons.</th>
<th>(\Delta p_i \times \frac{100}{\sigma})</th>
<th>(\Delta \text{Quality} \times \frac{100}{\sigma})</th>
<th>(\Delta CS \times \frac{100}{\sigma})</th>
<th>(\Delta SS \times \frac{100}{\sigma})</th>
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<td>(xy)</td>
<td>(xy)</td>
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<td>10.11</td>
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<td>-0.32</td>
<td>0</td>
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<tr>
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<td>(XY)</td>
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<td>0</td>
<td>+7.00</td>
<td>0</td>
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<td>(Xy)</td>
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<td>-</td>
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<td>-2.61</td>
<td>-3.45</td>
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**Table 2:** Changes in consumer surplus and social welfare (per type of consumer) resulting from the introduction of bundled discounts by firms with the same quality levels.

Clearly, the discount introduction will affect different groups of consumers in different ways. Consider first the consumers whose choices do not change with the discount introduction. Within this group of consumers, there are different cases to consider that we discuss in turn. Firstly, consumers who purchase one of the bundles \((xy\) or \(XY\)), which now entitles them to a discount, will be better off as the bundle net price decreases upon the discount introduction. Secondly, consumers who purchase a pair of products that does not entitle them to a discount will see the price of both products increase and will be worse-off. Finally, consumers who are
only interested in buying one of the products will also see the corresponding headline price increase and will therefore also be worse-off.

To complicate things further, some consumers will make different choices after the bundle is introduced and may pay a higher price for more quality or a lower price for lower quality than in the benchmark case. All these consumers become better off with the exception of those who only care for one of the products in the bundle. Overall, the aggregate effect is negative.

On a more general level, and also considering the cases presented in Table 1, whenever the pair of high quality firms offers a bundled discount, consumers’ surplus decreases, whereas if this pair does not offer a discount then consumer surplus will increase. In particular, in the case of unilateral bundling by the high quality firms, the price effects are strong and the increase in the headline price of the high quality product is large. This will hurt those consumers who will keep on purchasing the unbundled high quality product to such an extent that it will offset any benefits other consumers accrue. However, it is also possible that bundled discounts increase consumer surplus, as would be the case whenever the pair of high quality firms decides not to offer a bundled discount.

Now consider the effects on social surplus. Making use of Lemma 2 and Lemma 3, if both the high quality firms and the low quality firms offer a bundled discount, then, relative to the no discounting benchmark, social surplus falls by $\frac{2}{9} \left( 8S + s \right) - (1.74S + 0.26s) \simeq 3.45 \times 10^{-2}$. Notice that, in our setting, social surplus depends only on the quality of products purchased by consumers since: (i) prices and discounts are mere transfers between consumers and producers; and (ii) quality is valued by all consumers and produced with no additional costs. Hence, any change that makes consumers switch to pairs of products with more quality (such as switching from $xy$ to $Xy$ or from $Xy$ to $XY$) is welfare enhancing. The introduction of the two bundled discounts by producers of the same quality level will have a positive effect on welfare from those consumers who switch from $xY$, $Xy$ and $xy$ to $XY$ but a negative effect on welfare, which is dominant, from those consumers who switch from $xY$ and $Xy$ to $xy$ and from those who switch from $X$ or $Y$ to $x$ or $y$. Contrary to what happens regarding consumer surplus, social surplus decreases in all scenarios in which bundled discounts are introduced.

It should be noted that the total demand for the high quality firms’ products is not a sufficient statistic for evaluating the impact of bundled discounts on social surplus, simply because quality is not valued equally by all consumers. To illustrate this point, consider the (unilateral) introduction of a bundled discount by the high quality producers, which is the scenario in which the number of high quality sales would be the highest. Initially, the

\footnote{In fact, in the case of (bilateral) bundling by the two pairs of firms with the same quality, there is a decrease in social surplus, as illustrated above, despite the increase in the number of high quality units sold.}
consumer who is indifferent between the high and low quality versions of a given good takes
the same value both for product \(X\) and product \(Y\). In particular, for all types of consumers,
the indifferent consumer has a valuation for quality of \(\theta^k_j = 1/3\) (see Figure 1). When the price
of the high quality bundle decreases, the headline prices of the high quality producers increase
and the headline prices of the low quality producers decrease and the indifferent consumers
will change in opposite directions. More consumers who care for both products will purchase
higher quality products, but more consumers who care for only one of the available products
will purchase lower quality products. So, despite the possible increase in the aggregate number
of high quality sales, welfare may decrease because the consumers who will switch from the low
to the high quality products have a lower valuation for quality than those consumers who will
switch from the high to the low quality version of a given product.

**Robustness**

From the discussion above, one may be tempted to conclude that since reductions in both
consumer welfare and social surplus result from the impact of the bundled discounts on con-
sumers who only care for one of the available products, if these consumers were removed from
the model our welfare results would be reversed. However, we now show that this is not the
case. Assuming both pairs of producers of the same quality offer bundled discounts which they
fund equally, \(\alpha_{XY} = \alpha_{xy} = 1/2\), we show in the online Appendix that equilibrium discounts
are \(\beta_{XY} = 0.33\) and \(\beta_{xy} = 7.93 \times 10^{-2}\) (again, a corner solution in the case of \(\beta_{XY}\)), and that
the equilibrium headline prices are \(p_X = p_Y = 0.83\sigma\) and \(p_x = p_y = 0.334\sigma\). Table 3 presents
the impact of the introduction of these discounts on the different groups of consumers. As this
Table demonstrates, consumers whose surplus decreases upon the introduction of the discounts
are those who, before the discount introduction, were buying a pair of products with different
quality. This is true regardless of whether these consumers decide to change their consumption
pattern after the discounts are introduced or not. In addition, the welfare loss faced by these
consumers more than offsets any reduction in the bundle prices (and the associated welfare
gain) of the remaining groups of consumers who were buying a pair of products of the same
quality before the introduction of the discounts and keep on buying a bundle of products of
the same quality after the discounts are introduced, thereby benefiting from a decrease in the
price paid for the chosen bundle. As a result, overall social and consumer surpluses decrease
in this bilateral bundling scenario, confirming the qualitative result of the model presented in
the previous sections.
Table 3: Consumer surplus and welfare per type of consumer in the absence of consumers who value only one of the goods.

In the online Appendix we also show that the results do not depend on the specific definition of small discounts that we have used and that the results in terms of consumer and social surplus would not be qualitatively different in the case of large discounts, i.e. discounts that induce all consumers to purchase one of the two bundles.

V. Conclusion

Bundled discounts provide purchasers with the opportunity to pay less for a bundle of products than if they purchased each item in the package separately at the corresponding headline price. Despite the fact that this business practice is ubiquitous in today’s society, economic theory has devoted very little attention to this issue until recently.

This paper studies the consequences of bundled discounts in an oligopoly setting where pairs of firms sell vertically differentiated and otherwise unrelated products. More specifically, we investigate the effects induced by the introduction of bundled discounts by firms with similar quality levels or by firms supplying goods of different quality levels.

Some interesting results are obtained regarding the competitive effects of bundled discounts, shedding some light on the potential antitrust risks associated with this particular type of discount arrangement. First, whenever bundled discounts are offered by (one or two pairs of) firms, then, relative to the no-discounting benchmark case, the headline prices of the bundling firms always increase whereas the headline prices of the firms not involved in discounting (if
any) decrease in most cases. Second, bundled discounts are not free of antitrust concerns: bundled discounts may induce a decrease in consumer surplus and in social surplus. Third, when firms make simultaneous or sequential decisions regarding their possible participation in a bundled discount scheme, the robust outcome is that of bilateral bundling by firms with similar quality and the introduction of the discounts is, in equilibrium, profitable for some firms. In addition, consumer surplus is at the lowest level of all the cases considered.

Armstrong (2012) highlights that “when firms offer independent products (i.e. the purchase of one product has no impact on a consumer’s willingness to pay for a second product), Pareto improvements are possible if firms coordinate their pricing policies. This is achieved by means of bundling so that a consumer is offered a discount if she buys both products. Such bundling discounts, if they can be implemented by separate firms, can improve both profits and consumer surplus.” Our contribution to this discussion is the identification of a context – bundling by vertically differentiated competitors – in which bundling discounts by independent firms, while profitable for some firms, end up harming consumers. Hence, our results differ from the previous literature on bundling discounts by independent firms in two important ways. First, they embody different predictions regarding the induced impact of the adoption of these discounting schemes on both consumers’ and social surplus. Second, the typical prisoner’s dilemma identified in most of the existing literature is absent in our setting, meaning that the practice of mixed bundling is shown to be profitable for some firms.

In concluding, let us point out that our results are based on two important assumptions. Firstly, we assume that the market is always covered and that consumers have unit demands. Market demand is, therefore, inelastic and discounts do not bring additional consumers to the market or increase the quantity demanded by each individual consumer. The role of discounts in our setting is, therefore, to steal business from rivals. If the model was extended to include elastic demands, there would be two additional effects resulting from the discount introduction. Consumers with preferences that lead them to select a pair of products that entitles them to a discount (and who would, thus, benefit from a lower net bundle price) would be better off either by purchasing for the first time (entering the market) or by purchasing a larger number of units. However, consumers who only care for one of the available products or consumers whose preferences make them purchase products that do not entitle them to a discount may either decide not to purchase at all or to purchase a smaller quantity when they observe an increase in the headline prices upon the introduction of the discount. While this approach has generality on its side, it is considerably more difficult to derive results. On top of that, our conjecture is that even though the results under price elastic demand may change quantitatively, the qualitative
aspects of our discussion regarding why consumer and social surplus may decrease with the discount introduction would still be present. Secondly, we assume that the valuations for the quality of both products are not correlated. If there was a positive correlation in the valuation for quality of both products, the proportion of consumers choosing products of different quality would be smaller. As these are the consumers who are more likely to be adversely affected by the introduction of the discounts, our conjecture is that the consequences for consumers would be less grim. In any case, our results suggest that competition authorities should scrutinize in detail the use of bundled discounts in industries where independent suppliers offer vertically differentiated products, despite the fact that these types of agreements have so far received little attention.

References


