Welfare Decreasing Endogenous Mergers between Producers of Complementary Goods*

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Abstract

This paper investigates the competitive effects of mergers involving producers of complementary goods, which are usually considered to be welfare increasing, in a setting where: (i) consumers need to purchase two components to make up a system; and (ii) there is competition between two vertically differentiated producers of one of the components whereas the second (must-have) component is monopolized. We find that the (privately profitable) merger involving the low quality producer of one component and the monopolist producer of the other component may decrease both consumers' surplus and social welfare for parameter values such that this merger can endogenously occur.

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1 Introduction

Mergers between producers of complements are usually considered to be welfare increasing. With complementary goods, a reduction in the price of one of them increases the demand for its complement(s). Typically, when a merger involves the producers of complements that form a system, this effect is internalized by insiders and their prices end up being reduced. For example, when analyzing the merger involving General Electric (GE) and Honeywell,[1] Motta (2004) defended that "[a]nother reason why we should expect prices to move downwards, although it is impossible to quantify this effect without having more detailed information, is the fact that GE and Honeywell sell products that are complementary. Like for vertically related products (where this corresponds to the elimination of double marginalization), a firm that sells two or more complementary products will sell them at lower prices than if the same products were sold by independent firms" (p. 388).[2]

This type of merger has previously been analyzed in several different settings. Gaudet and Salant (1992) analyse the case in which each of several complements that make up a composite good (or a system) is produced by a different monopolist, whereas Economides and Salop (1992) study the case in which there are two differentiated brands of each of two components. An intermediate case, with competition in the production of one complement and monopoly in the production of the other(s) has been considered by Kim and Shin (2002). In all of these cases, the merger between producers of complements was shown to decrease prices and to increase consumers' welfare. More recently, however, Masson et al. (2014), Alvisi et al. (2011), and Choi (2008) have established that these mergers may have an ambiguous effect on both consumers and social welfare.

The present paper contributes to this debate. More specifically, we investigate the competitive effects of endogenous mergers involving producers of two complementary goods that are needed to make up a system, in a setting where there is competition between two vertically differentiated producers of the first component while the second must-have component is monopolized. A case in point is the 2011 acquisition of Skype by Microsoft. With its over 85% market share, Microsoft is in a position close to monopoly in the market for operating systems, the must-have component in our setting. The complementary good may then be any type of software application such as video, chat and voice (over IP, VoIP) call services. The latter are offered by several companies under brands

[1] *General Electric/Honeywell*, case COMP/M.2220.

[2] Along these lines, the European Commission argued that after the merger the resulting new entity would be able to make attractive package (or bundled) offers to airlines and to airframe manufacturers: "The complementary nature of the GE and Honeywell product offerings coupled with their respective existing market positions will give the merged entity the ability and the economically rational incentive to engage in bundled offers or cross-subsidisation across product sales to both categories of customers" (*General Electric/Honeywell*, para. 349).

like Skype, WhatsApp, Viber, etc., which arguably sell services of different quality levels. VoIP quality, for instance, can be measured by noise, distortion, or gaps in speech.

Another relevant example is the one regarding bundles in the pharmaceutical industry. As Armstrong (2013), highlights, “Pharmaceuticals are sometimes used in isolation and sometimes as part of an approved ‘cocktail’ with one or more drugs supplied by other firms. Drugs companies have the ability to set different prices depending on whether the drug is used on a stand-alone basis or in a cocktail”. (p.2) In this industry, branded products are usually protected by a patent and, thus, offered by a monopolist while the patent is active. However, generics enter the market if a patent-holder waives its rights or, more often, when the patent that protects a branded product expires. In addition, branded pharmaceuticals may be vertically differentiated toward generics, at least in terms of the way consumers perceive quality. Hence, if a given prescribed cocktail of drugs is composed of a combination of (say) two drugs, one of which is offered only by a branded firm while the other is offered by both a branded producer and generic producers, our setting applies.[3] These two motivating examples also illustrate that there is some ground for the assumption that the second component is monopolized. This may arise from legal barriers to entry (e.g. patents), natural entry barriers, or market based barriers to entry (e.g. network effects).

Within this theoretical structure, we find that a merger that involves the low quality producer of the first component and the monopolist producer of the second component may hurt consumers' surplus (as well as social welfare). The underlying intuition is the following. After the merger, the merged entity will internalize pricing externalities resulting from the complementarity of its controlled components and, as a result, the overall price of the merged entity (fully controlled) system is reduced below the level that the two firms involved in the merger were setting when acting independently. At the same time, however, the merged entity will increase the price of the stand-alone (must-have) second component, relative to its level before the merger. This will then have a negative impact on the sales of the system which is *not* fully controlled by the merged entity (composed of a competitor's component and the must-have component sold by the merged entity), strategically making it less attractive to consumers. Consequently, the overall impact of the merger on consumers' surplus is *a priori* ambiguous. On the one hand, some consumers who were not buying at all before the merger, start buying due to the decrease in the overall price of the merged entity fully controlled system while those that keep on purchasing this system benefit from a lower

[3] Note that the presence of generics is not necessary. Different patented therapeutic-substitute drugs can be vertically differentiated and used in conjunction with a third product that must be present. The existence of products of this sort is common, for example, in HIV treatment.

price. On the other hand, (i) some consumers previously purchasing the highest quality system will keep on doing it but paying a higher price, and (ii) other consumers will switch to the lower quality system (the one fully controlled by the merged entity) to benefit from its reduced price. In case (i) consumers' welfare is negatively affected and the same may happen in case (ii).[4] We find that under some circumstances effects (i) and (ii) more than compensate the positive effects, implying that the merger will be consumers'-welfare-reducing. On the other hand, we also conclude that this vertical merger's impact on aggregate profits is such that the industry prefers it to a strictly horizontal alternative merger involving the vertically differentiated producers of the first component needed to make up a system. As for the merger that involves the high quality producer of the first component and the monopolist producer of the second component, it does not have any effects on the market outcome because the marginal consumer for the high quality system does not consider the price of the must-have component. This consumer is indifferent between the high and low quality systems and, thus, will always have to purchase the must-have component. Likewise, the marginal consumer for the stand alone component is indifferent between purchasing the low quality system and not purchasing at all, and therefore does not consider the price of the high quality product in his decision.

Given these three alternative merger possibilities, we investigate whether the merger that involves the low quality producer of the first component and the monopolist producer of the second component can arise endogenously when it has a negative impact on consumers' surplus and social welfare. In order to do so, we present two alternative merger formation protocols: i) a standard sequential offers coalition formation game and ii) a model of mergers as a first-price auction game, and conclude that the merger in question can endogenously occur. The reason depends on the specific merger game adopted. In the first game, the key aspect is that, as described above, the mergers that include the low quality producer of the first component are more profitable for the insiders than the one that does not. In the second game, the two-firm merger in question occurs when it is the one that generates the highest industry-wide profits.

Both Alvisi et al. (2011) and Masson et al. (2014) also assume vertically differentiated components and show that some mergers between producers of complements may reduce consumers' surplus. In Alvisi et al. (2011), this happens when the mergers involve producers with the same (high or low) quality level. In their framework, however, although the goods are complements in the

[4] Tarola and Vergari (2015) analyse the related case of a merger between producers of vertically differentiated asymmetric complements, pointing out a "new positive quality effect, due to an increase in the average quality of the goods on sale." (p. 72)

sense that they are needed to make up a system, the cross-price elasticity of demand for the goods involved in the merger is positive, i.e. despite technical complementarity, consumers perceive the two components as substitute products, implying that prices will increase after a merger involving similar quality producers.[5] To better understand the intuition behind their result, consider the following theoretical framework. Suppose consumers need to purchase two components to make up a system. There are two alternatives for the first component, 1 and 2, and two alternatives for the second one, 3 and 4. Suppose also that the high quality versions are versions 1 and 3 and the low quality versions are 2 and 4. Assuming that consumers differ only with respect to their valuation for quality, the demand for each good depends on the consumers who are indifferent between the different alternatives. If the ranking of alternative systems, from the least to the most valued, is $2+4$, $2+3$, $1+4$, $1+3$ and assuming a covered market, the consumer who is indifferent between purchasing component 1 or 2 will depend on the price of systems $2+3$ and $1+4$. Now, if the price of component 3 increases, this will induce an increase in the demand for system $1+4$, meaning that although components 1 and 3 are complements, the price of component 3 positively affects the demand for component 1.[6] Likewise, the demand for system $2+3$ is positively affected by the price of component 1, which enters the competing system $1+4$.[7] The demand for component 3 is, therefore, positively affected by the price of the complement component 1 and vice-versa. In contrast, in our setting the complements sold by the firm that results from the merger do affect each others' demand functions as complements, rather than as substitutes, as in Alvisi et al. (2011).[8]

Masson et al. (2014), on the other hand, consider a setting in which all components are also fully compatible and are either high or low quality. In addition, the price-setting dominant producers sell the high quality version of each of the two components, and compete against many firms selling the low-quality versions of the two components with no individual market power (and, therefore, charging prices at the marginal cost level). They also consider three cases regarding consumers' willingness to pay for the quality of the two components: (1) perfectly positively correlated, (2) perfectly negatively correlated, and (3) imperfectly negatively correlated. While in case (1) the

[5] Increasing the price of one of them increases the demand for the other one and, when this effect is internalized due to the merger, prices will increase.

[6] The increase in the price of component 3 also makes some consumers switch from system $1+3$ to $1+4$. However, this does not affect the demand for component 1.

[7] The demand for system $1+3$ is not affected by the price of component 1 because it cancels out in the comparison between the prices of systems $1+3$ and $1+4$.

[8] Alvisi and Carbonara (2013) consider a similar structure in which the producer of one component is a monopolist and there is competition in the production of the other. The equilibrium welfare is compared to the one obtained in the cases of an integrated monopoly or a complementary monopoly. Note that the merger that leads to the former involves producers of substitutes and of complements, whereas the merger that leads to the latter is a purely horizontal merger. We, instead, focus on a merger between producers of complements.

merger involving the producers of the two high-quality components has no effect on consumers' surplus, in cases (2) and (3) this merger is shown to decrease consumers' surplus. In particular, in case (3) the integrated merged firm producing the high quality versions of both components is shown to maximize profits by engaging in mixed bundling, thereby setting prices for both the high-quality system and each of its individual components. Consumers are affected by the merger in two different ways. Those consumers who keep on buying the same system are affected only through price, whereas those who switch to the merged entity system are also affected by a change in quality. Overall, and as already mentioned, consumers' surplus is shown to reduce through post-merger mixed bundling.

Even though the paper by Masson et al. (2014) is closely related to ours, in terms of both assumptions and results, there are several important differences that should be highlighted. In terms of assumptions, the main differences are the following. First, while we model vertical product differentiation and consumers' preferences along the lines proposed by Gabszewicz and Thisse (1979), whereby consumers care only about the value of the system without any need to specify the value of each component, Masson et al. (2014), in the most relevant case for our paper, model consumers' preferences for the two components as imperfectly negatively correlated. Second, their paper assumes a high-quality and a low-quality version of each of the two components needed to make up the system. The present paper, however, assumes that one of the components, the must-have component, has a single version. Third, even though the merger discussed is a profitable one, Masson et al. (2014) assume that the low-quality outsiders play a very passive role and do not strategically react to the merger. In their setting, on the low-quality version (of both components) there is pure Bertrand competition, which implies that outsider firms exercise no market power and, therefore, both before and after the merger, price at the marginal cost level. The fact that the two low quality components are valued and priced at zero implies that the consumer's decision in this model is formally equivalent to deciding to buy either one or two (not necessarilly complementary) products with additive valuations. If a consumer's decision is to buy the two products he does purchase a "system" but if the decision is to buy only one, the "system" can then be completed by freely obtaining the low quality component.[9] In contrast, in the present paper, outsider firms do play an active role and react to mergers by strategically adjusting their price. In particular,

[9] If both products have the same value for all consumers (perfectly positively correlated valuations), the profit of the merged firm is the same regardless of whether it decides to bundle or not the two components. If there are negatively perfectly correlated preferences, the aggregate value of the two products is the same for all consumers, which implies that the merged firm can fully extract their willingness to pay for a "bundle" with both products, obtaining higher profits than when selling the two products separately. Thus, in these two cases, there is no mixed bundling.

we show that after a merger that involves the low quality producer of the first component and the monopolist producer of the second component, the outsider firm (the one producing the high-quality first component) will decrease its price so as to mitigate the effect of the increase in the price of the stand-alone (must-have) second component on the sales of the high-quality system (in which the outsider's product is present). In light of these differences, Masson et al. (2014) may, in some sense, have assumed more structure than is necessary to obtain their result about the likely negative welfare effects of mergers between complementary goods.

In terms of results, the differences between the two papers are threefold. First, there are significant differences with respect to the consumers' purchasing decisions after the merger, as will be described later on. Second, Masson et al. (2014) obtain a lower consumer welfare by assuming a merger between the two high-quality component producers, while the present paper shows that a merger that involves the producer of a low-quality component can also decrease welfare. Third, in Masson et al. (2014) only one type of merger may be profitable, whereas in our case there are several alternative mergers to be considered. Therefore, we also establish the circumstances under which the welfare decreasing merger may arise endogenously. This also contrasts with Choi (2008), who extends Economides and Salop (1992) to allow for mixed bundling by the merged firm. Choi (2008) shows that a "conglomerate" merger can harm social and consumers' surplus when the degree of substitutability between competing systems is high. However, the merger decision is not endogenous and the model does not consider vertical differentiation.

The remainder of the paper is organized as follows. Section 2 presents the model, characterizes the pre-merger equilibrium and investigates the effects resulting from a two-firm merger involving the low quality producer of the first component and the monopolist producer of the second one. Section 3 endogenizes merger decisions and, finally, Section 4 offers some concluding comments. All proofs of the formal results presented in the text are relegated to the Appendix.

2 The Model

Consumers purchase a composite good or a system, made up of two components. One of the components is vertically differentiated and sold by two firms (firms 1 and 2) that compete with one another, while the other must-have component is sold exclusively by firm 3. Firm 1 is assumed to be the high quality producer of the first component and faces a unit cost C, whereas firm 2, the low quality producer, faces no costs. Consumers choose between systems $1+3$ or $2+3$ or not purchasing at all. Prices are set simultaneously.

We further assume that N consumers value quality differently with s, uniformly distributed in $[0,1]$, measuring consumer valuation for quality. Without loss of generality, we normalize N to 1. The net valuation of system $i+3$, with $i=1,2$, is therefore given by

$$sv_i - p_i - p_3,$$

where v_i is the valuation for the system composed of firm i and firm 3's products, with $v_1 > v_2$, and p_i and p_3 are the corresponding unit prices. Not purchasing the system results in zero utility for the consumer. With respect to the model parameters, we normalize $v = v_2/v_1$ and $c = C/v_1$.

A consumer prefers system $1+3$ to system $2+3$ if and only if

$$sv_1 - p_1 - p_3 > sv_2 - p_2 - p_3 \Leftrightarrow s > s_{1,2} := \frac{p_1 - p_2}{v_1 - v_2}$$

and she prefers buying system $i+3$ to making no purchase if and only if

$$sv_i - p_i - p_3 > 0 \Leftrightarrow s > s_{i,0} := \frac{p_i + p_3}{v_i}.$$

The following lemma presents the demand for each component.

Lemma 1 *(a) If $s_{1,2} > s_{2,0}$ and $s_{1,2} < 1$, the demand for the components is given by:*

$$\begin{aligned} D_1(p_1,p_2,p_3) &= 1 - \frac{p_1 - p_2}{v_1 - v_2} \\ D_2(p_1,p_2,p_3) &= \frac{p_1 - p_2}{v_1 - v_2} - \frac{p_2 + p_3}{v_2} \\ D_3(p_1,p_2,p_3) &= 1 - \frac{p_2 + p_3}{v_2} \end{aligned}$$

(b) If $s_{1,2} > s_{2,0}$ and $s_{1,2} > 1$, the demand for the components is given by:

$$\begin{aligned} D_1(p_1,p_2,p_3) &= 0 \\ D_2(p_1,p_2,p_3) = D_3(p_1,p_2,p_3) &= \max\left\{1 - \frac{p_2 + p_3}{v_2}, 0\right\} \end{aligned}$$

(c) *If* $s_{1,2} < s_{2,0}$*, the demand for the components is given by*

$$D_1(p_1, p_2, p_3) = D_3(p_1, p_2, p_3) = \max\left\{1 - \frac{p_1 + p_3}{v_1}, 0\right\}$$

$$D_2(p_1, p_2, p_3) = 0.$$

Note that in the first case, D_1 (resp. D_3) does not depend on p_3 (resp. p_1). This happens because the marginal consumer for firm 1 is the one indifferent between system $1+3$ and system $2+3$ and the marginal consumer for firm 3 is the one indifferent between system $2+3$ and making no purchase. Hence, the former's decision does not involve p_3 (because it is buying component 3 in any case) and the latter's does not involve p_1. Moreover, $\partial D_2/\partial p_3 = \partial D_3/\partial p_2 = -1/v_2 < 0$: firms 2 and 3 sell complementary goods.

In the second case, prices are such that system $1+3$ would only be preferable to system $2+3$ for inadmissibly high values of the quality valuation parameter. Thus, there is no demand for component 1.

It should also be highlighted that in the third case the demand for component 2 is zero: for those consumers who prefer system $2+3$ to system $1+3$, buying system $2+3$ would result in a negative surplus. Hence, either system $1+3$ is bought or neither.

With respect to the model parameters, we assume the following throughout the paper:

Assumption 1: *Let* $v \in (0,1)$ *and* $c \in [c^-, c^+]$*, with* $c^- := \frac{1-v}{1+2v}$ *and* $c^+ := 1 - v$.

Recall that $v_1 > v_2$, i.e. component 1 has higher quality than component 2. Thresholds c^- and c^+ in Assumption 1 are functions of v that ensure, respectively, that all firms face a positive demand in the pre-merger equilibrium and that the additional valuation corresponding to a system involving the higher quality component exceeds the increase in its associated cost. In particular: (i) the upper bound on the cost of the high quality component, $c < c^+$, is needed to ensure that after a merger between the producers of (complementary) components 2 and 3 there is an equilibrium with positive sales of component 1; and (ii) the lower bound on the cost of the high quality component, $c > c^-$, is needed to ensure that after a merger between the producers of (substitute) components 1 and 2 there is no equilibrium with no sales of component 2. This is the simplest setting to obtain our main results and the corresponding intuitions.

2.1 Pre-merger equilibrium

We start by presenting the equilibrium prices regarding the benchmark no-merger scenario.[10]

Lemma 2 *Let Assumption* 1 *hold. The pre-merger equilibrium prices are:*

$$
\begin{aligned}
p_1^{\emptyset} &= \frac{c\,(v+3)+3\,(1-v)}{6}v_1 \\
p_2^{\emptyset} &= \frac{vc}{3}v_1 < p_1^{\emptyset} \\
p_3^{\emptyset} &= \frac{v\,(3-c)}{6}v_1.
\end{aligned}
$$

The price of firm 1 (that of the higher quality component) increases with its marginal cost c and the price of firm 2 (that of the lower quality component) also increases with c but by a smaller magnitude. The price of firm 3 (a complement to firm 1 and 2's products) decreases with c. Regarding the differentiation parameter v, p_1 decreases with the quality of the substitute sold by firm 2, whereas p_2 increases with its own quality. Finally, p_3 increases with the "average" quality of the systems due to the higher demand.[11]

2.2 The effects of the 2+3 merger

In this Section we investigate the competitive effects of a two-firm merger involving the low quality producer of the first component and the monopolist producer of the must-have (second) component.

After the merger the merged firm sets the price of both the 2+3 system, P_3, and the stand-alone price of the component for which there is no competition, p_3. We are assuming that it is illegal to tie the sale of this component with the other one, which would lead to the exclusion of the remaining independent firm, firm 1. It is straightforward to check that such exclusion would lead to lower consumers' surplus and welfare.

We further assume no cost savings from the merger and also that the merger does not change the quality of both systems, that is, all effects result solely from market interaction.

[10] Naturally there are other equilibria in which the three components are sold at high prices (say, all above v_1). Given that the price(s) of the other component is (are) so high that no consumer will buy any system, there is no profitable deviation. However, we do not consider these equilibria as they are Pareto dominated by the one presented in Lemma 1. The same applies to the post-merger equilibria discussed below.

[11] When v is high (close to 1), the lower quality component is almost as good as the higher quality component. So, the "average" quality of the available systems is high, which leads to more demand for both.

A consumer prefers system $1+3$ to system $2+3$ if and only if

$$sv_1 - p_1 - p_3 > sv_2 - P_3 \Leftrightarrow s > \frac{p_1 + p_3 - P_3}{v_1 - v_2}$$

and she prefers system $2+3$ to no purchase if and only if

$$sv_2 - P_3 > 0 \Leftrightarrow s > \frac{P_3}{v_2}.$$

Defining, in this case, $p_2 = P_3 - p_3$, the resulting demand functions are as in Lemma 1.

Now, Lemma 3 presents the post-merger equilibrium prices.

Lemma 3 *Let Assumption* 1 *hold. After the* $2+3$ *merger, the equilibrium prices are:*

$$\begin{aligned} P_3^{2+3} &= \frac{v}{2} v_1 \\ p_1^{2+3} &= \frac{1 - v + 2c}{3} v_1 \\ p_3^{2+3} &= \frac{v + 2\,(1-c)}{6} v_1. \end{aligned}$$

Combining Lemmas 2 and 3, the 2+3 merger decreases the price of system 2+3 and increases the price of system 1+3. In terms of component prices, this merger lowers the price of component 1 as well as the implicit price of component 2 (measured by $P_3 - p_3$), while increasing that of component 3. Before the merger, several price effects were not internalized: A reduction in p_2 increases the demand for component 3 because it brings new consumers to buy system 2+3. Likewise, a reduction on p_3 has a similar effect in the demand for component 2. The internalization of these effects alone would lead to a lower p_2 and p_3 after the $2+3$ merger. However, there is an additional effect. Before the merger, the price of component 3 was irrelevant for consumers when deciding to buy system $1+3$ or $2+3$. In other words, this price did not affect the consumer who is indifferent between the low and the high-quality system. After the merger, the pricing decisions by firm $2+3$ can affect this choice. Because the merged firm can keep $P_3 = p_2 + p_3$ constant while increasing p_3 (by lowering p_2 accordingly), it is now able to increase the relative price of the competing system, thereby inducing some consumers to switch from system $1+3$ to system $2+3$. This will not affect the sales of component 3 but will increase those of component 2.

Thus, the merged firm strategically uses its two prices as follows. By increasing p_3, the price of

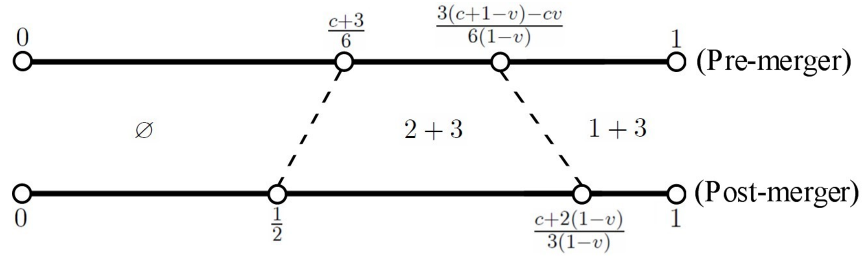

Figure 1: Pre- and post-merger consumer choices.

the monopolized component 3, it decreases the demand for the system that it does not fully own, system $1+3$. However, it offsets this increase in p_3 by reducing the implicit price p_2, the price of the component that faces competition from firm 1. Hence, after the $2+3$ merger, p_2 falls as does the price of its substitute, p_1. However, the price of the must-have component 3 rises.

As a result of these price changes, the $2+3$ merger will affect consumers' decisions in a way that is summarized in the following Table.

	After merger (1)	Before merger (2)	(1) − (2)
% buy $\varnothing$	$\frac{1}{2}$	$\frac{c+3}{6}$	$-\frac{1}{6}c<0$
% buy $2+3$	$\frac{2c+1-v}{6(1-v)}$	$\frac{c}{3(1-v)}$	$\frac{1}{6}>0$
% buy $1+3$	$\frac{1-c-v}{3(1-v)}$	$\frac{3(1-v)-c(3-v)}{6(1-v)}$	$-\frac{1}{6}\left(1-c\right)<0$

Table 1: Distribution of pre- and post-merger consumer choices.

Figure 1 illustrates pre- and post-merger consumer choices. The number of consumers who purchase any system increases with the merger, which means that those consumers who were not buying before the merger and now buy the merged entity fully controlled system will be better-off. However, part of the increase in the sales of this merged entity lower quality system, $2+3$, is obtained at the expense of the sales of the higher quality one, $1+3$, which may imply a negative effect on consumers' surplus. For those consumers who continue purchasing system $2+3$ (resp. $1+3$) consumer surplus will increase (resp. decrease).

The following table presents the change in consumers' surplus, insiders' profits, and welfare for the different groups of consumers.[12] Some consumers do not change their purchasing decision with the merger (these are denoted by $\varnothing \to \varnothing$, $2+3 \to 2+3$, $1+3 \to 1+3$). Others choose differently

[12] All values are multiplied by $72/v_1$.

after the merger (as indicated by $\varnothing \to 2+3$, $1+3 \to 2+3$).

s	$\left[0, \frac{1}{2}\right]$	$\left[\frac{1}{2}, \frac{c+3}{6}\right]$	$\left[\frac{c+3}{6}, \frac{3(c+1-v)-cv}{6(1-v)}\right]$	$\left[\frac{3(c+1-v)-cv}{6(1-v)}, \frac{c+2(1-v)}{3(1-v)}\right]$	$\left[\frac{c+2(1-v)}{3(1-v)}, 1\right]$
	$\varnothing \to \varnothing$	$\varnothing \to 2+3$	$2+3 \to 2+3$	$1+3 \to 2+3$	$1+3 \to 1+3$
ΔCS	0	$c^2 v$	$\frac{4c^2 v}{1-v}$	$-\frac{1-c-v-cv}{(1-c)^{-1}} \gtrless 0$	$-\frac{4(1-c-v)^2}{1-v}$
ΔW	0	$cv\,(c+6)$	0	$-\frac{7(1-c-v)-cv}{(1-c)^{-1}} \gtrless 0$	0
$\Delta(\pi_2+\pi_3)$	0	$6cv$	$-\frac{4c^2 v}{1-v}$	$2\,(1-c)\,vc$	$\frac{4(cv+2(1-c-v))}{(1-v)(1-c-v)^{-1}}$

Table 2: Changes in consumers' surplus, welfare, and insiders' profits as function of s

An interesting possibility is then the fact that consumers' surplus and welfare may decrease with a merger between producers of complements. Proposition 1, below, discusses the effects of the merger on consumers' surplus, profits, and welfare.

Proposition 1 *Let Assumption* 1 *hold.* (*i*) *The* 2+3 *merger is always profitable for the insiders and decreases the profit of the outsider, firm* 1. (*ii*) *Consumers' surplus, industry profits, and welfare increase with the* $2+3$ *merger if and only if* $c > 1 - \sqrt{v}$.

Figure 2 illustrates this result. The two black curves depict the admissible interval for c that is considered in Assumption 1. Below the red curve (that represents $1 - \sqrt{v}$), the merger between producers of complements decreases consumers' surplus, industry profits, and welfare.

Hence, when consumers' surplus decreases with the $2+3$ merger, i.e. when $c < 1 - \sqrt{v}$, it is because i) those consumers who keep on purchasing system $1+3$ will be worse off due to this system price increase and ii) consumers who change from system 1+3 to (the cheaper but of inferior quality) system $2+3$ will also be worse-off (because $c < 1 - \sqrt{v}$ implies $-(1-c-v-cv)/(1-c)^{-1} < 0$). Thus, a necessary condition for consumer surplus to decrease is that consumers who change from system $1+3$ to system $2+3$ become worse-off.

As for social welfare, the decrease occurs when sufficient consumers change from the higher quality system to the lower quality one.[13] For all those consumers who continue to purchase the

[13] In the merger analyzed in Masson *et al.* (2014) the insiders are the producers of the system with the higher quality. At the pre-merger equilibrium, around 10% of the consumers purchase the high-quality system whereas the

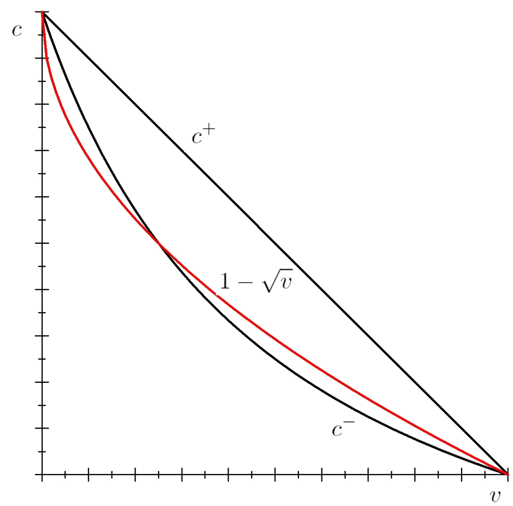

Figure 2: Welfare effects of the $2+3$ merger: Consumers' surplus, industry profits, and welfare increase if and only if $c > 1 - \sqrt{v}$.

same system after the merger, associated welfare does not vary with the merger. The reason is that these consumers purchase exactly the same quantity (one unit of the system) although at a different price. The change in price, therefore, merely affects how surplus is divided between consumers and producers, without affecting the total surplus.

As for insiders' aggregate profits, the merger enables firm $2+3$ to attract consumers who did not purchase before as well as consumers previously buying the high quality system. With respect to the consumers who continue buying the high quality system, the merged firm charges a higher stand-alone price for the must-have component and also obtains a higher profit. The exceptions are the consumers who purchase the low quality system both before and after the merger. The profit obtained from these consumers decreases with the merger as the system price decreases.

The condition presented in Proposition 1, under which the $2+3$ merger decreases consumers' surplus and industry profits, is easier to verify if c and v are both low. When this the case, system $1+3$, when compared to system $2+3$, is less costly to produce and more valued by consumers. As seen above, system $1+3$ loses consumers to system $2+3$ as a result of the $2+3$ merger. Therefore,

remaining 90% purchase a hybrid high-low quality system. Post-merger, the distribution of consumers among system types varies. Consumers buying the high-quality system (resp. hybrid system) represent 65% (resp. 35%). The 35% (resp. 10%) of consumers who always purchase the hybrid system (resp. high-quality system), observe a price increase (resp. decrease) and, thus, a decrease (resp. increase) in their surplus. For the 55% of consumers who change to the high-quality system after the merger, the authors conclude that those consumers made better off are the ones with relatively high valuations for both components. Note that in our setting, with the merger, consumers switch in the opposite direction: from the high-quality system to the low-quality one. Additionally, some consumers start buying the system after the merger takes place, an effect absent in Masson *et al.* (2014), which obviously has a positive impact on consumer surplus and welfare. Nonetheless, the $2+3$ merger may decrease both.

when $c < 1 - \sqrt{v}$, this flow of consumers lowers consumers' surplus and industry profits. As a low v also means that the consumers who start purchasing the $2 + 3$ system after the merger do not benefit much, the negative effects on consumers' surplus and industry profits dominate.

In concluding this section, it should be remarked that even though market shares are often used in merger analysis to predict the likely effects of mergers, in the case under analysis they do not represent a relevant indicator for that purpose. As shown in the Appendix, the *ex-ante* relationship between the consumer shares of firms 1 and 2 is not useful in predicting whether the $2 + 3$ merger will increase or decrease welfare. We formalize this in the following remark:

Remark 1 The $2 + 3$ merger may have negative (or positive) effects on welfare when firm 2 has a larger share than firm 1 or a smaller share than firm 1.[14]

3 Endogenous mergers

We have shown that the $2 + 3$ merger may be detrimental to consumers' welfare, despite the fact that it is a merger between producers of complements. In this section, we discuss whether this specific merger, when detrimental to consumers' welfare, may endogenously occur in equilibrium. To this end, we assume that a coalition formation game occurs before the price competition stage. We start by characterizing the effects of alternative mergers that may arise in our setting and then present the coalition formation stage.

3.1 Alternative mergers

Excluding the three-firm merger, in addition to the $2+3$ merger, there are two other possible (two-firm) mergers to consider: the $1+3$ merger between producers of complements and the $1+2$ merger, a horizontal merger between the two producers of the vertically differentiated component.[15] So, in what follows, we first analyse the effects of these alternative mergers and then characterize which mergers arise endogenously.

Lemma 4 *Let Assumption* 1 *hold.* (*i*) *After the* $1 + 2$ *merger, in (the unique) equilibrium prices*

[14] If instead one considers value market shares (the percentage of firm 1 and firm 2 aggregate revenue received by each firm) firm 1 always has a larger market share than firm 2 before the merger.

[15] Following Horn and Persson (2001), the exclusion of this merger may be justified by "organizational inefficiencies that increase with the absolute size of the firm or with a lack of competition" (p. 312).

are:

$$p_1^{1+2} = \frac{1}{6}\left(3c + 3 - v\right) v_1$$

$$p_2^{1+2} = p_3^{1+2} = \frac{1}{3} v v_1.$$

(ii) *After the* $1+3$ *merger, in equilibrium prices are as in Lemma* 2.

As expected, after a merger involving sellers of substitutes, the prices of components 1 and 2, which are now sold by a monopolist, increase, whereas the price of the complementary good 3 decreases. Moreover, both systems' aggregate prices increase. It is also straightforward to conclude that the $1+3$ merger does not have any consequences on prices, profits, and welfare.[16]

The following auxiliary Lemma, which will be useful later, explains how insiders and the outsider are affected by the mergers.

Lemma 5 *Let Assumption* 1 *hold. Then, i*) *no merger is strictly unprofitable; ii*) *no merger benefits the outsider firm; iii*) *Merger 2+3 is more (privately) profitable than merger 1+2 if and only if* $c < c^*$*; iv*) *The 2+3 merger is more profitable for the industry as a whole than the 1+2 merger.*

[16] The $1+3$ merger has no effects because p_1 does not affect the profit of firm 3 and p_3 does not affect the profit of firm 1. This follows from the demand functions presented in Lemma 1 and from the assumption that there are no cost savings from the merger and also that the merger does not change the quality of the systems.

The expression for c^* is presented in the Appendix.[17] Figure 3 illustrates this new threshold for c.

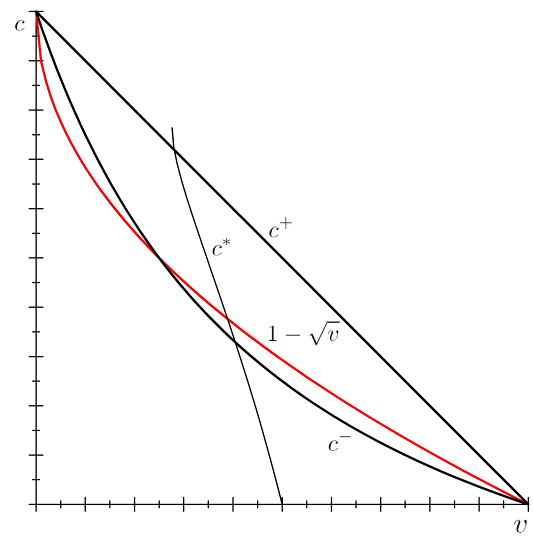

Figure 3: 2+3 vs 1+2 merger profitability:

Merger 2+3 is more profitable than merger

1+2 if and only if $c < c^*$.

Table 3 compares the industry profits in these two merger alternatives. It describes (i) consumers' optimal choices as a function of the quality valuation parameter s, and (ii) the associated profit margins obtained from each consumer (divided by v_1), for the two alternative mergers.

[17] For $v < \frac{1}{2} - \frac{1}{10}\sqrt{5}$, Assumption 1 implies that any admissible c is lower than c^*.

s	$\left[0,\frac{1}{2}\right]$	$\left[\frac{1}{2},\frac{2}{3}\right]$	$\left[\frac{2}{3},\frac{1+c-v}{2(1-v)}\right]$	$\left[\frac{1+c-v}{2(1-v)},\frac{c+2(1-v)}{3(1-v)}\right]$	$\left[\frac{c+2(1-v)}{3(1-v)},1\right]$
merger 2+3	∅	2+3	2+3	2+3	1+3
merger 1+2	∅	∅	2+3	1+3	1+3
margin 2+3 (a)	∅	$\frac{v}{2}$	$\frac{v}{2}$	$\frac{v}{2}$	$\frac{4-4c-v}{6}$
margin 1+2 (b)	∅	∅	$\frac{2v}{3}$	$\frac{3-3c+v}{6}$	$\frac{3-3c+v}{6}$
(a)-(b)	∅	$\frac{v}{2}>0$	$-\frac{v}{6}<0$	$-\frac{3(1-c-v)+v}{6}<0$	$\frac{1-c-2v}{6}\gtrless 0$

Table 3: Consumers' choices and associated profit margins under the two merger alternatives.

Even when all the profit margins are higher under the $1+2$ merger, the $2+3$ merger is more profitable for the industry. This happens because the number of consumers purchasing any system is larger with the $2+3$ merger. In fact, the consumer indifferent between purchasing a system or not purchasing at all depends on the price of the low quality system and this price decreases with the $2+3$ merger while it increases with the $1+2$ merger. This, and the possibility that the $1+3$ system may be more expensive after the $2+3$ merger than after the $1+2$ merger, explains why the former leads to higher industry profits.

Even though the effects of each merger on both insiders' profitability and industry profitability have been discussed in detail above, a remark is worth making about the mergers' effects on the outsider's profit. Clearly, the $1+3$ merger is (also) neutral in terms of the induced impact on the outsider's profit. Now, as far as the the $2+3$ and the $1+2$ mergers are concerned, in both cases the outsider's profit is always negatively affected by the merger. Consider the $2+3$ vertical merger first. After this merger and as explained in Section 2.2, the merged entity strategically increases the price of the stand-alone (must-have) component 3, which increases the relative price of the competing system $1+3$, making it less attractive to consumers. As a result, some consumers are induced to switch from system $1+3$ to system $2+3$, which obviously has a negative effect on the outsider firm 1's demand and profits. Consider now the horizontal merger $1+2$. As it is standard in mergers involving producers of substitutes when merger-specific efficiency gains are absent, after the merger the prices of the insiders' products will increase. This will then lead to a decrease in the demand of both components 1 and 2, which in turn will also hurt both the demand and profits of the must-have component sold by the outsider to this merger, firm 3.

3.2 Merger stage

We start by discussing whether the $2+3$ merger can survive counter-offers by the outsider firm seeking to break it apart, using Barros' (1998) notion of merger stability.

According to Barros (1998), "[t]he first step of the analysis is to identify under which conditions each merger is carried out. First, one must check that profits of merging firms do increase relative to the sum of pre-merger individual profits (participation constraint). (...) The second set of conditions requires stability of the merger, meaning that the firm external to the merger cannot offer to either of the firms participating in the merger a more profitable alternative."

Using this approach, we find that the only stable merger is the $2+3$ merger, but only for parameter values such that it increases welfare and consumers' surplus. For the other parameter values, the outsider, firm 1, may, by exerting its maximum effort, make an offer either to firm 2 or to firm 3 that breaks up the $2+3$ merger. However, the merger that would result from such an offer (merger $1+2$ or $1+3$) would never be itself stable. This simply means that there is no offer that firm 1 might make to firm 3, for instance, that leads to a merger that is itself immune to a counter-offer by firm 2 to either firm 1 or firm 3.

Given the fact that *there is a cycle of successive offers that break each possible merger*, this approach is not useful, in our particular setting, in predicting which mergers will occur. As more structure is needed to identify which mergers will take place, in what follows we consider two alternative merger protocols for the coalition formation game.[18] The first corresponds to a sequential offers game and the second models a merger as a first-price auction game. In both cases, (*i*) firms' incentives when participating in the acquisition game are investigated in detail, allowing an outsider that is potentially harmed by a merger or acquisition to preempt it by outbidding its rival; and (*ii*) the price of the target firm and the gain obtained by the acquirer are endogenously determined. The following two subsections present each of these games.

[18] We borrow both games from the literature. As the two rely on different but specific assumptions, we have opted to present both alternatives so as to show that the result that the $2+3$ merger occurs in equilibrium (when welfare detrimental) is robust to different assumptions regarding the coalition formation stage.

3.2.1 Sequential offers game

Following Bloch (1995, 1996) and Ray and Vohra (1999), in this section we model the merger formation process as a sequential game in which firms make a merger proposal to a single rival that includes a profit sharing rule. We assume that a ranking of firms is exogenously defined *ex-ante* by nature and that this ranking is common knowledge. The first firm in the ranking makes an offer, if any. In case no offer is made, it is the turn of the second firm in the ranking to make a decision. In case an offer is made, it can either be accepted, in which case the merger between the two firms takes place and the merger game ends, or rejected, in which case the firm that just rejected the offer can make an alternative proposal, if any. In addition, if the first two firms in the ranking make no offer when called to play, it is the turn of the last firm in the ranking to make an offer. This approach, which follows Gabszewicz et al. (2017), is in line with Bloch (1996) and Ray and Vohra (1999) but differs from them in the two following important respects: (*i*) the profit sharing rule is endogenous and is included in the merger formation game; and (*ii*) each firm is allowed to have just one shot at making an offer. This setting then allows all firms to make offers to each other. In addition, it should be highlighted that even when a coalition is formed before a given firm is able to make an offer, the specific offer that resulted in the merger formation and the corresponding decision to accept it have taken into consideration (i.e. have anticipated) all other offers that would have ensued in case of rejection, should the subsequent firms in the ranking be called to move and make a decision on whether to make a merger proposal.

The following Proposition presents the equilibrium of this endogenous merger game.

Proposition 2 *Let assumption* 1 *hold. When firm* 1 *is the initiator of the merger stage: i*) *if* $c > c^*$*, then the* $1 + 3$ *merger will take place; ii*) *if* $c < c^*$*, then the* $1 + 3$ *or the* $1 + 2$ *merger will take place. When firm* 2 *is the initiator of the merger stage, then the* $1 + 2$ *or the* $2 + 3$ *merger will take place. When firm* 3 *is the initiator of the merger stage: i*) *if* $c > c^*$*, then the* $1 + 3$ *or the* $2 + 3$ *merger will take place; ii*) *if* $c < c^*$*, then the* $1 + 3$ *merger will take place.*

In case a third firm is called to make a merger decision, this last mover will make an ultimatum offer to another firm, in which: (*i*) the target firm is, obviously, the one that leads to the most profitable merger involving the firm making the ultimatum offer; and (*ii*) the firm making the ultimatum offer captures the entirety of the incremental profit generated by the merger between the two.

Given the results in Lemma 5, firm 1 will always make the ultimatum offer to firm 2; firm 2 will make the ultimatum offer to firm 1 if $c > c^*$ and to firm 3 otherwise; firm 3 will always make the ultimatum offer to firm 2.

If the second firm is called to make a merger proposal, it always has the option to make a counter offer to the game initiator, which is shown to be better than making an offer to the last firm to move or than making no offer at all: an offer to the last one to move would be rejected unless it replicates the ultimatum offer. Making no offer would lead to the same outcome. By using the ultimatum offer by the last firm to move as a threat point, the second firm to move is then able to make an offer to the game initiator in which this firm is forced to accept either the no-merger status quo profit (if the target of the last ultimatum offer would be the game initiator) or, instead, and even worse, the outsider's profit (that the game initiator would get at the ultimatum offer in which it is not the target firm).

Anticipating this, the game initiator will try to avoid the worst possible ultimatum outcome from its perspective by making, at the beginning of the merger formation game, an offer to the firm that would be able to make this (worst) ultimatum offer. This preempts the possibility of such offer taking place. Put it another way, the game initiator makes an offer (that will be accepted) to the firm that is in a position to hurt it the most if given the opportunity to make the last offer of the merger formation game.

Table 4 illustrates the equilibrium mergers and the corresponding payoffs. As an illustration, take the case of firm 1 as the game initiator. If $c > c^*$, firm 2 would make the ultimatum offer to firm 1 and firm 3 would make the ultimatum offer to firm 2. In the first case, firm 1 will obtain $\pi_1^{\emptyset}$, which is larger than the profit it would obtain in the second case: $\pi_1^{2+3} < \pi_1^{\emptyset}$. Thus, firm 1 makes an offer to firm 3, which is equal to the counter-offer that firm 3 would find it optimal to make to firm 1, if it rejected firm 1's offer. If, however, $c < c^*$ firm 2 would make the ultimatum offer to firm 3 and firm 3 would make the ultimatum offer to firm 2. In either case, firm 1 will obtain π_1^{2+3}, so it will be indifferent between making an offer to firm 2 or to firm 3.

$c > c^*$	Merger	Firm 1	Firm 2	Firm 3
Firm 1 moves first	1+3	$\pi_1^{\emptyset}$	π_2^{1+3}	$\pi_{1+3}^{1+3} - \pi_1^{\emptyset}$
Firm 2 moves first	1+2 or 2+3	$\pi_{1+2}^{1+2} - \pi_2^{\emptyset}$ or π_1^{2+3}	$\pi_2^{\emptyset}$	π_3^{1+2} or $\pi_{2+3}^{2+3} - \pi_2^{\emptyset}$
Firm 3 moves first	1+3 or 2+3	$\pi_{1+3}^{1+3} - \pi_3^{1+2}$ or π_1^{2+3}	π_2^{1+3} or $\pi_{2+3}^{2+3} - \pi_3^{1+2}$	π_3^{1+2}
$c < c^*$	Merger	Firm 1	Firm 2	Firm 3
Firm 1 moves first	1+2 or 1+3	π_1^{2+3}	$\pi_{1+2}^{1+2} - \pi_1^{2+3}$ or π_2^{1+3}	π_3^{1+2} or $\pi_{1+3}^{1+3} - \pi_1^{2+3}$
Firm 2 moves first	1+2 or 2+3	$\pi_{1+2}^{1+2} - \pi_2^{\emptyset}$ or π_1^{2+3}	$\pi_2^{\emptyset}$	π_3^{1+2} or $\pi_{2+3}^{2+3} - \pi_2^{\emptyset}$
Firm 3 moves first	1+3	$\pi_{1+3}^{1+3} - \pi_3^{\emptyset}$	π_2^{1+3}	$\pi_3^{\emptyset}$

Table 4: Equilibrium merger(s) and corresponding firms' payoff

Assume now that i) all firms have the same probability of being the initiator of the merging stage and ii) in case of indifference, each alternative is chosen with the same probability. If $c < c^*$, then merger 2+3 takes place with a probability of 1/6. If $c > c^*$, merger 2+3 takes place with a probability of 1/3.

Hence, the $2 + 3$ merger may endogenously occur in equilibrium, even when it is harmful for consumers, i.e., when nature selects firm 2 as the initiator of the merging stage or when firm 3 is the initiator of the merging stage and $c > c^*$.

When firm 2 is the initiator, it will be indifferent between making an offer to either firm 1 or 3. This happens because the insiders' aggregate profits in the mergers in which firm 2 participates are higher than those when firm 2 is the outsider to merger $1 + 3$. Thus, firm 2 will be the target of an ultimatum offer made by firm 1 or firm 3. As such, firm 2's payoff will always be equal to $\pi_2^{\emptyset}$. Likewise, when firm 3 is the initiator and $c > c^*$, it anticipates that the ultimatum offers of firm 1 and firm 2 are respectively to firm 2 and firm 1. Again, the payoff of firm 3 will always be equal to π_3^{1+2} and hence firm 3 will be indifferent between making an offer to either firm 1 or 2.[19]

[19] The particular demand structure in Lemma 1, in which the demand of firms 1 and 3 does not depend on p_3 and p_1, respectively, implies that all alternative mergers are more profitable than merger $1 + 3$, (see Lemma 4). However, as long as, for some parameter values, the $1 + 3$ merger were the least profitable, the result would still hold.

3.2.2 Mergers as an auction

We now assume the following alternative game, which models a merger as an auction with externalities in the spirit of Jehiel and Moldovanu (1996, 2000).[20] Other works in the literature interpreting a merger as an auction of a target firm include Henkel et al. (2015) and Norbäck et al. (2014, 2010), to name a few.[21]

In formal terms, our proposed auction game is as follows. Initially, nature selects with equal probability a firm that will be the target of a merger or acquisition. Then, the two remaining firms submit simultaneous bids to acquire the target firm and the higher bid is accepted, provided that it is above the target's current profit.[22] The price competition stage then follows, as described in Section 2. This setting then allows the acquirer firms to play an active role in the acquisition game and, in particular, to offer the target firm a bigger share of the aggregate post-merger profits in order to preempt the alternative possible merger (by its rivals), which may decrease its profit substantially. The next Proposition identifies the Nash equilibrium in pure undominated strategies of this endogenous merger game.

Proposition 3 *Let assumption 1 hold. (i) If nature chooses firm* 1 *as the target firm, the equilibrium merger is* $1+3$*. (ii) If nature chooses firm* 2 *as the target firm, the equilibrium merger is* $2+3$*. (iii) If nature chooses firm* 3 *as the target firm, the equilibrium merger is* $2+3$ *if* $c > 1-\sqrt{v}$ *and it is* $1+3$ *otherwise.*

If nature chooses firm i as the target firm, the equilibrium merger will be the one that generates the highest industry-wide profits within the set of mergers that involve firm i. On the one hand, the merger must be sufficiently profitable for the insiders so that the acquiring firm is willing to make a high offer. On the other hand, since the merger never benefits the outsider, its impact on the outsider's profit should not be so harsh as to make the outsider embark on a winning counter-offer.

Hence, the $2+3$ merger may endogenously occur in equilibrium, even when it is harmful for consumers, i.e., when nature selects firm 2 as the target firm and when $c < 1-\sqrt{v}$. This happens because the $2+3$ merger is more profitable for the industry than the $1+2$ merger.[23,24] Above, we

[20] The externality in mergers arises because a merger affects the profits of agents not directly involved in the transaction, the non-merging rivals.

[21] In practice, bidding wars are common. For instance, Malmendier et al. (forthcoming) construct a data set of all U.S. mergers with overlapping bids between 1985 and 2012, identifying as many as 293 takeover contests.

[22] If the two bidders make the same bid, each one obtains the target firm with equal probability.

[23] In this case, the fact that the $1+3$ merger does not have any effect on the market outcome does not play any role, for obvious reasons.

[24] When firm 1 is the target firm, there are two alternative mergers: the $1+3$ merger which has no effects and the

have assumed that the target is any firm with equal probability. However, it can be argued that firm 2 is more likely to be the target firm: Liu and Qiu (2013) provide some evidence that targets tend to be firms that generally have poor performance measures (including profitability), compared with those of the acquirer firms. Following this empirical evidence, Baziki et al. (2017), who also model mergers as a first-price perfect information auction, assume that the firm that is up for sale is the (domestic) one for which the current fixed costs make it unable to operate profitably. Interestingly, in our model, if one takes profits as the measure for performance, then, before the merger firm 2 is the least profitable firm in the region of parameter values wherein the $2+3$ merger decreases both welfare and consumer surplus, as the following Remark highlights.

Remark 2 In the *status quo*, firm 2 is the one with lower profits when $c < 1 - \sqrt{v}$.

Hence, if a common fixed cost were introduced in our setting, in the spirit of Baziki et al. (2017), firm 2 would be a natural candidate firm to be the one up for sale, which would then lead to the $2+3$ merger.

4 Conclusions

This paper studies the welfare effects of mergers between producers of two complementary goods needed to make up a system. In our model there is competition between two vertically differentiated producers of the first component, while the second must-have component is produced by a monopolist.

We find that the merger involving the low quality producer of the first component and the monopolist producer of the second one may hurt consumers' surplus and welfare, a result that is in sharp contrast with most of the existing literature. This results from the impact of the merger on components' prices. After the merger, even though the overall price of the merged entity fully controlled system will be lower, the stand-alone price of the must-have component is strategically increased. Now, since the merged entity fully controlled system becomes cheaper, its market share naturally increases after the merger. However, there are two different sources justifying this market

$1+2$ merger, a merger between producers of substitutes. If the latter merger took place, the insiders would raise prices, which would lower the demand for the two systems and, hence, the demand for firm 3. Firm 3's profits would then decrease significantly, meaning that firm 3 is willing to outbid firm 2 and prevent this merger from taking place.

If the target is firm 3, it is possible that the $2+3$ merger occurs in equilibrium provided that industry profits increase when compared to the $1+3$ merger scenario (or, equivalently, to the pre-merger equilibrium payoffs). From Proposition 1, this happens if and only if the $2+3$ merger benefits consumers.

share increase, with opposing effects on consumers' welfare. First, some consumers who were not buying at all start consuming the merged entity fully controlled system, which enhances consumers' surplus. Second, some consumers previously purchasing the higher quality system (i.e. the system composed of the must-have component produced by the merged entity alongside a competitor's high quality first component) will continue doing so at a higher price or will switch to the lower quality merged entity (fully controlled) system so as to benefit from its reduced price, which negatively affects consumers' welfare. Overall, this second effect may more than offset for the first one, implying that consumers may be negatively affected by the merger. This happens for parameter values such that this merger may endogenously arise. Also worthy of note is the fact that the industry as a whole prefers this vertical (or quasi-vertical) merger to a strictly horizontal one.

In concluding let us point out that a limitation of our analysis is that there is no possibility of entry into the monopolized component industry following a merger. Even though we do believe that our setting constitutes a good approximation for the type of strategic interaction that takes place in some real world industries, clearly, in practice there may also be situations in which there are no substantial barriers to entry. Two main difficulties can be anticipated, however, if one extends our setting to allow for the possibility of entry. First, the entrant should be allowed to offer a vertically differentiated version of the must-have component. Otherwise, entry could rule out the exercise of market power by the producers of the must-have component as well as important strategic price effects that result from a merger. This would lead to multiple possibilities and to a proliferation in the number of parameters. Second, although the merger formation games we use could be extended to analyse a setting in which multiple mergers were possible, it would become considerably more difficult to solve them, since they would have to encompass the possibility of "defensive" mergers to occur, whereby the outsiders to a first merger would be allowed to merge subsequently. Hence, while a model allowing for entry and sequential mergers has generality on its side, it would also be significantly more difficult to obtain results. Hopefully, however, the above model can be seen as a stepping stone in the direction of a more complete analysis, which we plan to carry out in our future research.

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Appendix

Proof. (Lemma 1) Let p_1, p_2 and p_3 be the unit prices set by the three firms. A consumer prefers system $1+3$ to system $2+3$ if and only if

$$sv_1 - p_1 - p_3 > sv_2 - p_2 - p_3 \Leftrightarrow s > s_{1,2} := \frac{p_1 - p_2}{v_1 - v_2}.$$

A consumer prefers system $i+3$ to making no purchase if and only if

$$sv_i - p_i - p_3 > 0 \Leftrightarrow s > s_{i,0} := \frac{p_i + p_3}{v_i} \geq 0.$$

Hence:

The demand for system $1+3$ is given by consumers with $\max\{s_{1,2}; s_{1,0}\} < s < 1$, if any.

The demand for system $2+3$ is given by consumers with $s_{2,0} < s < \min\{s_{1,2}; 1\}$, if any.

Consumers with $0 < s < \min\{s_{2,0}; s_{1,0}\}$ do not purchase any system.

For any price levels p_1, p_2, and p_3, there are two relevant possibilities: (i) If $s_{1,2} > s_{2,0}$, then $s_{1,0} > s_{2,0}$ and $s_{1,2} > s_{1,0}$. Hence, $0 \leq s_{2,0} < s_{1,0} < s_{1,2}$. In addition, it is also relevant to consider whether $s_{1,2}$ is larger or smaller than 1. (ii) If $s_{1,2} < s_{2,0}$, then $s_{1,0} < s_{2,0}$ and $s_{1,2} < s_{1,0}$. Hence, $s_{1,2} < s_{1,0} < s_{2,0}$.

Overall there are three possibilities to consider:

(a) If $s_{1,2} > s_{2,0}$ and $s_{1,2} < 1$, the demand for the components is given by:

$$\begin{aligned} D_1(p_1, p_2, p_3) &= 1 - \frac{p_1 - p_2}{v_1 - v_2} \\ D_2(p_1, p_2, p_3) &= \frac{p_1 - p_2}{v_1 - v_2} - \frac{p_2 + p_3}{v_2} \\ D_3(p_1, p_2, p_3) &= 1 - \frac{p_2 + p_3}{v_2} \end{aligned}$$

(b) If $s_{1,2} > s_{2,0}$ and $s_{1,2} > 1$, the demand for the components is given by:

$$\begin{aligned} D_1(p_1, p_2, p_3) &= 0 \\ D_2(p_1, p_2, p_3) &= D_3(p_1, p_2, p_3) = \max\left\{1 - \frac{p_2 + p_3}{v_2}, 0\right\} \end{aligned}$$

(c) If $s_{1,2} < s_{2,0}$, the demand for the components is given by

$$D_1(p_1,p_2,p_3) = D_3(p_1,p_2,p_3) = \max\left\{1 - \frac{p_1+p_3}{v_1}, 0\right\}$$
$$D_2(p_1,p_2,p_3) = 0.$$

This completes the proof. ■

Proof. (Lemma 2): Let

$$c^+ := 1 - v$$
$$c^- := \frac{1-v}{1+2v}.$$

a) Assume initially case (a) in Lemma 1, i.e. $s_{1,2} > s_{2,0}$ and $s_{1,2} < 1$. The first-order conditions for profit maximization are

$$\frac{\partial\left((p_1 - cv_1)\left(1 - \frac{p_1-p_2}{v_1-v_2}\right)\right)}{\partial p_1} = 0$$
$$\frac{\partial\left(p_2\left(\frac{p_1-p_2}{v_1-v_2} - \frac{p_2+p_3}{v_2}\right)\right)}{\partial p_2} = 0$$
$$\frac{\partial\left(p_3\left(1 - \frac{p_2+p_3}{v_2}\right)\right)}{\partial p_3} = 0$$

that yield

$$p_1^{\emptyset} = \frac{c\,(v+3) + 3\,(1-v)}{6}v_1$$
$$p_2^{\emptyset} = \frac{vc}{3}v_1$$
$$p_3^{\emptyset} = \frac{v\,(3-c)}{6}v_1$$

We have $s_{1,2} > s_{2,0} \Leftrightarrow \frac{1}{3}\frac{c}{1-v} > 0$ and $s_{1,2} < 1 \Leftrightarrow c < \frac{3}{3-v}\,(1-v)$, which are both always true (the latter is implied by $c < c^+$).

The corresponding equilibrium profits are given by:

$$\pi_1^{\emptyset} = \frac{(3(1-v) - c(3-v))^2}{36(1-v)} v_1$$

$$\pi_2^{\emptyset} = \frac{vc^2}{9(1-v)} v_1$$

$$\pi_3^{\emptyset} = \frac{v(3-c)^2}{36} v_1.$$

We still need to check, however, that firms do not unilaterally profit from deviating to cases (b) or (c) in Lemma 1, i.e. that they do not unilaterally deviate and set prices such that $s_{1,2} > 1$ or $s_{1,2} < s_{2,0}$, respectively. This is done in what follows for each firm.

Deviation to case (b):

Only firm 2 might have an incentive to lower its price such that $s_{1,2} > 1$ or $p_2 < p_1 - v_1(1-v)$ with $p_1 = p_1^{\emptyset}$ so that

$$p_2 < v_1 \frac{c(v+3) - 3(1-v)}{6}.$$

With such a price, firm 2's first-order condition would be

$$\frac{\partial\left(p_2(1 - \frac{p_2+p_3}{v_2})\right)}{\partial p_2} = \frac{\partial\left(p_2(1 - \frac{p_2+\frac{v(3-c)}{6}v_1}{v_2})\right)}{\partial p_2} = 0.$$

The optimal p_2 is then given by

$$p_2 = \frac{1}{12} v v_1 (c+3)$$

with $s_{2,0} < 1 \Leftrightarrow -\frac{1}{12}(c+3) < 0$ which is always true.

Now, the constraint is not binding if and only if

$$\frac{1}{12} v v_1 (c+3) < v_1 \frac{c(v+3) - 3(1-v)}{6} \Leftrightarrow c > \frac{3(2-v)}{(v+6)}.$$

Recall that $c < c^+$. As $\frac{3(2-v)}{(v+6)} > c^+ \Leftrightarrow v\frac{v+2}{v+6} > 0$ is always true, $c > \frac{3(2-v)}{(v+6)}$ is impossible, implying that the constraint is binding and the deviation profits will be lower.

Deviation to case (c):

Firm 1: Given the equilibrium levels for p_2 and p_3, firm 1 may unilaterally set a price such that

$s_{1,2} < s_{2,0}$ or $p_1 < \frac{p_2 + p_3(1-v)}{v}$ with $p_2 = p_2^{\emptyset}$ and $p_3 = p_3^{\emptyset}$ so that

$$p_1 < \frac{c\,(v+1) + 3\,(1-v)}{6} v_1.$$

With such a price, firm 1's first-order condition would be

$$\frac{\partial\left((p_1 - cv_1)\left(1 - \frac{p_1+p_3}{v_1}\right)\right)}{\partial p_1} = \frac{\partial\left((p_1 - cv_1)\left(1 - \frac{p_1 + \frac{v(3-c)}{6}v_1}{v_1}\right)\right)}{\partial p_1} = 0$$
$$\Leftrightarrow p_1 = \frac{c\,(v+6) - 3v + 6}{12} v_1$$

with $s_{1,0} < 1 \Leftrightarrow c < \frac{6-3v}{6-v}$, which is implied by $c < c^+$. As $\frac{c(v+6)-3v+6}{12} > \frac{c(v+1)+3(1-v)}{6} \Leftrightarrow \frac{1}{12}c\,(4-v) + \frac{1}{4}v > 0$ is always true, the constraint is binding and deviation profits will be lower.

Firm 2: If $s_{1,2} < s_{2,0}$, firm 2 will have zero profits, implying that there is obviously no profit from deviation for this firm.

Firm 3: Given the equilibrium levels for p_1 and p_2, firm 3 may unilaterally set a price such that $s_{1,2} < s_{2,0}$ or $p_3 > \frac{vp_1 - p_2}{1-v}$ with $p_1 = p_1^{\emptyset}$ and $p_2 = p_2^{\emptyset}$ so that

$$p_3 > \frac{c\,(v+1) + 3\,(1-v)}{6(1-v)} vv_1.$$

With such a price, firm 3's first-order condition would be

$$\frac{\partial\left(p_3\left(1 - \frac{p_1+p_3}{v_1}\right)\right)}{\partial p_3} = \frac{\partial\left(p_3\left(1 - \frac{\frac{c(v+3)+3(1-v)}{6}v_1 + p_3}{v_1}\right)\right)}{\partial p_3} = 0$$
$$p_3 = \frac{3\,(v+1) - c\,(v+3)}{12} v_1$$

with $s_{1,0} < 1 \Leftrightarrow -3\,(1-c) - v\,(3-c) < 0$, which is always true.

If $\frac{3(v+1)-c(v+3)}{12}v_1 > \frac{c(v+1)+3(1-v)}{6(1-v)}vv_1 \Leftrightarrow c < \frac{3(1-v)^2}{v^2+3}$, the constraint is not binding and firm 3's profit becomes $\pi_3^D = \left(\frac{c(v+3)-3(v+1)}{12}\right)^2 v_1$. This deviation is unprofitable if and only if

$$\pi_3^D < \pi_3^{\emptyset} \Leftrightarrow c > \frac{3v^2 - 12\sqrt{v} + 9}{2v + v^2 + 9}$$

which is implied by $c > c^{-}$.

If, instead, $c > \frac{3(1-v)^2}{v^2+3}$, the constraint is binding and deviation profits will be lower.

b) Assume now case (b) in Lemma 1, i.e. $s_{1,2} > s_{2,0}$ and $s_{1,2} > 1$. Note that in this case firm 1 has zero profits. For any p_2 and p_3, firm 1 will always try to prevent $s_{1,2} > 1$ from taking place by lowering its price at most to C. When this is the case, we have $s_{1,2} = \frac{C-p_2}{v_1-v_2}$ which is always lower than 1 for any $p_2 > 0$:

$$\frac{C-p_2}{v_1-v_2} < 1 \Leftrightarrow p_2 > v_1\left(c+v-1\right)$$

which always holds, because $c < c^{+} \Leftrightarrow c+v-1 < 0$. Therefore, there is no equilibrium in this case.

c) Finally, assume now case (c) in Lemma 1, i.e. $s_{1,2} < s_{2,0}$. If $s_{1,0} < 1$, firm 2 has no demand and the first-order conditions for profit maximization are

$$\frac{\partial\left(\left(p_1 - cv_1\right)\left(1 - \frac{p_1+p_3}{v_1}\right)\right)}{\partial p_1} = 0$$
$$\frac{\partial\left(p_3\left(1 - \frac{p_1+p_3}{v_1}\right)\right)}{\partial p_3} = 0$$

that yield

$$p_1 = \frac{1}{3}\left(2c+1\right)v_1$$
$$p_3 = \frac{1}{3}\left(1-c\right)v_1$$

with $s_{1,0} = \frac{c+2}{3} < 1$. However, firm 2 will prefer to set any positive price such that $s_{1,2} > s_{2,0}$ or $p_2 < p_1 v - p_3\left(1-v\right)$, which is possible if $p_1 v - p_3\left(1-v\right) = \frac{1}{3}v_1\left(c\left(v+1\right)-\left(1-2v\right)\right) > 0$, or, equivalently, if $c > \frac{1-2v}{1+v}$, which is equivalent to $c > c^{-}$. Therefore, no equilibrium exists in this case. ■

Proof. (Lemma 3): After the $2+3$ merger, the merged firm sets both the price of the $2+3$ system, P_3, and the stand-alone price of the component for which there is no competition, p_3. We will refer to p_2, the implicit price, as $P_3 - p_3$.

a) Assume initially that $s_{1,2} > s_{2,0}$ and $s_{1,2} < 1$. The first-order conditions for profit maximization are

$$\frac{\partial\left((p_1 - cv_1)\left(1 - \frac{p_1 - p_2}{v_1 - v_2}\right)\right)}{\partial p_1} = 0$$
$$\frac{\partial\left(p_2\left(\frac{p_1 - p_2}{v_1 - v_2} - \frac{p_2 + p_3}{v_2}\right) + p_3\left(1 - \frac{p_2 + p_3}{v_2}\right)\right)}{\partial p_2} = 0$$
$$\frac{\partial\left(p_2\left(\frac{p_1 - p_2}{v_1 - v_2} - \frac{p_2 + p_3}{v_2}\right) + p_3\left(1 - \frac{p_2 + p_3}{v_2}\right)\right)}{\partial p_3} = 0$$

that result in

$$\begin{aligned} p_1^{2+3} &= \frac{1 - v + 2c}{3} v_1 \\ p_2^{2+3} &= \frac{v}{2} v_1 - \frac{v + 2(1-c)}{6} v_1 \\ p_3^{2+3} &= \frac{v + 2(1-c)}{6} v_1 \\ P_3^{2+3} &= \frac{v}{2} v_1 \end{aligned}$$

and the corresponding profits are

$$\begin{aligned} \pi_1^{2+3} &= \left(\frac{1 - c - v}{3}\right)^2 \frac{v_1}{1 - v} \\ \pi_{2+3}^{2+3} &= \frac{\left(4c^2 - 8c(1-v) + v(1-5v) + 4\right)}{36(1-v)} v_1. \end{aligned}$$

We have $s_{1,2} > s_{2,0} \Leftrightarrow \frac{2c+1-v}{6(1-v)} > 0$ and $s_{1,2} < 1 \Leftrightarrow c < c^+ := 1 - v$, which are both true.

We still need to check, however, that firms do not unilaterally profit from deviating to the cases (b) or (c) in Lemma 1, i.e. that they do not want to unilaterally deviate and set prices such that $s_{1,2} > 1$ or $s_{1,2} < s_{2,0}$. This will be done in what follows.

Deviation to case (b):

The merged entity might have an incentive to lower its (implicit) price for component 2 (or increase p_3) so that

$$s_{1,2} > 1 \Leftrightarrow P_3 - p_3 < p_1 - v_1 + v_2$$

where $p_1 = p_1^{2+3}$. When this is the case, firm $2+3$'s profit is

$$p_3 0 + P_3 \left(1 - \frac{P_3 - p_3 + p_3}{v_2}\right) = P_3 \left(1 - \frac{P_3}{v_2}\right).$$

It can then set p_3 to infinite and maximize profit in P_3, obtaining profit $\pi_{2+3}^D = \frac{1}{4}v_2$ at $P_3 = \frac{1}{2}v_2$. This corresponds to the profit and price set by a monopolist producing system $2+3$. We can now compare the merged entity profit under this deviation with its profit at the candidate equilibrium

$$\pi_{2+3}^D < \pi_{2+3}^{2+3} \Leftrightarrow -\frac{1}{9}v_1 \frac{(c+v-1)^2}{1-v} < 0.$$

As this is always true, this deviation from the candidate merger equilibrium is not profitable.

Deviation to case (c):

Firm 1: Given the equilibrium levels for p_2 and p_3, firm 1 may unilaterally set a price such that $s_{1,2} < s_{2,0}$ or $p_1 < \frac{p_2 + p_3(1-v)}{v}$ with $p_2 = p_2^{2+3}$ and $p_3 = p_3^{2+3}$ so that

$$p_1 < v_1 \frac{2c+1-v}{6}.$$

With such a price, firm 1's first-order condition would be

$$\frac{\partial\left((p_1 - cv_1)\left(1 - \frac{p_1+p_3}{v_1}\right)\right)}{\partial p_1} = \frac{\partial\left((p_1 - cv_1)\left(1 - \frac{p_1 + \frac{v+2(1-c)}{6}v_1}{v_1}\right)\right)}{\partial p_1} = 0$$
$$\Leftrightarrow p_1 = v_1 \frac{8c+4-v}{12}.$$

As $v_1 \frac{8c+4-v}{12} - v_1 \frac{2c+1-v}{6} = \frac{1}{12}v_1 (4c+v+2) > 0$, the constraint is binding and profits will be lower.

Firm $2+3$: Given the equilibrium levels for p_1, the merged firm may unilaterally set prices p_2 and p_3 such that $s_{1,2} < s_{2,0}$. As p_2 plays no role in case (c), firm $2+3$ can put p_2 at infinite and set any p_3 while still satisfying the constraint. The optimal p_3 is given by

$$\frac{\partial\left(p_3\left(1 - \frac{p_1+p_3}{v_1}\right)\right)}{\partial p_3} = \frac{\partial\left(p_3\left(1 - \frac{\frac{1-v+2c}{3}v_1 + p_3}{v_1}\right)\right)}{\partial p_3} = 0$$
$$\Leftrightarrow p_3 = v_1 \frac{v + 2(1-c)}{6}$$

and the corresponding profits are $\pi_{2+3}^{D} = v_1 \left(\frac{v+2(1-c)}{6}\right)^2$. Because

$$\pi_{2+3}^{D} < \pi_{2+3}^{2+3} \Leftrightarrow -\left(\frac{2c+1-v}{6}\right)^2 \frac{v}{1-v} v_1 < 0$$

the deviation is, however, not profitable.

b) Assume now that $s_{1,2} > s_{2,0}$ and $s_{1,2} > 1$. As seen above in the case in which the seller of system 2+3 is a monopolist, we have that p_3 is irrelevant and the optimal P_3 is $\frac{1}{2}v_2$. Having p_1 and p_3 sufficiently high so that no consumer buys system 1+3 along with $P_3 = \frac{1}{2}v_2$ is an equilibrium. However, it is Pareto dominated by the equilibrium in case $s_{1,2} > s_{2,0}$ and $s_{1,2} < 1$.

c) Finally, assume that $s_{1,2} < s_{2,0}$. Then, component 2 does not have any demand and we have the same candidate for equilibrium as before the merger because one of the components of the merged firm faces no demand (see proof of Lemma 2, case c)):

$$p_1 = \frac{1}{3} v_1 (2c+1)$$
$$p_3 = \frac{1}{3} v_1 (1-c)$$

and

$$\pi_1 = (p_1 - cv_1)\left(1 - \frac{p_1+p_3}{v_1}\right) = \pi_{2+3} = p_3\left(1 - \frac{p_1+p_3}{v_1}\right) = \frac{1}{9}(1-c)^2 v_1.$$

However, this is not an equilibrium, because given $p_1 = \frac{1}{3}v_1(2c+1)$, the merged firm can set p_2 and p_3 such that $s_{1,2} > s_{2,0} \Leftrightarrow v_2(p_1+p_3) - v_1(p_2+p_3) > 0$ and increase its profit. In that case, profits would be maximized when

$$\frac{\partial \left(p_2\left(\frac{\frac{1}{3}v_1(2c+1)-p_2}{v_1-v_2} - \frac{p_2+p_3}{v_2}\right) + p_3\left(1 - \frac{p_2+p_3}{v_2}\right)\right)}{\partial p_2} = 0$$
$$\frac{\partial \left(p_2\left(\frac{\frac{1}{3}v_1(2c+1)-p_2}{v_1-v_2} - \frac{p_2+p_3}{v_2}\right) + p_3\left(1 - \frac{p_2+p_3}{v_2}\right)\right)}{\partial p_3} = 0.$$

This results in

$$p_2 = \frac{1}{6} v_1 (2c + 3v - 2)$$
$$p_3 = \frac{1}{3} v_1 (1 - c)$$

and $v_2 (p_1 + p_3) - v_1 (p_2 + p_3) = \frac{1}{6} v_1^2 v (2c + 1) > 0$, with profit

$$\pi_{2+3}^D = \frac{1}{36} v_1 \frac{-8c - 3v + 12cv + 4c^2 + 4}{1 - v}.$$

This deviation is profitable if $\pi_{2+3}^D - \frac{1}{9} (1 - c)^2 v_1 = \frac{1}{36} v v_1 \frac{(2c+1)^2}{1-v} > 0$, which is always true. ■

Proof. (Proposition 1): In the pre-merger equilibrium, consumers' surplus is given by

$$\begin{aligned} CS^{\emptyset} &= \int_{s_{1,2}}^{1} (s v_1 - p_1 - p_3)\, ds + \int_{s_{2,0}}^{s_{1,2}} (s v_2 - p_2 - p_3)\, ds \\ &= \frac{c^2 (9 - 5v) + 9 (2c - 1)(v - 1)}{72 (1 - v)} v_1 \end{aligned}$$

whereas individual profits are:

$$\begin{aligned} \pi_1^{\emptyset} &= (p_1 - c v_1) \left(1 - \frac{p_1 - p_2}{v_1 - v_2} \right) = \frac{(3 (1 - c) - v (3 - c))^2}{36 (1 - v)} v_1 \\ \pi_2^{\emptyset} &= p_2 \left(\frac{p_1 - p_2}{v_1 - v_2} - \frac{p_2 + p_3}{v_2} \right) = \frac{v c^2}{9 (1 - v)} v_1 \\ \pi_3^{\emptyset} &= p_3 \left(1 - \frac{p_2 + p_3}{v_2} \right) = \frac{(3 - c)^2 v}{36} v_1. \end{aligned}$$

Finally, industry profits, Π, and social welfare, W, are:

$$\begin{aligned} \Pi^{\emptyset} &= \frac{1}{36} \frac{18cv - 9v - c^2 v - 18c + 9c^2 + 9}{1 - v} v_1 \\ W^{\emptyset} &= \frac{1}{72} \frac{-54c (1 - v) + 27 \left(c^2 + 1 - v \right) - 7c^2 v}{1 - v} v_1. \end{aligned}$$

In the post-merger equilibrium, consumers' surplus is given by

$$CS^{2+3} = \int_{s_{1,2}}^{1} (sv_1 - p_1 - p_3)\, ds + \int_{s_{2,0}}^{s_{1,2}} (sv_2 - P_3)\, ds$$
$$= \left(\frac{4c^2 - 8c(1-v) - v(5v-1) + 4}{72(1-v)} \right) v_1$$

whereas profits and welfare are

$$\pi_{2+3}^{2+3} = \frac{\left(4c^2 - c8(1-v) + v(1-5v) + 4\right)}{36(1-v)} v_1$$
$$\pi_1^{2+3} = \frac{(c+v-1)^2}{9(1-v)} v_1$$
$$\Pi^{2+3} = \frac{1}{36} \frac{-16c - 7v + 16cv + 8c^2 - v^2 + 8}{1-v} v_1$$
$$W^{2+3} = \frac{1}{72} \frac{-40c - 13v + 40cv + 20c^2 - 7v^2 + 20}{1-v} v_1.$$

The merger is profitable if and only if $\pi_{2+3}^{2+3} - \pi_2^{\emptyset} - \pi_3^{\emptyset} > 0$, which is equivalent to

$$c^2(v-4) - c(6v-8) - 4(1-v) < 0.$$

This is always true because the last expression is maximized at $c^* = \frac{4-3v}{4-v}$, which is larger than c^+. Thus, the maximum is obtained at $c = c^+$ and it is given by

$$(1-v)^2(v-4) - (1-v)(6v-8) - 4(1-v) = -v(1-v)(v+1) < 0.$$

The merger decreases firm 1's profits if and only if $\pi_1^{2+3} - \pi_1^{\emptyset} < 0$, which is equivalent to

$$\frac{-1}{36}(1-c)(cv + 5(1-c-v)) < 0$$

which is always true.

The merger benefits consumers if and only if $CS^{2+3} - CS^{\emptyset} > 0$, which is equivalent to

$$-\frac{5}{72}\left(-2c - v + c^2 + 1\right) > 0 \Leftrightarrow c > 1 - \sqrt{v}.$$

The merger increases industry profits if and only if $\Pi^{2+3} - \Pi^{\emptyset} > 0$, which is equivalent to

$$-\frac{1}{36}\left(-2c - v + c^2 + 1\right) > 0 \Leftrightarrow c > 1 - \sqrt{v}.$$

This completes the proof. Combining these results with those of Lemma 2 we obtain:

$$\begin{aligned}\frac{\Delta P_3}{v_1} &= -\frac{1}{6}vc < 0\\ \frac{\Delta p_1 + \Delta p_3}{v_1} &= \frac{1}{6}\left(1 - c - v\right) > 0\end{aligned}$$

or, in terms of component prices,

$$\begin{aligned}\frac{\Delta p_1}{v_1} &= -\frac{1}{6}\left(1 - c\right)\left(1 - v\right) < 0\\ \frac{\Delta p_2}{v_1} &= -\frac{1}{3}\left(1 - v\right)\left(1 - c\right) < 0\\ \frac{\Delta p_3}{v_1} &= \frac{1}{6}\left(v\left(c - 2\right) - 2c + 2\right) > 0\end{aligned}$$

where p_2 is the implicit price of component 2, equal to $P_3 - p_3$. ■

Proof. (Lemma 4):

(i) The 1+2 merger:

(a) Assume initially that $s_{1,2} > s_{2,0}$ and $s_{1,2} < 1$. The set of first-order conditions after the 1+2 merger is given by

$$\begin{aligned}\frac{\partial\left(\left(p_1 - cv_1\right)\left(1 - \frac{p_1 - p_2}{v_1 - v_2}\right) + p_2\left(\frac{p_1 - p_2}{v_1 - v_2} - \frac{p_2 + p_3}{v_2}\right)\right)}{\partial p_1} &= 0\\ \frac{\partial\left(\left(p_1 - cv_1\right)\left(1 - \frac{p_1 - p_2}{v_1 - v_2}\right) + p_2\left(\frac{p_1 - p_2}{v_1 - v_2} - \frac{p_2 + p_3}{v_2}\right)\right)}{\partial p_2} &= 0\\ \frac{\partial\left(p_3\left(1 - \frac{p_2 + p_3}{v_2}\right)\right)}{\partial p_3} &= 0\end{aligned}$$

from which we obtain

$$p_1^{1+2} = \frac{1}{6}\left(3c + 3 - v\right) v_1$$
$$p_2^{1+2} = p_3^{1+2} = \frac{1}{3} v v_1.$$

Note that $s_{1,2} > s_{2,0} \Leftrightarrow c > \frac{1-v}{3}$ and $s_{1,2} < 1 \Leftrightarrow c < 1 - v$, which are both always true under Assumption 1. Individual profits are given by

$$\pi_{1+2}^{1+2} = \frac{1}{36} \frac{-18c - 14v + 18cv + 9c^2 + 5v^2 + 9}{1 - v} v_1$$
$$\pi_3^{1+2} = \frac{1}{9} v v_1.$$

We still need to check, however, that firms do not unilaterally profit from deviating to the other cases in Lemma 1, i.e. to unilaterally deviate and set prices such that $s_{1,2} > 1$ or $s_{1,2} < s_{2,0}$.

Deviation to case (b):

The merged entity might have an incentive to change its prices such that

$$s_{1,2} > 1 \Leftrightarrow p_2 < p_1 - v_1 + v_2.$$

Given $p_3 = p_3^{1+2} = \frac{1}{3} v v_1$ and by setting p_1 to infinite, firm 1+2 can choose p_2 to maximize

$$p_2 \left(1 - \frac{p_2 + \frac{1}{3} v v_1}{v_2}\right)$$

while still verifying the constraint $s_{1,2} > 1$. The optimal price is $p_2 = \frac{1}{3} v v_1$ and the corresponding profit is $\pi_{1+2}^{D} = \frac{1}{9} v v_1$. Because

$$\pi_{1+2}^{1+2} - \pi_{1+2}^{D} > 0 \Leftrightarrow \frac{1}{4} \frac{(1 - c - v)^2}{1 - v} v_1$$

is always true, this deviation is not profitable.

Deviation to case (c):

Firm $1 + 2$: As p_2 plays no role in case (c), firm $1 + 2$ can put p_2 at infinite and set any p_1 that satisfies the constraint $s_{1,2} < s_{2,0}$. The optimal p_1 is given by

$$\frac{\partial\left((p_1 - cv_1)\left(1 - \frac{p_1+p_3}{v_1}\right)\right)}{\partial p_1} = \frac{\partial\left((p_1 - cv_1)\left(1 - \frac{p_1+\frac{1}{3}vv_1}{v_1}\right)\right)}{\partial p_1} = 0$$

$$\Leftrightarrow p_1 = \frac{1}{6}v_1\left(3c - v + 3\right)$$

and the corresponding profits are $\pi^D_{1+2} = \left(\frac{3c+v-3}{6}\right)^2 v_1$. Because

$$\pi^{1+2}_{1+2} - \pi^D_{1+2} > 0 \Leftrightarrow \frac{v}{1-v}\left(\frac{3c+v-1}{6}\right)^2 v_1 > 0$$

is always true, the deviation is not profitable.

Firm 3: Given the equilibrium levels for p_1 and p_2, firm 3 may unilaterally set a price such that $s_{1,2} < s_{2,0}$ or $p_3 > \frac{vp_1-p_2}{1-v}$ with $p_1 = p_1^{1+2}$ and $p_2 = p_2^{1+2}$ so that

$$p_3 > \frac{v}{1-v}\frac{3c+1-v}{6}v_1.$$

With such a price, firm 1's first-order condition would be

$$\frac{\partial\left(p_3\left(1 - \frac{p_1+p_3}{v_1}\right)\right)}{\partial p_3} = \frac{\partial\left(p_3\left(1 - \frac{\frac{1}{6}(3c+3-v)v_1+p_3}{v_1}\right)\right)}{\partial p_3} = 0$$

$$\Leftrightarrow p_3 = \frac{v+3-3c}{12}v_1.$$

If $\frac{v}{1-v}\frac{3c+1-v}{6}v_1 - \frac{v+3-3c}{12}v_1 = \frac{1}{12}v_1\frac{3(1+v)c-(1-v)(3-v)}{1-v} > 0$, the constraint is binding and profits will be lower. Thus, the constraint is binding if $c > \frac{1-v}{3}\frac{3-v}{1+v}$ and deviating is not profitable.

Otherwise, if $c < \frac{1-v}{3}\frac{3-v}{v+1}$, the profit from deviating is $\pi^D_3 = \frac{1}{144}v_1\left(3c-v-3\right)^2$ with

$$\pi^D_3 - \pi^{1+2}_3 < 0 \Leftrightarrow \frac{1}{144}\left(9c^2 - c\left(6v+18\right) + v^2 - 10v + 9\right)v_1 < 0.$$

Now, this deviation is not profitable if and only if $(1-\sqrt{v})\frac{3-\sqrt{v}}{3} < c < (1+\sqrt{v})\frac{3+\sqrt{v}}{3}$, which is implied by $c^- < c < \frac{1-v}{3}\frac{3-v}{v+1}$.

b) Assume now that $s_{1,2} > s_{2,0}$ and $s_{1,2} > 1$. The set of first-order conditions after the $1+2$

merger is the same as when there is no merger because one of the components sold by the merged firm, namely component 1, has no demand:

$$\frac{\partial\left(p_2\left(1-\frac{p_2+p_3}{v_2}\right)\right)}{\partial p_2}=0$$
$$\frac{\partial\left(p_3\left(1-\frac{p_2+p_3}{v_2}\right)\right)}{\partial p_3}=0$$

from which we obtain

$$p_2=p_3=\frac{1}{3}vv_1$$

with p_1 sufficiently high, such that $s_{1,2}>s_{2,0}$ and $s_{1,2}>1$. The corresponding profits are

$$\pi_{1+2}=\frac{1}{9}vv_1.$$

Given $p_3=\frac{1}{3}vv_1$, firm $1+2$ can deviate to the equilibrium in case (a) by keeping p_2 constant and lowering p_1 to $p_1^{1+2}=\frac{1}{6}\left(3c+3-v\right)v_1$ so that $s_{1,2}<1$ and thus obtaining profit π_{1+2}^{1+2}. As shown above, this is a profitable deviation.

c) Finally, assume now that $s_{1,2}<s_{2,0}$. Then, component 2 does not have any demand and we have the same candidate for equilibrium as before the merger (see Lemma 2),

$$p_1=\frac{1}{3}\left(2c+1\right)v_1$$
$$p_3=\frac{1}{3}\left(1-c\right)v_1$$

and

$$\pi_{1+2}=\left(p_1-cv_1\right)\left(1-\frac{p_1+p_3}{v_1}\right)=\pi_3=p_3\left(1-\frac{p_1+p_3}{v_1}\right)=\frac{1}{9}\left(1-c\right)^2v_1.$$

However, this is not an equilibrium because, given p_3, the merged firm can set p_1 and p_2 such that $s_{1,2}>s_{2,0}$ and increase its profit. The set of first-order conditions after the $1+2$ merger is given by

$$\frac{\partial\left((p_1-cv_1)\left(1-\frac{p_1-p_2}{v_1-v_2}\right)+p_2\left(\frac{p_1-p_2}{v_1-v_2}-\frac{p_2+\frac{1}{3}v_1(1-c)}{v_2}\right)\right)}{\partial p_1}=0$$
$$\frac{\partial\left((p_1-cv_1)\left(1-\frac{p_1-p_2}{v_1-v_2}\right)+p_2\left(\frac{p_1-p_2}{v_1-v_2}-\frac{p_2+\frac{1}{3}v_1(1-c)}{v_2}\right)\right)}{\partial p_2}=0.$$

The corresponding solution is

$$p_1=\frac{1}{3}v_1\left(2c+1\right)$$
$$p_2=\frac{1}{6}v_1\left(c+3v-1\right)$$

with $s_{1,2}>s_{2,0}\Leftrightarrow c>\frac{1-v}{1+2v}$, yielding profit

$$\pi_{1+2}^D=\frac{1}{36}\frac{-2c+2v+12cv^2+8c^2v-10cv+c^2-3v^2+1}{v\left(1-v\right)}v_1.$$

This deviation is profitable if $\pi_{1+2}^D>\frac{1}{9}\left(1-c\right)^2v_1\Leftrightarrow\frac{1}{36}v_1\frac{(c+v+2cv-1)^2}{v(1-v)}>0$, which is always true.

We still have to show that, after the 1+2 merger, the merged firm continues selling both versions of the first component. Assume that it sells only component i. Then there is only one system and consumers purchase it if $sv_i-p_i-p_3>0\Leftrightarrow s>\frac{p_i+p_3}{v_i}$. The demand for each component is then

$$D_i=D_3=\left(1-\frac{p_i+p_3}{v_i}\right)$$

and the first-order conditions for profit maximization by the two firms are

$$\frac{\partial\left((p_i-c_iv_i)\left(1-\frac{p_i+p_3}{v_i}\right)\right)}{\partial p_i}=0$$
$$\frac{\partial\left(p_3\left(1-\frac{p_i+p_3}{v_i}\right)\right)}{\partial p_3}=0$$

with $c_1 = c$ and $c_2 = 0$. The equilibrium prices are then equal to

$$p_i = \frac{2}{3}c_i v_i + \frac{1}{3}v_i$$
$$p_3 = \frac{1}{3}v_i - \frac{1}{3}c_i v_i$$

with $1 - \frac{p_i + p_3}{v_i} = \frac{1}{3}(1 - c_i) > 0$. Moreover, the equilibrium profit of the merged firm is

$$\pi_{1+2} = \pi_3 = (p_i - c_i v_i)\left(1 - \frac{p_i + p_3}{v_i}\right) = \frac{1}{9}v_i (1 - c_i)^2$$

and this firm prefers to sell the high quality component if $\frac{1}{9}v_1 (1-c)^2 > \frac{1}{9}v_2 1 \Leftrightarrow v < (1-c)^2$. We now consider two possibilities:

1. Let $v > (1-c)^2$. Then, $\frac{1}{9}vv_1 - \frac{1}{36}\frac{-18c-14v+18cv+9c^2+5v^2+9}{1-v}v_1 = -\frac{1}{4}\frac{(c+v-1)^2}{1-v}v_1 < 0$.
2. Let $v < (1-c)^2$. Then, it is easy to check that $\frac{1}{9}v_1 (1-c)^2 < \frac{1}{36}\frac{-18c-14v+18cv+9c^2+5v^2+9}{1-v}v_1$.

Thus, it is preferable to sell both products.

(ii) The 1+3 merger:

As for the 1+3 merger in cases (a) or (b), with $s_{1,2} > s_{2,0}$, the outcome is the same as in the pre-merger game because the price of firm 1 does not affect the demand for firm 3 and vice-versa. In case (c) we have $s_{1,2} < s_{2,0}$ and firm 2 does not face any demand. The profit of firm 1+3 is then

$$\pi_{1+3} = (p_1 - cv_1)\left(1 - \frac{p_1 + p_3}{v_1}\right) + p_3\left(1 - \frac{p_1 + p_3}{v_1}\right)$$

which is maximized at any p_1 and p_3 such that $p_1 + p_3 = \frac{1}{2}v_1 (c+1)$ because only the overall system price matters. The corresponding profit is $\frac{1}{4}(1-c)^2 v_1$. Given that only the system price matters, the merged firm could use p_3 to ensure that $s_{1,2} < s_{2,0}$ while adjusting p_1 to maximize profit. However, because

$$\frac{1}{4}v_1 (1-c)^2 < \pi_1^{\emptyset} + \pi_3^{\emptyset} \Leftrightarrow -\frac{1}{9}c^2 v \frac{v_1}{1-v} < 0$$

is always true, the merged firm prefers not to have $s_{1,2} < s_{2,0}$. ■

Proof. (Lemma 5): The 1+3 merger is not unprofitable because equilibrium profits do not change. The 1+2 merger is profitable if and only if

$$\pi_{1+2}^{1+2} - \pi_1^0 - \pi_2^0 > 0 \Leftrightarrow$$

$$\frac{1}{36} v v_1 \frac{4(1-v) - 6c(1-v) + c^2(2-v)}{1-v} > 0$$

which is equivalent to

$$c < \frac{3(1-v) - \sqrt{(1-5v)(1-v)}}{2-v} \text{ and } c > \frac{3(1-v) + \sqrt{(1-5v)(1-v)}}{2-v}.$$

If $v > \frac{1}{5}$ there are no real roots and the merger is profitable. If $v < \frac{1}{5}$, the assumption that $c < c^+ := 1 - v$ ensures that

$$c < \frac{3(1-v) - \sqrt{(1-5v)(1-v)}}{2-v}$$

is always true, because

$$1 - v < \frac{3(1-v) - \sqrt{(5v-1)(v-1)}}{2-v} \Leftrightarrow$$

$$-v(v+3)(1-v)(2-v) < 0.$$

The 2+3 merger is profitable if

$$\pi_{2+3}^{2+3} - \pi_2^0 - \pi_3^0 > 0 \Leftrightarrow$$

$$2c(3v-4) + 4(1-v) + c^2(4-v) > 0$$

which is equivalent to

$$c < \frac{4 - 3v - \sqrt{v(5v-4)}}{4-v} \text{ and } c > \frac{4 - 3v + \sqrt{v(5v-4)}}{4-v}.$$

If $v < \frac{4}{5}$ there are no real roots and the merger is profitable. If $v > \frac{4}{5}$, the assumption that

$c < c^{+} := 1 - v$ ensures that

$$c < \frac{4 - 3v - \sqrt{v(5v-4)}}{4 - v}$$

is always true, because

$$1 - v < \frac{4 - 3v - \sqrt{v(5v-4)}}{4 - v} \Leftrightarrow$$
$$-v(1-v)(4-v)(v+1) < 0.$$

As for outsiders, the 1+3 merger does not benefit outsiders because equilibrium profits do not change. The 1+2 and the 2+3 mergers have a negative impact on the outsider's profit:

$$\pi_1^{2+3} - \pi_1^{\emptyset} = -\frac{(1-c)(5(1-c-v)+cv)}{36} < 0$$
$$\pi_3^{1+2} - \pi_3^{\emptyset} = -\frac{v(1-c)(5-c)}{36} < 0.$$

The 2+3 merger is more profitable than the 1+2 merger if and only if

$$\pi_{2+3}^{2+3} - \pi_2^{\emptyset} - \pi_3^{\emptyset} > \pi_{1+2}^{1+2} - \pi_1^{\emptyset} - \pi_2^{\emptyset}$$
$$\Leftrightarrow \frac{c^2\left(2v^2 - 7v + 4\right) - c\left(12v^2 - 20v + 8\right) + 8v^2 - 12v + 4}{36(1-v)} > 0.$$

The two roots are $\frac{2(2-3v)(1-v)\pm 2\sqrt{v(1-v)(5v-5v^2-1)}}{4-7v+2v^2}$. If $5v - 5v^2 - 1 < 0$ there are no real roots. This happens for $v > \frac{1}{10}\sqrt{5}+\frac{1}{2}$ or $v < \frac{1}{2} - \frac{1}{10}\sqrt{5}$. Note that $2v^2 - 7v + 4 > 0$ is equivalent to $v < \frac{7}{4} - \frac{1}{4}\sqrt{17}$, which is implied by $v < \frac{1}{2} - \frac{1}{10}\sqrt{5}$. So, for $v < \frac{1}{2} - \frac{1}{10}\sqrt{5}$ the 2+3 merger is more profitable than the 1+2 merger. For $\frac{1}{2} - \frac{1}{10}\sqrt{5} < v < \frac{1}{10}\sqrt{5} + \frac{1}{2}$, the 2+3 merger is more profitable than the 1+2 merger if and only if

$$c < c^{*} := \frac{2(2-3v)(1-v) - 2\sqrt{v(1-v)(5v-5v^2-1)}}{4 - 7v + 2v^2}.$$

For $v > \frac{1}{10}\sqrt{5} + \frac{1}{2}$ it is impossible that the 2+3 merger is more profitable than the 1+2 merger. ■

Proof. (Proposition 2): Assume that the order of the offers is such that the first offer is made by firm k, the second by firm j and, the last one by firm i.[25]

Let $s_h^{g+h}(n)$ be an offer made by g to h when $n-1$ offers have previously been rejected, with $n = 1, 2, 3$.

3rd offer

If any firm h receives a "final" offer from firm i, $s_h^{i+h}(3)$, it should accept it if and only if $s_h^{i+h}(3)\pi_{i+h}^{i+h} \geq \pi_h^{\emptyset}$, because in case of rejection the game ends without any merger taking place. Anticipating this, firm i will make to firm h the lowest possible offer that is accepted, if any: we call this an ultimatum offer. Let $\pi_l^{U_{i\to h}}$ denote the profit to firm $l = i, g, h$ when firm i makes the ultimatum offer to firm h. Thus, $\pi_i^{U_{i\to h}} := \pi_{i+h}^{i+h} - \pi_h^{\emptyset}$, $\pi_h^{U_{i\to h}} = \pi_h^{\emptyset}$ and $\pi_g^{U_{i\to h}} = \pi_g^{i+h}$ with $i = 1, 2, 3$; $g = 1, 2, 3$; $h = 1, 2, 3$, and $i \neq g \neq h$.

Note that given the structure of the game, firm i is in a position to make an ultimatum offer if it has made no offer before and, on top of that, if one of two things happened: (i) if the second offer was addressed to firm i and firm i rejected it or (ii) if firm j and k's offers were made to one another and were both rejected. When this is the case, firm i has three possibilities: to make an offer to firm j, to make an offer to firm k, or to make no offer at all. So firm i will make the ultimatum offer to j if and only if $\pi_i^{U_{i\to j}} \geq \pi_i^{U_{i\to k}}$ and $\pi_i^{U_{i\to j}} \geq \pi_i^{\emptyset}$. The first condition ensures that the ultimatum offer to firm j is more profitable than the ultimatum offer to firm k, whereas the second condition guarantees that making the ultimatum offer is better than making no offer. That is, the ultimatum offer will be made to firm j if and only if:

$$\pi_{i+j}^{i+j} - \pi_j^{\emptyset} \geq \pi_{i+k}^{i+k} - \pi_k^{\emptyset} \Leftrightarrow \pi_{i+j}^{i+j} - \pi_i^{\emptyset} - \pi_j^{\emptyset} \geq \pi_{i+k}^{i+k} - \pi_i^{\emptyset} - \pi_k^{\emptyset}$$

and, simultaneously,

$$\pi_{i+j}^{i+j} - \pi_j^{\emptyset} \geq \pi_i^{\emptyset} \Leftrightarrow \pi_{i+j}^{i+j} \geq \pi_i^{\emptyset} + \pi_j^{\emptyset}.$$

Hence, when making the ultimatum offer, each firm will select the most profitable merger in which it participates (as long as it is profitable). In our specific case, no merger is strictly unprofitable, so the second inequality above always holds. In addition, given the asymmetry of the firms involved, there are three different ultimatum offers that we discuss in turn.

Let i be firm 1. Merger $1+2$ is profitable and merger $1+3$ does not increase profits. Thus, firm 1 will make the ultimatum offer to firm 2.

[25] Note that this is not necessarily the nature- selected ranking, because the first firm in the ranking may make an offer to the third one, who then becomes the second firm to make an offer (in case it rejects the first firm's offer)

Let i be firm 2. It will make an ultimatum offer to firm 3 (rather than to firm 1) if and only if:

$$\pi_{2+3}^{2+3} - \pi_2^{\emptyset} - \pi_3^{\emptyset} > \pi_{1+2}^{1+2} - \pi_1^{\emptyset} - \pi_2^{\emptyset} \Leftrightarrow$$

$$c < c^* := \frac{2\left(2-3v\right)\left(1-v\right) - 2\sqrt{v\left(1-v\right)\left(5v-5v^2-1\right)}}{4-7v+2v^2}.$$

So, if firm 1's costs c are relatively low, firm 2 prefers to make an ultimatum merger offer to firm 3. Otherwise, firm 2's ultimatum offer is made to firm 1.

Let i be firm 3. Merger $2+3$ is profitable and merger $1+3$ does not increase profits. Thus, firm 3 will make the ultimatum offer to firm 2.

Summing up, firm 1 will always make the ultimatum offer to firm 2. Firm 2 will make the ultimatum offer to firm 3 if $c < c^*$ and to firm 1 otherwise. Firm 3 will always make the ultimatum offer to firm 2. We denote by $\pi_l^{U_i}$ the profit of firm l when firm i makes its optimal ultimatum offer (the offer to the target as defined in the previous paragraphs). If $l = i$, this then represents the payoff that firm i can obtain if it rejects any offer that it received or if the two other firms have made their offers (if any) to one another.

2nd offer

We now address the second offer. This second offer can be made by firm j, if i) firm j is the second firm to move in nature's ranking and there was no offer by firm k (the first firm to move); or ii) firm j is the target of the first offer from firm k but rejected it.

Having received an offer, $s_j^{j+k}(1)$, firm j has four options:

i) accept the offer and end the game, receiving $s_j^{j+k}(1)\pi_{j+k}^{j+k}$.

ii) reject the offer and make an offer to firm i leading, in case of rejection, to an ultimatum offer by firm i. In this case, the lowest such offer that will be accepted is $s_i^{i+j}(2)\pi_{i+j}^{i+j} = \pi_i^{U_i}$. So firm j would get $\pi_{i+j}^{i+j} - \pi_i^{U_i}$.

iii) reject the offer and make a counter-offer, $s_k^{k+j}(2)$, to firm k. The lowest such offer that will be accepted is $s_k^{k+j}(2)\pi_{j+k}^{j+k} = \pi_k^{U_i}$, because after the rejection of a second offer, there will be an ultimatum game by the last firm to move, firm i. So firm j would get $\pi_{j+k}^{j+k} - \pi_k^{U_i}$.

iv) reject the offer and make no offer, leading to an ultimatum offer by firm i, in which case firm j gets $\pi_j^{U_i}$.

Having received no offer, firm j has only the last three options.

We now compare the payoffs associated with firm j's alternatives.

From firm j's perspective, Option iii) always dominates option iv), given that for any $i = 1, 2, 3$;

$j = 1, 2, 3$; $k = 1, 2, 3$; and $i \neq j \neq k$ we have that:

$$\pi_{j+k}^{j+k} - \pi_k^{U_i} \geq \pi_j^{U_i} \Leftrightarrow \pi_{j+k}^{j+k} \geq \pi_j^{U_i} + \pi_k^{U_i}.$$

For any $h \neq i$ we have $\pi_h^{U_i} \leq \pi_h^{\emptyset}$: firm h is either the target of the ultimatum offer by firm i (receiving $\pi_h^{\emptyset}$) or the outsider to the merger resulting from this ultimatum (receiving $\pi_h^{i+g} \leq \pi_h^{\emptyset}$ because a merger never benefits the outsider. Then, as $\pi_j^{U_i} \leq \pi_j^{\emptyset}$ and $\pi_k^{U_i} \leq \pi_k^{\emptyset}$ we have $\pi_j^{U_i} + \pi_k^{U_i} \leq \pi_j^{\emptyset} + \pi_k^{\emptyset}$. As no merger is unprofitable in our setting, we have $\pi_j^{\emptyset} + \pi_k^{\emptyset} \leq \pi_{j+k}^{j+k}$, implying that option *iii*) always dominates option *iv*).

Also, from firm j's perspective, Option *iii*) always dominates option *ii*) :

$$\pi_{j+k}^{j+k} - \pi_k^{U_i} \geq \pi_{i+j}^{i+j} - \pi_i^{U_i} \Leftrightarrow \pi_{j+k}^{j+k} - \pi_j^{\emptyset} - \pi_k^{U_i} \geq \pi_{i+j}^{i+j} - \pi_i^{U_i} - \pi_j^{\emptyset}.$$

Because $\pi_k^{U_i} \leq \pi_k^{\emptyset}$ and because no merger is unprofitable, we have $\pi_{j+k}^{j+k} - \pi_j^{\emptyset} - \pi_k^{U_i} \geq \pi_{j+k}^{j+k} - \pi_j^{\emptyset} - \pi_k^{\emptyset} \geq 0$. Also, $\pi_i^{U_i} \geq \pi_{i+j}^{i+j} - \pi_j^{\emptyset}$ (the profit when firm i makes the optimal ultimatum offer is not lower than the one it can get by making the ultimatum offer to firm j). So, $\pi_{i+j}^{i+j} - \pi_i^{U_i} - \pi_j^{\emptyset} \leq 0$, which implies that option *iii*) always dominates option *ii*).

Finally, we now compare Option *i*) and Option *iii*): Firm j accepts the offer (instead of rejecting it and making a counter-offer) if $s_j^{j+k}(1)\pi_{j+k}^{j+k} \geq \pi_{j+k}^{j+k} - \pi_k^{U_i}$. The lowest acceptable k's offer is then $\pi_{j+k}^{j+k} - \pi_k^{U_i}$. Firm k will profit $\pi_k^{U_i}$ when making this offer to firm j.

Likewise, if there were no offer targeting firm j, firm j will make an offer, $s_k^{k+j}(2)$, to firm k. The lowest such offer that will be accepted is $s_k^{k+j}(2)\pi_{j+k}^{j+k} = \pi_k^{U_i}$ and firm j would get $\pi_{j+k}^{j+k} - \pi_k^{U_i}$.

1st offer

If making any offer, firm k will make the offer to firm j if and only if $\pi_k^{U_i} \geq \pi_k^{U_j}$.

Let $k = 1$: when firm 1 is the first to move, its optimal choice depends on whether firm 2 makes the ultimatum offer to firm 1 ($c > c^*$) or to firm 3 ($c < c^*$). If $c > c^*$, firm 1 makes an offer to firm 3 if and only if

$$\pi_1^{U_2} \geq \pi_1^{U_3} \Leftrightarrow \pi_1^{\emptyset} \geq \pi_1^{2+3}$$

which is always true. If instead $c < c^*$, firm 1 is indifferent between making an offer to firm 2 or to firm 3 since

$$\pi_1^{U_2} = \pi_1^{U_3} \Leftrightarrow \pi_1^{2+3} = \pi_1^{2+3}.$$

Let $k=2$: firm 2 is indifferent between making an offer to firm 1 or to firm 3 since

$$\pi_2^{U_1} = \pi_2^{U_3} \Leftrightarrow \pi_2^{\emptyset} = \pi_2^{\emptyset}.$$

Let $k=3$: when firm 3 is the first to move, its optimal choice depends on whether firm 2 makes the ultimatum offer to firm 1 ($c > c^*$) or to firm 3 ($c < c^*$). If $c > c^*$, firm 3 is indifferent between making an offer to firm 1 or to firm 2 since

$$\pi_3^{U_1} = \pi_3^{U_2} \Leftrightarrow \pi_3^{1+2} = \pi_3^{1+2}$$

which is always true. If instead $c < c^*$, firm 3 makes an offer to firm 1 if and only if

$$\pi_3^{U_1} \leq \pi_3^{U_2} \Leftrightarrow \pi_3^{1+2} \leq \pi_3^{\emptyset}.$$

If firm k does not make an offer, firm j (the second to move) will embark on option iii) above, and firm k will end up receiving the ultimatum payoff $\pi_k^{U_i}$. This is dominated by making an offer (because in this case firm k can choose between $\pi_k^{U_i}$ and $\pi_k^{U_j}$). ■

Proof. (Proposition 3): Assume that nature selected firm $i = 1,2,3$ as the target firm and let B_j denote firm j's bid for firm i. If $B_j \geq \pi_i^0$ then firm j's bid, if it is the highest, will be accepted. Therefore, firm $k \neq j$ will best respond by outbidding firm j with $B_k = B_j + \varepsilon$ if and only if

$$\pi_{k+i}^{k+i} - B_k \geq \pi_k^{j+i}$$

which, for $\varepsilon \to 0$, is equivalent to

$$B_j \leq \pi_{k+i}^{k+i} - \pi_k^{j+i}.$$

Therefore, outbidding the rival is the best response by firm k to any $B_j \in \left[\pi_i^0, \pi_{k+i}^{k+i} - \pi_k^{j+i}\right]$. From Lemma 5 i) and ii), this interval is non-empty. Alternatively, if $B_j < \pi_i^0$ then firm j's bid will never be accepted. Therefore, firm k will best respond with $B_k = \pi_i^0$ if and only if

$$\pi_{k+i}^{k+i} - \pi_i^0 \geq \pi_k^0$$

which, from Lemma 5 i), is always true. Thus, outbidding with $B_k = \pi_i^0$ is the best response for $B_j < \pi_i^0$. Summing up, firm k outbids firm j if $B_j \leq \overline{B_j} \equiv \pi_{k+i}^{k+i} - \pi_k^{j+i}$. So, for any target firm i, the acquirer that is willing to outbid higher offers by its rival is firm k if and only if

$$\overline{B_j} > \overline{B_k} \Leftrightarrow \pi_{k+i}^{k+i} - \pi_k^{j+i} > \pi_{j+i}^{j+i} - \pi_j^{k+i} \Leftrightarrow \pi_{k+i}^{k+i} + \pi_j^{k+i} > \pi_{j+i}^{j+i} + \pi_k^{j+i}$$

or, in words, if the industry profits in the $k + i$ merger are higher than those in the $j + i$ merger. If the inequality above holds, firm k will acquire the target firm i with a bid equal to $B_k = \overline{B_k} + \varepsilon$ and firm j will not best-respond by outbidding this offer.

(i) If nature chooses firm 1 as the target firm, the equilibrium merger is 1+3 if and only if

$$\pi_{1+3}^{1+3} + \pi_2^{1+3} > \pi_{1+2}^{1+2} + \pi_3^{1+2} \Leftrightarrow \frac{1}{36} v v_1 \frac{v + c^2 - 1}{v - 1} > 0 \Leftrightarrow c < \sqrt{1 - v}$$

which is implied by $c < c^+ = 1 - v$.

(ii) If nature chooses firm 2 as the target firm, the equilibrium merger is 2+3 if and only if

$$\begin{aligned} \pi_{2+3}^{2+3} + \pi_1^{2+3} > \pi_{1+2}^{1+2} + \pi_3^{1+2} \Leftrightarrow \\ \frac{1}{36} v_1 \frac{-c^2 + 2c(1-v) - (1-2v)(1-v)}{1-v} > 0 \Leftrightarrow \\ 1 - v + \sqrt{v(1-v)} < c < 1 - v - \sqrt{v(1-v)}. \end{aligned}$$

As $c^+ < 1 - v + \sqrt{v(1-v)}$ and $c^- > 1 - v - \sqrt{v(1-v)}$, this is true under assumption 1.

(iii) If nature chooses firm 3 as the target firm, the equilibrium merger is 1+3 if and only if

$$\begin{aligned} \pi_{1+3}^{1+3} + \pi_2^{1+3} > \pi_{2+3}^{2+3} + \pi_1^{2+3} \Leftrightarrow \\ (1-v)\left((1-c)^2 - v\right) > 0 \Leftrightarrow c < 1 - \sqrt{v}. \end{aligned}$$

This completes the proof. ■

Proof. (of Remark 1): The volume market share of firm 2 is larger than the one of firm 1 if and only if $\frac{c}{3(1-v)} > \frac{3(1-v)-c(3-v)}{6(1-v)} \Leftrightarrow v > \frac{3-5c}{3-c}$. Depending on c, $\frac{3-5c}{3-c}$ this may be larger or smaller than $(1-c)^2$, meaning that the $2+3$ merger may reduce consumers' surplus and welfare when the market share of firm 2 is larger than the market share of firm 1 or smaller than the market share of firm 1. In terms of value market share, firm 1's market share always exceeds firm 2's:

$$\frac{vc}{3}\frac{c}{3(1-v)}v_1-\frac{c(v+3)+3(1-v)}{6}\frac{3(1-v)-c(3-v)}{6(1-v)}v_1<0$$
$$\frac{v_1}{36(1-v)}\left(-v^2\left(c^2-6c+9\right)+v\left(4c^2-6c+18\right)+9c^2-9\right)<0$$

which is implied by $c<1-v$. ■

Proof. (of Remark 2) Firm 2 is the one with lower profits in the status quo if

$$\pi_2^{\emptyset}<\pi_1^{\emptyset}\Leftrightarrow\frac{vc^2}{9(1-v)}<\frac{(3(1-v)-c(3-v))^2}{36(1-v)}$$
$$\pi_2^{\emptyset}<\pi_3^{\emptyset}\Leftrightarrow\frac{vc^2}{9(1-v)}<\frac{v(3-c)^2}{36}.$$

These inequalities can be respectively simplified to:

$$-c^2(9-v)+6c(3-v)+9v-9<0$$
$$c^2(v+3)+6c(1-v)+9v-9<0.$$

We have that $c<1-\sqrt{v}$ implies $-c^2(9-v)+6c(3-v)+9v-9<0$ because the expression is maximized at $c=3\frac{v-3}{v-9}>1-v$. So it is increasing in c for $c<c^+$ and, evaluated at $c=1-\sqrt{v}$, it is

$$-\left(1-\sqrt{v}\right)^2(9-v)+6\left(1-\sqrt{v}\right)(3-v)+9v-9$$
$$=\left(\sqrt{v}\right)^2\left(\sqrt{v}+5\right)\left(\sqrt{v}-1\right)<0.$$

We have that $6c(1-v)+c^2(v+3)+9v-9<0$ is always true for $c<1-v$ because

$$6c(1-v)+c^2(v+3)+9v-9<6(1-v)(1-v)+(1-v)^2(v+3)+9v-9$$
$$=-v(v+8)(1-v)<0.$$

This completes the proof. ■