An Optimal Control Approach to the Unit Commitment Problem

F. A. C. C. Fontes* and D. B. M. M. Fontes** and L. A. Roque***

Abstract— **The Unit Commitment (UC) problem is a well-known combinatorial optimization problem arising in operations planning of power systems. It is typically formulated as nonlinear mixed-integer programming problem and has been solved in the literature by a huge variety of optimization methods, ranging from exact methods (such as dynamic programming, branch-and-bound) to heuristic methods (genetic algorithms, simulated annealing, particle swarm). Here, we start by formulating the UC problem as a mixed-integer optimal control problem, with both binary-valued control variables and real-valued control variables. Then, we use a variable time transformation method to convert the problem into an optimal control problem with only real-valued controls. Finally, this problem is transcribed into a finite-dimensional nonlinear programming problem to be solved using an optimization solver.**

Index Terms— **optimal control; calculus of variations; maximum principle; normality; optimality conditions.**

I. INTRODUCTION

In this work, we address the Unit Commitment Problem using Optimal Control methodologies. Despite being an highly researched problem with dynamical and multi-period characteristics, it appears that it has not been addressed by optimal control methods before.

The Unit Commitment (UC) problem plays a key role in power system operations, not only because its optimal scheduling might provide huge gains, but also because it maintains system reliability by keeping a proper spinning reserve. The thermal UC problem can be divided into two subproblems: the mixed-integer nonlinear programming problem of determining the on/off state of the generation units for each time period over a scheduling horizon and the nonlinear programming problem of dispatching the load among on-line units. The UC objective is to minimize the total operating cost of the generating units during the scheduling horizon while satisfying a large set of system and operational constraints. Due to its combinatorial nature, multi-period characteristics, and nonlinearities, this problem is highly computational demanding and, thus, solving the UC problem for real sized systems is a hard optimization task. The UC problem has been extensively studied in the literature. Several numerical optimization techniques, based both on exact or on approximate algorithms have been reported.

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Several approaches based on exact methods have been used, such as dynamic programming, mixed-integer programming, benders decomposition, lagrangian relaxation and branch and bound methods, see e.g. [16], [8], [29], [3]. The main drawbacks of these traditional techniques are the large computational time and memory requirents for large complexity and dimensionality problems. Dynamic programming [16], [23] is a powerful and flexible methodology, however its suffers from the dimensionality problem, not only in computational time, but also in storage requirements. Recently a stochastic dynamic programming approach to schedule power plants was proposed [25]. In [3], a solution using lagrangian relaxation is proposed. However, the problem becomes too complex as the number of units increases and there are some difficulties in obtaining feasible solutions. The branch-and-bound method proposed in [8] uses a linear function to represent the fuel consumption and a time-dependent start-up cost, but has a exponential growth in the computational time with problem dimension.

More recently, several metaheuristic methods such as evolutionary algorithms and hybrids of the them have been proposed, see e.g. [31], [10], [27], [6], [2]. These approaches have, in general, better performances than the traditional heuristics. The most commonly used metaheuristic methods are simulated annealing [22], [27], evolutionary programming [14], [24], memetic algorithms [31], particle swarm optimization [35], tabu search [21], [32], and genetic algorithms [15], [28], [9], [26]. For further discussion and comparison of these methodologies, with special focus on metaheuristic methods see [26].

Although the UC problem is a highly researched problem with dynamical and multi-period characteristics, it appears that it has not been addressed before by optimal control methods, except trying to solve the dynamic programming recursion as mentioned. The nearest problems that we have found in the literature using optimal control methods are for the dispatch problem (see [34] and references therein) which considers that it is already decided which are the units that are on.

So, the formulation of the UC problem as a mixed-integer optimal control problem, given in section II, is novel. However, the main contribution is the variable time transformation method, described in Section III, which converts the mixed-integer optimal control problem (OCP) into one with only real-valued controls.

Most literature on optimal control deals with problems with only real-valued controls, both the analytical methods based on variational analysis (see e.g. [33], [7]) and also numerical schemes ([5], [4]). There are, however, some works that are able to address optimal control problems (OCP) with discrete control sets (see e.g. [11], [20]), although dealing directly with the discrete-valued controls is computationally demanding. The transformation of a mixed-integer optimal control problem into a problem with only real-valued controls is not new, nor is new the general idea of a variable time transformation method. See the classical reference [13] and also [30], [18], [19], [1], [17]. See also the recent work [12] for a discussion of several variable time transformation methods.

Although the approach in [12] could, in principle, be used for the UC problem, it would require explicit enumeration of all possible combinations of the control set, which in our case would lead to 2^N (with N being the number of thermal units) discrete control values and would be impractical to solve. So, we can consider our variable time transformation a suitable modification of [12] when the discrete-valued controls are not scalar, that is valid for the UC problem.

II. The Unit Commitment Problem

The Unit Commitment Problem involves both the scheduling of power units (i.e the decision when each unit is turned on or turned off along a predefined time horizon), and the economic dispatch problem (the problem of deciding how much each unit that is on should produce). The scheduling of thermal units is an integer programming problem and the economic dispatch problem is a nonlinear (real-valued) programming problem. The UC problem is then as a nonlinear, non-convex and mixed integer combinatorial optimization problem [9]. The objective of the UC problem is the minimization of the total operating costs over the scheduling horizon while satisfying the system demand, the spinning reserve requirements and other generation constraints such as the capacity limits, ramp rate limits and minimum up/down times. The objective function is expressed by the sum of the fuel and start-up costs. The model has two types of control variables. On the one hand, binary control variables $u_j(t)$, which are either set to 1, meaning that unit j is committed at time period t; or otherwise are set to zero. On the other hand, real valued variables $\Delta_j(t)$, which enable to control the amount of energy produced by unit j at time period t. Before giving the mathematical formulation let us introduce the parameters and variables notation.

Indexes:
t: Time period index;
j: Generation unit index;

For convenience, let us also define the index sets:
$\mathcal{T} := \{1, \dots, T\}$
$\mathcal{J} := \{1, 2, \dots, N\}$

Decision/Control Variables:
$\Delta_j(t)$: Amount of thermal generation of unit j to be incremented or decremented comparatively to the production at the previous time period ;
$\mathbf{u_j(t)}$: Status of unit j at time period t (1 if the unit is on; 0 otherwise);

State Variables:
$\mathbf{y_j(t)}$: Thermal generation of unit j at time period t, in $[MW]$;
$\mathbf{T_j^{on/off}(t)}$: Number of time periods for which unit j has been continuously on-line/off-line until time period t, in $[hours]$;

Parameters:
$\mathbf{T}$: Number of time periods (hours) of the scheduling time horizon;
$\mathbf{N}$: Number of generation units;
$\mathbf{R(t)}$: System spinning reserve requirements at time period t, in $[MW]$;
$\mathbf{D(t)}$: Load demand at time period t, in $[MW]$;
$\mathbf{Ymin_j}$: Minimum generation limit of unit j, in $[MW]$;
$\mathbf{Ymax_j}$: Maximum generation limit of unit j, in $[MW]$;
$\mathbf{T_{c,j}}$: Cold start time of unit j, in $[hours]$;
$\mathbf{T_{min,j}^{on/off}}$: Minimum uptime/downtime of unit j, in $[hours]$;
$\mathbf{S_{H/C,j}}$: Hot/Cold start-up cost of unit j, in [\$];
$\Delta_j^{dn/up}$: Maximum allowed output level decrease/increase in consecutive periods for unit j, in $[MW]$;

A. *Objective Function*

The objective of the UC problem is the minimization of the total cost for the whole planning period, in which the total cost is expressed as the sum of fuel and start-up costs of the generating units. Therefore, the objective function is as follows:

Minimize

$$\sum_{t=1}^{T}\left(\sum_{j=1}^{N}\{F_j(y_j(t))u_j(t)+S_j(t)(1-u_j(t-1))u_j(t)\}\right) \tag{1}$$

where, the generation costs, i.e. the fuel costs, are conventionally given by a quadratic cost function as follows:

$$F_j(y_j(t)) = a_j \cdot (y_j(t))^2 + b_j \cdot y_j(t) + c_j, \tag{2}$$

with the cost coefficients a_j, b_j, c_j of unit j. The start-up costs, that depend on the number of time periods during which the unit has been off, are given by

$$S_j(t) = \begin{cases} S_{H,j}, & \texttt{if } T_{min,j}^{off} \le T_j^{off}(t) \le T_{min,j}^{off} + T_{c,j}, \\ S_{C,j}, & \texttt{if } T_j^{off}(t) > T_{min,j}^{off} + T_{c,j}. \end{cases} \tag{3}$$

B. *The state dynamics*

The state dynamics in this model are as follows:

The thermal production of each unit, at time period t, depends of the amount of thermal production in previous time period and is limited by the maximum allowed decrease and increase of the output that can occur during one time period

$$y_j(t) = [y_j(t-1) + \Delta_j(t)]\,.u_j(t), \tag{4}$$

for $t \in \mathcal{T}$ and $j \in \mathcal{J}$, with $\Delta_j(t) \in \left[-\Delta_j^{dn}, \Delta_j^{up}\right]$. The number of time periods continuously on-line until time period t is given by

$$T_j^{on}(t) = \left[T_j^{on}(t-1)+1\right].u_j(t), \qquad (5)$$

for $t \in \mathcal{T}$ and $j \in \mathcal{J}$ The number of time periods continuously off-line until time period t is given by

$$T_j^{off}(t) = \left[T_j^{off}(t-1)+1\right].(1-u_j(t)), \qquad (6)$$

for $t \in \mathcal{T}$ and $j \in \mathcal{J}$.

C. Constraints

The constraints can be classified into two sets: the demand constraints and the operational generator constraints. The first set of constraints is composed by the load requirements and spinning reserve requirements, which can be written as follows:

1) Power Balance Constraints

The total amount of power generated at each time period must meet the load demand.

$$\sum_{j=1}^{N} y_j(t)\cdot u_j(t) \geq D(t), t \in \{1,2,...,T\}. \qquad (7)$$

2) Spinning Reserve Constraints

The spinning reserve is the total amount of real power generation available from on-line units net of their current production level. The reserve power available is used when a unit fails or an unexpected increase in load occurs.

$$\sum_{j=1}^{N} Ymax_j(t)\cdot u_j(t) \geq R(t)+D(t), t \in \{1,2,...,T\}. \qquad (8)$$

The second set of constrains includes unit capacity limits and the minimum number of time periods that the unit must be continuously in each status (on-line or off-line).

3) Unit Capacity Limits

Each unit has a maximum and minimum output limits.

$$\begin{aligned} &Ymin_j \cdot u_j(t) \leq y_j(t) \leq Ymax_j \cdot u_j(t), \\ &\text{for } t \in \{1,2,...,T\} \text{ and } j \in \{1,2,...,N\}. \end{aligned} \qquad (9)$$

4) Minimum Uptime/Downtime Constraints

The unit cannot be shut down or started-up instantaneously once it is committed or decommitted. The minimum up-time/downtime constraints indicate that there is a minimum time that each unit must be on-line or off-line, respectively.

$$\begin{aligned} &\left(T_j^{on}(t-1) - T_{min,j}^{on}\right)\cdot(u_j(t-1)-u_j(t)) \geq 0, \\ &\text{for } t \in \{1,2,...,T\} \text{ and } j \in \{1,2,...,N\}. \end{aligned} \qquad (10)$$

$$\begin{aligned} &\left(T_j^{off}(t-1) - T_{min,j}^{off}\right)\cdot(u_j(t)-u_j(t-1)) \geq 0, \\ &\text{for } t \in \{1,2,...,T\} \text{ and } j \in \{1,2,...,N\}. \end{aligned} \qquad (11)$$

5) Initial state constraints

The values of $T_j^{off}(0)$ and $T_j^{on}(0)$ are given for the initial time. The values of $u_j(0)$ are defined accordingly (i.e. set to one if $T_j^{on}(0)$ is positive, set to zero otherwise). The values of $y_j(0)$ can be chosen satisfying

$$Ymin_j \cdot u_j(0) \leq y_j(0) \leq Ymax_j \cdot u_j(0). \qquad (12)$$

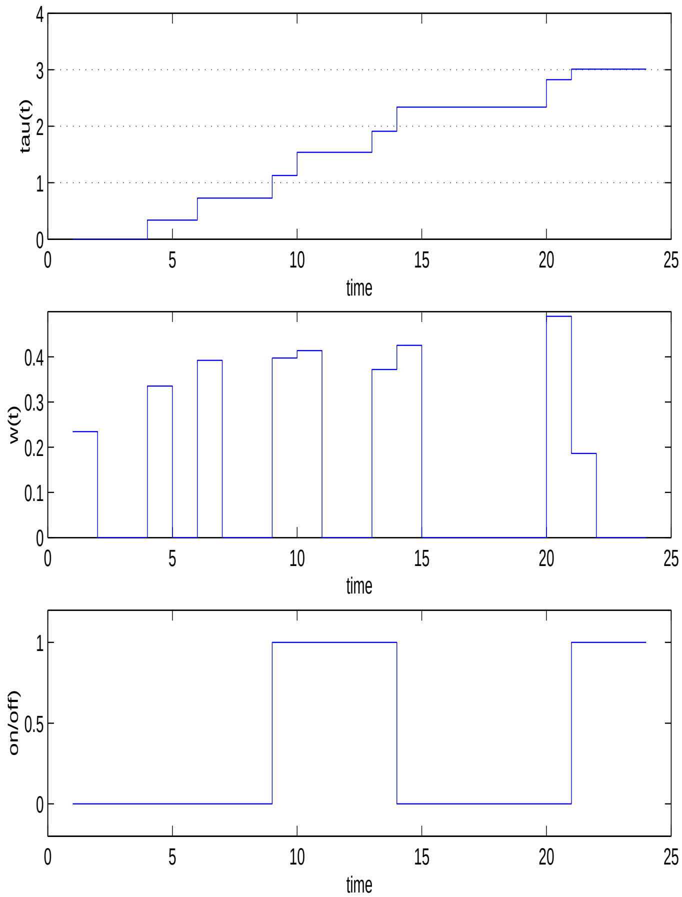

Fig. 1. Example of a function τ, the corresponding on/off status, and the corresponding function w

III. The Variable Time Transformation Method

The idea here is to develop a variable time transformation in order to convert the mixed-integer OCP into an OCP with only real-valued controls.

Consider, for each unit j, a non-decreasing real-valued function $t \mapsto \tau_j(t)$. Consider also a set of values $\bar{\tau}_1, \bar{\tau}_2, \ldots$ such that when $\tau_j(t) = \bar{\tau}_k$ for odd k we have a transition from off to on in unit j, and when $\tau_j(t) = \bar{\tau}_k$ for even k we have a transition from on to off. So, we consider that unit j is:

- on if $\tau_j(t) \in [\bar{\tau}_1, \bar{\tau}_2) \cup [\bar{\tau}_3, \bar{\tau}_4) \cup \ldots \cup [\bar{\tau}_{2k-1}, \bar{\tau}_{2k})$;
- off if $\tau_j(t) \in [0, \bar{\tau}_1) \cup [\bar{\tau}_2, \bar{\tau}_3) \cup \ldots \cup [\bar{\tau}_{2k}, \bar{\tau}_{2k+1})$.

An illustrative example is shown in Fig. 1.

It might help to interpret τ_j to be a transformed time scale and that the values $\bar{\tau}_1, \bar{\tau}_2, \ldots$ are switching "times" in the transformed time scale. We can consider, without loss of generality, that the values $\bar{\tau}_k$ are equidistant. Nevertheless, in real time t, the distance between two events $\bar{\tau}_k$ and $\bar{\tau}_{k+1}$ can be stretched or shrunk to any non-negative value, including zero, depending on the shape of the function $t \mapsto \tau_j(t)$.

To simplify exposition, and without loss of generality, let us consider that $\bar{\tau}_k - \bar{\tau}_{k-1}$ is constant and equal to 1, for all $k = 1, 2, \ldots$. In such case, unit j is:

- on if $\tau_j(t) \in [1,2) \cup [3,4) \cup \ldots \cup [2k-1, 2k)$;
- off if $\tau_j(t) \in [0,1) \cup [2,3) \cup \ldots \cup [2k, 2k+1)$.

Now, consider that we have the controls

$$w(t) \in [0,1], \quad t = 0, 1, \ldots, T-1,$$

that represent the increment from $\tau(t)$ to $\tau(t+1)$ such that

$$\tau(t) = \tau_0 + \sum_{k}^{t-1} w_(k)$$

or

$$w(t) = \tau(t+1) - \tau(t), \quad \text{with} \quad \tau(0) = \tau_0.$$

Possible values of w in our example are shown in Fig. 1.

A. UC problem as an Optimal Control Problem with real-valued controls

We recall the index set $\mathcal{J}$ and redefine $\mathcal{T}$ to be more consistent with usual discrete-time control formulations.

$\mathcal{T} := \{0, \ldots, T-1\}$ and $\mathcal{J} := \{1, 2, \ldots, N\}$.

In the same spirit, we redefine the control $\Delta_j(t)$ for $t \in \{0, \ldots, T-1\}$ to be the amount of thermal generation incremented or decremented in the next time period (rather than comparatively to the previous period).

We should note that our controls are all real-valued and comprise:
$\Delta_j(t) \in \left[-\Delta_j^{dn}, \Delta_j^{up}\right]$,
$w_j(t) \in [0,1]$.

Define the sets of time periods:

$I_j^{on} := \{t \in \mathcal{T} : \tau_j(t) \in [2k-1, 2k), k \geq 1\}$,
$I_j^{off} := \mathcal{T} \setminus I_j^{on}$,
$I_j^{off>on} := \{t \in \mathcal{T} : \tau_j(t) \geq 2k+1, \tau_j(t-1) < 2k+1, k \geq 0\}$,
$I_j^{on>off} := \{t \in \mathcal{T} : \tau_j(t) \geq 2k, \tau_j(t-1) < 2k, k \geq 1\}\}$.

Finally, we are able to formulate our OCP:

Minimize

$$\sum_{j=1}^{N} \left(\sum_{t \in I_j^{on}} F_j(y_j(t)) + \sum_{t \in I_j^{off>on}} S_j(t) \right), \tag{13}$$

subject to the dynamic constraints

$$\tau_j(t+1) = \tau_j(t) + w_j(t) \quad j \in \mathcal{J},\ t \in \mathcal{T}, \tag{14}$$

$$T_j^{on}(t+1) = \begin{cases} T_j^{on}(t) + 1 & j \in \mathcal{J},\ t \in I_j^{on}, \\ 0 & j \in \mathcal{J},\ t \in I_j^{off}, \end{cases} \tag{15}$$

$$T_j^{off}(t+1) = \begin{cases} T_j^{off}(t) + 1 & j \in \mathcal{J},\ t \in I_j^{off}, \\ 0 & j \in \mathcal{J},\ t \in I_j^{on}, \end{cases} \tag{16}$$

$$y_j(t+1) = \begin{cases} y_j(t) + \Delta_j(t) & j \in \mathcal{J},\ t \in I_j^{on}, \\ 0 & j \in \mathcal{J},\ t \in I_j^{off}, \end{cases} \tag{17}$$

the initial state constraints

$$T_j^{on}(0) = T_{j,0}^{on} \quad \text{(given)}, \tag{18}$$

$$T_j^{off}(0) = T_{j,0}^{off} \quad \text{(given)}, \tag{19}$$

$$\tau_j(0) = \begin{cases} 0 & \text{if } T_{j,0}^{on} = 0 \\ 1 & \text{if } T_{j,0}^{on} > 0, \end{cases} \tag{20}$$

$$y_j(0) = \begin{cases} 0 & \text{if } T_{j,0}^{on} = 0 \\ y_{j,0} \in [Ymin_j, Ymax_j] & \text{if } T_{j,0}^{on} > 0, \end{cases} \tag{21}$$

the control constraints

$$\Delta_j(t) \in \left[-\Delta_j^{dn}, \Delta_j^{up}\right], \tag{22}$$

$$w_j(t) \in [0,1], \tag{23}$$

and the pathwise state constraints

$$y_j(t) \in [Ymin_j, Ymax_j] \quad j \in \mathcal{J},\ t \in I_j^{on}, \tag{24}$$

$$\sum_{j \in \mathcal{J}} y_j(t) \geq D(t) \quad t = 1, 2, \ldots, T, \tag{25}$$

$$\sum_{j \in \mathcal{J}} Ymax_j(t) \geq R(t) + D(t) \quad t = 1, 2, \ldots, T, \tag{26}$$

$$y_j(t) \in [Y_{min_j}, \max\{Y_{min_j}, \Delta_j^{up}\}] \quad j \in \mathcal{J}, t \in I_j^{off>on}, \tag{27}$$

$$T_j^{on}(t-1) \geq T_{min,j}^{on} \quad j \in \mathcal{J},\ t \in I_j^{on>off}, \tag{28}$$

$$T_j^{off}(t-1) \geq T_{min,j}^{off} \quad j \in \mathcal{J},\ t \in I_j^{off>on}. \tag{29}$$

An example of a possible realization of the functions $\Delta_j, w_j, \tau_j, T_j^{on}, T_j^{off}$, and y_j for a specific unit j is shown in Fig. 2.

IV. Convertion into a Nonlinear Programming Problem

To construct our nonlinear programming problem (NLP), we start by defining the optimization variable x containing both the control and state variables. That is

$$x = [\Delta, w, \tau, T^{on}, T^{off}, y]$$

with dimension $(6T+1) * N$). (We could have considered just the controls Δ, w together with the free initial state $y(0)$. An option which, despite having the advantage of a lower dimensional decision variable, is known to frequentely have robustness problems, specially in OCPs with pathwise state constraints such as ours. For further dicussion see e.g. Betts [4].)

To facilitate the optimization algorithm, we separate the constraints that are simple variable bounds, linear equalities, linear inequalities and the remaining:

- upper/lower bounds: equations (22)-(25);
- linear equalities: (15);
- linear inequalities: (26);
- nonlinear equalities: (16)-(18); and
- nonlinear inequalities: (27)-(30).

Note that (19)-(21) are not implemented as constraints since the initial values of these state variables are considered as parameters and not variables.

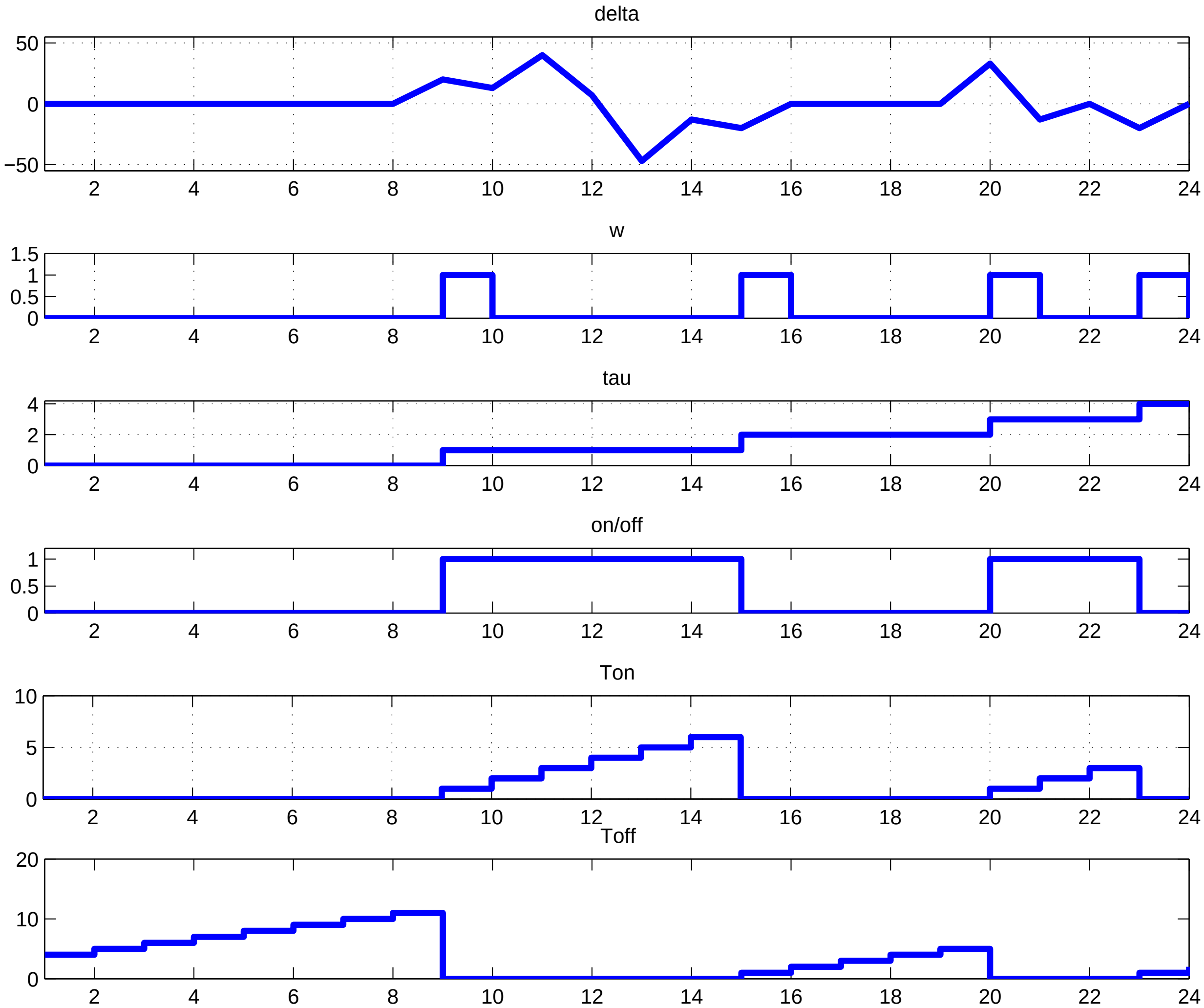

Fig. 2. Example of possible functions $\Delta_j, w_j, \tau_j, T_j^{on}, T_j^{off}$, and y_j

With these considerations the problem is formulated as the following NLP

$$
\begin{aligned}
&\text{Minimize}_{x\in\mathbb{R}^{(6T+1)\times N}} J(z)\\
&\text{subject to}\\
&\quad LB \leq x \leq UB\\
&\quad A_{eq}x = b_{eq}\\
&\quad A_{ineq}x \leq b_{ineq}\\
&\quad g(z) = 0\\
&\quad h(z) \leq 0.
\end{aligned}
$$

Of course, since this (real-valued) NLP is a problem that originally was a MI-NLP, it is still a very hard problem. Namely, it is a nonconvex problem and standard NLP solvers will find just a local, not necessarily global, optimum. The numerical solution of this NLP for some benchmark UC problem instances will be reported in another publication.

V. CONCLUSIONS

We have addressed the Unit Commitment problem using optimal control methods, which appears that has not been done previously. In order to solve the mixed-integer optimal control problem (OCP), we have converted it into another OCP with only real-valued controls. This process required a novel variable time transformation that was able to address adequately several discrete-valued control variables arising in the original problem formulation. The transformed real OCP was transcribed into a nonlinear programming problem to be solved by a nonlinear optimization solver.

VI. ACKNOWLEDGMENTS

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