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REGULAR ARTICLE

Innovation diffusion with heterogeneous networked agents: a computational model

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Abstract It is well established that S-shaped curves describe the diffusion processes of many innovations quite well, but little insight on the mechanics of diffusion is achieved by simple curve fitting. We propose an evolutionary model of the diffusion process, focusing on the characteristics of economic agents and on the interactions among them, and relate those determinants with the observed shape of the diffusion curve. Using simulation techniques, we show that the proposed model is able to explain why an innovation may not diffuse globally across an economy/region, even when it faces no rival innovations. Moreover, we show how network size, informational spillovers, and the behavior of innovation prices shape the diffusion process. The results regarding network size and informational spillovers rationalize the importance of informational lock-outs, proving they can influence both the aggregate adoption rate and the speed of the diffusion process. With respect to innovation prices, simulation results show that faster price decline leads to higher aggregate adoption rates, and that the diffusion process is more sensitive to the pricing dynamics than to the network size or the behavior of spillovers.

Keywords Innovation diffusion · Evolutionary agent-based models · Network effects · Knowledge spillovers

JEL Classification O33

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"…without diffusion, innovation would have little social or economic impact" (Hall 2005: 459).

1 Introduction

Diffusion has been defined as a process by which the market for a new technology changes over time, and from which production and usage patterns of new products/services and production processes result (Geroski 2000; Stoneman and Battisti 2010). In other words, diffusion is the mechanism that makes innovations useful by spreading them throughout a population (Rogers 1985). It is an intrinsic part of the innovation process, given that the learning, imitation, and feedback effects that arise during the spread of a new technology contribute to enhance the original innovation (Hall 2005).

Although interrelated, adoption and diffusion are distinct concepts (Metcalfe 1988). The former involves the decision taken by agents to incorporate a new technology into their activities—it is concerned with the process of decision making. The latter is concerned with how the economic significance of a new technology (e.g., market share) changes over time. Nevertheless, both concepts integrate the diffusion process. Comprehending this process is the key to understand how conscious innovative activities carried out by firms and governmental institutions—activities such as funding research and development, transferring technology, launching new products or creating new processes—produce the improvements in economic and social welfare that are usually the end goal of such activities.

Recently Peres et al. (2010) suggest that the diffusion framework, if it is to remain a state-of-the-art paradigm for market evolution, must broaden in scope as to encompass social influences, which include all the inter-dependencies among agents that affect various market players, with or without their explicit knowledge. According to these authors the role of network structure in diffusion is still in its infancy, and from a modeling perspective the key issue is how to incorporate the social network into the diffusion model. Earlier models (e.g., Bass 1969) assumed that the social system is homogenous and fully connected, and thus could be adequately represented at the aggregate level. These aggregate models, although parsimonious (Parker 1994), provide little intuition as to how individual market interactions are linked to global market behavior.

Recent empirical research on social networks revealed that social systems are neither homogenous nor fully connected (Kossinets and Watts 2006), urging diffusion research to gradually extend its focus from the aggregate level to an individual-level perspective (Peres et al. 2010). Agent-based modeling or multi-agent simulation, which describes the market as a collection of individual elements (units, agents, or nodes) interacting with each other through connections (links), emerges as a key tool for overcoming some of the limitations of aggregate-level diffusion models. First, by establishing a connection between individual-level influences and aggregate effects, it enables the researcher to better relate individual-level activities to firm/agents performance, which is measured at the aggregate level. Second, this modeling approach enables one to distinguish between inter-dependencies, namely to explore network externalities (Goldenberg et al. 2010). Third, this type of model allows for heterogeneity by enabling individual susceptibility to influences to differ across units, or by setting different link structures for each unit.

Therefore, using simulation methods to study innovation diffusion is an approach capable of shedding new light on the mechanics underlying the diffusion processes observed in reality. Through simulation we are able to understand how a set of factors (e.g., network size, knowledge externalities, prices) influences the diffusion of an innovation when all the other remain constant, hence gaining a deeper knowledge of what really shapes the diffusion process. The works of Deroïan (2002) and Lim et al. (2003) are examples of this line of research. The present paper draws upon this spirit, intentionally adopting an evolutionary perspective, and its contribution is twofold. First, we put forth a variation of the epidemic model which is able to generate—and explain—situations in which an innovation is not adopted by all the agents of an economy/region. This brings into the epidemic models an outcome typically found in evolutionary multi-innovation diffusion models (e.g. Silverberg et al. 1988). Second, it also provides an insight on how the diffusion process of an innovation is shaped by the size of the agents" contact networks, by the formation and absorption of knowledge spillovers, and by the innovation pricing dynamics.

In Sect. 2 we frame our model into the relevant literature, providing a full description of its assumption and characteristics in Sect. 3. Section 4 presents the simulation results, and in our concluding remarks we highlight the main contributions of the research.

2 Relevant literature

Since the pioneering studies of Griliches (1957) and Mansfield (1961), the analysis of the diffusion of innovations within an economic system has been recognized by economists and other scientists (e.g., Marchetti 1991) as central to understand the contribution of technical progress to economic growth. A relevant body of research has investigated the issue, focusing on the theoretical arguments explaining “S-shaped” diffusion curves and on the determinants of the speed of diffusion. The traditional theories have been the subject of growing criticism (Davies 1979; Gold 1981; Cainarca et al. 1989). Alternative models have been suggested, some trying to revitalize the neoclassical tradition (e.g., Reinganum 1981; Balcer and Lippman 1984; Ireland and Stoneman 1986; Stoneman and Ireland 1986), some others instead emphasizing uncertainty, bounded rationality and disequilibrium (e.g., Abernathy and Utterback 1975; Rosenberg 1976; Metcalfe 1981; Dosi 1982; Nelson and Winter 1982; Gibbons and Metcalfe 1986; Silverberg et al. 1990; Loch and Huberman 1999). A distinguishing feature of these latter contributions is their sharing of a Schumpeterian view of the dynamics of the innovation-diffusion process.

In the present paper we put forward a computational diffusion model with heterogeneous agents, living on (random) networks, with informational externalities and learning-by-using on the demand side and with learning-by-doing on the supply side. Our research brings together several strands of the economics and agent-based modeling literature, including evolutionary models of disequilibrium diffusion, information contagion, informational cascades, network externalities, and dynamic increasing returns. Our model follows the tradition of evolutionary models of technological change introduced by Nelson and Winter (1982).

Building on Nelson and Winter (1982), Silverberg et al. (1990: 75) have defined technology diffusion in the light of the evolutionary approach as "the diffusion of techniques and new products under conditions of uncertainty, bounded rationality and endogeneity of market structures as a disequilibrium process". The model presented in this paper is similar in spirit to the one developed (and analyzed via simulation) by those authors. Their model focuses on firm market shares and pioneer advantage, depending on the amount of external (industry-wide) learning, whereas our model emphasizes the social interactions between agents who are heterogeneous in their capacity to manage networks, being network sizes agent-specific and randomly determined for each agent.

Our model comprises a single technology being diffused, in line with earlier work by Lekvall and Wahlbin (1973), and in clear contrast with other evolutionary models which analyze a scenario of competing technologies (e.g., Silverberg et al. 1990; Loch and Huberman 1999). Nevertheless, our model is capable of explaining why an innovation may not diffuse globally across an economy/region, even when it faces no rival innovations. In short, within a parsimonious framework our model can account for the lock-in (Arthur 1989) and path dependence (David 1994) in technology adoption that evolutionary economists call for. Thus, our model is also related to path dependence models with positive externalities, or increasing returns (Arthur 1989, 1994; David 1994).

Path dependence models emphasize that small initial advantages may determine which one of (possibly several) competing technologies is chosen by the user community. Path dependence may have several roots (David 1994): it may result from historically grown mutually consistent expectations, from "sunk" investments in channels and coordination protocols, or, as in our model, from positive externalities (benefits resulting from other users adopting the same technology, see Granovetter 1978; Cusumano et al. 1992). Such positive externalities are also referred to as "bandwagon" adoptions (Abrahamson and Rosenkopf 1997).

Additionally, our work contributes to bridge economics and sociology literature in what regards the diffusion of technologies, and it does so by taking into account the contagion process ocurring in the innovation diffusion, in line with Shih (2008) and Cantono and Silverberg (2009). It is known that agents tend to be affected by the opinions and behaviors of other agents belonging to a cohesive group or occupying a position of structural equivalence (Burt 1987). Such influence process is referred to as the contagion effect (Huang et al. 2011). Diffusion, being an exceptional form of communication, involves participants providing and sharing information; that is, it involves spreading certain innovation information by participants in a social system through particular channels (Rogers 1985).

As referred by Lundvall (1992), the production and diffusion of new knowledge occurs in the mutual learning of members, and that is conducive to the development and diffusion of new technology. On the other hand, social influence occurs when actor behavior, attitudes, or beliefs involuntarily follow those of others in the same social system (Leenders 1995), and contagion is often used to describe the processes involved in social influence. In other words, social contagion arises from actor proximity in social structure using one another to manage the uncertainty of innovation (Burt 1987). Thus, social contagion is based on the interpersonal synapse over which innovation is transmitted.

The related ideas of contagion effects in innovation diffusion have been extensively employed to study electronic commerce diffusion (Shih 2008), science and engineering technology diffusion (Harkola and Greve 1995; Cantono and Silverberg 2009), efficiency learning and diffusion networks (Chiffoleau 2005). The present study deploys a model with (randomly) networked, interacting heterogeneous agents who—although exhibiting limited capacity to manage information gathered from their networks—obtain knowledge about the features of the innovation and how to use it efficiently from the informational externalities that spill over from other agents.

Our work is also related to the theoretical literature on informational cascades (Bikhchandani et al. 1992; Spiwoks et al. 2008) according to which agents who make decisions based on their own private information are, nevertheless, influenced by others" decisions too. Due to agents" tendency to derive information from the behavior of prior adopters, agents converge on adopting a behavior with increasing momentum and declining individual evaluation of the merits of the behavior (Golder and Tellis 2004).

Thus, our model combines two types of social influences that have garnered recent interest (Peres et al. 2010): social signals related with informational cascades (social information that individuals infer from adoption of an innovation by others) and network externalities, which occur when the utility of a technology to an agent increases as more agents adopt the new technology (Rohlfs 2001).

Although network influences and network externalities have been extensively researched in economics, the emphasis has been mainly on the state of equilibrium rather than on the dynamic path towards that steady state (Jackson 2008). Most evolutionary economists would react with discomfort to this notion as a characteristic of real markets. This is not to say that the equilibrium concept is unimportant, but rather that it is incomplete. First, the path to equilibrium is just as important as the equilibrium itself, especially when the path might take an inordinate amount of time. Second, because of the Schumpeterian "creative destruction" process, in which an innovation destructively changes economic and business environments, equilibrium is never reached in most scenarios.

Summing up, our model follows the tradition of evolutionary models of technological change—characterized by uncertainty, bounded rationality, increasing returns, path dependency, and far from equilibrium dynamics—but enhances that framework with: (i) the structure and dynamics of social networks composed of heterogeneous agents who are not fully interconnected; (ii) learning-by-using and learning-by-doing in demand and supply sides, respectively; (iii) the existence of informational cascades that result from the network effects.

3 A model of innovation diffusion: accounting for a kaleidoscope of influences

In our model, a single innovation diffuses across a population of N heterogeneous agents who establish connections among themselves. Agents are heterogeneous along many dimensions: they differ in their initial wealth, in their ability to manage networks, and also in their ability to process any information they collect from those networks.

Agents derive utility from using the innovation and from holding wealth. The utility each agent obtains by using the innovation is linked to the knowledge he/she has about it. As such, agents who have not yet adopted the innovation turn to their networks to obtain information about it. Because agents have to purchase the innovation in order to benefit from it, innovation adoption entails a loss of utility on the amount of wealth spent to pay for the price asked. To make their adoption decisions, agents consider both the loss of utility on wealth and the potential utility gain associated with using the innovation.

3.1 Network formation and management

Agents in the economy live on variable-sized random networks. The networks are agent-specific, in the sense that if agent j is a node in the network of agent i, the reverse is not necessarily true. Agent j may or may not choose to include agent i in his/her network.[1]

In line with much of the evolutionary economics literature, we assume that agents have a limited capacity to manage information (e.g., Simon and Egidi 1992). The number of nodes in the network of agent i is bounded and determined by his/her capacity to manage a network, s_i. For example, if $s_i = 2$ then agent i will establish connections with two other distinct agents. They become the nodes of his/her network. The discrete-valued capacity s_i is different across agents, and it is randomly drawn from the interval $[1, S]$ according to a discrete uniform distribution, $s_i \sim U(1, S)$. Network sizes in the economy therefore vary from 1 node to S nodes, and the average network size increases with S.

Agents can update their networks at any moment, and there is no limit to the number of times they may do so. We assume that agents have a probability Φ of updating their networks, and we take Φ as being time and state-invariant, and equal for all agents. Whenever agent i performs a network update, he disconnects from one randomly-selected node of his/her network and uses the spare capacity to establish a new link. The new node is selected from the set of agents that were not part of agent i's network before the update.

3.2 Learning-by-using and knowledge spillovers

Agents begin to use the innovation once they adopt it. We assume that as a byproduct of that usage, agents accumulate knowledge about how to be more efficient in handling the innovation. As this learning-by-using process develops, the experience level of agent i increases monotonically with the amount of time that has passed since he/she adopted the innovation, Z_i. The self-acquired experience level of an adopter, $E_{i,t}$, is normalized to take values over the interval $[0, 1]$, and starting from 0 it approaches the value 1 asymptotically. More specifically, we assume that at time t the experience of an agent i who has adopted the innovation is given by

$$E_{i,t} = 1 - \gamma \frac{1}{Z_i\left(t, \overline{t_i}\right)}, \quad t > \overline{t_i}, \tag{1}$$

[1] In other words, agents establish one-way observation networks.

where $\gamma \leq 1$ is a constant common to all agents, t is the current period, $\overline{t_i}$ is the period when agent i made the decision to adopt the innovation, and $Z_i\left(t, \overline{t_i}\right) = t - \overline{t_i}$.

We assume that an adopter generates knowledge spillovers to other agents, and that such spillovers originate in his/her own experience with the innovation. This assumption reflects the idea that other agents may, if they choose to do so, try to mimic the knowledge displayed by those who are already adopters. For simplicity, we assume that the spillover generated by agent i is equal to his/her own self-acquired experience level, $E_{i,t}$, gained through the learning-by-using process. We assume that knowledge obtained by observing other agents is not suitable to be transformed into a valid spillover.[2]

The total information available in agent i's network, A_i, is therefore the sum of the spillovers generated by each one of the nodes,

$$A_i = \sum_{i=j_1}^{j_{s_i}} E_{i,t}, \tag{2}$$

where $\left\{j_1, j_2, \ldots, j_{s_i}\right\}$ are the nodes that constitute agent i's network, and $E_{i,t}$ is the spillover being generated by each one of those nodes at time t. If the agent in one node has not yet adopted the innovation, his/her own experience with the innovation is none, and as a result his/her spillover takes the value 0.

Agents are heterogeneous in their ability to process the information that is available through their networks. Some agents are capable of processing all the information available, while others may only be capable of processing a fraction of it. Denoting the information-processing capacity by l_i, the *effective* amount of knowledge agent i obtains from his/her network, A_i^*, is given by

$$A_i^* = l_i \sum_{i=j_1}^{j_{s_i}} E_{i,t}, \tag{3}$$

where l_i takes values over the interval $[0, 1]$ according to a uniform distribution, $l_i \sim U(0, 1)$. We assume that adopting and using the innovation helps agents to consolidate the knowledge they obtain from their networks. As such, agents permanently store the knowledge they obtained from their networks at the time of adoption. Those who do not adopt the innovation at time t forget such knowledge, and in period $t + 1$ they have to re-process the pool of spillovers that is available through their networks.

If agent i has not yet adopted the innovation, then the total knowledge he/she has about it, $K_{i,t}$, comes only from processing information that spills over from the nodes in his/her network. If that agent has already adopted the innovation, then his/her total knowledge is the sum of the self-acquired experience and of the knowledge he/she kept from his network

[2] An agent needs to have "hands-on" experience with the innovation before he can serve as a reference to other agents.

$$K_{i,t} = E_{i,t} + l_i \sum_{i=j_1}^{j_{s_i}} E_{i,\overline{t_i}}. \tag{4}$$

3.3 Adoption and usage of the innovation

Agents are endowed with an initial level of wealth which they can use to purchase the innovation, or simply keep from period to period. We assume that cash balances do not earn interest. The initial wealth of agent i is denoted by w_i, and it is randomly drawn from the interval [0, 1] according to a uniform distribution, $w_i \sim U(0, 1)$. Agents derive an utility flow $U_{i,t}^1$ from any unspent wealth according to

$$U_{i,t}^1 = g\left(\lambda_{i,t}\right) = \overline{g}\lambda_{i,t}, \tag{5}$$

where $\lambda_{i,t}$ denotes the amount of unspent wealth of agent i at time t, and we set $\overline{g} = 1$. If agent i has not yet purchased the innovation, $\lambda_{i,t} = w_i$. If he/she has already purchased the innovation then $\lambda_{i,t} = w_i - P_{\overline{t_i}}$, where $P_{\overline{t_i}}$ denotes how much agent i paid for the innovation at the time of adoption.

In addition to utility from holding wealth, agents also derive utility from using the innovation, which we denote by $U_{i,t}^2$. The utility flow agent i obtains from the innovation depends on his/her knowledge of its features, and on his/her knowledge about how to use it efficiently. For simplicity, we specify a linear relationship between the utility derived from the innovation and the stock of knowledge about it

$$U_{i,t}^2 = f\left(K_{i,t}\right) = \overline{f}K_{i,t}, \tag{6}$$

where we set $\overline{f} = 1$. As explained before, for agents who have not yet adopted the innovation K_i is simply equal to the knowledge they obtain from their network, according to Eq. (3).

Thus, considering the utility functions on cash balances and on innovation usage described above, for a non-adopter agent i the total utility flow at time t, $U_{i,t} = U_{i,t}^1 + U_{i,t}^2$, is summarized by

$$U_{i,t}\left(w_i, s_i, l_i, X_i, P_t\right) = \left(w_i - X_i P_t\right) + X_i \left(l_i \sum_{i=j_1}^{j_{s_i}} E_{i,t}\right), \tag{7}$$

where P_t is the price of the innovation at time t, and X_i is a dummy variable that takes the value 1 if the agent decides to adopt the innovation, and the value 0 otherwise. Agents maximize utility period-by-period, and as such agent i will decide to adopt the innovation at time t if the condition

$$U_{i,t}\left(w_i, s_i, l_i, X_i = 1, P_t\right) > U_{i,t}\left(w_i, s_i, l_i, X_i = 0, P_t\right) = w_i \tag{8}$$

holds, subject to $P_t \leq w_i$. Note that there is no inter-temporal inconsistency in the period-by-period utility maximization because: (i) agents keep the knowledge they obtained from their networks at the time of adoption; (ii) the self-acquired experience $E_{i,t}$ is strictly increasing with time. To see this, let $\overline{t_i}$ denote the period in which agent i chooses to adopt the innovation. Then condition (8) held at time $\overline{t_i}$, and condition

$$\left(w_i - P_{\overline{t_i}}\right) + \left(E_{i,\overline{t_i}+k} + l_i \sum_{i=j_1}^{j_{s_i}} E_{i,\overline{t_i}}\right) > w_i \tag{9}$$

will hold for all $k > 0$, where the left-hand side of this inequality is utility at time $\overline{t_i} + k$ for the agent that made the decision to adopt the innovation at time $\overline{t_i}$, and the right-hand side is the utility he/she would have enjoyed at time $\overline{t_i} + k$ had he/she remained a non-adopter.

Finally, we assume that agents who have adopted the innovation face a time and state-invariant probability δ of abandoning it. Once they do so, they stop producing information spillovers to other agents. This reflects the idea that an agent may stop using the innovation and substitute it for a different one, in a process that may be thought of as a sequential innovation path.

In such scenario, agent i receives a wealth endowment $w_{i,k}$ which he/she can use to adopt innovation k, and once that happens he/she receives a new wealth endowment $w_{i,k+1}$ which he/she can use to adopt innovation $k+1$. Considering, for example, the existence of technological requirements (i.e., adopting $k+1$ requires prior adoption of k) allows us to treat the adoption processes for different innovations individually.[3]

3.4 Supply and pricing of the innovation

There is a single supplier of the innovation, and that at each time t the supplier is willing to sell any number of units at the current price, P_t. Prices may take values over the interval [0, 1], and initially units are priced at the top end of that interval, $P_1 = 1$. Furthermore, we assume that the price of the innovation decreases with the number of units produced in the past, as a result of decreasing production costs. This assumption reflects the existence of learning-by-doing in the supply side, in line with Arrow (1962) work.

The price of the innovation declines according to a power law, and with Q_t denoting the number of units supplied at time t we have that P_t is given by

$$P_t\left(Q_{t-1}, Q_{t-2}, \ldots, Q_1\right) = 1 - a\left(\sum_{k=1}^{t-1} Q_k\right)^b, \quad t > 1, \tag{10}$$

[3] This kind of rank-dependance in adoption may also stem from financial constraints. For example, the non-normalized price of innovation $k+1$ may exceed, at all times, the maximum wealth available for the adoption of k, and a proper wealth level may only be obtained/earned once k is adopted.

where $a > 0$ and $b > 0$ are constants. We set the parameter a as a function of parameter b and the number of individuals in the economy, N. More specifically, we set $a = \frac{1}{N^b}$. This assumption causes the price of the innovation to be zero once all agents in the economy adopt it. To see this, note that each agent may only purchase one unit of the innovation, and thus N units are sold when all of them become adopters. This means the term $\left(\sum_{k=1}^{t-1} Q_k\right)^b$ in Eq. 10 is equal to N^b once the last agent adopts the innovation. Hence, $P_t = 1 - \frac{1}{N^b} N^b = 0$.

Given the constraint imposed on a, the parameter b fully governs the behavior of the price P_t: (i) if $b = 1$ then Eq. 10 becomes a linear function, and the price P_t decreases at a constant rate as the number of adopters grows; (ii) if $0 < b < 1$ then Eq. 10 becomes a convex function, and the price P_t declines with the number of adopters at a decreasing rate; (iii) if $0 < b < 1$ then Eq. 10 becomes a concave function, and the price P_t declines with the number of adopters at an increasing rate. In our model we use $0 < b < 1$.

3.5 Presence of early adopters

We assume that a number of agents, Y, are gifted with the innovation once it starts to be supplied in the market. This reflects the fact that the supplier is aware of the positive effects that network externalities have on agents' decisions to adopt the innovation, and for that reason the supplier is willing to incur in a sunk cost to stimulate the initial steps of the diffusion process.

4 Simulation results

In this section we run simulations for different calibrations of our model, in order to analyze how model parameters influence the diffusion process of the innovation. In our analysis we focus on the influence of network size, knowledge spillovers and price behavior. For each model calibration, we take 200 randomly-generated economies and simulate a 100-period adoption process for each one of them. We then use the results from each simulation to compute the mean adoption rate at each time t, as well as its standard deviation across simulations. We also compute the 10th and 90th percentiles for the adoption rate across simulations.

In all simulations we consider economies populated by 1,000 individuals, and we set the number of early adopters to be 10% of the total population ($Y = 100$). We set $\Phi = 0.05$ for all simulations, thus at any given period t there is a 5% probability of agent i updating one node in his/her network. In addition, we consider that at each period the agents who have adopted the innovation may stop using it with a 5% probability ($\delta = 0.05$). The remaining parameters are set according to Table 1, which describes the plan of simulations.

4.1 The effects of network size

The mean adoption rate at the end of the simulation period is positively related to the value of parameter S, as depicted in Fig. 1. Because higher values of S imply larger

Table 1 Parameter values used in the simulations (N = 1,000)

Effect studied	S	γ	$(a;\ b)$
Network size	Variable	$\gamma = 1$	$a = \frac{1}{N^b}$; $b = 0.5$
Knowledge spillovers	$S = 10$	Variable	$a = \frac{1}{N^b}$; $b = 0.5$
Price behavior	$S = 10$	$\gamma = 1$	$a = \frac{1}{N^b}$; b variable

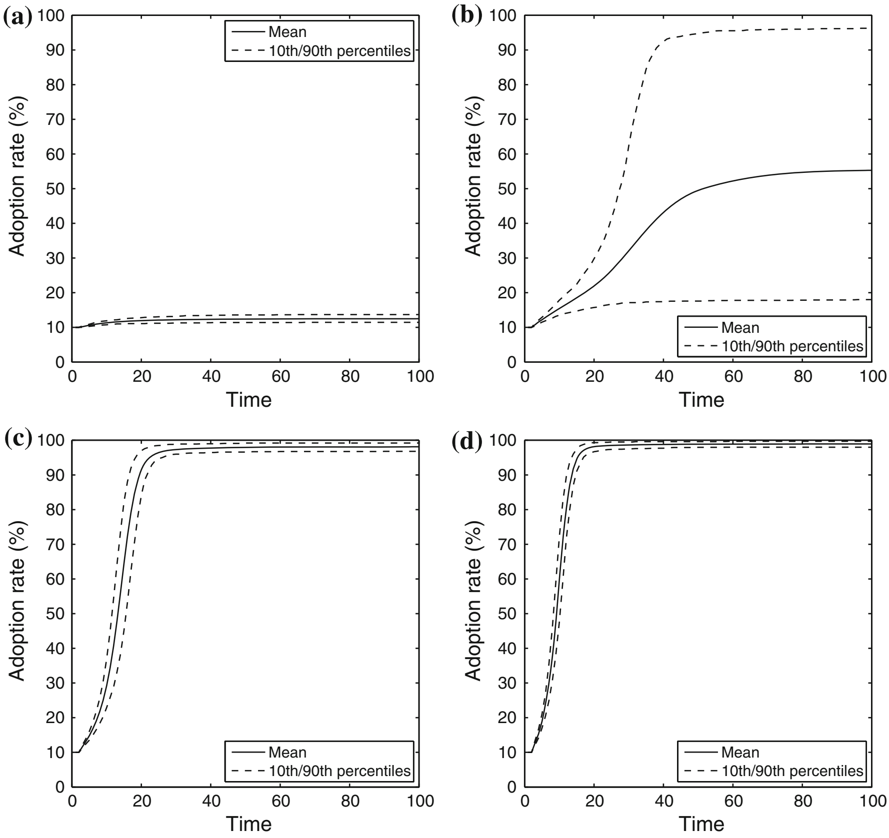

Fig. 1 Diffusion curves and the effect of network size: **a** $S = 5$; **b** $S = 10$; **c** $S = 15$; and **d** $S = 20$

network sizes, this result shows that the more inter-connected economies tend to experience higher adoption rates. Table 2 shows that for sufficiently large network sizes the average adoption rate approaches 100%, but in economies with smaller networks the outcome is significantly different, as full innovation diffusion is less likely to occur. In fact, given the values of the remaining parameters ($b = 0.5$, $\gamma = 1$), diffusion is practically nonexistent when networks have at most 5 nodes ($S = 5$). Network size also influences the speed and timing of the diffusion process. With larger networks the innovation diffuses more swiftly, and convergence towards the final value of the adoption rate is achieved earlier (see Fig. 1). Three mechanisms contribute to these

Table 2 Mean adoption rates and standard deviation at $t = 100$

	Mean (%)	Standard deviation
Network size		
$S = 5$	12.50	0.87
$S = 10$	55.30	35.35
$S = 15$	98.11	0.97
$S = 20$	98.93	0.67
Knowledge spillovers		
$\gamma = 0.1$	98.43	0.76
$\gamma = 0.4$	97.41	5.53
$\gamma = 0.7$	89.23	21.45
$\gamma = 1.0$	55.30	35.35
Price behavior		
$b = 0.25$	99.10	0.43
$b = 0.50$	55.30	35.35
$b = 0.75$	11.43	0.57
$b = 1.00$	10.56	0.29

results regarding the the mean diffusion rate and the speed at which the diffusion process develops.

First, larger networks contain a higher number of nodes, all of which can potentially produce information about the innovation. On average, non-adopters will be able to access a bigger pool of knowledge spillovers, and it is easier for them to collect the information they need to enhance the utility attached to the innovation. Then, given a higher level of utility derived from using the innovation, the choice of adopting it becomes attractive for a larger range of prices P_t. The decision to adopt the innovation is more likely to occur, and more likely to take place earlier on in the simulation, when the price P_t has not yet declined by much. For this reason, the pool of knowledge spillovers grows faster and this helps to further increase adoption.

Second, larger networks facilitate the coordination among adopters and non-adopters, thereby improving the knowledge transfer process. Given that at any moment adopters may stop using the innovation with a probability Φ, non-adopters must contact them while they are still "active" in order to benefit from the knowledge spillovers.[4] In addition, once contact is established it is in the best interest of non-adopters to maintain such links as long as the adopters remain "active". When the average network size is small, non-adopters are less likely to contact "active" adopters because they have fewer nodes to allocate. Furthermore, with smaller networks agents are more likely to disconnect themselves from "active" adopters whenever there is network update.[5]

[4] We will refer to adopters that are still using the innovation as "active", and use the term "inactive" to refer to those who have stopped using the innovation.

[5] For example, when a random network update takes place in a 5-node network, an existing link to an "active" adopter has a 20% probability of being disconnected, whereas in a 20-node network that probability is just 5%.

Given that "active" adopters may end up being replaced by "inactive" ones, or even by agents who have not yet adopted the innovation, small networks display a lower capacity to retain a stable pool of knowledge spillovers. Large networks, on the other hand, promote greater stability in knowledge transfers, and through this mechanism they promote higher innovation diffusion.

Third, because the two effects mentioned above contribute to increase the number of agents who turn into adopters, larger networks also cause a faster decline of the price P_t. As a result, there is a faster decline in the loss of utility associated with paying the price at the moment of adoption. Non-adopters are therefore more likely to adopt the innovation earlier on, starting to produce knowledge that spills over to other agents at an earlier stage. The end result is an acceleration of the diffusion process. Thus, in addition to the direct effects on information availability described before, larger networks produce a second round effect on knowledge production through faster price declines.

Looking at mean diffusion curves, however, does not fully reveal the influence that network size exerts on the diffusion process. Considering the 10th and 90th percentile lines in Fig. 1, and the standard deviation data in Table 2, it is clear that the variability of outcomes across simulations is related to the value of parameter S. More specifically, there seems to be a non-monotonous relationship between adoption outcome variability and the size of networks.

Given the values for the other model parameters ($\gamma = 1$, $b = 0.5$), small networks (e.g., $S = 5$) do not seem capable of promoting the knowledge exchanges necessary to fuel the diffusion process, and as such all (simulated) economies converge to a low adoption rate. As S increases, the network size effect begins to exert a bigger influence and some economies reach very high adoption rates. Accordingly, the mean adoption rate, the standard deviation across simulations, and the 90th percentile line all shift upwards. As S increases even further, the network size effect completely takes over the diffusion process and all economies converge to adoption rates close to 100, increasing the mean adoption rate and decreasing dispersion across simulations. The data on Tables 3 and 4 suggests that such non-monotonous relationship holds with other values of b and γ, and that the precise value of S at which the dispersion is higher depends on the values of b and γ.

4.2 The effects of knowledge spillovers

Simulation results suggest that the behavior of knowledge spillovers also plays a role in shaping the diffusion process. Figure 2 shows that when adopters accumulate knowledge and generate spillovers at a slower pace (high γ), the mean adoption rate at the end of the diffusion process is lower. Similarly to what we have seen for the network size effect, Table 2 shows that certain values of the parameter that governs the build-up of spillovers, γ, are compatible with adoption rates close to 100%, given the remaining model parameters ($S = 10$, $b = 0.5$). The explanation for this result is straightforward. Because at any time adopters may stop using the innovation with probability δ, slow knowledge accumulation leads to smaller spillovers to other agents.

Table 3 Mean adoption rates and standard deviation (in parenthesis) for different combinations of S and γ (at $t = 100$, with $b = 0.5$)

	$\gamma = 0.1$	$\gamma = 0.4$	$\gamma = 0.7$	$\gamma = 1$
$S = 5$				
Mean	16.20	13.91	12.99	12.50
SD	(3.09)	(1.46)	(1.07)	(0.87)
$S = 10$				
Mean	98.43	97.41	89.23	55.30
SD	(0.76)	(5.53)	(21.45)	(35.35)
$S = 15$				
Mean	99.05	98.90	98.96	98.11
SD	(0.66)	(0.69)	(0.75)	(0.97)
$S = 20$				
Mean	99.44	99.31	99.18	98.93
SD	(0.41)	(0.53)	(0.56)	(0.67)

Table 4 Mean adoption rates and standard deviation (in parenthesis) for different combinations of S and b (at $t = 100$, with $\gamma = 1$)

	$b = 0.25$	$b = 0.50$	$b = 0.75$	$b = 1.00$
$S = 5$				
Mean	97.40	12.50	10.41	10.16
SD	(0.93)	(0.87)	(0.22)	(0.13)
$S = 10$				
Mean	99.10	55.30	11.43	10.56
SD	(0.43)	(35.35)	(0.57)	(0.29)
$S = 15$				
Mean	99.49	98.11	13.58	11.16
SD	(0.32)	(0.97)	(1.27)	(0.41)
$S = 20$				
Mean	99.63	98.93	27.76	12.11
SD	(0.23)	(0.67)	(22.28)	(0.80)

This happens because the knowledge accumulation process may stop before adopters reach their full potential. Non-adopters then have access to a smaller pool of spillovers, and all else equal this reduces the attractiveness of adopting the innovation.

However, the outcome of the diffusion process seems to be less sensitive to how fast agents generate knowledge spillovers than to how large networks are. According to the simulation results in Table 3, and holding constant the value the pricing parameter b, the difference in the mean adoption rate between simulations with $\gamma = 0.1$ and simulations with $\gamma = 1$ is typically less than 5 percentage points. The exception is when $S = 10$, the in which case the mean adoption rate ranges from 55.3 up to 98.43 ($\gamma = 1$ and $\gamma = 0.1$, respectively). In contrast, changes to the network size parameter S typically result in variations of the mean diffusion rate that may exceed 80% points (e.g., the difference between the results with $S = 5$ and $S = 20$). Table 5 shows that a similar comparison can be drawn between the influence of how fast knowledge spillovers increase (parameter γ) and the influence of price behavior (parameter b). The mean adoption rate at the end of the simulations can vary more than 80 percentage

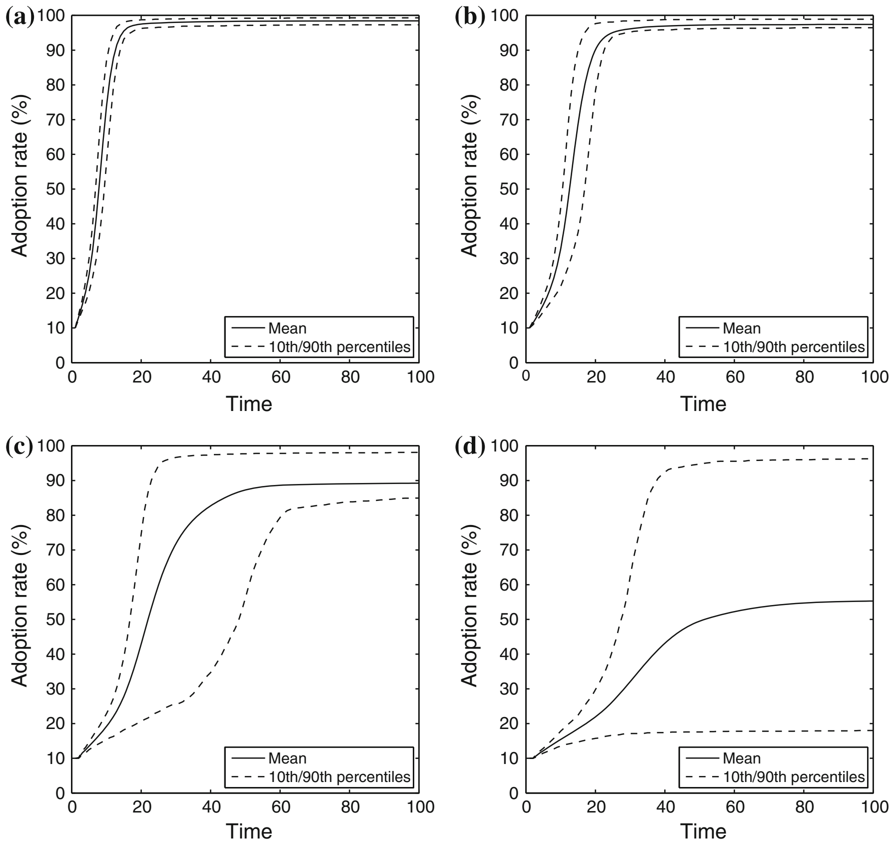

Fig. 2 Diffusion curves and the effect of knowledge spillovers: **a** $\gamma = 0.1$; **b** $\gamma = 0.4$; **c** $\gamma = 0.7$; and **d** $\gamma = 1.0$

points in response to changes in b, whereas changes in γ cause the mean adoption rate to shift by less than 2 percentage points (considering $S = 10$).

The 10th and 90th percentile lines in Fig. 2 seem to suggest that, holding S and b constant, the variability of outcomes across simulations is monotonously related to γ, the parameter governing the spillovers. In the case depicted in Fig. 2 ($S = 10$ and $b = 0.5$), slower generation of spillovers (high γ) causes higher dispersion of the final adoption rate across simulations. The sign of this relation, however, is not always the same. As the the data from Tables 3 and 5 show, higher values of γ may sometimes be associated with lower dispersion across simulations. In the presence of large networks (high values for S), increases in γ generically lead to increases in the standard deviation of the adoption rate at $t = 100$, whereas with smaller networks (e.g., the $S = 5$ case in Table 3) the increase in γ actually reduces the dispersion in the results across simulations.

The results relating γ with the mean adoption rate at $t = 100$ the variability of outcomes across simulations are relatively simple to explain, and they provide some guidance on how important the behavior of knowledge spillovers is. When S is very

Table 5 Mean adoption rates and standard deviation (in parenthesis) for different combinations of S and γ (at $t = 100$, with $S = 10$)

	$\gamma = 0.1$	$\gamma = 0.4$	$\gamma = 0.7$	$\gamma = 1$
$b = 0.25$				
Mean	99.58	99.51	99.42	99.10
SD	(0.27)	(0.31)	(0.33)	(0.43)
$b = 0.50$				
Mean	98.43	97.41	89.23	55.30
SD	(0.76)	(5.53)	(21.45)	(35.35)
$b = 0.75$				
Mean	12.71	12.09	11.73	11.43
SD	(0.98)	(0.79)	(0.60)	(0.57)
$b = 1.00$				
Mean	10.78	10.75	10.62	10.56
SD	(0.36)	(0.33)	(0.29)	(0.29)

low (small networks) or very high (large networks), the influence of the network size effect is so large that it locks economies into diffusion processes with "low adoption" or "high adoption" paths, respectively.[6] In those cases, the pace at which knowledge spillovers are being generated does little to influence the final outcome. Similar conclusions surface when we consider different combinations of b and γ. When b takes a very high or very low value, the effect of price behavior dominates the diffusion process and economies are locked into a "low adoption" or "high adoption" path, respectively. Even if the pace at which agents generate spillovers unequivocally alters the mean adoption rate at $t = 100$, γ exerts only a small influence over its value.

Nevertheless, in economies that are already locked into a "low adoption" path because of slow price decline or small networks, slower spillover generation (high γ) contributes to drag down the diffusion process even further. The "low adoption" path becomes harder to escape from, hence the decrease in both the mean and the standard deviation of the adoption rate. In economies locked into a "high adoption" path because of fast price decline or large networks, slower spillover generation acts against the positive influence of price reductions and network size. In such cases we observe a decrease in the mean adoption rate, and an increase in the variability of outcomes across simulations, as the slow build-up of spillovers may affect some (simulated) economies more than others.

4.3 The effects of price behavior

The simulation results shown in Fig. 3 indicate that the outcome of the diffusion process is extremely sensitive to how fast the innovation price decreases, with the mean adoption rate moving close to 100% when prices fall at a higher rate (low b). As explained

[6] The threshold that defines what "very low" and "very high" values of S are is dependent on the third parameter being studied, b. For the results reported in Table 3, "very low" corresponds to $S = 5$ and "very high" corresponds to $S = 15$ and above. Nevertheless, for a other values of b there are values of S from which the same results emerge.

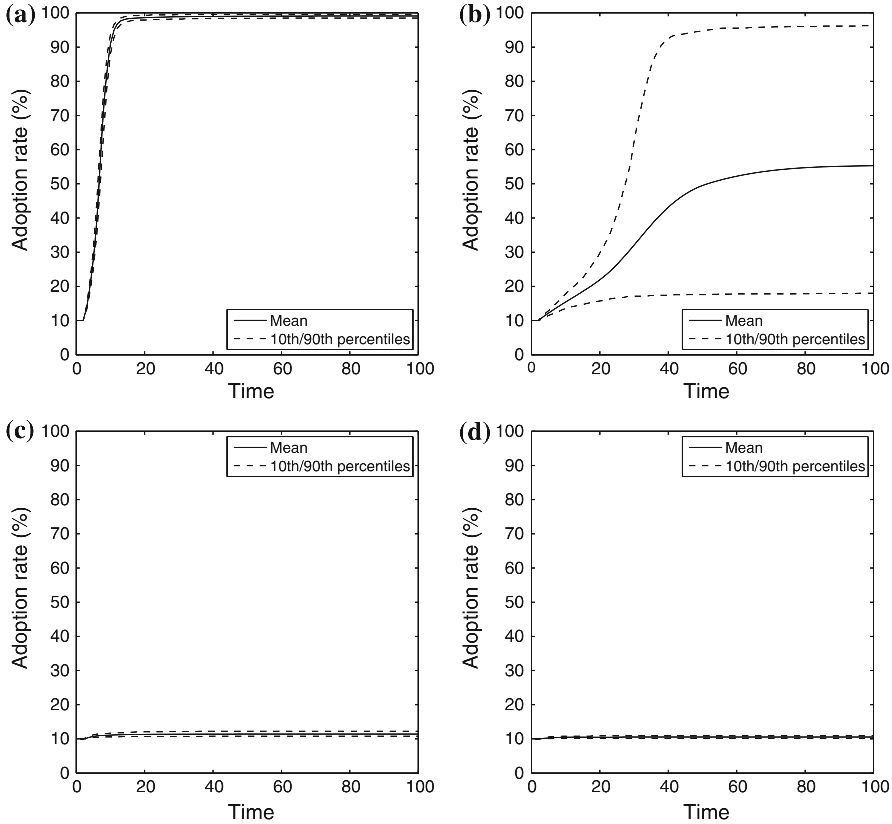

Fig. 3 Diffusion curves and the effects of price behavior: **a** $b = 0.25$; **b** $b = 0.5$; **c** $b = 0.75$; and **d** $b = 1$

before, when the price declines more quickly, so does the loss of utility from spending wealth to purchase the innovation. For some agents this makes adoption attractive earlier on, and the number of adopters increases at a faster pace. Consequently, the pool of spillovers in the economy also increases more quickly, contributing to increase the attractiveness of the innovation for even more non-adopters, thereby pushing the adoption rate upwards. In addition, it is important to note that for the lower ($\leq$0.25) and higher ($\geq$0.75) values of parameter b, the standard deviation across simulation drops to very small values (Table 2), and the 10th and 90th percentile lines stay very close to the mean diffusion curve.

Such results suggest that in our model price dynamics have a profound influence over the diffusion process, and simulation data using different combinations of S, γ and b confirms this. As shown in Table 4, when parameter b takes a very low value ($\leq$0.25) the effects of price declines completely dominate the diffusion process, and the network size effect does little to influence the final value of the adoption rate—even if it shifts between extremes ($S = 5$ and $S = 20$). For very high values of b ($0.75 \leq b \leq 1.00$), the effects of price behavior also dominate the network size effects, although the latter manage to retain some influence when $b = 0.75$. As discussed earlier, the more the two effects work against each other (i.e., small networks and fast price decline, or large networks and slow price decline), the higher is the variability of outcomes across simulations.

Data from the simulations with different values of b and γ (holding $S = 10$) yields similar results. Except for the case in which $b = 0.5$, the value of the parameter governing knowledge accumulation and spillover creation, γ, has little influence over the mean adoption rate observed at time $t = 100$. And like in the simulations involving b and the network size parameter S, here the dispersion in the adoption rate across simulations increases when the effects of spillovers run against the effects of price behavior (e.g., slow build-up of spillovers and fast price decline, or vice versa). Furthermore, comparing the data in Tables 4 and 5 we see that given any value for parameter b, parameter S has more influence over the final value of the mean adoption rate than parameter γ. This confirms the idea that the average size of networks matters more than how fast agents generate knowledge spillovers. Pricing dynamics, however, seem to dominate both effects.

5 Concluding remarks

In this paper we have proposed a theoretical framework which is able to explain why an innovation may not diffuse globally across an economy, even when it faces no rival innovations. Furthermore, we used computational simulations to demonstrate that network size, knowledge spillovers and the dynamics of innovation prices all play a role in shaping the diffusion process. The results regarding network size and information spillovers support the idea that informational lock-outs are capable of exerting significant influence over the aggregate adoption rate and the speed of the diffusion process. However, simulation results also show that the influence of network size and spillover effects on the diffusion prices is not as pronounced as that of pricing dynamics. In fact our results show that under certain circumstances—very fast or very slow price decline—the influence of pricing dynamics completely dominates the diffusion process, and even sizable increases (or decreases) in the size of agents networks produce little effect over the final adoption rate.

Many paths for research remain open to study through the use of simulations such as those we have presented in this paper. Remaining within the realm of innovation diffusion, it would be interesting to know, for example, how the diffusion process is affected when agents evaluate the adoption decision at different times. Such approach could shed additional light on the evolutionary concept of path dependence, and maybe enhance our understanding of how relevant the timing of adoption decisions is for the existence (or absence) of informational lock-outs. Further research could also consider how the distribution of information-processing capacity in the economy influences the adoption of new technologies, and perhaps study how the costs of changing such distribution—through schooling or training, for example—compare to potential benefits that come from moving to the new technologies.

On the other hand, the informational cascades mechanisms contained in our model may also have applications outside the scope of innovation diffusion. One possible field of application would be finance, where the propagation of information may affect asset prices. And in macroeconomics, for example, information diffusion and its effect on how agents update their expectations could, potentially, yield additional insights about phenomena such as that of nominal and real rigidities.

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