The Price Level and the Money Demand After an Interest Rate Shock

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Abstract

I obtain a slow reaction of prices and money demand after an interest rate shock. I study two shocks: a permanent and a temporary increase in the interest rate. The price level drops after the shocks and a period of six months of low inflation follows. The real and nominal money demands adapt slowly to the shocks. I obtain the short and the long run behavior of prices and money in line with the empirical evidence with the same model. I calibrate the model with U.S. data from 1900 to 1997. Agents decide the time to exchange bonds for money. This is equivalent to allow endogenous segmentation. The framework is a general equilibrium Baumol-Tobin model with focus on the transition.

JEL classification: E3, E4, E5.

Keywords: price level, money demand, interest rate shocks, monetary policy, transfer costs, endogenous segmentation.

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1. INTRODUCTION

I obtain a slow response of prices and money demand after a monetary shock. The key aspect of the model is a transfer cost whenever agents exchange bonds for money. Several empirical studies report slow adjustment of the price level and of the monetary aggregates after a monetary shock. For example, Cochrane (1994), Christiano, Eichenbaum and Evans (1999), and Uhlig (2005). I offer a dynamic monetary model to explain these facts.

I study two different shocks from a steady state with zero inflation: a permanent and a temporary increase of one percentage point in the nominal interest rate. Figures (1) and (2) show the simulation results for the behavior of the price level and the money demand after the shocks. I calibrate the parameters of the model using U.S. data from 1900 to 1997.

The main contributions of this paper are to find persistent real effects after monetary shocks and to describe the transition with a general equilibrium model with the following two characteristics. (1) In the short run, slow responses of the price level and money. (2) In the long run, growth rates of the price level and of the nominal money demand equal to the steady state inflation rate, and real money demand decreasing with the interest rate. These two characteristics of the transition are in accordance with the empirical evidence on the short and long run behavior of prices and money.

The results in figures (1) and (2) are the following. The price level, and the nominal and real money demands decrease after the shocks and they adapt slowly to the new interest rate. There is a price overshooting in six months. For the permanent shock, the real money demand decreases gradually to a lower value. The short-run interest elasticity of the money demand is smaller than its long-run elasticity. The nominal money demand initially decreases and later grows at its steady state growth rate of
Fig. 1. Results of a permanent interest rate shock from 3 to 4 percent per year. Annual mean is the geometric mean of the variable, centered in July 1st. The first point is the value before the shock. Detailed analysis in section four.

For the temporary shock, the real and nominal money demands initially decrease, but later increase towards their initial levels. The nominal money demand returns to a little higher level to account for a temporary inflation. I analyze the effects of the shocks in more detail in section four.

The transfer cost makes agents decide when to exchange bonds for money. Therefore, agents decide when they have access to the bonds market and when they do not have. Markets are endogenously segmented. This has substantial effects in the response of prices and money.

First, the model implies that the real money demand decreases when there is a
permanent increase in the nominal interest rate. With fixed transfer periods, in contrast, the real money demand increases with the interest rate. This is a counterfactual result not present with endogenous decisions, as we have in this paper. The empirical fact that the long-run real money demand decreases with the interest rate is a long established result. See, for example, Meltzer (1963) and Lucas (1988). Second, we could expect the real effects to decrease or to vanish once agents adjusted their transfer intervals. Surprisingly, the effects of a change in the nominal interest rate last longer. I obtain the slow reaction and the long run behavior of prices and money demand, in line with the empirical evidence, with the same model.
The payment of the transfer cost is in goods. This is important for the convergence of the price level and of the money demand. With transfer cost in utility terms, the model lacks a price mechanism and we do not have convergence. I let prices and the real interest rate change during the transition. Romer (1987), and Fusselman and Grossman (1989) also study the effects of interest rate shocks in economies with transfer costs. Romer (1987) assumes that the government chooses the interest rate and prices simultaneously to keep the real interest rate constant in an overlapping generations model. He obtains convergence as old generations are removed from the economy and new ones face the new interest rate. Fusselman and Grossman (1989) are closer to the present model as they let prices change and they assume infinite-lived agents. But their economy does not converge after the shock as they assume transfer cost in utility terms.

Agents visit infrequently the bond market because of the transfer cost. Although bonds receive interest and money does not, it is optimal to sell a large quantity of bonds for money and purchase goods with this money for a certain period. Agents have different money balances. When there is a shock, those agents with little money visit earlier the bond market and transfer the amount of money already taking into account the new interest rate path. Those agents with more balances take longer to make their first transfer after the shock. This generates the lagged responses of prices and money.

In the usual cash-in-advance model, in contrast, there is no cost to exchange bonds for money in every period and it is optimal to do so. All agents behave as a representative agent and the response of prices and money demand to changes in the interest rate are instantaneous or last one period.

Grossman and Weiss (1983) and Rotemberg (1984) proposed the first models with market segmentation to study the liquidity effect after monetary changes. In these models, agents participate in open market operations in fixed time intervals. Fuerst
(1992), Lucas (1990), Alvarez and Atkeson (1997), and Alvarez, Lucas and Weber (2001) have further contributions on the effects of monetary changes with models of exogenous market segmentation or fixed transfer times. Alvarez, Atkeson and Edmond (2003) are closer to the present paper. However, they keep the interval between transfers fixed and let the interest rate follow a stochastic process. The effects of shocks to the interest rate last for only one holding period in their model. I find more persistent effects. Moreover, the real money demand implied by their model increases if they subject their economy to a permanent increase in the interest rate.

Alvarez, Atkeson and Kehoe (2002) have endogenous segmentation with a different nature. Agents in their model start each period with some stochastic money balances. They decide whether to use all of their money balances to consume or to pay a fee and also buy contingent bonds. Agents do not keep money balances for the next period. As a result, the inflation rate is always equal to the money growth rate. In the present model, the inflation rate and the money growth rate are different in the short run.

The framework is a general equilibrium version of Baumol (1952) and Tobin (1956). I focus on the transition. Jovanovic (1982) and Romer (1986) focus on the steady state. Jovanovic assumes constant consumption within holding periods. I let consumers optimize their consumption profile. Romer assumes overlapping generations and zero intertemporal discount. I assume infinite-lived agents and positive intertemporal discount. The difficulty of allowing the transfer times to change optimally is the relation between aggregate variables and individual behavior. See, for example, Caplin and Leahy (1991, 1997).

I structure the paper in five sections: (1) introduction, (2) the model, (3) steady state, (4) transition after the shocks, and (5) conclusions. All proofs are in the appendix.
2. THE MODEL

There is a continuum of agents who need to use money to trade goods. The agents trade goods in the goods market and bonds in the asset market. They incur a transfer cost when they sell bonds for money in the asset market and transfer the proceeds to the goods market. The transfer cost is paid in goods. It covers administrative, opportunity, and non-pecuniary costs involved in making a transfer, as motivated by Baumol (1952) and Tobin (1956). Time is continuous. This is a simplifying assumption. It allows us to ignore integer constraints. The model is adapted from Grossman (1987)\(^1\).

Consumers are infinitely lived and they discount the future at the rate \( \rho > 0 \). They choose how much to consume and the time of each transfer. Let \( c(t) \) denote consumption at time \( t \) and \( N_j \geq 0 \) \( j = 1, 2, \ldots \) denote the interval between transfers. The time of each transfer is given by the summation of the intervals \( N_j, T_j \equiv \sum_{s=1}^{j} N_j, T_0 \equiv 0 \).

Consumers have preferences given by

\[
U(c) = \sum_{j=0}^{\infty} \int_{T_j}^{T_{j+1}} e^{-\rho t} \log c(t) \, dt.
\]  

(1)

The transfer cost does not enter in the utility function. The logarithmic utility is not essential for the results. We need homothetic preferences to obtain a real money demand linear in income. See the appendix for the case of a constant relative risk aversion utility in the general form.

Each agent owns two accounts: a brokerage account and a bank account, as in

\(^{1}\)I include the decision of the optimal interval between transfers and remove the possibility of paying a proportional transfer cost within holding periods. Grossman assumes that agents equalize their marginal utility of wealth whereas I characterize the steady state as a function of the endowments. I use the same notation of Grossman whenever possible.
Alvarez, Atkeson and Edmond (2003). The brokerage account contains all resources used in the asset market. The bank account contains the monetary resources used for goods purchases.

There is a single and nonstorable good. Each consumer produces \( Y \) units of the good in every period. The price level at time \( t \) is given by \( P(t) \) and inflation is denoted by \( \pi(t) \equiv \dot{P}(t)/P(t) \), where \( \dot{x}(t) \equiv \partial x(t)/\partial t \). At each time, the agents sell the production in the goods market and deposit the proceeds in the brokerage account. The agents cannot use the money from the sale of goods in the same period\(^2\). Production is constant. This is not a restrictive assumption. It allows us to isolate the effects of market segmentation on the behavior of prices and money after an interest rate shock. It also simplifies the computation of the transition.

Agents have to pay a fee in goods in order to transfer resources from the brokerage account to the bank account. The transfer cost is the crucial assumption of the paper. It generates a nondegenerate distribution of money holdings across agents, a slow response of prices after a monetary shock, and a propagation mechanism after the shock. The transfer cost is proportional to income, it is given by \( \gamma Y, \gamma > 0 \). This is a technical assumption. Consumption and money demand in the steady state will be linear in income with this assumption. A value \( \gamma = 1 \) means that the transfer cost is equal to one working day per transfer.

Government bonds are traded in the asset market. Let \( Q(t) \) be the value at time zero of one dollar to be received at time \( t \). Denote the nominal interest rate by \( r(t) \).

Let \( B_0 \) denote bond holdings held by each agent at time zero and let \( W_0 \) denote total wealth initially available in the brokerage account. The initial resources in the brokerage account are equal to the initial bond holdings plus the present value of

\(^2\)We may view an agent as a family composed of two types of individuals, a worker and a shopper, as in Lucas (1990). The worker produces the consumption good in each period and deposits the proceeds in a brokerage account. The shopper decides when to sell bonds for money and how to use the money to buy goods.
production,

\[ W_0 = B_0 + \int_0^\infty Q(t) P(t) Y dt. \]  

(2)

Any difference in the initial wealth in the brokerage account across consumers is given by the initial amount of bonds held, \( B_0 \). The present value of production is the same for all agents. The agents need to sell bonds and transfer the monetary proceeds to the goods market in order to buy goods.

The agents have \( M_0 \) in money holdings in the bank account at time zero. \( M_0 \) can be used promptly for good transactions.

Each agent is identified by the pair \((M_0, W_0)\). There is a given distribution \( F \) of \( M_0 \) and \( W_0 \) along with its density function \( f \).

Let \( M(t, M_0, W_0) \) denote the quantity of money held at time \( t \) by consumer \((M_0, W_0)\). The amount of money for a purchase of goods is subtracted from the bank account whenever there is a purchase. Therefore, the cash in advance constraint in this economy is

\[ \dot{M}(t, M_0, W_0) = -P(t) c(t, M_0, W_0), \quad t \neq T_1, T_2, \ldots \]  

(3)

where \( c(t, M_0, W_0) \) denotes consumption at time \( t \) of consumer \((M_0, W_0)\). The cash in advance constraint stresses the transactions role of money: agents need to use money in order to buy goods.

After the first transfer, at time \( T_1 \), the agents withdraw the exact amount of money necessary to consume until the next transfer. When it happens, they make another transfer.

The agents have to decide consumption and money holdings for each time \( t \), and when they will transfer resources between their accounts. This decision is done at time zero given the path of the nominal interest rate and of the price level. If it were
costless to transfer resources, the agents would hold only the quantity of money to buy goods for each particular time. This is because bonds accrue interest whereas money does not.

The initial value of individual money holdings, \( M_0 \), is exogenously given. It is not necessarily the amount of money required for consumption between 0 and \( T_1 \). Agents are, therefore, allowed to transfer an amount \( K \geq 0 \) from the bank account to the brokerage account at time \( T_1 \).³

Each consumer faces two constraints. From time zero until the first transfer, the money balances for consumption and any nonspent balance must be financed by the initial money holdings \( M_0 \). After the first transfer, the money balances used for consumption and to pay the transfer fee must be financed by the resources \( W_0 \) at the brokerage account and by any nonspent balances \( K \). By the cash in advance constraint, the money balances necessary to purchase goods during a holding period \( T_j \) to \( T_{j+1} \) must be equal to the consumption spending during the same period.

Therefore, the individual maximization problem is to maximize (1), subject to

\[
\sum_{j=1}^{\infty} Q (T_j) \int_{T_j}^{T_{j+1}} P (t) c (t) \, dt + \sum_{j=1}^{\infty} Q (T_j) P (T_j) \gamma Y \leq W_0 + Q (T_1) K, 
\]

and

\[
\int_0^{T_1} P (t) c (t) + K \leq M_0 \tag{5}
\]

plus the nonnegativity constraints for \( c (t) \), \( N_j \), and \( K \). I remove the reference to \((M_0, W_0)\) of these variables to simplify notation when it does not lead to ambiguity.

³\( K \) is the quantity of money not used in \([0, T_1)\) and deposited in the brokerage account at \( t = T_1 \). We have \( K > 0 \) if \( M_0 \) is higher than the value otherwise chosen by the agent. In principle we would need a variable \( K \) for each holding period, \( K_j \). But we know that, with positive nominal interest rates, the quantity of money will be chosen to make \( K_j = 0 \) for \( j \geq 2 \).
The constraint (4) states that the present value of all money transfers is equal to the present value of the deposits in the brokerage account. It already uses the cash-in-advance constraint and the fact that money holdings are exhausted in the end of each holding period. The constraint (5) states that consumption until the first transfer must be financed from $M_0$.

The first order condition with respect to consumption within a holding period is

$$e^{-\rho t} u'(c(t, M_0, W_0)) = \lambda (M_0, W_0) Q(T_j) P(t), \quad T_j < t < T_{j+1}, \ j = 1, 2, \ldots, \quad (6)$$

for consumer $(M_0, W_0)$. $\lambda (M_0, W_0)$ is the Lagrange multiplier of the present value budget constraint (4). Consumption within a holding period is decreasing if inflation is greater than or equal to $-\rho$, as $u$ is concave. Agents concentrate consumption in the beginning of a holding period to avoid losing resources for inflation. See the appendix for the full characterization of the first order conditions.

Denote $c^+(t)$ and $c^-(t)$ as consumption respectively in the beginning and in the end of a holding period. Combining the first order conditions with respect to the time $T_j$ and consumption yields

$$\gamma Y \left[ r(T_j) - \pi(T_j) \right] + r(T_j) \int_{T_j}^{T_{j+1}} \frac{P(t) c(t)}{P(T_j)} dt = c^+(T_j) \left[ u(c^+(T_j)) - u(c^-(T_j)) \right], \quad (7)$$

for $j = 2, 3, \ldots$

The left hand side of (7) is the marginal gain of delaying the transfer while its right hand side is the marginal loss. The marginal gain is given by postponing the payment of the transfer cost at time $T_j$ and decreasing the amount of real balances needed to purchase goods from $T_j$ to $T_{j+1}$. It also takes into account the net effect of increasing the period from $T_{j-1}$ to $T_j$ and decreasing the period from $T_j$ to $T_{j+1}$. But this net
effect is equal to zero with log utility. The marginal loss in the right hand side is the net effect in utility of the change in the length of the holding periods $T_{j-1}$ to $T_j$ and $T_j$ to $T_{j+1}$.

From the first order conditions, we have that the optimal values $c(t, M_0, W_0; Y)$ are homogeneous of degree one in $(M_0, W_0, Y)$ and the optimal values of the intervals between transfers $N_j(M_0, W_0; Y)$ are homogeneous of degree zero in $(M_0, W_0, Y)$. If money, bonds and production are multiplied by the same factor, then all consumers maintain their transfer times and multiply their consumption levels by this factor. Since production is equal for all agents, I drop this identifier and index the individual solutions by $(M_0, W_0)$.

The government issues money in order to finance the initial quantity of bonds held by the public, $B_0^S$. Therefore, the government budget constraint is

$$B_0^S = \int_0^\infty Q(t) P(t) \frac{M^S(t)}{P(t)} dt,$$

where $M^S(t)$ denotes the money supply at time $t$. If the government wants to inject money, it needs to exchange bonds for money with the consumers in the asset market. The present value of these open market operations is reflected by the term $B_0^{S4}$.

We have to take into account the transfer cost to write the market clearing condition for goods. Let

$$A(t, \delta) \equiv \{(M_0, W_0) : T_j(M_0, W_0) \in [t, t+\delta)\},$$

for a certain $j = 1, 2, \ldots$, denote the set of consumers making a transfer during the interval $[t, t+\delta)$. The measure of $A(t, \delta)$ gives the number of transfers from time $t$ to time $t+\delta$.

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4 Bonds are financed only by seigniorage. We could also include taxes and government purchases. This would not change the results as far as taxes and government purchases are set exogenously.
The market clearing condition for goods is therefore

\[ \int c(t, M_0, W_0) \, dF(M_0, W_0) + \gamma Y \lim_{\delta \to 0} \int_{A(t, \delta)} \frac{1}{\delta} \, dF(M_0, W_0) = Y, \]  

(9)

where the second term in the left-hand side gives the total amount of resources directed to financial transfers at time \( t \). The market clearing conditions for money and bonds are

\[ \int M(t, M_0, W_0) \, dF(M_0, W_0) = M^S \]  

and

\[ \int B_0(M_0, W_0) \, dF(M_0, W_0) = B^S_0. \]

The definition of equilibrium is standard. Equilibrium is defined as prices \( P(t) \), \( Q(t) \), demands \( c(t, M_0, W_0) \), and interval between transfers \( N_j(M_0, W_0) \) such that (i) \( c(t, M_0, W_0) \) and \( N_j(M_0, W_0) \) solve the maximization problem of each agent \( (M_0, W_0) \), (ii) the government budget constraint holds, and (iii) the market clearing conditions for goods, money and bonds hold.

3. THE ECONOMY IN THE STEADY STATE

It is optimal in this economy to maintain money balances sufficient to purchase goods for several periods. The transfer cost makes agents exchange bonds for money infrequently. Some will have just exchanged bonds for money and others will be about to replenish their money balances.

Suppose that an economy has not been hit by a monetary shock for several years. In this economy, the interest rate is constant and the inflation rate is constant. The interval between transfers is the same for all agents. The individual consumption, money and bond holdings are different at a particular time, but they have the same behavior through time. The economy is in the steady state. It looks the same at each time.

In this section, I first obtain the interval between transfers in the steady state. I then obtain the equilibrium real money demand. Finally, I write the cross section
of money and bonds necessary to imply the steady state. All of these variables are functions of the interest rate, the transfer cost, and the preference discount factor $\rho$. The steady state is unique. In the next section, a monetary shock hits the economy initially in the steady state.

In order to calibrate the model, I use data for the U.S. during the period 1900-1997. The value of the transfer fee $\gamma$ is found so that the theoretical money-income ratio passes through the geometric mean of the data. See figure (3).

![Figure 3](image_url)

**Figure 3.** Money-income ratio in the steady state. The data points are for the U.S. economy in the period 1900-1997. The circle o marks the geometric mean of the data.

Formally, I say that an economy is in the steady state if it has the following char-
acteristics: constant interest rate and inflation; all agents with the same interval between transfers, \( N_j (M_0, W_0) = N \) for all \((M_0, W_0)\) and \( j = 2, 3, \ldots \); and all agents with the same behavior of consumption along holding periods, \[ c (T_1 (M_0, W_0) + t, M_0, W_0) = c (T_1 (M_0', W_0') + t, M_0', W_0') , \] \( i = 0 \)

for all pairs \((M_0, W_0), (M_0', W_0')\) where \( T_1 \) is the time of the first transfer. As we should have aggregate money demand and consumption constant over time, I also require that the same number of consumers exchange bonds for money at each time. Therefore, the distribution \( F \) of \((M_0, W_0)\) is such that \( T_1 (M_0, W_0) \) is uniformly distributed along \([0, N]\)^5.

I first characterize the decision of \( N_j (M_0, W_0) \), \( j \geq 2 \), and then proceed to the choice of \( T_1 (M_0, W_0) \). I write \( N (r, \gamma, \rho) \) to stress the dependence of the steady state interval between transfers to the parameters of the model.

Denote the value of consumption just after a transfer by \( c_0 \). This is the highest level of consumption within a holding period. In the steady state, consumption within a holding period is given by \( c(t) = c_0 e^{-rt} \), \( T_j \leq t < T_{j+1} \), by the first order condition for consumption in (6). The market clearing condition for goods implies

\[
\frac{1}{N (r, \gamma, \rho)} \int_0^{N(r,\gamma,\rho)} c_0 e^{-rx} dx + \frac{\gamma Y}{N (r, \gamma, \rho)} = Y. \tag{11}
\]

The terms in the left hand side refer respectively to the aggregate consumption and to the total amount of resources used in transfers. The integral is over time rather

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5We could maintain constant aggregate consumption with different consumption patterns. For example, some consumers could have lower consumption in all periods and we could adjust \( F \) to have a higher number of them. However, for the stationary equilibrium, it is natural to require uniform consumption patterns. A proof that in these conditions the only distribution of transfers compatible with the steady state is the uniform distribution is given by Kocherlakota in the appendix of Grossman (1985).
than over agents because consumption is the same for all agents along holding periods in the steady state and the distribution of agents is uniform.

Rewriting (11) yields the value of $c_0$ as a function of $N (r, \gamma, \rho)$,

$$c_0 = Y \left(1 - \frac{\gamma}{N (r, \gamma, \rho)}\right) \frac{r N (r, \gamma, \rho)}{1 - e^{-r N (r, \gamma, \rho)}}.$$  \hspace{1cm} (12)

According to this expression, consumption is homogeneous of degree one in income. We need $NY > \gamma Y$ to have positive consumption: production during a holding period must be greater than the transfer cost paid to obtain money holdings for the same period.

Notice the effect of the payment of financial transfers. We must take into account transfers in the market clearing. Thus, consumption can be less than $Y$ during the entire holding period. When the cost is in utility terms, in contrast, then the term $1 - \gamma/N$ vanishes and so $c_0$ is always greater than $Y$.

We can now use the first order conditions with respect to the transfer times in (7) to obtain the optimal transfer period in the steady state. In particular, $r = \rho + \pi$ and $N_j = N$ in the steady state.

**Proposition 1.** The optimal interval between transfers in the steady state, $N (r, \gamma, \rho)$, is given by the positive root of the equation

$$r N - \frac{r}{\rho} (1 - e^{-\rho N}) = \rho \gamma \left[ \frac{c_0 (N)}{Y} \right]^{-1}$$  \hspace{1cm} (13)

where $c_0 (N)$ is given by equation (12).

*Proof.* See appendix.

**Proposition 2.** A positive solution for equation (13), $N (r, \gamma, \rho)$, exists and is unique for all $r > 0$, $\rho > 0$ and $\gamma > 0$.

*Proof.* See appendix.

Beyond the logarithmic case, we could expect that the equilibrium would not exist.
for small values of the coefficient of relative risk aversion. This is not the case. The proof in the appendix is for all positive values of relative risk aversion.

The interval between transfers \( N \) decreases with the interest rate and increases with the transfer cost. It decreases to zero when the transfer cost goes to zero. These and other properties are in the following proposition.

**Proposition 3.** The optimal value of the interval between transfers in the steady state is such that (i) \( \frac{\partial N}{\partial r} < 0 \); (ii) \( \frac{\partial N}{\partial \gamma} > 0 \); (iii) \( \lim_{\gamma \to 0} N = 0 \); (iv) \( \frac{\partial N}{\partial \rho} > 0 \); (v) \( N > \gamma \); and (vi) \( \lim_{r \to 0} rN = \varepsilon > 0 \).

**Proof.** See appendix.

About properties iii-vi. If the value of the intertemporal discount \( \rho \) is higher for a fixed nominal interest rate then, in the steady state, inflation is smaller as \( \rho \) is equal to the real interest rate in this case. In turn, the equilibrium interval is higher. Property v implies that consumption is always positive within a holding period, see equation (12). As property vi states, the product \( rN \) converges to a small positive constant when \( r \) goes to zero, although the transfer period increases as the interest rate decreases. This will make the real money demand bounded when \( r \) goes to zero.

The terms \( rN \) and \( \rho N \) are close to zero for a broad range of parameters\(^6\). With a second-order Taylor expansion of \( e^{-\rho N} \) and \( e^{rN} \) in equation (13) we obtain

\[
N \approx \sqrt{\frac{2\gamma}{r}}.
\]

We have the square-root formula for the interval between transfers\(^7\).

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\(^6\)For example, with \( r = 4\% \text{ p.a.} \) and \( \rho = 3\% \text{ p.a.} \) we have \( rN = 0.02 \) and \( \rho N = 0.01 \) for \( \gamma = 1.791 \), see the calibration of \( \gamma \) below in this section. We have \( \lim_{r \to 0} rN \approx \rho \gamma / (1 + \rho \gamma / 2) = 1.5 \times 10^{-4} \). See the proof of proposition 3 in the appendix for details.

\(^7\)The true value of \( N \) is higher than the approximation by 0.3% for \( \gamma = 1.791 \), \( r = 4\% \text{ p.a.} \) and \( \rho = 3\% \text{ p.a.} \) Jovanovic (1982) also obtains the square-root formula as an approximation of his model. Lucas (2000) obtains the square-root formula with the McCallum-Goodfriend framework, where time and real money balances interact via a transactions technology.
For the money demand, index consumers by their position in one holding period, \( n \in [0, N) \). Later, we will make this more precise by characterizing the values of \( M_0 \) and \( W_0 \) such that \( T_1 (M_0, W_0) = n \). Aggregate money demand is, therefore,

\[
M(t) = \frac{1}{N} \int_{0}^{t-jN} M(t, n) \, dn + \frac{1}{N} \int_{t-jN}^{N} M(t, n) \, dn,
\]

where \( M(t, n) \) denotes the individual money demand. Consumers with \( n \in [0, t-jN) \) will be in their \((j+1)\)th holding period, and consumers with \( n \in [t-jN, N) \) will be in their \( j \)th holding period, \( t > jN, j = 1, 2, \ldots \) Solving the integrals above yields the following proposition.

**Proposition 4.** In the steady state, the real money demand is given by

\[
m(r, Y, \gamma, \rho) = \frac{c_0(r, Y, \gamma, \rho)}{\rho} \left[ 1 - e^{-rN} \frac{1 - e^{-(r-\rho)N}}{(r-\rho)N} \right],
\]

where \( c_0(r, Y, \rho, \gamma) \) is given by equation (12) and \( N = N(r, \gamma, \rho) \) is given by proposition 2.

*Proof.* See appendix \(^8\).

The real money demand is homogeneous of degree one in \( Y \), as \( c_0 \) is homogeneous of degree one in \( Y \). Therefore, the elasticity of the money demand with respect to income is equal to one. The real money demand is decreasing in the interest rate. In models with fixed interval between transfers, in contrast, the real money demand is increasing in the interest rate.

Figure (3) shows the steady-state equilibrium values of the money-income ratio \( m/Y \) given by equation (14) along with U.S. annual data during 1900 to 1997. The income \( Y \) is normalized to one. The figure has the long-run equilibrium values of the money-income ratio when the interest rate is constant compared to the annual

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\(^8\)We can also use the symmetry of the steady state and the cash in advance constraint to derive the money demand. The proof in the appendix also contains this alternative derivation.
data. If the interest rate increases permanently, the money-income ratio eventually decreases. In the next section we study the transition after interest rate changes, or the short-run money-income ratio.

During the period 1900-1997, the nominal interest rate varied in the U.S. from around 0.5 to 15 percent per year and velocity, i.e., the inverse of the money-income ratio, varied from around 2 to 8 per year. The mean interest rate in the period is around 4 percent and the mean velocity is 0.25. I calibrate the model following the procedure in Lucas (2000). The parameter $\rho$ is set so that a nominal interest rate of 3 percent per year implies zero inflation in the steady state. Therefore, $\rho = 3$ percent per year. The value of $\gamma$ is chosen so that the money-income ratio passes through the geometric mean of the data. This implies $\gamma = 1.791$. This value means that the average consumer pays the equivalent of roughly 1.8 working days per transfer. If the interest rate is equal to 4 percent, this implies about 2 transfers per year. Therefore, with 5 working days per week and 52 weeks per year, the average consumer devotes around 1.38 percent of the total working time for financial transfers. According to OECD data, the workers in U.S. worked around 1,900 hours per year on average from 1950 to 1997. Therefore, the model estimates on average about 30 minutes per week devoted to financial transactions when inflation is equal to one percent per year.

I calculated numerically the interest rate elasticities of the real money demand and of the interval between transfers. The interest rate elasticities are approximately constant and close to $-1/2$. Therefore, the money demand is close to the Baumol-Tobin money demand\textsuperscript{9}. The calculations for the elasticities of the real money demand and of the interval $N$ with respect to the transfer cost $\gamma$ also yield values around 0.5.

\textsuperscript{9}The interest rate elasticity is higher in absolute value if the coefficient of relative risk aversion is smaller, but the differences are small. If the CRRA is sufficiently small, for example, equal to 0.01, we can have interest-rate elasticities close to $-1$. But these values of the CRRA are below the usual estimates. The results are similar for various values of the intertemporal discount $\rho$. The value of $\rho$ has a small effect on the steady-state money demand.
When the nominal interest rate is positive, agents use part of their resources to manage the optimal money balances. Welfare is maximized when the nominal interest rate is equal to zero and inflation is equal to $-\rho$, as in Friedman (1969). To calculate the quantity of money when $r$ is close to zero, consider the limit of the real money demand in (14) when $r \to 0$. The value of the interval between transfers $N$ increases without bound and the product $rN$ converges to a positive constant, therefore, the money demand converges to $Y/\rho$ when $r \to 0$. Hence, the optimal quantity of money is equal to the present value of production, discounted by the intertemporal discount rate$^{10}$.

We obtain the price level at time zero, $P_0$, with the real money balances $m$ and the initial supply of money $M_0S$. It is immediate to verify that, in the steady state, inflation is equal to the growth rate of money and hence $r = \rho + \pi$. Given that the goods and the money markets clear, the bonds market also clears by Walras’ Law.

The steady state interval between transfers is the same for all agents. It does not depend on the initial conditions $M_0$ and $W_0$. The time of the first visit to the asset market, $T_1$, depends on these values, as it is in the following proposition.

**Proposition 5.** In the steady state, the first transfer from the brokerage account to the bank account, $T_1(r, \rho, \gamma, M_0, W_0)$, is given by the solution of

$$rT_1 = \log \frac{\lambda}{\mu} + rN,$$

where $\lambda(r, \gamma, \rho, W_0, T_1)$ and $\mu(r, \gamma, \rho, M_0, T_1)$ are respectively the Lagrange multipliers of the budget constraints (4) and (5).

**Proof.** See appendix.

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$^{10}$The behavior of the money demand is similar when $r$ decreases towards zero and when $r = 0$. The model, however, is not well defined when $r = 0$ because the value of $N$ increases towards infinity. In Jovanovic (1982), the welfare cost is positive when $r$ decreases towards zero because he assumes constant consumption. See Silva (2004) for a more extensive analysis of the welfare cost of inflation using the present model.
In the steady state, individual consumption is periodic. Let $\hat{c}(x)$ denote consumption at the position $x \in [0, N(r, \gamma, \rho))$ of a holding period. The relation between $\hat{c}(x)$ and $c(t; M_0, W_0)$ is

$$\hat{c}(x) = c[x + T_1 + (j - 1)N; M_0, W_0]$$

for each $(M_0, W_0)$ in the support of $F$, for a certain $j = 1, 2, \ldots$. The function $\hat{c}(x)$ is given by $\hat{c}(x) = c_0 e^{-rx}$, $0 \leq x < N(r, \gamma, \rho)$. Figure (4) shows the behavior of consumption within a holding period of an arbitrary consumer. Consumption can be lower than income in all points of time. In particular, $c_0 = Y$ in the case of zero inflation: $r = \rho$. The ratio of consumption in the beginning of a holding period to consumption in the end of a holding period is approximately equal to $1 + \sqrt{2\gamma r}$. If $r$ or $\gamma$ are small then consumption is approximately constant.

The pattern of consumption is approximately linear. It is not linear by the compounding of interest. As a result, although the distribution of agents along the transfer interval is uniform, the distribution of real money holdings is not uniform\footnote{We can find the distribution of real money holdings across agents analytically. The calculations and results are available upon request.}. The distribution of the money demand across consumers is nondegenerate, as we need in order to have real effects after a monetary shock.

We now write the values of $M_0$ and $W_0$ that imply a uniform distribution of consumers along the interval $[0, N)$. Let $n \in [0, N)$ index consumers by the time of the first transfer. Define the functions $\tilde{M}_0(n)$ and $\tilde{W}_0(n)$ respectively as the initial amounts of deposits in the bank and brokerage accounts such that a consumer with $M_0 = \tilde{M}_0(n)$ and $W_0 = \tilde{W}_0(n)$ transfers resources at $t = n, n + N, n + 2N$ and so on.

$\tilde{M}_0(n)$ must be exactly enough to allow the consumer to consume at the steady
Fig. 4. Individual consumption within a holding period. Nominal interest rate equal to 4% p.a., inflation rate equal to 1% p.a. Transfer fee equal to 1.8 working days per transfer. These parameters imply around 180 days of interval between transfers.

state rate in the interval \([0, n]\). On the other hand, \(\tilde{W}_0(n)\) is equal to the present value of all future transfers from \(t = n\) and on, plus the present value of the total transfer cost. Proposition 6 gives the values of \(\tilde{M}_0(n)\) and \(\tilde{W}_0(n)\).

**Proposition 6.** Given the values of \(N(r, \gamma, \rho)\) and \(P_0\), the values of the initial money holdings in the bank account, \(\tilde{M}_0(n)\), and of the initial wealth in the brokerage...
account, $\tilde{W}_0(n)$, such that the consumer chooses $T_1 = n$ are

$$\tilde{M}_0(n) = P_0 c_0(N) e^{rn} e^{-rN} \frac{1 - e^{-\rho n}}{\rho}$$

and

$$\tilde{W}_0(n) = \frac{e^{-\rho n}}{1 - e^{-\rho N}} \left( P_0 c_0(N) \frac{1 - e^{-\rho N}}{\rho} + P_0 \gamma Y \right),$$

for $n \in [0, N(r, \gamma, \rho))$, where $c_0(N)$ is given by (12).

Proof. See appendix.

$\tilde{M}_0(n)$ is increasing in $n$ while $\tilde{W}_0(n)$ is decreasing in $n$. Hence, a consumer with more initial money holdings and less initial bond holdings will make the first transfer later than a consumer with less money and more bonds. The values of $\tilde{M}_0(n)$ will be useful in the calculation of the equilibrium path after a shock.

4. INTEREST RATE SHOCKS

Suppose that the nominal interest rate has been constant for a long time and changes suddenly. What will be the effects on the price level, on the nominal money demand, and on the real money demand?

I study two changes in the interest rate. In the first, the interest rate goes up permanently from 3 percent to 4 percent per year. In the second, the interest rate goes up from 3 percent to 4 percent per year but gradually decreases back to 3 percent per year. I call the two changes a permanent and a temporary shock to the interest rate respectively.

Why changing the interest rate and not the money supply? First, for a technical reason, there is the need to decrease the dimensionality of the problem. If we fix a path for the interest rate, we obtain the price level from the problem for each agent
and the market clearing condition for goods. We then obtain the money supply with
the market clearing for money. The problem would be intractable if we had to find
the price level in conjunction to the interest rate with two market clearing conditions.
Second, Central Banks usually track the interest rate in their daily operations rather
than the quantity of money. Implicitly, they assume that the quantity of money will
change according to the interest rate.

I now define the analytical structure to study the shock. The objective is to ap-
proximate a situation in which the economy is initially in the steady state and the
interest rate changes unexpectedly. The money holdings at the time of the shock are
equal to the values compatible with the economy in the first steady state.

Suppose the existence of two possible states denoted by \( s = 1, 2 \). In state 1 the
government sets the nominal interest rate at \( r_1 \) for all periods. In state 2, the govern-
ment sets the path of the nominal interest rate at \( r(t) \) for each time. The realization
of the state occurs at time zero.

Agents trade bonds contingent on the realization of the states. Thus, the deposits
in the brokerage account change according to the state. The quantity of money is not
contingent on the state.

Consumers have to use the quantity of money \( M_0 \) available in the bank account from
time zero until the first transfer. Therefore, we now have two budget constraints from
time zero until the first transfer in the problem (1), (4) and (5). One for each price
level path in the respective state. On the other hand, we need only one present value
budget constraint after the first transfer as consumers use the contingent bonds to
transfer resources between states. The budget constraint after the first transfer must
be augmented by the expenditures in each state. Finally, consumers now maximize
the expected value of utility weighted by the probabilities of each state\(^{12}\).

The economy behaves as one with constant interest rate if the probability of a shock

\(^{12}\)See appendix C for the analytical statement of the problem.
is very small. In particular, the cross section of money holdings is close to the values with only one state.

Therefore, proceed in the following way to obtain the values of optimal consumption and transfer times. First, calculate the money and bond holdings such that the economy is in the steady state under the initial nominal interest rate $r_1$. The money holdings are given by $\tilde{M}_0(n)$ in proposition 6, where $n \in [0, N)$, $N$ is the holding period under the interest rate $r_1$, and $P_0 = M_0^S/m$. Second, calculate the new optimal individual consumption and the new transfer times given the interest rate path $r(t)$ and the initial money holdings $\tilde{M}_0(n)$.

Using this procedure, the consumption at time $t$ and transfer times $T_j \equiv N_1 + \ldots + N_j$ for each agent $n \in [0, N)$ are

$$c(t, n) = \frac{e^{-\rho t}}{\lambda Q(T_j) P(t)}, \quad t \in (T_j(n), T_{j+1}(n)), \quad j = 1, 2, \ldots$$  \hspace{1cm} (15)

and

$$-r(t) N_j(n) + \frac{\gamma Y [r(t) - \pi(T_j(n))]}{c^+(T_j(n))} = -r(t) \frac{1 - e^{-\rho N_{j+1}(n)}}{\rho}, \quad j = 2, 3, \ldots$$  \hspace{1cm} (16)

where $Q(t)$ is the bond price $\exp \left( - \int_0^t r(s) \, ds \right)$, $\pi(t)$ is the rate of inflation, $c^+(T_j(n)) = \left[ \lambda e^{\rho T_j(n)} Q(T_j(n)) P(T_j(n)) \right]^{-1}$ is the level of consumption just after the $j$th transfer, and $\lambda$ is the Lagrange multiplier of the budget constraint after the first transfer$^{13}$. Equation (16) states how the transfer interval $N_j$ relates to the next transfer interval $N_{j+1}$ during the transition.

The monetary authority chooses the path of the nominal interest rate $r(t)$. We obtain the individual consumptions and transfer times with equations (15) and (16). We obtain the equilibrium price level with the market clearing for goods. Finally, the

$^{13}$The equation for the first transfer time $T_1 (= N_1)$ is in the appendix.
market clearing for money gives the equilibrium quantity of money.

Different monetary shocks are described by the path of the interest rate after the shock. I assume that the economy is initially in equilibrium with a constant nominal interest rate equal to 3 percent per year. This implies zero inflation before the shock as the intertemporal rate of discount is also equal to 3 percent per year. For the permanent shock, the monetary authority sets the interest rate \( r(t) \) at 4 percent per year for all periods. For the temporary shock, the monetary authority sets the interest rate \( r(t) \) at 4 percent per year at time zero and then decreases the interest rate towards 3 percent per year at a constant rate. The process is, therefore, \( r(t) = r_1 + (r_2 - r_1) e^{-\eta t} \) where \( r_1 \) is 3 percent per year, \( r_2 \) is 4 percent per year and \( \eta \) gives the persistency of the shock. I follow Alvarez, Atkeson and Edmond (2003) and choose \( \eta \) to approximate the response of the nominal interest rate to a shock similar to the response shown in Christiano, Eichenbaum and Evans (1999) and Uhlig (2005)\(^{14}\).

I proceed numerically in order to obtain the equilibrium path of the price level and the other equilibrium values. For each agent, we have a system of equations in the form \( h(N_j, N_{j+1}) = 0 \). In order to have a finite system, I assume that after the \( J \)th transfer each agent chooses \( N_{j+1} = N' \), where \( N' \) is the steady state transfer interval under the new interest rate. That is, if agents are under the new interest rate for long enough, they choose a transfer interval equal to the new steady state interval. The value of \( J \) should be large to approximate the solution. We then have a system of \( J \) equations and \( J \) unknowns \( N_1, \ldots, N_J \) for each agent.

The interval \([0, N]\) is discretized as \( \{n_1, n_2, \ldots, n_{\text{max}}\} \) where \( n_1 = 0 \) and \( n_{\text{max}} \) is smaller than \( N \) but sufficiently close. In equilibrium, we must have, for each time \( t \),

\[
\frac{1}{n_{\text{max}}} \sum_n c(t, n; P) + \frac{1}{n_{\text{max}}} \gamma Y \times \text{Number of Transfers} (t; P) = Y, \quad (17)
\]

\(^{14}\)For the time in days, \( \eta = \frac{-12 \log 0.87}{365} \).
where $P$ stands for the path of the price level. The left hand side of this equation is equal to the aggregate demand. Their components are consumption and resources used to transfer assets from the brokerage account to the bank account. The right hand side is equal to the aggregate supply.

The procedure to find the price level during the transition is the following. (i) Start with a guess for the price level during the transition. (ii) Calculate the transfer times and consumption for each agent. (iii) Check the equilibrium condition (17). (iv) If the difference between demand and supply is smaller than a preestablished value for every $t$, stop. If not, change $P(t)$ and repeat steps (i)-(iii). In the simulations, $J = 40$ and the number of consumers is such that $n_{t+1} - n_t$ is equal to 0.10 day. This implies 2,094 agents for the parameters used. I follow the calibration of section 3. The number of transfers at $t$ is calculated summing the agents with $T_j(n)$ such that $t \leq T_j(n) < t + 1$, that is, the unit of time is one day\textsuperscript{15}.

The results of the simulations for the permanent and the temporary shocks are in figures (1) and (2). The liquidity effect is common to both shocks. In the short run, after an increase in the interest rate, the price level adapts slowly and the nominal and the real money demands decrease. After six months, there is an overshooting in the price level for both the permanent and the temporary shocks. The figures make reference to the money-income ratio $M/(PY)$. The behavior of this ratio is equal to the behavior of the real money demand in this model as $Y$ is exogenous and constant, $Y$ is normalized to one.

I discuss first the results for the price level and for the annual money demand. Then, I discuss the intuition of the model and the results for the daily values of the

\textsuperscript{15}The maximum difference between demand and supply after the last iteration is equal to 2% for the permanent shock and 1.4% for the temporary shock for a total of 600 iterations. Several other simulations were performed with different number of intervals (different $J$’s), number of consumers, transfer costs, and initial guesses for prices. These changes do not affect the qualitative behavior of the price level or of the other equilibrium variables.
money demand.

Figure (5) shows the price level in detail for both shocks during the first quarter after the shock. The price level drops for both shocks and returns to its initial level only after around 30 days. During the first quarter, the price level is relatively fix and close to the initial price level. The difference between the geometric average of the price level during the first quarter and the initial price level is only 0.01 percent for the permanent shock and 0.02 percent for the temporary shock. A researcher with access to the average of the price level during this period would probably conclude that the price level is sticky after the shock.

![Price Level After the Shocks](image)

**Fig. 5.** Price level after the shocks as a percentage of the price level before the shock.

For the permanent shock, the annual nominal money demand decreases about 11
percent during the first two years. The price level increases at a rate lower than its long-run growth rate for the first six months. The real money demand decreases slowly towards its new steady state. After one year, its annual mean decreases about 12 percent from its initial level and it is 2 percent higher than the new steady state level.

In the long run, the price level and the nominal money demand grow at a constant rate. Moreover, the real money demand is smaller than its initial value as the nominal interest rate is higher. The constant rate of growth of the price level and of the nominal money demand is equal to the inflation rate, one percent per year, equal to the difference between the nominal and the real interest rate.

For the temporary shock, the annual mean of the nominal and the real money demands decrease to about 5 percent of their initial value during the first year. The price level increases towards its new steady state level. It does not jump to a higher value as we would have in a model without transfer costs.

The price level just after the temporary shock behaves within the ranges of the empirical estimation in Uhlig (2005). In the long run, however, the model predicts an increase in the price level\textsuperscript{16}. The lower quantity of money is compatible with the higher interest rate only temporarily. The time with relatively low quantity of money lasts while there are only consumers doing the first transfer. After this period, no consumer will be willing to hold as much real balances as the opportunity cost of money increases with the interest rate. As a result, the price level changes, although with a lag, to a higher level.

If the shock is temporary, the inflation rate returns to zero, its initial value, as the nominal interest rate decreases back to the value of the real interest rate. Hence, in

\textsuperscript{16}It is difficult to use the estimation in Uhlig (1995) for the long run. The estimation is not completely comparable. As it is common in this literature, Uhlig restricts the behavior of the price level: he only considers dynamics with a decreasing price level.
the long run, the price level and the nominal money demand are constant, and the real money demand returns to its initial level. The nominal money demand is higher than its initial level in this case. This occurs because there is inflation during the transition. As the real money demand returns to its initial level, there must be an increase in the quantity of money to offset the increase in prices.

Friedman (1969) studies the effect of a once-and-for-all change in the quantity of money and of a continuous increase in the quantity of money. I relate the two exercises with the temporary and the permanent increase in the interest rate respectively. The reason is because the final effect of a temporary increase in the interest rate is a once-and-for-all increase in the quantity of money, and the final effect of a permanent change is a continuous increase in the quantity of money.

The present model gives an analytical justification to the expected effects of monetary shocks. The advantage is that now we can quantify the changes in the price level and the money demand, and we can give a meaning to what we understand by the short run and the long run. With the calibration of section 3, the long run stands for the behavior after two years, as the price level and the money demand are close to their new steady state values, and most of the effects of the policy shock occur within the first six months. We can also be more specific about the predictions. For example, the path of the price level after a permanent change is close to the price level C in figure 4 of Friedman (1969). But he is not able to predict the path of the price level in his analysis. Friedman also predicts the overshooting in the price level.

The reason for the effects in the short run is the different behavior of agents according to their initial money balances. The transfer cost makes agents economize in the use of money to avoid making a transfer too soon. The agents with more money balances will not increase their rate of consumption within holding periods as they would without the transfer cost. The agents with little money balances make a transfer sooner and they can readjust their money balances and consumption pat-
terns. They consume at a faster rate because they decrease the interval between 
transfer to reflect the higher interest rate. Initially, the number of agents that have 
made a transfer after the shock is relative small. Consequently, prices do not change 
instantaneously with the change in the nominal interest rate.

After about six months, two groups of agents with different consumption patterns 
meet. The first group is composed of the agents who had little money balances and 
were about to make a transfer when the shock hit the economy. They are now doing 
their second transfer. The second group is composed of those who had substantial 
money holdings at the time of the shock and have not made a transfer since that 
time. When the two groups meet there is a fast increase in the price level, otherwise, 
demand would be much higher because the first group consumes at a faster rate and 
are now doing the second transfer. Six months is approximately the new steady state 
interval between transfers for a four percent interest rate.

In the long run, with an increase in the nominal interest rate, agents have more 
real money balances than they would like to have. As a consequence, each agent will 
try to use their money and spend more than with a lower interest rate. As total 
demand cannot be higher than total output, and output is constant, the price level 
increases. This explanation for the long-run effects of an increase in the interest rate 
is standard. It is also present in Friedman (1969).

Prices become smooth and we have convergence because of the change in the trans-
fer times. It is more costly to make a transfer when prices are high because agents 
pay the transfer cost in goods. When the interest rate increases, the number of trans-
fers concentrates in certain periods as agents anticipate their transfer times. In these 
periods, the price level increases. This generates the cyclical variation best seen in 
the daily values for the money demand. As it is more costly to make a transfer when 
prices are higher, agents change their transfer times to periods in which prices are 
lower. This behavior makes the number of transfers per day eventually constant and
the economy converges to the new steady state. The redistribution of the transfers is slow. The economy experiences changes in the price level five years after the shock.

With transfer cost in utility terms, as in Fusselman and Grossman (1989), the economy lacks this price incentive to make agents change transfer times. The price level disappears from the first order conditions. As a result, in contrast to the results in the present paper, prices and the money demand do not converge with transfer cost in utility terms.

The results are also different from models with fixed transfer periods as in Grossman and Weiss (1983) and Rotemberg (1984). Alvarez, Atkeson and Edmond (2003) introduce a more recent model with this feature\textsuperscript{17}. In these models, the effects of a change in the interest rate last for only one holding period. I find more persistent effects as agents adapt to the new interest rate. The holding period under zero inflation is around 200 days for the calibrated parameters in this model. The policy would be felt for only this period for constant transfer periods. This is a surprising result because the real effects could vanish as agents adjusted their transfer times optimally. We have fluctuations after the shock because of the slow change in the transfer times. They work as a the propagation mechanism.

Another difference from models with constant transfer periods is that, in these models, the long-run real money demand \textit{increases} after a permanent increase in the nominal interest rate. In contrast, in the present model the real money demand \textit{decreases} after a permanent increase in the nominal interest rate. For higher nominal interest rates we expect agents to reduce their real money balances. This is an well established empirical fact clear in figure (3) and discussed, for example, in Meltzer (1963) and Lucas (1988). The present model generates this fact.

\textsuperscript{17}Alvarez, Atkeson and Edmond study the behavior of prices when the nominal interest rate has a random component. They simplify the decision of the transfer timings but introduce a more complex path for the interest rate.
The price level and the rate of inflation after the shock for the whole simulation are in figure (6). The shock to the interest rate causes a strong variation in the inflation rate. Gradually, the peaks and troughs of the inflation rate converge towards its steady state level of 1 percent per year or zero in the case of the permanent or temporary shock respectively. There is an inflation overshooting in about six months both for the permanent and for the temporary shocks.

\[
\text{Inflation} = (\log P_t + 30 - \log P_t) \times 12.
\]

For the permanent shock, the price level grows at a slower rate than the long-run inflation rate in the first six months. We can see that by the initial decrease in the difference between the price level and the trend. The price level gradually increases.
at the same rate of the trend.

For the temporary shock, the price level increases gradually towards its higher steady state value. The new steady state value of the price level is higher for the temporary shock because agents try to decrease their money balances during the period in which the interest rate is higher. The agents with little money balances just before the shock are successful in doing this and the price level initially does not react much. But later, as more agents try to decrease their money balances, there is a temporary inflation and the price level in the new steady state increases.

5. CONCLUSIONS

I present a monetary model to calculate the effects of interest rate shocks on the price level and money. The model has only one departure from the cash in advance model: a transfer cost whenever agents exchange bonds for money. I study two shocks: a permanent and a temporary increase in the nominal interest rate. The implications of the model are in accordance with the empirical evidence on the short and the long run behavior of prices and money. The price level drops just after the shocks and slowly adjusts to the new steady state.

The model also predicts the duration of the effects of the shock. The main effects happen in the first six months after the shocks. At about that time, the inflation rate overshoots its steady state level. The price level and the real and nominal money demands behave close to their steady state behavior two years after the shock. But the model still predicts some variation after that period.

I let agents adjust optimally their transfer intervals. This assumption changes results in important ways. First, the results for the long-run effects of the real and nominal demands after a permanent increase in the interest rate agrees with the empirical evidence. The real money demand decreases when the interest rate increases. A model with fixed transfer periods predicts an increase in the real money demand
when the interest rate increases. Second, the effects of the shocks last for much longer. With fixed transfer periods, the effects last for only one holding period, about 200 days in the calibrated model. With the present model, in contrast, there is still variation in the equilibrium values five years after the shock.

The key parameter obtained with the calibration is the transfer cost value. We need a high transfer cost to obtain money demand levels compatible with the values for the U.S. This implies a large support for the distribution of money holdings and, in turn, increases the persistence of the shock.

We obtain convergence because the transfer cost is paid in goods and not in utility terms. Agents avoid selling bonds for money when prices are relatively high because the transfer cost is higher during these periods. With cost in utility terms, in contrast, the transfer cost does not change. The variation in the price level makes the number of transfers in each day converge towards a constant. This mechanism acts slowly because the calibration implies that agents make about two transfers per year in the steady state.

In the steady state, the interval between transfers and the aggregate money demand are close to the ones in Baumol and Tobin. This happens although utility maximization implies that consumption changes over time in this model. The picture is very different out of the steady state.

We need to keep track of each consumer because the distribution of money holdings has important effects on equilibrium values. This increases the complexity of the problem and makes difficult the introduction of a stochastic component. However, the analysis of a stochastic economy in a similar setting could reveal other features of the link between interest rates, prices, and money demand. This is left as a suggestion for future research.
APPENDIX A - FIRST ORDER CONDITIONS

This appendix shows the first order conditions in the general case, when the utility function $u$ has the constant relative risk aversion form,

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & \text{for } \sigma \neq 1, \sigma > 0, \\ \log c & \text{for } \sigma = 1. \end{cases}$$

The body of the text treats the log case.

The Lagrangian of the problem in (1), (4) and (5) is

$$\mathcal{L} = \sum_{j=0}^{\infty} \int_{T_j}^{T_{j+1}} e^{-\rho t} u(c(t)) \, dt + \lambda(M_0, W_0) \left[ W_0 + Q(T_1) K(M_0, W_0) \right. \\
- \sum_{j=1}^{\infty} Q(T_j) \int_{T_j}^{T_{j+1}} P(t) c(t, M_0, W_0) \, dt - \sum_{j=1}^{\infty} Q(T_j) P(T_j) Y \gamma] \\
+ \mu(M_0, W_0) \left[ M_0 - \int_{0}^{T_1} P(t) c(t, M_0, W_0) \, dt - K(M_0, W_0) \right],$$

where $T_j = T_j(M_0, W_0)$.

The first order conditions are given by

$$e^{-\rho t} u'(c(t, M_0, W_0)) = \lambda(M_0, W_0) Q(T_j) P(t), \text{ for } t \in (T_j, T_{j+1})$$

$$e^{-\rho T_j} u'(c^+(T_j, M_0, W_0)) = \lambda(M_0, W_0) Q(T_j) P(T_j),$$

$$e^{-\rho T_{j+1}} u'(c^-(T_{j+1}, M_0, W_0)) = \lambda(M_0, W_0) Q(T_j) P(T_{j+1});$$
and, for $t \in [0, T_1]$, 

$$e^{-\rho t} u' (c (t, M_0, W_0)) = \mu (M_0, W_0) P (t), \quad t \in (0, T_1),$$

$$u' (c^+ (0, M_0, W_0)) = \mu (M_0, W_0) P (0),$$

$$e^{-\rho T_1} u' (c^- (T_1, M_0, W_0)) = \mu (M_0, W_0) P (T_1).$$

$T_1$:

$$e^{-\rho T_1} u (c^- (T_1)) - e^{-\rho T_1} u (c^+ (T_1))$$

$$= \lambda \left[ -\dot{Q} (T_1) K + \dot{Q} (T_1) \int_{T_1}^{T_2} c (t) P (t) dt - Q (T_1) c^+ (T_1) P (T_1) dt \right]$$

$$+ \lambda Y \gamma \left[ \dot{Q} (T_1) P (T_1) + Q (T_1) \dot{P} (T_1) \right]$$

$$+ \mu P (T_1) c^- (T_1);$$

$T_j, \ j = 1, 2, ...$:

$$e^{-\rho T_j} u (c^- (T_j)) - e^{-\rho T_j} u (c^+ (T_j)) = \lambda \left[ \dot{Q} (T_j) \int_{T_j}^{T_{j+1}} P (t) c (t) dt \right.$$

$$- \dot{Q} (T_j) P (T_j) c^+ (T_j) + Q (T_{j-1}) P (T_j) c^- (T_j)$$

$$\left. + \lambda Y \gamma \left[ \dot{Q} (T_j) P (T_j) + Q (T_j) \dot{P} (T_j) \right] \right].$$

$K$:

$$Q (T_1) \lambda (M_0, W_0) - \mu (W_0, M_0) \leq 0 \ (= 0 \text{ if } K > 0);$$

and the budget constraints.

With CRRA utility, $u' (c (t)) c (t) = (1 - \sigma) u (c (t))$ for $\sigma \neq 1$. Using this in the
first order condition for $T_j$, $j = 2, 3, \ldots$, we have, after simplification

$$
\gamma Y [r(T_j) - \pi(T_j)] + \frac{1}{1 - \sigma} \left[ \frac{Q(T_{j-1})}{Q(T_j)} c^-(T_j) - c^+(T_j) \right] \\
= \left[ \frac{Q(T_{j-1})}{Q(T_j)} c^-(T_j) - c^+(T_j) \right] - r(T_j) \int_{T_j}^{T_{j+1}} \frac{P(t)c(t)}{P(T_j)} dt.
$$

If $\sigma = 1$, we obtain

$$
\gamma Y [r(T_j) - \pi(T_j)] + r(T_j) \int_{T_j}^{T_{j+1}} \frac{c(t) P(t)}{P(T_j)} dt = c^+(T_j) \log \frac{Q(T_{j-1})}{Q(T_j)}.
$$

Using the budget constraint and the first order condition with respect to consumption, the value of $\lambda(M_0, W_0)$ is given by

$$
\lambda(M_0, W_0) = \sum_{j=1}^{\infty} \int_{T_j}^{T_{j+1}} e^{-\rho t} [c(t)]^{1-\sigma} dt \\
\times \left[ W_0 + Q(T_1) K - \gamma Y \sum_{j=1}^{\infty} Q(T_j) P(T_j) \right]^{-1}
$$

for $\sigma \neq 1$, and

$$
\lambda(M_0, W_0) = \frac{e^{-\rho T_1}}{\rho} \left[ W_0 + Q(T_1) K - \gamma Y \sum_{j=1}^{\infty} Q(T_j) P(T_j) \right]^{-1}
$$

(18)

for $\sigma = 1$. The value of $\lambda$ in the steady state is $\lambda = \frac{1}{P_0 e^{\sigma}}$. Working analogously for $\mu(M_0, W_0)$ using the budget constraint for $0 \leq t < T_1$, we obtain

$$
\mu(M_0, W_0) = \frac{1}{M_0 - K} \left[ \int_{0}^{T_1} e^{-\rho t} [c(t)]^{1-\sigma} dt \right]
$$

(19)
for \( \sigma \neq 1 \), and

\[
\mu(M_0, W_0) = \frac{1 - e^{-\rho T_j}}{M_0 - K} \frac{1 - e^{-\rho T_j}}{\rho}
\]

for \( \sigma = 1 \).

**APPENDIX B - PROOFS**

**Proposition 1**

*Proof.* The first order condition with respect to \( T_j \) in the steady state implies, after simplification,

\[
-\sigma \frac{1}{1 - \sigma} \left[ c^+ (T_j) - c^- (T_j) e^{r N_j} \right] = -r \int_{T_j}^{T_j+1} c(t) \frac{P(t)}{P_0 e^{\pi T_j}} dt + \gamma Y \left( r + \pi \right).
\]

Using \( c(t) = c_0 e^{-\pi (t-T_j)} \), \( j = 1, 2, ..., r = \rho + \pi \), and simplifying, yields

\[
-\frac{1}{1 - 1/\sigma} \left[ 1 - e^{r N_j (1-1/\sigma)} \right] - \frac{\rho}{c_0} Y = r \frac{1 - e^{-\rho N_j + 1} e^{r N_j + 1} (1-1/\sigma)}{\rho - r (1 - 1/\sigma)}.
\]

With \( N_j = N_{j+1} = N \) we have the desired result. The steps for \( \sigma = 1 \) are analogous.

We can also obtain the expression for \( \sigma = 1 \) if we note that

\[
\lim_{\sigma \to 1} -\frac{1}{1 - 1/\sigma} \left[ 1 - e^{r N (1-1/\sigma)} \right] = r N.
\]

**Proposition 2**

*Proof.* Define the functions \( a, b, G : (\gamma, +\infty) \to R \) for \( \sigma \neq 1 \) by

\[
a(N) = \left( \frac{1 - e^{-r N / \sigma}}{r N / \sigma} \right) \left( 1 - \frac{\gamma}{N} \right)^{-1},
\]
\[ b(N) = -\frac{1}{1 - 1/\sigma} (1 - e^{\rho N(1 - 1/\sigma)}) - r \frac{1 - e^{\rho N(1 - 1/\sigma)}e^{-\rho N}}{\rho - r (1 - 1/\sigma)}, \]

and

\[ G(N) = b(N) - \rho \gamma a(N). \]

Note that \( a = Y/c_0 \). The optimal interval \( N^* \) is such that \( G(N^*) = 0 \).

We have \( \lim_{N \to \gamma_+} a(N) = +\infty \) and \( \lim_{N \to \gamma_+} b(N) = 0 \). Therefore,

\[ \lim_{N \to \gamma_+} G(N) = -\infty. \]

We also have that \( \lim_{N \to \infty} a(N) = 0 \). Intuitively, if \( N \to \infty \) then the consumer is consuming almost his total present value in the beginning of the holding period. So \( c_0 \) is very large, and \( a = Y/c_0 \to 0 \). Moreover, \( b'(N) > 0 \), and \( a'(N) < 0 \) because

\[ e^{\rho N/\sigma} > 1 + Nr/\sigma - \gamma r/\sigma. \]

Hence,

\[ G'(N) = b'(N) - \rho \gamma a'(N) > 0. \]

Even though \( G \) is always increasing, it can be the case that \( \lim_{N \to +\infty} G(N) < 0 \). This possibility is ruled out for \( \sigma \geq 1 \) because \( \lim_{N \to \infty} b'(N) = +\infty \) for \( \sigma > 1 \) and \( \lim_{N \to \infty} b'(N) = r \) for \( \sigma = 1 \). On the other hand, \( \lim_{N \to \infty} b'(N) = 0 \) for \( 0 < \sigma < 1 \). In this case, \( \lim_{N \to +\infty} G(N) = \lim_{N \to +\infty} b(N) = \sigma \rho > 0 \).

**Proposition 3**

Proof.

(i) The proof of \( \partial N/\partial r < 0 \) is for \( \sigma \geq 1 \). However, as detailed below, I could not find a counterexample for \( \sigma < 1 \) even for \( \sigma \) as small as 0.001.
\[
\frac{\partial N}{\partial r} = -\frac{\partial G(r;N)/\partial r}{G'(N)}. \quad \text{We know that } G'(N) > 0. \text{ On the other hand,}
\]

\[
a_r (r; N) = \frac{(1 + rN/\sigma) - e^{rN/\sigma}}{e^{rN/\sigma}r^2 (N/\sigma)} (1 - \gamma/N)^{-1} < 0.
\]

Write \(b(N)\) as

\[
b(r; N) = rN \left[ f (g_1 (r)) - f (g_2 (r)) \right]
\]

where \(f (x) \equiv \frac{1-e^{-x}}{x} \), \(g_1 (r) \equiv N(r(1/\sigma - 1)) \) and \(g_2 (r) \equiv N(\rho + r(1/\sigma - 1))\). Note that \(f (x)\) is decreasing and convex. Using the chain rule, we have that \(\frac{\partial f(g(r))}{\partial r} = \frac{\partial f(g(r))}{\partial g(r)} \frac{\partial g(r)}{\partial r}\). Therefore,

\[
b_r (r; N) = N \left[ f (g_1) - f (g_2) \right] - Nr(1/\sigma - 1) [f' (g_2) - f' (g_1)].
\]

We have always \(g_2 > g_1\) because \(\rho, N > 0\). Therefore \(f (g_1) > f (g_2)\) and \(f' (g_2) > f' (g_1)\).

If \(\sigma > 1\) then \((1/\sigma - 1) < 0\) and so \(b_r (r; N) > 0\). If \(\sigma = 1\) then \(b_r (r; N) = N \left( 1 - \frac{e^{-N\rho}}{N\rho} \right) > 0\) because \(0 < f (x) < 1\). Hence, \(G_r (r; N) = b_r (r; N) - \rho \gamma a_r (r; N) > 0\) and \(\partial N/\partial r < 0\).

If \(0 < \sigma < 1\) then the second term of the expression for \(b_r (r; N)\) is negative. For small values of \(\sigma\) and large values of \(r\) it can be the case that \(b_r (r; N) < 0\), even for \(N = N^*\). However, it was found numerically that the term \(a_r (r; N)\) is large enough in absolute value when \(N = N^*\) to imply a positive value of \(G_r (r; N^*)\). Various combinations of \(\sigma\) and \(r\) were tested, even for values of \(\sigma\) as small as 0.001.

(ii) \(G_{\gamma} (\gamma; N) = -\rho a (N) - \rho \gamma a (N) / \partial \gamma, \partial a (N) / \partial \gamma = \frac{1-e^{-N\rho}}{rN/\sigma} \frac{N}{(N-\gamma)^2} > 0\). Thus, \(G_{\gamma} (\gamma; N) < 0\) and \(\partial N/\partial \gamma = -G_{\gamma} (\gamma; N^*) / G'(N^*) > 0\).

(iii) We saw that \(N\) decreases when \(\gamma\) decreases and that \(N > \gamma\). The equation
that determines $N$ converges to

$$\frac{-1}{1 - 1/\sigma} \left(1 - e^{rN(1-1/\sigma)}\right) = r \frac{1 - e^{-N(\rho-r(1-1/\sigma))}}{\rho - r(1-1/\sigma)}$$

when $\gamma \to 0$. This expression holds if and only if $N = 0$.

(iv) We have $a_\rho (\rho; N) = 0$. Using the definitions of $f$, $g_1$ and $g_2$ as above, we have

$$b_\rho (\rho; N) = rN \left[ f' (g_1 (\rho)) \frac{\partial g_1 (\rho)}{\partial \rho} - f' (g_2 (\rho)) \frac{\partial g_2 (\rho)}{\partial \rho} \right] = -rN^2 f (g_2 (\rho)) < 0.$$ 

We then have $G_\rho (\rho; N) = -rN^2 f (g_2 (\rho)) - \gamma a (N)$. The value of $G_\rho$ is negative if and only if $-rN^2 f' (g_2 (\rho)) < \gamma a (N)$. At $N = N^*$, $\rho \gamma a (N^*) = b (N^*)$, therefore, the inequality is true if and only if

$$-\rho rN^2 f' (g_2) < rN \left[ f (g_1 (r)) - f (g_2 (r)) \right].$$

This can be written as

$$\frac{f (g_2 (r)) - f (g_1 (r))}{g_2 - g_1} < f' (g_2),$$

where we used the fact that $g_2 - g_1 = \rho N$. But the last inequality is always true because $f$ is convex and $g_2 > g_1$. Therefore, $G_\rho (\rho; N^*) < 0$. Thus, $\partial N/\partial \rho = -G_\rho (\rho; N^*) / G' (N^*) > 0$.

(v) The optimal $N$ is given by $G (N) = 0$. As $G$ is increasing and continuous in $N$, and $\lim_{N \to \gamma^+} G (N) = -\infty$ then it must be the case that the optimal $N$ is higher than $\gamma$.

(vi) When $r$ decreases, $N$ increases. But $\lim_{r \to 0} rN$ is bounded because $|\partial N/\partial r| < 1$. Define $x \equiv \lim_{r \to 0} rN$. Using the equation that defines $N$ and $\lim_{r \to 0} N = +\infty$,
the value of $x$ is given by

$$\frac{1 - e^{-x(1/\sigma-1)}}{1/\sigma - 1} = \rho\gamma \left( \frac{1 - e^{-x/\sigma}}{x/\sigma} \right).$$

In order to have an idea of the magnitude of this number, consider this equation for $\sigma = 1$,

$$x = \rho\gamma \left( \frac{1 - e^{-x}}{x} \right).$$

This value of $x$ is approximately equal to $\rho\gamma/(1 + \rho\gamma/2).$ ■

**Proposition 4**

*Proof.* We provide two proofs for this proposition.

(1) For $t > jN$, consumers will be in their $j$th or $(j+1)$th holding period, $j = 1, 2, \ldots$. Individual money demand is given by

$$M(t, n) = \begin{cases} \int_t^{T_{j+2}(n)} P(t) \ c_0 e^{-\pi(t-T_{j+1}(n))} \ dx, & n \in [0, t-jN), \\ \int_t^{T_{j+1}(n)} P(t) \ c_0 e^{-\pi(t-T_{j}(n))} \ dx, & n \in [t-jN, N). \end{cases}$$

where $T_{j}(n) \equiv n + (j-1)N$. Solving the integrals and with a change of variables, aggregate money demand is given by

$$M(t) = \frac{1}{N} \int_{t-N}^{t} P_0 c_0 \frac{e^{\pi s + N} e^{-rN/\sigma} - e^{\pi t e^{-r(s-t)/\sigma}}} {e^{-r(s-t)/\sigma}} ds$$

or,

$$\frac{M(t)}{P(t)} = \frac{c_0}{N} \int_{0}^{N} \frac{e^{(r-\rho-r/\sigma)N} e^{-(r-\rho)x} - e^{-rx/\sigma}}{(r-\rho-r/\sigma)} dx.$$

Solving the integral, with the value of $c_0$ given by equation (12) and with $m \equiv M(t)/P(t)$, we obtain the real money demand in the text. ■
(2) Real money demand for any consumer in the stationary equilibrium is such that

$$m^n(t) = -c^n(t) - \pi m^n(t),$$

where $m$ denotes real money demand and the superscript refers to the consumer with $n \in [0, N)$. For $n = 0$, the boundary condition for this differential equation is $m(N) = 0$. Solving the differential equation, we obtain

$$m^0(t) = c_0 e^{-rt} \frac{e^{\rho(t-N)}e^{r(1-1/\sigma)N} - e^{r(1-1/\sigma)t}}{r (1 - 1/\sigma) - \rho}.$$ 

By the symmetry of the steady state, aggregate real money demand is given by

$$m(t) = \frac{1}{N(r, \rho, \sigma, \gamma)} \int_0^{N(r, \rho, \sigma, \gamma)} m^0(t) \, dt.$$ 

Substituting $m^0(t)$ and solving the integral yields the desired result. 

**Proposition 5**

Proof. By the first order condition for $T_1$ and $e^{-\mu t} u(c) = c \mu (M_0, W_0) P(t) / (1 - \sigma).$ We obtain, after rearranging,

$$\frac{\sigma}{1 - \sigma} \frac{c^{-}(T_1)}{Q(T_1)} - \frac{\sigma}{1 - \sigma} \lambda c^{+}(T_1)$$

$$= \lambda \left[ -\frac{Q(T_1)}{Q(T_1) P(T_1)} \frac{\dot{Q}(T_1)}{\dot{P}(T_1)} \int_{T_1}^{T_2} c(t) \frac{P(t)}{P(T_1)} \, dt \right] + \lambda Y \gamma \left[ \frac{\dot{Q}(T_1)}{Q(T_1)} + \frac{\dot{P}(T_1)}{P(T_1)} \right].$$

In the steady state, we know that $c(t) = c_0 e^{-\tilde{\phi}(T_2-t)}, T_1 \leq t < T_2$, $c^{-}(T_1) = c_0 e^{-\tilde{\phi}N}$, and $c^{+}(T_1) = c_0$. Also, inflation is constant and $r = \rho + \pi$. Then, with $K = 0$ and
after simplification

\[
\frac{\sigma}{1 - \sigma} e^{rT_1} e^{-\frac{\sigma}{\mu} T_1} e^{-\frac{\sigma}{\mu} T_N} - \frac{\sigma}{1 - \sigma} \frac{\lambda}{\mu} = \frac{\lambda}{\mu} \left[ -r \frac{1 - e^{-(\rho - r)(\sigma - 1)/\sigma}}{\rho - r (\sigma - 1) / \sigma} - \rho \frac{Y}{c_0} \right].
\]

With the optimality condition for \( N \), we obtain

\[
e^{rT_1} e^{-\frac{\sigma}{\mu} T_N} = \frac{\lambda}{\mu} e^{-\frac{\sigma}{\mu} T_N} e^{rN}.
\]

Taking logs on both sides yields the desired result. \( \blacksquare \)

**Proposition 6**

*Proof.* \( M_0 (n) \) is exactly enough to allow the consumer to consume at the steady state rate in the interval \([0, n)\). This value is such that

\[
\tilde{M} (n) = \int_0^n P(t) c(t) \, dt.
\]

c(0) is not necessarily equal to the level of consumption when a transfer is made. This is only true for the consumer \( n = 0 \). We know that

\[
c^-(n) = c_0 e^{-\frac{\sigma}{\mu} T_N},
\]

for all \( n \in [0, N) \), and that

\[
\frac{\dot{c}}{c} = -\frac{r}{\sigma}.
\]

Solving this differential equation yields

\[
c(x, n) = c_0 e^{\frac{\sigma}{\mu} T_n} e^{-\frac{\sigma}{\mu} T_N} e^{-\frac{\sigma}{\mu} x}, \quad 0 \leq x < n.
\]
Therefore, solving the integral

\[
\tilde{M}(n) = \int_0^n P_0 e^{\pi t} c_0 e^{\frac{\pi n}{N}} e^{-\frac{\pi n}{N}} e^{-\frac{\pi n}{N}} dt
\]

we have the desired result for \( \tilde{M}(n) \).

For \( \tilde{W}_0(n) \). First, the value of money needed in each holding period is given by

\[
M_j = \int_{n+(j-1)N}^{n+jN} P(t) c_0 e^{-\frac{\pi n}{N}(t-T_j)} dt,
\]

where \( j = 1, 2, \ldots \) and \( T_j = n + (j - 1) N \). So,

\[
M_j = P_0 c_0 e^{\pi n} e^{\frac{(\pi - r) N}{(\pi - r/\sigma)}} e^{\pi(j-1)N} \equiv \bar{M} e^{\pi(j-1)N}
\]

The value at \( t = n \) of these transfers is \( A_M \equiv \bar{M} \frac{1}{1-e^{\pi n}} \). For the transfer cost, we have

\[
TC_j = \gamma Y P(n + (j - 1) N), \ j = 1, 2, \ldots
\]

\[
= P_0 \gamma Y e^{\pi(n+(j-1)N)}.
\]

Working analogously, \( A_{TC} \equiv P_0 \gamma Y e^{\pi n} \frac{1}{1-e^{-\pi n}} \). Finally, the value of \( \tilde{W}(n) \) is given by

\[
\tilde{W}(n) = e^{-rn} A_M + e^{-rn} A_{TC}.
\]
APPENDIX C - EQUATIONS FOR SECTION 4, INTEREST RATE SHOCKS

The maximization problem of each agent is

\[
\max \theta \sum_{j=0}^{\infty} \int_{T_j(1)}^{T_{j+1}(1)} e^{-\rho t} u \left( c(t, 1) \right) dt + (1 - \theta) \sum_{j=0}^{\infty} \int_{T_j(2)}^{T_{j+1}(2)} e^{-\rho t} u \left( c(t, 2) \right) dt
\]

subject to

\[
\sum_{s=1,2} \left[ \sum_{j=1}^{\infty} Q \left( T_j(s), s \right) \int_{T_j(s)}^{T_{j+1}(s)} P(t, s) c(t, s) dt \right. \\
\left. + \sum_{j=1}^{\infty} Q \left( T_j(s), s \right) P \left( T_j(s), s \right) \gamma Y \right] = W_0 + \sum_{s=1,2} Q \left( T_1(s), s \right) K(s),
\]

\[
\int_{0}^{T_1(1)} P(t, 1) c(t, 1) dt + K(1) = M_0,
\]

\[
\int_{0}^{T_1(2)} P(t, 2) c(t, 2) dt + K(2) = M_0.
\]

\(W_0\) denotes deposits in the brokerage account,

\[
W_0 \equiv B_0 + \sum_{s=1,2} \int_{0}^{\infty} Q(t, s) P(t, s) Y dt.
\]

To obtain the levels of consumption and the transfer times, use the first order
conditions of the utility maximization problem with respect to \( c(t) \) and \( T_j \). We have

\[
c^+(T_j) \left[ \frac{Q(T_j-1)}{Q(T_j)} c^-(T_j) - 1 \right] \frac{\sigma}{1-\sigma} + \gamma Y [r_2 - \pi(T_j)] = -r_2 \int_{T_j}^{T_{j+1}} \frac{P(t) c(t)}{P(T_j)} dt,
\]

with \( \left( \frac{c^+(T_j)}{c^-(T_j)} \right)^{-\sigma} = \frac{Q(T_j)}{Q(T_{j-1})} = e^{-r_2 T_j} \), \( j = 2, 3, ..., \frac{c(t)}{c^-(T_j)} = \left[ \frac{e^{-\rho T_j} P(t)}{e^{-\rho T(T_j)}} \right]^{-1/\sigma} \), and substituted \( Q(t) = e^{-r_2 t} \). With further simplification we obtain the formula in the body of the text. The logarithmic case is analogous.

For the first transfer time, the first order conditions of the utility maximization problem with respect to \( c(t) \) and \( T_1 \) imply

\[
c^+(T_1) \left[ \frac{\mu(n)}{\lambda} \frac{1}{Q(T_1)} c^-(T_1) - 1 \right] \frac{\sigma}{1-\sigma} - r_2 \frac{K}{P(T_1)} = -r_2 \int_{T_1}^{T_2} \frac{P(t) c(t)}{P(T_1)} dt - \gamma Y [r_2 - \pi(T_1)],
\]

with \( \left( \frac{c^+(T_1)}{c^-(T_1)} \right)^{-\sigma} = \frac{\lambda}{\mu(n)} \) and \( Q(T_1) = \frac{\lambda}{\mu(n)} e^{-r_2 N_1} \). \( \mu(n) \) is the Lagrange multiplier associated to the budget constraint until the first transfer. In the logarithmic case, we obtain

\[
- r(t) T_1(n) - \log \frac{\mu(n)}{\lambda} + \gamma Y [r(t) - \pi(T_1(n))] - \frac{r(t) K(n)}{P(T_1(n)) c^+(T_1(n))} = -r(t) \frac{1 - e^{-\rho N_2(n)}}{\rho}.
\]

**APPENDIX D - DATA**

I am using a similar data set as the one used in Lucas (2000).

**GDP**

From 1900 to 1928 it is from the Bureau of the Census (1975), *Historical Statistics of the United States: Colonial Times to 1970*. Series F1, Nominal GDP. From 1929
to 2000 it is from NIPA, Tables 1.1.5, 1.1.6.

**Interest Rate**

The nominal interest rate is the short commercial paper rate. From 1900 to 1975 it is from Friedman and Schwartz (1982), *Monetary trends in the United States and the United Kingdom: their relation to income, prices and interest rates, 1875-1975*, Chicago: University of Chicago Press, Table 4.8, column 6, p. 122, “Interest Rate, Annual Percentage, Short-Term, Commercial Paper Rate”. From 1976 to 1997 it is from the *Economic Report of the President*, Table B-73 “Bond Yields and Interest rates”. In Friedman and Schwartz, the data are for commercial paper 60 to 90 days before 1924, and 4 to 6 months thereafter. In the Economic Report of the President, the data are for commercial paper 4 to 6 months before 1980, and 6 months thereafter.

**Money**

From 1900 to 1913, it is from the Bureau of the Census (1960), *Historical Statistics of the United States: colonial times to 1957*, Series X-267, “demand deposits adjusted plus currency outside banks”. From 1914 to 1958 it is from Friedman and Schwartz (1963), *A Monetary History of the United States, 1867-1960*, December of each year, seasonally adjusted. For M1, I used column 7, sum of currency and demand deposits. From 1959 to 1997 it is from the Federal Reserve Bank of St. Louis, FRED Database, series M1SL, December of each year, seasonally adjusted.

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