Calling Clubs: Network Competition with Non-Uniform Calling Patterns*

Steffen Hoernig† Roman Inderst‡ Tommaso Valletti§

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Abstract

The paper introduces a flexible model of telecom network competition that allows for *non-uniform* calling patterns. We analyze the implications of *skewed* calling patterns on off- and on-net prices as well as equilibrium reciprocal access charges in a model with two-part tariffs. Contrary to existing results in the literature, call prices are not set at marginal cost, and when calling patterns are very skewed, networks benefit from above-cost access charges.

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†Universidade Nova de Lisboa and CEPR.

‡University of Frankfurt (IMFS), Imperial College London and CEPR.

§Imperial College London, University of Rome II and CEPR.

1 Introduction

A common assumption in the literature on competition between telecommunications networks is that of uniform calling patterns: Each customer is equally likely to call any other customer in the market (e.g., Armstrong, 1998; Laffont et al., 1998a, 1998b). Amongst other things, this generates balanced calling patterns, whereby any given customer is equally likely to make or receive on-net (or, likewise, off-net) calls. We show below that the assumption of uniform calling patterns drives some of the key results in the literature, such as that with two-part tariffs marginal call prices should be set at perceived cost, or that firms should optimally negotiate below-cost access charges so as to reduce competition in the market (Gans and King, 2001). Off-net prices should then be cheaper than on-net prices, which is a pattern not often observed in practice.

This paper introduces a flexible model allowing for non-uniform calling patterns. Customers differ in their preferences for a particular network, e.g., how strongly they are attracted by its brand appeal. Instead of stipulating that a customer is equally likely to call any other customer, we allow for the fact that a given customer is more likely to call other customers who have similar preferences ("calling clubs"). For instance, the brand positioning of a network may be more appealing to a particular age group. Likewise, differences in local network coverage could generate similar patterns of call preferences. Both under general distributional assumptions and for a flexible linear specification, capturing both the workhorse models of a Hotelling line and a Salop circle, we investigate the implications of skewed versus uniform calling patterns for on- and off-net prices, equilibrium profits, as well as equilibrium access charges.

In our model, firms offer customers on-net and off-net calls, while customers differ in their anticipated usage of these two services at either network. Furthermore, given that the offers of both networks jointly determine their respective subscriber base, for each user the volume of on-net and off-net calls depends both on the tariff of the chosen network and on the tariff of competitors. When calling patterns are not uniform, firms practice price discrimination by using off-net and on-net prices as metering devices, thereby extracting more of the information rent of consumers who have a strong preference for the respective network (and therefore make more on-net calls).[1] This feature will lead to a distortion

[1]Given that in our model a customer's private information at the stage of contracting relates only to his "horizontal preferences", there is only a superficial relation to the literature on multidimensional price

of marginal prices away from the perceived marginal cost of calls: On-net prices are set above and off-net price are set below their respective perceived marginal cost. This does not mean, though, that off-net prices are set below their economic cost, or below on-net prices, for that matter. In fact, their equilibrium level will depend on the access charge.

Indeed, our second key result is that the jointly profit-maximizing reciprocal access charge depends crucially on the skewness of calling patterns. When calling patterns are sufficiently localized, an access charge above termination cost is chosen. This also implies that off-net calls will become more expensive than on-net calls. Both results contrast with the opposite predictions under uniform calling patterns.

In our model, as in most of the literature, customers do not realize positive utility from received calls. Jeon et al. (2004) show that a change in this assumption can also overturn some of the results in the received literature. In particular, off-net prices can become much more expensive than on-net prices in order to penalize rival customers that will not receive many off-net calls. However, Cambini and Valletti (2008) show that the resultig off-net "connectivity breakdown" disappears when access charges are chosen endogenously. Further work has attempted to explain the apparent puzzle of why reciprocal access charges are typically set above cost in practice, especially in mobile telephony where theoretical models of network competition are more immediately applicable. Armstrong and Wright (2009) show that above-cost access charges can result when mobile firms are also able to set monopoly access charges for calls from fixed lines, and differential access charges for termination of calls from fixed and from mobile networks would otherwise generate arbitrage opportunities. Lopez and Rey (2009) study when high access charges can be used as a foreclosure device. Jullien et al. (2009) show how above-cost access charges may relax competition when there are two different groups of users, heavy users and light users, and where the light users have a elastic subscription demand.[2]

Arguably, allowing for non-uniform calling patterns adds realism to models of network competition. Even when customers are more likely to call those with similar preferences, our modelling approach still allows both for the case where any customer is called with positive probability and for the case where only a strict subset of customers will be called with positive probability. Even in the latter case, the subsets of calling partners will be

discrimination under competition (Armstrong and Vickers, 2001, 2008; Rochet and Stole, 2002).

[2]Elastic subscription of demand is also analyzed by Hurkens and Jeon (2009), though they find that below-cost access charges should be set to relax the intensity of competition.

overlapping, which differs from the analysis of "calling clubs" in Gabrielsen and Vagstad (2008) and Calzada and Valletti (2008), where only customers with identical preferences form perfectly closed and non-overlapping calling clubs.[3]

The rest of this paper is organized as follows. Section 2 introduces the model. In Section 3 we solve, for a given network size, for the optimal off- and on-net pricing structure. Section 4 determines equilibrium tariffs for a given reciprocal access charge, which is in turn solved for in Section 5. Section 6 offers some concluding remarks.

2 A Model of Competition with "Calling Clubs"

Firms' Costs. We consider competition between two firms, $i = 1, 2$. Both firms incur a fixed cost f to serve each subscriber. The marginal cost of providing a telephone call consists in the terminating and originating cost, which for simplicity is assumed to be symmetric and equal to c_0, and the conveying cost, c_1. As a result, the total marginal cost of an on-net call initiated and terminated on the same network i is $c_{ii} \equiv 2c_0 + c_1$. Firms pay each other a reciprocal termination access charge a when a call initiated on network i is terminated on a different network j. Thus, for an off-net call, the economic marginal cost is still c_{ii}, but the "perceived" marginal cost for the network that initiates the call is $c_{ij} \equiv c_1 + c_0 + a$.

Consumers' Preferences. The market consists of a mass one of consumers, which differ in their preferences for the two firms. As is standard, any given consumer is characterized by his preferred characteristics, where we normalize the space of characteristics to $x \in X := [0, 1]$. Consumers' preferences are distributed according to some atomless CDF $H(x)$ with density $h(x) > 0$ over X, while the two firms' own "attributes" are represented by their respective locations at the two extremes. (Note, however, that below we comment on how our example with a linear (Hotelling) specification extends to the case where the space of preferences is represented, instead, by a Salop circle.)

With some abuse of notation, it is convenient to refer to the locations for firms as x_i, such that $x_1 = 0$ and $x_2 = 1$ respectively. Preferences and firms' locations may relate to

[3] While we allow for differences in on- and off-net prices, in this paper we do not consider the *endogenous* formation of such "calling clubs" through a pricing structure that could, for instance, condition on the identity of the called persons ("friends and family").

the brand image that a given firm has created through its marketing. We keep our analysis symmetric by stipulating that $H(x) = 1 - H(1 - x)$, which implies, in particular, that $h(x) = h(1 - x)$ and $H(1/2) = 1/2$.

A consumer subscribed to firm i obtains the quasi-linear utility

$$y + v_0 + v_i(\cdot) - \widetilde{d}(x_i, x),$$

where y is the income of the consumer, v_0 is a fixed utility term derived from subscription that is assumed to be high enough to guarantee full market coverage, $v_i(\cdot)$ denotes the net indirect utility from making calls and is discussed below, and where $\widetilde{d}(x_i, x) \geq 0$ captures the disutility of a consumer with the preferred characteristics x when subscribing to a network with characteristics x_i. We assume for simplicity that $\widetilde{d}(x_i, x) = d(|x - x_i|)$ with $d(0) = 0$ and $d' > 0$.

Subscription Contracts. Firms offer multi-part tariffs and can discriminate between on-net and off-net calls. As a result, consumers pay a tariff with the following structure:

$$T_i(q_{ii}, q_{ij}) = F_i + p_{ii}q_{ii} + p_{ij}q_{ij},$$

where F_i is the fixed subscription fee that consumers pay to firm i, p_{ii} and q_{ii} are the price and quantity for on-net calls, e.g., in terms of minutes, and p_{ij} and q_{ij} are the respective price and quantity for off-net calls from network i to network $j \neq i$. The level of consumer surplus associated with the demand function $q_{ii}(p_{ii})$ is denoted as $v_{ii} = v(p_{ii})$ for calls on-net, and similarly $v_{ij} = v(p_{ij})$ for calls off-net. This indirect utility function $v(\cdot)$ has standard properties. In particular, it holds for the respective price p and quantity q that $dv/dp = -q$. As in much of the literature, in what follows it will be convenient to stipulate that this indirect utility is independent of the identities of the caller and recipient, though the identity of the recipient of the call and its length, i.e., either q_{ii} or q_{ij}, in general will differ.[4]

Local Calling Preferences. The novel ingredient in our model is that consumers differ in their individual calling patterns. Denote by $G(x'' \mid x')$ the likelihood with which a

[4]With this simple tariff structure we can, as we show, go a long way in characterizing the equilibrium even with general demand and calling patterns. In future work, the analysis could be extended to allow for a menu of tariffs (or, for a single non-linear tariff), which would offer firms more scope to price discriminate.

consumer with preference ("address") x' will choose to call consumers $x \leq x''$. Depending on whether the chosen recipient belongs to the same network or not, the respective call minutes will then equal q_{ii} or q_{ij}. $G(x'' \mid x')$ is assumed to be absolutely continuous in x'' with density $g(x'' \mid x')$. We do not require that $g(x'' \mid x') > 0$ for all $x', x'' \in X$.

A weak assumption that ensures that, without further specification of $G(\cdot)$, consumers are more likely to call consumers with similar preferences is that $G(x'' \mid x')$ is ordered in the sense of strict First-Order Stochastic Dominance: The higher is x', the less likely is the consumer to contact other consumers with a low address x'' and the more likely is the consumer to contact other consumers with a high address. Formally, we thus stipulate that, for all $x \in X$, $G(x \mid x')$ is (weakly) decreasing in x', while it is strictly decreasing for a subset of X that has positive measure. Note again that this allows for the case with a changing support of $G(x \mid x')$.[5]

We invoke also a symmetry assumption with respect to the local call preferences of consumers closer to firm 1 and consumers closer to firm 2:

$$G(x'' \mid x') = 1 - G(1 - x'' \mid 1 - x'). \tag{1}$$

This implies, in particular, that $g(x'' \mid x') = g(1-x'' \mid 1-x')$ and symmetric call preferences at $x = 1/2$: $g(x'' \mid 1/2) = g(1 - x'' \mid 1/2)$ and $G(1/2 \mid 1/2) = 1/2$.

A uniform calling pattern obtains when $G(x \mid x') = H(x)$ holds for *all* x and x': The probability that consumers to the left of location x are called is strictly proportional to their number, and independent of the identity of the caller. As a result, at the network level calling patterns will be balanced, i.e., the number of incoming and outgoing calls are equal.

Market Game. At $t = 1$, firms start by determining a reciprocal access charge. At $t = 2$, they compete for consumers by simultaneously making offers $T_i(\cdot)$. At $t = 3$, consumers subscribe and place calls. At this stage, all payoffs are realized.

In what follows, we analyze this game by backward induction. Section 3 solves for firms' optimal pricing policies, where we take market shares as given, such that the focus

[5] It should be noted that with this setup the choices of the distribution of consumers' preferences, $H(x)$, and that of the local calling patterns of each consumer, $G(x'' \mid x')$, are independent. As a consequence, if for given x' the area X' on which $G(x'' \mid x')$ puts "most of the mass" is not "densely populated" (in terms of low probability mass of $H(x)$ over X'), then the likelihood with which a given subset of X' receives a call is relatively high, compared to the case where the area X' is more "densely populated".

is on firms' optimal price discrimination strategy for on-net and off-net calls. Section 4 solves for the equilibrium offers, which determine equilibrium market shares. Finally, in Section 5 we solve for the equilibrium access charge chosen by the firms in the first stage.

Example. We conclude the description of the model with the specification of a particular, tractable example. For this we suppose that consumers are uniformly distributed over X: $h(x) = 1$ for all $x \in X$. In addition, we specify that the local calling patterns take on the following particular form. With probability $1 - \lambda$ a consumer calls someone in $x \in X$ (uniformly) at random.

Let $b \in [0, 1]$ be the size of "calling clubs". If the consumer has location $b/2 \leq x' \leq 1 - b/2$, then with the residual probability λ he calls only, though again randomly, consumers with closer-by location $x \in N(x') = [x' - b/2, x' + b/2]$. In this example, λ represents how relevant the calling club is, while a smaller value of b indicates more concentrated calling clubs. It remains to specify how consumers with $x < b/2$ or $x > 1 - b/2$ choose their local calling partner (with probability λ). It turns out that for our further analysis, provided that b is not too large, this is inconsequential. To complete the specification, we thus presume that all consumers with $x' < b/2$ will, with probability λ, choose any consumer $x \in N(x') = [0, b]$ with equal probability, while those with $x' > 1 - b/2$ choose any consumer $x \in N(x') = [1 - b, 1]$ with equal probability.

More formally, we have in this example

$$g(x \mid x') = 1 - \lambda + \frac{\lambda}{b} I_{N(x')}(x),$$

where $I_S(.)$ is the indicator function for the set S. If we let $J_S(x) = \int_0^x I_S(y)\,dy$, then

$$G(x \mid x') = (1 - \lambda)x + \frac{\lambda}{b} J_{N(x')}(x). \tag{2}$$

For $x \leq \hat{x}$, calling patterns are then given by

$$G(\hat{x} \mid x) = \begin{cases} (1-\lambda)\hat{x} + \lambda\left(\frac{1}{2} + \frac{\hat{x}-x}{b}\right) & \text{for } \hat{x} - b/2 \leq x \leq \hat{x} \\ (1-\lambda)\hat{x} + \lambda & \text{for } x < \hat{x} - b/2 \end{cases}.$$

Clearly, this example obeys the first-order stochastic dominance criterion, since $G(\hat{x} \mid x)$ decreases in x for $\hat{x} - b/2 \leq x \leq \hat{x}$ and otherwise is constant.

For $\lambda = 0$ we have $g(x \mid x') = 1$ and $G(x \mid x') = x$. In this case, the example reduces to a standard uniform calling pattern as in Armstrong (1998), Laffont et al. (1998a, 1998b) or Gans and King (2001).

Finally, for the example we specify that $d(x) = \tau x > 0$, where τ is the unit transportation cost.

3 Pricing Structure

Utilities. As we will establish, in equilibrium firm 1 serves all consumers $x \leq \hat{x}$ and firm 2 all consumers $x \geq \hat{x}$. The marginal consumer $\hat{x}$ is just indifferent between the two offers. Given $\hat{x}$, for any consumer x the net utility from subscribing to firm 1 is then given by

$$U_1(x, \hat{x}) = u_1(x, \hat{x}) + v_0 - F_1 - d(x)$$

with

$$u_1(x, \hat{x}) = G(\hat{x} \mid x)v(p_{11}) + [1 - G(\hat{x} \mid x)]\, v(p_{12}).$$

If the consumer subscribes, instead, to network 2, then the respective utility equals

$$U_2(x, \hat{x}) = u_2(x, \hat{x}) + v_0 - F_2 - d(1 - x),$$

with

$$u_2(x, \hat{x}) = [1 - G(\hat{x} \mid x)]\, v(p_{22}) + G(\hat{x} \mid x)v(p_{21}).$$

Profits and Surplus. Given the marginal consumer $\hat{x}$ and a contract T_1, from each subscribing consumer at location x firm 1 makes expected profits equal to the sum of the fixed part F_1 plus the expected call charges

$$\pi_1(x, \hat{x}) = G(\hat{x} \mid x)q(p_{11})(p_{11} - c_{11}) + [1 - G(\hat{x} \mid x)]\, q(p_{12})(p_{12} - c_{12}) \tag{3}$$

plus the expected access revenues

$$R_{12}(x, \hat{x}) = (a - c_0)q(p_{21}) \int_{\hat{x}}^{1} \frac{g(x \mid x')}{h(x)} dH(x').$$

We can thus write the total expected profits that firm 1 obtains from a given consumer at location x as

$$\Pi_1(x, \hat{x}) = \pi_1(x, \hat{x}) + F_1 + R_{12}(x, \hat{x}) - f.$$

For given cutoff $\hat{x}$, firm 1 thus obtains the total expected profits

$$\overline{\Pi}_1 (\hat{x}) = \int_0^{\hat{x}} \Pi_1(x, \hat{x}) dH(x). \tag{4}$$

It is useful for what follows to express the joint surplus that firm 1 realizes with a given consumer by

$$\omega_1(x,\hat{x}) = v_0 - d(x) + u_1(x,\hat{x}) + \pi_1(x,\hat{x}) + R_{12}(x,\hat{x}) - f. \tag{5}$$

The respective definitions for firm 2 are symmetric:

$$\begin{aligned}
\pi_2(x,\hat{x}) &= [1 - G(\hat{x} \mid x)]\, q(p_{22})(p_{22} - c_{22}) + G(\hat{x} \mid x) q(p_{21})(p_{21} - c_{22}), \\
R_{21}(x,\hat{x}) &= (a - c_0) q(p_{12}) \int_0^{\hat{x}} \frac{g(x \mid x')}{h(x)} dH(x'), \\
\Pi_2(x,\hat{x}) &= \pi_2(x,\hat{x}) + F_2 + R_{21}(x,\hat{x}) - f, \\
\overline{\Pi}_2(\hat{x}) &= \int_{\hat{x}}^1 \Pi_2(x,\hat{x}) dH(x), \\
\omega_2(x,\hat{x}) &= v_0 - d(1-x) + u_2(x,\hat{x}) + \pi_2(x,\hat{x}) + R_{21}(x,\hat{x}) - f.
\end{aligned}$$

3.1 Optimal Price Discrimination

In this Section, we take the firms' market shares $\alpha_1 = H(\hat{x})$ and $\alpha_2 = 1 - H(\hat{x})$, and thus the cutoff $\hat{x}$, as given. We consider the firms' program to optimally choose on- and off-net prices so as to maximize profits.

More precisely, we consider the following program. We take as given the gross utility level that the marginal consumer must obtain: $U_1(\hat{x},\hat{x}) \geq \overline{U}$. (In equilibrium, this will be given from the offer of the competing firm, such that $\overline{U} = U_2(\hat{x},\hat{x})$.) For given $\hat{x}$ and $\overline{U}$ we then solve for the optimal choices p_{11} and p_{22} to maximize Π_1. We relax this program by only considering the participation constraint of the marginal consumer $x = \hat{x}$ but not that of consumers $x \leq \hat{x}$. Further below we impose sufficient conditions for when (both on and off equilibrium) the solution to the relaxed program is indeed a solution to the original one.

To characterize the optimal prices, it is convenient to introduce the elasticity (of per-minute demand) $\eta(p) = -\frac{q'(p)p}{q(p)} > 1$. Furthermore, to ensure that the firm's program has a unique solution, a sufficient condition is that the demand function satisfies $q'' < 0$ (where $q > 0$). In what follows, we assume that this holds.

Proposition 1 *For given market share $\alpha_1 = H(\hat{x})$ and given utility of the marginal consumer $U_1(\hat{x},\hat{x}) \geq \overline{U}$, the optimal on- and off-net prices of firm 1 under localized calling* *patterns, solving the (relaxed) program, satisfy*

$$\frac{p_{11}-c_{11}}{p_{11}}=\frac{1}{\eta(p_{11})}\left[\frac{\int_0^{\hat{x}}\left[G(\hat{x}\mid x)-G(\hat{x}\mid\hat{x})\right]dH(x)}{\int_0^{\hat{x}}G(\hat{x}\mid x)dH(x)}\right]\geq 0 \tag{6}$$

and

$$\frac{p_{12}-c_{12}}{p_{12}}=-\frac{1}{\eta(p_{12})}\left[\frac{\int_0^{\hat{x}}\left[G(\hat{x}\mid x)-G(\hat{x}\mid\hat{x})\right]dH(x)}{\int_0^{\hat{x}}\left[1-G(\hat{x}\mid x)\right]dH(x)}\right]\leq 0. \tag{7}$$

Likewise, the symmetric program for firm 2 *yields*

$$\frac{p_{22}-c_{22}}{p_{22}}=\frac{1}{\eta(p_{22})}\left[\frac{\int_{\hat{x}}^1\left[G(\hat{x}\mid\hat{x})-G(\hat{x}\mid x)\right]dH(x)}{\int_{\hat{x}}^1\left[1-G(\hat{x}\mid x)\right]dH(x)}\right]\geq 0 \tag{8}$$

and

$$\frac{p_{21}-c_{21}}{p_{21}}=-\frac{1}{\eta(p_{21})}\left[\frac{\int_{\hat{x}}^1\left[G(\hat{x}\mid x)-G(\hat{x}\mid\hat{x})\right]dH(x)}{\int_{\hat{x}}^1G(\hat{x}\mid x)dH(x)}\right]\leq 0. \tag{9}$$

Proof. See Appendix.

Note that with a uniform calling pattern, when $G(\hat{x}\mid x)=H(\hat{x})$, we have $p_{ij}=c_{ij}$ in all four cases, i.e., the standard marginal cost pricing result. The intuition for why with a skewed calling pattern marginal prices are above marginal cost for on-net calls and below marginal cost for off-net calls is the following: Both prices, say p_{12} and p_{11} for firm 1, serve as metering devices. Higher on-net prices extract more of the "information rent" for customers that have a stronger preference for firm 1 than the marginal customer, $\hat{x}$. On the other hand, this distortion in on-net prices reduces the joint surplus $\omega_1(\hat{x},\hat{x})$, which is then shored up by decreasing p_{12} accordingly.

Importantly, note that $p_{ii}>c_{ii}$ and $p_{ij}<c_{ij}$ do not imply that off-net calls are cheaper than on-net calls, given that the relation of the respective costs, c_{ii} and c_{ij}, depends also on the endogenous access charge. We will return to this below after having solved for the equilibrium access charge.

Note also that with symmetry, i.e., if the marginal consumer is located at $\hat{x}=1/2$, the prevailing marginal prices $p_{11}=p_{22}=p_{ii}$ and $p_{12}=p_{21}=p_{ij}$ are obtained simply from substitution of $\hat{x}=1/2$ into Proposition 1 and using symmetry of $H(x)$ and $G(\cdot)$ (cf. condition (1)). For what follows, it is convenient to adopt a more concise notation. Let

$$\Gamma=\int_{1/2}^1 G(\frac{1}{2}\mid x)dH(x)=\int_0^{1/2}\left(1-G(\frac{1}{2}\mid x)\right)dH(x),$$

where the second equality follows from symmetry. Note that Γ is the share of total calls that are made off-net to an individual network in a symmetric equilibrium. With a uniform calling pattern, it is

$$\Gamma = \int_0^{1/2} \left(1 - H\left(\frac{1}{2}\right)\right) dH(x) = \frac{1}{4},$$

while when the calling pattern is skewed then $\Gamma < 1/4$, i.e., more calls are made on-net than off-net.

In a symmetric equilibrium, on- and off-net prices in Proposition 1 simplify to

$$\begin{aligned} \frac{p_{ii} - c_{ii}}{p_{ii}} &= \frac{1}{\eta(p_{ii})} \frac{1/4 - \Gamma}{1/2 - \Gamma}, \\ \frac{p_{ij} - c_{ij}}{p_{ij}} &= -\frac{1}{\eta(p_{ij})} \frac{1/4 - \Gamma}{\Gamma}, \end{aligned}$$

which indeed results in marginal-cost pricing if and only if $\Gamma = 1/4$.

Finally, we comment on our restriction to only consider the participation constraint at the marginal type $\hat{x}$. The proof of Proposition 1 derives a sufficient condition for when the solution to the relaxed programme is a solution to the original problem. Focusing, given symmetry, on the condition for the program of firm $i = 2$, this sufficient condition is that for all $x \leq \widehat{x}$ it is true that

$$\frac{\partial G(\hat{x} \mid x)}{\partial x} [v(p_{11}) - v(p_{12}) + v(p_{22}) - v(p_{21})] < d'(x) + d'(1-x), \tag{10}$$

which at the symmetric equilibrium outcome simplifies to

$$[v(p_{ii}) - v(p_{ij})] \frac{\partial G(1/2 \mid x)}{\partial x} < \frac{d'(x) + d'(1-x)}{2}. \tag{11}$$

Recall that $\frac{\partial G(1/2|x)}{\partial x}$ is negative, and therefore if $p_{ii} \leq p_{ij}$ the condition above certainly holds. In the example, where $d'(x) = \tau$, condition (11) and, more generally, condition (10) are surely satisfied whenever τ is not too low.[6] In what follows, we stipulate that this is the case. The joint assumption that the level of horizontal differentiation, as captured by $d' > 0$ or, more specifically, $\tau > 0$ and v_0 are both not too low, ensuring full coverage, is also typically invoked in the literature (e.g., Laffont et al., 1998b).

[6]Recall also that with a uniform calling pattern it holds that $\frac{\partial G(x'|x)}{\partial x} = 0$, such that the left-hand side in either condition is equal to zero.

Optimal Uniform Price. It seems useful to derive briefly the optimal uniform prices, i.e., the equilibrium in the case where networks do not price discriminate between on- and off-net calls. Note that in this case, given the uniform price p_i, each customer independently of his location realizes the utility (gross of fees) $v(p_i)$ at the respective network i.

Proposition 2 *When firms are constrained to charge a uniform price p_i, then this is optimally set equal to the "average marginal cost", i.e.,*

$$\begin{aligned} p_1 &= c_{ii} + (a - c_0) \int_0^{\hat{x}} \left[1 - G(\hat{x} \mid x)\right] \frac{h(x)}{H(\hat{x})} dx, \\ p_2 &= c_{ii} + (a - c_0) \int_{\hat{x}}^1 G(\hat{x} \mid x) \frac{h(x)}{1 - H(\hat{x})} dx. \end{aligned} \tag{12}$$

Proof. See Appendix.

The above prices consist of the on-net cost plus the termination surcharge on the average number of off-net calls for the network's customers. Therefore these prices are clearly equal to perceived marginal cost, given the calling pattern of an average consumer of either network.

In a symmetric equilibrium, we obtain

$$p_1 = p_2 = c_{ii} + 2\Gamma(a - c_0). \tag{13}$$

Therefore, skewed calling patterns lead to lower uniform prices due to their larger on-net share of calls.

3.2 Example

We now apply Proposition 1 to the example. For this note first that $\alpha_1 = \hat{x}$ and $\alpha_2 = 1 - \hat{x}$. We also restrict consideration to the case where $b/2 \leq \hat{x} \leq 1 - b/2$.

Proposition 3 *In the linear example, on-net and off-net prices, as characterized in Proposition 1, satisfy for firm* 1

$$\begin{aligned} \frac{p_{11} - c_{11}}{p_{11}} &= \frac{1}{\eta(p_{11})} \frac{\lambda}{2} \frac{\hat{x} - \frac{b}{4}}{(1-\lambda)\hat{x}^2 + \lambda\left(\hat{x} - \frac{1}{8}b\right)}, \\ \frac{p_{12} - c_{12}}{p_{12}} &= -\frac{1}{\eta(p_{12})} \frac{\lambda}{2} \frac{\hat{x} - \frac{b}{4}}{\hat{x} - \left[(1-\lambda)\hat{x}^2 + \lambda\left(\hat{x} - \frac{1}{8}b\right)\right]}, \end{aligned}$$

and for firm 2

$$
\begin{aligned}
\frac{p_{22}-c_{22}}{p_{22}} &= \frac{1}{\eta(p_{22})}\frac{\lambda}{2}\frac{1-\hat{x}-\frac{b}{4}}{(1-\lambda)(1-\hat{x})^2+\lambda\left(1-\hat{x}-\frac{1}{8}b\right)}, \\
\frac{p_{21}-c_{21}}{p_{21}} &= -\frac{1}{\eta(p_{21})}\frac{\lambda}{2}\frac{1-\hat{x}-\frac{b}{4}}{(1-\hat{x})-\left[(1-\lambda)(1-\hat{x})^2+\lambda\left(1-\hat{x}-\frac{1}{8}b\right)\right]}.
\end{aligned}
$$

Proof. See Appendix.

For a discussion of the characterization in Proposition 3 it is convenient to take the symmetric case, where $\hat{x}=1/2$, such that

$$
\begin{aligned}
\frac{p_{ii}-c_{ii}}{p_{ii}} &= \frac{1}{\eta(p_{ii})}\left[\frac{\lambda\left(1-\frac{b}{2}\right)}{1+\lambda\left(1-\frac{b}{2}\right)}\right], \\
\frac{p_{ij}-c_{ij}}{p_{ij}} &= -\frac{1}{\eta(p_{ij})}\left[\frac{\lambda\left(1-\frac{b}{2}\right)}{1-\lambda\left(1-\frac{b}{2}\right)}\right].
\end{aligned}
$$

In both cases, the last term is zero at $\lambda=0$. With uniform calling patterns, marginal prices are not distorted. Further, in both cases, the respective term in rectangular brackets is strictly increasing in λ. In addition, when $\lambda>0$, it is strictly decreasing in b. Hence, prices become more distorted the more relevant the "club" is, and the more (locally) skewed customers' calling preferences become.

4 Equilibrium

We now determine the market equilibrium for given access charge a. Throughout the subsequent analysis we assume existence of a unique equilibrium in pure strategies. This equilibrium will then be symmetric with $\hat{x}=1/2$.

4.1 First-Order Condition

Now that we have derived p_{11} and p_{12} as functions of $\hat{x}$, we will derive firm 1's profit-maximizing market share. Firm 1's fixed fee F_1 is defined by the condition that the marginal consumer must be indifferent between the offers of the two firms, i.e., $U_1(\hat{x},\hat{x})=U_2(\hat{x},\hat{x})$, with

$$F_1=u_1(\hat{x},\hat{x})-u_2(\hat{x},\hat{x})+d(1-\hat{x})-d(\hat{x})+F_2$$

and thus

$$\frac{dF_1}{d\hat{x}} = \frac{d}{d\hat{x}}\left(u_1(\hat{x},\hat{x}) - u_2(\hat{x},\hat{x})\right) - d'(\hat{x}) - d'(1-\hat{x}).$$

Taking into account this dependence of F_1 on market share $\hat{x}$, together with the envelope theorem with respect to p_{11} and p_{12}, the maximization of profits

$$\overline{\Pi}_1(\hat{x}) = \int_0^{\hat{x}} \left[\pi_1(x,\hat{x}) + F_1 + R_{12}(x,\hat{x}) - f\right] dH(x)$$

over $\hat{x}$ leads to the following first-order condition:

$$\begin{aligned} \frac{d\overline{\Pi}_1(\hat{x})}{d\hat{x}} &= \int_0^{\hat{x}} \left(\frac{\partial \pi_1(x,\hat{x})}{\partial \hat{x}} + \frac{\partial R_{12}(x,\hat{x})}{\partial \hat{x}}\right) dH(x) \\ &\quad + \left(\omega_1(\hat{x},\hat{x}) - U_1(\hat{x},\hat{x})\right) h(\hat{x}) \\ &\quad + H(\hat{x})\frac{dF_1}{d\hat{x}}. \end{aligned} \tag{14}$$

Here, the first term captures the resulting change in call and termination profits per (inframarginal) customer; the second term captures the change in value of the marginal customer (and thus the gain or loss of their resulting profits $\omega_1(\hat{x},\hat{x}) - U_1(\hat{x},\hat{x})$); and the third term captures the change in the fixed fee obtained from inframarginal customers.[7]

It is convenient to refer to the these three terms as CPC ("change in call and termination profits per consumer", CNC ("change in the number of consumers"), and CFI ("change in the rent of infra-marginal consumers"). We are interested in a symmetric equilibrium and, therefore, restrict the exposition to the equilibrium outcome with symmetry, $\hat{x} = 1/2$. We then find

$$\begin{aligned} CPC &= (r_{ii} - r_{ij}) \int_0^{1/2} g(\frac{1}{2} \mid x) dH(x) - \frac{1}{2} t_{ij} h(\frac{1}{2}) \\ &= \left[(r_{ii} - r_{ij})\gamma - \frac{1}{2} t_{ij}\right] h(\frac{1}{2}), \end{aligned}$$

where we use $r_{ij} := q(p_{ij})(p_{ij} - c_{ij})$, $t_{ij} := (a - c_0)q(p_{ji})$, and

$$\gamma := \frac{1}{h(1/2)} \int_0^{1/2} g(\frac{1}{2} \mid x) dH(x).$$

Intuitively, as customers with location $\hat{x} = 1/2$ switch to network 1, the calls from customers $x < \hat{x}$ to $\hat{x} = 1/2$ become on-net instead of off-net calls, which results in a profit difference of $r_{ii} - r_{ij}$. In addition, access charges that would otherwise be earned from all $x < \hat{x}$ who are called by $\hat{x} = 1/2$ are now lost.

[7] Note that the second and third terms together constitute the standard price-vs-quantity trade-off.

As 2γ denotes the probability with which any customer located at 1/2 will be called in equilibrium, when calling patterns are balanced, it must hold that $\gamma = 1/2$.

Next, using that $u_1(\frac{1}{2}, \frac{1}{2}) = u_2(\frac{1}{2}, \frac{1}{2})$, we have

$$\begin{aligned} CNC &= \left(\pi_1(\frac{1}{2}, \frac{1}{2}) + R_{12}(\frac{1}{2}, \frac{1}{2}) + F - f\right) h(\frac{1}{2}) \\ &= \left[\frac{1}{2}(r_{ii} + r_{ij}) + t_{ij} \int_{1/2}^{1} \frac{g(\frac{1}{2} \mid x)}{h(\frac{1}{2})} dH(x) + F - f\right] h(\frac{1}{2}). \end{aligned}$$

As, by symmetry,

$$\int_{1/2}^{1} g(\frac{1}{2} \mid x) dH(x) = \int_{0}^{1/2} g(\frac{1}{2} \mid x) dH(x) = \gamma h(\frac{1}{2}),$$

this becomes

$$CNC = \left[\frac{1}{2}(r_{ii} + r_{ij}) + \gamma t_{ij} + F - f\right] h(\frac{1}{2}).$$

Finally, making use of $dF_1/d\hat{x}$ as mentioned above, we have at $\hat{x} = 1/2$ that

$$CFI = \chi (v_{ii} - v_{ij}) h(\frac{1}{2}) - d'(\frac{1}{2}),$$

where we have defined

$$\chi := \frac{1}{h(1/2)} \left[g(\frac{1}{2} \mid \frac{1}{2}) + \left.\frac{\partial G(1/2 \mid x)}{\partial x}\right|_{x=1/2} \right].$$

Also χ has an intuitive interpretation. Expressed in terms of partial derivatives, the term in rectangular brackets is equal to $G_1(1/2 \mid 1/2) + G_2(1/2 \mid 1/2)$. Here, $G_1(1/2 \mid 1/2)/h(1/2)$ captures the "local skewness" of the calling pattern of the marginal customer, which is equal to one under a uniform calling pattern and strictly larger otherwise. Instead, $G_2(1/2 \mid 1/2)$ captures how calling patterns between customers differ at 1/2. With a uniform calling pattern we clearly have that $G_2(1/2 \mid 1/2) = 0$, while it is a negative term for skewed distributions. As we elaborate below in more detail in the example, both terms $G_1(1/2 \mid 1/2)$ and $G_2(1/2 \mid 1/2)$ are jointly influenced when, across all customers, the calling pattern becomes more skewed.

Summing up, using the newly introduced parameters, the first-order condition that $CPC + CNC + CFI = 0$ thus becomes

$$\begin{aligned} &\left[(r_{ii} - r_{ij})\gamma - \frac{1}{2}t_{ij}\right] h(\frac{1}{2}) \\ &+ \left[\frac{1}{2}(r_{ii} + r_{ij}) + \gamma t_{ij} + F - f\right] h(\frac{1}{2}) \\ &+ \chi (v_{ii} - v_{ij}) h(\frac{1}{2}) - d'(\frac{1}{2}) = 0. \end{aligned}$$

Symmetric Fixed Fee. Solving this condition for F leads to the following intermediary result.

Proposition 4 *In a symmetric equilibrium, fixed fees are equal to*

$$F = f + \frac{d'(1/2)}{h(1/2)} - \left(\frac{1}{2} + \gamma\right) r_{ii} - \left(\frac{1}{2} - \gamma\right) (r_{ij} - t_{ij}) - \chi \left(v_{ii} - v_{ij}\right). \qquad (15)$$

It is useful to note that when the calling pattern is balanced, at least at the marginal customer, such that $\gamma = 1/2$, then expression (15) simplifies to

$$F = f + \frac{d'(1/2)}{h(1/2)} - r_{ii} - \chi \left(v_{ii} - v_{ij}\right). \qquad (16)$$

Finally, a further simplification is obtained when the calling pattern is uniform, such that also $\chi = 1$ and we thus have that

$$F = f + \frac{d'(1/2)}{h(1/2)} - r_{ii} - \left(v_{ii} - v_{ij}\right). \qquad (17)$$

We return to these expressions later. When the calling pattern is balanced (but not necessarily uniform), inspection of (16) reveals that the access charge has a feedback effect on the fixed fee only through the off-net indirect utility, v_{ij}, as this depends on the off-net price p_{ij}. This is frequently referred to as a "waterbed effect". It is important to stress that the magnitude of this waterbed effect is diluted by the factor χ which is 1 only when the calling pattern is uniform. When instead the calling pattern becomes more skewed, then χ decreases and the feedback effect of a change in the access charge will be much diluted.

Example. In our example, after substituting the simplified values for γ and $\chi = 1 - \lambda$ in (15), or also in (16) as incoming and outgoing calls are balanced at each customer, we obtain

$$F = f + d'(\frac{1}{2}) - r_{ii} - (1 - \lambda) \left(v_{ii} - v_{ij}\right).$$

Note here that when consumers make only local calls ($\lambda = 1$), then the fixed fee is independent of the access charge, such that there is no "waterbed effect" on the fixed fee. The access charge will obviously always have an effect on off-net charges though. Recalling from Proposition 3 the expression for the off-net price, $\frac{p_{ij} - c_{ij}}{p_{ij}} = -\frac{1}{\eta(p_{ij})} \frac{\lambda\left(1 - \frac{b}{2}\right)}{1 - \lambda\left(1 - \frac{b}{2}\right)}$, we obtain immediately the following result.

Proposition 5 *In the example, in a symmetric equilibrium and for a constant elasticity demand, we have for the "waterbed effect":*

$$\begin{aligned}\frac{dp_{ij}}{da} &= \frac{\left[1-\lambda\left(1-\frac{b}{2}\right)\right]\eta}{\eta-\lambda\left(1-\frac{b}{2}\right)(\eta-1)} > 0,\\ \frac{dF}{da} &= -(1-\lambda)q(p_{ij})\frac{dp_{ij}}{da} \leq 0,\end{aligned}$$

such that the bill of the "marginal" customer at $\hat{x} = 1/2$ *always decreases with an increase of the access charge:*

$$\frac{d\left(F + p_{ij}q_{ij}(p_{ij})/2\right)}{da} < 0.$$

Proof. See Appendix. ■

As calls are priced in the elastic portion of the demand function, an increase in the off-net price due to an increase of the access charge will always cause off-net call expenditure to decrease. The waterbed effect on the fixed fee goes in the same direction to reduce the total bill. This effect disappears when most calls are made to the calling club ($\lambda \rightarrow 1$), while the off-net impact of an increase of the access charge is always present. With skewed calling patterns, though, the impact on the bill of the marginal consumer does not coincide with the impact on the bill of inframarginal consumers. Thus we study next the optimal choice of the access charge when consumers have skewed calling patterns.

4.2 Equilibrium Profits

Substituting the symmetric equilibrium fee into profits, we obtain the following:

Proposition 6 *Profits in a symmetric equilibrium are* $\bar{\Pi}_1\left(\frac{1}{2}\right) = \bar{\Pi}_2\left(\frac{1}{2}\right) = \bar{\Pi}^*$*, with*

$$\bar{\Pi}^* = \frac{d'(1/2)}{2h(1/2)} + \frac{\chi}{2}(v_{ij} - v_{ii}) + \left(\Gamma + \frac{\gamma}{2} - \frac{1}{4}\right)(r_{ij} + t_{ij} - r_{ii}) + \left(\frac{1}{2} - \gamma\right)t_{ij}. \qquad (18)$$

Proof. See Appendix. ■

Equilibrium profits thus depend, in general, on the following terms. The first term captures the way profits depend on the substitutability of goods and the density for marginal consumers $\frac{d'\left(\frac{1}{2}\right)}{2h\left(\frac{1}{2}\right)}$. The second term $\frac{\chi}{2}(v_{ij} - v_{ii})$ is related to price-generated network externalities as perceived by customers. The third term, reflects the extent to which revenues from calls and termination $(r_{ij} + t_{ij} - r_{ii})$ are passed back to customers. Finally, the last term $\left(\frac{1}{2} - \gamma\right)$ depends on whether calling patterns are balanced at the marginal customer.

If the calling pattern is balanced, i.e., $\gamma = \frac{1}{2}$, expression (18) simplifies to

$$\bar{\Pi}^* = \frac{d'(1/2)}{2h(1/2)} + \frac{\chi}{2}(v_{ij} - v_{ii}) + \Gamma(r_{ij} + t_{ij} - r_{ii}). \tag{19}$$

With a uniform calling pattern, $\Gamma = 1/4$ and $\chi = 1$, this further reduces to

$$\bar{\Pi}^* = \frac{d'(1/2)}{2h(1/2)} + \frac{1}{2}(v_{ij} - v_{ii}) + \frac{1}{4}(r_{ij} + t_{ij} - r_{ii}). \tag{20}$$

5 Access Charge

In this section we determine the jointly profit-maximizing access charge under price discrimination between on- and off-net calls. Taking into account only the off-net terms that depend on a in (18), and noting that, given symmetry and thus $p_{ij} = p_{ji}$,

$$r_{ij} + t_{ij} = q(p_{ij})(p_{ij} - c_1 - 2c_0) = q(p_{ij})(p_{ij} - c_{ii}),$$

the jointly profit-maximizing access charges are found by solving

$$\max_{p_{ij}} \left\{ \frac{\chi}{2} v(p_{ij}) + \left(\Gamma + \frac{\gamma}{2} - \frac{1}{4}\right)(p_{ij} - c_{ii})\, q(p_{ij}) + \left(\frac{1}{2} - \gamma\right)(a - c_0)\, q(p_{ij}) \right\}. \tag{21}$$

This obtains the following characterization:

Proposition 7 *The jointly profit-maximizing access charge is given by*

$$a = c_0 + c_{ii} \left(\frac{\left(2\Gamma + \gamma - \frac{1}{2}\right)\frac{1}{4\Gamma} - \chi}{\left(2\Gamma + \gamma - \frac{1}{2}\right)(\eta(p_{ij}) - 1) + \chi} \right). \tag{22}$$

Proof. See Appendix. ■

Recall that for the network that initiates but does not terminate a call, economic marginal cost are equal to "perceived" marginal cost only when $a = c_0$, i.e., only when the term in brackets in (22) is equal to zero.

Before returning to the example, expression (22) also allows to obtain some general insights. For this it is, however, convenient to restrict attention to the case where the calling pattern is balanced at least at $x = 1/2$. That is, with $\gamma = 1/2$ the marginal customer at $x = 1/2$ makes and receives the same number of calls. Further, suppose that the elasticity is constant (or, likewise, that we can momentarily ignore changes in the elasticity as they are sufficiently small). Then, (22) becomes

$$a = c_0 + c_{ii} \left(\frac{1/2 - \chi}{2\Gamma (\eta - 1) + \chi} \right). \tag{23}$$

With a uniform calling pattern, we had $\Gamma = 1/4$ next to $\chi = 1$, implying that

$$a = c_0 - c_{ii} \frac{1}{\eta + 1}.$$

As noted in the Introduction, this is just a restatement of the result in Gans and King (2001). As is argued there, when calling patterns are uniform, firms can dampen competition and, thereby, increase joint profits by setting the reciprocal access charge below economic cost of connection.

Further, as long as $\chi \geq 0$, we have that the access charge lies below economic cost whenever $\chi > 1/2$, while the opposite holds when $\chi < 1/2$. In our example we make precise how this case arises if and only if calling patterns are sufficiently skewed. Again, the driving force is here is to dampen competition, albeit now this is done through setting the price above economic cost. This is a "standard" result for competition in non-network industries, e.g., when we consider two firms that can, through a two-part tariff, licence access to some essential inputs: Setting the – now reciprocal and linear – "wholesale" price above cost dampens competition.

Concentrating on the case of balanced calling patterns ($\gamma = 1/2$), the solution comes from simplifying expression (21) to

$$\max_{p_{ij}} \left\{ \frac{\chi}{2} v(p_{ij}) + \Gamma (p_{ij} - c_{ii}) q(p_{ij}) \right\}$$

Whether the jointly profit-maximizing off-net price is above or below the true marginal cost c_{ii} depends on whether the "weight" $\frac{\chi}{2}$ on consumer call surplus is smaller or larger than the "weight" on call profits Γ. Again, with a uniform calling pattern (i.e., as in Gans and King, 2001), the corresponding weights are 1/2 and 1/4, resulting — as we have already observed — in off-net prices below marginal cost. In general, if the weight on v_{ij} is larger than the weight on $r_{ij} + t_{ij} = (p_{ij} - c_{ii}) q(p_{ij})$, then p_{ij} will be lower than socially optimal, i.e., below c_{ii}. This is achieved through $a < c_0$. Otherwise the access charge will be set above cost.

Example. In our example, the profit-maximizing access charge can be readily obtained as

$$a = c_0 - \frac{2(1-2\lambda)\,c_{ii}}{2(1-\lambda)(1+\eta) + \lambda b(\eta - 1)}. \tag{24}$$

This expression is strictly increasing in λ. It takes its lowest value with uniform calling pattern ($\lambda = 0$), in which case we obviously obtain again the below-cost result that $a = c_0 - \frac{c_{ii}}{\eta+1}$. The access charge then increases, and it is endogenously set at cost, $a = c_0$, if and only if $\lambda = 1/2$. When calls are only made to the local calling club ($\lambda = 1$), the access charge then takes its highest value $a = c_0 + \frac{2c_{ii}}{b(\eta-1)} > c_0$. Also, for a given proportion λ of calls to a calling club, the lower is b (i.e., the more concentrated the calling club is) the higher is the access charge. (When calls are only to calling clubs, i.e.., $\lambda = 1$, and in addition, $b \to 0$, then we can perform the same limit analysis as we did, more generally, above.)

While with no calling clubs the off-net price turns out to be cheaper by the on-net price because the access charge is chosen below cost, this is not necessarily true with the presence of local calling clubs. When $\lambda = 1$, the off-net price is actually set at the monopoly level, which is therefore higher than the on-net price. This is achieved by setting an access charge well above cost.

In general, by substituting (24) into the expressions for the on-net and off-net mark-ups obtained in Section 3, we get

$$\begin{aligned} p_{ii} &= \frac{\eta c_{ii}}{\eta - \frac{(2-b)\lambda}{2(1+\lambda)-b\lambda}}, \\ p_{ij} &= \frac{\eta c_{ii}}{\eta + \frac{2(1-\lambda)-b\lambda}{2(1-\lambda)+b\lambda}}. \end{aligned}$$

It is then the case that, at the equilibrium access charge, off-net prices are higher than on-net prices when

$$p_{ii} > p_{ij} \Longrightarrow \lambda > \lambda^* = \frac{2 - 3b + \sqrt{36 - 28b + 9b^2}}{4(2-b)},$$

i.e., when a significant portion of calls are placed to the local "calling club". It is thus key to have a sufficiently relevant "calling club" to generate this result, which instead in absent in the received literature with uniform calling patterns.

Equivalently, the condition for off-net prices to be higher than on-net prices can also be written as

$$p_{ii} > p_{ij} \Longrightarrow b > b^* = \frac{2(1+\lambda-2\lambda^2)}{\lambda(3-2\lambda)}.$$

This way of expressing the condition may seem a bit surprising at first sight, since, for a given proportion λ of calls to a "calling club", we need the club to be not "too" concentrated. This result, though, is understood by noting that b plays two roles. On the one hand, as shown by (24), the tighter the club the higher the access charge, which tends to raise the off-net price directly. On the other hand, we must recall from Proposition 3 that b also plays a role in the metering problem: For a given a, the on-net mark up increases and, at the same time, the off-net mark-up decreases the more concentrated the club becomes. Thus, if the club is "too" concentrated, the off-net negative mark-up will more than compensate for the high access charge compared to on-net calls.

6 Conclusion

We introduce a flexible model of network competition that allows for non-uniform calling patterns. The model allows us to analyze, both generally and in a tractable example, the implications of skewed calling patterns ("calling clubs") on off- and on-net prices, as well as equilibrium reciprocal access charges.

This extension is particularly relevant to capture the empirical observation that consumers seem to have non-uniform calling patterns as they call much more selected numbers.[8] This in turn generates results that are close to stylized facts, namely we can explain

[8] Take, for instance, the following data for Italy, where there are four mobile operators (source: AGCOM, 2008)

	TIM	Vodafone	Wind	H3G
Sub (000s)	29,450	27,595	16,202	7,922
Mkt share (%)	*0.36*	0.34	0.20	0.10
Calls on net (millions)	19,795	25,025	16,831	2,050
Calls off net (millions)	8,518	7,211	4,001	4,100
Calls on net (%)	*0.70*	0.78	0.81	0.33
Calls off net (%)	*0.30*	0.22	0.19	0.67
Imbalance ratio	**4.08**	**6.74**	**16.87**	**4.62**

The numbers above refer only to mobile-to-mobile calls, while calls to fixed lines have been removed. If on- and off-net prices were identical, and if calls were made proportional to market shares, then calling patterns would be uniform, and the "imbalance" ratio would be 1. The customers of TIM, for instance, should make 36% of their mobile calls on net, and 64% off net. Instead the percentages are 70% and 30% respectively, making an on-net call 4.08 times more likely than an off-net call. Of course this is a crude aggregate measure, since we do not have information on prices and individual behavior, but it is important to recall that in Gans and King (2001), for instance, the off-net calls should cheaper than on-net calls resulting in an imbalance ratio *below* 1. It is also quite difficult to reconcile the imbalance ratios above with a story purely based on on-net prices cheaper than off-net prices (but still within uniform calling patterns), as the call demand elasticities would have to take implausibly large values. Instead, why access charges are set above cost, which, in turn, can generate off-net prices more expensive than on-net prices.

In this work, we have taken local “calling clubs” as exogenous. Endogenizing their formation represents a relevant next step to be investigated in future research.

7 Appendix: Omitted Proofs

Proof of Proposition 1. Given constant market shares, firm 1's fixed fees are determined by the condition $U_1(\hat{x},\hat{x}) = \bar{U}$, i.e., $F_1 = u_1(\hat{x},\hat{x}) + v_0 - d(\hat{x}) - \bar{U}$. Substituting these into firm 1's profits leads to

$$\begin{aligned}\overline{\Pi}_1(\hat{x}) &= \int_0^{\hat{x}} \left[\pi_1(x,\hat{x}) + F_1 + R_{12}(x,\hat{x}) - f\right] dH(x) \\ &= \int_0^{\hat{x}} \left[\pi_1(x,\hat{x}) + u_1(\hat{x},\hat{x})\right] dH(x) + const,\end{aligned}$$

where the last term on the right-hand side does not depend on p_{11} and p_{12}. After substituting for u_1 and π_1, we obtain from the maximization of the relevant terms

$$\int_0^{\hat{x}} \left[G(\hat{x} \mid x)q(p_{11})(p_{11} - c_{11}) + G(\hat{x} \mid \hat{x})v(p_{11})\right] dH(x)$$

with respect to p_{11} the first-order condition

$$q'(p_{11})(p_{11} - c_{11}) \int_0^{\hat{x}} G(\hat{x} \mid x) dH(x) + q_{11} \int_0^{\hat{x}} \left[G(\hat{x} \mid x) - G(\hat{x} \mid \hat{x})\right] dH(x) = 0,$$

which solves to (6). Proceeding analogously for p_{12} yields (7), which completes the characterization for firm 1. The respective characterization for firm 2, (8) and (9), is perfectly symmetric.

We finally check when it is indeed feasible to ignore the participation constraint of all customers with location $x < \hat{x}$, i.e., when indeed, as presumed in the relaxed program, $U_1(x,\hat{x}) \geq U_2(x,\hat{x})$. We have

$$U_1(x,\hat{x}) - U_2(x,\hat{x}) = \left[u_1(x,\hat{x}) - u_2(x,\hat{x})\right] - \left[d(x) - d(1-x)\right] - \left[F_1 - F_2\right],$$

where

$$\begin{aligned}u_1(x,\hat{x}) - u_2(x,\hat{x}) &= \left\{G(\hat{x} \mid x)v(p_{11}) + \left[1 - G(\hat{x} \mid x)\right] v(p_{12})\right\} \\ &\quad - \left\{\left[1 - G(\hat{x} \mid x)\right] v(p_{22}) + G(\hat{x} \mid x)v(p_{21})\right\}.\end{aligned}$$

a combination of on-net prices cheaper than off-net, *and* skewed calling patterns due to "calling clubs" seems to have quite a realistic appeal.

In case of symmetry, this becomes

$$u_1(x,1/2) - u_2(x,1/2) = [v(p_{ii}) - v(p_{ij})]\,[2G(1/2 \mid x) - 1]\,.$$

A sufficient condition for $U_1(x,\hat{x}) > U_2(x,\hat{x})$ holding strictly for all $x < \hat{x}$ is that

$$\frac{\partial}{\partial x}\left[U_1(x,\hat{x}) - U_2(x,\hat{x})\right] < 0,$$

which generally holds when

$$\frac{\partial G(\hat{x} \mid x)}{\partial x}\left[v(p_{11}) - v(p_{12}) + v(p_{22}) - v(p_{21})\right] < d'(x) + d'(1-x). \qquad (25)$$

(Cf. also the further discussion in the main text following the Proposition.) **Q.E.D.**

Proof of Proposition 2. We have, as in the proof of Proposition 1, that

$$\overline{\Pi}_1(\hat{x}) = \int_0^{\hat{x}} \left[\pi_1(x,\hat{x}) + u_1(\hat{x},\hat{x})\right] dH(x) + const.$$

With uniform prices, i.e., $p_1 := p_{11} = p_{12}$, we have $u_1(\hat{x},\hat{x}) = v(p_1)$ and

$$\begin{aligned} \pi_1(x,\hat{x}) &= q(p_1)(p_1 - [G(\hat{x} \mid x)c_{11} + [1 - G(\hat{x} \mid x)]\,c_{12}]) \\ &= q(p_1)\,(p_1 - c_1(x,\hat{x}))\,, \end{aligned}$$

with

$$c_1(x,\hat{x}) = c_{11} + [1 - G(\hat{x} \mid x)]\,(a - c_0)$$

(and likewise for firm 2). The problem of firm 1 is thus to choose p_1 to maximize

$$\overline{\Pi}_1(\hat{x}) = \int_0^{\hat{x}} \left[q(p_1)\,(p_1 - c_1(x,\hat{x})) + v(p_1)\right] dH(x) + const,$$

which has the unique solution (12). **Q.E.D.**

Proof of Proposition 3. For $x \leq \hat{x}$, we have from (2) that local calling patterns are given by

$$G(\hat{x} \mid x) = \begin{cases} (1-\lambda)\hat{x} + \lambda\left(\frac{1}{2} + \frac{\hat{x}-x}{b}\right) & \text{for } \hat{x} - b/2 \leq x \leq \hat{x} \\ (1-\lambda)\hat{x} + \lambda & \text{for } x < \hat{x} - b/2 \end{cases}.$$

Note also that $G(\hat{x} \mid \hat{x}) = (1-\lambda)\,\hat{x} + \lambda\frac{1}{2}$. We thus have that

$$\int_0^{\hat{x}} G(\hat{x} \mid x)dH(x) = (1-\lambda)\hat{x}^2 + \lambda\left(\hat{x} - \frac{1}{8}b\right).$$

Next, we have, after some transformations,

$$\int_0^{\hat{x}} \left[G(\hat{x} \mid x) - G(\hat{x} \mid \hat{x}) \right] dH(x) = \frac{\lambda}{2} \left(\hat{x} - \frac{b}{4} \right),$$

which with symmetry simplifies to $\frac{\lambda}{4}\left(1 - \frac{b}{2}\right)$. With these preliminary calculations at hand, the characterization for p_{11} follows from application of Proposition 1.

Next, for the calculation of p_{12}, note that $\int_0^{\hat{x}} \left[1 - G(\hat{x} \mid x)\right] dH(x) = H(\hat{x}) - \int_0^{\hat{x}} G(\hat{x} \mid x) dH(x)$. Again, the characterization for p_{12} follows then simply from application of Proposition 1. The same holds for p_{21} and p_{22}. **Q.E.D.**

Proof of Proposition 5. Recall from Proposition 3 the expression for the off-net price, $\frac{p_{ij} - c_{ij}}{p_{ij}} = -\frac{1}{\eta(p_{ij})} \frac{\lambda\left(1 - \frac{b}{2}\right)}{1 - \lambda\left(1 - \frac{b}{2}\right)}$. Recalling that $c_{ij} = c_1 + c_0 + a$, it is thus

$$\begin{aligned} \frac{dp_{ij}}{da} &= \frac{\left[1 - \lambda\left(1 - \frac{b}{2}\right)\right] \eta}{\eta - \lambda\left(1 - \frac{b}{2}\right)(\eta - 1)} > 0, \\ \frac{dF}{da} &= -(1 - \lambda) v'(p_{ij}) \frac{dp_{ij}}{da} = (1 - \lambda) q(p_{ij}) \frac{dp_{ij}}{da} < 0. \end{aligned}$$

The marginal consumer located at 1/2 makes exactly 1/2 of the calls off-net. The parts of the bill that may be affected by a change in a are therefore equal to $F + p_{ij} q_{ij}(p_{ij})/2$, with:

$$\frac{d\left(F + p_{ij} q_{ij}(p_{ij})/2\right)}{da} = \frac{-\left[1 - \lambda\left(1 - \frac{b}{2}\right)\right](\eta + 1 - 2\lambda)\eta q_{ij}(p_{ij})}{2\left[\eta - \lambda\left(1 - \frac{b}{2}\right)(\eta - 1)\right]} < 0.$$

Q.E.D.

Proof of Proposition 6. We have

$$\begin{aligned} \bar{\Pi}_1\left(\frac{1}{2}\right) &= \int_0^{\frac{1}{2}} \left[\pi_1(x, \frac{1}{2}) + R_{12}(x, \frac{1}{2})\right] dH(x) + \frac{1}{2}(F - f) \\ &= \int_0^{\frac{1}{2}} \left[G(\frac{1}{2} \mid x) r_{11} + \left[1 - G(\frac{1}{2} \mid x)\right] r_{12}\right] dH(x) \\ &\quad + t_{ij} \int_0^{1/2} \int_{1/2}^{1} \frac{g(x \mid x')}{h(x)} dH(x') dH(x) + \frac{1}{2}(F - f). \end{aligned}$$

This simplifies to

$$\begin{aligned} \bar{\Pi}_1\left(\frac{1}{2}\right) &= \left(\frac{1}{2} - \Gamma\right) r_{ii} + \Gamma\left(r_{ij} + t_{ij}\right) + \frac{1}{2}(F - f) \\ &= \frac{d'\left(\frac{1}{2}\right)}{2h(\frac{1}{2})} + \frac{\chi}{2}(v_{ij} - v_{ii}) + \left(\Gamma + \frac{\gamma}{2} - \frac{1}{4}\right)(r_{ij} + t_{ij} - r_{ii}) + \left(\frac{1}{2} - \gamma\right) t_{ij}. \end{aligned}$$

Q.E.D.

Proof of Proposition 7. Starting from (21), the resulting FOC is

$$\left\{-\frac{\chi}{2}q\left(p_{ij}\right)+\left(\Gamma+\frac{\gamma}{2}-\frac{1}{4}\right)\left[q\left(p_{ij}\right)+\left(p_{ij}-c_{ii}\right)q'\right]\right\}\frac{dp_{ij}}{da}+\left(\frac{1}{2}-\gamma\right)\left[q\left(p_{ij}\right)+\left(a-c_0\right)q'\frac{dp_{ij}}{da}\right].$$

Assuming constant elasticity demand, we can deduce from (7) that

$$p_{ij}=c_{ij}\frac{\eta\Gamma}{1/4+\left(\eta-1\right)\Gamma},\ \frac{dp_{ij}}{da}=\frac{\eta\Gamma}{1/4+\left(\eta-1\right)\Gamma}.$$

Now, substuting also $c_{ij}=c_{ii}+\left(a-c_0\right)$ and $q'=-\eta q\left(p_{ij}\right)/p_{ij}$ in the FOC above which finally solves for (22). **Q.E.D.**

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