Competition between multiple asymmetric networks: A toolkit and applications

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Abstract

This paper presents a model of competition between an arbitrary number of telecommunications networks, in the presence of tariff-mediated network externalities, call externalities, and cost and surplus asymmetries. We determine the Nash equilibria in linear and two-part tariffs, provide an appropriate stability criterium, and show how the model can be calibrated to existing market outcomes. As an application, we reconsider the setting of mobile termination rates for calls from the fixed network, and between mobile networks, in the presence of many asymmetric networks.

Keywords: Telecommunications network competition, on/off-net pricing, asymmetry, call externality, multiple networks, linear tariffs, two-part tariffs

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1 Introduction

Two great obstacles of applying models of telecommunications competition to real-world markets are that most either assume symmetric firms and / or consider a duopoly. To our knowledge, there are few or no realistic cases that can be portrayed as a symmetric duopoly, since most telecommunications markets are characterized either by at least three firms which have entered at different points in time, as in mobile telephony, or by one large incumbent and several smaller rivals using different technologies, as is often the case in fixed telephony. One reason for the assumptions of symmetry and duopoly that is usually advanced is that models with several asymmetric networks are not tractable. Here we attempt to show otherwise.

While a series of recent papers has presented models of network competition with more than two networks, as listed below, all either have assumed symmetry or have not been able to give closed-form solutions for the equilibrium. In this paper we set out to develop and solve a rather general model of competition between interconnected telecommunications networks. As in Hoernig (2007) for two networks, there are tariff-mediated network externalities, i.e. networks price discriminate between on- and off-net calls, and call externalities, i.e. receiving calls conveys utility, and networks can be asymmetric in size. Still, we go beyond the scope of that paper by allowing for an arbitrary number of networks and asymmetries in network and per-customer fixed costs. While being at the centre of the ongoing debate about the regulation of mobile termination rates (MTRs) in the European Union, cost differences seem to have been largely ignored in the economic literature on network competition.

We show how to set up and solve network competition models with many asymmetric firms, both for competition in linear and two-part tariffs. The model and most of the calculations are rendered in matrix notation, exploiting maximally the underlying quadratic functional form of profits and the linearity of the demand structure. This vastly reduces the complexity of the derivations and leads to equilibrium conditions in the form of one-liners.

As a first step, we propose a generalization of the condition of stability in expectations introduced by Laffont, Rey and Tirole (1998b) to multiple networks. Effectively, it imposes an upper limit on the intensity of preferences, as a function of tariff-mediated network externalities. This stability condition assumes that networks compete in prices, but is independent of whether networks compete in linear or two-part tariffs.

We then derive the socially optimal prices and market shares in the presence of asymmetric cost and perceived consumer surplus. As expected, efficient prices reflect the true costs of origination and termination on the originating and terminating networks, respectively. Our main finding concerning market shares is that the socially optimal outcome can be implemented by setting fixed fees that reflect exactly the differences in fixed cost (net of fixed-to-mobile termination profits), if and only if network usage costs are symmetric. With cost differences, these fixed fees must be corrected for the effects of differing retail prices.

In the main part of the paper, we derive the Nash equilibria in the price competition games with linear and two-part tariffs. As concerns off-net prices, we allow networks to set a uniform off-net price to all other networks, or to set different prices to groups of other networks. With linear tariffs, we show that the condition in Hoernig (2007) which links the level of the off-net price to the level of the on-net price in the case of two networks, continues to hold "on average" in the case of many networks. If there is a uniform off-net price to all competing networks then this price is set based on average perceived off-net cost.

With two-part tariffs, we show that identical off-net prices to a group of competing networks are set based on average perceived off-net cost, and as if all competitors had the same average market share as the members of this group. Rather unsurprisingly, the on-net prices continue to be set at the efficient levels independently of cost asymmetries and the number of networks.

For the case of two-part tariffs we show how to calibrate the model to real-market cases by computing the fundamental differences in consumer surplus that give rise to the asymmetry in the first place, over and above any cost differences. This exercise is becoming ever more useful for academics and regulators as many countries in Europe, such as France and Portugal, decided recently to make available more spectrum for the entry of additional (fourth or fifth) mobile networks. In these cases it is essential to be able to model asymmetries in the presence of many networks.

As a final exercise, we explore the implications of our results for the setting of mobile termination rates. We first consider the "waterbed effect" in fixed-to-mobile interconnection, i.e. the phenomenon where profits from fixed-to-mobile termination are handed on to consumers, leading to lower retail prices. With linear tariffs, we predict this effect to exist, since both on- and off-net prices decrease with higher fixed-to-mobile termination profits, yet without being able to determine its strength. On the other hand, with two-part tariffs we show that even in the presence of many asymmetric networks the waterbed effect is full at the level of each individual network, as all of the termination profit is handed over to consumers through lower fixed fees. These findings imply that it continues to be true in the general case that higher fixed-to-mobile MTRs amount to a transfer of surplus from the customers of fixed networks to those of mobile networks (and in the case of linear tariffs, to mobile networks themselves).

Concerning mobile-to-mobile termination rates, we generalize the result of Gans and King (2001), with competition in two-part tariffs, to the case of many symmetric networks. Their finding was that networks maximize join profits by setting off-net prices below the efficient level and therefore MTRs below the true cost of termination. We show that as the number of networks increases, joint profit-maximizing off-net prices converge towards the efficient price. The corresponding MTRs only converge to termination cost in the absence of call externalities, otherwise they remain bounded further below cost.

Related literature: There is now a vast amount of work that has sprung from the seminal contributions of Armstrong (1998) and Laffont, Rey and Tirole (1998a,b). In the following we will mostly concentrate on the papers that consider price discrimination between on- and off-net prices, in the tradition of the third paper just mentioned. See the Laffont and Tirole (2000), Armstrong (2002) and Vogelsang (2003) for surveys about the literature on network competition. < check for multiple >

Duopoly network competition in linear tariffs has been considered by Doganoglu and Tauman (2002), Berger (2004), de Bijl and Peitz (2004), DeGraba (2004), Hoernig (2007), and Geoffron and Wang (2008). Duopoly equilibrium results under two-part tariffs have been derived by Gans and King (2001), Peitz (2005), Berger (2005), and Hoernig (2007).

Call externalities have been considered in Jeon, Laffont and Tirole (2004), Berger (2004, 2005), Hoernig (2007), and Armstrong and Wright (2007). < check HermalinKatz01 WP, KimLim01 IEP, CambiniValletti08 JIE> Our modeling of asymmetries based on differences in surplus that consumers derive directly from pertaining to one or the other network has been introduced by Carter and Wright (1999, 2003), and has been taken up in de Bijl and Peitz (2004), Peitz (2005) and Hoernig (2007).

Several papers on mobile-to-mobile interconnection have considered more than two competing networks, in different models where all firms directly compete with each other. Symmetric networks are assumed by: Calzada and Valletti (2008), and Armstrong and Wright (2007).[1] Dewenter and Haucap (2005) consider more than two asymmetric networks, but they take market shares as given and thus do not close the model. Closest to our paper is

[1] In an extension section, Calzada and Valletti (2008) consider asymmetric calling patterns with three networks.

Thompson, Renard and Wright (2007), in using a similar demand specification and considering an arbitrary number of networks which can differ in subscription surplus. Yet, networks in their model do not price-discriminate between on- and off-net calls, and no closed-form solution for the equilibrium is derived.[2]

Gans and King (2000) analyse how mobile networks set fixed-to-mobile termination rates under customer ignorance about which mobile network they are calling. Under the assumption that mobile networks' market shares are fixed they consider an arbitrary number of asymmetric networks. On the other hand, they assume symmetric duopoly when modeling competition in two-part tariffs between networks. Wright (2002) considers the setting of fixed-to-mobile termination rates by an arbitrary number of competing symmetric mobile networks. While he abstracts from mobile-to-mobile calls and uses a more general formulation of subscription demand, his pricing structure is equivalent to two-part tariffs with call prices set at cost. Thus his results can be compared to the ones derived in our framework. He shows that all profits from fixed-to-mobile termination are passed on to mobile customers, i.e. there is a full "waterbed effect", if a common shift in the cost of signing up subscribers does not change equilibrium profits. This is the case for example in Hotelling models under full market coverage. The waterbed effect is less than full for example if the market is less than fully covered.[3] Armstrong (2002, section 3), elaborating on Armstrong (1997), models setting of fixed-to-mobile termination rates by an indeterminate number of symmetric mobile networks under perfect competition.

This paper has the following structure: Section 2 presents the model, discusses stability in consumer expectations and derives socially optimal prices and market shares. Section 3 presents the Nash equilibrium solutions in linear and nonlinear tariffs, while Section 4 considers the symmetric case. Finally, Sections 5 and 6 present results on fixed-to-mobile and mobile-to-mobile termination, while Section 7 concludes.

[2] Other models of competition between multiple symmetric networks under non-discriminatory pricing are Jeon and Hurkens (2008), Stennek and Tangerås (2008) and Tangerås (2009). On the other hand, Hurkens and Jeon (2008) only consider two networks under termination-based price discrimination.

[3] Genakos and Valletti (2008) perform a similar analysis assuming a logit demand structure.

2 Model Setup

2.1 Demand, Market Shares and Consumer Surplus

The following model is a generalization of the network competition models of Laffont, Rey, Tirole (1998) and Carter and Wright (1999, 2002) to many asymmetric networks. It leads to a demand formulation that is similar to those of Armstrong and Wright (2007, AW below) and the "spokes model" by Chen and Riordan (2007, CR below),[4] but allows explicitly for exogenous asymmetry between networks. All networks directly compete against each other, which for more than three networks is different from the mostly used generalization of the Hotelling model to multiple firms, the circular city model of Salop. The equilibrium concept we employ is static Nash equilibrium of the pricing game between networks in either linear or two-part tariffs with price discrimination between on- and off-net prices.[5]

There are $n \geq 2$ networks, and consumers are located on $n(n-1)/2$ segments of individual length $l(n)$ which link all networks to each other (thus networks compete on a complete graph). The total mass of consumers is 1, thus each segment has $\frac{2}{n(n-1)}$ consumers. Let $d(n) = \frac{2}{n(n-1)}/l(n)$ be the density of consumers in preference space.

Transport cost are linear, with unit cost $t > 0$. As $t \to \infty$ each network becomes a local monopoly, while for $t \to 0$ transport cost disappear and we approach perfect competition. Let $\sigma = d(n)/2t$.

Market shares are $\alpha_i > 0$ with $\sum_{i=1}^{n} \alpha_i = 1$. All networks are interconnected, thus consumers can make calls to any one of them.

A client of network i receives surplus $w_i + A_i/\sigma$, where A_i is a measure of the consumer's fixed surplus from being connected to network i (which may include brand value, trust etc.), and w_i is the surplus arising from making calls, defined below. We assume $A_1 \geq A_2 \geq \ldots \geq A_n = 0$, i.e. the lowest surplus level is normalized to zero since only the differences $A_i - A_j$ will matter. Network i charges a two-part tariff consisting of a fixed charge F_i, and prices per minute of p_{ii} for on-net calls and p_{ij} for off-net calls to network j.

Consumers' utility of calls is $u(q)$, with indirect utility $v(p) = \max_q u(q) - pq$ (with $v'(p) = q(p)$). Below we will denote the price elasticity of demand as $\eta = -pq'/q$, but note that we never assume it to be constant. Let v_{ij}, q_{ij}, u_{ij} be defined as $v(p_{ij})$, $q(p_{ij})$, $u(q_{ij})$. The utility of receiving calls is $\gamma u(q)$

[4]The spokes model is built from specific graphical foundations, while Armstrong and Wright's demand formulation is presented *ad hoc.*

[5]Some authors refer to these tariffs as "multipart tariffs", but we employ here the original terminology used in Laffont, Rey and Tirole (1998b).

where $\gamma \in [0,1)$. Assuming an *ex-ante* balanced calling pattern, w_i is given by

$$w_i = \sum_{j=1}^{n} \alpha_j \left(v_{ij} + \gamma u_{ji}\right) - F_i = \sum_{j=1}^{n} \alpha_j h_{ij} - F_i \tag{1}$$

Defining the $(n \times n)$ −matrix $h = (h_{ij})_{ij}$ and the $(n \times 1)$ −vectors $F = (F_i)_i$ and $\alpha = (\alpha_i)_i$, we can restate the above in matrix form as

$$w = h\alpha - F. \tag{2}$$

The matrix h is a function of prices, and will therefore be indirectly a function of costs, MTRs and market shares.

We assume throughout that on each segment both adjoining networks have clients, thus the indifferent consumer on segment ij is located at the distance x_j from network i, defined by

$$w_i + 2tA_i - tx_{ij} = w_j + 2tA_j - t\left(l - x_{ij}\right). \tag{3}$$

Solving for x_{ij} yields network i's market share on segment ij as

$$\begin{aligned} x_{ij} &= \frac{l}{2} + \frac{1}{2t}\left(w_i - w_j\right) + \left(A_i - A_j\right) \\ x_{ij} &= \frac{1}{n\left(n-1\right)} + A_i - A_j + \sigma\left(w_i - w_j\right). \end{aligned}$$

Summing over segments yields network i's total market share:

$$\alpha_i = d\left(n\right)\sum_{j\neq i} x_{ij} = \frac{1}{n} + \left(\left(n-1\right)A_i - \sum_{j\neq i} A_j\right) + \sigma\left(\left(n-1\right)w_i - \sum_{j\neq i} w_j\right) \tag{4}$$

$$\alpha_i = \sum_{j\neq i} x_{ij} = \frac{1}{n} + \left(n-1\right)A_i - \sum_{j\neq i} A_j + \sigma\left(\left(n-1\right)w_i - \sum_{j\neq i} w_j\right) \tag{5}$$

Thus $\sigma = \frac{1}{ln(n-1)t}$. In the symmetric case, this expression for market shares is equivalent to AW (p. 31) with $\sigma = 1/2t\left(n-1\right)$ or $l = \frac{2}{n}$ and CR (p. 902) under maximum variety and after setting $\sigma = 1/n\left(n-1\right)$ or $l = 1$.[6]

In matrix notation we have

$$\alpha = \alpha_0 + B\left(A + \sigma w\right), \tag{6}$$

[6]In Section 4 we will come back to the economic significance of these transformations of the differentiation parameter.

where α_0 is the $(n \times 1)$ vector of symmetric market shares $1/n$ and B is an $(n \times n)$ matrix with the values $(n-1)$ on the diagonal and -1 elsewhere. Market shares in a fully covered market must add up to 1, which is the case here: Let E be the $(n \times 1)$ vector of ones, then

$$\sum_{i=1}^{n} \alpha_i = E'\alpha = E'\alpha_0 + E'B\left(A + \sigma w\right) = n \times \frac{1}{n} = 1, \tag{7}$$

because $E'B = 0$.

Plugging (2) into (6) leads to

$$\left(I - \sigma Bh\right)\alpha = \alpha_0 + B\left(A - \sigma F\right), \tag{8}$$

and solving for α leads to

$$\begin{aligned} \alpha &= \left(I - \sigma Bh\right)^{-1}\left[\alpha_0 + B\left(A - \sigma F\right)\right] \\ &= G\alpha_0 + H\left(A - \sigma F\right), \end{aligned} \tag{9}$$

where I is the $(n \times n)$ identity matrix, $G = \left(I - \sigma Bh\right)^{-1}$ and $H = \left(I - \sigma Bh\right)^{-1} B$. Thus we have found a simple unique solution for market shares *given prices*.[7] The following Lemma states some properties of G and H which will be useful later on.

Lemma 1 *We have: $E'G = E'$, $E'H = 0$ and $HE = 0$. In particular, $\sum_{i=1}^{n} H_{ij} = 0$ for all j, and $\sum_{j=1}^{n} H_{ij} = 0$ for all i.*

Proof. First note that $E'\left(I - \sigma Bh\right) = E' - \sigma 0h = E'$ since $E'B = 0$. Therefore

$$E'G = E'\left(I - \sigma Bh\right)\left(I - \sigma Bh\right)^{-1} = E'.$$

Note that $GE \neq E$ in general. Furthermore, $E'H = \left(E'G\right)B = E'B = 0$ and $HE = G\left(BE\right) = 0$ since $BE = 0$. ■

Consumer surplus: Total consumer surplus consists of the difference between the surplus from pertaining to networks and making calls, and "transport cost" which measures the welfare cost of a less than perfect fit with

[7] This solution does not yield equilibrium market shares explicitly since both H_0 and H may depend on market shares indirectly through prices. We study price choice in the next section.

preferences:

$$
\begin{aligned}
S &= \sum_{i=1}^{n}\left[\alpha_i\left(w_i+2tA_i\right)-\sum_{j\neq i}\int_0^{x_{ij}} tzdz\right] \\
&= \sum_{i=1}^{n}\left[\alpha_i\left(w_i+\frac{A_i}{\sigma}\right)-\frac{1}{4\sigma}\sum_{j\neq i}x_{ij}^2\right] \qquad (10)\\
&= \alpha'\left(h\alpha-F+\frac{1}{\sigma}A\right)-\frac{1}{4\sigma}\sum_{i,j\neq i}x_{ij}^2.
\end{aligned}
$$

2.2 Stability

One important technical aspect, discussed at length in Laffont, Rey, Tirole (1998b) for the duopoly case, is the stability of equilibrium in consumer expectations. In this section we show how this stability condition can be generalized to the presence of an arbitrary number of firms.

Lemma 2 *The Nash equilibrium in the price competition game, no matter whether in linear or in two-part tariffs, is stable in consumer expectations if and only if $\alpha_i \geq 0$ for all $i = 1, ..., n$ and $\sigma \in (0, 1/\kappa)$, where κ is the largest eigenvalue of Bh.*

Proof. The condition that all α_i are non-negative is a pre-condition for a well-defined equilibrium candidate. Now consider, similar to Laffont, Rey, Tirole (1998b), a virtual tâtonnement process where consumers observe market shares α_{t-1} and then join networks based on the resulting welfare. This leads to market shares

$$\alpha_t = \alpha_0 + B\left(A+\sigma\left(h\alpha_{t-1}-F\right)\right) = \left[\alpha_0 + B\left(A-\sigma F\right)\right]+\sigma Bh\alpha_{t-1}.$$

The effect of market shares at $t-1$ on market shares at time t is given by $d\alpha_t/d\alpha_{t-1} = \sigma Bh$. For this tâtonnement process to converge, it is necessary that the largest eigenvalue of σBh be less than 1, which is equivalent to the condition stated in the Lemma. ∎

Since B has rank $(n-1)$, one eigenvalue of Bh is zero. With symmetric prices, we have $h_{ii} \equiv h_{on}$, $h_{ij} \equiv h_{off}$, and the other $(n-1)$ eigenvalues of Bh are all equal to $n\left(h_{on}-h_{off}\right)$. Thus under symmetry equilibrium is stable if

$$\sigma < \bar{\sigma} = \frac{1}{n\left(h_{on}-h_{off}\right)}.$$

This leads to some straight-forward implications for market stability:

Proposition 1 *With symmetric networks competing in linear or two-part tariffs, the symmetric market equilibrium is less likely to be stable*

1. *for a higher number of firms, for given per-minute prices;*
2. *for a higher mobile termination rate a;*
3. *for a higher competitive intensity σ.*

Proof. 1. $\bar{\sigma}$ decreases in n for given $(h_{on} - h_{off})$. 2. h_{off} decreases in a, and $\bar{\sigma}$ increases in h_{off}. 3. Higher σ more likely violates the stability condition. ■

It is of interest to note that the corresponding stability conditions, by virtue of the transformations of the differentiation parameter σ indicated above, for AW and CR would be $\frac{1}{2t} < \frac{n-1}{n(h_{on}-h_{off})}$ and $1 < \frac{n-1}{h_{on}-h_{off}}$. In both cases we obtain the counter-intuitive result that market stability increases (rather than decreases) with the number of networks. Maybe more worryingly, instability is more likely to occur in market with few networks.

2.3 Profits

Networks incur fixed cost per customer of f_i, and have on-net cost $c_{ii} = c_{oi} + c_{ti}$, where the indices o and t stand for origination and termination, respectively. The mobile termination charge on network i is a_i, so that costs of off-net calls from network i to network $j \neq i$ are $c_{ij} = c_{oi} + a_j$. The mobile termination margin is $m_i = a_i - c_{ti}$. Networks' profits are

$$\pi_i = \alpha_i \left(\sum\nolimits_{j=1}^{n} \alpha_j R_{ij} + F_i + Q_i - f_i \right), \tag{11}$$

where $R_{ij} = (p_{ij} - c_{oi} - a_j)\, q_{ij} + (a_i - c_{ti})\, q_{ji}$ are the profits from calls between networks i and j. Note that this simplifies to $R_{ii} = (p_{ii} - c_{ii})\, q_{ii}$, and $R_{ij} = (p_{ij} - c_{ij})\, q_{ij} + m_i q_{ji}$ for $j \neq i$. Furthermore, $Q_i = m_i q_{fi}$ are fixed-to-mobile termination profits.

Let J^{ij} be the matrix with entry 1 at position (i, j) and zero elsewhere, R be the $(n \times n)$ matrix with entries R_{ij}, and F, Q, f be the $(n \times 1)$-vectors with entries F_i, Q_i, and f_i, respectively. We can express network i's profits in matrix notation as

$$\pi_i = \alpha' J^{ii} (R\alpha + F + Q - f), \tag{12}$$

and, since $\sum_{i=1}^{n} J^{ii} = I$, joint profits of all networks as

$$\sum_{i=1}^{n} \pi_i = \alpha' \left(R\alpha + F + Q - f \right). \tag{13}$$

Total welfare in the market for mobile telephony is given by

$$\begin{aligned} W &= S + \sum_{i=1}^{n} \pi_i & (14) \\ &= \alpha' \left[\left(R + h \right) \alpha + \frac{1}{\sigma} A + Q - f \right] - \frac{1}{4\sigma} \sum_{i,j \neq i} x_{ij}^2 & (15) \end{aligned}$$

We can now describe first-best prices and market shares:

Proposition 2 *1. First-best per-minute prices are* $p_{ij} = \frac{c_{oi}+c_{tj}}{1+\gamma}$ *for all* $i, j = 1, ..., n$.

2. Let $M \equiv R+h$ *at first-best prices. Then socially optimal market shares in the mobile telephony market are*

$$\alpha^* = \left(I - \sigma B \left(M' + M \right) \right)^{-1} \left[\alpha_0 + B \left(A + \sigma \left(Q - f \right) \right) \right], \tag{16}$$

if asymmetries are small enough. With symmetric network cost, optimal market shares become

$$\alpha^* = \alpha_0 + B \left(A + \sigma \left(Q - f \right) \right). \tag{17}$$

Proof. In the expression for aggregate profits the terms corresponding to mobile-to-mobile termination costs and profits cancel, so that after some re-ordering of terms with indices ij and ji,

$$\alpha' \left(R + h \right) \alpha = \sum_{i,j} \alpha_i \alpha_j \left[\left(p_{ij} - c_{oi} - c_{tj} \right) q_{ij} + v_{ij} + \gamma u_{ij} \right].$$

Thus for each pair ij the same surplus maximization problem is posed, with first-order condition

$$q_{ij} + \left(p_{ij} - c_{oi} - c_{tj} \right) q'_{ij} - q_{ij} + \gamma u'_{ij} q'_{ij} = 0.$$

Since $u'_{ij} = p_{ij}$ at the consumer's optimal choice of call minutes the above result obtains.

Let $M \equiv R + h$ at the socially optimal prices. Then we need to maximize social surplus

$$W = \alpha' M \alpha + \alpha' \left(\frac{1}{\sigma} A + Q - f \right) - \frac{1}{4\sigma} \sum_{i,j \neq i} x_{ij}^2$$

subject to the conditions $x_{ji} = \frac{2}{n(n-1)} - x_{ij}$ and $x_{ij} \geq 0$ for all $j \neq i$, $i = 1, ..., n$. Omitting for the moment the non-negativity constraints, and substituting out x_{ji} in $\alpha_j = \sum_{k \neq j} x_{jk}$, we have $\frac{d\alpha}{dx_{ij}} = (e_i - e_j)$, where e_i and e_j are $(n \times 1)$ vectors with value 1 at position i and j, respectively, and zeros elsewhere. Thus, maintaining the substitution of x_{ji}, we have the first-order conditions, for all i and $j \neq i$,

$$\begin{aligned}
\frac{dW}{dx_{ij}} &= (e_i - e_j)' M \alpha + \alpha' M (e_i - e_j) + (e_i - e_j)' \left(\frac{1}{\sigma} A + Q - f \right) \\
&- \frac{1}{2\sigma} x_{ij} + \frac{1}{2\sigma} \left(\frac{2}{n(n-1)} - x_{ij} \right) = 0.
\end{aligned}$$

Taking into account that $\alpha' M (e_i - e_j) = (e_i - e_j)' M' \alpha$, and summing the conditions over $j \neq i$, we obtain

$$B_i (M' + M) \alpha + B_i \left(\frac{1}{\sigma} A + Q - f \right) - \frac{1}{\sigma} \alpha_i + \frac{1}{\sigma n} = 0,$$

where B_i is row i of the matrix B. Stacking these equations leads to

$$B (M' + M) \alpha + B \left(\frac{1}{\sigma} A + Q - f \right) - \frac{1}{\sigma} \alpha + \frac{1}{\sigma} \alpha_0 = 0$$

and the condition

$$(I - \sigma B (M' + M)) \alpha = \alpha_0 + B (A + \sigma (Q - f)).$$

Note that $B (M' + M) = 0$ with symmetric network cost. These results hold as long as all $x_{ij} \geq 0$, which holds if and only if the asymmetries in network cost and $A + \sigma (Q - f)$ are small enough. ■

< comment, interpret >

With symmetric network cost, since $Bh^* = 0$, by (9) the socially optimal market shares α^* can be induced by introducing fixed fees equal to $F = f - Q + k$, where k has the same value in each component. That is, not the absolute value of fixed fees is relevant, only the differences between firms.

What needs to be signalled to consumers is the difference in net fixed cost $(f - Q)$ per consumer. If on the other hand network costs are not symmetric, then there is no longer a simple correspondence between conditions (9) and (16), and fixed fees must be chosen such that

$$\sigma BF = \alpha_0 + BA - (I - \sigma Bh)\left(I - \sigma B\left(M' + M\right)\right)^{-1}\left[\alpha_0 + B\left(A + \sigma\left(Q - f\right)\right)\right].$$

Note, though, that our result on the first-best market share, and the resulting fixed fees, considers the transfer Q from the fixed telephony market as given. In particular, it does not take into account the welfare loss caused by this transfer. These optimal market shares also take the surplus asymmetry A, which distinguishes networks in the eyes of consumers, as given. Indeed, if consumers as a whole prefer some networks then at the social optimum these networks' market shares should be higher.

3 Pricing Equilibrium

In this section we will describe equilibrium prices and market shares under both linear and nonlinear pricing. As some results concerning the effects of mobile termination rates are known to differ between these two types of strategies (see e.g. Laffont, Rey and Tirole (1998a,b), it seems useful to consider the case of many firms for both of them.

3.1 Linear Tariffs

With linear tariffs, let $F = 0$. Each network chooses the prices p_{ii} and p_{ij} in order to maximize its profits

$$\pi_i = \alpha_i\left(\sum\nolimits_{j=1}^{n}\alpha_j R_{ij} + Q_i - f_i\right) = \alpha' J^{ii}\left(R\alpha + Q - f\right).$$

We first state a central result about how per-minute prices affect market shares.

Lemma 3 *For any price p, we have $\frac{d\alpha}{dp} = \sigma H \frac{dh}{dp}\alpha$, with $\sum_{i=1}^{n}\frac{d\alpha_i}{dp} = 0$.*

Proof. From condition (8) we have $(I - \sigma Bh)\,\alpha = \alpha_0 + BA$. Taking derivatives on both sides leads to

$$-\sigma B\frac{dh}{dp}\alpha + (I - \sigma Bh)\frac{d\alpha}{dp} = 0,$$

from which the result follows. Furthermore, $\sum_{i=1}^{n} \frac{d\alpha_i}{dp} = E' \frac{d\alpha}{dp} = \sigma\left(E'H\right) \frac{dh}{dp}\alpha = 0$ since $E'H = 0$. ■

As is common in models network competition in linear prices, we cannot give explicit expressions for the equilibrium prices. Still, we can show how the equilibrium off-net prices relate to on-net prices. For the sake of generality, we consider the case where network i divides its competitors into separate groups K and charges a uniform off-price p_{iK} to each group. Extreme cases are where each group contains a single member (in which case there is price discrimination between all networks), or where all other networks are in the same group (the case of a uniform off-net price). We obtain the following results on equilibrium prices:

Proposition 3 *1. Network i's equilibrium on-net price satisfies the following condition:*

$$L_{ii} = \frac{p_{ii} - c_{ii}}{p_{ii}} = \frac{1}{\eta} - \frac{\sigma\left(1+\gamma\eta\right) H_{ii}}{\eta} \left(\frac{\pi_i}{\alpha_i^2} + \sum_{j=1}^{n} R_{ij} \frac{H_{ji}}{H_{ii}} \right). \tag{18}$$

2. If network i sets uniform prices p_{iK} to different groups K of competing networks, its average off-net Lerner index

$$\bar{L}_{ij} = \frac{\sum_K \sum_{j \in K} \alpha_j \left(p_{iK} - c_{ij}\right)/p_{iK}}{1 - \alpha_i} \tag{19}$$

satisfies the condition

$$\bar{L}_{ij} = \frac{1}{\eta} + \frac{\left(1+\gamma\eta\right)^{-1} - \alpha_i}{1 - \alpha_i} \left(L_{ii} - \frac{1}{\eta} \right). \tag{20}$$

3. Network i's profits are given by

$$\pi_i = \alpha_i^2 \left(\frac{1}{\sigma H_{ii}} \frac{1 - \eta L_{ii}}{1 + \gamma\eta} - \sum_{j=1}^{n} R_{ij} \frac{H_{ji}}{H_{ii}} \right). \tag{21}$$

Proof. For on-net prices we obtain

$$\frac{dh}{dp_{ii}} = \left(\gamma p_{ii} q'_{ii} - q_{ii}\right) J^{ii} = -\left(1+\gamma\eta\right) q_{ii} J^{ii}.$$

Thus

$$\frac{d\alpha}{dp_{ii}} = -\sigma\left(1+\gamma\eta\right) q_{ii} H J^{ii} \alpha = -\sigma\left(1+\gamma\eta\right) q_{ii} \alpha_i H_{.i},$$

where $H_{\cdot i}$ is the ith column of H. Furthermore, $\frac{dR}{dp_{ii}} = (1 - \eta L_{ii}) q_{ii} J^{ii}$, where $L_{ii} = (p_{ii} - c_{ii}) / p_{ii}$ is the Lerner index for on-net calls. The first-order condition for profit-maximization with respect to the on-net price is

$$\frac{d\alpha'}{dp_{ii}} J^{ii} (R\alpha + Q - f) + \alpha' J^{ii} \frac{dR}{dp_{ii}} \alpha + \alpha' J^{ii} R \frac{d\alpha}{dp_{ii}} = 0,$$

which simplifies to

$$-H_{ii} (R_{i\cdot}\alpha + Q_i - f_i) - \alpha_i R_{i\cdot} H_{\cdot i} + \frac{\alpha_i (1 - \eta L_{ii})}{\sigma (1 + \gamma\eta)} = 0 \tag{22}$$

or

$$\pi_i = \alpha_i^2 \left(\frac{1}{\sigma H_{ii}} \frac{1 - \eta L_{ii}}{1 + \gamma\eta} - \sum_{j=1}^{n} R_{ij} \frac{H_{ji}}{H_{ii}} \right). \tag{23}$$

Solving for L_{ii} leads to the condition on the on-net price.

2. Assume that network i sets a uniform off-net price p_{iK} to a set K of other networks. We have

$$\frac{dh}{dp_{iK}} = -q_{iK} J^{iK} + \gamma p_{iK} q'_{iK} J^{Ki} = -q_{iK} \left(J^{iK} + \gamma\eta J^{Ki} \right),$$

where J^{iK} and J^{Ki} are matrices with ones at locations ij and ji where $j \in K$, respectively, and zeros elsewhere. Thus

$$\frac{d\alpha}{dp_{iK}} = -\sigma q_{iK} H \left(J^{iK} + \gamma\eta J^{Ki} \right) \alpha = -\sigma q_{iK} \left(\sum_{j \in K} \alpha_j H_{\cdot i} + \gamma\eta\alpha_i \sum_{j \in K} H_{\cdot j} \right)$$

The first-order condition for a profit maximum becomes

$$\frac{d\alpha'}{dp_{iK}} J^{ii} (R\alpha + Q - f) + \alpha' J^{ii} \frac{dR}{dp_{iK}} \alpha + \alpha' J^{ii} R \frac{d\alpha}{dp_{iK}} = 0,$$

where $\frac{dR}{dp_{iK}}$ has elements $q_{iK} (1 - \eta L_{ij})$, where $L_{ij} = (p_{iK} - c_{ij}) / p_{iK}$ at locations ij, $j \in K$, and $m_j q'_{iK}$ at locations ji, $j \in K$. Note that off-net costs c_{ij} may differ between receiving networks j. This first-order condition can be rewritten as

$$\begin{aligned} 0 \ = \ & \left(\sum_{j \in K} \alpha_j H_{ii} + \gamma\eta\alpha_i \sum_{j \in K} H_{ij} \right) (R_{i\cdot}\alpha + Q_i - f_i) \\ & + \alpha_i R_{i\cdot} \left(\sum_{j \in K} \alpha_j H_{\cdot i} + \gamma\eta\alpha_i \sum_{j \in K} H_{\cdot j} \right) - \frac{\alpha_i}{\sigma} \sum_{j \in K} \alpha_j (1 - \eta L_{ij}). \end{aligned}$$

Summing over all sets K, and making use of $\sum_{j \neq i} H_{\cdot j} = -H_{\cdot i}$ from Lemma 1, leads to

$$-H_{ii}\left(R_{i\cdot}\alpha + Q_i - f_i\right) - \alpha_i R_{i\cdot} H_{\cdot i} + \frac{\alpha_i \left(1-\alpha_i\right)\left(1-\eta \bar{L}_{ij}\right)}{\sigma\left(1-\alpha_i-\gamma\eta\alpha_i\right)} = 0, \tag{24}$$

where $\bar{L}_{ij} = \sum_{j \neq i} \alpha_j L_{ij} / \left(1-\alpha_i\right)$ is the weighed average Lerner index of off-net prices, or

$$\pi_i = \alpha_i^2 \left(\frac{\left(1-\alpha_i\right)\left(1-\eta \bar{L}_{ij}\right)}{\sigma\left(1-\alpha_i-\gamma\eta\alpha_i\right) H_{ii}} - \sum_{j=1}^{n} R_{ij} \frac{H_{ji}}{H_{ii}} \right). \tag{25}$$

Taking the difference between (25) and (23) we obtain

$$\frac{\alpha_i^2 \left(1-\alpha_i\right)\left(1-\eta \bar{L}_{ij}\right)}{\sigma\left(1-\alpha_i-\gamma\eta\alpha_i\right) H_{ii}} = \frac{\alpha_i^2 \left(1-\eta L_{ii}\right)}{\sigma\left(1+\gamma\eta\right) H_{ii}},$$

from which the above result follows. ■

Condition (18) is the generalization to n asymmetric networks of condition (12) in Laffont, Rey and Tirole (1998b). The result on off-net prices is the generalization to n networks with asymmetric costs, and up to n different off-net prices to groups of networks, of equations (6) in Laffont, Rey and Tirole (1998b) and (11) in Hoernig (2007) for two networks. It is remarkable that the relationship between the average level of off-net prices, as measured by $\bar{L}_{ij}$, and on-net prices remains the same even as the number of networks increases.

If network i charges a uniform off-net price p_{iu} to all other networks, then we can reformulate $\bar{L}_{ij}$ as follows:

$$\bar{L}_{ij} = \frac{\sum_{j \neq i} \alpha_j \left(p_{iu} - c_{ij}\right)/p_{iu}}{1-\alpha_i} = \frac{p_{iu} - \bar{c}_{iof}}{p_{iu}}, \tag{26}$$

where $\bar{c}_{iof} = \left(\sum_{j \neq i} \alpha_j c_{ij}\right) / \left(1-\alpha_i\right)$ is the weighted average off-net cost faced by network i. Thus for a uniform off-net price, $\bar{L}_{ij}$ simply becomes the Lerner index relative to weighted average off-net cost.

< interpretation for on/off-net differential >

As we will see below, the expression (21) uncovers a previously overlooked link between the equilibrium profits under competition in linear and two-part tariffs. This link may aid future research into the relationship between the two modes of competition.

3.2 Two-Part Tariffs

In this section, we determine the equilibrium prices, fixed fees and market shares for the case of competition in two-part tariffs. We find the following:

Proposition 4 *If networks compete in two-part tariffs,*

1. *On-net prices are set efficiently at* $p_{ii} = c_{ii}/\left(1+\gamma\right)$*.*
2. *The uniform off-net price to a group* K *of competing networks is*

$$p_{iK} = \frac{\sum_{j\in K} \alpha_j c_{ij}}{\sum_{j\in K} \alpha_j - \frac{|K|}{n-1}\gamma\alpha_i}. \tag{27}$$

3. *Equilibrium fixed fees are given by*

$$F = f - Q - \left(\hat{R} + R\right)\alpha, \tag{28}$$

where $\hat{R}$ *is an* $(n \times n)$ *matrix with elements* $\hat{R}_{ii} = \sum_{j=1}^{n} \frac{H_{ji}}{H_{ii}} R_{ij} - \frac{1}{\sigma H_{ii}}$ *and* $\hat{R}_{ij} = 0$ *for* $j \neq i$*.*

Proof. 1. In order to determine equilibrium call prices, we follow the standard procedure of first keeping market shares α constant and solving (4) for F_i,

$$F_i = \sum_{j=1}^{n} \alpha_j v_{ij} + \alpha_i \gamma u_{ii} - \frac{\alpha_i}{n-1} \sum_{j\neq i} u_{ij} + const,$$

where "*const*" denotes terms that do not depend on network i's prices. Substituting this into profits leads to

$$\pi_i = \alpha_i \left(\sum_{j=1}^{n} \alpha_j \left(R_{ij} + v_{ij}\right) + \alpha_i \gamma u_{ii} - \frac{\alpha_i}{n-1} \sum_{j\neq i} u_{ij} \right) + const.$$

This expression can now be maximized over call prices. As concerns the on-net price, network i solves

$$\max_{p_{ii}} \left\{R_{ii} + h_{ii}\right\} = \left\{\left(p_{ii} - c_{ii}\right) q_{ii} + v_{ii} + \gamma u_{ii}\right\},$$

which has first-order condition

$$q_{ii} + \left(p_{ii} - c_{ii}\right) q_{ii}' - q_{ii} + \gamma u_{ii}' q_{ii}' = 0.$$

Since $u'_{ij} = p_{ij}$ at the consumer's optimal choice for all $i, j = 1, ..., n$, we obtain

$$p_{ii} = \frac{c_{ii}}{1+\gamma}, \tag{29}$$

i.e. on-net prices are set at the efficient level.

2. Assume now that network i wants to set a uniform off-net price p_{iK} towards a group K of other networks, solving

$$\max_{p_{iK}} \left\{ \sum_{j \in K} \left(\alpha_j \left(\left(p_{iK} - c_{ij} \right) q_{iK} + v_{iK} \right) - \frac{\alpha_i}{n-1} \gamma u_{iK} \right) \right\}.$$

Here $q_{iK} = q\left(p_{iK}\right)$, $v_{iK} = v\left(p_{iK}\right)$ and $u_{iK} = u\left(q_{iK}\right)$. Performing similar calculations as above leads to

$$p_{iK} = \frac{\sum_{j \in K} \alpha_j c_{ij}}{\sum_{j \in K} \alpha_j - \frac{|K|}{n-1} \gamma \alpha_i}. \tag{30}$$

3. Now we determine the equilibrium fixed fees. Take the call prices and fixed fees of networks $j \neq i$ as given, and consider the first-order condition of network i's profit maximum in (11) with respect to its fixed fee:

$$\frac{\partial \pi_i}{\partial F_i} = \frac{\partial \alpha_i}{\partial F_i} \left(\sum_{j=1}^{n} \alpha_j R_{ij} + F_i + Q_i - f_i \right) + \alpha_i \left(\sum_{j=1}^{n} \frac{\partial \alpha_j}{\partial F_i} R_{ij} + 1 \right) = 0.$$

From (9), for all $i, j = 1, ..., n$ we have $\frac{\partial \alpha_j}{\partial F_i} = -\sigma H_{ji}$, where H_{ji} is the ji-element of matrix H. The first-order condition can then be solved for F_i as

$$F_i = f_i - Q_i - \alpha_i \left(\sum_{j=1}^{n} \frac{H_{ji}}{H_{ii}} R_{ij} - \frac{1}{\sigma H_{ii}} \right) - \sum_{j=1}^{n} \alpha_j R_{ij}. \tag{31}$$

Letting $\hat{R}$ be an $(n \times n)$ matrix with $\hat{R}_{ii} = \sum_{j=1}^{n} \frac{H_{ji}}{H_{ii}} R_{ij} - \frac{1}{\sigma H_{ii}}$ and $\hat{R}_{ij} = 0$ if $j \neq i$, we can write

$$F = f - Q - \left(\hat{R} + R \right) \alpha. \ \blacksquare \tag{32}$$

Thus we confirm the standard result of the efficiency of on-net prices under two-part tariffs for the case of many asymmetric networks. If there are no call externalities ($\gamma = 0$) then $p_{ii} = c_{ii}$, while in the presence of the latter the efficient on-net price is below cost.

As concerns the off-net prices, in the absence of call externalities they are equal to weighted average off-net cost:

$$p_{iK} = \frac{\sum_{j\in K} \alpha_j c_{ij}}{\sum_{j\in K} \alpha_j}.$$

This is a natural generalization of the result for two firms. Furthermore, as in Jeon *et al.* (), Berger (2005) and Hoernig (2007), the off-net prices increase in γ and are above (weighted average) off-net cost if $\gamma > 0$. Expression (27) shows that network i sets its off-net price to a set K of networks as if it was setting a uniform off-net price to all networks, assuming they all have the same average market share as those in the set K.

Two special cases of off-net prices are a uniform off-net price

$$p_{iu} = \frac{\sum_{j\neq i} \alpha_j c_{ij}}{1 - \alpha_i - \gamma\alpha_i},$$

and price discrimination between all networks, with

$$p_{ij} = \frac{\alpha_j c_{ij}}{\alpha_j - \frac{1}{n-1}\gamma\alpha_i}.$$

We now consider equilibrium profits and market shares.

Proposition 5 *Equilibrium profits and market shares are, respectively,*

$$\pi_i^* = \alpha_i^2 \left(\frac{1}{\sigma H_{ii}} - \sum_{j=1}^{n} R_{ij} \frac{H_{ji}}{H_{ii}} \right), \tag{33}$$

$$\alpha^* = \left(I - \sigma B \left[h + \hat{R} + R \right] \right)^{-1} \left(\alpha_0 + B \left[A + \sigma \left(Q - f \right) \right] \right). \tag{34}$$

Proof. The expression for profits results from substituting equilibrium fixed fees into (11). Finally, substituting fixed fees into (8) yields the condition for the equilibrium market share. ■

One should take note that the expression for equilibrium profits (33) is every similar to the one in (21). Indeed, this similarity is no coincidence:

Corollary 1 *At efficient on-net prices, the expressions for equilibrium profits (21) under linear tariffs and (33) under two-part tariffs are formally identical.*

Proof. With $p_{ii} = \frac{c_{ii}}{1+\gamma}$, we have $\frac{1-\eta L_{ii}}{1+\gamma\eta} = 1$. Thus the additional term in (21) disappears. ■

The same argument holds for expression (25), by the way, since at the off-net prices (27) the average Lerner index has value $\bar{L}_{ij} = \gamma\frac{\alpha_i}{1-\alpha_i}$, which again makes the additional term disappear. These observations imply that the fundamental difference between competition in linear and two-part tariffs lies in how usage prices are set, rather than in the existence or not of a fixed fee. Maybe surprisingly, the expression for equilibrium profits under linear tariffs turns out to be more general than the one under two-part-tariffs, rather than less, as it applies to both cases (with different retail prices, sure enough). < explore relationship with DeGraba (2004) >

Note that an alternative expression for equilibrium profits under two-part tariffs is $\pi_i^* = -\alpha' J^{ii}\hat{R}\alpha$, which leads to the handy expression for joint equilibrium profits of

$$\sum_{i=1}^{n} \pi_i = -\alpha'\hat{R}\alpha. \tag{35}$$

The right-hand side of (34) in general depends indirectly on α through $h+\hat{R}+R$ and off-net prices. Contrary to the two-firm case, this is true even if there are no call externalities, since in this case the off-net prices are equal to off-net costs weighted by market shares. Only if off-net costs (including mobile termination rates) are symmetric will the dependence on α disappear. In the latter case (34) gives an explicit solution for market shares, but otherwise numerical methods need to be employed.

Calibration: If we want to calibrate the model using observed market shares and prices, the fixed surplus A can be calculated from (34), starting by the normalization $A_n = 0$:

$$BA = \left(I - \sigma B\left(\hat{R} + R + h\right)\right)\alpha - \alpha_0 - \sigma B\left(Q - f\right). \tag{36}$$

The matrix B cannot be inverted, but using $A_n = 0$ and solving the first $(n-1)\times(n-1)$ dimensions of this system yields the unique $A_1, ..., A_{n-1}$ that give rise to the observed market shares α.

4 The Special Case of Symmetric Networks

4.1 Preliminaries

In this section we will shortly resume the outcomes of our models under symmetric networks, i.e. with equal network and fixed costs, the same surplus

A (normalized to zero) and the same mobile-to-mobile and fixed-to-mobile termination charges for all networks.

In a symmetric equilibrium market shares are $\alpha_i \equiv \frac{1}{n}$. Let network costs including MTRs be c_{on} and c_{off} for on- and off-net calls, respectively. The equilibrium surplus from on- and off-net calls is $h_{ii} \equiv h_{on}$ and $h_{ij} \equiv h_{off}$, with $H_{ii} = H_{on}$ and $H_{ij} = H_{off}$. Let $R_{ii} \equiv R_{on} = (p_{on} - c_{on})\, q_{on}$ and $R_{ij} = R_{off} = (p_{off} - c_{on})\, q_{off}$ for $j \neq i$,[8] and $F_i \equiv F_0$, $Q_i \equiv Q_0$ and $f_i = f_0$. Thus profits from (11) become

$$\pi = \frac{1}{n}\left(\frac{1}{n}R_{on} + \frac{n-1}{n}R_{off} + F_0 + Q_0 - f_0\right). \tag{37}$$

Our first result is of technical nature and applies to both linear and two-part tariffs:

Lemma 4 *With n symmetric networks,*

$$H_{on} = \frac{n-1}{1 - n\sigma\left(h_{on} - h_{off}\right)}, \quad H_{off} = -\frac{1}{1 - n\sigma\left(h_{on} - h_{off}\right)}. \tag{38}$$

If the symmetric equilibrium is stable in customer expectations, i.e. $n\sigma\left(h_{on} - h_{off}\right) < 1$, then $H_{on} > 0$ and $H_{off} < 0$.

An important characteristic of our n-firm Hotelling model, which as Chen and Riordan (2007, p. 898) have pointed out is shared by the Salop circular city model (1979), is that each network's demand elasticity converges to infinity as the number of networks grows. For example, assuming $\gamma = 0$ and $a = c_t$ in order to guarantee stability for all n,[9] we have in our model

$$\frac{d\alpha_i}{dF_i} = -\sigma H_{on} = -\left(n-1\right)\sigma.$$

Demand elasticity becomes

$$\varepsilon^H = -\frac{d\alpha_i}{dF_i}\frac{F_i}{\alpha_i} = n\left(n-1\right)\sigma F_i \to \infty.$$

The main implication of elasticity becoming infinite is that as the number of networks becomes large not only do networks' profits converge to zero, but so do profits per subscriber (see below) and also industry profits.

[8] The occurrence of c_{on} in R_{off} is not a typo — it is due the cancelling-out of mobile-to-mobile interconnection profits.

[9] Evidently, the following discussion does not depend on this assumption for small n.

This is rather different with the logit model employed by Calzada and Valletti (2008) and the spokes model in CR. As for the former, assuming $\gamma = 0$ and $a = c_t$, and denoting the differentiation parameter τ with $\tau \to \infty$ indicating convergence to homogeneity, market shares are given by

$$\alpha_i = \frac{\exp(-\tau F_i)}{\sum_{j=1}^{n} \exp(-\tau F_j)}, \tag{39}$$

with

$$\begin{aligned} \frac{d\alpha_i}{dF_i} &= -\tau \alpha_i (1 - \alpha_i) = -\frac{(n-1)\tau}{n^2} \\ \varepsilon^L &= -\frac{d\alpha_i}{dF_i}\frac{F_i}{\alpha_i} = \frac{(n-1)\tau}{n} \to \tau F_i. \end{aligned}$$

Thus in the logit model, as the number of networks becomes large, demand elasticity remains finite, and therefore networks' profits per subscriber (and industry profits) do not converge to zero, though individual networks' profits still do. The same holds true for the spokes model (p. 914). Thus both are models of "monopolistic competition".

The above expressions actually provide a decisive hint as to how to compare the symmetric equilibrium outcomes of the Hotelling and logit models: The outcomes are equivalent for $\sigma = \tau/n^2$. As we have noted above, a similar expression holds directly for market shares in the spokes model, $\sigma = \frac{1}{n(n-1)}$, which is of the same magnitude in n as for the logit model, and AW with $\sigma = \frac{1}{2t(n-1)}$. Going back to the definition of the space of product differentiation in the Hotelling model as a complete graph with lines linking all networks, the logit and spokes models behave like a Hotelling model where the length of lines is not decreasing in n, but rather constant (with consumer density decreasing accordingly). Thus the remaining local market power in both models can be understood as arising from a "stretching" of the product space in the presence of more varieties. AW occupies an intermediate ground where line length decreases slower than in our model (while total preference space increases), but does so fast enough to make local market power evaporate as $\varepsilon = nF_i/2t \to \infty$.

4.2 Symmetric equilibrium with linear tariffs

We will now consider linear and two-part tariffs in turn. With linear tariffs, the condition describing symmetric off-net prices becomes (let $L_{ii} \equiv L_{on}$ and $L_{ij} = \bar{L}_{ij} = L_{off}$)

$$L_{off} = \frac{1}{\eta} + \frac{n(1+\gamma\eta)^{-1} - 1}{n-1}\left(L_{on} - \frac{1}{\eta}\right). \tag{40}$$

As for the case with two networks (see Hoernig 2007), the relation between off-net and on-net Lerner indices is given by a line that passes through the monopoly point $\left(\frac{1}{\eta}, \frac{1}{\eta}\right)$. With $\gamma = 0$ both Lerner indices are equal, while $L_{off} > L_{on}$ for $\gamma > 0$ as long as p_{on} is smaller than the monopoly price. As a result, we have $p_{off} > p_{on}$ whenever either $\gamma > 0$ or $c_{off} > c_{on}$. Actually, the off-net price is above the monopoly price based on the perceived off-net cost if and only if $\gamma\eta > n - 1$ (generalizing Berger (2004) where $n = 2$).

Define the monopoly prices $p_{on}^m = \frac{\eta c_{on}}{\eta - 1}$ and $p_{off}^m = \frac{\eta c_{off}}{\eta - 1}$, and let $R_{off}^m = \left(p_{off}^m - c_{on}\right) q\left(p_{off}^m\right)$.

Proposition 6 *Assume demand elasticity η is constant and consider linear tariffs.*

1. *If $\sigma \approx 0$ and $R_{off}^m > f$, or if $\gamma \approx 0$ and $a \approx c_t$, both on- and off-net prices decrease in the number of networks n.*

2. *If $\gamma \approx 0$, and either $\sigma \approx 0$ or $a \approx c_t$, then the on/off-net differential decreases in n if mobile-to-mobile termination rates are above cost. If $\gamma > 0$ or $a > c_t$, the off-net price remains bounded away from the on-net price even with a large number of networks.*

3. *As the number of networks becomes large, the off-net price converges to the (Ramsey) break-even price.*

Proof. In the following assume that the stability condition holds for the n considered. In particular, for very large n this implies that either σ, or γ and $a - c_t$, are very small.

1. With symmetric networks and constant-elasticity demand, the equilibrium is given by the two conditions

$$S_{on} = 2R_{on} + (n-2) R_{off} - \frac{1 - n\sigma\left(h_{on} - h_{off}\right)}{\sigma(n-1)} \frac{1 - \eta L_{on}}{1 + \gamma\eta} - n\left(f_0 - Q_0\right) \tag{41}$$

$$S_{off} = \frac{p_{off} - c_{off}}{p_{off}} - \frac{1}{\eta} - \frac{n(1+\gamma\eta)^{-1} - 1}{n-1} \left(\frac{p_{on} - c_{on}}{p_{on}} - \frac{1}{\eta}\right) = 0. \tag{42}$$

At $\sigma = 0$, networks set monopoly prices p_{on}^m and p_{off}^m. Solving (42) for p_{off}, letting $p_{on} = p_{on}^m - \delta\sigma$ and expanding (41) around $\sigma = 0$ results in

$$\phi_n - \delta \frac{(\eta - 1)^2}{\eta c_1 (n-1)(1+\gamma\eta)} + O(\sigma) = 0,$$

where $\phi_n = 2R_{on}(p_{on}^m) + (n-2) R_{off}^m - nf$ is positive by $R_{on}(p_{on}^m) \geq R_{off}^m > f$. Solving for δ implies that for small σ we have

$$p_{on} = p_{on}^m - \frac{p_{on}^m (1+\gamma\eta)}{\eta - 1}(n-1)\phi_n \sigma + O(\sigma^2). \tag{43}$$

Expanding p_{off} about $\sigma = 0$ leads to

$$\begin{aligned} p_{off} &= p_{off}^m - \frac{(n-1-\gamma\eta) c_2}{(n-1)(1+\gamma\eta) c_1} \delta\sigma + O(\sigma^2) \\ &= p_{off}^m - \frac{p_{off}^m}{\eta - 1}(n-1-\gamma\eta)\phi_n \sigma + O(\sigma^2) \end{aligned} \tag{44}$$

Since both $(n-1)\phi_n$ and $(n-1-\gamma\eta)\phi_n$ are increasing in n, both prices decrease in n for small σ.

On the other hand, at $\gamma = 0$ and zero termination margin we have $p_{off} = p_{on}$ (42). Applying the implicit function theorem to (41) we obtain

$$\frac{dp_{on}}{dn} = \frac{dp_{off}}{dn} = -\frac{n(n-1)^2 \sigma\pi + (1 - \eta L_{on})}{n(n-1)^2 \sigma R_{on}' + (n-1)\frac{\eta c_{on}}{p_{on}^2}} < 0.$$

The result then follows by continuity around the limit points.

2. If $\gamma = 0$ then $p_{off} = \frac{c_{off}}{c_{on}} p_{on}$, and we have

$$\frac{d(p_{off} - p_{on})}{dn} = \left(\frac{c_{off}}{c_{on}} - 1\right)\frac{dp_{on}}{dn},$$

which is negative in both cases considered above if $c_{off} > c_{on}$. As $n \to \infty$ the next point shows that p_{off} converges to the (positive) break-even price p_b, and p_{on} then converges to $\frac{c_{on}}{c_{off}} p_b$. The latter remains bounded away from p_b if $c_{off} > c_{on}$.

If $\gamma > 0$, we have

$$\begin{aligned} \frac{d}{dn}\left(\frac{n(1+\gamma\eta)^{-1} - 1}{n-1}\right) &= \frac{\gamma\eta}{(1+\gamma\eta)(n-1)^2} > 0, \\ \lim_{n\to\infty} \frac{n(1+\gamma\eta)^{-1} - 1}{n-1} &= \frac{1}{1+\gamma\eta}. \end{aligned}$$

Thus as n increases the difference in Lerner indices becomes smaller and converges to a limit value. Even if termination is priced at cost the off-net price will remain above the on-net price by the factor δ given by

$$\frac{p_{on}\delta - c}{p_{on}\delta} - \frac{1}{\eta} = \frac{1}{1+\gamma\eta}\left(\frac{p_{on} - c}{p_{on}} - \frac{1}{\eta}\right),$$

or $\delta = c\frac{1+\gamma\eta}{(\eta-1)\gamma p_{on}+c} > 1$ since p_{on} is below the monopoly price $\frac{\eta c}{\eta-1}$.

3. As $n \to \infty$, (41) becomes $R_{off} = f_0 - Q_0$. ■

While it is not surprising that the prices decrease as the number of networks increases, it is not obvious that the off-net price decreases faster than the on-net price. The latter occurs because off-net calls make up a larger and larger portion of calls on each network, increasing the cost of setting high off-net prices for strategic reasons.

As we have seen, this strategic incentive does not disappear in the limit in the sense that the on-net price will remain below the off-net price. On the other hand, since in the limit all call revenue is brought in through off-net calls, in the limit competition drives the price for these calls to the break-even level. Since in this model consumers do not differ in their demands for calls, this price is also the (second-best) Ramsey price which maximizes total surplus under the restriction of linear pricing.[10]Note that in our model call revenue just equals net fixed cost, i.e. per-subscriber fixed cost minus profits from fixed-to-mobile termination. Thus "efficiency" in this context does not take into account effects on the fixed network.

If both on- and off-net prices decrease then clearly the profits of individual networks and total industry profits (which are equal to profits per subscriber) decrease with n, as can be readily seen from (37) (with $F_0 = 0$). Actually, per-subscriber profit converges to zero in the limit, as we just have seen.

It may be helpful to consider a numerical example. Let $c_{on} = 1$, $c_{off} = 1.5$, $f_0 - Q_0 = 0$, constant elasticity call demand with $\eta = 2$, $\gamma = 0$. Note that with these parameter values the monopoly on- and off-net prices are 2 and 3, respectively, and the break-even price is equal to 1. Varying the number of firms results in the following equilibrium values:

	p_{on}	p_{off}	mq_{off}	$n\pi$	$1 - \sigma n\,(h_{on} - h_{off})$
$n = 2$	1.549	2.323	0.093	0.237	0.785
$n = 3$	1.121	1.682	0.177	0.193	0.554
$n = 4$	0.922	1.383	0.262	0.127	0.277
$n = 5$	0.833	1.250	0.320	0.080	0

The market remains stable in customer expectations while $n < 5$. It is remarkable that as the number of firms increases both the on- and off-net prices may fall below their respective marginal cost, while the equilibrium continues to be stable. This is made possible by the additional profit contribution from mobile-to-mobile termination, which actually rises due to lower off-net prices.

[10]See Laffont, Rey and Tirole (1998b), p. 42.

4.3 Symmetric equilibrium with two-part tariffs

Since Nash equilibria with two-part tariffs are more amenable to analysis, the case of symmetric equilibria with many firms has already been considered by several authors, such as Calzada and Valletti (2008) under logit demand and AW with Hotelling demand. The latter also considered call externalities, but of a different function form. Thus our results here complement both previous papers.

With two-part tariffs and symmetric networks, the on-net price remains equal to $p_{on} = \frac{c_{on}}{1+\gamma}$. The off-net price becomes

$$p_{iu} = p_{iK} = p_{ij} = \frac{n-1}{n-1-\gamma} c_{off}. \tag{45}$$

This off-net price is decreasing with n and converges to perceived marginal cost c_{off} as the number of networks becomes large. As with linear tariffs (at least under certain conditions on model parameters), the on/off-net differential decreases if more networks are present since the on-net price remains constant at the efficient level.

Fixed fees are

$$F_0 = f_0 - Q_0 + \frac{1 - n\sigma \left(h_{on} - h_{off} \right)}{\sigma n \left(n-1 \right)} - \frac{2}{n} R_{on} - \frac{n-2}{n} R_{off},$$

which approaches $F_0 = f_0 - Q_0 - R_{off}$ (i.e. zero profits per subscriber) as n becomes large. Equilibrium profits can be written as

$$\pi = \frac{1}{n^2} \left(\frac{1 - n\sigma \left(h_{on} - h_{off} \right)}{\sigma \left(n-1 \right)} - R_{on} + R_{off} \right), \tag{46}$$

where $R_{on} = -\frac{\gamma}{1+\gamma} c_{on} q_{on}$ and $R_{off} = \left(\frac{n-1}{n-1-\gamma} c_{off} - c_{on} \right) q_{off}$. Profits per subscriber and industry profits $n\pi$ converge to zero as n becomes large, as do evidently individual networks' profits.

AW, assuming call externalities given by bq_{off} rather than $\gamma u \left(q_{off} \right)$, obtain equilibrium call prices of (in our notation):

$$p_{on} = c_{on} - b,\ p_{off} = c_{off} + \frac{b}{n-1}.$$

They do not state the equilibrium value of fixed fees, while individual networks' profits are (their Π/K from (21), in our notation)

$$\pi = \frac{1}{n^2} \left(2t - \frac{n}{n-1} \left(h_{on} - h_{off} \right) - R_{on} + R_{off} \right).$$

This is formally equal to (46) with $\sigma = 1/2t\,(n-1)$. As in our model, profits per subscriber converge to zero.

The corresponding expressions for equilibrium fixed fees and profits in Calzada and Valletti (2008) are (using our notation in (39), with $R_{on} = 0$ and $R_{off} = mq_{off}$ due to the absence of call externalities)

$$\begin{aligned} F_0 &= f_0 - Q_0 + \frac{n - \tau\,(v_{on} - v_{off})}{\tau\,(n-1)} - \frac{n-2}{n} m q_{off} \\ \pi &= \frac{1}{n}\left(\frac{n - \tau\,(v_{on} - v_{off})}{\tau\,(n-1)} + m q_{off}\right) \end{aligned}$$

These expressions are equal to ours for $\sigma = \tau/n^2$. As we have pointed out above, when the number of networks becomes large one should expect profits per subscriber to converge to a finite value in the logit model, and this is indeed what we find: $n\pi \to \frac{1}{\tau} > 0$.

5 Fixed-To-Mobile Termination and the Waterbed effect

In this section we will state what our previous results imply for the fixed-to-mobile "waterbed effect", i.e. the phenomenon according to which termination profits accruing from interconnection to the fixed network lead to reductions in prices for mobile retail customers. We will not consider in detail the individual incentives of networks to set higher fixed-to-mobile termination rates, as has been done by Gans and King (2000) and by Wright (2002). Rather, we are interested in the equilibrium effect of changes in the (regulated) termination rate.

Linear Tariffs: With linear tariffs, by (22) and (24), $\frac{\partial \pi_i}{\partial p_{ii} \partial Q_i}$ and $\frac{\partial \pi_i}{\partial p_{ij} \partial Q_i}$ both have the sign of $-H_{ii}$, which is negative at least if the market is close enough to the symmetric equilibrium. Thus higher fixed-to-mobile profits Q_i lowers both p_{ii} and p_{ij}, and network i's market share will increase. As long as prices are strategic complements, all equilibrium prices will fall. Therefore consumers of all networks will receive at least part of the rent due to higher fixed-to-mobile termination charges. Thus a fixed-to-mobile waterbed effect exists even with linear tariffs, but at this level of generality we cannot determine its extent. Furthermore, while it is clear that each single network prefers to have a high fixed-to-mobile MTR Q_i, the total effect on aggregate equilibrium profits is unclear at this level of generality. Even so, we can show the following:

Proposition 7 *Assume that demand elasticity η is constant. If networks are sufficiently symmetric, and call externalities and mobile-to-mobile termination rates are small, then under linear tariffs equilibrium call prices decrease and profits increase in fixed-to-mobile termination profits.*

Proof. We show that the result holds for symmetric networks, zero call externalities and cost-based termination. The general case then follows by continuity. Assume $Q_i = Q_0$ for all i, and consider a change in Q_0. With n symmetric networks, no call externalities and cost-based mobile-to-mobile termination, the symmetric equilibrium linear tariff involves $p_{off} = p_{on}$. Thus $R_{off} = R_{on}$ and $v_{on} = v_{off}$, and the on-net price is given by condition (41) through

$$nR_{on} - \frac{1 - \eta L_{on}}{\sigma (n-1)} + n (Q_0 - f_0) = 0.$$

Letting $R'_{on} = dR_{on}/dp_{on} = q_{on} (1 - \eta L_{on})$, we have

$$\frac{dp_{on}}{dQ_0} = -\frac{n}{nR'_{on} + \frac{\eta}{\sigma(n-1)} \frac{c_{on}}{p_{on}^2}} < 0,$$

and the effect on equilibrium profits is given by

$$\frac{d\pi}{dQ_0} = \frac{d}{dQ_0} \frac{1}{n} (R_{on} + Q_0 - f_0) = \frac{1}{n} \frac{\eta c_{on}}{\sigma (n-1) np_{on}^2 R'_{on} + \eta c_{on}} > 0.$$

■

Thus at least if market shares are not too asymmetric, call externalities small enough, and mobile-to-mobile MTRs are small enough, we see that the waterbed effect is not full: Networks retain a share of fixed-to-mobile termination profits. From the proof, we can see that this share is given by <attention: approximation!>

$$\omega \approx \frac{\eta c_{on}}{\sigma (n-1) np_{on}^2 R'_{on} + \eta c_{on}}. \tag{47}$$

If the market is very little competitive ($\sigma \approx 0$ or n small) then firms retain almost all termination profits and the waterbed effect disappears. In the other extreme, if σ or n are very large (and if the Nash equilibrium still exists) then the waterbed effect is almost full. Thus the extent of the waterbed effect under linear tariffs depends decisively on the intensity of competition.

Two-part tariffs: With two-part tariffs the outcome is much easier to establish: Remember from (28) that equilibrium fixed fees are given by

$$F = f - Q - \left(\hat{R} + R\right) \alpha,$$

where Q is the vector of per-customer profits from fixed-to-mobile termination. Thus under the assumptions of this model, including Hotelling demand and full market coverage, with two-part tariffs all termination profits are handed over to mobile consumers, i.e. there is a full waterbed effect, even in the case of a Nash equilibrium with many asymmetric networks. Furthermore, the waterbed effect is full at the level of each single network. Wright (2002) showed the corresponding result for n symmetric networks in his Proposition 3. He also made clear that the result of a full waterbed effect is an artifact of the Hotelling model where market-wide cost increases do not feed through into lower profits. If costs do feed through, for example because subscription demand is elastic, then networks retain some termination revenue and the waterbed effect again is not full. Genakos and Valletti (2008) show this directly for a model with a logit demand structure.

As concerns the effect of different individual fixed-to-mobile termination charges on market shares, consider condition (34) defining equilibrium market shares under two-part tariffs:

$$\alpha^* = \left(I - \sigma B\left[h + \hat{R} + R\right]\right)^{-1}\left(\alpha_0 + B\left[A + \sigma\left(Q - f\right)\right]\right).$$

A higher Q_i has a similar effect as a higher perceived surplus A_i or lower per-subscriber cost f_i, and thus increases network i's market share.

6 Mobile-to-mobile termination

In this section we consider separately the effects of mobile-to-mobile termination on prices and profits. For simplicity, we assume that firms are symmetric.[11]

Linear tariffs: The results that we derive below generalize those in the duopoly models of Laffont, Rey and Tirole (1998b) and Berger (2004), the latter with call externalities, to an arbitrary number of symmetric networks.[12]

Proposition 8 *Assume demand elasticity η is constant and that networks are symmetric.<verify increasing on-net prices with $n > 2$! >*

1. *If either σ is small, or if γ is small and $a \approx c_t$, then the on-net price decreases and the off-net price increases in a.*

[11] If sufficiently clear analytical results on the asymmetric case become available these will be included in later versions.

[12] Berger's graphical approach cannot be applied with more than two networks because prices can no longer be isolated in the first-order conditions.

2. *If γ and σ are small then profits are increasing in a at $a = c_t$, and the per-customer profit increase is higher for a larger number of networks.* <*adapt*>

Proof. For small σ, we have from (43) and (44) that

$$\begin{aligned}\frac{dp_{on}}{da} &\approx -\frac{p_{on}^m(1+\gamma\eta)}{\eta-1}(n-1)(n-2)\frac{dR_{off}\left(p_{off}^m\right)}{dp_{off}}\frac{dp_{off}}{da}\sigma + O\left(\sigma^2\right) \\ \frac{dp_{off}}{da} &\approx \frac{\eta}{\eta-1} + O(\sigma)\end{aligned}$$

The off-net price increases with the MTR to first-order, while the on-net price increases if $n > 2$ and $a > c_t$ because $dR_{off}\left(p_{off}^m\right)/dp_{off} < 0$.

Assume $\gamma = 0$, with $p_{off} = \frac{c_{on}+a-c_t}{c_{on}} p_{on}$. The first-order condition for the on-net price becomes

$$2R_{on} + (n-2)R_{off} - \frac{1-n\sigma(v_{on}-v_{off})}{\sigma(n-1)}(1-\eta L_{on}) = n(f_0 - Q_0),$$

implying the effect on the on-net price at $a = c_t$ of

$$\left.\frac{dp_{on}}{da}\right|_{a=c_t} = -\frac{\sigma\left((n-1)^2+1\right)R'_{on}}{\sigma(n-1)nR'_{on} + \eta\frac{c_{on}}{p_{on}^2}}\frac{p_{on}}{c_{on}} < 0,$$

and on the off-net price of

$$\left.\frac{dp_{off}}{da}\right|_{a=c_t} = \left(\frac{\sigma(n-2)R'_{on} + \eta\frac{c_{on}}{p_{on}^2}}{\sigma(n-1)nR'_{on} + \eta\frac{c_{on}}{p_{on}^2}}\right)\frac{p_{on}}{c_{on}} > 0.$$

The results for small but positive γ follow by continuity. ■

< discuss >

Two-part tariffs: As concerns two-part tariffs, for now we quickly consider the symmetric equilibrium. We derive a generalization of the result of Gans and King (2001) to n networks, and compare our results with those of Calzada and Valletti (2008) who made a similar analysis for symmetric networks under a logit demand specification.[13] Joint profits are

$$n\pi = \frac{1}{n}\left(\frac{1-n\sigma(h_{on}-h_{off})}{\sigma(n-1)} - R_{on} + R_{off}\right). \tag{48}$$

[13] Calzada and Valletti also consider competition in utilities, which we will not pursue here.

The effect of the mobile-to-mobile MTR on profits is indirect, through the effect of the off-net price p_{off} on h_{off} and R_{off}. As we have seen in the proof of point 1 of Proposition 4, if both h_{off} and R_{off} had the same relative weight in profits then p_{off} would be set efficiently. As it happens, though, with n networks h_{off} has weight $\frac{n}{n-1}$ relative to R_{off}, which implies that networks want to set an off-net price that is lower than the socially optimal value. This is what Gans and King have shown. On the other hand, our result implies that this effect becomes less strong as n becomes large since $\frac{n}{n-1} \to 1$. Formally, when choosing their jointly profit-maximizing off-net price, networks maximize

$$\frac{n}{n-1}\left(v_{off} + \gamma u_{off}\right) + \left(p_{off} - c_{on}\right) q_{off}.$$

The maximum is obtained at

$$p_{off} = \frac{(n-1)\, c_{on}}{n\gamma + n - 1 + 1/\eta} < \frac{c_{on}}{1+\gamma} = p_{on}. \tag{49}$$

This in particular implies that at the jointly profit-maximizing offnet- price the equilibrium is stable in customer expectations for all n, since $h_{off} > h_{on}$. The above expression for p_{off} and (45) imply

$$a = c_t - \frac{(n+1)\,\gamma\eta + 1}{n\gamma\eta + (n-1)\,\eta + 1} c_{on}. \tag{50}$$

The above discussion is summed up in the following Proposition:

Proposition 9 *If networks compete in two-part tariffs, the joint profit-maximizing is set below the efficient level. It decreases in γ and increases in η and n.*

Thus a will be lower in the presence of call externalities. This is intuitive, as the aim of setting a low MTR is to reduce network effects which make networks compete harder. These network effects are stronger in the presence of call externalities.

AW's profit-maximizing off-net price and MTR can be rewritten as

$$\begin{aligned} p_{off} &= \frac{(n-1)\, c_{on} - nb}{n - 1 + 1/\eta} \\ a &= c_t - \frac{c_{on}}{(n-1)\,\eta + 1} - \frac{(n^2-1)\,\eta + 1}{\eta n - \eta + 1} \frac{b}{n-1}. \end{aligned}$$

While the expressions differ somewhat from those in our model, the implied economic effects are the same.

For $\gamma = b = 0$, (50) and AW's result on MTRs can be rewritten as

$$\frac{a - c_t}{c_{on}} = -\frac{1}{(n-1)\,\eta + 1}, \tag{51}$$

which is identical to Calzada and Valletti (2008, p. 1231). Thus this result seems fairly robust to different specifications of demand which assume full coverage.

As the number of networks increases, the joint profit-maximizing MTR converges towards $a = c_t - \frac{\gamma}{\gamma+1} c_{on}$ while $p_{off} \to \frac{c_{on}}{1+\gamma}$ (in AW, $p_{off} \to c_{on} - b$ and $a \to c_t - b$). The MTR remains below cost because the joint profit-maximizing off-net price converges to the efficient price. Therefore only in the absence of call externalities will networks want to set MTRs at the true cost of termination in the limit.

7 Conclusions

In this paper we have presented a tractable extension of network competition models with tariff-mediated externalities to an arbitrary number of asymmetric firms (surplus and cost asymmetry), and derived Nash equilibria under both linear and two-part tariffs. We derived a generalized stability condition and determined the first-best prices and market shares, and showed how to calibrate the model to markets with more than two networks under two-part tariffs. Finally, we uncovered an interesting new link between equilibrium profits under linear and two-part tariffs, and reconsidered the implications of multiple networks for the effect and choice of fixed-to-mobile and mobile-to-mobile termination rates.

Future versions of this paper will explore the effects of asymmetries for mobile termination, and present some actual calibrations for mobile telephony markets with at least four networks.

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