New Network Goods*

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Abstract

New horizontally-differentiated goods involving product-specific network effects are quite prevalent. Consumers' preferences for each of these new goods often are initially unknown. Later, as sales data begin to accumulate, agents learn market-wide preferences which thus become common knowledge. We call network goods' markets showing these two features "new network markets." For such markets, we pinpoint the factors determining whether the market-wide preferred firm reinforces its lead as time elapses, both when market-wide preferences are time invariant and when they may change. In the former case, whether a firm that leads after the first period subsequently reinforces such a lead depends on the relative strength of the network effects vs. the degree of horizontal differentiation between goods. In stark contrast, the leading firm always reinforces its lead when it enjoys a sustained market-wide preference but market-wide preferences can vary. Moreover, we show that new network markets are more prone to increased sales dominance of the leading firm than are regular network markets. Finally, we characterize the social-welfare maximizing allocation of consumers to networks and use it to evaluate from a social-welfare viewpoint the market outcomes of both types of new network goods as well as regular network goods.

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1 Introduction

A PERENNIAL ISSUE in markets involving network effects is whether the firm that finds itself with the largest installed base systematically oversells its competitors, thereby eventually yielding disproportionate market power or even becoming a monopolist. The following quotation from Varian and Shapiro (1999, p. 179) summarizes the issue: "The new information economy is driven by the economics of networks (...) positive feedback makes the strong get stronger and the weak grow weaker." The idea is that consumers may wish to buy the good that most others end up buying in order to reap the most benefits from the network effect. An important related question is whether and to what extent such markets yield outcomes differing from the socially-optimal one.

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We study these issues for what we term "new network goods." These are new horizontally-differentiated goods involving product-specific network effects that reach the market almost simultaneously such that: (i) when the new goods are introduced, neither consumers nor firms know which one most consumers prefer; (ii) yet, as sales data accumulate, market-wide preferences become common knowledge. One can think of the former as the *launch* phase of the industry and of the latter as the *mature* phase.

A current example of a new network goods' market is that for HDTV DVDs where two alternative data storage formats are vying for consumers' preferences: Blu-ray (backed by, among others, Sony) and HD-DVD (backed by Toshiba and NEC).[1] Other recent examples are the consoles market where Microsoft, Nintendo and Sony compete by simultaneously launching new generations of game consoles, and the storage-media market were Imation and Iomega used to compete with the SuperDisk and Zip formats.[2,3] These examples suggest that many network markets are indeed "new network markets."

In these markets consumer preferences involve not only an idiosyncratic term specific to each consumer that models the extent of horizontal differentiation between goods, but also a factor common to all consumers buying in each period that captures market-wide preferences. These may be permanent or temporary, i.e., they may stay constant over all time periods or vary from one period to the next. In fact, a good may be preferred by the majority of consumers because of physical differences intrinsic to the goods, in which case such an advantage *lasts over time.* On the other hand, the majority of consumers may prefer one good to others because of, say, a superior brand image or a particularly successful advertising and marketing campaign at the time the product was launched, but such a preference may later be reversed, for instance because, after a while, it became apparent that the initially-preferred good proves to be more prone to breakdown than its competitors. In this case, an initial market-wide advantage may vanish or even be reversed once the market matures. If we regard market-wide preferences as introducing an element of vertical differentiation into preferences, we can think of this case as involving *reversible* vertical differentiation.

We study whether a firm that finds itself leading at the end of the launch phase (i.e., with a larger installed base) will milk such an advantage by subsequently charging a high price, thereby diluting its initial installed-base advantage, or, instead, will price more moderately and use its initial lead as a lever for further increasing its market share. Moreover, we compare market outcomes to the socially-optimal allocation of goods to consumers. We investigate these issues for new network markets, both when market-wide preferences are permanent and when they may vary, and compare them with "regular" network markets where market-wide preferences are common knowledge from the outset.

In order to treat these issues, we need a model with several features: (i) early buyers should be forward looking and try to estimate the total (current plus future) sales of each good, since network benefits are proportional to them; (ii) late buyers should be backward

[1] See *The Economist*, November 3, 2005.

[2] Network effects arise due to game sharing (a direct network effect) and variety (an indirect network effect), and movie and file swapping.

[3] Imation discontinued the production of its SuperDisk drives perhaps as a consequence of learning through sales data that most consumers preferred the Zip format. Recently so did the consortium backing HD-DVD.

looking insofar as installed base is itself directly relevant for network size, and indirectly so through its influence on buying decisions of current and future consumers. This is the case because a firm's large installed base favors its current and future sales (all else equal) and, hence, its final network size; (iii) moreover, because early sales are beneficial for late sales, firms should be allowed to dynamically price, i.e., to initially offer bargains with the aim of obtaining a large installed base that will later permit the setting of higher prices.[4] One should thus allow for penetration and under-cost pricing; (iv) horizontal differentiation should also be present since consumers idiosyncratically differ in their valuation of the competing goods' characteristics. Thus, one explicitly captures in a dynamic setting the tension between horizontal differences that tend to split the market among firms, and network effects that have the opposite effect. A truly dynamic model of network goods should encompass all these features.

Besides the previous characteristics, in order to model "new network goods," we allow either product to be preferred by the majority of consumers due to the non-observable realization of a random variable common to all consumers. This unobservable common term adds to the usual idiosyncratic horizontal-differentiation term to determine gross surplus which, added to the network benefit, yields willingness to pay. Thus, initial consumers who enjoy one good more than the other do not know if the majority of other consumers also show the same relative preference, or if this is instead an idiosyncratic trait. Afterwards, second-period consumers, as well as firms, infer which product enjoys a market-wide preference upon observing first-period sales. Thus, with time and through learning, permanent market-wide preferences become common knowledge.

By the very nature of the issues that it addresses, our analysis has to involve several driving forces that concurrently shape equilibrium behavior. One may then legitimately wonder how different modeling details would impact and, perhaps, alter our results. In order to allay this concern we (i) have opted for as standard a modeling as we possibly can, (ii) always spell out the full intuition of the results in a manner that is independent of how the various driving factors figure in the detailed model and (iii) are candid about it in the few cases where this is not the case.

We find that when a good's market-wide preference springs from differences inherent to the goods, in which case such a preference is lasting, the firm that obtains the larger market share in the first period reinforces its lead in the following period *if and only if* the network effect is significant enough compared to the degree of product differentiation. This finding contrasts sharply with Arthur and Ruszczynski's (1992), who show that a firm's sustained increase in market share, when it finds itself with a larger installed base, depends on the discount rate: when the future is significantly discounted, the leading firm prefers to milk its initial advantage; otherwise, it builds on its initial installed-base lead and further increases it.

Strikingly, in the case of *reversible* vertical differentiation—which we address by con-

[4] Dynamic pricing is well understood in the literature. What we wish to emphasize is that it must be allowed by the modeling, at least if the goods are "sponsored" by profit-maximizing firms, rather than available at marginal cost ("unsponsored").

sidering a variant of the model with two independent realizations of the non-observable random variable, each affecting consumers buying in one period—when a firm obtains the same market-wide preference in both periods, it *always* reinforces its lead. When taken together, these results make it clear that minute differences in the structure of a network goods' market can have a striking influence on its dynamic path toward monopolization or away from it.

We use this variant of the model to treat the effect of consumer fads—defined as a fleeting market-wide preference for a product that neither consumers nor firms anticipate—on new network markets. We show that, surprisingly, when one firm is preferred by the majority of consumers in one period while the other firm benefits from the very same advantage in the following period, the latter obtains a higher profit over the two periods in spite of the presence of network effects and regardless of their strength, a result that runs counter to the prevailing intuition.[5] As we point out later, this result may depend on the specific modeling adopted and, insofar as it is unexpected, indicates that further (future) research of the issue may fruitfully be carried out.

We also compare "new" with "regular" network markets, where any advantage of one product over the other is known from the outset. This could result, for instance, from advanced testing of the new goods reported in the media that makes market-wide preferences common knowledge from the outset, say, by making apparent a good's superior features. We show that the parameters' range for which the firm with a larger installed base after the first period increases its dominance in the second period is smaller in the case of regular network markets. Thus, increased dominance is more likely in new than in regular network markets.

Finally, we characterize how a social planner would assign consumers to networks in order to compare market outcomes with socially-optimal ones. We show that in new network markets the smaller network is too big compared with the socially-optimal outcome, and that such a bias is generally more pronounced, and thus welfare is lower, when market-wide preferences are immutable. Moreover, we show that this bias is also present in the case of regular network markets and that these yield the least welfare when network effects are not strong, i.e., the newness of network markets attenuates their welfare sub-optimality when network effects are not too strong.

Though the literature on markets displaying network effects is by now quite extensive, fully dynamic models addressing these issues are scarce.[6] Arthur and Ruszczynski (1992) is a notable exception already mentioned.[7] Keilbach and Posch (1998) model a market as a generalized urn scheme encompassing sequential buying decisions on the part of consumers, and firms' exogenous (and, thus, not necessarily optimal) adjustments of price to market share. They consider the limit behavior of market shares as successive consumers make their buying decisions and show how different price-adjustment rules on the part of firms lead to one, several or all firms surviving in the long run.

[5]See Liebowitz and Margolis (1994, p. 143) who criticize this type of result.
[6]See Farrell and Klemperer (2007), subsection 3.7.4.
[7]See also Hansen (1983).

More recently, Mitchell and Skrzypacz (2006) have discussed this issue in the context of a dynamic model, while also discussing social-welfare issues. They treat a particular type of regular network goods such that consumers care only about current and previous-period sales while not trying to estimate each network's final size. In line with Arthur and Ruszczynski (1992), they conclude that when firms heavily discount the future, a leading firm tends to dissipate its lead. On the other hand, if the future is lightly discounted, the leading firm tends to build on its early lead by continuing to charge low prices. In this case, leaders tend to extend their advantage. Moreover, Mitchell and Skrzypacz discuss how the quantitative extent of leadership affects firms' pricing. In sum, they analyze rather carefully the impact of the discount factor and relative size of installed bases on pricing and market-share paths of regular network goods' markets. The effect of consumers rationally forecasting the future (final) installed bases of each product is not present in the analysis, and learning about market-wide preferences and hence new network goods cannot be treated in their framework.

Argenziano (2008) treats preferences that resemble ours insofar as the gross surplus excluding the network effect consists of the sum of two components which consumers cannot disentangle. She assumes that these terms are both ruled by the normal distribution while we assume that they are governed by the uniform distribution. Like us, she assumes that consumers' expectation of the idiosyncratic term is nil at the outset. Unlike us, she assumes that consumers' expectation of the common term may differ from zero at the outset, i.e., consumers may *ex ante* receive a signal concerning the relative quality of the goods, which may then be confirmed or disproved by the actual realization of the common term. Moreover, she models an increase in horizontal differentiation as an increase in the variance of the distribution ruling the idiosyncratic term (we instead model it in the usual manner as an increase in each consumer's welfare cost of not being able to consume its most-preferred variety). Thus, the models differ in their informational assumptions and modeling of horizontal differentiation.

More importantly, Argenziano's model is static and, as such, learning is absent. Therefore, new network goods are not discussed. She studies the static competition between networks, i.e., how consumers partition themselves between the two networks in a single period whereas we instead deal with the dynamic evolution of the two networks while also modeling the effects of consumer learning about initially-unknown market-wide preferences. Finally, she too compares market and socially-optimal outcomes, and highlights a pricing effect underscored by Mitchell and Skrzypacz which our analysis also encompasses.

More recently, Cabral (2009) has developed a model where consumers with idiosyncratic preferences for either network sequentially enter a market involving product-specific network effects which he then applies to create a dynamic version of Laffont, Rey and Tirole (1998a,b) static model of network competition. His model has the important advantage of considering the successive entry of many consumers (rather than just two successive cohorts of consumers, as we do) while also allowing for their random exit (death). His model, unlike ours, in not fully analytically solvable, and thus requires study by numerical simulation. Moreover, his framework is not suited to studying learning of market-wide preferences, the phenomenon that underlies new network goods. Interestingly, his results regarding network size dynamics, namely the possibility that the leading firm may want to milk such an advantage, thereby diluting it, or instead rely on its market share lead to further increase it, complement ours. He concludes that leading firms will further increase their lead unless their market share is already very high, i.e., monopoly is not expected to be the long-run equilibrium of such a market.

The paper is organized as follows. We describe the model in Section 2 and solve it in Section 3. Section 4 presents results regarding the evolution of market shares and market fads. Section 5 characterizes the social-welfare maximizing allocation of goods to consumers, which is then used to compare new and regular network markets from a social-welfare viewpoint. Finally, Section 6 briefly concludes. All material not needed for a quick understanding of the model, its solution and main results is found in several appendices.[8]

2 The Model

We consider a model with two periods. In each period, unit-demand consumers uniformly distributed along a unit-length linear city reach the market and decide which good to buy.[9] Two firms, A and B, located at the endpoints of the linear city sell differentiated goods endowed with product-specific network effects, i.e., incompatible, which are also denoted A and B, respectively. We assume that firms compete in prices, which they set in each period. Let both firms' marginal cost be constant and equal and, without loss of generality, nil.

The total (two-period) sales of good A is given by $x_1 + x_2$, where $x_i \in [0,1]$ is the measure of consumers who choose good A in period $i = 1,2$. Each consumer enjoys a surplus resulting from the network effect which increases linearly at rate $e > 0$ with the good's total (two-period) sales, i.e., good A's network benefit equals $e \times (x_1 + x_2)$ while B's equals $e \times (2 - (x_1 + x_2))$.[10] Hence, e is a constant that measures the intensity of the network effect.

In each period, consumers choose the good that offers the greatest expected net surplus. To determine it, consumers must consider (i) the gross surplus excluding the network effect, (ii) the expected network benefit, which depends on the good's total sales, and (iii) the price.

For each consumer, the *difference* between the gross surplus yielded by good A and that yielded by good B is given by random variable $v(\cdot,\cdot)$. A consumer with a positive value of

[8]We have tried to keep all appendices as self-contained as possible. As such, cross-references were kept to a minimum. We ask for the reader's understanding for the few that remain.

[9]This straightforwardly models situations where the purchase of the new network good is triggered by the breakdown of an older one (a DVD player, say). Before breakdown, the *additional* utility brought about by the purchase of a new appliance is too small compared to its price, and consumers do not buy. This decision is reversed by the occurrence of a breakdown. Staggered breakdown leads successive cohorts of consumers (two in our stylized model) to immediately acquire a new network good (thus having to choose between, for instance, a Blu-Ray and an HD-DVD enabled DVD player), while being aware that buying at a later moment would involve better information on the relative quality of the goods on offer and better coordination with the (by then) larger installed base. This is rationally the case when consumers' disutility of going a single period without the appliance (without watching DVDs, to continue with the example) is very significant when compared to the informational and coordination gains and the possible advantageous price variation arising from an ulterior purchase.

Technically, we avoid the durable goods' issue, not juxtaposing it to the coordination problem at the root of network goods' markets. Needless to say, this modeling option is widespread in the literature to which we are trying to contribute.

[10]Thus, we adhere to Metcalfe's law.

$v(\cdot,\cdot)$ obtains a larger gross surplus by choosing good A rather than B. Otherwise, it obtains a larger gross surplus by choosing good B.

Let us understand how $v(\cdot,\cdot)$ is built. Take a consumer located at $j \in [0,1]$. Random variable $v(j,z)$ equals the sum of two components, random variable z, common to all consumers, and random variable $a(j)$, specific to each consumer, i.e., idiosyncratic:[11]

$$v(j,z) = a(j) + z.$$

The realization of z determines how much, on average, *all* consumers prefer good A to B. We assume it to have uniform distribution with support $[-w,w]$:

$$z \rightsquigarrow U(-w,w).$$

The uniform distribution depicts maximal ignorance (in a Bayesian sense) on the part of consumers and firms concerning the market-wide relative valuation of the two goods.

Random variable $a(j)$ measures how much a particular consumer idiosyncratically prefers good A to B or vice versa. It is constructed as follows. Recall that each period's consumers are uniformly distributed along the interval $[0,1]$ with A located at 0 and B located at 1. Let t measure the degree of product differentiation between the two goods. A consumer located at 0, *ceteris paribus*, idiosyncratically prefers good A to B by an amount t, while a consumer located at 1 idiosyncratically prefers good B to A by the same amount. Therefore, $a(j)$ is uniformly distributed with support $[-t,t]$. Formally,

$$j \rightsquigarrow U(0,1) \wedge a(j) = t - 2tj \Rightarrow a \rightsquigarrow U(-t,t).$$

We assume that the density functions of j and z, as well as the equalities $v(j,z) = a(j) + z$ and $a(j) = t - 2tj$ are common knowledge. Moreover, each consumer privately observes the realization of $v(j,z)$ in its particular case, i.e., knows how much it prefers one good to the other, all else equal. Take a consumer whose realization of $v(\cdot,\cdot)$ is positive. Though it therefore prefers good A to B by the amount $v(\cdot,\cdot)$, all else equal, it does not know if this is caused by a high realization of z, in which case most consumers also prefer good A to B, or a low realization of j, in which case it is she or he that idiosyncratically enjoys good A more than B. In plain words, each consumer knows which good it prefers and by how much, but does not know to what extent such preference is shared by all other consumers.[12]

For first-period consumers, the expected net surplus of acquiring good A equals

$$C + v(j,z) + e \times (\tilde{x}_1(v(j,z)) + \tilde{x}_2(v(j,z))) - p_1^A,$$

while the expected net surplus of buying good B is given by

$$C + e \times (2 - (\tilde{x}_1(v(j,z)) + \tilde{x}_2(v(j,z)))) - p_1^B,$$

[11]By assuming that the realization of z is common to all consumers, first- as well as second-period ones, we are modeling the case when market-wide preferences are immutable. Later we will tackle the case when first-period consumers are affected by the realization of a random variable, z_1, while second-period ones are affected by the realization of another random variable, z_2, thus modeling the case of market-wide preferences that may vary as time elapses.

[12]Needless to say, a first-period consumer cannot deduce where it is located along the linear city since it does not know the realization of z.

where $\tilde{x}_1(v(j,z))$ and $\tilde{x}_2(v(j,z))$ represent the estimates of good A's first- and second-period market shares after the consumer has privately observed its realization of $v(j,z)$, p_1^A and p_1^B represent the prices charged by firms A and B in period 1, and C is a constant sufficiently large for all the market to be covered in equilibrium. Second-period consumers have similar expressions except that $\tilde{x}_1(v(j,z))$ is replaced by firm A's observed first-period sales, x_1^*.

3 Solving the Model

This section solves the model for the case when market-wide preferences are irreversibly fixed. Readers interested only in results can skim the computations and retain only equations (13), (14) and (15), which represent first- and second-period equilibrium prices, and equations (16) and (17), which represent first- and second-period equilibrium quantities.

In order to compare new network goods when market-wide preferences are fixed with the case where these preferences can vary, we solve (in Appendix D) a variant of the model with two random variables akin of z, each one impacting one period. In this case, first- and second-period equilibrium prices are given by (18), (19) and (20), and first- and second-period equilibrium quantities are described by (21) and (22).

Finally, in order to compare new to regular network goods, we solve (in Appendix E) yet another variant of the model where the realization of z is assumed to be common knowledge from the outset. In this case, first- and second-period equilibrium quantities are given by (23) and (24).

In sum, equations (13) to (24) are all that readers concerned only with results and their intuition need to bear in mind. These readers may thus skim the next section without having to dwell on the details.

3.1 Fixed market-wide preferences

Let us solve the model for the case of immutable market-wide preferences. In order to choose a good, first-period consumers must compare the expected net surpluses yielded by goods A and B. Denote by x_1 the location of first-period consumers *indifferent* between the two goods and, hence, first-period demand. It is implicitly defined by:

$$\begin{aligned} C + v(x_1,z) + e(\tilde{x}_1(v(x_1,z)) + \tilde{x}_2(v(x_1,z))) - p_1^A = \\ = C + e(2 - (\tilde{x}_1(v(x_1,z)) + \tilde{x}_2(v(x_1,z)))) - p_1^B. \end{aligned}$$

Replacing $v(x_1,z)$ by its components, $t - 2tx_1 + z$, and solving for x_1 yields the location of first-period indifferent consumers and, simultaneously, good A's first-period demand:

$$x_1 = \frac{p_1^B - p_1^A + z + t - 2e + 2e(\tilde{x}_1(v(x_1,z)) + \tilde{x}_2(v(x_1,z)))}{2t}.$$

Assume that consumers estimate demand as equaling expected demand conditional on their observation of $v(\cdot,z)$. From the previous expression, we get, for an indifferent first-period consumer:

$$\begin{aligned}\tilde{x}_1\left(v\left(x_1,z\right)\right) &= E\left[x_1|v\left(x_1,z\right)\right] = \\ &= \frac{p_1^B - p_1^A + E\left[z|v\left(x_1,z\right)\right] + t - 2e + 2e\left(\tilde{x}_1\left(v\left(x_1,z\right)\right) + \tilde{x}_2\left(v\left(x_1,z\right)\right)\right)}{2t} \\ &= \frac{p_1^B - p_1^A + E\left[z|v\left(x_1,z\right)\right] + t - 2e + 2e\tilde{x}_2\left(v\left(x_1,z\right)\right)}{2\left(t-e\right)},\end{aligned}$$

where $E\left[a|v\left(\cdot,z\right)\right]$ is the expected value of variable a by a first-period consumer who has observed realization $v\left(\cdot,z\right)$.

Because the expected value of z is not the same for all consumers, they can have different expectations of the demand for good A in the first and second periods. For instance, a consumer who privately observes a high value of $v\left(\cdot,z\right)$ will abandon its null prior on z in favor of a positive posterior. This, in turn will lead him to form high (i.e., greater than $\frac{1}{2}$) estimates for $\tilde{x}_1\left(v\left(\cdot,z\right)\right)$ and $\tilde{x}_2\left(v\left(\cdot,z\right)\right)$. Thus, a first-period consumer who has privately observed $v\left(\cdot,z\right)$ takes first-period demand to be given by

$$x_1 = \frac{p_1^B - p_1^A + z + t - 2e + 2e\left(\tilde{x}_1\left(v\left(\cdot,z\right)\right) + \tilde{x}_2\left(v\left(\cdot,z\right)\right)\right)}{2t}, \tag{1}$$

and, recalling that all consumers estimate demand as equaling expected demand conditional on their observation of $v\left(\cdot,z\right)$, we have:

$$\tilde{x}_1\left(v\left(\cdot,z\right)\right) = E\left[x_1|v\left(\cdot,z\right)\right] = \frac{p_1^B - p_1^A + E\left[z|v\left(\cdot,z\right)\right] + t - 2e + 2e\tilde{x}_2\left(v\left(\cdot,z\right)\right)}{2\left(t-e\right)}. \tag{2}$$

This expected demand results in a unique and stable equilibrium when t exceeds e. If instead $e > t$, this expected demand is based on a non-unique and unstable equilibrium, in which case there are two other stable equilibria where *all* consumers choose one of the two goods. The reason is that when $e > t$, the network effect dominates product differentiation to such an extent that consumers may prefer to coordinate on all buying the same good rather than splitting. In the end, the equilibrium turns out to be similar to one in which there is no product differentiation at all. Since we want to analyze the case where product differentiation also drives the results, we assume that $t > e$ for now. However, once we take into account the interaction between periods, this restriction will be strengthened.[13]

In order to determine first-period demand, first-period consumers also need to compute the expected second-period demand, $\tilde{x}_2\left(v\left(\cdot,z\right)\right)$. For that, one must model second-period consumers' behavior as well as firms' optimal second-period pricing.

Second-period consumers and firms, having observed actual first-period quantity demanded x_1^*, i.e., sales of both products, correctly infer the value of z.[14] Therefore, they exactly determine second-period demand.

In order to choose a good, second-period consumers compare the net benefit of adopting each of the two goods. A consumer indifferent between the two goods is such that:

$$C + v\left(x_2,z\right) + e\left(x_1^* + x_2\left(v\left(x_2,z\right)\right)\right) - p_2^A = C + e\left(2 - \left(x_1^* + x_2\left(v\left(x_2,z\right)\right)\right)\right) - p_2^B,$$

[13] See Appendix A for details.

[14] Appendix B explains this inference process in detail.

which yields, after substitution of $v(x_2, z)$ by its components, $t - 2tx_2 + z$,

$$x_2 = \frac{p_2^B - p_2^A + z + t - 2e + 2ex_1^*}{2(t-e)}, \tag{3}$$

where x_1^* is the observed market share of good A at the end of the first period. All r.h.s. variables are either observable or exactly inferred.[15] Hence, second-period consumers exactly estimate second-period demand, x_2^*.

To obtain second-period prices, p_2^A and p_2^B, consider firm A's profit-maximization problem in the second period, while bearing in mind that firms, too, have inferred the realization of z at the end of the first period upon observing actual first-period sales by reasoning exactly like second-period consumers. Therefore, they too exactly estimate second-period demand as did second-period consumers.[16] Thus, making use of (3), we have

$$\underset{p_2^A}{\text{Max}} \qquad p_2^A x_2 = p_2^A \frac{p_2^B - p_2^A + z + t - 2e + 2ex_1^*}{2(t-e)}.$$

The f.o.c. equals

$$p_2^B + z + t - 2e + 2ex_1^* = 2p_2^A,$$

whereas the s.o.c. equals $-\frac{1}{t-e}$ and thus is strictly negative due to the assumption that $t > e$. By the same token, we have for firm B:

$$p_2^A - z + t - 2ex_1^* = 2p_2^B.$$

By solving the system of equations formed by these first-order conditions, we obtain the prices charged in the second period:

$$\begin{cases} p_2^A &= \frac{1}{3}z + t + \frac{2}{3}ex_1^* - \frac{4}{3}e \\ p_2^B &= -\frac{1}{3}z + t - \frac{2}{3}e - \frac{2}{3}ex_1^*. \end{cases} \tag{4}$$

Replacing these in (3), one has

$$x_2 = \frac{t - \frac{4}{3}e + \frac{1}{3}z + \frac{2}{3}ex_1^*}{2(t-e)}. \tag{5}$$

First-period consumers do not know the realization of z and x_1^*. Thus, they cannot determine the actual second-period demand, and must make use of (5) to compute expected demand:

$$\tilde{x}_2(v(\cdot, z)) \quad = \quad \frac{t - \frac{4}{3}e + \frac{1}{3}E[z|v(\cdot, z)] + \frac{2}{3}e\tilde{x}_1(v(\cdot, z))}{2(t-e)}. \tag{6}$$

We now have two equations, (2) and (6), which together determine $\tilde{x}_1(v(\cdot, z))$ and $\tilde{x}_2(v(\cdot, z))$ as a function of all known parameters, first-period prices and $E[z|v(\cdot, z)]$. We can replace them in (1) to finally obtain first-period demand

$$x_1 \quad = \quad \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{E[z|v(\cdot, z)]\,e(2t-e)}{t(3t^2 - 6te + 2e^2)}. \tag{7}$$

[15] Recall that z was exactly inferred by second-period consumers (and firms) upon observation of first-period sales.

[16] As Appendix B makes clear.

Appendix A makes it plain that only for $t > 1.577e$ do we have a unique and stable intermediate equilibrium without all consumers bunching on a good. Thus, we tighten the previously made assumption $t > e$ to this more stringent inequality.

At this point, one must tackle the inference problem encapsulated in $E[z|v(\cdot, z)]$, i.e, compute the expectation of z by a consumer who observed a given realization of $v(\cdot, z)$. The assumptions made on the supports of $a(\cdot)$ and z yield $[-t-w, t+w]$ as the support of v. We now postulate that there are always some consumers who value good A more than B, while others have the opposite valuation ordering when firms charge the same price. This amounts to assuming that, whatever the realization of z, variable $v(\cdot, z)$ can assume positive *and* negative values depending on the realization of $a(\cdot)$. This is tantamount to imposing $t > w$.[17] By doing so, we are essentially guaranteeing that horizontal differentiation always plays a role as a determinant of behavior, i.e., it is never overwhelmed by a strong market-wide preference for a good.

We show in Appendix C how, given their private signal $v(\cdot, z)$, first-period consumers form their expectation of z. Also, Appendix C makes it clear that first-period demand is estimated by first-period consumers as follows:

(i) For consumers who observe a realization of $v \in [t-w, t+w]$:

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{(v+w-t)\,e\,(2t-e)}{2t\,(3t^2-6te+2e^2)}.$$

(ii) For consumers who observe a realization of $v \in [-t+w, t-w]$:

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}. \tag{8}$$

(iii) For consumers who observe a realization of $v \in [-t-w, -t+w]$:

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{(v+t-w)\,e\,(2t-e)}{2t\,(3t^2-6te+2e^2)}.$$

Appendix C demonstrates that (8) is the relevant demand curve. This has a very intuitive explanation. Begin by viewing the first case above as representing consumers who are quite "optimistic" about good A's market prospects because, having observed a high realization of $v(\cdot, z)$, i.e., having found good A to be so superior to good B, their posterior concerning z no longer equals the prior, 0, but is positive instead. The intermediate case comprises the "middle grounders," whose posterior for z equals the prior, 0. Finally, the last equation represents the "pessimists." Appendix C shows that "middle grounders" always determine market demand.[18]

To determine optimal first-period prices, firms have to take into account their effect on second-period demand and optimal prices. Hence, we now determine then as a function of first-period prices only.

[17] Thus ensuring, as we will see, that the equilibrium value of x_1 lies on $(0, 1)$.

[18] Interestingly enough, even though "middle grounders" always determine actual demand—i.e., indifferent consumers are necessarily "middle grounders"—they may be wrong in their estimate of z. To see this, consider the case where the realization of z is extreme, namely w, in which case "optimists" are nearer to correctly estimating market-wide preferences than "middle grounders" (see Appendix C for details).

By replacing (8) in (4) and (5), we obtain

$$p_2^A = \frac{1}{3}z + t - e + \frac{1}{3}\frac{ez}{t} + \frac{e\,(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}, \tag{9}$$

$$p_2^B = -\frac{1}{3}z + t - e - \frac{1}{3}\frac{ez}{t} - \frac{e\,(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}, \tag{10}$$

and

$$x_2 = \frac{1}{2} + \frac{\frac{1}{3}z + \frac{1}{3}\frac{ez}{t}}{2\,(t-e)} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}. \tag{11}$$

The profit maximization problem of firm A is[19]

$$\underset{p_1^A}{\text{Max}} \quad \Pi^A = E\left[x_1\left(p_1^A, p_1^B\right)p_1^A\right] + E\left[x_2\left(p_1^A, p_1^B\right)p_2^A\right].$$

By replacing (8), (9) and (11) in the objective function and bearing in mind that p_1^A is not a random variable, but p_2^A is because its value depends on the realization of z, we can now easily compute a symmetric equilibrium.[20]

$$\begin{aligned}\Pi^A = {} & E\left[\frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}\right]p_1^A + \\ & + E\left[\left(\frac{1}{2} + \frac{\frac{1}{3}z + \frac{1}{3}\frac{ez}{t}}{2\,(t-e)} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}\right) \times \right. \\ & \left. \times \left(\frac{1}{3}z + t - e + \frac{1}{3}\frac{ez}{t} + \frac{e\,(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}\right)\right].\end{aligned}$$

Computing the f.o.c. of this problem and using symmetry, $p_1^A = p_1^B$, we have

$$p_1^A = p_1^B = t - \frac{5}{3}e - \frac{1}{3}\frac{e^2}{t-e}. \tag{12}$$

Equilibrium first-period prices depend positively on the degree of product differentiation and negatively on the extent of the network effect. A decrease in price increases expected sales and, thus, expected network size. Therefore, the stronger is the network effect, the greater is the impact of a decrease in price on each period's demand, and so the lower is the first-period price that firms want to charge.[21]

[19] Though it would be easy to introduce a discount factor affecting the second period, we do not do so since the role of discounting in determining the dynamic path of network goods' markets is already well understood—see the Introduction for a discussion of Arthur and Ruszczynski (1992) and Mitchell and Skrzypacz (2006). Moreover, the absence of discounting of second-period profits is in keeping with the remarks made above in fn. 9.

The reader may have noticed that equilibrium second-period sales (unlike first-period ones) are not necessarily strictly between 0 and 1. To see it, suppose that $t < 2e$ and take a realization of z close to t, i.e., $z \lessapprox t$. Then, since $\frac{\frac{1}{3}z+\frac{1}{3}\frac{ez}{t}}{2(t-e)}$ in (11) equals $\frac{1}{2}\frac{1}{3}\frac{t+e}{t-e}\frac{z}{t} \lessapprox \frac{1}{2}\frac{1}{3}\frac{t+e}{t-e}$, and noting that $\frac{t+e}{t-e} > 3$ for $t < 2e$, we have $\frac{\frac{1}{3}z+\frac{1}{3}\frac{ez}{t}}{2(t-e)} > \frac{1}{2}$. This finally yields $x_2 = \frac{1}{2} + \frac{\frac{1}{3}z+\frac{1}{3}\frac{ez}{t}}{2(t-e)} > 1$. In this case, x_2 would equal 1 in a symmetric equilibrium where $p_1^A = p_1^B$. This possibility can be excluded by assuming that the support of z is small compared to t, i.e., by assuming that $w \ll t$—in which case $\frac{z}{t} \ll 1$—implying that second-period equilibrium sales are always strictly between 0 and 1. In sum, explicitly dealing with this mathematical issue would further clutter the analysis without shedding any further light on the economic problem under examination. Moreover, as we saw above, limiting the support of z relative to the value of t would avoid this complication. Thus, we will proceed without *explicitly* introducing it while bearing in mind where appropriate that x_2 may indeed equal 0 or 1.

[20] Note that firms are symmetric at the beginning of the game. This is the case since, even though one of them may be favored by the majority of consumers, neither one yet knows it *and* demand is determined by "middle grounders" whose posterior on market-wide preferences, z, equals 0.

[21] We have assumed that costs are nil, i.e., that prices are already net of marginal costs. One may then be concerned with the possible negativity of prices arising from high values of e. We assume that negative prices are possible to avoid having to deal with another constraint that would make the presentation more complicated. As the reader may want to check as she or he reads on, doing so would not add to the results' intuition.

The second derivative of the problem at hand equals $\frac{1}{2}(t-e)\frac{-18t^2+36te-11e^2}{(3t^2-6te+2e^2)^2}$. This second derivative is negative if $t < 0.376e$ or $t > 1.623e$. Since we have already seen that only for $t > 1.577e$ do we have a unique and stable equilibrium without full bunching on a good, we must retain $t > 1.623e$ as the relevant constraint.

In sum, from (9), (10) and (12), first- and second-period equilibrium prices equal

$$p_1^A = p_1^B = t - \frac{5}{3}e - \frac{1}{3}\frac{e^2}{t-e}, \tag{13}$$

$$p_2^A = \frac{1}{3}z + t - e + \frac{1}{3}\frac{ez}{t}, \tag{14}$$

$$p_2^B = -\frac{1}{3}z + t - e - \frac{1}{3}\frac{ez}{t}, \tag{15}$$

whereas, from (8) and (11), first- and second-period equilibrium quantities equal

$$x_1 = \frac{1}{2} + \frac{z}{2t}, \tag{16}$$

$$x_2 = \frac{1}{2} + \frac{\frac{1}{3}z + \frac{1}{3}\frac{ez}{t}}{2(t-e)}. \tag{17}$$

3.2 Varying market-wide preferences

In Appendix D we solve a variant of the model involving two random variable akin to z, namely z_1 and z_2, each affecting one period, in order to address the case of varying market-wide preferences. From (D.16), (D.14) and (D.15) we obtain first- and second-period equilibrium prices for the case where market-wide preferences may vary,

$$p_1^A = p_1^B = t - \frac{5}{3}e - \frac{1}{3}\frac{e^2}{t-e}, \tag{18}$$

$$p_2^A = t - e + \frac{1}{3}\frac{ez_1}{t}, \tag{19}$$

$$p_2^B = t - e - \frac{1}{3}\frac{ez_1}{t}, \tag{20}$$

and, from (D.12) and (D.13), first- and second-period equilibrium quantities,

$$x_1 = \frac{1}{2} + \frac{z_1}{2t}, \tag{21}$$

$$x_2 = \frac{1}{2} + \frac{z_1 e}{6t(t-e)} + \frac{z_2}{2t}. \tag{22}$$

3.3 Regular network goods

In Appendix E we solve a variant of the model where the realization of z is common knowledge from the outset. In this case, from (E.8) and (E.9), first- and second-period equilibrium quantities equal

$$x_1 = \frac{1}{2} + \frac{1}{2}\frac{9zt - 2ez}{14e^2 - 54te + 27t^2} \tag{23}$$

$$x_2 = \frac{1}{2} + \frac{1}{2}\frac{-4e^2z + 15ezt - 9zt^2}{(e-t)(14e^2 - 54te + 27t^2)}. \tag{24}$$

4 Market shares' evolution

4.1 Irreversible vertical differentiation (time-invariant market-wide preferences)

We want to check whether in a market for new network goods, i.e., involving initial uncertainty concerning the market-wide preferences of consumers that gets resolved as sales data accumulate, the firm that obtains the larger market share in the first period tends to increase it in the next period. Let us begin with the case where one good enjoys an irreversible market-wide preference.

Proposition 1 *Under the conditions of our model, when one product enjoys a time-invariant market-wide preference, it reinforces its market dominance if and only if network effects are strong enough* vis-à-vis *the degree of product differentiation.*

Proof Take good A's sales in both periods, given by (16) and (17). Simple computations show that a firm increases its market share in the second period, $x_2 > x_1$, iff $t < 2e$. Recalling that we restrict our analysis to $t > 1.623e$, we conclude that the firm that obtains the larger market share in the first period will increase it in the second period iff $t \in (1.623e, 2e)$. In this case, the leader opts for building up its lead. If $t > 2e$, a first-period installed-base advantage—despite the favorable market-wide preference being permanent and becoming common knowledge—is subsequently milked and, thus, reduced. ■

Let us explain the result intuitively. At the beginning of the second period, the value of z becomes known and firms as well as consumers learn which good benefits from a market-wide preference. If there where no network effects, $e = 0$, second-period consumers' behavior would not be affected by this information. Firms, on the other hand, would alter their pricing according to z's realization, with the market-wide favored firm charging a higher price than its competitor, whereas in the first period they both charged the same price. These pricing responses *lead the market-wide preferred firm to sell less and its opponent to sell more than in the first period.* More formally, $e = 0$ and $z \gtreqless 0$ yield $x_1 \gtreqless x_2$ in equilibrium. Consider now the impact of the network effects, $e > 0$. Second-period consumers flock towards the market-wide preferred firm whose demand thus increases while its competitor's decreases *vis-à-vis* the first-period demands. *This leads the market-wide preferred firm to sell more and its opponent to sell less than in the first period.* Besides e, the degree of horizontal differentiation, t, affects the strength of this last effect. The higher t, the fewer consumers shift to the market-wide preferred good because doing so entails a higher horizontal-differentiation welfare cost. Hence, the lower t, i.e., the less differentiated are the goods, the higher the sales of the market-wide preferred firm in the second period (for a given value of e). Why? Not only will more second-period consumers opt for the market-wide preferred good because the horizontal-differentiation cost of doing so is smaller, but more first-period consumers will buy the market-wide preferred good in the first period (recall that $x_1 = \frac{1}{2} + \frac{z}{2t}$ in equilibrium) which, by further increasing the overall (two-period) market share of the preferred firm, also induces more second-period consumers to buy the market-wide preferred good.

This result qualifies Shapiro and Varian's increased-dominance assertion for new network markets insofar as it shows that increased dominance may not occur depending on the relative strength of two structural parameters. As we will see next, this conclusion does not extend to a market with similar structural parameters but displaying *reversible* market-wide preferences: in such a case, Shapiro and Varian's assertion of ever increasing dominance is restored.

4.2 Reversible vertical differentiation (unknown time-variable market-wide preferences)

We now address new network markets where the market-wide advantage initially enjoyed by one firm may be non-permanent. In order to compare this case with the one discussed previously, let one firm enjoy the same advantage in both periods, i.e., $z_1 = z_2 = z$. Then,

Proposition 2 *When one product enjoys a sustained though reversible market-wide preference, it* always *reinforces its market dominance.*

Proof From (21) and (22), equilibrium sales equal

$$\begin{aligned} x_1 &= \frac{1}{2} + \frac{z_1}{2t} \\ x_2 &= \frac{1}{2} + \frac{z_1 e}{6t\,(t-e)} + \frac{z_2}{2t}. \end{aligned}$$

If $z_1 = z_2 > 0$, and since $t > e$, then $x_1 < x_2$. Thus, if the market-wide valuation of the two goods is the same in both periods, i.e., if $z_1 = z_2$, the firm with the larger market share in the first period will always increase it in the following period. ■

In this case, learning the realization of z_1 does not yield any information concerning z_2. Both firms and consumers hold a nil prior on z_2 and expect second-period consumers' valuations $v_2(\cdot, z_2)$, based on their posteriors, to be distributed as they expected first-period consumers' valuations $v_1(\cdot, z_1)$, based on similarly distributed posteriors arising from an equally nil prior on z_1. One difference remains. Whereas initially both firms were on an equal footing regarding installed base, now the firm benefiting from a market-wide advantage in the first period starts the second one with a larger installed base. Thus, the leading firm obtains an even larger percentage of second-period consumers than it did of first-period ones.

4.3 Regular network markets (known time-invariant market-wide preferences)

We now address regular network markets and use them as a term of comparison for new network markets. To address regular network markets, in Appendix E we solve another variant of the model where the realization of z is common knowledge from the outset, and thus immediately observable by first-period consumers. This accounts for the possibility that, for example, reviews of the new products appearing in the press prior to their launch may make it apparent that one good is vertically better than the other. The main conclusion is that z being initially observable decreases the range of circumstances under which the firm that gains a larger installed base is able to increase its market share subsequently.

Proposition 3 *Increased market dominance is less likely in regular network markets (where market-wide preferences are common knowledge) than in new network markets (where market-wide preferences become common knowledge only after initial sales are observed).*

Proof Proposition 1's proof showed that increased market dominance occurred in new network markets with an irreversible market-wide preference iff $t < 2e$. Appendix E shows that increased market dominance occurs in regular network markets iff $t < 1.694e$. ■

Intuitively, when z is common knowledge from the outset, final sales of each good are known in advance by all consumers. Thus, the estimates of final network size (total sales) are the same for first- and second-period consumers. Therefore, there is no reason for the firm that obtains the greater market share in the first period to increase it in the final period due to the network effect. The reason why we may still have a positive trend in market share is firms' strategic pricing. To see this, suppose that we also impose that prices should be time invariant. Then, prices, as well as expected final sales, are the same in both periods, and so consumers will split between goods in the same manner in both periods. Therefore, each firm will have the same market share in both periods. In this case and despite the network effect, the firm that obtains the larger market share in the first period will neither increase it, nor decrease it in the following period.

4.4 Consumer fads

By their very nature, new network goods' market where market-wide preferences are reversible may by subject to consumer fads. By this we mean unanticipated market-wide preferences that prove fleeting: one product may initially be preferred by most consumers who, after a while, may prefer another one *without firms being able to anticipate such preferences and their swings.* The prevailing intuition would suggest that the firm that initially benefits from a consumer fad would fare better overall due to the network effect since it can make it apparent to late buyers that its installed base is bigger, whereas its competitor cannot benefit from a similar mechanism based on a favorable market-wide preference that will only materialize later on.[22] As we will see, when markets are subject to consumer fads, this intuition is only partial.

Consider a scenario where one firm benefits from a given market-wide advantage (favorable consumer fad) in the first period, whereas its opponent enjoys the same advantage in the second period. One concludes that even though the firm benefiting from the initial consumer fad ends up selling more than its opponent, surprisingly the latter fares better in terms of profit. Formally,

[22] This intuition is well summarized by the following quotation from Klemperer (2008): "Firms promoting incompatible networks compete to win the pivotal early adopters, and so achieve ex post dominance and monopoly rents. Strategies such as penetration pricing and pre-announcements (see, e.g., Farrell and Saloner (1986)) are common. History, and especially market share, matter because an installed base both directly means a firm offers more network benefits and boosts expectations about its future sales (...) late developers struggle while networks that are preferred by early pivotal customers thrive."

Proposition 4 *Let there be network effects, $e > 0$. Under the conditions of our model, when one firm benefits from a consumer fad in the first period while its opponent benefits from an equal-strength consumer fad in the second, the latter firm obtains a higher profit despite the fact that the first firm ends up selling more.*

Proof See Appendix F. ■

This result is predicated on the interplay of a quantity and a price effect. On the one hand, the firm that benefits from an early installed-base advantage arising from being initially preferred by consumers will attain higher overall sales because this firm's early sales result in a large installed base that is observable by late buyers, whereas the opponent firm cannot benefit from a similar installed-based effect when it benefits from a late consumer fad. Regarding total *quantity* sold, an early market-wide advantage is desirable insofar as it leads to higher sales. However, the firm that benefits from an early market-wide advantage ends up selling more in the first period *when penetration pricing is depressing prices*, whereas its opponent, benefiting from a late market-wide advantage, sells more when the market is mature and prices are higher. This pricing effect overcomes the quantity effect described above, and so a firm that benefits from a late advantage in market-wide preferences ends up faring better than the its opponent. In sum, in new network markets subject to consumer fads, the firm benefiting from such a fad in the mature phase of the industry may earn a higher profit than a competitor benefiting from an equal-strength fad in the launch phase.

As pointed out, this result deserves mention because of its counterintuitive nature. Its robustness with respect to other model specifications deserves further investigation. For instance, our modeling does not involve discounting, a fact that implicitly increases the relative importance of second-period profits. Moreover, we have assumed that exactly half the market buys initially at low (penetration) prices whereas the other half buys subsequently at high (ripoff) prices. Other partitions would impact the result not only quantitatively but presumably also qualitatively. Obviously, for low discounting and consumer partitions close to parity, the result would still go through due to continuity arguments. In sum, further analysis of this issue seems to be useful.

5 Social welfare

By studying how an omniscient and benevolent social planner would allocate goods to consumers, we can compare market allocations with the socially optimal one. As before, the reader who wishes to concentrate solely on results and their intuition can retain the characterization of the social-welfare maximizing allocation of goods to consumers featured in (28) and proceed to the next subsection.

The social welfare resulting from an allocation of goods to consumers, (x_1, x_2), is given by

$$\begin{aligned} W \quad = \quad & \underbrace{(x_1 + x_2)\, z}_{\text{vertical differentiation}} + \underbrace{t x_1 - t x_1^2 + t x_2 - t x_2^2}_{\text{horizontal differentiation}} + \\ & + \underbrace{e\,(x_1 + x_2)^2 + e\,[2 - (x_1 + x_2)]^2}_{\text{network effects}} + 2C. \end{aligned} \tag{25}$$

To understand this expression, begin by recalling that first-period consumers obtain a payoff of

$$C + v\left(x_1, z\right) + e\left(x_1 + x_2\right) - p_1^A$$

if they consume good A, or

$$C + e\left(2 - \left(x_1 + x_2\right)\right) - p_1^B$$

if they opt for good B. Similar expressions apply to second-period consumers.

It is easy to see that, in both periods, the social-welfare maximizing allocation must be such that all consumers assigned to A must be the ones lying closest to its location, in which case consumers assigned to B are also those located closest to it. Otherwise one could reduce horizontal-differentiation welfare costs by relocating consumers without changing the measure of consumers assigned to each network (i.e., $x_1 + x_2$), thus keeping constant the value of the welfare terms associated with vertical differentiation (because the measure of consumers benefiting from the better vertically-differentiated product would stay constant) and network effects (because the measure of consumers assigned to each good would not vary).

First-period consumers opting for good A altogether obtain a payoff arising from $v\left(x_1, z\right)$ amounting to

$$\int_0^{x_1} v\left(x, z\right) \mathrm{d}x = \int_0^{x_1} \left(t - 2tx + z\right) \mathrm{d}x = \left. tx - tx^2 + zx \right]_0^{x_1} = tx_1 - tx_1^2 + zx_1.$$

A similar expression applies to second-period consumers, giving rise to the first two terms in (25) measuring the impact of vertical and horizontal differentiation on welfare. The first one simply says that consumers opting for A benefit (or suffer) from the vertical-differentiation gain (loss) yielded by a positive (negative) realization of z. The second term, associated with horizontal differentiation, is also intuitive if one minimizes it with respect to both variables and notes that the minimum is reached when $x_1 = x_2 = \frac{1}{2}$, i.e., horizontal differentiation costs are minimized if consumers are equally split between goods in both periods. Moreover, note that a measure of consumers $x_1 + x_2$ who opt for A *each* obtains $e\left(x_1 + x_2\right)$ through the network effect while, similarly, each of those who opt for B obtains $e\left(2 - \left(x_1 + x_2\right)\right)$. This gives rise to the third term in (25). Also, all consumers obtain C regardless of which good they buy. This, in turn, gives rise to the fourth term in (25). Finally, since we have assumed unit demand and full coverage, prices are purely a transfer from consumers to firms devoid of any impact on social welfare.

The partial derivatives of W with respect to x_1 and x_2 are

$$\begin{aligned} \frac{\partial W}{\partial x_1} &= z + t - 2tx_1 + 4e\left(x_1 + x_2\right) - 4e \\ \frac{\partial W}{\partial x_2} &= z + t - 2tx_2 + 4e\left(x_1 + x_2\right) - 4e. \end{aligned} \tag{26}$$

Take (26) and note that a symmetric allocation $x_1 = x_2$ constitutes a solution of the problem at hand if an interior solution exists, i.e., $0 < x_1, x_2 < 1$, as well as if it does not, in which case $x_1 = x_2 = 0$ or $x_1 = x_2 = 1$. Hence, we may write $x_1 = x_2 = x$ and simply study

$$\begin{aligned} \frac{\partial W}{\partial x_1} = \frac{\partial W}{\partial x_2} &= z + t - 2tx + 8ex - 4e \\ &= z + \left(2x - 1\right)\left(4e - t\right). \end{aligned} \tag{27}$$

It is easy to see that when $z > 0$, one must have $x_1 = x_2 \in \left[\frac{1}{2}, 1\right]$. To see it, assume, to the contrary, that $x_1 = x_2 < \frac{1}{2}$ characterizes the social-welfare maximizing allocation when $z > 0$. Then, the allocation $(1 - x_1, 1 - x_2)$ would yield exactly the same network-effect benefits and horizontal differentiation costs while allowing a larger measure of consumers to benefit from the better (vertically-differentiated) network. Thus, from now on, we will analyze the case $z > 0$, which restricts the socially optimal values of x_1 and x_2 to the interval $\left[\frac{1}{2}, 1\right]$. The case $z < 0$ is similar, *mutatis mutandis*.

We are now ready to compute the socially optimal allocation of consumers to networks. If $4e - t \geq 0$, from (27) we have $\frac{\partial W}{\partial x_i} > 0, \forall x_i \in \left[\frac{1}{2}, 1\right]$ with $i = 1, 2$. Hence, social welfare is maximized when $x_1 = x_2 = 1$, i.e., all consumers belong to the network benefiting from a vertical differentiation advantage. Intuitively, when network effects, which require that consumers all belong to the same network, are strong enough *vis-à-vis* horizontal-differentiation welfare costs, which require that consumers split up, social welfare is maximized when all consumers are allocated to the same network. Which one? The network benefiting from a positive realization of z, i.e., the one that is (vertically) better.

Take the case $4e - t < 0$, i.e., $t > 4e$. Two sub-cases arise: either (i) $0 \leq t - 4e \leq z$ or (ii) $t - 4e > z$. In sub-case (i), simple computations involving (27) show that, similarly to the previous paragraph, $\frac{\partial W}{\partial x_i} > 0, \forall x_i \in \left[\frac{1}{2}, 1\right)$ with $i = 1, 2$. Again, social welfare is maximized when $x_1 = x_2 = 1$, i.e., when all consumers belong to the network benefiting from a vertical differentiation advantage. Here, the strength of the network effects *together* with the difference in (vertical) quality between the two goods *vis-à-vis* the strength of the horizontal-differentiation costs makes it optimal to assign all consumers to one network. In sub-case (ii), we reach an interior solution for the social-welfare maximization problem, $\frac{\partial W}{\partial x_i} = 0, i = 1, 2$, in which case one has $x_1 = x_2 = \frac{1}{2} + \frac{z}{2t - 8e}$.[23] In contrast with the previous cases, here horizontal-differentiation welfare costs are so marked that society is better off when consumers with a significant preference for the worse good buy it even though they form a small network.

In sum, the social-welfare maximizing allocation of consumers to networks, (x_1, x_2), is as follows:[24]

$$x_1 = x_2 = \begin{cases} 1 & t - 4e \leq z \\ \dfrac{1}{2} + \dfrac{z}{2t - 8e} & t - 4e > z. \end{cases} \tag{28}$$

[23]From (26) we have

$$\frac{\partial^2 W}{\partial x_1^2} = 4e - 2t < 0$$
$$\frac{\partial^2 W}{\partial x_2^2} = 4e - 2t < 0$$
$$\frac{\partial^2 W}{\partial x_1 \partial x_2} = 4e > 0,$$

where the first two inequalities arise from the fact that $t > 4e$. Moreover, $\frac{\partial^2 W}{\partial x_1^2} \frac{\partial^2 W}{\partial x_2^2} = (4e - 2t)^2 = (2t - 4e)^2 > (8e - 4e)^2 = (4e)^2 = \left(\frac{\partial^2 W}{\partial x_1 \partial x_2}\right)^2$, where again we have made use of the fact that $t > 4e$. Hence, the second-order conditions for a maximum are fulfilled.

[24]In the case where good A benefits from a market-wide preference, i.e., $z > 0$.

5.1 Results

One must compare the market equilibria arising in new network markets (both when market-wide preferences are immutable and when they can vary) with the socially-optimal allocation of goods to consumers.

Proposition 5 *The least-preferred good obtains a larger market share than is socially optimal, both when one good enjoys a time-invariant market-wide preference and when market-wide preferences may vary and one good enjoys the same market-wide preference in both periods. Moreover, this social-welfare sub-optimality is (weakly) greater when market-wide preferences are time invariant.*

Proof See Appendix F. ■

This result is easy to understand. Product-specific network effects give rise to an externality since consumers do not take into account the welfare loss that they impose on the remaining consumers when deciding which good to acquire, namely when they opt for a good bought by a minority of consumers rather than the one that is favored by most. Hence, the market-wide less-preferred good ends up being sold to too many consumers from a social welfare viewpoint. Why is this issue (weakly) augmented when market-wide preferences are time invariant? In this case, as seen above, which good benefits from a market-wide preference becomes known after the first period in the case of a time-invariant market-wide preference, prompting (i) consumers to flock to the better (vertically-differentiated) good and (ii) firms to price accordingly. These two effects run counter to each other as far as second-period sales of the market-wide preferred firm are concerned. In the context of our model, the net result of these two effects is a social-welfare reduction compared to the case where they are not present because market-wide preferences may vary over time. Hence, the conclusion that social-welfare sub-optimality is (weakly) greater when preferences are time invariant.[25]

One may wonder about the extent to which the previous results are attributable to the fact that we are dealing with new network goods. For regular network goods, too, the least-preferred good attracts too many buyers from a social-welfare viewpoint.

Proposition 6 *The least-preferred good obtains in both periods a larger market share than is socially optimal when market-wide preferences are common knowledge from the outset.*

Proof See Appendix F. ■

All the effects associated with newness of network goods' markets are absent in this case. Thus, this result arises solely due to the externality mentioned before: consumers do not take into account the welfare loss imposed on the majority of consumers when they buy the market-wide less-preferred good.

The last two propositions make it instant that one compares the extent of the social-welfare sub-optimality of regular and new network goods. We do so next while including the proofs in the main text because they contain important intuitive reasoning involving the driving forces that shape the welfare performance of network goods' markets.

[25]See below for a detailed discussion of these countervailing effects.

Proposition 7 *When network effects are weak, social welfare is maximal when network goods are new and market-wide preferences can vary, intermediate when they are fixed and minimal when they are known from the outset, i.e., in the case of regular network goods.*

Proof Begin by considering a scenario without network effects, $e = 0$. From (28), the socially-optimal allocation of consumers to networks equals

$$x_1 = x_2 = \frac{1}{2} + \frac{z}{2t}.$$

Intuitively, social welfare is maximized when both goods sell the same quantity, $\frac{1}{2}$, in each period if z's realization equals 0, because neither good is vertically better than the other and splitting consumers equally between goods minimizes horizontal-differentiation welfare costs. On the other hand, when $z \neq 0$, the good that proves to be better should attain sales in excess of $\frac{1}{2}$ by the amount $\frac{z}{2t}$. Intuitively, when $z \neq 0$ there is a tradeoff between having more consumers buying the better (vertically-differentiated) good and thus benefiting from a welfare increase of z as a result of doing so, and these very same consumers suffering increased horizontal-differentiation welfare costs, proportional to t, as a result of consuming a good that is less to their idiosyncratic liking. This tradeoff is optimally balanced when a measure $\frac{z}{2t}$ of consumers in excess of $\frac{1}{2}$ consume the better (vertically-differentiated) product.

Now, take the case of time-invariant market-wide preferences. From (16) and (17), we have

$$\begin{aligned} x_1 &= \frac{1}{2} + \frac{z}{2t} \\ x_2 &= \frac{1}{2} + \frac{z}{6t}. \end{aligned}$$

In this case, in the first period, consumers are optimally divided between goods whereas in the second-period too few consumers are assigned to the better (vertically-differentiated) good. Why? In the first-period, both firms charge the same price since they share the same prior on market-wide preferences, $E[z] = 0$. As such, consumers split between the two goods on the basis of their relative preference for either one, namely, by taking into account their privately-observed $v(\cdot, z)$. Thus, they privately weight their choice of which good to buy as would a benevolent dictator, therefore reaching the socially-optimal outcome. However, in the second-period, firms already know the realization of z and their second-period pricing reflects this: the firm benefiting from a market-wide preference increases its price and its opponent lowers its. This distorts consumers' choices away from the social optimum, inducing them to buy less of the better (vertically-differentiated) good.

Consider now the case of new network goods with time-variant market-wide preferences. From (21) and (22), we have

$$\begin{aligned} x_1 &= \frac{1}{2} + \frac{z}{2t} \\ x_2 &= \frac{1}{2} + \frac{z}{2t}. \end{aligned}$$

Now, even in the second period, socially-optimal quantities of both goods are bought. Why? Once the second-period begins, firms again must choose price on the basis of their prior on market-wide preferences, $E[z_2] = 0$, rather than their knowledge of the realization of z_1 (as in the previous case). Hence, they charge the same price in the second period despite the asymmetric installed base, which is rendered irrelevant to second-period pricing decisions by the absence of network effects.[26] This, in turn, implies that consumers again make their choice of which good to buy on the basis of $v(\cdot, z_2)$, a choice aligned with that of a social planner.

Finally, in the case of regular network goods, market-wide preferences are common knowledge from the outset. From (23) and (24), one has

$$\begin{aligned} x_1 &= \frac{1}{2} + \frac{z}{6t} \\ x_2 &= \frac{1}{2} + \frac{z}{6t}. \end{aligned}$$

Here the pricing-induced distortion affecting the second-period of the time-invariant market-wide preferences' case is present in both periods. Hence, the socially sub-optimal equilibrium quantities.

Thus, we conclude that new "network" goods (involving immutable as well as time-variable market-wide preferences) yield higher social welfare than regular "network" goods, social welfare being maximal when market-wide preferences may vary. Finally, continuity on e of the equilibrium quantities and the social-welfare maximizing allocation of consumers to goods, yields the conclusion that this result also applies when network effects are weak, $e \gtrapprox 0$. ■

We now consider the case when network effects are not weak.

Proposition 8 *When network effects are strong, social welfare in new network goods is higher when market-wide preferences can vary than when they are fixed.*

Proof Suppose that $e > 0$ and consider the socially optimal allocation of goods to consumers. The stronger are network effects, the more consumers it is socially optimal to assign to the good benefiting from a market-wide preference. This much underlies the term $8e$ in the socially-optimal allocation $x_1 = x_2 = \frac{1}{2} + \frac{z}{2t-8e}$ if $e > 0$ and $x_1 = x_2 = 1$ if $e \gg 0$. In plain words, the emergence of network effects makes it socially optimal to allocate more, or even all, consumers to the market-wide preferred good.

Take market allocations. In the case of time-invariant as well as time-variable market-wide preferences, first-period consumers will be unaffected in their choices by the emergence of network effects' considerations. Why? On the one hand, firms' pricing, though affected by the emergence of network effects (see (13) and (18)), remains symmetric, i.e., even though both firms reduce the price they charge to (try to) increase their installed base at the end of the first period, they do so by the same amount. Thus, consumers will not change their choices on account of prices *vis-à-vis* the case without network effects. Moreover, by directly considering network effects, consumers either reinforce their decision of which good to

[26] This is the one instance where the model collapses to a sequence of totally unrelated markets involving two cohorts of consumers. In this case, neither network effects, nor learning generate interactions between periods, while in all other cases one or both of these factors relate them.

buy (this being the case of "optimistic" and "pessimistic" consumers who have observed "extreme" values of $v(\cdot, z)$) or see no reason to change it ("middle grounders"). Hence, the first-period equilibrium quantities are unaffected by the emergence of network effects.

On the contrary, second-period equilibrium quantities will be affected by the emergence of network effects through three channels. (i) On the one hand, the good that benefited from a market-wide preference in the first period benefits from an asymmetric installed base which, due to the network effect, increases its second-period demand and reduces its opponent's. This effect is present regardless of whether market-wide preferences are immutable or not. (iia) On the other hand, in the case of immutable market-wide preferences, second-period consumers know which good benefits from a market-wide preference and flock towards it. Moreover, (iib) because the firm benefiting from a market-wide preference in the first period knows that it will also benefit from the same advantage in the second period, its pricing will be less aggressive. By the same token, its opponent's will be more so. Effects (iia) and (iib) countervail each other. *Thus, in contrast to all the previous effects, whose impact on equilibrium sales was unequivocal, market-wide preferences becoming common knowledge may either increase or decrease second-period quantity sold compared to the case where market-wide preferences may vary.* In the latter case, only effect (i) is present, and second-period sales of the market-wide preferred firm exceed first-period ones (Proposition 2). In the case of immutable market-wide preferences, effects (iia) and (iib) are additionally present and the leading firm may sell either more or less in the second period than it did in the first one (Proposition 1).

One can quantify these three effects by comparing second-period sales when market-wide preferences are immutable, as given by (17),

$$x_2 = \frac{1}{2} + \frac{ze}{6t(t-e)} + \frac{z}{6(t-e)},$$

with the case when they can vary, as given by (22),

$$x_2 = \frac{1}{2} + \frac{z_1 e}{6t(t-e)} + \frac{z_2}{2t},$$

while bearing in mind the case $e = 0$. The terms $\frac{ze}{6t(t-e)}$ and $\frac{z_1 e}{6t(t-e)}$ are similar, reflecting the fact that a larger installed base benefits the firm that obtained a market-wide preference in the first period, regardless of whether that advantage is permanent or not. These terms capture effect (i) described above.

When market-wide preferences are time variable, effects (iia) and (iib) are absent. Hence, firms approach competition for second-period consumers on the basis of a common null prior concerning z_2. In this case, second-period consumers are disputed as first-period ones were, as the term $\frac{z_2}{2t}$ indicates. To see it, recall that first-period sales equal $x_1 = \frac{1}{2} + \frac{z_1}{2t}$ and note the similarity between $\frac{z_1}{2t}$ and $\frac{z_2}{2t}$. Hence, second-period sales of the market-wide preferred firm necessarily increase (because of effect (i)) and more second-period consumers end up buying the market-wide preferred good, a social-welfare increasing change.

Consider the case when market-wide preferences are time invariant. Effects (iia) and (iib) are present. Hence, they account for the difference between $\frac{z_2}{2t}$ observed in the case of time-variant market-wide preferences and the present case where the corresponding term is

$\frac{z}{6(t-e)}$. The difference between these two expressions can be decomposed into two terms. First, the ratio $\frac{1}{6}$ appears instead of $\frac{1}{2}$ as a result of the less-aggressive pricing of the market-wide preferred firm and the more aggressive pricing of its opponent *excluding the impact on consumers' decisions of their consideration of network effects as a result of market-wide preferences having become common knowledge.* To see it, simply compare the two expression while assuming that network effects are nil, $e = 0$ and recall the previous proposition's proof. Second, when this impact is factored in, the ratio $\frac{1}{t-e}$ emerges instead of $\frac{1}{t}$, reflecting the fact that some consumers now opt for the market-wide preferred good in spite of their idiosyncratic preference for the other good. The fact that the two ratios' changes are opposite in sign implies that equilibrium second-period sales of the market-wide preferred firm may be smaller or larger than those observed in the first period (as Proposition 1 states). In the former case, the social-welfare sub-optimality resulting from the excessive sales of the worse vertically-differentiated good is augmented and so, unequivocally, new network goods' markets perform worse when market-wide preferences are time invariant than when they can vary. In the latter case, one must compare second-period sales of the market-wide preferred firm under time varying and time invariant market-wide preferences, i.e., one must compare $2t$ with $6\,(t-e)$. The former is less than the latter the latter for $t > 1.5e$ and, recalling our assumption that $t > 1.623e$, one concludes that, in the conditions of our model, second-period sales of the leading firm when market-wide preferences may vary over time exceed those prevailing under time invariance. Hence, under the conditions of our model, socially sub-optimal sales of the market-wide preferred firm is more pronounced when market-wide preferences are fixed. ■

Regrettably, one cannot compare the social-welfare performance of new network goods' markets with their regular counterpart when network effects are strong. To see it note that when market-wide preferences are known from the outset, effects (iia) and (iib) are present not only in the second but also in the first period. Moreover, effect (iia) reinforces itself across periods because increased sales in each period brought about the fact that market-wide preferences are common knowledge from the outset leads more consumers in the other period to buy the market-wide preferred good due to the network effect. This, in turn, makes effect (iib) stronger in both periods. These facts make it impossible to compare regular network goods' equilibrium quantities with their counterparts for new network goods when network effects are strong, as visual comparison of (16) and (17), and (21) and (22) with (23) and (24) suggests.

6 Conclusion

We developed a model of what we have termed "new network markets," i.e., a differentiated-goods model of a market with network effects and consumers' and firms' initial uncertainty concerning consumers' overall valuation of the goods that becomes resolved as sales data accumulate. We show that the firm that obtains the larger market share in the first period increases its market share in the last period if and only if the network effect is significant enough compared to the degree of product differentiation, as long as market-wide preferences are time invariant (irreversible vertical differentiation). Strikingly, if market-wide preferences can vary over time (reversible vertical differentiation), then the firm with a larger installed base will always reinforce its lead if it keeps enjoying the same market-wide preference.

The idea that in a market with network effects, the firm that obtains a larger market share in the initial period tends to subsequently increase its dominance is widely held. We qualify this observation by showing that it is not always true, depending on the relative strength of the network effect *vis-à-vis* product differentiation, as well as whether market-wide advantages (vertical differentiation) are irreversible or not. The latter qualification underscores the importance of apparently minor industry-structure details in determining the industry's long-run path toward or away from monopolization. Also, we show that uncertainty over market-wide preferences enlarges the set of circumstances under which leaders amplify their market-share advantage.

The version of the model allowing for variable market-wide preferences allows for the study of consumer fads, i.e., fleeting market-wide preferences that agents cannot anticipate. On the one hand, the firm that initially benefits from consumers' preferences sells more overall than a competitor benefiting from a similar consumer fad at a latter stage. However, this favorable quantity effect may be overcome by a price effect: the initially-preferred firm makes the bulk of its sales at the first-period (bargain) price whereas its competitor sells mostly at the second-period (ripoff) prices. This result is important because it shows that in network markets subject to consumer fads, contrary to intuition, benefiting from a late fad may be better than benefiting from an earlier one. Whether this result is robust to other model specifications seems to be a topic worth analyzing.

We also show that the least-preferred good obtains too many sales from a social-welfare viewpoint in new network markets. Moreover, this sub-optimality is generally more serious when market-wide preferences are time invariant, i.e., when late consumers' market-wide preferences become common knowledge. Also, by studying regular network markets where market-wide preferences are known from the outset, we are able to show that these generate less welfare than new network markets if network effects are relatively unimportant, a result that does not necessarily apply when network effects are strong.

In our model, uncertainty concerning market-wide preferences is resolved immediately after the first period: half the consumers (period-1 early buyers) buy before market-wide preferences become common knowledge whereas the other half (period-2 late buyers) do so fully informed. In reality, we would expect that information concerning sales (and, thus, market-wide preferences) would percolate before fifty percent of potential consumers have purchased, but also that many late buyers would pick a good while still not knowing which product is actually favored by the majority of consumers—either because they do not follow sales data, talk to friends about hot products that everyone seems to be acquiring or for other such reason. A more realistic scenario would involve the sequential entry of successive cohorts of consumers, in each co-existing consumers who are aware of market-wide preferences with those who are not. Our modeling avoids these complications in favor of tractability.

Appendix A

In this appendix we show that a unique and stable equilibrium without bunching of all consumers on a good exists if and only if $t > 1.577e$, i.e., iff the degree of product differentiation is large enough compared to the intensity of the network effect.

For expositional clarity, we begin by showing that in a model with only one period, a unique and stable equilibrium without full bunching exists if and only if $t > e$.[27] The result for the two-period model in the main text then follows easily by analogy. In this appendix, we ignore the dependency of $\tilde{x}_1$ and $\tilde{x}_2$ on $v(\cdot, z)$ since this dependency plays no role in the argument.

In a one-period model, the indifferent consumer is given by

$$C - tx_1 + z + e\tilde{x}_1 - p^A = C - t(1 - x_1) + e(1 - \tilde{x}_1) - p^B,$$

from which we obtain the following demand function

$$x_1 = \frac{p^B - p^A + z + t - e}{2t} + \frac{e}{t}\tilde{x}_1. \tag{A.1}$$

A consumer's estimate of x_1 is then given by:

$$\tilde{x}_1 = \frac{p^B - p^A + E[z|v(\cdot, z)] + t - e}{2t} + \frac{e}{t}\tilde{x}_1 \tag{A.2}$$

$$= \frac{1}{2} + \frac{p^B - p^A + E[z|v(\cdot, z)]}{2(t - e)}. \tag{A.3}$$

If $t < e$, the intermediate expectation of x_1 given by equation (A.3), namely $0 < \tilde{x}_1 < 1$, is not the only one possible. Two other extreme expectations concerning x_1, namely $\tilde{x}_1 = 0$ and $\tilde{x}_1 = 1$, can consistently be entertained by consumers as part of an equilibrium. This is so because $t < e$ implies that all consumers—including those located at the far-off end of the horizontal-differentiation line—attach a higher value to buying the same good as do all other consumers rather than their idiosyncratically preferred good. In this case, equilibria involving complete bunching on a good may occur.

Moreover, the intermediate equilibrium is unstable when $t < e$. If consumers hold an expectation slightly different from that given by (A.3), they will all buy one good. Equation (A.1) makes this clear if one notes that $t < e \Rightarrow \frac{e}{t} > 1$—the latter being the coefficient affecting $\tilde{x}_1$ on the r.h.s. of (A.1)—implies $\frac{\partial x_1}{\partial \tilde{x}_1} > 1$.

The extreme cases—in which all consumers are driven by the network effect to coordinate on consuming the same good—are tantamount to having no product differentiation at all.

We now consider the two-period model treated in the main text. Here, first-period consumers take into consideration the impact of their decisions on their second-period counterparts. The condition for a unique and stable intermediate equilibrium is now more demanding since an increase in the expected value of x_1 leads to an increase in the expected value of x_2 due to the network effect. This, in turn, leads to an increase of the expected value of x_1. Thus, the incentives for all consumers to choose the same good are stronger, and so the condition for a unique and stable intermediate equilibrium is more demanding.

[27] This is also the relevant interval in a model with two periods in which first-period consumers do not take into account the impact of their decisions on second-period consumers.

The first-period indifferent consumer is determined by

$$C - tx_1 + z + e(\tilde{x}_1 + \tilde{x}_2) - p_1^A = C - t(1 - x_1) + e(2 - (\tilde{x}_1 + \tilde{x}_2)) - p_1^B,$$

from which we obtain

$$x_1 = \frac{p_1^B - p_1^A + z + t - 2e + 2e(\tilde{x}_1 + \tilde{x}_2)}{2t},$$

and finally

$$\tilde{x}_1 = \frac{p_1^B - p_1^A + E[z|v(\cdot,z)] + t - 2e + 2e\tilde{x}_2}{2(t-e)}. \tag{A.4}$$

Equation (6) in the main text states that

$$\tilde{x}_2 = \frac{t - \frac{4}{3}e + \frac{1}{3}E[z|v(\cdot,z)] + \frac{2}{3}e\tilde{x}_1}{2(t-e)}.$$

Replacing it in (A.4), we obtain

$$\tilde{x}_1 = \frac{p_1^B - p_1^A + E[z|v(\cdot,z)] + t - 2e + 2e\frac{t - \frac{4}{3}e + \frac{1}{3}E[z|v(\cdot,z)]}{2(t-e)}}{2(t-e)} + \frac{\frac{4}{3}e^2}{4(t-e)^2}\tilde{x}_1.$$

Now, analogously to (A.2), the intermediate equilibrium is unique and stable iff the coefficient affecting $\tilde{x}_1$ on the r.h.s. of the previous equality is less than 1, i.e., $\frac{\frac{4}{3}e^2}{4(t-e)^2} < 1$. This is the case iff $t < 0.423e$ or $t > 1.577e$.[28] Hence, a unique and stable equilibrium without bunching of all consumers on a good exists if and only if $t > 1.577e$.

Appendix B

In this appendix we show that second-period consumers and firms deduce the realization of z upon observing x_1^*. Recall that first-period demand equals

$$x_1 = \frac{p_1^B - p_1^A + z + t - 2e + 2e(\tilde{x}_1(v(x_1,z)) + \tilde{x}_2(v(x_1,z)))}{2t}. \tag{B.1}$$

From (B.1), a first-period consumer who has observed realization $v(\cdot,z)$, takes first-period demand as being given by

$$x_1 = \frac{p_1^B - p_1^A + z + t - 2e + 2e(\tilde{x}_1(v(\cdot,z)) + \tilde{x}_2(v(\cdot,z)))}{2t}. \tag{B.2}$$

From (B.2), the estimate of x_1 by a first-period consumer who has observed realization $v(\cdot,z)$ equals

$$\tilde{x}_1(v(\cdot,z)) \equiv E[x_1|1,v(\cdot,z)] = \frac{p_1^B - p_1^A + E[z|1,v(\cdot,z)] + t - 2e + 2e\tilde{x}_2(v(\cdot,z))}{2(t-e)}, \tag{B.3}$$

where $E[a|1,v(\cdot,z)]$ denotes the expected value of random variable a by a first-period consumer who has observed realization $v(\cdot,z)$.

[28] The very same conclusion can be obtained by solving the whole model and noting that the expression $3t^2 - 6te + 2e^2$ appears in the denominator of the terms determining $\tilde{x}_1$ and $\tilde{x}_2$, where it plays a role akin to $t - e$ in (A.3) above. Than, by checking that $3t^2 - 6te + 2e^2$ is convex and the roots of $3t^2 - 6te + 2e^2 = 0$ are 0.423 and 1.577, we conclude that $3t^2 - 6te + 2e^2 > 0$ for $t < 0.423$ and $t > 1.577$.

A second-period indifferent consumer is such that

$$\begin{aligned} C + a\left(x_2\right) + z + e\left(x_1^* + E\left[x_2|2, v\left(x_2, z\right)\right]\right) - p_2^A &= \\ = C + e\left(2 - \left(x_1^* + E\left[x_2|2, v\left(x_2, z\right)\right]\right)\right) - p_2^B, \end{aligned}$$

where $E\left[a|2, v\left(\cdot, z\right)\right]$ denotes the expected value of random variable a by a second-period consumer who has observed realization $v\left(\cdot, z\right)$. Thus, the second-period demand curve equals

$$x_2 = \frac{p_2^B - p_2^A + z + 2eE\left[x_2|2, v\left(x_2, z\right)\right] + t - 2e + 2ex_1^*}{2t}.$$

Hence, a second-period consumer who has observed realization $v\left(\cdot, z\right)$, takes second-period demand as being given by

$$x_2 = \frac{p_2^B - p_2^A + z + 2eE\left[x_2|2, v\left(\cdot, z\right)\right] + t - 2e + 2ex_1^*}{2t}. \tag{B.4}$$

Thus, for such a consumer, expected second-period demand is given by

$$E\left[x_2|2, v\left(\cdot, z\right)\right] = \frac{p_2^B - p_2^A + E[z|2, v\left(\cdot, z\right)] + t - 2e + 2ex_1^*}{2\left(t - e\right)}. \tag{B.5}$$

Substituting (B.5) in (B.4), we obtain

$$x_2 = \frac{p_2^B - p_2^A + z + t - 2e + 2ex_1^*}{2\left(t - e\right)} + \frac{eE\left[z|2, v\left(\cdot, z\right)\right] - ez}{2t\left(t - e\right)}. \tag{B.6}$$

Assume that first-period consumers act based on the expectation that second-period consumers correctly infer z after observing x_1^*, i.e., that $E\left[z|2, v\left(j, z\right)\right] = z, \forall j \in [0, 1]$.[29] Then, (B.6) collapses to

$$x_2 = \frac{p_2^B - p_2^A + z + t - 2e + 2ex_1^*}{2\left(t - e\right)}.$$

First-period consumers need to compute the expected value of x_2:

$$\begin{aligned} \tilde{x}_2\left(v\left(\cdot, z\right)\right) &= E\left[x_2|1, v\left(\cdot, z\right)\right] = \\ &= \frac{E\left[p_2^B \middle| 1, v\left(\cdot, z\right)\right] - E\left[p_2^A \middle| 1, v\left(\cdot, z\right)\right] + E\left[z|1, v\left(\cdot, z\right)\right]}{2\left(t - e\right)} + \\ &\quad + \frac{t - 2e + 2e\tilde{x}_1\left(v\left(\cdot, z\right)\right)}{2\left(t - e\right)}. \end{aligned} \tag{B.7}$$

From (4) in the main text, we have

$$E\left[p_2^A \middle| 1, v\left(\cdot, z\right)\right] = \frac{1}{3}E\left[z|1, v\left(\cdot, z\right)\right] + t + \frac{2}{3}e\tilde{x}_1\left(v\left(\cdot, z\right)\right) - \frac{4}{3}e \tag{B.8}$$

$$E\left[p_2^B \middle| 1, v\left(\cdot, z\right)\right] = -\frac{1}{3}E\left[z|1, v\left(\cdot, z\right)\right] + t - \frac{2}{3}e - \frac{2}{3}e\tilde{x}_1\left(v\left(\cdot, z\right)\right). \tag{B.9}$$

By solving the equation system formed by (B.3), (B.7), (B.8) and (B.9), we conclude that

$$\tilde{x}_1\left(E\left[z|1, v\left(\cdot, z\right)\right], t, e, p_1^A, p_1^B\right),$$

and

$$\tilde{x}_2\left(E\left[z|1, v\left(\cdot, z\right)\right], t, e, p_1^A, p_1^B\right).$$

[29] Note that this implies that second-period consumers do not use their private signal, $v\left(j, z\right)$, to deduce the realization of z. All they need to know, besides structural parameters, are first-period sales.

By replacing these expressions in (B.1), we obtain

$$\begin{aligned} x_1 &= \frac{p_1^B - p_1^A + z + t - 2e}{2t} + \\ &+ \frac{2e\left\{\tilde{x}_1\left(E\left[z|1, v\left(\cdot, z\right)\right], t, e, p_1^A, p_1^B\right) + \tilde{x}_2\left(E\left[z|1, v\left(\cdot, z\right)\right], t, e, p_1^A, p_1^B\right)\right\}}{2t}. \end{aligned}$$

Appendix C shows that first-period *indifferent* consumers are such that their posterior after observing their realization of $v\left(x_1, z\right)$, namely $E\left[z|1, v\left(x_1, z\right)\right]$, equals their prior, $E\left[z\right] = 0$, in a symmetric equilibrium, a fact known to second-period consumers as, again, Appendix C makes plain. Thus, we have

$$x_1 = \frac{p_1^B - p_1^A + z + t - 2e + 2e\left\{\tilde{x}_1\left(0, t, e, p_1^A, p_1^B\right) + \tilde{x}_2\left(0, t, e, p_1^A, p_1^B\right)\right\}}{2t}. \tag{B.10}$$

Solving the system of equations formed by (2) and (6) yields

$$\begin{aligned} \tilde{x}_1 &= \frac{1}{2} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right) + E\left[z|v\left(\cdot, z\right)\right]\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2} \\ \tilde{x}_2 &= \frac{1}{2} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right) + E\left[z|v\left(\cdot, z\right)\right]t}{3t^2 - 6te + 2e^2}, \end{aligned}$$

which, for a consumer such that $E\left[z|v\right] = 0$ and a symmetric equilibrium, $p_1^A = p_1^B$, yields $\tilde{x}_1\left(0, t, e, p_1^A, p_1^B\right) = \tilde{x}_2\left(0, t, e, p_1^A, p_1^B\right) = \frac{1}{2}$, i.e., an indifferent first-period consumer holding a posterior of 0 for z estimates final sales as being equal for both goods in a symmetric equilibrium. Thus, (B.10) collapses to

$$x_1 = \frac{p_1^B - p_1^A + z + t}{2t}. \tag{B.11}$$

Finally, a symmetric equilibrium, $p_1^A = p_1^B$, yields

$$x_1 = \frac{z + t}{2t}. \tag{B.12}$$

It is clear from (B.12) that x_1 is monotone in z. Hence, by observing first-period sales, x_1^*, second-period consumers do infer the realization of $z = 2tx_1^* - t$. So do firms by following this very same reasoning. To see it, note that even though second-period consumers do receive a private signal—their realization of $v\left(\cdot, z\right)$—whereas firms do not, second-period consumers do not make use of it in deducing z.

Appendix C

Determination of $E\left[z|v\left(\cdot, z\right)\right]$

From

$$\begin{aligned} &v = a + z \\ &a \rightsquigarrow U\left(-t, t\right) \\ &z \rightsquigarrow U\left(-w, w\right), \end{aligned}$$

we have that v is itself a random variable with support $[-t-w, t+w]$. Moreover, it was also assumed in the main text that $t > w$.

Divide the support of v into three intervals.

(i) Intermediate values: $v \in [-t+w, t-w]$.

When $v \in [-t+w, t-w]$, for a given value of v, variable z can assume all values in the interval $[-w, w]$. Also, for a given value of v, to each value of z corresponds a unique value of a.[30] Since a and z are both uniformly distributed random variables, we conclude that for each value of v, variable z can assume all values in its support with the same probability. Therefore, the density function of z, given the realization of v, is

$$f[z|v] = \frac{1}{w-(-w)}, \qquad -w \leq z \leq w.$$

Thus, the posterior density function of z once a given value of $v(\cdot, z)$ has been observed, equals the prior density function of z:

$$E[z|v] = E[z] = 0.$$

For intermediate values of v, consumers cannot infer anything new about the expected value of z by observing their own relative valuation of the two goods as given by v.

In the extreme cases—high or low values of v—consumers can infer something new about the expected value of z by observing their own relative valuation of the two goods. For instance, if a consumer observes a high value of v, it infers that this value cannot be associated with a low value of z and so the posterior expected value of z exceeds zero.

(ii) High values: $v \in [t-w, t+w]$.

If $v \in [t-w, t+w]$, then variable z cannot assume all values in $[-w, w]$. In particular, z cannot assume values toward the low end of its support, its posterior expected value no longer being zero, but exceeding it instead. For a given value of $v \in [t-w, t+w]$, z can assume values in the interval $[v-t, w]$. Thus, the density function of variable z, given the realization of v, is

$$f[z|v] = \frac{1}{w-(v-t)}, \qquad v-t \leq z \leq w.$$

Therefore, the posterior expected value of z equals

$$E[z|v] = \frac{w+(v-t)}{2}.$$

Therefore, $E[z|v]$ can assume values between 0 (when $v = t-w$) and w (when $v = t+w$).

(iii) Low values: $v \in [-t-w, -t+w]$.

[30] To see this, consider the following example. If $v = 0$, then $z = w \Rightarrow a = -w$, and $z = 0 \Rightarrow a = 0$, and $z = -w \Rightarrow a = w$.

Similar computations yield

$$f[z|v] \quad = \quad \frac{1}{v+t-(-w)}, \qquad\qquad -w \le z \le v+t,$$

and

$$E[z|v] \quad = \quad \frac{v+t+(-w)}{2}.$$

Therefore, $E[z|v]$ can assume values between $-w$ (for $v = -t - w$) and 0 (for $v = -t + w$).

Figure 1 depicts in its lower panel the inference process leading to the posterior $E[z|v]$ for the assumption made in the main text, $t > w$, as well as, in the upper panel, for $t = w$, a benchmark case used in the next appendix's discussion. Crucially for what follows, regardless of the relative values of t and w, a consumer who observes $v = 0$ must form a posterior $E[z|v] = 0$.

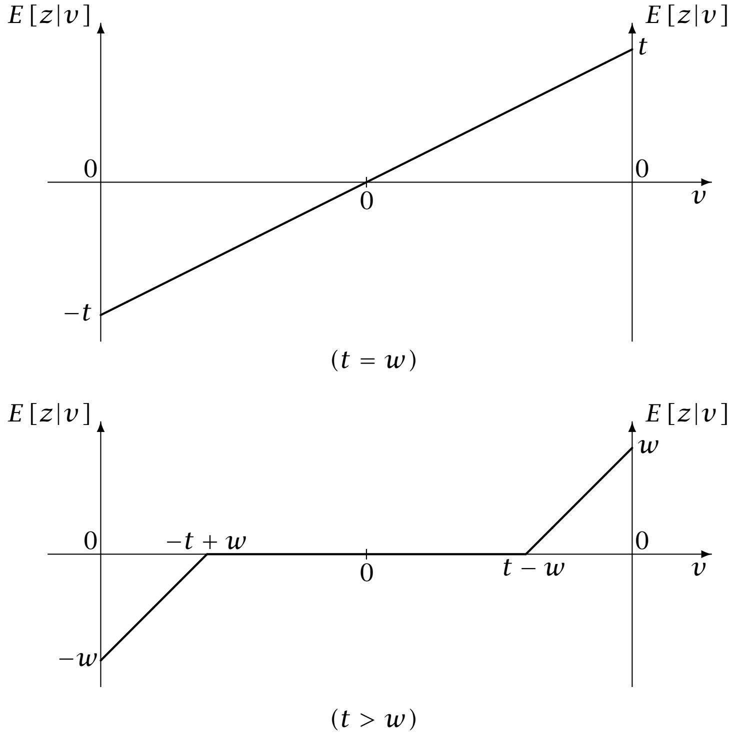

FIGURE 1: *Posterior on z as a function of observed $v(\cdot,\cdot)$.*

First-period demand curve as a function of $E[z|v(\cdot, z)]$

For intermediate values of v, i.e., $v \in [-t+w, t-w]$, we have $E[z|v] = 0$. Then, (7) collapses to

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}.$$

For high values of v, i.e., $v \in [t-w, t+w]$, we have $E[z|v] = \frac{w+(v-t)}{2}$ which, inserted in (7), yields

$$x_1 \quad = \quad \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{(v+w-t)\,e\,(2t-e)}{2t\,(3t^2 - 6te + 2e^2)}.$$

For low values of v, i.e., $v \in [-t-w, -t+w]$, we have $E[z|v] = \frac{v+t+(-w)}{2}$ which, inserted in (7), yields

$$x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} + \frac{(v+t-w)\,e\,(2t-e)}{2t\,(3t^2 - 6te + 2e^2)}.$$

First-period demand curve

We now show that a first-period *indifferent* consumer has $E[z|v(x_1, z)] = 0$ and thus $x_1 = \frac{1}{2} + \frac{z}{2t} + \frac{3}{2}\frac{(t-e)(p_1^B - p_1^A)}{3t^2-6te+2e^2}$ is the first-period demand function.

Take any realization of z, say, $\overline{z}$. By definition, $v = z + a$, $a \in [-t, t]$ and $z \in [-w, w]$. This, together with the assumption $t > w$, implies that $\exists \overline{x}_1, 0 < \overline{x}_1 < 1 : \overline{z} + a(\overline{x}_1) = 0$. Thus, for such a consumer located at $\overline{x}_1$, we have $v = 0$. Trivially, $v = 0 \in [-t+w, t-w]$. From the first subsection of this appendix, this implies $E[z|v] = E[z] = 0$.

Moreover, solving the system of equations formed by (2) and (6) yields

$$\begin{aligned}
\tilde{x}_1 &= \frac{1}{2} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right) + E[z|v(\cdot, z)]\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2} \\
\tilde{x}_2 &= \frac{1}{2} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right) + E[z|v(\cdot, z)]\,t}{3t^2 - 6te + 2e^2},
\end{aligned}$$

which, for a consumer such that $E[z|v] = 0$, yields

$$\begin{aligned}
\tilde{x}_1 &= \frac{1}{2} + \frac{3}{2}\frac{(t-e)\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2} \\
\tilde{x}_2 &= \frac{1}{2} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right)}{3t^2 - 6te + 2e^2}.
\end{aligned}$$

Now take $p_1^A = p_1^B$, i.e., a symmetric equilibrium and note that these expressions collapse to $\tilde{x}_1 = \tilde{x}_2 = \frac{1}{2}$. Thus, such a consumer fulfills the equality $C + v(a(\overline{x}_1), \overline{z}) + e(\tilde{x}_1 + \tilde{x}_2) - p_1^A = C + e(2 - (\tilde{x}_1 + \tilde{x}_2)) - p_1^B$. Consumers slightly to the right of $\overline{x}_1$, such that $x_1 > \overline{x}_1$ while $v \in [-t+w, t-w]$, strictly prefer good B because $v < 0$ and $\tilde{x}_1 = \tilde{x}_2 = \frac{1}{2}$. Consumers further to the right, such that $x_1 > \overline{x}_1$ and $v \in [-t-w, -t+w]$, strictly prefer good B because $v < 0$ and $\tilde{x}_1 = \tilde{x}_2 < \frac{1}{2}$. A similar argument establishes that consumers to the left of $\overline{x}_1$ strictly prefer good A.

Appendix D

The main text treats the case of an immutable vertical-differentiation advantage. In this appendix, we solve a variant of the model that accounts for the possibility that one good may benefit from a market-wide preference early on whereas the opponent may benefit from such a market-wide preference later, i.e., the realization of z may differ between periods. To this effect, define variables $v_l(\cdot, \cdot)$ as the sum of two random variables, $a(\cdot)$ and z_l, where $l = 1, 2$ denotes the period. We assume that z_1 and z_2 are independent, so that nothing can be inferred about z_2 after agents infer the realization of z_1 from first-period sales (an inference process described in Appendix B). Summarizing,

$$\begin{aligned}
&v_l\left(j, z_l\right) = a\left(j\right) + z_l \\
&z_l \rightsquigarrow U\left(-w, w\right) \qquad l = 1, 2 \\
&a\left(j\right) = t - 2t\, j \\
&j \rightsquigarrow U\left(0, 1\right) \Rightarrow a \rightsquigarrow U\left(-t, t\right).
\end{aligned}$$

Since the first-period demand is similar to the one obtained in the main text, a first-period consumer who has observed $v_1\left(\cdot, z_1\right)$ takes demand to be given by

$$x_1 = \frac{p_1^B - p_1^A + z_1 + t - 2e + 2e\left(\tilde{x}_1\left(v_1\left(\cdot, z_1\right)\right) + \tilde{x}_2\left(v_1\left(\cdot, z_1\right)\right)\right)}{2t}. \tag{D.1}$$

The expected demand is thus:

$$\tilde{x}_1\left(v_1\left(\cdot, z_1\right)\right) = \frac{p_1^B - p_1^A + E\left[z_1 | v_1\left(\cdot, z_1\right)\right] + t - 2e + 2e\tilde{x}_2\left(v_1\left(\cdot, z_1\right)\right)}{2\left(t - e\right)}. \tag{D.2}$$

The second-period demand function is determined as in the main text, except that now the realization of z_2 is unknown at the beginning of the second period. Thus, a second-period consumer who has observed $v_2\left(\cdot, z_2\right)$ takes second-period demand to be given by

$$x_2 = \frac{p_2^B - p_2^A + z_2 + t - 2e + 2ex_1^* + 2eE\left[x_2 | v_2\left(\cdot, z_2\right)\right]}{2t}. \tag{D.3}$$

Taking expectations on both sides, the second-period demand expected by a second-period consumer who has observed $v_2\left(\cdot, z_2\right)$ equals

$$E\left[x_2 | v_2\left(\cdot, z_2\right)\right] = \frac{p_2^B - p_2^A + E\left[z_2 | v_2\left(\cdot, z_2\right)\right] + t - 2e + 2ex_1^*}{2\left(t - e\right)}. \tag{D.4}$$

By replacing (D.4) in (D.3), we obtain

$$x_2 = \frac{p_2^B - p_2^A + z_2 + t - 2e + 2ex_1^*}{2\left(t - e\right)} + \frac{eE\left[z_2 | v_2\left(\cdot, z_2\right)\right] - ez_2}{2t\left(t - e\right)}.$$

Thus, second-period demand equals

$$x_2 = \frac{p_2^B - p_2^A + z_2 + t - 2e + 2ex_1^*}{2\left(t - e\right)} + \frac{eE\left[z_2 | v_2\left(x_2, z_2\right)\right] - ez_2}{2t\left(t - e\right)}. \tag{D.5}$$

Firms in the second period do not know the realization of z_2 and act on the basis of its expected value, namely 0. Thus, from (D.5), second-period demand as expected by firms equals

$$E\left[x_2 | 0\right] = \frac{p_2^B - p_2^A + t - 2e + 2ex_1^*}{2\left(t - e\right)} + \frac{eE\left[E\left[z_2 | v_2\left(x_2, z_2\right)\right] | 0\right]}{2t\left(t - e\right)}, \tag{D.6}$$

where, with a slight abuse of notation, $E\left[a | 0\right]$ denotes the expectation of random variable a conditional on the null prior on z_2. Because the realization of z_1 will likely differ from 0, not only will x_1^* likely differ from $\frac{1}{2}$ but, as a consequence, second-period prices will also likely differ. In such cases, the observed realization of $v_2\left(x_2, z_2\right)$ of an *indifferent* second-period consumer may differ from zero. Therefore, $E\left[z_2 | v_2\left(x_2, z_2\right)\right]$ may or may not differ from zero for indifferent second-period consumers depending on the inference process described in Appendix C (see Figure 1). If t exceeds w enough, the range of realizations of $v_2\left(\cdot, z_2\right)$

leading to a posterior $E[z_2|v_2(\cdot,z_2)] = 0$ is wide enough for an indifferent second-period consumer to hold a zero expectation concerning z_2 even when $x_1 \neq \frac{1}{2}$ and second-period prices differ. To see this, consider the lower graph in Figure 1 and note that the expectation of z_2 formed by consumers who have observed the most extreme values of $v_2(\cdot,z_2)$—namely, $v_2(0,z_2)$ and $v_2(1,z_2)$—approaches 0 as w approaches 0. Hence, we assume that t exceeds w enough to ensure that $E[z_2|v_2(x_2,z_2)]$ does indeed equal zero for an indifferent second-period consumer. Then, (D.6) collapses to

$$E[x_2|0] \quad = \quad \frac{p_2^B - p_2^A + t - 2e + 2ex_1^*}{2(t-e)}. \tag{D.7}$$

The profit maximization problem of firm A in the second period is

$$\underset{p_2^A}{\text{Max}} \quad E\left[p_2^A x_2 \middle| 0\right].$$

Since p_2^A is not a random variable, we can write

$$\underset{p_2^A}{\text{Max}} \quad p_2^A E[x_2|0] = p_2^A \frac{p_2^B - p_2^A + t - 2e + 2ex_1^*}{2(t-e)}.$$

The f.o.c. equals

$$p_2^B + t - 2e + 2ex_1^* = 2p_2^A.$$

The s.o.c. equals

$$-\frac{1}{t-e} < 0.$$

By the same token, we have for firm B

$$p_2^A + t - 2ex_1^* = 2p_2^B.$$

We can now solve the system of equations encompassing these first-order conditions, obtaining

$$\begin{cases} p_2^A &= \quad t + \frac{2}{3}ex_1^* - \frac{4}{3}e \\ p_2^B &= \quad t - \frac{2}{3}e - \frac{2}{3}ex_1^*. \end{cases} \tag{D.8}$$

Replacing these equalities in (D.5), we obtain

$$x_2 = \frac{z_2 + t - \frac{4}{3}e + \frac{2}{3}ex_1^*}{2(t-e)} + \frac{eE[z_2|v_2(x_2,z_2)] - ez_2}{2t(t-e)}. \tag{D.9}$$

Hence, the second-period demand expected by a first-period consumer with valuation $v_1(\cdot,z_1)$ is

$$\begin{aligned} &\tilde{x}_2(v_1(\cdot,z_1)) = E[x_2|v_1(\cdot,z_1)] = \\ &= \quad \frac{E[z_2|v_1(\cdot,z_1)] + t - \frac{4}{3}e + \frac{2}{3}e\tilde{x}_1(v_1(\cdot,z_1))}{2(t-e)} + \\ &\quad + \frac{eE[E[z_2|v_2(x_2,z_2)]|v_1(\cdot,z_1)] - eE[z_2|v_1(\cdot,z_1)]}{2t(t-e)}, \end{aligned}$$

which immediately simplifies to

$$\begin{aligned} &\tilde{x}_2(v_1(\cdot,z_1)) = E[x_2|v_1(\cdot,z_1)] = \\ &= \quad \frac{t - \frac{4}{3}e + \frac{2}{3}e\tilde{x}_1(v_1(\cdot,z_1))e + E[E[z_2|v_2(x_2,z_2)]|v_1(\cdot,z_1)]}{2(t-e)}, \end{aligned} \tag{D.10}$$

because $E[z_2|v_1(\cdot, z_1)] = 0$, since z_1 and z_2 are independent and first-period consumers must thus rely on their prior on z_2, namely, $E[z_2] = 0$.

We are left with computing $E[E[z_2|v_2(x_2, z_2)]|v_1(\cdot, z_1)]$, i.e., the estimate that a first-period consumer who has observed $v_1(\cdot, z_1)$ forms of a second-period indifferent consumer's estimate of z_2. Consider a first-period *indifferent* consumer. Besides holding a posterior on z_2 also equal to the prior, $E[z_2|v_1(x_1, z_1)] = E[z_2] = 0$, because z_1 and z_2 are independent, it must hold a posterior on z_1 equal to the prior, $E[z_1|v_1(x_1, z_1)] = E[z_1] = 0$, by the argument of the last subsection of Appendix C.[31] Thus, an indifferent first-period consumer should expect both goods to attain the same first-period sales, $\tilde{x}_1 = \frac{1}{2}$, unless it expects second-period prices to differ as this would induce excess sales of one firm over the other in the second period and hence overall. In fact, a first-period *indifferent* consumer expects second period-prices to be equal: from (D.8), $E\left[p_2^A \middle| v_1(x_1, z_1)\right] = E\left[p_2^B \middle| v_1(x_1, z_1)\right] = t - e$ for $\tilde{x}_1 = \frac{1}{2}$. Since a first-period indifferent consumer expects both goods to sell equally in the first period, $\tilde{x}_1 = \frac{1}{2}$, and second-period prices to be equal, it also expects a second-period *indifferent* consumer to have a posterior equal to its prior, $E[z_2|v_2(x_2, z_2)] = E[z_2] = 0$, by the argument presented in the last subsection of Appendix C. Hence $E[E[z_2|v_2(x_2, z_2)]|v_1(x_1, z_1)] = E[z_2|v_2(x_2, z_2)] = E[z_2] = 0$. The same argument applies to all other first-period consumers who, while not indifferent, hold a null posterior on z_1, i.e., "middle-grounders."

On the contrary, first-period consumers who hold a non-zero posterior on z_1, namely "optimists" and "pessimists," may or may not expect $E[z_2|v_2(x_2, z_2)]$ to equal 0, depending on the inference process described in Appendix C (see Figure 1), an issue also faced by firms at the beginning of the second period, as already discussed above.[32] If t exceeds w enough, the range of realizations of $v_2(\cdot, z_2)$ leading to a posterior $E[z_2|v_2(\cdot, z_2)] = 0$ is wide enough for a first-period consumer who observed an extreme value of $v_1(\cdot, z_1)$ to expect an indifferent second-period consumer to hold a zero expectation concerning z_2 despite the fact that the first-period consumer expects z_1 to differ from 0. To see this, consider the lower graph in Figure 1 and note that the expectation of z_1 formed by consumers who have observed the most extreme values of $v_1(\cdot, z_1)$—namely, $v_1(0, z_1)$ and $v_1(1, z_1)$—approaches 0 as w approaches 0. Hence, even these "extreme" first-period consumers expect both goods to attain sales close to $\frac{1}{2}$ in both periods and second-period prices not to differ significantly. This, together with their zero prior on z_2, in turn implies that they expect *indifferent second-period consumers to be located close to the mid-point of the linear city and thus hold a null posterior on z_2.*

On the other hand, when w equals t, only those first-period consumers who have observed $v_1(\cdot, z_1) = 0$ expect indifferent second-period consumers to hold an expectation

[31] As far as indifferent first-period consumers are concerned, the only informational difference between this case and the one treated in Appendix C lies in the fact that, when z is time invariant, the posterior $E[z_1|v_1] = E[z_1] = 0$ applies to both periods, whereas here it is replaced by an equally null posterior $E[z_2|v_1] = 0$ for the second period. Hence, first-period indifferent consumers form the same expectation of equilibrium variables in both cases.

[32] The difference between the two sets of agents lies in that firms observe (possibly very) assymetric first-period sales at the end of the first period, $x_1^* \neq \frac{1}{2}$, whereas first-period consumers may expect them as a result of having observed a realization of $v_1(\cdot, z_1)$ fairly different from 0.

$E\left[z_2|v_2\left(x_2,z_2\right)\right]=0$. All other first-period consumers, who hold a non-zero posterior on z_1, expect one of the goods to begin the second-period with an installed base advantage and, as a consequence, second-period prices to differ. This, in turn, implies that all these first-period consumers must expect indifferent second-period consumers to have observed a realization of $v_2\left(x_2,z_2\right)\neq 0$ and thus also to hold a non-zero posterior on z_2.[33] We assume that the case described in the previous paragraph applies, i.e., t exceeds w enough so that $E\left[E\left[z_2|v_2\left(x_2,z_2\right)\right]|v_1\left(\cdot,z_1\right)\right]=0, \forall v_1\left(\cdot,z_1\right)\in\left[-t-w,t+t\right]$.[34]

Thus, (D.10) simplifies to

$$\tilde{x}_2\left(v_1\left(\cdot,z_1\right)\right) \quad = \quad \frac{t-\frac{4}{3}e+\frac{2}{3}e\tilde{x}_1\left(v_1\left(\cdot,z_1\right)\right)}{2\left(t-e\right)}. \tag{D.11}$$

By replacing (D.11) in (D.2) and the resulting equality in (D.1), we obtain

$$x_1 \quad = \quad \frac{1}{2}+\frac{z_1}{2t}+\frac{3}{2}\frac{\left(t-e\right)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}+\frac{1}{2}\frac{e\left(3t-2e\right)E\left[z_1|v_1\left(\cdot,z_1\right)\right]}{t\left(3t^2-6te+2e^2\right)}.$$

As explained above, indifferent first-period consumers are such that $E\left[z_1|v_1\left(x_1,z_1\right)\right]=E\left[z_1\right]=0$. So, the previous expression collapses to

$$x_1=\frac{1}{2}+\frac{z_1}{2t}+\frac{3}{2}\frac{\left(t-e\right)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}. \tag{D.12}$$

Finally, by replacing (D.12) in (D.9), we obtain

$$x_2=\frac{1}{2}+\frac{z_1e}{6t\left(t-e\right)}+\frac{z_2}{2t}+\frac{1}{2}\frac{e\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}+\frac{1}{2t}\frac{E\left[z_2|v_2\left(x_2,z_2\right)\right]e}{t-e}.$$

Finally, using the fact that $E\left[z_2|v_2\left(x_2,z_2\right)\right]=0$, we have

$$x_2=\frac{1}{2}+\frac{z_1e}{6t\left(t-e\right)}+\frac{z_2}{2t}+\frac{1}{2}\frac{e\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}. \tag{D.13}$$

By replacing (D.12) in (D.8), we obtain

$$p_2^A \quad = \quad t-e+\frac{1}{3}\frac{ez_1}{t}+\frac{e\left(t-e\right)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}, \tag{D.14}$$

and

$$p_2^B \quad = \quad t-e-\frac{1}{3}\frac{ez_1}{t}-\frac{e\left(t-e\right)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}. \tag{D.15}$$

The profit maximization problem of firm A is[35]

$$\underset{p_1^A}{\text{Max}} \qquad \Pi^A=E\left[x_1\left(p_1^A,p_1^B\right)p_1^A+x_2\left(p_1^A,p_1^B\right)p_2^A\right],$$

[33]Intuitively, the symmetry of the problem that we described above for indifferent first-period consumers and, more generally, first-period consumers with a null posterior on z_1, does not hold for first-period consumers who have observed realizations of $v_1\left(\cdot,z_1\right)$ such that their posterior on z_1 differs from zero. These first-period consumers expect an indifferent second-period consumer to be such that its observed realization of $v_2\left(\cdot,z_2\right)$ compensates for the facts that $x_1\neq\frac{1}{2}$ and second-period prices differ as a result of the realization of $z_1\neq 0$, doing so both directly and through its effect on $E\left[x_2|v_2\left(x_2,z_2\right)\right]$ via $E\left[z_2|v_2\left(x_2,z_2\right)\right]$. For this to be the case, $v_2\left(x_2,z_2\right)$ must necessarily differ from 0, implying $E\left[z_2|v_2\left(x_2,z_2\right)\right]\neq 0$ as Figure 1's upper graph makes clear.

[34]This issue did not arise in the main text because the realization of z was deduced by all second-period consumers upon observing first-period sales.

[35]Recall fn. 19.

Replacing (D.12), (D.13) and (D.14) in the profit maximization problem, we obtain

$$
\begin{aligned}
\underset{p_1^A}{\text{Max}} \quad \Pi^A = \quad & E\left[\frac{1}{2}+\frac{z_1}{2t}+\frac{3}{2}\frac{(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\right]p_1^A+ \\
& +E\left[\left(\frac{1}{2}+\frac{z_1 e}{6t\,(t-e)}+\frac{z_2}{2t}+\frac{1}{2}\frac{e\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\right)\times\right. \\
& \left.\times\left(t-e+\frac{1}{3}\frac{ez_1}{t}+\frac{e\,(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\right)\right]= \\
= \quad & \left[\frac{1}{2}+\frac{3}{2}\frac{(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\right]p_1^A+E\left[\frac{1}{2}\left(t-e+\frac{1}{3}\frac{ez_1}{t}\right)+\right. \\
& +\frac{1}{2}\frac{e\,(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}+\left(\frac{z_1 e}{6t\,(t-e)}+\frac{z_2}{2t}\right)\left(t-e+\frac{1}{3}\frac{ez_1}{t}\right)+ \\
& +\left(\frac{z_1 e}{6t\,(t-e)}+\frac{z_2}{2t}\right)\frac{e\,(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}+ \\
& +\frac{1}{2}\frac{e\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\left(t-e+\frac{1}{3}\frac{ez_1}{t}\right) \\
& \left.+\frac{1}{2}\frac{e\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\frac{e\,(t-e)\left(p_1^B-p_1^A\right)}{3t^2-6te+2e^2}\right)\Bigg].
\end{aligned}
$$

Computing the f.o.c. and using symmetry, $p_1^A = p_1^B$, we have

$$
p_1^A = p_1^B = t - \frac{5}{3}e - \frac{1}{3}\frac{e^2}{t-e}. \tag{D.16}
$$

Thus, equilibrium first-period prices are the same as in the previous section. As to the s.o.c., we have

$$
(t-e)\,\frac{-3\,(3t^2-6te+2e^2)+e^2}{(3t^2-6te+2e^2)^2},
$$

which is negative if $t > \frac{5}{3}e$, a restriction we now retain.

Appendix E

In this appendix we develop a model similar to the one in the main text except that random variable z is no longer unknown in the first period.

The first-period demand function is determined as in the main text. The only difference is that now the exact value of z is common knowledge:

$$
x_1 = \frac{p_1^B - p_1^A + z + t - 2e + 2e\,(\tilde{x}_1 + \tilde{x}_2)}{2t}.
$$

The expected value of x_1 is now equal to its actual value, i.e., $x_1 = \tilde{x}_1$:

$$
\begin{aligned}
x_1 &= \frac{p_1^B - p_1^A + z + t - 2e + 2e\left(x_1 + \tilde{x}_2\right)}{2t} \\
&= \frac{p_1^B - p_1^A + z + t - 2e + 2e\tilde{x}_2}{2\left(t-e\right)}. \qquad \text{(E.1)}
\end{aligned}
$$

The second-period demand function and prices are determined as in the main text:

$$
\begin{aligned}
x_2 &= \frac{p_2^B - p_2^A + z + t - 2e + 2ex_1}{2\left(t-e\right)} \qquad \text{(E.2)} \\
p_2^A &= \frac{1}{3}z + t + \frac{2}{3}ex_1 - \frac{4}{3}e \qquad \text{(E.3)} \\
p_2^B &= -\frac{1}{3}z + t - \frac{2}{3}e - \frac{2}{3}ex_1. \qquad \text{(E.4)}
\end{aligned}
$$

In contrast to the main text, since z is known from the outset, the expectations of x_2, p_2^B and p_2^A are equal to their actual value. By inserting (E.3) and (E.4) into (E.2), we obtain

$$
x_2 = \frac{\frac{1}{3}z + t - \frac{4}{3}e + \frac{2}{3}ex_1}{2\left(t-e\right)}. \qquad \text{(E.5)}
$$

By substituting (E.5) in (E.1), bearing in mind that $\tilde{x}_2 = x_2$, we obtain:

$$
x_1 = \frac{1}{2} + \frac{3}{2}\frac{\left(t-e\right)\left(p_1^B - p_1^A\right) + z\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}. \qquad \text{(E.6)}
$$

By substituting (E.6) in (E.5), we obtain:

$$
x_2 = \frac{1}{2} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right) + zt}{3t^2 - 6te + 2e^2}. \qquad \text{(E.7)}
$$

By substituting (E.6) in (E.3) and (E.4), we obtain:

$$
p_2^A = \frac{1}{3}z + t - e + \frac{e\left(t-e\right)\left(p_1^B - p_1^A\right) + ez\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2},
$$

and

$$
p_2^B = -\frac{1}{3}z + t - e - \frac{e\left(t-e\right)\left(p_1^B - p_1^A\right) + ez\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}.
$$

The first-period profit-maximization problem of firm A is

$$
\underset{p_1^A}{\text{Max}} \quad \Pi^A = \left(x_1\left(p_1^A, p_1^B\right)p_1^A + x_2\left(p_1^A, p_1^B\right)p_2^A\right),
$$

or

$$
\begin{aligned}
\underset{p_1^A}{\text{Max}} \quad \Pi^A = & \left(\frac{1}{2} + \frac{3}{2}\frac{\left(t-e\right)\left(p_1^B - p_1^A\right) + z\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}\right)p_1^A + \\
& + \left(\frac{1}{2} + \frac{1}{2}\frac{e\left(p_1^B - p_1^A\right) + zt}{3t^2 - 6te + 2e^2}\right) \times \\
& \quad \times \left(\frac{1}{3}z + t - e + \frac{e\left(t-e\right)\left(p_1^B - p_1^A\right) + ez\left(t - \frac{2}{3}e\right)}{3t^2 - 6te + 2e^2}\right).
\end{aligned}
$$

The f.o.c. equals

$$\frac{1}{2}\frac{54p_1^At^2e-46p_1^Ate^2-27p_1^Bt^2e+22p_1^Bte^2-26zt^2e+20zte^2+9t^4+8e^4}{\left(3t^2-6te+2e^2\right)^2}+$$
$$+\frac{1}{2}\frac{-18p_1^At^3+10p_1^Ae^3-42t^3e+66t^2e^2-40te^3+9p_1^Bt^3-4p_1^Be^3+9zt^3-4ze^3}{\left(3t^2-6te+2e^2\right)^2}=0.$$

The second derivative equals

$$-\frac{-27t^2e+23te^2+9t^3-5e^3}{\left(3t^2-6te+2e^2\right)^2}=\frac{-3t+e}{3t^2-6te+2e^2}+\frac{e^2\left(e-t\right)}{\left(3t^2-6te+2e^2\right)^2}.$$

As in the main text, one must have $t>1.577e$ in order to have a unique and stable equilibrium without full bunching on one good. For $t>1.577e$, the expression immediately above is negative, ensuring that the s.o.c. is verified.

The problem facing firm B is

$$\underset{p_1^B}{\text{Max}}\qquad \Pi^B=\left(1-x_1\left(p_1^A,p_1^B\right)\right)p_1^B+\left(1-x_2\left(p_1^A,p_1^B\right)\right)p_2^B,$$

or

$$\begin{aligned}\underset{p_1^B}{\text{Max}}\qquad \Pi^B=&\left(\frac{1}{2}-\frac{3}{2}\frac{(t-e)\left(p_1^B-p_1^A\right)+z\left(t-\frac{2}{3}e\right)}{3t^2-6te+2e^2}\right)p_1^B+\\ &+\left(1-\frac{1}{2}-\frac{1}{2}\frac{e\left(p_1^B-p_1^A\right)+zt}{3t^2-6te+2e^2}\right)\times\\ &\times\left(-\frac{1}{3}z+t-e-\frac{e\left(t-e\right)\left(p_1^B-p_1^A\right)+ez\left(t-\frac{2}{3}e\right)}{3t^2-6te+2e^2}\right).\end{aligned}$$

The f.o.c. for firm B's problem equals

$$-\frac{1}{2}\frac{27p_1^At^2e-22p_1^Ate^2-54p_1^Bt^2e+46p_1^Bte^2-26zt^2e+20zte^2-9t^4-8e^4}{\left(3t^2-6te+2e^2\right)^2}-$$
$$-\frac{1}{2}\frac{-9p_1^At^3+4p_1^Ae^3+42t^3e-66t^2e^2+40te^3+18p_1^Bt^3-10p_1^Be^3+9zt^3-4ze^3}{\left(3t^2-6te+2e^2\right)^2}=0.$$

Solving the system of equations formed by the two first-order conditions, we obtain the optimal prices charged in the first period:

$$\begin{aligned}p_1^B&=-\frac{1}{3}\frac{56e^4-328te^3+12ze^3-60zte^2+582t^2e^2-378t^3e+78zt^2e-27zt^3+81t^4}{(e-t)\left(14e^2-54te+27t^2\right)}\\ p_1^A&=-\frac{1}{3}\frac{56e^4-328te^3-12ze^3+60zte^2+582t^2e^2-378t^3e-78zt^2e+27zt^3+81t^4}{(e-t)\left(14e^2-54te+27t^2\right)}.\end{aligned}$$

By replacing these in (E.6) and (E.7), we obtain

$$x_1=\frac{1}{2}+\frac{1}{2}\frac{9zt-2ez}{14e^2-54te+27t^2}\tag{E.8}$$
$$x_2=\frac{1}{2}+\frac{1}{2}\frac{-4e^2z+15ezt-9zt^2}{(e-t)\left(14e^2-54te+27t^2\right)}.\tag{E.9}$$

If $z>0$, x_1 and x_2 exceed $\frac{1}{2}$, as was to be expected. Moreover, $x_2>x_1$ if and only if $t\in(1,577e,1.694e)$.

Appendix F

Proof of Proposition 4

Take the model involving time-varying market-wide preferences and consider two particular realizations of the common terms such that in the first period, A benefits from a consumer fad, i.e., $z_1 = K > 0$, whereas in the second period the symmetric case occurs, $z_2 = -K$, and compare it to the opposite case where B is preferred in the first period, i.e., $z_1 = -K < 0$, whereas in the second period the symmetric case occurs, $z_2 = K$. Take the first scenario, $(z_1, z_2) = (K, -K)$. From (18), (19), (21) and (22), A's profit equals:

$$\Pi^A\Big|_{(K,-K)} = p_1^A x_1 + p_2^A x_2 =$$
$$= \left[t - \frac{5}{3}e - \frac{1}{3}\frac{e^2}{t-e}\right] \cdot \left[\frac{1}{2} + \frac{K}{2t}\right] + \left[t - e + \frac{1}{3}\frac{eK}{t}\right] \cdot \left[\frac{1}{2} + \frac{Ke}{6t\,(t-e)} - \frac{1}{2}\frac{K}{t}\right].$$

Similarly, under the second scenario, $(z_1, z_2) = (-K, K)$, A's profit equals:

$$\Pi^A\Big|_{(-K,K)} = p_1^A x_1 + p_2^A x_2 =$$
$$= \left[t - \frac{5}{3}e - \frac{1}{3}\frac{e^2}{t-e}\right] \cdot \left[\frac{1}{2} - \frac{K}{2t}\right] + \left[t - e - \frac{1}{3}\frac{eK}{t}\right] \cdot \left[\frac{1}{2} - \frac{Ke}{6t\,(t-e)} + \frac{1}{2}\frac{K}{t}\right].$$

Simple computations yield

$$\Pi^A\Big|_{(K,-K)} - \Pi^A\Big|_{(-K,K)} = -\frac{Ke^2}{3t\,(t-e)} < 0.$$

Thus, the firm that benefits from a consumer fad in the second period in better off whenever network effects are felt.

This is true despite the fact that the firm that benefits from a consumer fad in the first period ends up selling more than its opponent. To see it, take the first scenario, $(z_1, z_2) = (K, -K)$ and note that firm A's total sales exceed 1 iff $e > 0$:

$$x_1 + x_2 > 1 \Leftrightarrow \frac{1}{2} + \frac{1}{2}\frac{K}{t} + \frac{1}{2} + \frac{Ke}{6t\,(t-e)} - \frac{1}{2}\frac{K}{t} > 1 \Leftrightarrow \frac{Ke}{6t\,(t-e)} > 0.$$

Proof of Proposition 5

Let us begin with the case when one product benefits from a time-invariant market-wide preference. From (16) and (17), the equilibrium quantities for each good in a symmetric equilibrium equal

$$\begin{aligned} x_1 &= \frac{1}{2} + \frac{z}{2t} \\ x_2 &= \frac{1}{2} + \frac{\frac{1}{3}z + \frac{1}{3}\frac{ez}{t}}{2\,(t-e)}. \end{aligned}$$

We had assumed that the support of z, namely $[-w, w]$, was such that $t > w$. Thus, mere inspection of x_1 shows that good A's first-period equilibrium quantity is always less than 1. Consider now A's second-period sales, $x_2 = \frac{1}{2} + \frac{\frac{1}{3}z+\frac{1}{3}\frac{ez}{t}}{2(t-e)} = \frac{1}{2} + \frac{1}{2}\frac{1}{3}\frac{t+e}{t-e}\frac{z}{t}$. It may equal 1 or fall short of it. On the one hand, when z's realization is close to t, i.e., $z \lessapprox t$, the term $\frac{z}{t} \lessapprox 1$. Moreover, $\frac{1}{3}\frac{t+e}{t-e} = 1$ when $t = 2e$ and exceeds 1 when $t < 2e$. Thus, when $z \lessapprox t$ and $t < 2e$, all second-period consumers opt for the market-wide preferred good. Intuitively, when the market-wide advantage of one firm over the other is quite marked ($z \lessapprox t$), and horizontal-differentiation welfare costs, as measured by t, are not too significant when compared to the strength of the network effects, e, then second-period consumers, upon observing the extreme market-wide preference for one good as revealed by first-period sales, will all buy it in the second-period. On the other hand, from (17), when $z \approx 0$, second-period consumers split between goods. In sum, the market outcome when market wide preferences are immutable is such that $x_1 < 1$ while $x_2 \leq 1$.

We can now compare the market outcome with the socially-optimal allocation of consumers to goods. From (28), the latter is as follows:

$$x_1 = x_2 = \begin{cases} 1 & t - 4e \leq z \\ \dfrac{1}{2} + \dfrac{z}{2t - 8e} & t - 4e > z. \end{cases}$$

First, take the case $t - 4e \leq z$. Social welfare is maximized when the good that benefits from a market-wide preference is adopted by all consumers in both periods, whereas the market outcome splits them between networks in either the first or both periods. Second, when $t - 4e > z$, the fact that $\frac{z}{2t-8e} > \frac{z}{2t}$ implies that in the first period the market assigns fewer consumers than is socially optimal to the good that benefits from a market-wide preference. Moreover, $\frac{\frac{1}{3}z+\frac{1}{3}\frac{ez}{t}}{2t-2e} < \frac{\frac{1}{3}z+\frac{1}{3}\frac{ez}{t}}{2t-8e} < \frac{z}{2t-8e}$ where the last inequality results from the fact that $t - 4e > z > 0$ implies $\frac{e}{t} < 1$ which, in turn, implies $\frac{1}{3}z + \frac{1}{3}\frac{ez}{t} < z$. Thus, in the second period the market assigns fewer consumers than is socially optimal to the good that benefits from a market-wide preference. All this shows that the market outcome when market-wide preferences are immutable assigns more consumers to the worse (vertically-differentiated) good than is socially optimal.

Let us now perform a similar analysis for the case when market-wide preferences may vary over time and one product enjoys the same preference in both periods. From (21) and (22), the equilibrium quantities in a symmetric equilibrium when one good benefits from the same market-wide advantage in both periods equal

$$\begin{aligned} x_1 &= \frac{1}{2} + \frac{z}{2t} \\ x_2 &= \frac{1}{2} + \frac{ze}{6t\,(t-e)} + \frac{z}{2t}. \end{aligned}$$

From $x_1 = \frac{1}{2} + \frac{z}{2t}$ we conclude that first-period consumers always split between goods since, by assumption, $w < t$ and this implies $z < t$. From $x_2 = \frac{1}{2} + \frac{ze}{6t(t-e)} + \frac{z}{2t}$ we conclude that second-period consumers may all want to buy the market-wide preferred good if $z \lessapprox t$. Again, from (28), when $t - 4e \leq z$, social welfare is maximized when the good that benefits from a market-wide preference is adopted by all consumers in both periods, whereas the market splits them between goods in either the first or both periods. When $t - 4e > z$, the fact that $\frac{z}{2t-8e} > \frac{z}{2t}$ implies that in the first period the market assigns fewer consumers to the good that benefits from a market-wide preference than is socially optimal. Moreover, the fact that $\frac{ze}{6t(t-e)} + \frac{z}{2t} < \frac{z}{2t-8e}$ emerges if one bears in mind that $\frac{ze}{6t(t-e)} + \frac{z}{2t} = \frac{ze+3(t-e)z}{6t(t-e)} =$

$\frac{(3t-2e)z}{6t(t-e)} = \frac{\left(1-\frac{2e}{3t}\right)z}{2(t-e)} < \frac{z}{2t-2e} < \frac{z}{2t-8e}$, where we made use of the fact that $t - 4e > z > 0$ implies $\frac{2e}{3t} < 1$. Thus, in the second period the market outcome assigns fewer consumers than is socially optimal to the good that benefits from a market-wide preference. In sum, the market outcome when market-wide preferences may vary assigns more consumers to the worse (vertically-differentiated) good than is socially optimal.

Let us show that this welfare sub-optimality is generally more accentuated when a market-wide advantage is immutably fixed. To see it, note that first-period equilibrium sales are the same regardless of whether market-wide preferences are time invariant or not. On the other hand, when market-wide preferences are time invariant the second-period equilibrium quantity equals $\frac{1}{2} + \frac{\frac{1}{3}z+\frac{1}{3}\frac{ez}{t}}{2(t-e)} = \frac{1}{2} + \frac{ze}{6t(t-e)} + \frac{z}{6(t-e)}$, whereas we have $\frac{1}{2} + \frac{ze}{6t(t-e)} + \frac{z}{2t}$ for the opposite case. All that remains to be shown is that $\frac{z}{2t} > \frac{z}{6(t-e)}$. This inequality amounts to $2t < 6(t-e) \Leftrightarrow 4t > 6e \Leftrightarrow t > 1.5e$, which is indeed the case in view of the conditions previously imposed. Thus, unless the realization of z and the values of t and e are such that good A's second-period sales equal 1 in both cases, the social welfare sub-optimality is greater when preferences are time invariant.

Proof of Proposition 6

From (23) and (24), the equilibrium quantities for each good in a symmetric equilibrium when market-wide preferences are known from the outset equal

$$\begin{aligned} x_1 &= \frac{1}{2} + \frac{1}{2}\frac{9zt - 2ez}{14e^2 - 54te + 27t^2} \\ x_2 &= \frac{1}{2} + \frac{1}{2}\frac{-4e^2z + 15ezt - 9zt^2}{(e-t)(14e^2 - 54te + 27t^2)}, \end{aligned}$$

whereas, from (28), the social-welfare maximizing allocation of consumers to networks is as follows:

$$x_1 = x_2 = \begin{cases} 1 & t - 4e \le z \\ \frac{1}{2} + \frac{z}{2t-8e} & t - 4e > z. \end{cases}$$

Again, when $t - 4e \le z$, social welfare is maximized when the good that benefits from a market-wide preference is adopted by all consumers in both periods, whereas the market outcome may split them between goods in both periods for low values of e. To see it, consider a realization of $z \lessapprox t$ and $e \approx 0$ such that $t - 4e \le z$. Take $\lim_{e\to 0} \frac{9zt-2ez}{14e^2-54te+27t^2} = \lim_{e\to 0} \frac{-4e^2z+15ezt-9zt^2}{(e-t)(14e^2-54te+27t^2)} = \frac{z}{3t} \approx \frac{1}{3}$, since $z \lessapprox t$. Hence, both market equilibrium quantities will be approximately equal to $\frac{1}{2} + \frac{1}{2}\frac{1}{3} = \frac{2}{3}$ and will thus fall short of 1, whereas the social-welfare maximizing allocation of consumers to goods has all consumers buying the market-wide preferred good.

When $t - 4e > z$, we have $\frac{1}{2}\frac{9zt-2ez}{14e^2-54te+27t^2} < \frac{1}{2}\frac{9zt}{27t^2-54te} < \frac{1}{2}\frac{z}{3t-6e} < \frac{z}{3t-6e} < \frac{z}{2t-8e}$ since $3t - 6e > 2t - 8e$. This implies that the market outcome assigns fewer consumers in the first period to the good that benefits from a market-wide preference than is socially optimal. Moreover, simple computations show that $\frac{1}{2}\frac{9zt-2ez}{14e^2-54te+27t^2} > \frac{1}{2}\frac{-4e^2z+15ezt-9zt^2}{(e-t)(14e^2-54te+27t^2)}$ for $t > 1.694e$. Thus, for $t - 4e > z > 0$ implying $t > 4e$, we have $x_2 < x_1 < \frac{1}{2} + \frac{z}{2t-8e}$.

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