When does the high quality firm drive compatibility?*

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October 27, 2009

Abstract

We analyse firms' incentives to provide compatibility between two network goods with different intrinsic qualities. We consider the case in which both firms have the power to veto compatibility and the case in which none has this power. We show that in some equilibria, it is the high quality that invests in compatibility, whereas in others, the more intuitive result of the low quality firm triggering compatibility holds.

Compatibility is always underprovided from the social point of view.

Keywords: Compatibility, vertical differentiation, network effect.

JEL Classification: L13, L15.

1 Introduction

Since Rohlfs (1974) first described his theory of the interdependent demand for communication services that economists have studied different aspects of the so-called network

*We are grateful to Oscar Amerighi, Rabah Amir, Emanuele Bacchiega, Paul Belleflamme, Giuseppe De Feo, Vincenzo Denicolò, Mario Denni, Jean Gabszewicz, Luca Lambertini, Martin Peitz, Mario Tirelli, Vincent Vannetelbosch, Xavier Wauthy, and seminar audiences at EARIE 2008 (Toulouse University), at ESEM 2008 (Bocconi University), at the University of Bologna, at FUSL and CREI for useful comments and discussion about an earlier draft.

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industries. These are characterized by the existence of positive externalities in consumption i.e. the utility of agents is increasing in the number of users of the goods. The consequence of this is that consumers choosing one such good always forego the benefits of interacting with the agents that chose other goods. A possible solution that firms may envisage to increase the utility of their products and hence the willingness to pay of the consumers is to establish some form of compatibility. Indeed, firms decide whether to make their goods compatible with those of their rivals, thus competing *in the market*, or to make them incompatible thus competing *for the market* (standard war). As Besen and Farrell (1994) put forward, "there is no general answer to the question of whether firms will prefer competition for the potentially enormous prizes under inter-technology competition, or the more conventional competition that occurs when there are common standards." [1]

The objective of this paper is to study the compatibility choice of vertically differentiated firms operating in a network market. Although our analysis is not intended to suit any particular industry, examples we can think of are: the game console industry, the operating systems industry and the high definition DVD industry.[2] In these industries, typically one firm is perceived by the consumers as being the high quality firm.[3]

[1]A recent example, is Microsoft and Sony Team on Digital Entertainment Content Management System: though rivals in the gaming-console market, both companies find they have much to gain from working closely to integrate the new Sony VAIO XL1 Digital Living System with Microsoft Windows XP Media Center Edition 2005, http://www.microsoft.com/presspass/features/2005/oct05/10-18Sony.mspx.

[2]We should stress that these industries can be classified as *two-sided markets* in the sense that consumers' valuation of the goods depends positively on the number of users of the other side of the market. Game players enjoy the benefits of a large number of game developers, as the reverse is also true. Obviously this structure plays a role in the standardization process. However, for the sake of simplicity we disregard this characteristic. For an analysis of competition in two-sided markets see Armstrong (2006).

[3]Think of MAC vs Microsoft Windows operating systems, where it is widely recognized that Microsoft has captured the largest market share with lower prices, whereas Apple Macintosh has proved to be a higher quality product (for instance in terms of high resolution graphics, anti-virus etc.). Another example is provided by the new generation of high definition DVD, HD-DVD produced by Toshiba vs Blu-Ray DVD produced by Sony: the storage potential is around 40 percent lower for HD-DVD disks.

Compatibility can be achieved either by *standardization*, or by the introduction of a *converter*, a device which allows consumers from one side of the market to enjoy (partially or fully) the network of the other side.[4] Often, the converter device represents a compromise of quality, in the sense that compatibility may be imperfect. Likewise, standardization is a costly requirement because it limits product variety. In our model, we assume that compatibility is a feature of the product itself, a characteristic that enhances the network effect.

Our paper is the first to concentrate on the incentives of vertically differentiated firms to provide partial or full compatibility, thus contributing to the ongoing discussion in the literature on network externalities. We also extend this literature by studying compatibility incentives under strong network effects, that is when the importance of the network for consumers is so high to induce multiple equilibria in demands.

In general, the level of compatibility that can be achieved depends on the allocation of the intellectual property rights between the firms and on technical specificities of the products. Concentrating on property rights, as pointed out by de Palma et al. (1999), two situations can arise: i) no firm can veto the move towards compatibility, in which case the final level of compatibility is given by the maximal between the levels chosen noncooperatively by the firms; ii) both firms have veto power over compatibility, or, in other words, the final level of compatibility is the minimal between the levels chosen by the firms. We examine these two possibilities and also compare the private and social incentives towards compatibility. To this end, we develop a two-stage game where firms first choose the degree of compatibility and then the price of their products. Finally, consumers buy one unit of either good.

We identify the conditions under which different incentives to provide compatibility arise as a function of the intrinsic quality differentiation and the importance of the network effect. Under weak network effect, i.e., when the weight of the network effect relative to the vertical differentiation is not very strong, we can observe full compatibility

[4] For instance, in the HD-DVD/Blu-Ray case, producers of DVD discs have come out with a disc which had the HD-DVD version of the movie on one side and the Blu-Ray version on the other.

at equilibrium. In this case, where both firms may remain active in the market, they are willing to offer it because an increase in the conversion level softens competition. However, the low quality firm has higher incentives to provide full compatibility in order to avoid the possibility of being stranded out of the market. On the other hand, under strong network effect, i.e., when the network effect dominates the vertical differentiation, we observe multiple equilibria for consumers' demands. Namely, as consumers value very highly the network, they can all coordinate on either the high quality or the low quality good. However, in any equilibrium of the full game, coordination takes place on the high quality good as long as it maintains its overall quality dominance. This also provides a theoretical rationale for the empirical evidence that a less efficient technology is not likely to conquer the market and lock out a more efficient one.[5] To sum up, we show that both firms may have incentives to provide compatibility. In spite of that, as long as the network effect is not high enough to allow a switch in the overall quality differential, the low quality firm is willing to pay more for compatibility. The contrary may hold when the network effect is strong enough for the switch to occur. Finally, the status of firms' property rights is crucial for the compatibility result. Concerning the social incentives, we find that compatibility is always underprovided.

Related Literature.

Several papers examine the issue of compatibility from different points of view. Baake and Boom (2001) study the decision of full or zero compatibility in a context of vertically differentiated products competing simultaneously in the market, not allowing for partial compatibility, as we do in our paper. Crémer et al. (2000) consider an extension of the seminal paper by Katz and Shapiro (1985) to study compatibility decisions in a Cournot oligopoly with homogeneous goods and heterogeneous consumers, where firms differ in their installed base of consumers. The standard result predicts that smaller firms always have higher incentives for product compatibility than bigger firms. De Palma et al. (1999) focus on the possibility that consumers achieve full compatibility through

[5] This empirical evidence is in contrast with many theoretical models. Among others, Farrel and Saloner (1985) and (1986) represent two examples of inefficient technology adoption due to the so-called *bandwagon effect*.

multi-homing in a context where goods are homogeneous except for the dimension of the network. They provide a thorough discussion of the impact of different property rights on the provision of compatibility. More recently, Doraszelski et al. (2009) analyse the long run stability of compatible products when differentiation is also derived from the size of the network alone and not from the intrinsic qualities of the products.

Also, a number of articles discuss the alternative path towards compatibility, which, as mentioned before, is standardization. These include Farrell and Saloner (1985, 1992), and more recently Ostrovsky and Schwarz (2005) and Alexandrov (2008).

Finally, it is worth mentioning a strand of the literature which focuses on firms' compatibility strategies towards vertically related firms. Theoretical models distinguish according to whether each component is sold by an independent firm or each firm produces everything necessary to form the final good (system). As an example of the first context, Church and Gandal (1992) study the software provision decision of software firms to hardware firms. As for the case of firms supplying all the necessary components, Matutes and Regibeau (1992) study firms' incentives to standardize components in industries where consumers try to assemble a number of components into a system that meets their specific needs.

The outline of the paper is as follows. Section 2 describes the model. Section 3 provides the results on the price competition and compatibility choice by firms and present the comparison with the socially optimal compatibility level. Section 4 concludes the paper with a discussion of the results. The main proofs appear in Appendix A (Section 5), while the more technical proofs are relegated to Appendix B (Section 6).

2 Model

Two firms, A and B, produce competing technologies at constant marginal cost set to zero. These technologies are vertically differentiated and characterized by network externalities in consumption, i.e. consumer's utility is increasing in the number of consumers that choose the same technology. Firms may decide to render the technologies compatible through a converter whose quality determines the degree of network benefits that the consumers enjoy from the rival technology. Hence, consumers' utility is a function of the intrinsic quality of the technology, of the size of the network and of the compatibility that can be achieved with the rival network. We assume that there is a continuum of consumers indexed by x which is uniformly distributed in the interval $[0,1]$. Thus, x measures consumers' valuation of the quality: high consumer types value quality improvements more than low consumer types. Each consumer has a unit demand and buys either one unit of good A or one unit of good B. We rule out the possibility of no purchase, that is we concentrate on the situation in which the market is fully covered.[6][7]

We assume that consumer's utility takes the following form:

$$U_A(x) = \beta_A x + \alpha \left[D_A + \tau D_B\right]$$

$$U_B\left(x\right) = \beta_B x + \alpha \left[D_B + \tau D_A\right]$$

The first term of the utility function, $\beta_i x$ is the stand alone value of the technology for consumer type x. The parameter β_i represents the quality of technology i and we assume throughout that $\beta_B > \beta_A$, i.e., the intrinsic quality of technology B is higher than that of technology A. The second term in the utility is the network benefit, where the parameter $\alpha > 0$ denotes the intensity of the network effect and D_i is the demand of technology i. Therefore, consumers differ in their valuation of the intrinsic quality but value equally the network effect. The latter consists of the externality coming from the interaction with consumers that buy the same technology, (D_i), and the externality resulting from the existence of a converter, which allows consumers to partially benefit from the rival network (τD_j).

The final quality of conversion is endogenous and given by $\tau \in [0,1]$ which is a function of the degrees of conversion chosen by each firm, τ_A and τ_B, respectively. In order to model compatibility, we consider two possibilities: either $\tau = \max\{\tau_A, \tau_B\}$ or $\tau = \min\{\tau_A, \tau_B\}$. Underlying the first formulation is the idea that no firm can

[6] We also exclude the possibility for consumers to join both networks. This could be an alternative way to achieve compatibility as studied by de Palma et al. (1999).

[7] Covered markets occur in the case of mature and widespread industries.

prevent a move towards compatibility. As such, the final compatibility is the maximum of the levels chosen by the firms. In other words, each firm can unilaterally provide compatibility not being necessary that both contribute to the quality of the device. In contrast, underlying the second hypothesis is the idea both firms have veto power over compatibility decisions: to achieve (at least partial) compatibility both firms have to agree and contribute to the quality of the device. There is a linear cost of producing the converter which is increasing in τ_i and given by $c\tau_i$. We assume that firms are equally efficient in producing the converter and thus face the same cost function.

The final level of conversion influences the overall quality differential between the technologies, which is then determined by two sources of quality differentiation. The first one is exogenous and given by $k \equiv \beta_B - \beta_A$ and the other, endogenous, is proportional to the difference in the networks' size and given by $\alpha(D_B - D_A)(1-\tau)$. The endogenous source of differentiation can be manipulated by the firms through the choice of prices and through the choice of the conversion level (τ). We define the overall quality differential as:

$$k + \alpha(D_B - D_A)(1 - \tau).$$

We can interpret this expression as follows: when either the two networks have the same size ($D_A = D_B$) or compatibility is perfect ($\tau = 1$), consumers perceive the technologies as being identical in terms of the network effect.

The overall quality of good B is higher than that of good A if $k \geq \alpha(1 - \tau)$, i.e. even in the extreme case that the network benefit for firm A is the highest ($D_A = 1$ and $D_B = 0$), good B maintains its quality dominance. Thus a switch in the overall differentiation takes place only if consumers value very highly the network, i.e. for $k < \alpha(1 - \tau)$.[8]

Both firms decide first the quality of the conversion that they are willing to offer their consumers and then compete in prices.[9] We model their decisions as a two stage

[8] In Appendix B (Section 6.3) we discuss about asymmetric distributions for consumer types alternative to the uniform distribution. The intuitive result is that the more consumers are concentrated around zero (one), the more firm A (B) has a demand advantage.

[9] This sequence is standard in the literature: compatibility is a technological decision and hence fixed

game and as such the solution concept that we will be using is the subgame perfect Nash equilibrium.

Consumers choose between the technologies maximising their net surplus. In this maximization problem they take as given the decisions of the others and have rational expectations about the size of the networks. Consumer x buys technology A if and only if $U_A(x) - p_A > U_B(x) - p_B$ and $U_A(x) - p_A > 0$. Denote $\hat{x}$ the consumer type which is indifferent between the two technologies and assume that the type $x = 0$ has positive net utility from buying product A, i.e. $U_A(0) - p_A = \alpha [D_A + \tau D_B] - p_A$ is nonnegative.[10] Demands are then given by

$$D_B = 1 - \hat{x},$$
$$D_A = \hat{x}.$$

We analyse the situation where both firms face a nonnegative demand, $\hat{x} \in [0, 1]$. There are three possible market configurations.

1. $D_A = 1$ and $D_B = 0$: the utility of consumers becomes $U_A = \beta_A x + \alpha - p_A$ and $U_B = \beta_B x + \alpha\tau - p_B$. For this market configuration to be possible, all consumers, even consumer type $x = 1$ should prefer to buy good A, i.e. $p_B - p_A \geq k - \alpha(1-\tau)$.

2. $D_A = 0$ and $D_B = 1$: the utility of consumers becomes $U_A = \beta_A x + \alpha\tau - p_A$ and $U_B = \beta_B x + \alpha - p_B$. For this market configuration to be possible, all consumers, even consumer type $x = 0$ should be interested in buying good B, i.e. $p_B - p_A \leq \alpha(1 - \tau)$.

3. D_A, $D_B \in (0, 1)$ and $D_A + D_B = 1$. In this market configuration both firms set positive prices and obtain positive profits.[11]

in the short run.

[10]The market coverage assumption in this model where $x \in [0, 1]$ is only possible thanks to the presence of positive network effects. Indeed, with $\alpha > 0$, consumer type zero may prefer buying because even if its valuation of the intrinsic quality is zero he benefits from the network of consumers buying the same good or compatible goods. See Gabszewicz and Garcia (2007) for a discussion.

[11]The market configurations just described occur under certain conditions on prices. A complete analysis of the feasible price regions can be found in Appendix A (5.1).

The indifferent consumer is:

$$\hat{x} = \alpha(1-\tau)\frac{D_A - D_B}{k} + \frac{p_B - p_A}{k}, \tag{1}$$

which implies that demands, in the interior solution case (market configuration 3.), are given by:

$$D_A = \frac{-\alpha(1-\tau)}{k-2\alpha(1-\tau)} + \frac{p_B - p_A}{k-2\alpha(1-\tau)}, \tag{2}$$

$$D_B = \frac{k-\alpha(1-\tau)}{k-2\alpha(1-\tau)} - \frac{p_B - p_A}{k-2\alpha(1-\tau)}. \tag{3}$$

Observing these expressions, we see that depending on the sign of $k - 2\alpha(1-\tau)$ they are either decreasing or increasing in own price. In what follows we distinguish the two cases.

- Weak Network effect. Assume first that $k > 2\alpha(1-\tau)$, such that D_A and D_B are decreasing in own price, as depicted in Figure 1:

$$D_B(p_A, p_B) = \begin{cases} 1, \quad p_B - p_A \leq \alpha(1-\tau) \\ \frac{k-\alpha(1-\tau)}{k-2\alpha(1-\tau)} - \frac{p_B-p_A}{k-2\alpha(1-\tau)}, \alpha(1-\tau) < p_B - p_A \leq k - \alpha(1-\tau) \\ 0, \quad p_B - p_A > k - \alpha(1-\tau) \end{cases} \tag{4}$$

$$D_A(p_A, p_B) = \begin{cases} 1, \quad p_B - p_A > k - \alpha(1-\tau) \\ \frac{-\alpha(1-\tau)}{k-2\alpha(1-\tau)} + \frac{p_B-p_A}{k-2\alpha(1-\tau)}, \alpha(1-\tau) < p_B - p_A \leq k - \alpha(1-\tau) \\ 0, \quad p_B - p_A \leq \alpha(1-\tau) \end{cases} \tag{5}$$

- Strong Network effect. Assume now that $0 < k < 2\alpha(1-\tau)$. The network effect plays a dominant role in the differentiation among products. As such, multiple equilibria in the consumers' game arise. In particular, as illustrated in Figure 2

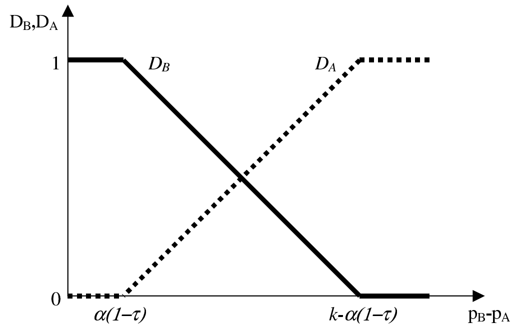

Figure 1: Demand functions: $k > 2\alpha(1-\tau)$.

for good A, the demands for the network goods are correspondences:

$$D_B\left(p_A, p_B\right) = \begin{cases} 1, \quad p_B - p_A \leq \alpha(1-\tau) \\ \frac{-k+\alpha(1-\tau)}{2\alpha(1-\tau)-k} + \frac{p_B - p_A}{2\alpha(1-\tau)-k}, k - \alpha\left(1-\tau\right) \leq p_B - p_A \leq \alpha(1-\tau) \\ 0, \quad p_B - p_A \geq k - \alpha\left(1-\tau\right) \end{cases} \tag{6}$$

$$D_A\left(p_A, p_B\right) = \begin{cases} 1, \quad p_B - p_A \geq k - \alpha\left(1-\tau\right) \\ \frac{\alpha(1-\tau)}{2\alpha(1-\tau)-k} - \frac{p_B - p_A}{2\alpha(1-\tau)-k}, k - \alpha\left(1-\tau\right) \leq p_B - p_A \leq \alpha(1-\tau) \\ 0, \quad p_B - p_A \leq \alpha\left(1-\tau\right) \end{cases} \tag{7}$$

For the range of prices such that $p_B - p_A \in [k - \alpha(1-\tau), \alpha(1-\tau)]$ there are three possible equilibria: either all consumers coordinate on good A or they all coordinate on good B or some consumers prefer good A and others prefer good B. Notice that in the last case, demands are increasing in own price. This is due to the fact that when deciding between A and B consumers value mostly the dimension of the network that they will enjoy. Thus, as demands increase, also the value of the goods does and in turn the consumer's willingness to pay increases. In all cases, consumers' expectations are rational and none of these equilibria is Pareto dominant. Therefore, we cannot select any of them.

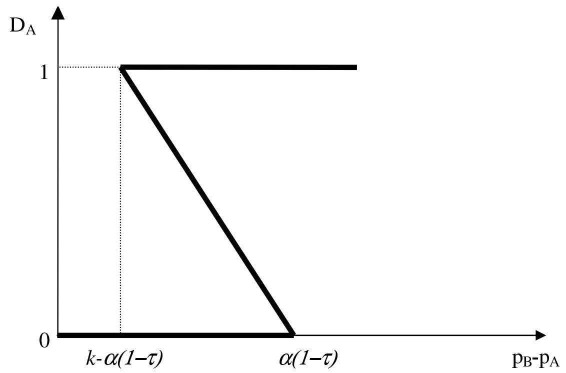

Figure 2: Demand for network good A: $k < 2\alpha(1-\tau)$.

3 The characterization of equilibria

3.1 Price competition under weak network effect

In the second stage of the game, firm i chooses its price p_i so as to maximize its profit Π_i:[12]

$$\Pi_i\left(p_i, p_j\right) = p_i D_i\left(p_i, p_j\right), \text{ with } i \neq j \text{ and } i, j = A, B$$

When $k > 2\alpha(1-\tau)$, the demands for the network goods are well defined functions, in particular they are linear and decreasing in own price. Given demands (4) and (5), the profits are:

$$\Pi_B = \begin{cases} p_B, & p_B - p_A \leq \alpha(1-\tau) \\ \left(\frac{k-\alpha(1-\tau)}{k-2\alpha(1-\tau)} + \frac{p_A - p_B}{k-2\alpha(1-\tau)}\right) p_B, & \alpha(1-\tau) < p_B - p_A \leq k - \alpha\left(1-\tau\right) \\ 0, & p_B - p_A > k - \alpha\left(1-\tau\right) \end{cases}$$

$$\Pi_A = \begin{cases} p_A, & p_B - p_A > k - \alpha\left(1-\tau\right) \\ \left(\frac{-\alpha(1-\tau)}{k-2\alpha(1-\tau)} + \frac{p_B - p_A}{k-2\alpha(1-\tau)}\right) p_A, & \alpha(1-\tau) < p_B - p_A \leq k - \alpha\left(1-\tau\right) \\ 0, & p_B - p_A \leq \alpha\left(1-\tau\right) \end{cases}$$

Price competition leads to the following results.

[12] We here forgo the compatibility costs which are constant in the price-setting stage. We introduce them in the compatibility choice stage (next subsection).

Proposition 1 *1. When $k > 3\alpha(1-\tau)$, there exists a unique Nash equilibrium of the price game, given by:*

$$p_A = \frac{1}{3}k - \alpha(1-\tau), \tag{8}$$
$$p_B = \frac{2}{3}k - \alpha(1-\tau). \tag{9}$$

The corresponding equilibrium demands are[13]

$$D_A = \frac{1}{3}\frac{k - 3\alpha(1-\tau)}{k - 2\alpha(1-\tau)}, \tag{10}$$
$$D_B = \frac{1}{3}\frac{2k - 3\alpha(1-\tau)}{k - 2\alpha(1-\tau)}. \tag{11}$$

2. When $k \in (2\alpha(1-\tau), 3\alpha(1-\tau)]$, there exists a unique Nash equilibrium of the price game, given by

$$p_A = 0,$$
$$p_B = \alpha(1-\tau),$$

where $D_A = 0$ and $D_B = 1$.

Proof. Follows from the observation of the reaction functions obtained in the following Lemma.

Lemma 2 *For $k > 3\alpha(1-\tau)$, the price reaction function of firm B is given by,*

$$p_B(p_A) = \begin{cases} \frac{1}{2}(k - \alpha(1-\tau)) + \frac{p_A}{2}, \text{ if } p_A \leq k - 3\alpha(1-\tau) \\ \alpha(1-\tau) + p_A, \text{ if } p_A > k - 3\alpha(1-\tau). \end{cases}$$

Whereas for $2\alpha(1-\tau) < k \leq 3\alpha(1-\tau)$, the price reaction function is:

$$p_B(p_A) = p_A + \alpha(1-\tau).$$

[13] Notice that in this price equilibrium, a necessary condition for the market coverage assumption to hold is $k \leq 3.46\alpha$. When the vertical differentiation is very high ($k > 3.46\alpha$), consumer type zero prefers not buying rather buying good A whose quality is relatively very low.

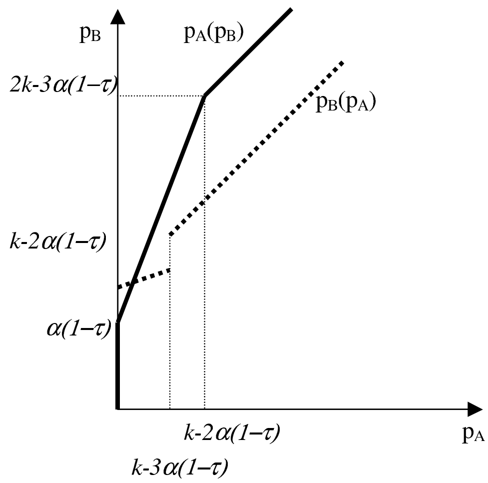

Figure 3: Price reaction functions: $k > 3\alpha(1-\tau)$.

As for firm A, the price reaction function, for $k > 2\alpha(1-\tau)$, *is*

$$p_A(p_B) = \begin{cases} p_A = 0,\ p_B \leq \alpha(1-\tau) \\ \frac{p_B - \alpha(1-\tau)}{2},\ \textit{if}\ \alpha(1-\tau) < p_B \leq 2k - 3\alpha(1-\tau) \\ p_B - k + \alpha(1-\tau),\ \textit{if}\ p_B > 2k - 3\alpha(1-\tau)\,. \end{cases}$$

Proof. See Appendix B (6.1). ■ ■

The reaction curves are depicted in Figure 3 for the case $k > 3\alpha(1-\tau)$, and in Figure 4 for the case $2\alpha(1-\tau) < k \leq 3\alpha(1-\tau)$. It is easy to see that the computed price equilibrium is the unique intersection of the price reactions functions in the relevant domain.

As in the classical model of vertical product differentiation the firm that produces the high quality good charges a higher price. For high intrinsic quality differences, $k > 3\alpha(1-\tau)$, prices are increasing in the degree of conversion and in the intrinsic vertical differentiation, k. When consumers value highly the network, or in other words, when α is large, firms behave more competitively in order to gain network advantage. This implies that prices are decreasing in α. This effect becomes milder in the presence of a converter. Compatibility renders the network size less important for consumers and

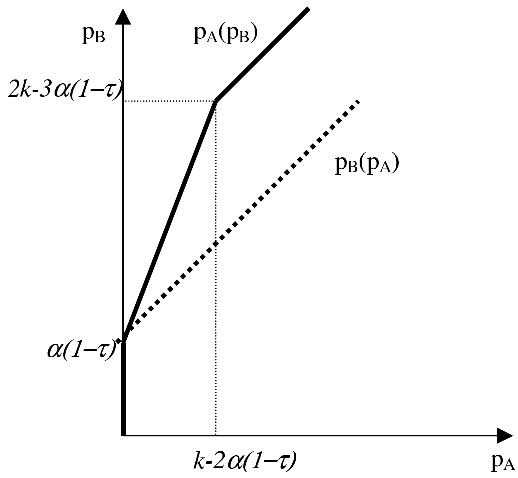

Figure 4: Price reaction functions: $2\alpha(1-\tau) < k \leq 3\alpha(1-\tau)$.

therefore prices increase with τ.

On the contrary, when the intrinsic quality difference is lower, $k \in (2\alpha\,(1-\tau)\,,\ 3\alpha\,(1-\tau)]$, the high quality firm is the only active firm in the market. In that case a higher valuation of the network, i.e. a higher α, allows the firm to extract a higher consumer surplus by setting a higher price. Also, as τ increases, the overall quality differential becomes lower and as such price competition intensifies. In order to maintain the whole market, firm B needs to set a lower price.

3.2 Price competition under strong network effect

For the strong network effect case, $0 < k < 2\alpha(1-\tau)$, we need to consider the demands (6) and (7). Then, profits are given by

$$\Pi_B = \begin{cases} 0, \quad p_B - p_A \geq k - \alpha(1-\tau) \\ \left(\frac{\alpha(1-\tau)-k}{2\alpha(1-\tau)-k} + \frac{p_B - p_A}{2\alpha(1-\tau)-k}\right) p_B, k - \alpha(1-\tau) \leq p_B - p_A \leq \alpha(1-\tau) \\ p_B, \quad p_B - p_A \leq \alpha(1-\tau) \end{cases}$$

$$\Pi_A = \begin{cases} 0, \quad p_B - p_A \leq \alpha(1-\tau) \\ \left(\frac{\alpha(1-\tau)}{2\alpha(1-\tau)-k} - \frac{p_B - p_A}{2\alpha(1-\tau)-k}\right) p_A, k - \alpha(1-\tau) \leq p_B - p_A \leq \alpha(1-\tau) \\ p_A, \quad p_B - p_A \geq k - \alpha(1-\tau) \end{cases}$$

Profits are nondecreasing in own price, hence firms have incentive to set prices as high as possible. Given the market coverage assumption, prices are bounded from above. Moreover, the existence of multiple consumer partition equilibria suggests for multiple price equilibria. In order to solve the price competition we follow a reasoning which is similar to that developed by Grilo et al. (2001) and we define an equilibrium of the price subgame as a price pair associated with a consumer partition. We obtain the following results.

Proposition 3 *1. For $\alpha(1-\tau) \leq k < 2\alpha(1-\tau)$, there exist multiple equilibria for the price subgame. Namely, any price pair (p_A, p_B) such that $p_B = \alpha(1-\tau)$ and $0 \leq p_A \leq 2\alpha(1-\tau) - k$ associated with $D_B = 1$ is an equilibrium.*

2. For $0 < k < \alpha(1-\tau)$, there exist multiple equilibria for the price subgame. Namely, there are two sets of corner equilibria:

(a) any price pair (p_A, p_B) such that $p_B = \alpha(1-\tau)$ and $p_A \geq 0$ associated with $D_B = 1$;

(b) any price pair (p_A, p_B) such that $p_A = \alpha(1-\tau) - k$ and $p_B \geq 0$ associated with $D_A = 1$.[14]

[14] We also find the following interior equilibrium: the price pair (p_A, p_B) such that $p_A = p_B =$

Proof. See Appendix A (5.2.1). ■

In the strong network effect case, consumers exhibit what is known as *strong conformity* (Grilo et al. 2001). This means that consumers would like to coordinate their choices on the same good in order to enjoy the maximum network effect because the difference in intrinsic qualities is not relevant. This is what our results predict. However, as long as the overall quality of good B is superior, i.e., $k > \alpha(1-\tau)$ at equilibrium, coordination takes place on the high quality good B. In contrast, if we let $0 < k < \alpha(1-\tau)$, a switch in the overall quality occurs and coordination could also take place on good A.

Selection of price-stage equilibrium. In order to solve the compatibility stage, and in turn the full game, in what follows, we need to select a particular price-stage equilibrium for the range of k such that $0 < k < 2\alpha(1-\tau)$. We consider both sets of corner equilibria in turn:

1. we select the price-stage equilibrium such that $p_A = 0$ and $p_B = \alpha(1-\tau)$ with $D_A = 0$ and $D_B = 1$;
2. we select the price-stage equilibrium in 1. for $\alpha(1-\tau) < k < 2\alpha(1-\tau)$ and the price-stage equilibrium such that $p_A = \alpha(1-\tau) - k$ and $p_B = 0$ associated with $D_A = 1$ for $0 < k < \alpha(1-\tau)$.[15] This second conjecture would allow us to investigate the interesting situation in which all consumers coordinate on the good with the lowest intrinsic quality.

In the following, we focus on the first price-stage equilibrium, and we discuss the second at the end of the compatibility stage.

3.3 Compatibility choice

From the price competition stage we see that the relative weight of the network effect affects the prevailing market configuration. Namely, under weak network effect, where

$\alpha\frac{\alpha(1-\tau^2)-k\tau}{2\alpha(1-\tau)-k}$ associated with $D_A = \frac{\alpha(1-\tau)}{2\alpha(1-\tau)-k} \in (\frac{1}{2}, 1)$. However, we exclude this second-stage price equilibrium because it locates in the boundary of the market coverage condition. Hence, it is safe to assume that without the market coverage assumption this equilibrium would not exist.

[15] We can exclude the price equilibria in which a firm sets a positive price when its demand is nil.

both firms attain to stay in the market, they both benefit from compatibility so that their preferences towards τ are analogous. In contrast, under strong network effect, as consumers coordinate on the same good, firms' incentives are opposite. Thus, the role of firms' veto power is crucial for the outcome of the compatibility game.

In this section we first formally analyse the choice of compatibility in the case in which no firm can veto compatibility; and then discuss the case in which both firms can veto compatibility.

3.3.1 No firm can veto compatibility

In the first stage of the game firms choose their compatibility levels non-cooperatively. Assume that the global conversion is given by $\tau = \max\{\tau_A, \tau_B\}$. As seen in the price competition, there are different price equilibria depending on the relative weight of the intrinsic quality and the network effect. Accordingly, we have the three following cases for the overall *first-stage profits* of firms.

Case 1 *(Unique) Interior solution in prices* $k > 3\alpha(1-\tau) \iff \tau \in (\frac{3\alpha-k}{3\alpha}, 1]$

$$\Pi_A^I = \begin{cases} \frac{\left(\alpha(\tau_A-1)+\frac{1}{3}k\right)^2}{k-2\alpha(1-\tau_A)} - c\tau_A \text{ if } \tau_A \geq \tau_B \\ \frac{\left(\alpha(\tau_B-1)+\frac{1}{3}k\right)^2}{k-2\alpha(1-\tau_B)} - c\tau_A \text{ if } \tau_A < \tau_B \end{cases} \tag{12}$$

$$\Pi_B^I = \begin{cases} \frac{\left(\alpha(\tau_A-1)+\frac{2}{3}k\right)^2}{k-2\alpha(1-\tau_A)} - c\tau_B \text{ if } \tau_A \geq \tau_B \\ \frac{\left(\alpha(\tau_B-1)+\frac{2}{3}k\right)^2}{k-2\alpha(1-\tau_B)} - c\tau_B \text{ if } \tau_A < \tau_B \end{cases} \tag{13}$$

Case 2 *(Unique) Corner solution in prices* $k \in [2\alpha(1-\tau), 3\alpha(1-\tau)] \iff \tau \in [\frac{2\alpha-k}{2\alpha}, \frac{3\alpha-k}{3\alpha}]$

$$\Pi_A^C = 0 - c\tau_A \tag{14}$$

$$\Pi_B^C = \begin{cases} \alpha(1-\tau_A) - c\tau_B \text{ if } \tau_A \geq \tau_B \\ \alpha(1-\tau_B) - c\tau_B \text{ if } \tau_A < \tau_B \end{cases} \tag{15}$$

Case 3 *Strong network effect case* where $k \in (0, 2\alpha(1-\tau)]$ or $\tau \in [0, \frac{2\alpha-k}{2\alpha})$. In order to illustrate such a possibility characterized by multiple price equilibria, we select the one where $p_A = 0$ and $p_B = \alpha(1-\tau)$ and consumers coordinate on good B which implies

$$\Pi_A^S = 0 - c\tau_A \;, \Pi_B^S = \begin{cases} \alpha\left(1-\tau_A\right) - c\tau_B \text{ if } \tau_A \geq \tau_B \\ \alpha\left(1-\tau_B\right) - c\tau_B \text{ if } \tau_A < \tau_B \end{cases}$$

To analyse the compatibility game we must consider three regions for the parameters (as Figure 5 illustrates):

i) $\frac{3\alpha-k}{3\alpha} \leq 0$, in which case for all values of τ_A and τ_B, the outcome of the pricing game is the unique price competition stage interior solution.

ii) $\frac{2\alpha-k}{2\alpha} < 0 < \frac{3\alpha-k}{3\alpha}$, in which case, for values of $\tau_A, \tau_B \in [0, \frac{3\alpha-k}{3\alpha})$, we have the corner solution in the price competition stage and for values of $\tau_A, \tau_B \in (\frac{3\alpha-k}{3\alpha}, 1]$ we have the interior solution in the price competition stage.

iii) $0 \leq \frac{2\alpha-k}{2\alpha} < \frac{3\alpha-k}{3\alpha}$, in this case, we have that for values of $\tau_A, \tau_B \in [0, \frac{2\alpha-k}{2\alpha})$ the outcome of the price competition stage is the one assumed in the strong network effect case; for $\tau_A, \tau_B \in [\frac{2\alpha-k}{2\alpha}, \frac{3\alpha-k}{3\alpha})$, the outcome of the price competition stage is the corner solution and for $\tau_A, \tau_B \in [\frac{3\alpha-k}{3\alpha}, 1]$, the outcome is the interior solution.

Notice that selecting the price-stage equilibrium such that $p_A = 0$ and $p_B = \alpha(1-\tau)$ with $D_A = 0$ and $D_B = 1$ for the range of k such that $0 < k < 2\alpha(1-\tau)$, then the corner solution of the price competition coincides with the strong network effect solution of the price competition, i.e., $\Pi_A^C = \Pi_A^S$ and $\Pi_B^C = \Pi_B^S$, therefore the last two regions can collapse in one. This is what we do to solve the compatibility stage.

The following Proposition presents the results of the compatibility game for each partition of the parameter space defined above.

Proposition 4 *Assume $\tau = \max\{\tau_A, \tau_B\}$. When the intrinsic quality differentiation is low, $(0 < k \leq 3\alpha)$, market coverage is the equilibrium outcome and the compatibility game yields full compatibility $(\tau = 1)$ if and only if the cost is low, namely, $c < \frac{k}{9}$. For higher conversion costs, the compatibility between the two network goods is zero.*

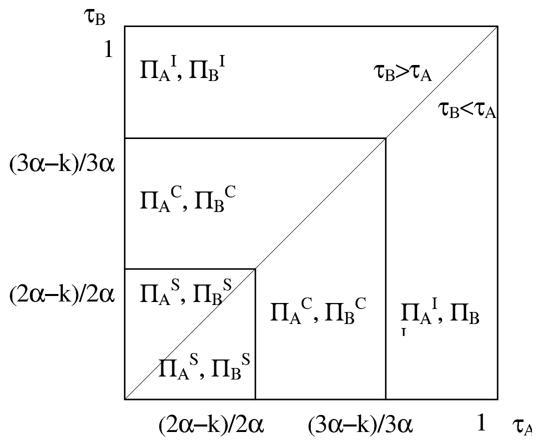

Figure 5: Parameters space for the compatibility

Proof. See Appendix A (5.2.2). ■

Proposition 4 highlights firms' incentives to provide compatibility depending on the relative importance of the network effect versus the intrinsic quality. It deserves a closer analysis. First, notice that we focus on the case $0 < k \leq 3\alpha$. Indeed also when $k > 3\alpha$, firms have the incentive to choose full compatibility, as long as the cost is not too high. This is due to the fact that as the degree of compatibility increases, the price competition softens. However, when the result is either full or no compatibility, low consumer types prefer not to buy anything. Namely, when compatibility is absent the quality of good A is so low with respect to the quality of good B that low consumer types do not buy it. On the other hand, when compatibility is full, although the overall differentiation decreases, the price increase prevents some consumers from buying.

As for the case of $0 < k \leq 3\alpha$, we find that firms have incentive to provide full compatibility for small levels of its cost. However, the lower intrinsic quality differentiation allows for a market coverage equilibrium.

	$0 \leq \frac{3\alpha-k}{3\alpha}$
$c = 0$	$\left.\begin{array}{l}\tau_A = 1, \tau_B \in [0,1] \\ \tau_B = 1, \tau_A \in \left[\frac{9\alpha-4k}{9\alpha}, 1\right]\end{array}\right\} \tau = 1$
$0 < c < \frac{(4k-9\alpha)}{9}$	$\left.\begin{array}{l}\tau_A = 1, \tau_B = 0 \\ \tau_B = 1, \tau_A = 0\end{array}\right\} \tau = 1$
$\frac{(4k-9\alpha)}{9} < c < \frac{k}{9}$	$\tau_A = 1, \tau_B = 0\} \, \tau = 1$
$c > \frac{k}{9}$	$\tau_A = 0, \tau_B = 0\} \, \tau = 0$

Table 1: Compatibility and costs

Looking at the particular behavior of each firm for different levels of c, summarized in Table 1, we can see that as long as $0 < c < \frac{(4k-9\alpha)}{9}$, the game has two Nash equilibria: $(\tau_A = 1, \tau_B = 0)$ and $(\tau_A = 0, \tau_B = 1)$. This is due to the fact that reaction functions are discontinuous and have a unique downward jump. Intuitively, when the opponent chooses a level of conversion high enough (not necessarily $\tau_i = 1$), the firm prefers to enjoy this level of conversion rather than paying for the converter. Furthermore, when $\frac{(4k-9\alpha)}{9} < c < \frac{k}{9}$, in equilibrium, only the low quality firm has incentive to offer compatibility. This is so, because, otherwise, firm B would be in a position to dominate completely the market. By offering compatibility, firm A attracts consumers and becomes active in the market.

From Table 1, we can also notice that whenever $c > 0$, firms never incur in wasteful duplication of compatibility costs. Indeed, at any equilibrium, there is only one firm providing a converter device.

In the following, we present the equilibrium results for all the relevant variables, given the price-stage equilibrium selection. When the vertical differentiation is such that $0 < k \leq 3\alpha$, equilibrium prices demands and profits depend on the compatibility cost in the following way. If $c < \frac{k}{9}$, which implies $\tau = 1$ and in turn the interior solution in prices,

$$D_A^* = \frac{1}{3},\ D_B^* = \frac{2}{3}$$
$$p_A^* = \frac{k}{3},\ p_B^* = \frac{2k}{3}.$$

Profits are then either, $\Pi_A^* = \frac{k}{9} - c$ and $\Pi_B^* = \frac{4k}{9}$ or $\Pi_A^* = \frac{k}{9}$ and $\Pi_B^* = \frac{4k}{9} - c$. Notice that also for low values of vertical product differentiation both firms are active in the market as long as the conversion cost is sufficiently low. If $c \geq \frac{k}{9}$, which implies $\tau = 0$ and in turn the corner (or the strong network effect) solution in prices,

$$D_A^* = 0,\ D_B^* = 1$$

$$p_A^* = 0,\ p_B^* = \alpha$$

$$\Pi_A^* = 0,\ \Pi_B^* = \alpha.$$

We conclude this characterization of the equilibria by briefly discussing the possibility of selecting the price-stage equilibrium such that $p_A = \alpha(1-\tau)-k$ and $p_B = 0$ associated with $D_A = 1$ for the range of $0 < k < \alpha(1-\tau)$. Our results show that in this case, the outcome of the compatibility game yields positive compatibility for a wider range of costs, namely, for $0 < c < \alpha k/(\alpha - k)$. However, compatibility is always partial $\tau = (\alpha - k)/\alpha$. The intuition is the following. For such low intrinsic quality differentiation, it is possible for the low quality firm to dominate completely the market if the overall differentiation is sufficiently high, or equivalently, if the compatibility level, τ, is low. As a result, firm A prefers zero compatibility. In contrast firm B, in order to prevent the solution in which its demand is zero, chooses partial compatibility. Notice that the specific equilibrium value of $\tau_B = (\alpha - k)/\alpha$ is the boundary under which demand of B is zero and over which it is one.

Welfare We next investigate whether the equilibrium compatibility level is optimal from a social welfare point of view. That is, we let the social planner choose the compatibility level, τ at a cost $c\tau$ and firms compete in prices, as before.[16] This implies that now firms' profits do not include the compatibility cost, as it is incurred only by the social planner.

Define, as usual, the social welfare by the following expression:

[16] This means that we consider a second best situation.

$$SW=\int_0^{\widehat{x}}\left(U_A\left(x\right)-p_A\right)dx+\int_{\widehat{x}}^1\left(U_B\left(x\right)-p_B\right)dx+\Pi_A+\Pi_B-c\tau. \tag{16}$$

We need to distinguish two cases according to the price competition outcome.

- When $k>3\alpha\left(1-\tau\right) \iff \tau\in(\frac{3\alpha-k}{3\alpha},1]$,

$$SW^I=\beta_B\frac{1}{2}-\frac{\tau^3-k^2 18\alpha\left(4-3\tau\right)+9k\alpha^2\left(1-\tau\right)\left(17-9\tau\right)}{18\left(k-2\alpha+2\alpha\tau\right)^2}-c\tau.$$

- When $k\in(0,3\alpha\left(1-\tau\right)]$ or $\tau\in[0,\frac{3\alpha-k}{3\alpha}]$,

$$SW^C=\frac{1}{2}\beta_B+\alpha\tau+\alpha\left(1-\tau\right)-c\tau=\frac{1}{2}\beta_B+\alpha-c\tau.$$

The following Proposition summarizes the results for the optimal compatibility choice of the social planner.

Proposition 5 *When the intrinsic quality differentiation is not too high with respect to the network effect* ($0<k\leq 3\alpha$)*, market coverage is the equilibrium outcome: full compatibility is the optimal solution from the social point of view when the cost is low. When cost is intermediate, the social planner chooses partial compatibility* $\tau=\frac{3\alpha-k}{3\alpha}$*. Finally for high costs, no compatibility is the preferred solution.*

Proof. See Appendix A (5.2.3). ■

We can now discuss how the welfare maximising solution differs from the private optimum. We always observe underprovision of compatibility. Firms are willing to offer full compatibility for a smaller range of the costs than the social planner. This result is rather intuitive. Compatibility in our context is a public good as both firms attain the same level of compatibility even if the investment is provided unilaterally. Figure 6 illustrates the private and social choice of compatibility for different levels of cost.

3.3.2 Both firms can veto compatibility

We here discuss the case in which the compatibility levels are complementary, i.e., the global conversion is given by $\tau=\min\{\tau_A,\tau_B\}$.[17] As before, the results depend on

[17] We provide our results in an informal and intuitive way, as the formal analysis is similar to that above.

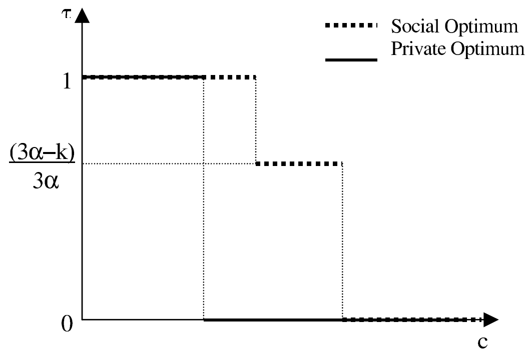

Figure 6: Private versus Social Choice, $\alpha < k < 3\alpha$.

the relative weight of the network effect which in turn affects the prevailing market configuration. Under weak network effect, both firms benefit from compatibility so that they have similar preferences towards τ. However, as they do not want to bear wasteful compatibility costs, their reaction functions are of the form: $\tau_i(\tau_j) = \tau_j$, with $i \neq j$. This implies multiple equilibria. As long as the compatibility cost is sufficiently low, the Pareto dominant equilibrium is full compatibility, i.e. $\tau_A = \tau_B = 1$. This result is in line with Crémer et al. (2000) which restrict their analysis to the weak network effect case. In contrast, under strong network effect, as consumers coordinate on the same good, firms' incentives are opposite. In particular, the firm ending up with zero demand (either the low or the high quality) is never willing to invest in compatibility, so that the unique equilibrium is $\tau_A = \tau_B = 0$.

The nature of this case implies that each firm has a veto right in the overall compatibility level. As a result, compatibility is less likely to be provided at equilibrium with respect to the case of substitute compatibility levels. Therefore, concerning the welfare analysis, we conclude that *a fortiori* compatibility is underprovided at equilibrium.

4 Conclusion

In this paper, we have analysed firms' incentives to provide compatibility between two network goods with different intrinsic qualities. We have provided a complete analysis by studying how the relative importance of vertical differentiation with respect to the network effect influences the price competition as well as the compatibility choice. Indeed, the final degree of compatibility allows firms to manipulate the overall product differentiation. Namely, when consumers' valuation of the network is not high enough to allow an overall quality switch of the vertically differentiated goods, full compatibility may arise: the low quality firm has higher incentives to offer it in order to prevent the rival from dominating the market. In contrast, when a quality switch occurs, that is the overall quality of the high quality good is lower than that of the low quality good thanks to the magnitude of the network effect, the firm with lower intrinsic quality could conquer the market thus being the high quality firm who has more to gain from compatibility. Comparing firms' compatibility decisions with the social optimum, we show that compatibility is always underprovided.

Our analysis points out new interesting results about firms' incentives to offer compatibility. Indeed, as Besen and Farrell (1994) describe, firms' horizontal compatibility strategies determine the form of competition in the market. In particular, with two firms, there are three combinations of such strategies: both firms choose incompatibility which results in a standard war; both firms prefer compatibility; and finally, one firm chooses incompatibility whereas the other prefers compatibility. The last is the only case where firms choose different strategies. This seems reasonable when firms are asymmetric. For example, Katz and Shapiro (1985) show how a larger firm is more likely to prefer incompatibility than a smaller firm. Similarly, Baake and Boom (2001) show that the high quality firm, in contrast with the rival firm, is against compatibility. However, we show that this need not be the case. In fact, for high levels of quality differential both firms have incentives to provide compatibility while for low levels of quality differential they may have asymmetric preferences. When vertical differentiation is high, competition is mild and both firms manage to stay in the market: offering compatibility they can further soften competition because consumers' valuation of the goods increases. In contrast, for low levels of vertical differentiation, either the high quality firm or the low quality firm are likely to conquer the market thus having opposite incentives for compatibility. Under strong network effects, expectations about the network size are crucial in determining the dominant firm in the market.

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5 Appendix A

5.1 Price regions

The fully covered market configurations described in the model occur when the following conditions hold:

$$p_A - p_B \leq \alpha\left(1-\tau\right)\left(D_A - D_B\right), \tag{17}$$

$$p_B - p_A \leq \alpha(1-\tau)(D_B - D_A) + k, \tag{18}$$

$$p_A \leq \alpha\left(D_A + \tau D_B\right). \tag{19}$$

Given the demands (2) and (3), these conditions can be reduced to:

$$p_A - p_B \leq \alpha (1-\tau) \frac{(2(p_B - p_A) - k)}{(k - 2\alpha + 2\alpha\tau)},$$
$$p_B - p_A \leq -\alpha(1-\tau)\frac{(2(p_B - p_A) - k)}{(k - 2\alpha + 2\alpha\tau)} + k,$$
$$p_A \leq \alpha \frac{\left(-\alpha\left(1-\tau^2\right) + k\tau - (p_A - p_B)(1-\tau)\right)}{(k - 2\alpha + 2\alpha\tau)}.$$

Assume first $k > 2\alpha(1-\tau)$, then we have

$$p_B \geq \alpha(1-\tau) + p_A, \tag{20}$$

$$p_B \leq k - \alpha(1-\tau) + p_A, \tag{21}$$

$$p_B \geq p_A \frac{(k - \alpha + \alpha\tau)}{\alpha(1-\tau)} - \frac{\left(k\tau - \alpha\left(1-\tau^2\right)\right)}{(1-\tau)} \tag{22}$$

The lines underlying condition 20 and 21 are parallel and their slope is 1. The intercept of the first line is smaller than that of the second line. As for condition 22, we have that the intercept is smaller than the other 2 (and possibly negative) and its slope is higher than 1, therefore, the space defined by these three lines is nonempty and given either by the light grey area or the light and dark grey areas as in Figure 7.

Now assume $k < 2\alpha(1-\tau)$, then we have

$$p_B \leq p_A + \alpha(1-\tau) \tag{23}$$

$$p_B \geq p_A + (k - \alpha + \alpha\tau) \tag{24}$$

$$p_B \leq \frac{\alpha\left(1-\tau^2\right) - k\tau}{1-\tau} + p_A \frac{k - \alpha(1-\tau)}{\alpha(1-\tau)} \tag{25}$$

The lines underlying condition 23 and 24 are parallel and their slope is 1. The intercept of the first line is bigger than that of the second line. As for condition 25, we have that the intercept is bigger than the other two and its slope is positive and lower than 1. Therefore, the space defined by these three lines is nonempty and given by the dark grey area as in Figure 8.

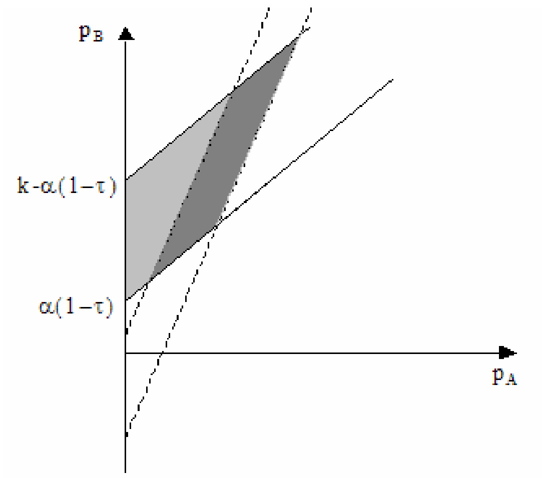

Figure 7: Price region: $k > 2\alpha(1-\tau)$.

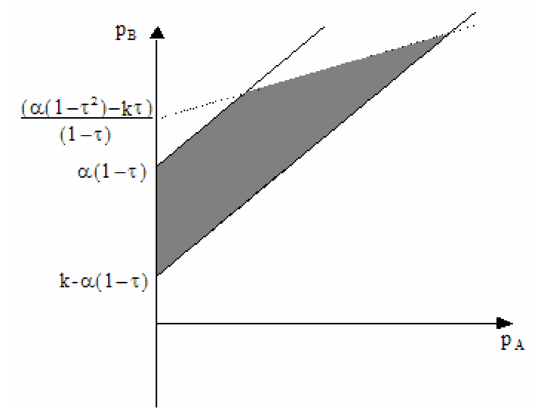

Figure 8: Price region: $k < 2\alpha(1-\tau)$.

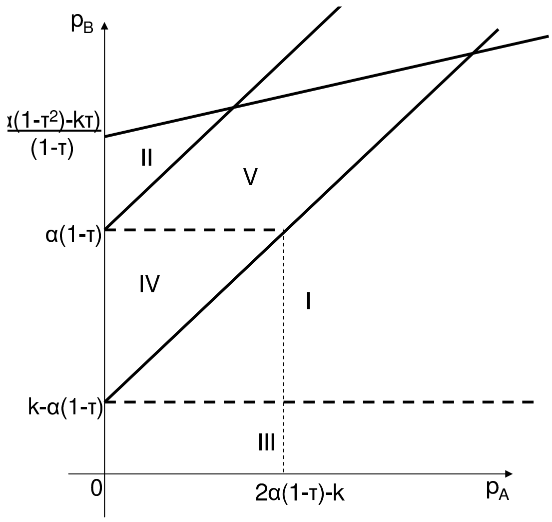

Figure 9: Price space for $\alpha(1-\tau) < k < 2\alpha(1-\tau)$.

5.2 Proofs

5.2.1 Proof of Proposition 3

Consider first the relevant price region depicted in Figure 9 for $\alpha(1-\tau) \leq k < 2\alpha(1-\tau)$. We divide this price space in 5 regions that we analyse in turn. In regions I and II, there is a unique consumer partition equilibrium: $(D_A = 0, D_B = 1)$ and $(D_A = 1, D_B = 0)$, respectively. In these regions, no price equilibrium exists: in region I (II), firm A (B) can profitably deviate by reducing its price p_A (p_B).

In regions IV and V, all three consumer partitions $D_A = 1$, $D_A = 0$ and $D_A \in (0, 1)$ are equilibria of the consumers' choice. We proceed by first eliminating the consumer partitions which are not compatible with a price equilibrium in these regions. Concerning region IV, $D_A = 1$ can never be part of the equilibrium of the full game because firm B can always reduce its price p_B and conquer a positive market share (going to region I). In contrast, for $D_B = 1$, given any p_B belonging to region IV, firm A cannot profitably deviate as its profit is zero in any case. Therefore, in region IV any price pair (p_A, p_B) such that $p_A \geq 0$ and p_B is $\alpha(1-\tau)$ is an equilibrium, whatever the consumer partition which actually realizes. Concerning region V, no price equilibrium can be associated with either $D_A = 1$ or $D_B = 1$ because the firm with no consumers can always reduce its price and obtain positive profit, (p_A can decrease and reach region II and similarly p_B can decrease and reach region I). We conclude by considering the possibility of $D_A \in (0,1)$ in regions IV and V.

Lemma 6 *Let $\alpha(1-\tau) < k < 2\alpha(1-\tau)$, the reaction functions of firms A and B in regions IV and V with $D_A \in (0,1)$, are given by,*

$$p_B(p_A) = \begin{cases} \alpha(1-\tau) + p_A, \; \text{if } p_A \leq \alpha\tau \\ \frac{\alpha(1-\tau^2)-k\tau}{1-\tau} + p_A \frac{(k-\alpha(1-\tau))}{\alpha(1-\tau)}, \; \text{if } p_A > \alpha\tau \end{cases}$$

$$p_A(p_B) = \begin{cases} p_B - k + \alpha(1-\tau), \; \text{if } p_B \geq k - \alpha(1-\tau) \\ 0, \; \text{if } p_B < k - \alpha(1-\tau) \end{cases}$$

Proof. See Appendix B (6.2). ■

Drawing these reaction functions, we can easily see that they intersect only once at $p_A = \alpha$ and $p_B = k + \alpha\tau$. However, such a price pair is incompatible with the consumer partition $D_A \in (0,1)$ which we can thus exclude. Indeed, as Figure 2 illustrates, at $p_B - p_A = k - \alpha(1-\tau)$, the equilibrium consumers' choice is either $D_A = 0$ or $D_A = 1$.

Finally, in region III, we have a unique equilibrium of the consumers' choice: any price pair (p_A, p_B) such that $p_B \leq k - \alpha(1-\tau)$ and $p_A \geq 0$ is associated with $D_A = 0$, $D_B = 1$. Profits are then $\Pi_A = 0$ for firm A and $\Pi_B = p_B$ for firm B: thus, firm A will be indifferent between any $p_A \geq 0$; however, firm B would always have an incentive to increase p_B so as to move to region IV (as long as p_A is sufficiently low), where it can set a higher price.

2. Consider now the relevant price region depicted in Figure 10 for $0 < k < \alpha(1-\tau)$. We divide this price space in 6 regions that we analyse in turn. We use a proof similar to that provided above.[18]

5.2.2 Proof of Proposition 4

i) $\frac{3\alpha-k}{3\alpha} < 0$

[18]The formal proof is available from the authors upon request.

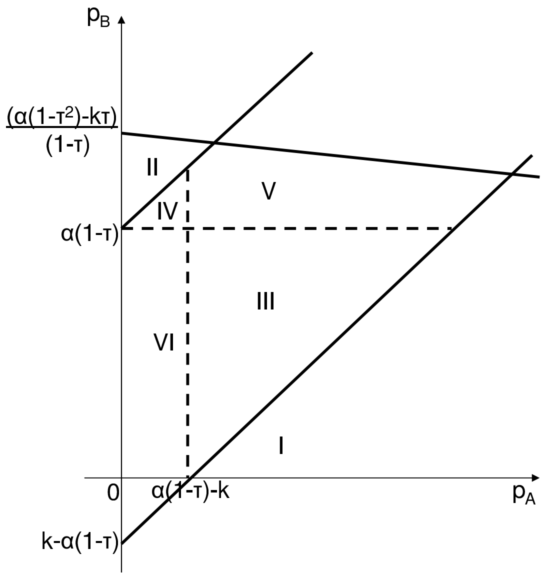

Figure 10: Price space for $k < \alpha(1-\tau)$.

Remember that for $\frac{3\alpha-k}{3\alpha} \leq 0$, the unique outcome of the price stage is the interior solution. As such, profits are given by (12) and (13). The revenues are either constant in τ_i (when $\tau_i < \tau_j$) or convex in τ_i, (when $\tau_i > \tau_j$). Consequently, the profit is maximized either at $\tau_i = 1$ or $\tau_i = 0$. Therefore overall compatiblity is either $\tau = 0$ or $\tau = 1$. Checking the condition for market coverage (19) it is easy to verify that it does not hold whenever $k > 3\alpha$, given the second stage equilibrium prices and demands, (8), (9), (10) and (11).

ii) $0 < \frac{3\alpha-k}{3\alpha}$

In this case, for values of $\tau_A, \tau_B \in [0, \frac{3\alpha-k}{3\alpha})$, we have the corner (or the strong network effect) solution in the price competition stage and profits are given by (14) and (15); and for values of $\tau_A, \tau_B \in (\frac{3\alpha-k}{3\alpha}, 1]$ we have the interior solution in the price competition stage, with profits (12) and (13). Let us consider first the revenue function of firm B. If $\tau_A \leq \frac{3\alpha-k}{3\alpha}$, as long as $\tau_B \leq \tau_A$, firm B's revenue is constant and equal to $\alpha(1-\tau_A)$; for $\tau_A < \tau_B < \frac{3\alpha-k}{3\alpha}$, its revenue is decreasing in τ_B and equal to $\alpha\,(1-\tau_B)$; finally, for $\tau_B > \frac{3\alpha-k}{3\alpha}$, the revenue is convex in τ_B.

If $\tau_A > \frac{3\alpha-k}{3\alpha}$, then, the revenue of firm B is constant and equal to $\frac{\left(\alpha(\tau_A-1)+\frac{2}{3}k\right)^2}{k-2\alpha(1-\tau_A)}$, as long as $\tau_B \leq \tau_A$; and convex increasing otherwise. If $\tau_A = 1$, then the revenue is constant and equal to $\frac{4k}{9}$. Similarly, for firm A, when $\tau_B \leq \frac{3\alpha-k}{3\alpha}$, its revenue is zero as long as $\tau_A \leq \frac{3\alpha-k}{3\alpha}$, and positive and convex otherwise. When $\tau_B > \frac{3\alpha-k}{3\alpha}$, firm A's revenue is constant and equal to $\frac{\left(\alpha(\tau_B-1)+\frac{1}{3}k\right)^2}{k-2\alpha(1-\tau_B)}$, as long as $\tau_A \leq \tau_B$; and convex increasing otherwise. When $\tau_B = 1$, then the revenue is constant and equal to $\frac{k}{9}$.

If $c = 0$, then, firm A always prefers $\tau_A = 1$, being indifferent in case $\tau_B = 1$. As for firm B, she will choose, $\tau_B = 0$ if $\alpha(1-\tau_A) > \frac{4k}{9} \iff \tau_A < \frac{9\alpha-4k}{9\alpha}$, and $\tau_B = 1$, otherwise. When $\tau_A = 1$, τ_B is indifferent. Formally,

$$\tau_B(\tau_A) = \begin{cases} 0 \text{ if } \tau_A \in \left[0, \frac{9\alpha-4k}{9\alpha}\right] \\ 1, \text{if } \tau_A \in [\frac{9\alpha-4k}{9\alpha}, 1) \\ [0,1], \text{if } \tau_A = 1 \end{cases},$$

$$\tau_A(\tau_B) = \begin{cases} 1, \text{if } \tau_B \in [0,1) \\ [0,1], \text{if } \tau_B = 1 \end{cases}$$

It is straightforward to see that there are multiple pure strategy Nash equilibria in the compatibility game, namely $\tau_A = 1$ and $\tau_B \in [0,1]$, and $\tau_B = 1$ and $\tau_A \in \left[\frac{9\alpha-4k}{9\alpha}, 1\right]$. The overall compatibility level is $\tau = 1$. There is an upward jump in the reaction function of firm B at $\tau_A = \frac{9\alpha-4k}{9\alpha}$ (which is positive for $k \in \left(2\alpha, \frac{9}{4}\alpha\right)$, even if it is negative, equilibrium is the same). The overall compatibility level is $\tau = 1$. This equilibrium respects the conditions (17)-(19) for the partition of the parameter space in which it arises.[19]

If $c \in (0, \frac{k}{9}]$, we must consider three subsets of $(0, \frac{k}{9}]$.[20] Let first $c < \frac{4k-9\alpha}{9}$, then firm B prefers $\tau_B = 1$ to $\tau_B = 0$, if $\tau_A \leq \frac{3\alpha-k}{3\alpha}$; otherwise, for $\tau_A > \frac{3\alpha-k}{3\alpha}$ she prefers

[19]The market coverage condition is satisfied for $k < 3\alpha$. When $\tau \to 1$, the RHS of condition (19)$\to -\infty$. Therefore, as p_B is finite and positive, the condition always holds.

[20]We assume that the boundaries of these subsets are positive. In the case in which they are negative, only the last subset is valid. Nevertheless, results are not affected.

$\tau_B = 0$ to $\tau_B = 1$, if $\frac{\left(\alpha(\tau_A - 1) + \frac{2}{3}k\right)^2}{k - 2\alpha(1-\tau_A)} > \frac{4k}{9} - c$. This inequality holds if and only if[21]

$$\tau_A > \frac{1}{9\alpha}\left(-9c + (9\alpha - 2k) + \sqrt{(k - 9c)(4k - 9c)}\right) \equiv \widetilde{\tau}. \tag{26}$$

Likewise, firm A prefers $\tau_A = 1$ to $\tau_A = 0$, if $\tau_B \leq \frac{3\alpha - k}{3\alpha}$; otherwise, if $\tau_B > \frac{3\alpha-k}{3\alpha}$ she prefers $\tau_A = 0$ to $\tau_A = 1$, if and only if $\tau_B > \widetilde{\tau} > \frac{3\alpha-k}{3\alpha}$. The reaction functions are, thus,

$$\tau_B(\tau_A) = \begin{cases} 1 \text{ if } \tau_A \in [0, \widetilde{\tau}] \\ 0\text{,if } \tau_A \in [\widetilde{\tau}, 1] \end{cases},$$

$$\tau_A(\tau_B) = \begin{cases} 1 \text{ if } \tau_B \in [0, \widetilde{\tau}] \\ 0 \text{ if } \tau_B \in [\widetilde{\tau}, 1] \end{cases}.$$

Therefore, there are two asymmetric pure strategy Nash equilibria in the compatibility game, namely, $(\tau_A, \tau_B) = (1, 0)$ and $(\tau_A, \tau_B) = (0, 1)$. Moreover there exists a unique level of τ_i such that firms are indifferent between $\tau_i = 1$ and $\tau_i = 0$, that is $\widetilde{\tau}$, defined by (26). Now, let $\frac{4k-9\alpha}{9} < c < \frac{\alpha}{9}\left(\frac{4k-9\alpha}{k-2\alpha}\right)$, then, if $\tau_A < \frac{3\alpha-k}{3\alpha}$, firm B prefers $\tau_B = 0$ to $\tau_B = 1$, if $\alpha(1 - \tau_A) > \frac{4k}{9} - c \iff \tau_A < \frac{9\alpha - 4k + 9c}{9\alpha} < \frac{3\alpha-k}{3\alpha}$; otherwise, for $\tau_A > \frac{3\alpha-k}{3\alpha}$, firm B prefers $\tau_B = 0$ to $\tau_B = 1$, if $\tau_A > \widetilde{\tau}$. Firm A prefers $\tau_A = 1$ to $\tau_A = 0$, if $\tau_B \leq \frac{3\alpha-k}{3\alpha}$; otherwise, if $\tau_B > \frac{3\alpha-k}{3\alpha}$ she prefers $\tau_A = 0$ to $\tau_A = 1$, if and only if $\tau_B > \widetilde{\tau}$. The reaction functions are, thus,

$$\tau_B(\tau_A) = \begin{cases} 0 \text{ if } \tau_A \in \left[0, \frac{9\alpha-4k+9c}{9\alpha}\right] \\ 1 \text{ if } \tau_A \in \left[\frac{9\alpha-4k+9c}{9\alpha}, \widetilde{\tau}\right] \\ 0 \text{ if } \tau_A \in [\widetilde{\tau}, 1] \end{cases},$$

$$\tau_A(\tau_B) = \begin{cases} 1 \text{ if } \tau_B \in [0, \widetilde{\tau}] \\ 0 \text{ if } \tau_B \in [\widetilde{\tau}, 1] \end{cases}.$$

[21] Define $\Phi(\tau_A) = \frac{\left(\alpha(\tau_A-1)+\frac{2}{3}k\right)^2}{k-2\alpha(1-\tau_A)} - \left(\frac{4k}{9} - c\right)$. $\Phi(\tau_A)$ has two real roots, τ^+ and τ^-. It is straightforward to see that $\tau^- < 0 < \tau^+ < 1$. We denote $\tau^+ = \frac{1}{9\alpha}\left(-9c + (9\alpha - 2k) + \sqrt{(k-9c)(4k-9c)}\right) \equiv \widetilde{\tau}$. This is positive for $c < \frac{\alpha(4k-9\alpha)}{9(k-2\alpha)}$.

Then there is a unique asymmetric pure strategy Nash equilibrium in the compatibility game, namely, $(\tau_A, \tau_B) = (1, 0)$. Also the reaction function of firm B has two jumps: one upwards at $\tau_A = \frac{9\alpha-4k+9c}{9\alpha}$, and one downwards at $\tau_A = \widetilde{\tau}$. Finally, let $\frac{\alpha}{9}\left(\frac{4k-9\alpha}{k-2\alpha}\right) < c < \frac{k}{9}$, in this case, $\widetilde{\tau} < 0$, and therefore the reaction functions become

$$\tau_B(\tau_A) = \begin{cases} 0 \text{ if } \tau_A \in \left[0, \frac{9\alpha-4k+9c}{9\alpha}\right] \\ 1 \text{ if } \tau_A \in \left[\frac{9\alpha-4k+9c}{9\alpha}, \frac{3\alpha-k}{3\alpha}\right] \\ 0 \text{ if } \tau_A \in \left[\frac{3\alpha-k}{3\alpha}, 1\right] \end{cases},$$

$$\tau_A(\tau_B) = \begin{cases} 1 \text{ if } \tau_B \in \left[0, \frac{3\alpha-k}{3\alpha}\right] \\ 0 \text{ if } \tau_B \in \left[\frac{3\alpha-k}{3\alpha}, 1\right] \end{cases}.$$

Notice that, given the increase in the cost with respect to the previous range, both firms start choosing zero compatibility for lower levels of the rival's choice, $(\frac{3\alpha-k}{3\alpha} < \widetilde{\tau})$. Then, there is a unique asymmetric pure strategy Nash equilibrium in the compatibility game, namely, $(\tau_A, \tau_B) = (1, 0)$. Also the reaction function of firm B has two jumps: one upwards at $\tau_A = \frac{9\alpha-4k+9c}{9\alpha}$, and one downwards at $\tau_A = \frac{3\alpha-k}{3\alpha}$. Independently of the cost subsets, the overall compatibility is $\tau = 1$. Equilibria, then respect conditions (17)-(19).

If $c \in [\frac{k}{9}, \infty)$, there is a unique symmetric pure strategy Nash equilibrium in the compatibility game, namely $\tau_A = 0$ and $\tau_B = 0$. Both for $\tau_A > \frac{3\alpha-k}{3\alpha}$ and $\tau_A < \frac{3\alpha-k}{3\alpha}$, the best reply of firm B is to choose $\tau_B = 0$. The overall compatibility level is $\tau = 0$. This equilibrium respects conditions (17)-(19).

5.2.3 Proof of Proposition 5

Consider the following relevant regions for the parameters:

i) $\frac{3\alpha-k}{3\alpha} < 0$

When $\frac{3\alpha-k}{3\alpha} < 0$, the unique equilibrium of the competition stage is the interior equilibrium. Then, the social welfare (16) net of costs is a convex and increasing function. For high values of c, this function is maximized at $\tau = 0$ and for low values, it is maximized at $\tau = 1$. Nevertheless, as shown in the proof of Proposition 4, these compatibility levels do not comply with markets being covered.

ii) $0 < \frac{3\alpha - k}{3\alpha}$

For this range of parameters, the social welfare (16) is defined by:

$$SW(\tau) = \begin{cases} \frac{1}{2}\beta_B + \alpha - c\tau, \ \tau \leq \frac{3\alpha-k}{3\alpha} \\ \beta_B \frac{1}{2} - \frac{\tau^3 - k^2 18\alpha(4-3\tau)+9k\alpha^2(1-\tau)(17-9\tau)}{18(k-2\alpha+2\alpha\tau)^2} - c\tau, \ \tau > \frac{3\alpha-k}{3\alpha} \end{cases}$$

The candidate maxima for this function are: $\tau = 0$, $\tau = \frac{3\alpha-k}{3\alpha}$, or $\tau = 1$. We must, then compare $SW(1)$ with $SW\left(\frac{3\alpha-k}{3\alpha}\right)$ and with $SW(1)$. Simple algebra allows us to conclude that

$$\begin{aligned} &\tau = 1 \text{ if } c < \frac{2}{3}\alpha \\ &\tau = \frac{3\alpha - k}{3\alpha} \text{ if } \frac{2}{3}\alpha < c < \frac{\left(3\alpha\left(4\alpha - 5k\right) + 5k^2\right)\alpha}{6\left(2\alpha - k\right)^2} \\ &\tau = 0, \text{ if } c > \frac{\left(3\alpha\left(4\alpha - 5k\right) + 5k^2\right)\alpha}{6\left(2\alpha - k\right)^2}. \end{aligned}$$

6 Appendix B

6.1 Proof of Lemma 2

Let us start by solving the maximization problem of firm B. In the first domain of the profit function, the profit is increasing in the price, as as such, it attains its maximum in the border of the interval in which this branch of the profit is defined, i.e. $p_B = p_A + \alpha(1-\tau)$. The second branch of the profit function is concave and it attains its maximum at $p_B = \frac{1}{2}(k - \alpha(1-\tau)) + \frac{p_A}{2}$. Whenever this maximum falls outside the relevant domain, i.e. $\frac{1}{2}(k - \alpha(1-\tau)) + \frac{p_A}{2} \leq p_A + \alpha(1-\tau)$, or equivalently $p_A \geq k - 3\alpha(1-\tau)$ the optimum is $p_B = p_A + \alpha(1-\tau)$. Whenever $p_A \leq k - 3\alpha(1-\tau)$, the optimum is $p_B = \frac{1}{2}(k - \alpha(1-\tau)) + \frac{p_A}{2}$. Evidently, given that the optimal solution for firm depends on whether p_A is superior or inferior to $k - 3\alpha(1-\tau)$, we must guarantee that this value is positive. In case $k < 3\alpha(1-\tau)$, then p_A is always higher than $k - 3\alpha(1-\tau)$ and as such, the only relevant best reply for firm B is $p_B = p_A + \alpha(1-\tau)$.

Let us now solve the maximization problem of firm A. In the first domain of the profit function, the profit is increasing in the price, therefore it attains its maximum in the border of the interval in which this branch of the profit is defined, i.e. $p_A = p_B - (k - \alpha(1-\tau))$. The second branch of the profit function is concave and it attains its maximum at $p_A = \frac{1}{2}(p_B - \alpha(1-\tau))$. When $\frac{1}{2}(p_B - \alpha(1-\tau)) \in (0, p_B - (k - \alpha(1-\tau))]$, or equivalentely, $\alpha(1-\tau) < p_B \leq 2k - 3\alpha(1-\tau)$, the optimum obtains at $p_A = p_B - (k - \alpha(1-\tau))$, when $\frac{1}{2}(p_B - \alpha(1-\tau)) < 0$, or equivalently, $p_B \leq \alpha(1-\tau)$ then $p_A = 0$. Finally, when $\frac{1}{2}(p_B - \alpha(1-\tau)) > p_B - (k - \alpha(1-\tau))$, that is, $p_B > 2k - 3\alpha(1-\tau)$, the global maximum obtains at $p_A = \frac{1}{2}(p_B - \alpha(1-\tau))$.

6.2 Proof of Lemma6

Consider the situation in which for $p_B - p_A \in [k - \alpha(1-\tau), \alpha(1-\tau)]$ the equilibrium demand is such that $D_A \in (0,1)$ and $D_B = 1 - D_A$. Looking at the profit function of firm B overall it is easy to see that: it is nondecreasing as long as $p_B < p_A + k - \alpha(1-\tau)$, it has a downward jump to zero at $p_B = p_A + k - \alpha(1-\tau)$, after that it starts increasing again as long as $p_B < p_A + \alpha(1-\tau)$ and it is zero otherwise. As such, the maximum is attained at $p_B = p_A + \alpha(1-\tau)$ if this price is lower than the limits imposed by conditions (17)-(19) on the prices. Otherwise, the reaction function of firm B is the upperbound of the price region, which for $k < 2\alpha(1-\tau)$ is defined by

$$\begin{aligned}
&p_B < p_A + \alpha(1-\tau), \\
&p_B > p_A + k - \alpha(1-\tau), \\
&p_B < \frac{\alpha\left(1-\tau^2\right) - k\tau}{1-\tau} + p_A \frac{k - \alpha(1-\tau)}{\alpha(1-\tau)},
\end{aligned}$$

A similar reasoning applies to firm A.

6.3 Consumer types distribution

Throughout the model we assume that consumers indexed by x are uniformly distributed in the interval $[0,1]$. The uniform distribution can be seen as a particular *Beta* distribution with parameters a and b such that $a = b = 1$. Formally, the Beta cumulative distribution function is

$$F(x; a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x u^{a-1}(1-u)^{b-1} du,$$

with $a > 0$, $b > 0$ and $x \in [0, 1]$. Solving consumers' maximization problem, in the interior solution case, we find the indifferent consumer, $\hat{x}$, defined by 1. Assuming rational expectations, we can then set $D_A = F(\hat{x})$ and $D_B = 1 - F(\hat{x})$. It is easy to see that under the uniform distribution, $F(x; 1, 1) = x$, which in turn implies that $D_A = \hat{x}$ and $D_B = 1 - \hat{x}$. As a robustness check, we here consider two alternative asymmetric distributions for consumers: $Beta(1, 2)$ and $Beta(2, 1)$, with $F(x; 1, 2) = 2x(1 - (x/2))$ and $F(x; 1, 2) = x^2$. In the first case, consumers concentrate more around zero; whereas in the second case, consumers concentrate more around one. Solving for the indifferent consumer, and in turn for the demands, we find that for any given price pair, D_A under the $Beta(1, 2)$ distribution is higher than D_A under the $Beta(1, 1)$ distribution, which is higher than D_A under the $Beta(2, 1)$ distribution. The result is rather intuitive, the more consumers are concentrated around low types which have a low valuation of quality, the higher the demand for the low quality good. In turn, this affects the results of the model since the demands influence the overall quality differential, measured by $k + \alpha(D_B - D_A)(1 - \tau)$. Indeed, the relative weight of the exogenous quality differential k with respect to the network effect determines the outcome of the full game. Namely, the higher D_A, the smaller the high quality firm advantage. Thus, for a given k, it is more likely for good A to become the good with higher overall quality and to conquer the market.

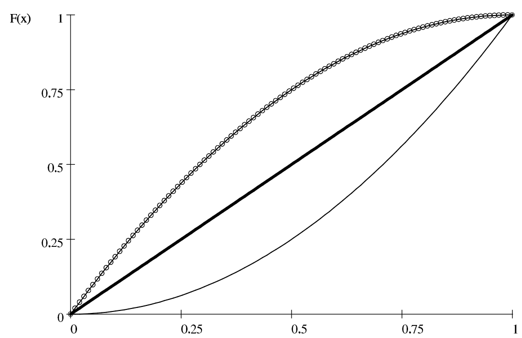

Figure 11: $Beta(1,1)$, solid thick line; $Beta(2,1)$, (lower) solid think line; $Beta(1,2)$, (upper) dots line.