Regulatory design under asymmetric information about demand

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Abstract

In this paper we compare the costs of two regulatory policies about the entry of new firms. We consider an incumbent firm that has more information about the market demand than the regulator. Then, the incumbent firm can use this advantage to persuade the regulator to make entry more difficult. With the first regulatory policy the regulator uses the incumbent price pre-regulation to get information about the demand. With the second regulatory policy the regulator design a mechanism to motivate the incumbent firm to price truthfully. We conclude that, for enough high values of the probability of low demand, the welfare is higher with the second (more active) regulatory policy.

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1. Introduction

To enter in the markets can be a difficult process for the firms as a result of institutional or economic reasons. Entry regulation is one of these reasons. In many countries there are a lot of administrative and bureaucratic requirements that make entry a long and hard way. Sometimes (for instance, in the mobile phone market, due to the limitations associated with the spectrum management), the governments fix the number of entrants. This is what we call here direct entry regulation.

Several authors have studied the effects of entry regulation on the performance of the markets. Djankov et al. (2002) for instance have analyzed "data on the regulation of entry of start-up firms in 85 countries". They concluded that "countries with heavier regulation of entry have higher corruption and larger unofficial economies, but not better quality of public and private goods. Countries with more democratic and limited governments have lighter regulation of entry".

Here we show that the cost of entry regulation can also result from the strategic behavior of the incumbent firms when they use its superior knowledge of the market to persuade the regulator to make entry more difficult or to prevent entry. Asymmetric information is considered by the literature and by the regulatory authorities an important feature that must be taken into consideration to the design of the regulatory policy. Regulated firms have many times more information about themselves and the market than the regulator. This informational advantage can be strategically exploited by the incumbent firms to avoid greater competition in the market. As in many cases the authorities have a direct or indirect influence on the number of firms that can enter the market the regulated incumbent firm can use its knowledge about the market to persuade the regulator to prevent the entry of some new firms, for instance, maintaining the level of entry barriers. One of the market characteristics that generally the regulated incumbent firm knows better than the regulator is the market demand. Firms have superior knowledge of the quality of the products and of the expected reaction of the consumers to that quality and they have closer and more frequent contact with their customers than the regulator (Tracy and Sappington, 1988).

One way how the regulator can get information about the demand size is by observing the price fixed by the incumbent firm. Although the regulator has other resources that it can use to measure the demand size, the price fixed by the incumbent
firm is an important piece of information. Then, the regulated incumbent firm can fix the price in order to mask the true size of demand in the spirit of Milgrom and Roberts (1982) model. The differences with Milgrom and Roberts is that here we consider that the incumbent firm tries to prevent entry using the regulator policy more than the expectations of the entrant. We show here that such a strategic behavior by the incumbent firm can be part of a pooling equilibrium following the work of Kim (2001). Differently from Kim we consider asymmetric information about demand and not about costs. Then, we ask if the regulator that seeks welfare maximization would benefit from having more initiative in the relation with the incumbent firm. Even if it has some degree of lack of information about the demand size the regulator can design a mechanism that will motivate the incumbent firm to price truthfully. Is it a better solution? The problem is that it results in some additional costs to the regulator in order to motivate the incumbent to accept the best contract. Our main concern here is to understand if more initiative by the regulator is rewarding compared to the pooling equilibrium in the signaling game. This is an important issue to understand the best behavior of the regulator in situations where some kind of entry regulation is necessary.

The tendency for liberalization and deregulation of several utilities observed in most European countries in the last few decades has reduced the direct intervention of the authorities in the definition of the number of firms that participate in each industry. The air transport sector is an example of this trend. However, we can advance several arguments, either theoretical or resulting from empirical observation, that support the present importance of entry regulation. In some sectors, a large number of firms might decrease social welfare, because of scale economies, network externalities or entry costs but, from an individual standpoint, the industry can be attractive. This is the Excess Entry Theory applied by Mankiw and Winston (1997) to oligopoly markets. Additionally, in industries with partial liberalization often entry is gradual and controlled. This is happening, for instance, in mobile telecommunications where, due to the scarcity of a vital input, the radio spectrum, new firms need the regulator's approval to enter. Before conceding licenses, regulators define the number of firms that can operate in the industry. The policy of the British regulator in the mobile segment of the telecommunications industry provides an example of entry regulation. In 1985, the regulator authorized the entry of two firms (Cellnet and Vodafone), following the model applied in the United States for the mobile telephone market. In 1991, two further mobile operators were licensed with the restriction of no further entry before 2005
(Newbery, 2000, p.323). Also, we can give a broader interpretation of entry regulation and consider that it means the public authority's actions that make entry easier or more attractive. In this context the public authority's decision is about the administrative and bureaucratic procedures that must be accomplished to enter the market or about the intensity of the entry promotion policy, as happens, for instance, in the definition of the remedies that accompany merger authorizations. Then, the motivation for entry regulation is the promotion of entry.

The structure of the paper is the following: section 2.1 describes the model with symmetric information. Then, section 3 analyses the regulatory policies under asymmetric information. Two hypothesis are considered: a regulatory policy under which the regulator clearly defines the number of entrants after observing the price fixed by the incumbent firm and a regulatory policy under which, before the observation of the incumbent's price, the regulator gives information about how it will decide the maximum number of entrants saying that this decision depends on the incumbent's price. Then, the incumbent chooses the price and, after observing this decision, the regulator applies the regulatory rule. These two different regulatory policies are modelized with a signalling game for the first case (described on section 2.2.1) and with an adverse selection game for the second case (described on section 2.2.2). Section 2.3 compares the two regulatory policies regarding its regulatory costs. Section 3 presents the main conclusions.

2. The Model

We assume a framework with linear demand, no variable costs, a fixed cost of $F$ for each firm and $n$ potential entrants. Demand can be low ($D^L$), represented by $P=1-Q$ or can be high ($D^H$) represented by $P=a-Q$ with $a>1$. At first there is only one firm in the market that defines the price and obtains profits. Then, after entry, there is the definition of another price and profits for all firms. The incumbent firm wants to maximize the sum of its profits of the two periods and the regulator wants to maximize social welfare, defined as the sum of consumer surplus and firms’ profits, in the second period.

In the description of the model we assume the following notation:
a) \( \pi_i^p(p^k) \) represents the monopolist profit when demand is of type \( i \) and the price is \( p^k \) (k=L,H). If \( i=k \), the monopolist has the maximum profit; if \( i\neq k \), the monopolist is giving up some profit.

b) \( \pi_i^i(n^i) \) represents the incumbent firm’s profit after entry when demand is of type \( i \) and there are \( n^i \) new firms. If \( i=j \) social welfare is maximized.

### 2.1 The Model with Symmetric Information

The time of the game with perfect information is the following:

- At stage 0 Nature chooses the demand size, \( D^L \) or \( D^H \) with probability \( r \) and \( 1-r \), respectively. All the agents, incumbent firm, entrants and regulator, observe the demand size.

- At stage 1 the incumbent firm chooses the price that maximizes its profit. The optimal monopolist price is represented by \( p^L \) or \( p^H \) if demand is \( D^L \) or \( D^H \), respectively.

- At stage 2 there are \( n \) firms that want to enter in the market regardless of demand size. However, the regulator defines the maximum number of new firms in order to maximize the social welfare. These numbers are represented by \( n^L \) or \( n^H \) for \( D^L \) or \( D^H \), respectively.

- At stage 3 the oligopolist interaction between the firms leads to the establishment of another price and of the corresponding profits.

Considering Cournot competition at the stage 3 we have the following results:

**Stage 1:**

\[ p^L = \frac{1}{2} \] and \( \pi_i^L(p^L) = \frac{1}{4} - F \) if \( D^L \) or \( p^H = \frac{a}{2} \) and \( \pi_i^H(p^H) = \frac{a^2}{4} - F \) if \( D^H \).

**Stage 2:**

\[ n^L = \frac{1}{\sqrt{F}} - 1 \] if \( D^L \) and \( n^H = \frac{1}{\sqrt{F}} a^2 - 1 \) if \( D^H \).

**Stage 3:**

\[ \pi_i^L(n^L) = \frac{1}{(1/\sqrt{F})^2} - F \] ; \( W^L(n^L) = \frac{(1/\sqrt{F} - 1)^2}{2(\frac{1}{\sqrt{F}})^2} + \frac{3/\sqrt{F} - 1}{(3/\sqrt{F})^2} - (3/\sqrt{F} - 1)F \) if \( D^L \)

and
It is important to notice that the regulatory problem described only exists if the number of firms that wish to enter in the market is high. This requires that the entrants profit is positive, meaning that entry is not blocked by the fixed costs. Considering the low demand, this condition requires $F<1$.

### 2.2 The Model with Asymmetric Information

We consider that the regulator has less information than the incumbent firm regarding the demand size.

To study the effects of asymmetric information we consider two scenarios that represent two different regulatory policies. One regulatory policy corresponds to the situation where the regulator, without knowing the demand size, sets the maximum number of entrants after observing the price defined by the incumbent firm. This regulatory policy can be modelized by a signalling game, where the regulator considers the incumbent's price as a signal about the demand size.

The other regulatory policy demands a more active attitude from the regulator. Before the definition of the price by the incumbent firm, the regulator let the incumbent firm know how many firms will enter the market depending on the incumbent price. Then, after observing that price, the regulator decides definitively the number of firms that will in fact enter the market. This regulatory policy is approached with an adverse selection model.

#### 2.2.1 The signalling model

The time of the signalling game is the following:
- At stage 0 Nature chooses the demand size, \( D_L \) or \( D_H \) with probability \( r \) and \( 1-r \), respectively. Only the incumbent firm observes this choice.
- At stage 1 the incumbent firm chooses the price and obtains the corresponding profits.
- At stage 2 the regulator observes the incumbent’s price and updates the beliefs about the demand size. Then, it decides the maximum number of entrants.
- At stage 3, the oligopolist interaction between the firms leads to the establishment of another price and of the corresponding profits.

In the description of the signalling model we assume the following notation:

a) The parameters \( p \) and \( q \) represent the regulator’s updated beliefs about the size of demand after observing the price. Hence, \( p \ (q) \) represents the probability of low demand if the incumbent’s price is \( P_H \ (P_L) \).

b) The strategies of the players (incumbent firm and regulator) have to specify how they behave in every possible scenario in which they are called to act. Therefore, in the signalling game the players’ strategies are represented by a pair of values. For the incumbent firm the pair \((p^x, p^y)\) means that it chooses price \( p^x \) if demand is \( D_L \) and it chooses price \( p^y \) if demand is \( D_H \). For the regulator the pair \((n^w, n^z)\) means that the regulator chooses to allow the entry of \( n^w \) new firms if the incumbent firm has chosen a price equal or below \( p^x \), and the regulator chooses to allow the entry of \( n^z \) new firms otherwise.

This signalling game has a pooling Perfect Bayesian Equilibrium (PBE)\(^1\) described by Proposition 1:

**Proposition 1:** The strategies and beliefs represented by

\[
\left( \frac{1}{2}, \frac{1}{2}, (n^p, n^H), (q = r, p = 0) \right)
\]

with \( n^p = \frac{1}{F} r + \frac{1}{F} a^2 - \frac{1}{F} a^2 r - 1 \) are a Perfect Bayesian Equilibrium if

\[
\pi_1^H (p^L) + \pi_2^H (n^p) \geq \pi_1^H (p^H) + \pi_2^H (n^H)
\]

**(Proof of Proposition 1)**: See Appendix A.

\(^1\) For a discussion about the existence of other equilibria (pooling and separating) see Sarmento and Brandão (2003).
The pooling PBE can be described in the following way: whatever the demand size, the incumbent firm chooses the price that maximizes the profit when demand is low. The objective of this strategy is to keep unclear to the regulator whether demand size is enough to accommodate many new firms. The regulator observes this price and updates its beliefs: the probability of low demand if the observed price is $p^L$ becomes $r$ and the probability of low demand if the observed price is $p^H$ becomes $p$. Then, the regulator allows the entry of $n^p$ firms if the price is $p^L$, or $n^H$ firms if the price is $p^H$. Notice that $n^p$ is an intermediate value between $n^L$ and $n^H$. At the equilibrium described, the incumbent firm strategically uses the entry regulation and the private information about demand to induce the regulator to protect its market from competition.

The existence of this pooling PBE requires the verification of condition (1) which has an intuitive explanation. We can interpret condition (1) by saying that the limit price strategy is attractive to the incumbent firm when demand is $D^H$, that is, the incumbent firm prefers to lose some profit initially and share the market with few firms, than to maximize the profit initially and after share the market with many firms.

At this equilibrium, the regulatory policy described by the signalling game has a regulatory cost that results from the fact that the number of authorized entrants is not the one that maximizes social welfare. If demand is $D^L$ ($D^H$) the regulator authorize $n^p$ and the social optimal number is $n^L$ ($n^H$).

The expected value of the regulatory cost at the pooling PBE is given by:

$$r[(W^L(n^L) - W^L(n^p)) + (1 - r)(W^H(n^H) - W^H(n^p))]$$

### 2.2.2 Adverse selection model

The adverse selection model represents the regulatory policy with more initiative by the regulator. With this policy the regulator gives information about how it will decide the maximum number of entrants dependent on the incumbent's price. Then, after observing the incumbent's price, the regulator applies the regulatory rule. The regulator's objective with this design is to induce the incumbent firm to truthfully reveal the demand size through its decision about the price. The information given by the regulator about how it will decide the maximum number of entrants is represented by a contract described as following: if the incumbent's price is equal or lower than $p^L$ the regulator authorizes $n^L$ entrants, otherwise the regulator authorizes $n^2$ entrants. We can
represent this policy saying that the regulator offers two contracts: \((p \leq p_L, n_L)\) and \((p > p_L, n_2)\). The design of these contracts is made in order that if demand is \(D_L\) the incumbent firm has an incentive to choose the first contract, and if demand is \(D_H\) the incumbent firm has an incentive to choose the second contract.

The time of the adverse selection game is the following:
- At stage 0 Nature chooses the demand size, \(D_L\) or \(D_H\) with probability \(r\) and \(1-r\), respectively. The incumbent firm observes this choice.
- At stage 1 the regulator announces the regulatory policy about entry.
- At stage 2 the incumbent firm chooses the price and obtains the corresponding profits.
- At stage 3 the regulator applies the regulatory policy deciding how many new firms can enter the market.
- At stage 4 the oligopolist interaction between the firms leads to the establishment of another price and of the corresponding profits.

The regulatory policy announced by the regulator can be represented by:

\[
   n = \begin{cases} 
   n_L & \text{if } p \leq p_L \\
   n_2 & \text{if } p > p_L 
   \end{cases}
\]

where \(n_2\) maximizes the expected value of the social welfare \(E(W) = rW^L(n^L) + (1-r)W^H(n_2)\) subject to the following conditions (incentive compatibility conditions):

i) \(\pi^I_1(p^L) + \pi^I_2(n^L) \geq \pi^I_1(p > p^L) + \pi^I_2(n_2)\)

ii) \(\pi^I_1(p > p^L) + \pi^I_2(n_2) \geq \pi^I_1(p^L) + \pi^I_2(n^L)\)

Notice that if \(n_2\) maximizes the expected value of social welfare, then \(n_2 > n^L\). Therefore, if demand is \(D_L\) the incumbent firm maximizes its profit choosing \(p^L\) and it also induces the authorization of few firms. Therefore, the first condition is verified.

If, on the contrary, demand is \(D_H\), the incumbent firm only chooses the second contract if \(\pi^I_1(p^H) + \pi^I_2(n_2) \geq \pi^I_1(p^L) + \pi^I_2(n^L)\) as the choice of \(p > p^L\) implies that the incumbent firm chooses \(p^H\) because with this price it maximizes the profit. This condition represents the incentive that the regulator must give to the incumbent firm with \(D_H\) in order to avoid that the incumbent's firm will choose \(p^L\) with the objective of restricting entry.

The regulator's problem can be written as:
Maximize \( rW^L(n_L) + (1 - r)W^H(n_2) \)

s.a. \( \pi^H_1(p^H) + \pi^H_2(n_2) \geq \pi^H_1(p_L) + \pi^H_2(n_L) \)

The solution for \( n_2 \) is

\[
\frac{1}{3} \left( \frac{a^2}{2} - \frac{a}{4} \right) \leq \frac{a^2}{4} - \frac{1}{3}.
\]

The value of \( n_2 \) must be between \( n^L \) and \( n^H \). Notice that \( n^L \) and \( n^H \) maximize the functions \( W^L(n) \) and \( W^H(n) \), respectively, and \( n_2 \) is the value that maximizes a linear combination of \( W^L(n) \) and \( W^H(n) \). This conclusion is synthesized in Proposition 2.

**Proposition 2.** The menu of contracts \((p^L,n^L)\) and \((p^H,n_2)\) with

\[
n_2 = \frac{1}{3} \left( \frac{a^2}{2} - \frac{a}{4} \right) \leq \frac{a^2}{4} - \frac{1}{3},
\]

solves the regulator's maximization problem described by the adverse selection model.

What is the cost of this regulatory policy?

The regulatory cost results from allowing \( n_2 \) entrants instead of \( n^H \) when demand is high. However, the commitment to \( n_2 < n^H \) is necessary to induce the incumbent firm to reveal the truth. The incumbent firm with \( D^H \) only chooses \( p^H \) if it has the promise of \( n_2 < n^H \) entrants.

The implementation of this revelation mechanism has the following expected cost:

\[
(1 - r)[W^H(n^H) - W^H(n_2)].
\]

### 2.3 Comparison of the regulatory policies

The regulatory policy described by the signalling model is preferred to the regulatory policy described by the adverse selection model if the expected costs of the former are lower than the expected costs of the latter, that is, if

\[
r[W^L(n_L) - W^L(n^p)] + (1 - r)[W^H(n^H) - W^H(n^p)] < (1 - r)[W^H(n^H) - W^H(n_2)]
\]

which is equivalent to
After substituting in this inequality for the values of \( n^L, n^p \) and \( n_2 \) this gives origin to a long inequality that we could not solve for \( r \). Nevertheless, we can establish a range of values of \( r \) for which the left hand side of the inequality is not lower than the right hand side. For that we proceed in the following way: first we note that the left hand side is always positive, as \( n^L \) is the value that maximizes \( W^L \). Then, for the cases where the right hand side is negative the inequality does not hold. After we note that the function \( W^H(n) \) is monotonically increasing in \( n \), reaching a maximum for \( n^H \). Also, we note that \( n^p < n^H \) and \( n_2 < n^H \). Then, when \( n^p < n_2 \), the right hand side of the inequality is negative. Therefore we proceed studying the expression \( n^p - n_2 \). We have seen by the value of the derivative that this expression decreases with \( r \). Finally, we calculate the value of \( r \) that turns \( n^p - n_2 \) into 0. With these two last results we prove Proposition 3.

**Proposition 3**: If \( r > r^* \), with \( r^* = \frac{F}{27(1-a^2)} \left(\frac{4a^2(1/F)^2}{4a^2 - (1/F)^3(1-a)^2} + 2\right)^3 - \frac{a^2}{1-a^2} \), the right hand side of the inequality (2) is negative. Then, the regulatory policy described by the adverse selection solution has a lower expected regulatory cost than the regulatory policy described by the pooling equilibrium of the signalling game.

**Proof of Proposition 3**: See Appendix B.

Therefore, if the probability of having low demand is high, the regulatory policy described by the adverse selection model is preferred to the regulatory policy described by the signalling model. This conclusion goes as expected because if demand is low the regulatory policy described by the adverse selection model does not have any regulator cost.

To illustrate this result we consider an example with \( F=0.1 \) and \( a=2 \). Figure 1 represents \( n^p - n_2 \). The value of \( r \) that turns \( n^p - n_2 \) into 0 is \( r^* = 0.56486 \). Then, for \( r > 0.56486 \) the regulatory policy approached by the adverse selection model is preferred by the regulator.
Although we can not know what is the better policy when the right hand side is positive we can argue that in this case there are also values of $r$ for which the policy represented by adverse selection is preferred by the regulator, depending on the value of $r$. In general the greater is $r$ the greater is the probability that the policy represented by adverse selection is preferred. As an illustration, we show what happens with $F=0.1$ and $a=2$. For this example, the policy approached by adverse selection is preferred to the policy approached by signaling as long as long as $r > 0.26$. Then, when the probability of low demand is high it is better for the regulator to adopt a more active behavior represented by the adverse selection game.

3. Conclusions

As we have shown here the incumbent firm can use its superior knowledge about the market demand to influence the regulatory policy about the entry of firms in the market. We have compared two kinds of answers from the regulatory authority to the strategic behavior of incumbent firms. In one of them the regulator has a passive behavior looking to the price fixed by the incumbent and taking it as a signal of the size of demand. We have shown that an equilibrium for such a game could be one where even if demand is high the incumbent firm fixes the price corresponding to low demand in order to send a signal to the regulator that will persuade him to make difficult the entry of new firms. The other way of response from the regulator that we have considered here demands a more active attitude. The regulator proposes a menu of contracts to the incumbent firm to create a mechanism that motivates the incumbent to tell the truth when it fixes the price.
We have compared from the point of view of the welfare the two types of response from the regulator. More specifically we analyzed the lost of welfare in each of the situations of equilibrium in relation to the symmetric information equilibrium and we called the difference as the cost of each of the asymmetric situations considered. Then, we have compared the two costs. The conclusion is that, in general, for high values of the probability of low demand the welfare is higher with the more active regulatory policy.

References

Appendix A

The proof of proposition 1 is done in three steps:

1. Let \( \left( \frac{1}{2}, \frac{1}{2} \right) \) be the best reply of the incumbent firm. Then, the regulator's information set corresponding to \( p^L \) is on the equilibrium path. Hence, the regulator's belief is updated by Bayes' rule and the incumbent firm's strategy, being \( q=r \), the prior belief. This means that after observing the price \( p^L \), the regulator has no additional information about demand. Then, the regulator chooses the value of \( n \) that maximizes the expected value of social welfare represented by:

\[
E(W(n)) = rW^d(n) + (1-r)W^H(n)
\]

The result is:

\[
\frac{d(E(W))}{dn} = 0 \Rightarrow n^p = \sqrt{\frac{1}{F} a^2 + \frac{1}{F} r - \frac{1}{F} r a^2} - 1
\]

Notice that this proof is independent of the value of \( P^L \).

2. It is necessary to prove that \( \left( \frac{1}{2}, \frac{1}{2} \right) \) is the incumbent's best response when \( (n^p,n^H) \) and \( (q=r,p=0) \). For that it is necessary to demonstrate that choosing \( p^L \) is the best option for the incumbent firm.

If the incumbent firm chooses \( p^L \) the payoff is:
\[
\pi_i^L(p^L) + \pi_i^L(n^p) = \frac{1}{4} - F + \frac{1}{\left(\frac{1}{F} (a^2 + r - ra^2)\right)^2} - F 
\] (1) \text{ if } D^L

or

\[
\pi_i^U(p^L) + \pi_i^U(n^p) = \frac{2a-1}{4} - F + \frac{a^2}{\left(\frac{1}{F} (a^2 + r - ra^2)\right)^2} - F 
\] (2) \text{ if } D^H

It is necessary to compare these payoffs with the payoffs of choosing \(p^H\). For this we must specify how the regulator would reply to \(p^H\). Let us assume, by now, that the regulator's reply to \(p^H\) is \(n^H\) and afterwards, at step 3, we present the proof of this statement.

Then, the incumbent payoff is:

\[
\pi_i^L(p^H) + \pi_i^L(n^H) = \frac{a(2-a)}{4} - F + \frac{1}{\left(\frac{1}{F} a^2\right)^2} - F 
\] (3) \text{ if } D^L

or

\[
\pi_i^U(p^H) + \pi_i^U(n^H) = \frac{a^2}{4} - F + \frac{a^2}{\left(\frac{1}{F} a^2\right)^2} - F 
\] (4) \text{ if } D^H

As \(n^p < n^H\) and as the profit function is monotonically decreasing to the right of \(n^L\) then \(1) \geq (3)\).

Also, \(2) \geq (4)\) as long as
\[
F \geq \frac{(a-1)^3(a^2 + r - ra^2)}{4^2 a \left[ (a^2)^3 - (a^2 + r - ra^2)^3 \right]^{2/3}} = F^*. 
\]

3. To demonstrate that choosing \(n^H\) after observing \(p^H\) is a regulator's best reply it is necessary to prove that the expected value of social welfare is greater with \(n^H\) than with \(n^p\). If \(p=0\) this condition is verified.

Appendix B
Proof of Proposition 3:

From the model with symmetric information we know that

\[ W^H(n) = \frac{a^2n^2}{4n + 2n^2 + 2} + \frac{a^2n}{2n + n^2 + 1} - Fn \]

is a quadratic function on n with a maximum for \( n = n^H \). From the signal and the adverse selection solutions we know that \( n^p < n^H \) and \( n_2 < n^H \). Therefore, if \( n^p < n_2 \) then \( W^H(n^p) - W^H(n_2) < 0 \), which means that condition (2) is false.

Knowing that \( n^p = \sqrt{\frac{1}{F} a^2 + \frac{1}{F} r - \frac{1}{F} ra^2 - 1} \) and \( n_2 = \frac{1}{3} a - \frac{1}{4} a \sqrt{a^2 + 1 - \frac{1}{4}} \) it is easier to conclude that \( n^p - n_2 \) is monotonically decreasing on r and it is negative for

\[ r > r^* \text{ with } r^* = \frac{F}{27(1-a^2)} \left( \frac{4a^2(\frac{1}{F})^\frac{2}{3}}{4a^2 - (\frac{1}{F})^\frac{2}{3}(1-a)^2} + 2 \right)^3 - \frac{a^2}{1-a^2} \]