Public Deficits and Economic Growth

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Within the discussion on how to implement fiscal discipline in the European Monetary Union, the adequacy of using the same fiscal rules for different Member-Countries has been challenged by some authors, calling for a temporary higher degree of fiscal flexibility in the case of small and less developed countries.

We develop a model of a monetary union between two countries that may differ in (i) economic dimension; (ii) exogenous levels of productivity directly related with the quality of domestic institutions; (iii) technological knowledge stocks; (iv) Research and Development (R&D) capacity.

By using comparative statics, the model allows us to examine the impact of fiscal shocks leading to public deficits and thus analyse (i) the eventual different implications of such deficits whether they occur in the small or in the big country; and (ii) whether an excessive deficit should (or not) be allowed in order to permit the catching up by the less developed country; (iii) whether some expenses related to public investment (incentives to R&D) should (or not) be considered as “pertinent” factors as considered in the recent SGP reform.

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1. Introduction

With the creation of the European Monetary Union (EMU), the framework for the definition and implementation of macroeconomic policies has dramatically changed. Member-Countries have lost their exchange rate and money supply instruments and the use of budgetary measures has been restrained by binding rules aimed at avoiding the creation and maintenance of excessive public deficits.

In 1992, the Maastricht Treaty set up the framework for the definition and implementation of national fiscal policies, limiting them by binding fiscal rules, including maximum ceilings of 3% and 60%, respectively for the public deficit to GDP ratio and for the public debt to GDP ratio, and urging for policy coordination. In 1997, the Stability and Growth Pact (SGP) reinforced the restrictive option taken in Maastricht, introducing “automatic” sanctions and the medium-term objective of budgetary equilibrium.

This solution has been subject of deep discussion and criticism in political and academic circles, mainly before 1995 and after 2000. The debate is focused on the way in which fiscal discipline should be implemented and controlled (e.g., Buiter et al., 1993; Rubio and Figueras, 1998) and not on the need for fiscal discipline.

In fact, fiscal discipline is generally taken as an essential means of avoiding negative external effects resulting from deficient budgetary behaviour (e.g., De Grauwe, 2005), namely a possible increase in the interest rate of the EMU, leading to possible pressures on the European Central Bank (ECB) to implement a more expansionist monetary policy, thus leading to an increase in inflation. Fundamentally, the European framework for fiscal policies has been put in place because of the need to keep price stability (e.g., Baldwin and Wyplosz, 2004).

To date, the literature presents an interesting discussion on the definition and implementation of fiscal rules, namely in the case of the EMU, positions varying from those who support current rules (Begg et al., 2004; Buti et al., 2005) to those who claim for
a significant reform (Casella, 1999; Creel, 2003; Pisani-Ferry, 2004; Wyplosz, 2005). In particular, it shows several topics where divergence is more profound, two of them being: the possibility of considering some expenses concerning public investment (namely incentives to R&D) as exceptions regarding the application of binding fiscal rules; and the possibility of differentiation of fiscal rules among countries, by taking into account the levels of economic divergence and the economic dimension of each country.

The debate is not yet closed, but it may already have made a relevant contribution to the recent SGP reform (European Council, 2005). The reformed SGP allows for a growing number of circumstances that lead to a non-automatic application of the sanctions, namely considering a diversified kind of public expenses that may justify the non-compliance to the “3 per cent” rule. As far as the present paper is concerned, it is relevant to note that, within that set, expenses regarding R&D are included.

In order to analyse whether or not such kind of public expenses should be treated differently and whether or not European fiscal rules should differ between countries, we consider a standard economic structure in endogenous R&D-growth theory, for two countries that compose a monetary union. In each country, the production of perfectly competitive final goods uses institutions and labour together with a continuum set of quality-adjusted intermediate goods. Intermediate goods, in turn, use designs (resulting from R&D activities) under monopolistic competition. The production function, adapted from the horizontal R&D growth models developed by Kiley (1999) and Acemoglu and Zilibotti (2001), incorporates substitutability between countries in the production of final goods.

As a result of the close relationship between the production of intermediate goods and R&D, this one can be encouraged either by a direct subsidy or through a subsidy to the production of intermediate goods. Such policies have a negative impact on the fiscal
budget of each country and that situation may lead to adverse consequences, such as those prevented by the SGP.

However, these policies may reduce the technological-knowledge gap between the two countries. In this case, they would be fundamental for an increase in the economic convergence within the union and, in particular, for the economic growth performance of the poorer country, which could justify different fiscal rules among countries. As apparent above, this will be the focus of the present work.

By assumption, countries differ in four features. The first one relates to economic dimension, measured by labour endowments: the one with higher active population is called Big, the other one is called Small. The second feature concerns domestic institutions, which are more advanced in the Big-country. The third feature is related to R&D capacities, the Big country being an innovator and the Small country being an imitator. The last feature is an endogenous consequence of the others and relates to the domestic quality indexes measuring technological knowledge, which are higher in the Big-country.

In a previous work (Afonso and Alves, 2006), we analysed a similar problem, but by taking into consideration complementarity between country-specific inputs and substitutability between countries. In this context, both countries were innovators, although the more developed being the more efficient in R&D activity.

The paper is structured as follows. Section 2 describes the model. Section 3 determines the equilibrium conditions. Section 4 analyses the effects of a governmental intervention. Finally, section 5 offers some concluding remarks.

2. The model

2.1. Final-goods sector
Each final good \( n \in [0, 1] \) is produced by one of two countries, the Small-country, \( S \), and the Big-country, \( B \). The former (latter) brings institutions, \( A_S (A_B) \), and labour, \( L_S (L_B) \), together with a continuum set of quality-adjusted intermediate goods, indexed by \( j \in [0, J] \) produced either in \( S \) or \( B \). The output of \( n \), \( Y_n \), at time \( t \) is,

\[
Y_n(t) = \left[ \int_0^J z_n(k, j, t) \left( 1 - \alpha \right) dj \right] \left[ \left( 1 - n \right) A_S^{\frac{1}{\alpha}} L_S, n \right] + \left[ n A_B^{\frac{1}{\alpha}} L_B, n \right]^\alpha.
\] (1)

By considering \( z_n(k, j, t) = q^{k,j,n} x_n(k, j, t) \), the integrals denote the contribution of intermediate goods to production. In the Schumpeterian tradition, each \( j \) used in the production of the final good \( n \) is quality-adjusted; i.e., the quality upgrade is \( q > 1 \), and \( k \) is the top-quality rung at time \( t \). The quantity of \( j \) with quality \( k \) at \( t \) can be produced by either \( B \), \( x_{B,n}(k, j, t) \), after a successful innovation or by \( S \), \( x_{S,n}(k, j, t) \), after a lower-priced successful imitation of the leading quality.\(^1\) Thus, the quality level of \( j \) rises entirely due to the R&D innovative activity. However, both countries use the state-of-the-art intermediate goods in their final goods production; i.e., \( k_B \geq k_S \), which can be produced domestically or not. In the latter case, countries import the higher quality of \( j \) for domestic use. The term \( 0 < (1 - \alpha) < 1 \) is the aggregate intermediate-goods input share.

The second and third terms on the right-hand side of (1) represent the role of the labour to production in \( S \) and in \( B \), respectively. Thus, \( 0 < \alpha < 1 \) is the labour input share. These terms include the labour levels of each country, where, by assumption, \( L_B > L_S \). The term \( A \) represents the level of exogenous productivity reliant on country’s institutions. As \( B \)’s institutions are, by hypothesis, more advanced, we consider \( A_B > A_S > 1 \), which means that an absolute productivity advantage of \( L_B \) over \( L_S \) is accounted. A relative productivity

\(^1\) That is, we are in the presence of a Vernon-type intermediate good cycle with production shifts from \( B \) to \( S \) with successful imitation and back with next successful innovation.
advantage of either type is captured by \((1-n)\) and \(n\), which implies that \(L_S\) (\(L_B\)) is relatively more productive in final goods indexed by smaller (larger) \(n\).

Thus, the production function of final goods (1) merges complementarity between inputs, (worldwide) intermediate goods and (country-specific) labour, with substitutability between countries, \(B\) and \(S\). The optimal choice of the producer country is reflected in the equilibrium threshold \(\bar{\pi}\),

\[
\bar{\pi} = \left\{ 1 + \left[ \frac{A_B^{1-\alpha} L_B}{A_S^{1-\alpha} L_S} \right]^{\frac{1}{2}} \right\}^{-\frac{1}{2}}.
\]  

(i) which comes from profit maximization (by perfectly competitive final goods producers and by monopolist producers of intermediate goods) and full employment equilibrium in factor markets, given the labour supply and the technological knowledge;

(ii) where the switch from one country to the other becomes advantageous and an increase in \(\bar{\pi}\) would mean a larger space for production in \(S\), thus appearing as a measure of its relative competitiveness. For example, a larger relative supply of labour, \(L_B/L_S\), and/or a higher relative productivity concerning the quality of national institutions, \(A_B/A_S\), results in a small \(\bar{\pi}\) and so in higher fraction of final goods produced in \(B\). Hence, optimally only \(S\) produces final goods indexed by \(n \leq \bar{\pi}\) and only \(B\) produces final goods with \(n > \bar{\pi}\); i.e.,

\[
L_B = x_n(k, j, t) = 0 \text{ for } \forall 0 < n \leq \bar{\pi} \text{ and } L_S = x_n(k, j, t) = 0 \text{ for } \forall \bar{\pi} \leq n \leq 1,
\]  

and the demand for each intermediate good by the producer of \(n\) is:

\[
x_n(k, j, t) = \left[ p_n(t) (1-\alpha) q_{(1-n)} \left( \frac{1}{\pi} \right) \right] \left\{ (1-n) A_B^{1-\alpha} L_S(t), \forall 0 < n \leq \bar{\pi}; \text{if } n \text{ is produced in } S \right\}

\[ n A_B^{1-\alpha} L_B(t), \forall \bar{\pi} \leq n \leq 1; \text{if } n \text{ is produced in } B \right\}.
\]  

where \(p_n\) and \(p(k, j, t)\) are prices of final good \(n\) and of intermediate good \(j\), respectively.

Both prices are given for the perfectly competitive producers of final goods.

Bearing in mind (4) the production function (1) can be written as:
\[ Y_a(t) = \left[ \frac{p_n(t)(1 - \alpha)}{p(k, j, t)} \right]^{1/\alpha} Q(t) \left[ (1-n) A_3^{1/\alpha} L_{S,H} + n A_1^{1/\alpha} L_{B,H} \right], \text{ where } Q(t) = \int_0^1 q^{1/\alpha} dj \] (5)

is an aggregate quality index of the stock of technological knowledge.

Since it is indifferent to produce \( \pi \) in \( B \) or \( S \), it can be used to express price indexes:

\[ \frac{p_B}{p_S} = \left[ \frac{\pi}{1 - \pi} \right]^\alpha, \] (6)

where \( p_B \) and \( p_S \) are the index prices of final goods produced by \( B \) and \( S \), respectively. In addition, since the aggregate or composite output of the union is obtained by integration over final goods and by normalizing its price at each time \( t \) to one (numeraire) – i.e., that

\[ Y(t) = \int_0^1 p_n(t) Y_a(t) \, dn = \exp \left[ \int_0^1 \ln p_n(t) \, dn \right] \exp \left[ \int_0^1 \ln Y_a(t) \, dn \right] = \exp \left[ \int_0^1 \ln Y_a(t) \, dn \right], \] (7)

which are the resources of the union that are available for consumption, \( C \), production of intermediate goods, \( X \), and R&D, \( R \): \( Y = C + X + R \) – and by considering (2), we can write

\[ p_S(t) = p_n(t)(1-n)^\alpha = \exp(-\alpha) \pi(t)^{-\alpha} \quad \text{and} \quad p_B(t) = p_n(t)n^\alpha = \exp(-\alpha)(1 - \pi(t))^{-\alpha}. \] (8)

The equilibrium aggregate resources devoted to intermediate-goods production, \( X = X_B + X_S \), and the equilibrium aggregate output, \( Y = Y_B + Y_S \), i.e., the composite final good in the union (7), are expressible as a function of the currently given factor levels:\(^2\)

\[ X(t) = \int_0^1 \int_0^1 x_n(k, j, t) dj \, dn = \begin{cases} X_S(t) = \left[ \frac{p_S(t) A_3^{1/\alpha}}{p(j, k, t)} \right]^{1/\alpha} L_S Q(t) + \left[ \frac{p_B(t) A_3^{1/\alpha}}{p(j, k, t)} \right]^{1/\alpha} L_B Q(t) \end{cases}, \] (9a)

\[ Y(t) = \int_0^1 p_n(t) Y_a(t) \, dn = \begin{cases} Y_S(t) = \left[ \frac{1 - \alpha}{p(j, k, t)} \right]^{1/\alpha} p_S^{1/\alpha}(t) A_3^{1/\alpha} L_S Q(t) + \left[ \frac{1 - \alpha}{p(j, k, t)} \right]^{1/\alpha} p_B^{1/\alpha}(t) A_3^{1/\alpha} L_B Q(t) \end{cases}. \] (9b)

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\(^2\) As it will be clear in next section, \( p(k, j, t) \) is independent of \( j \).
Equation (9b) shows clearly that: (i) economic growth is driven by the technological-knowledge progress in \( B \), reflected in the aggregate quality index; (ii) the contribution of \( B \) for the composite final good is higher than the contribution of \( S \), since, by assumption, \( L_B > L_S \) and \( A_B > A_S \).\(^3\)

The price paid for a unit of labour, \( w \), is equal to its marginal product. From (9b), the equilibrium measure of inter-country wage inequality, \( W \), is, at each time \( t \):

\[
W = \frac{w_B}{w_S} = \left( \frac{A_B^{q-1} L_S}{A_S^{q-1} L_B} \right)^{\frac{1}{2}},
\]

i.e., inter-country wage inequality can increase due to an exogenous improvement in \( B \)'s institutions and/or due to an exogenous decrease in labour endowments of \( B \).\(^4\)

2.2. Intermediate-goods sector

Each intermediate good is produced either in \( B \) or \( S \). In the former case, it embodies the latest innovation and in the latter it arises from the imitation, at a lower cost, of the latest innovation. In either case, intermediate goods used in final goods production embody the state-of-the-art technological knowledge accumulated in \( B \) and summarized in \( Q \).

Level effects in \( S \)

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\(^3\) Since \( S \) is not too backward (i.e., an appropriate taxonomy for our \( B \) and \( S \) countries would be developed versus developing, rather than developed versus underdeveloped), it is predictable that inter-country differences in prices of final goods are of second order. Moreover, in the context of a monetary union, with single currency and common market, prices of tradable goods tend to be very similar, as well as national inflation rates.

\(^4\) Since we consider inter-Union labour immobility, it is not possible to solve the problem of inter-country wage inequality through immigration from \( S \) to \( B \).
When compared with a pre-union situation, the improvement in the level of technological knowledge available to S – by access to the state-of-the-art intermediate goods – is a static gain of the union. Indeed, the technological-knowledge gap is always favourable to B; i.e.
\[ Q(t) > Q_b(t) = \int_0^t q_{2(t)} \left( \frac{1-a}{a} \right) dj, \]
and in a pre-union the technological knowledge level available to S is just the domestic.\(^5\)
Thus, S enjoys an immediate absolute and relative (to B) benefit in terms of aggregate output and factor prices since the marginal productivity of labour increase with \( Q \).

Moreover, the structure of final goods production is also affected, but in both countries. In fact, in pre-union each country produces all final goods, while after the union there is a final goods specialisation determined by differences in domestic labour endowments and domestic quality of institutions – see (2).

**Limit pricing of intermediate goods**

Since, by assumption, the production of intermediate goods and R&D are financed by the resources saved after consumption of the composite final good, the simplest hypothesis is to consider that, in each country, the production function of intermediate goods is identical to the composite final good specified by equations (1) and (7).\(^6\) Given this convenient simplification, the marginal cost of producing each \( j \) equals the marginal cost of producing the aggregate output, which, due to perfect competition in the final-goods sector, equals the price of the aggregate output; i.e., 1 (numeraire). Thus, the marginal cost of producing \( j \) is independent of its quality level and is identical across all domestic industries.

In order to allow for the entry of S’s intermediate goods in the union market; i.e., to allow producers in S of the same quality rung \( k \) to under-price its competitor in B, we

\(^5\) Note that, even under the union, not all innovations have been imitated yet at each \( t \).

\(^6\) Or, equivalently, that the composite final good is the input in the production of each intermediate good, as in Barro and Sala-i-Martin (1997), for example.
assume that the government in \( S \) subsidises the production of intermediate goods by paying a fraction ad-valorem, \( z_s \), of each firm’s production cost. Thus, the after subsidy marginal cost of intermediate goods production in \( S \) is \((1-z_s)\).\(^7\)

The production of an intermediate good involves a start-up cost of R&D, either in a new design invented in \( B \) or in its imitation (by reverse engineering) in \( S \). This investment can only be recovered if profits are positive within a certain period in the future. This is assured by costly R&D together with domestically enforced patents, which protect, inside but not outside the country, the leader firm’s monopoly of that quality good, while at the same time disseminating obtained knowledge to other domestic firms. Thus, knowledge of how to produce the latest quality good is public (non-rival and non-excludable) inside and semi-public (non-rival and partially non-excludable) outside each country.

Even without inter-country protection of patents, the current producer of \( j \) enjoys some inter-country monopoly power: for example, if s/he is from \( B \), thus being challenged by either another producer in \( B \) or by an imitator is \( S \), monopoly is temporarily assured by enforced patents in \( B \) and by costly imitation in \( S \). However, the length and magnitude (measured by the mark-up) of the monopoly power are shortened by the union – in pre-union situation the producer in \( B \) can be only challenged by other producer in \( B \) and not by an imitator in \( S \) with lower effective marginal cost (due to the subsidy).

Following Grossman and Helpman (1991, ch. 12), we consider that limit pricing by each leading monopolist is optimal. In general, depending on whether \( q(1-\alpha) \) is greater or lesser than 1, the leader of each \( j \) would, respectively, use the monopoly pricing, \( 1/(1-\alpha) \), or the limit pricing, \( q \), to capture the entire domestic market (e.g., Barro and Sala-i-Martin, 2004, ch. 7). To rule out monopoly pricing, we assume that the size of each quality, \( q \), is

\(^7\) Alternatively, we could consider that any of the governments can subsidise intermediate goods production, by means of an ad-valorem subsidy \( z_r \), which can be country-specific: i.e., \( z_{r,S} \) in \( S \) and \( z_{r,B} \) in \( B \) and \( z_{r,S} > z_{r,B} \).
not large enough. Under the union case, there are three possible sequences of successful R&D outcomes and their limit pricing consequences at $t$, given quality $k$ at $t-dt$, are:

**Table 1. Limit pricing of each intermediate good**

<table>
<thead>
<tr>
<th>$t-dt$</th>
<th>$t$</th>
<th>Share in $j \in [0,J]$ production at $t$</th>
<th>$p(j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$ produces and exports quality $k$</td>
<td>$B$ innovates, produces and exports quality $k+1$</td>
<td>$\Xi (1-\gamma)$</td>
<td>$p_{B,B}(j) = q$</td>
</tr>
<tr>
<td>$B$ produces and exports quality $k$</td>
<td>$S$ imitates, produces and exports quality $k$</td>
<td>$1-\Xi$</td>
<td>$p_{S,B}(j) = 1$</td>
</tr>
<tr>
<td>$S$ produces and exports quality $k$</td>
<td>$B$ innovates, produces and exports quality $k+1$</td>
<td>$\Xi \gamma$</td>
<td>$p_{B,S}(j) = q (1-z_s)$</td>
</tr>
</tbody>
</table>

The first mark-up is the highest – the entrant in $B$ competes with an incumbent in $B$ at the same (effective) marginal cost but with better quality. The second one is smaller – the entrant in $S$, with lower effective marginal cost, competes in the same quality rung with an incumbent in $B$. Compared with the first, the third mark-up is again smaller, but due to a different reason – the entrant in $B$ improves quality as in the first case, but competes with an incumbent with lower effective marginal cost.

In order to pin down which intermediate goods are produced in each country at each $t$, let: (i) $\Xi$ and $(1-\Xi)$ be the share of intermediate goods with production in $B$ and in $S$, respectively; (ii) $\gamma$ be the share of intermediate goods produced in $B$ having overcome imitator competition; (iii) $(1-\gamma)$ be the share of intermediate goods produced in $B$ having overcome innovator competition. The specification of these shares as functions of the probabilities of successful R&D follows Dinopoulos and Segerstrom (2006), such that the share of intermediate goods produced in $B$ increases with the probability of successful innovation and decreases with the probability of successful imitation.
We can now define a price index for intermediate goods, at each $t$, as a weighted average of the limit prices in Table 1:

$$\bar{p}(j) = 1 + \sum (q - 1) - \sum \gamma q z_x.$$  \hfill (12)

2.3. R&D sector

As suggested by (9b) R&D drives economic growth in $B$ and in $S$. A more detailed description of the technology of R&D activities is thus in order with the purpose of closing the characterization of $B$ and $S$ domestic economies.

R&D activities in $B$ result in innovative designs for the production of intermediate goods, which increase their quality. The designs are domestically patented and the leader in each $j$, which produces according to the latest patent, uses limit pricing to assure monopoly. The value of the leading-edge patent relies on the profit-yields accruing during each $t$ to the monopolist, and on the duration of the monopoly power. The duration, in turn, depends (i) on the probability of a new innovation, which creatively destroys the current leading-edge design in the lines of the Schumpeterian models (e.g., Aghion and Howitt, 1992); or (ii) on the probability of an imitation in $S$ (e.g., Grossman and Helpman, 1991, ch. 12). The probabilities of successful innovation and imitation are, thus, at the heart of R&D.

Let $I_B(k, j, t)$ denote the instantaneous probability at $t$ – a Poisson arrival rate – of successful innovation in the next higher quality $[k(j, t) + 1]$ in $j$,

$$I_B(k, j, t) = y_B(k, j, t) \beta_B q^{k(j, t) - 1} q^{-a_k} q^{-a_{k(j, t)}},$$  \hfill (13)

(i) $y_N(k, j, t)$ is the flow of final-good resources devoted to R&D in $j$ in $B$;
(ii) $\beta_B q^{k(j, t)}$, $\beta_B > 0$, is the positive learning effect of accumulated public knowledge from past successful R&D – e.g., Grossman and Helpman (1991, ch. 12) and Connolly (2003);
(iii) $\xi_s^{-1} q^{-\sigma k(j,t)}$, $\xi_s > 0$, is the adverse effect caused by the increasing complexity of quality improvements in $j$ (e.g., Kortum, 1997, and Dinopoulos and Segerstrom, 2006).

The positive learning effect, (ii), is thus modelled in such a way that, together with the adverse effect, (iii), it totally offsets the positive influence of the quality rung on the profits of each intermediate good leader firm, as we can see below.

In the absence of the union, the R&D process in $S$ mimics the R&D process in $B$, but less efficiently, i.e., with $k_s \leq k$ in (12). Since $S$ is less developed, but not too backward, we assume that there are some intermediate goods, but not all, for which $k_s < k$, implying that even in the absence of trade there are some state-of-the-art intermediate goods produced in both countries (i.e., for which $k_s = k$). Once $S$ has access to all the best quality intermediate goods due to the union, it becomes an imitator, improving the probability of successful R&D. Thus, R&D activities in $S$, when successful, result in imitation of current worldwide best qualities. Denoting the probability of successful imitation by $I_s(k, j,t)$ – the instantaneous probability of successful imitation of the current higher quality $k(j,t)$ in $j$,

$$I_s(k, j,t) = y_s(k, j,t) \beta_s q^{k_s(j,t)} \xi_s^{-1} q^{-\sigma k(j,t)} f\left(\tilde{Q}(t)\right)^{-\sigma+\tilde{Q}(t)}$$

where:

(i) $y_s(k, j,t)$ is the flow of final-good resources devoted to R&D in $j$ in $S$;

(ii) $\beta_s q^{k_s(j,t)}$, $0 < \beta_s < \beta_B$, $k_s \leq k$; i.e., we consider the learning effect of accumulated imitations lower than the learning effect of accumulated innovations;

(iii) $\xi_s^{-1} q^{-\sigma k(j,t)}$, $\xi_s > 0$; i.e., we consider that the adverse effect caused by the increasing complexity of quality improvements in $j$ is lower in the imitation situation;

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8 Since the Big-country is more developed, it can be alternatively considered that $\beta_B \xi_B > \beta_s \xi_s$; i.e., that $B$ has a better innovation capacity than $S$. 
(iv) \( f\left( \tilde{Q}(t) \right)^{-\sigma \tilde{Q}(t)} = \left( \tilde{Q}(t)(1-\tilde{Q}(t)) \right)^{-\sigma \tilde{Q}(t)} \), is a catching-up term, specific to \( S \), which includes positive effects of technological knowledge backwardness, since \( \sigma > 0 \) and 0 < \( \tilde{Q}(t) \equiv \frac{Q_S(t)}{Q_B(t)} \equiv \frac{Q_S(t)}{Q(t)} < 1 \) is the relative technological knowledge level of \( S \). That is, function \( f \) is quadratic over the range of interest, and, once affected by the exponent function \( -\sigma + \tilde{Q}(t) \), yields an increasing (in the technological-knowledge gap) advantage of backwardness – where the size of \( \sigma \) affects how quickly the probability of successful imitation falls as the technological-knowledge gap falls.

In addition to the direct effect of the union on the capacity of imitation, the level effect increases the aggregate income in \( S \) and thus more resources are available for R&D.

As mentioned earlier, we will allow any of the governments to subsidise R&D activities directly, by means of an ad-valorem subsidy \( z_r \), which can be country-specific (i.e., \( z_{r,S} \) in \( S \) and \( z_{r,B} \) in \( B \)).

2.4. Consumers

A time-invariant number of heterogeneous individuals in the union (also as in each country) – continuously indexed by \( a \in [0, 1] \) – decide the allocation of income, which is partly spent on consumption of the composite final good, and partly lent in return for future interest. For simplicity, we consider an exogenous threshold individual \( \overline{a} \), smaller than 0.5, since \( L_B = \int_{\overline{a}}^1 da \) > \( L_S = \int_{0}^{\overline{a}} da \): individuals \( a > \overline{a} \) are located in \( B \), whereas individuals \( a \leq \overline{a} \) are located in \( S \).

The infinite horizon lifetime utility of the individual \( a \) is the integral of a discounted constant elasticity of substitution (CIES) utility function,

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\(^9\) Consumers and government’s behaviour is similar to that described in Afonso and Alves (2006).
where: (i) $c(a, t)$ is the amount of consumption of the composite final good by the individual $a$, at $t$; (ii) $\rho > 0$ is the homogeneous subjective discount rate; and (iii) $\theta > 0$ is the inverse of the inter-temporal elasticity of substitution.

The budget constraint of individual $a$ equalises income earned to consumptions plus savings, at each $t$. Savings consist of accumulation of financial assets – $K$, with return $r$ – in the form of public debt owned by individuals and in the form of ownership of the firms that produce intermediate goods in monopolistic competition.\(^{10}\) The budget constraint, expressed as savings $+$ consumptions $=$ income, is

$$\dot{K}(a, t) + c(a, t) = [1 - \tau_K] r(t) K(a, t) + \begin{cases} [1 - \tau_{w, s}] w_s(a, t), & \text{if } a \leq \overline{a} \\ [1 - \tau_{w, b}] w_b(a, t), & \text{if } a > \overline{a} \end{cases},$$

where:

(i) $K(a, t)$ is thus the total asset holdings of the individual $a$, with return $r$; (ii) $w(a, t)$ is the wage of the individual $a$, at $t$; (iii) $\tau_K$ and $\tau_w$ are the ad-valorem taxes on assets and wages, respectively, which may be used by the government for fiscal policies purposes (in particular, as a means of financing, at least partially, the costs of the above mentioned subsidies); (iv) $w$ and $\tau_w$ may differ between countries, but not $\tau_K$;\(^{11}\) (v) $r$ is the same within the union, as a natural consequence of the monetary union.\(^{12}\)

\(^{10}\) The value of these firms, in turn, corresponds to the value of patents in use.

\(^{11}\) Note that wages are endogenously determined, while taxes are exogenous parameters. We assume that $\tau_K$ is the same in the two countries while $\tau_w$ may differ, as in the case of the European Union, the mobility of capital is largely greater than the mobility of labour, thus determining a higher degree of harmonisation in the case of taxation over financial assets revenues.

\(^{12}\) Also note that due to arbitrage in the domestic assets market, $r$ depends on $t$, but is independent from the type of labour.
Maximising (15) subject to (16) yields the growth rate of consumption, which is independent of the individual and is the standard Euler equation:

\[
\dot{c}(a,t) = \dot{\hat{c}}(t) = \hat{C}(t) = \frac{1}{\theta} \left[ \left( 1 - \tau_k \right) r(t) + \rho \right], \text{ where } C(t) \equiv \int_0^t c(a,t) \, da.
\] (17)

Since, in addition, to firms and individuals, both economies can also be influenced by domestic government policies, in order to finalise the characterisation of both economies, a description of the government’s budget is in order.

2.5. Government

In this model, the government of each country may intervene by imposing taxes on wages and/or on financial assets and by subsidising the production of intermediate goods and/or R&D activities. If necessary, the government may run a public deficit by issuing public debt sold to individuals.

The budget superavit, \( Bu_S \), of \( S \) and \( B \) is given respectively by:

\[
(1 + r(t)) Bu_{S}(t) = \tau_k \, r(t) \int_0^\pi K(a,t) \, da + \tau_{a, S} \int_0^\pi w(a,t) \, da - z_{s, S} \, X_S(t) - z_{r, S} \, R_S(t) - r(t) D_S(t); \tag{18a}
\]

\[
(1 + r(t)) Bu_{B}(t) = \tau_k \, r(t) \int_0^\pi K(a,t) \, da + \tau_{a, B} \int_0^\pi w(a,t) \, da - z_{s, B} \, X_B(t) - z_{r, B} \, R_B(t) - r(t) D_B(t). \tag{18b}
\]

Where \( D_S \) and \( D_B \) represent the public debt in \( S \) and \( B \), respectively. Thus, the first and second terms on the right-hand side represent government tax revenue from assets income and from labour income, respectively, while the third and fourth terms represent government expenditure on subsidies for intermediate goods and for R&D, respectively, and the last term relates to interest paid on public debt.\(^{13}\)

\(^{13}\) We know that \( Bu_S(t) = Taxes(t) - Subsidies(t) - r(t) \, D(t - dt) \) and \( D(t) = D(t - dt) - Bu_S(t) \), which gives \( D(t - dt) = D(t) + Bu_S(t) \), leading to equations (18a, b). For a matter of simplification and in order to focus the discussion just on direct subsidies to R&D, we will assume \( z_{s, B} = 0 \) and no taxes.
We will be particularly interested in the effects of higher levels of subsidies to R&D in the less developed country, regarding an eventual convergence towards the level of development of the other country. Such an effect would become an argument in favour of different fiscal rules among countries in the union, namely a temporary authorisation for higher ratios between public deficit and GDP in the less developed countries. In the same context, this would also be an argument in favour to certain “pertinent” factors included in the SGP reform.

3. Equilibrium

Once characterised the countries’ economic structure for given states of technological knowledge, we proceed to include the equilibrium dynamics of technological knowledge, which drives economic growth. The interaction effects between $B$ and $S$, arising from the union, play a crucial role in the dynamic general equilibrium.

3.1. Equilibrium R&D

Given the functional forms (13) and (14) of the probabilities of success in R&D, free-entry equilibrium is defined by the equality between expected revenue and resources spent. Taking, for example, the case of imitation; i.e., $S$, such equality takes the form

$$I_s(k, j,t) V_s(k, j, t) = (1 - z_{r,S}) y_s(k, j, t),$$

where:

$$V_s(k, j, t)$$

is the expected current value of the flow of profits to the monopolist producer of $j$ – the value of the monopolist firm owned by consumers in $S$ or the market value of the patent. The expected flow of profits depends on the amount in each $t$, the interest rate, and the expected duration of the flow, which is the expected duration of technological-knowledge leadership. Such duration, in turn, depends on the probability of a successful innovation in $B$, which is the potential challenger. The expression for $V_s(k, j, t)$ is
The amount of profits at \( t \), for the monopolist producer of \( j \), using an imitation of quality \( k \), \( \Pi_s(k, j, t) \), depends on the marginal cost, the mark-up, and the world demand for intermediate good \( j \) by the final-goods producers. Its expression is – bearing in mind the second sequence in Table 1:

\[
\Pi_s(k, j, t) = z_{s,S} (1 - \alpha) a^{-1} q^{(j,j)(1-\alpha)a^{-1}} \left[ L_S \left( A_s p_s \right)^{1-a^{-1}} + L_B \left( A_B p_B \right)^{1-a^{-1}} \right].
\]  

(21)

Plugging (21) into (20) and then (20) and (14) into (19) and solving for \( I_B \), the equilibrium probability of a successful innovation in \( B \) – given the interest rate and the price indexes of final goods – is

\[
I_B(t) = \beta_s \xi_s^{-1} f(\tilde{Q}(t))^{-\sigma+\tilde{Q}(t)} \tilde{Q}(t) \left( \frac{z_{s,S}}{1-z_{r,S}} \right) (1-\alpha)^{a^{-1}} \left[ L_S \left( A_s p_s \right)^{1-a^{-1}} + L_B \left( A_B p_B \right)^{1-a^{-1}} \right] - r_s(t), \forall k.
\]

(22)

The equilibrium \( I_B \) turns out to be independent of \( j \) and \( k \), due to the removal of scale of technological-knowledge effects – the positive influence of the quality rung on profits and on the learning effect is exactly offset by its influence on the adverse effect induced by increasing complexity (see the exponents of \( q \) in (21) and in (14)-(ii) and (iii)).

Since the probability of successful innovation, as a Poisson arrival rate, determines the state-of-the-art technological-knowledge progress, equilibrium can be translated into the path of technological knowledge in \( B \), which allows \( S \) to benefit as well. The relationship turns out to yield the following expression – where (22) is plugged in – for the equilibrium rate of growth of technological knowledge:

\[
V_s(k, j, t) = \frac{\Pi_s(k, j, t)}{r_s(t) + I_B(k, j, t)}.
\]

(20)
\[
\hat{Q}(t) = \left[ \beta_s \xi_{s}^{-1} f\left(\hat{Q}(t)\right)^{\sigma+\hat{r}(t)} \left(1 - \frac{z_{s,T}}{1 - z_{r,s}}\right) \right]^{1-\hat{r}(t)}
\]

\[
(1-\alpha)^{\hat{r}(t)} \left[ L_s \left( A_s, p_s \right)^{\alpha-1} + L_B \left( A_B, p_B \right)^{\alpha-1} \right] - r_s(t) \left[q^{(1-\alpha)\hat{r}(t)} - 1\right].
\]  

It is clear in (23) that there are feedback effects from imitation to innovation. That is, the positive level effect from B to S – the access to the state-of-the-art intermediate goods increases production and thus resources available to imitation – feeds back into the innovator, affecting its technological knowledge through creative destruction. Since subsidies in S improve technological knowledge in B, they improve not only the domestic level of development but the level of development of the union.

The equilibrium aggregate resources devoted to R&D, \(R\), at each time \(t\), are

\[
R(t) \equiv \begin{cases} 
R_s(t) = \int_{0}^{t} y_s(k, j, t) \, dj = \xi \beta^{-1} f(\hat{Q}(t))^{\sigma-\hat{r}(t)} I_s(t) Q(t) \\
R_B(t) = \int_{0}^{t} y_B(k, j, t) \, dj = \xi \beta^{-1} I_B(t) Q(t)
\end{cases}
\]

Like for \(Y\) and \(X\) – see (9a) and (9b) –, equation (24) shows that also resources devoted to R&D are positively related with the technological knowledge in B.

3.3. Steady state

In each county and thus in the union, the aggregate final good, \(Y\), is used for consumption, \(C\), and savings, which in turn are allocated between production of intermediate goods, \(X\), and R&D, \(R\). Since both countries have the access to the state-of-the-art intermediate goods and they have the same technology of production of final goods – except for the levels of exogenous productivity and labour endowments –, in steady state they present differences in the levels but not in the growth rates. The common and stable steady state growth rate is thus equal to growth rate of the technological knowledge in \(B\), because \(Y, X, \ldots\)
\( R \) and \( C \) are all constant multiples \( Q \). Through the Euler equation (17), the steady state interest rates, \( r^* (= r_s^* = r_B^*) \), are also equalized between countries.

The common and stable steady state growth rate, designed by \( g^* (= g_s^* = g_B^*) \) is thus:

\[
g^* = \ddot{Q}^* = \ddot{Y}^* = \ddot{X}^* = \ddot{R}^* = \ddot{C}^* = \ddot{c}^* = \frac{1}{\theta} \left[ 1 - \tau_k \right] r^* - \rho , \tag{25}
\]

implying constant steady-state levels of Big-Small gap in technological knowledge. Indeed, while entire convergence in available technological knowledge is instantaneous with the union (level effect), domestic levels may not converge completely; that is, \( \ddot{Q} \), which remains constant in steady state, may remain below one.

Clearly, R&D drives steady-state endogenous growth. The intensity of the driving force is, in turn, influenced by the union. In order to look at the steady-state effects of the union we must investigate \( g^* \) further. To this end, since \( g^* \) results directly from plugging \( r^* \) into the Euler equation, it is sufficient to compare the steady-state interest rate:

\[
r^* = \left( \left( 1 - \tau_k \right) + \theta \left[ q^{(1-\alpha)x^{-1}} - 1 \right] \right)^{-1} \left\{ \theta \beta_s \xi_s^{-1} f(\dot{Q})^{-\sigma + \sigma^*} \ddot{Q} \left( \frac{z_{s,s}}{1 - z_{r,s}} \right) \right\} , \tag{26}
\]

obtained by setting the growth rate of consumption in (17) equal to the technological knowledge growth rate in \( B \) given by (23), with the one that would prevail in a pre-union steady state. Bearing in mind that the advantages of backwardness term vanish from the probability of successful imitation (14) in the absence of the union,\(^ {14} \) the increment in the steady-state interest rate, from pre-union to union, depends on the difference

\(^ {14} \) In pre-union the R&D process in \( S \) mimics the R&D process in \( B \), but less efficiently (see section 2.3).
While evaluation of equation (27) requires solving for transitional dynamics through calibration and simulation, we can, however, emphasize five ways, in addition to the level effects, through which the union influences, in opposite directions, steady-state growth.

The first way in which the union influences steady-state growth is the positive catching-up effect on the probability of successful imitation. The advantages of backwardness are only obtained in the presence of the union (or alternatively under trade). Through the feedback effect described above, the probability of successful innovation is also affected and thus the steady-state growth rate – see equations (22) and (23).

The second way is the positive spillovers from B to S. Each innovation in B tends to lower the cost of imitation by S because the backwardness advantage is strengthened with each improvement of the technological-knowledge frontier.

The third way is the positive effect arising from market enlargement, which encourages R&D activities by effecting the respective profitability, as (21) shows.

The fourth – counteracting – channel is the monopolistic competition mark-up. The monopolist in B loses profits with the entry into the union: the average mark-up between the first and third situations in Table 1 above is smaller than $q$, which is the pre-union mark-up. The reason for this is that in pre-union successful innovators are protected from international competition. Once engaged in the union and imitation becomes profitable, profit margins in B are reduced, which discourages R&D activities.

The fifth – counteracting as well – way through which the union affects steady-state growth, is that firms in S have to support the R&D cost of state-of-the-art intermediate goods, possibly several quality rungs above (and thus more complex) their own experience level in pre-union. This is reflected in the presence of the gap $\tilde{Q}$ in (27).

\[ f(\tilde{Q}^*)^{-\rho} - \tilde{Q}^* \left[ L_s \left( A_s p_s^* \right)^{\alpha-1} + L_b \left( A_b p_b^* \right)^{\alpha-1} \right] - (1 - z_{x,s})^{\alpha-1} \left( \frac{q-1}{q} \right) L_s A_s^{x-1}, \]
The effect of the union on the steady-state growth rate is, thus, ambiguous. However, the comparative statics of the steady state interest rate $r^*$ in (26) – or alternatively of $g^*$ – is not affected by such ambiguity because the reported changes refer to steady-state under union. $r^*$ or $g^*$ are affected by the levels of exogenous variables and parameters, which is to be expected in an endogenous growth model. In particular, both countries’ exogenous levels of productivity ($A_N$ and $A_S$) and parameters of R&D technology ($\beta_S$ and $\xi_S^{-1}$) improve the common growth rate through their positive effect on the profitability of R&D, as (21) and (22) demonstrate. The impact on steady-state growth of an increase in the subsidy towards the production of intermediate goods in $S$, $z_{c,S}$, results from the combination of typical Schumpeterian-R&D effects: it implies a smaller effective marginal cost of production for the intermediate-goods producers in $S$, thereby encouraging imitative R&D and innovative R&D (feedback effect).

4. Government intervention

In this section, which is currently under construction, we solve numerically the transitional dynamics towards the steady state in order to illustrate the effect of government intervention on the country-specific technological knowledge. Using such results, we analyse whether different fiscal rules may be needed (or not) in order to offset divergences in development among member-countries within a monetary union.

5. Concluding remarks

The main purpose of this paper is to analyse if there are sustainable reasons for the existence of different fiscal rules among different Member-Countries and for the consideration of some kind of public expenses, namely incentives to R&D as exceptions in what concerns the application of the SGP rules.
To this purpose, we develop a dynamic general-equilibrium growth model with two countries forming a monetary union. Growth is driven by Schumpeterian-R&D applied to intermediate goods which complement labour in each country. In this context, we analyse the effects of a governmental intervention through subsidising (directly or indirectly) R&D activities and compare them to a situation with no governmental intervention.

For the moment, such model highlights the following main results:

(i) there is a level effect when the monetary union is created, which improves resources available for consumption and investment in the less developed country;

(ii) these resources have feedback effects on the other country and thus affecting the endogenous force of growth within the union, which is R&D innovative activity;

(iii) in spite of this, comparing the situation of pre-union and post-union, the effects on overall growth rate seem to be ambiguous;

(iv) in the context of existence of a monetary union, both countries’ exogenous levels of productivity and parameters of R&D technology improve the common growth rate through their positive effect on the profitability of R&D;

(v) similarly, an increase in direct or indirect subsidies to R&D also increase the common growth rate.

This last result suggests the pertinence of making some exceptions to the European framework for fiscal discipline, namely concerning to expenses leading to more resources devoted to R&D activities. In particular, this would apply to measures taken by the small and less developed countries, in this case possibly also leading to an easier catching-up. This will be analysed through transitional dynamics towards the steady state by solving numerically the model.
References


