This paper proposes a theoretical model of spacial duopoly, where the location, on the one hand, and the absorptive capacity of firms as function of their internal R&D investment, on the other hand, endogenously determine the maximum level of knowledge spillovers they might absorb. Our goal is to test whether this new modelling of spillovers modifies the traditional conclusions on firm location choices.

In this three-stage game, firms choose first their geographical location, then their level of internal R&D expenditures, and finally their prices.

We found that, at the optimum, firms location is symmetrical and tends to the centre of the market as the transportation costs increase, knowledge spillovers being in this case maximum. Moreover firms choose the same level of internal R&D and set the same price independently of their location.

Keywords: spatial agglomeration; endogenous spillovers; absorptive capacity; non-cooperative game

JEL classification: O33, R12, L13

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1 Introduction

This paper proposes a theoretical model of firms location with endogenous knowledge spillovers. In this model of spatial duopoly, the location of firms, on the one hand, and their absorptive capacity, as function of their internal R&D, on the other hand, endogenously determine the maximum level of knowledge spillovers firms might absorb. The introduction of an access to spillovers resulting from a deliberate investment decision aims to complete the existing literature on locational choices.

If the beneficial impact of absorptive capacities is already widespread in the empirical literature (Griffith, Reading, and Van Reenen, 2004; Giuliani and Bell, 2005; Scott, 2003), it remains rarely incorporated within theoretical models of location choices. When spillovers are considered in such models, their presence is assumed as “floating in the air” (Marshall, 1890; Breschi and Lissoni, 2001b; Breschi and Lissoni, 2001a) and their access is not function of firms behaviour. Within this literature, geographical distance is the only barrier to the diffusion of knowledge spillovers (Long and Soubeyran, 1998).

It is, however, widely accepted in the literature that spillovers are endogenous by nature (Rondé and Hussler, 2005; Breschi and Lissoni, 2004; Singh, 2005), suggesting that spatial proximity is neither a necessary nor a sufficient condition of larger knowledge spillovers between actors. Spatial models with exogenous spillovers exclusively generated by geographical distance (Piga and Poyago-Theotoky, 2005) seem, moreover, unable to explain the agglomeration of innovative activities as empirically observed (Catin and Hendrickx-Candela, 2003; Rosenthal and Strange, 2004). Agglomeration of firms towards the center never comes out as an explicit solution of these theoretical models.

We propose here a model that aims to take into account the endogenous nature of spillovers. Through deliberate choice mechanisms, firms define both their location and absorptive capacities. Firms hence determine, through these decisions, the intensity of external knowledge flow they might benefit from. In this sense, we match two areas of applied microeconomics: R&D strategic choices, on the one hand, that largely integrates the endogeneity of spillovers (Cohen and Levinthal, 1989; Kamien and Zang, 2000; Katsoulacos and Ulph, 1998; Kultti and Takalo, 1998); locational choices in spatial competition models, on the other hand, that often neglect the existence of spillovers, or reduce them to an exogenous phenomenon (Mai and Peng, 1999).

The novelty here is to consider that not only spillovers are constrained by geographical distance but also by the internal R&D efforts of each firm. In other words, the extent to which the knowledge generated by a firm affects
the other depends on both its location choice and investment decision. Hence, without an individual R&D effort, a firm does not benefit from large amount of spillovers, as already suggested by Kamien and Zang (2000): “[T]here is no manna from heaven”.

According to Long and Soubeyran (1998) the way one formalises knowledge spillovers has a significant impact on the results one obtains. In our model, the beneficial impact of geographical proximity is constrained by the absorptive capacity built by the firms. Our aim here is to test whether this affects the traditional outcome in terms of location choice found in the literature (i.e. firms meet halfway). The model we propose consists of a three-stage game where firms choose their geographical location, prior to their internal R&D expenditures and finally compete in prices. Section 2 presents the theoretical background of this model. Sections 3 presents the model. We, respectively, solve each stage of the game in sections 4, 5 and 6.

2 Theoretical Background

The introduction of knowledge spillovers in models of location choice is rather recent in the literature and mainly draws on the conclusion found in industrial organisation. In this stream of literature several attempts to endogenise spillovers are considered, starting with Cohen and Levinthal (1989). These papers, mainly focused on R&D strategies (competition vs. cooperation), introduce internal R&D efforts as a catalyst of spillover effects.

Cohen and Levinthal (1989) consider the following preconditions to benefit from knowledge spillover: (i) external knowledge has to be able to diffuse; (ii) firm has to develop a sufficient level of cognitive capacities (absorptive capacity). The more complex the external knowledge, the higher the level of absorptive capacity required.

Since Cohen and Levinthal (1989) pioneering works, the nature of spillovers and the role of absorptive capacity has been refined. Kamien and Zang (2000), hence, explicitly integrate in their model the absorptive capacity into the R&D activity. The effective level of R&D of a given firm depends on its actual R&D, the degree of specificity of its research activities, the intensity of exogenous spillovers and the characteristics of R&D undertaken in competing firms.

Alternative ways to account for the links between firms decisions and the amount of spillovers they can capture have been proposed, among others, in Katsoulacos and Ulph (1998), Kultti and Takalo (1998) and Maret (2003). All these models only consider the adoption of technological strategies (i.e. the nature or specificity of the knowledge firms might consciously
or not disclose to their competitors) and level of R&D expenditures, prior to the competition onto oligopolistic markets. None of these models explicitly accounts for the spatial dimension of spillovers.

The widespread literature on locational choices (Gabzewicz and Thisse, 1992), on the other hand, rarely consider the effects linked to R&D activity and knowledge spillovers, concentrating on the principle of minimal differentiation (Hotelling, 1929). Long and Soubeyran (1998), however, proposes a noticeable exception, developing a model of locational choice in a symmetric Cournot duopoly with knowledge spillovers function of the firms’ locations.

More recently, Piga and Poyago-Theotoky (2004; 2005) release the assumption of symmetric duopoly, complementing the previous models, providing the following expression for spillovers:

$$X_i = x_i + (1 - y_2 + y_1) x_j \forall i, j = \{1; 2\}; i \neq j$$

where $y_i$ is the location of a firm $i$ on a unitary-length interval, $x_i$ is the internal R&D effort of firm $i$ and $X_i$ is the (“effective”) R&D of firm $i$.

The solution found by Piga and Poyago-Theotoky (2005) shows that the lower the transportation costs, the closer the firms and the higher the spillovers. The centrifugal forces of price competition, leading the firms to locate at the market end-points, seems therefore counterbalanced by the centripetal forces generated by spillovers. Agglomeration, however, do not appear as an explicit solution of this model.

This somehow suggests that geographic proximity and its effect on knowledge spillovers are not sufficient to explain the actual phenomenon of agglomeration in innovative activities (Catin and Hendrickx-Candéla, 2003; Rosenthal and Strange, 2004).

The present paper proposes to cover this gap building upon a model of spatial duopoly where the choices in terms of location and absorptive capacities endogenously determine the maximum level of spillovers. The next section presents the main assumptions of our model. Solving our model consists in finding the equilibrium of a non-cooperative game in three stages where the players are the firms, the strategies concerns price levels, R&D expenditures and locations. The payoffs are defined by the profit levels. In a first step, firms select their location. Then they fix their level of R&D expenditures. In the last step they compete in prices.
3 The model

This model builds upon the original prototype of spatial competition model developed by Hotelling (1929) and includes refinements introduced in the literature since 1929.

Two firms are producing an homogenous good at an equal and constant marginal cost $\bar{c} \geq 0$. To simplify the analysis, without loss of generality, we set these marginal costs as null: $\bar{c} = 0$. Consumers are uniformly distributed along the unit interval $[0,1]$ and $y_i$ denotes the location of firm $i$ along this interval: $y_i \in [0;1]$, $\forall i = 1,2$ with $y_2 \geq y_1$.

Firms invest in R&D in order to improve the quality of their products. Moreover, the R&D undertaken by a firm might benefit to the other firm through knowledge spillovers. Formally, the effective R&D effort ($X_i$) of firm $i$ can be represented as follows:

$$X_i = x_i + (1 + y_1 - y_2)x_j^{1-\alpha}x_i^{\alpha}(1 - \alpha) \quad i, j = 1,2 ; i \neq j$$

where $x_i$ refers to firm $i$’s internal R&D effort, and $\alpha$ represents the intensity of exogenous spillovers in the sector under study.

Equation (1) constitutes the main originality of the paper. Indeed, it captures the relationship between spillovers, the geographic distance between firms ($1 + y_1 - y_2$) and firms’ absorptive capacities. In this expression, spillovers depend not only on distance between firms locations, but also on firms internal R&D efforts ($x_i, x_j$). In other words, and contrary to existing literature, we consider absorptive capacities on the one hand and geographic location on the other hand as strategic variables of choice to catch knowledge. Without a minimum internal R&D effort, firm $i$ can not benefit from knowledge created by firm $j$.

As soon as a firm engages in a minimum of internal R&D ($x_i \neq 0$), it benefits from knowledge spillovers. The latter are maximum when firms share the same location ($y_1 = y_2$). Conversely, spillovers are the lower when firms locate at the market endpoints ($y_1 = 0$ and $y_2 = 1$).

The exogenous parameter $\alpha$ refers to the level of exogenous spillovers within the industry. Put differently, it captures the degree of generality of the created knowledge. If $\alpha = 1$, independently from their internal R&D, firms are not able to absorb external knowledge. Knowledge is too specific to diffuse and to benefit to an external agent. R&D expenditures supported by firm $i$ do not benefit to firm $j$. On the contrary when $\alpha = 0$, the knowledge

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1According to Kamien and Zang (2000): “[T]here is no manna from heaven”. 

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created is so common that they do not require any specific capacity to be absorbed. One can think of the public knowledge as diffused through the educational system, for instance. Firm $i$ internal R&D effort benefits in that case to all firms in the industry. The level of this parameter $\alpha$ is therefore linked to institutional factors such as the national patent policy (Cohen and Levinthal, 1989), or strategic factors, such as the firms propensity to codify the knowledge they create (Maret, 2003). According to equation (1), the marginal effect of knowledge absorption decreases with the level of internal R&D efforts: the marginal increase of capacities to take advantage of others ideas is larger when firm $i$ undertakes a low level of internal R&D than when it already invests a lot in R&D.

A consumer located at $s$ ($s \in [0; 1]$), who decides to buy from firm $i$, derives the following utility:

$$u_s = v + X_i - p_i - t (s - y_i)^2$$

(2)

where $v \geq 0$ is the reservation utility obtained by any consumer for any purchases (regardless the identity of the selected producer), $p_i$ is the mill price of firm $i$’s product, and $t > 0$ the transportation cost per unit.

The transportation costs are borne by the consumer as in Hotelling (1929). But contrary to Hotelling (1929) we choose a quadratic function of transportation costs as D’Aspremont, Gabszewicz, and Thisse (1979) show that this shape guarantees the existence of an equilibrium in the price sub-game. We assume high transaction costs for the resale of goods between consumers to avoid such behaviour. Lastly, following Harter (1993), we assume that $v \geq 3t$ to ensure that at equilibrium, each consumer purchases one unit of good from the firm with the lowest price.

This game is solved using backward induction. We start solving the price game (section 4). We then solve the R&D game (section 5), and finally identify the equilibrium in the location game (section 6). The last part of the paper summarises the main conclusions of the paper.

4 The Price Game

To solve the price game, we first have to characterise the demand addressed to both firms. To do so we first have to identify the location ($s$) of the marginal consumer, for whom the utility of buying a product to firm 1 equals the utility of buying to firm 2. The marginal consumer ($\hat{s}$) is identified by the following identity:
\[ v + X_1 - p_1 - t (\hat{s} - y_1)^2 = v + X_2 - p_2 - t (\hat{s} - y_2)^2 \]  

which yields:

\[ \hat{s} = \frac{(p_2 - p_1) - (X_2 - X_1)}{2t(y_2 - y_1)} + \frac{y_2 + y_1}{2} \]  

We can then derive the demand addressed to both firms as follows:

\[ D_1 = \int_0^{\hat{s}} dx = \hat{s} \]  
\[ D_2 = \int_{\hat{s}}^1 dx = 1 - \hat{s} \]  

The profit functions are then given by:

\[ \pi_i = p_i D_i - c(x_i) \]  

where \( c(x_i) \) are the level of R&D expenditures, with \( c' > 0 \) and \( c'' > 0 \) to account for decreasing returns in R&D.

In what follows, our cost function adopts the same shape as the one used in Piga and Poyago-Theotoky (2005) in order to insure the comparison with their model:

\[ c(x_i) = \frac{1}{2} x_i^2 \]  

After taking the first order conditions\(^2\) we obtain the equilibrium prices:

\[ p_1 = \frac{1}{3} \left[ (X_1 - X_2) + t(y_2 - y_1)(2 + y_2 + y_1) \right] \]  
\[ p_2 = \frac{1}{3} \left[ (X_2 - X_1) + t(y_2 - y_1)(4 - y_2 - y_1) \right] \]  

Substituting equations (9) and (10) into the equation (7), we respectively obtain:

\[ \pi_1 = \frac{[X_1 - X_2 + t(y_2 - y_1)(2 + y_2 + y_1)]^2}{18t(y_2 - y_1)} - \frac{x_1^2}{2} \]  
\[ \pi_2 = \frac{[X_2 - X_1 + t(y_2 - y_1)(4 - y_2 - y_1)]^2}{18t(y_2 - y_1)} - \frac{x_2^2}{2} \]  

In the remainder of the paper we concentrate on the case where both firms are active on the market. For only one or less firms active on the market, the second order condition is also satisfied as \( \frac{\partial^2 \pi_i}{\partial p_i^2} = -\frac{1}{t(y_2 - y_1)} < 0 \).
analysis of the interplay between endogenous spillovers and location choices would not make sense.

The equilibrium of the price game being calculated, the next step of the analysis rests in solving the R&D subgame.

5 The R&D Game

In this second step, firms choose their level of internal R&D \( (x) \) in a non-cooperative way, their locations being given. Substituting the expression of profits (11) and (12), into the expression for effective R&D (1) we can deduce the best-response functions from the first-order conditions. Unfortunately, an analytical solution can not be found in the general case \( (\forall \alpha; t) \). We choose to compute the numerical equilibrium of the model for \( \alpha = 0.5 \) (i.e. for an intermediate level of exogenous spillovers).

We obtain,

\[
\begin{align*}
  x_1(x_2) &= \frac{x_2 - t(y_2 - y_1)(2 + y_2 + y_1)}{1 - 9t(y_2 - y_1)} \quad (13) \\
  x_2(x_1) &= \frac{x_1 - t(y_2 - y_1)(4 - y_2 - y_1)}{1 - 9t(y_2 - y_1)} \quad (14)
\end{align*}
\]

And after solving the system of equation, the equilibrium R&D efforts:

\[
\begin{align*}
  x_1 &= \frac{3t(y_2 - y_1)(2 + y_2 + y_1) - 2}{3[9t(y_2 - y_1) - 2]} \quad (15) \\
  x_2 &= \frac{3t(y_2 - y_1)(4 - y_2 - y_1) - 2}{3[9t(y_2 - y_1) - 2]} \quad (16)
\end{align*}
\]

At this stage, one can already point that if firms choose to locate at the same point \( (y_1 = y_2) \), the optimal R&D effort undertaken is constant and non-null for both firms:

\[
x_1 = x_2 = \frac{1}{3}
\]

Hence, even when the distance between firms, and therefore the geographical constraint on knowledge spillovers, is minimal, a fixed R&D effort is required. Put differently, firms choose the same (non null) quality improvement.

Moreover, this result holds when considering symmetrical locations \( (y_1 + y_2 = 1) \) at the first stage. The optimal R&D effort remains constant, non-null \( (x_1 = x_2 = \frac{1}{3}) \) and independent from the distance between firms \( (y_2 - y_1) \). These results confirm the principle of minimal differentiation.
Using equations 15 and 16, we obtain the following profit functions:

\[
\pi_1 = \frac{[3t(y_2 - y_1)(2 + y_2 + y_1) - 2]^2[9t(y_2 - y_1) - 1]}{18[9t(y_2 - y_1) - 2]^2}
\]

(17)

\[
\pi_2 = \frac{[3t(y_2 - y_1)(4 - y_2 - y_1) - 2]^2[9t(y_2 - y_1) - 1]}{18[9t(y_2 - y_1) - 2]^2}
\]

(18)

This concludes the R&D game.

6 The Location Game

At this first stage of the game, firms select their location. They take into account the consequences of their choice on their subsequent choices of R&D and price. We deduce the best response functions for the location game from the first order derivatives of the profit functions (17) and (18).

We obtain:

\[
y_1(y_2) = -1 \pm \sqrt{(y_2 + 1)^2 - \frac{2}{3t}}
\]

(19)

\[
y_2(y_1) = 2 \pm \sqrt{(y_1 - 2)^2 - \frac{2}{3t}}
\]

(20)

We rule out the other sets of reaction functions are they implied at least one of the two firms to be out of the market. \(^3\) Our aim here is to concentrate on the duopoly situations. We then consider the following best responses:

\[
y_1(y_2) = -1 + \sqrt{(y_2 + 1)^2 - \frac{2}{3t}}
\]

(21)

\[
y_2(y_1) = 2 - \sqrt{(y_1 - 2)^2 - \frac{2}{3t}}
\]

(22)

Solving this system of equations we obtain the following results for the location game:

\[
y_1 = \frac{1}{2} - \frac{1}{9t}
\]

(23)

\(^3\) As:

\[
y_1(y_2) = -1 - \sqrt{(y_2 + 1)^2 - \frac{2}{3t}} < 0
\]

\[
y_2(y_1) = 2 + \sqrt{(y_1 - 2)^2 - \frac{2}{3t}} > 1
\]
The optimal location by firms is symmetrical and converges to the center as the transportation costs increase (as shown by Figure 1). Moreover, none of the firms enters the market if transportation costs \( t \) are too low.\(^4\)

The optimal level of effective R&D effort is then the following:

\[
X_1 = X_2 = \frac{1}{2} - \frac{1}{27t} \tag{25}
\]

As \( t \) increases the level of effective R&D at the optimum increases (see Figure 2). Hence, as firms tend to join halfway, the knowledge stock of each firms increases.

We can now deduce the optimal values for profits an prices for each firms. Profits are the following if locations are symmetrical:

\[
\pi_1 = \pi_2 = \frac{9t(y_2 - y_1) - 1}{18} \tag{26}
\]

Substituting equations (23) and (24) into (26) we then obtain the profit levels at the equilibrium:

\[
\pi_1 = \pi_2 = \frac{1}{18} \tag{27}
\]

Prices are also symmetrical and independent from the transportation costs. Substituting equations (23), (24) and (25) into equations (9) and (10), we obtain the optimal price:

\[
p_1 = p_2 = \frac{2}{9} \tag{28}
\]

\(^4\)As:

\[
\lim_{t \to \infty} y_1(t) = \lim_{t \to \infty} y_2(t) = \frac{1}{2} \quad \frac{\partial y_1}{\partial t} > 0 ; \quad \frac{\partial^2 y_1}{\partial t^2} < 0 ; \quad \frac{\partial^2 y_2}{\partial t^2} > 0
\]

\(^5\)As:

\[
y_1 < 0 \quad y_2 > 1 \quad \text{if } t < \frac{2}{9}
\]
Figure 1: Firms’ optimal locations as function of transportation costs

Figure 2: Firms’ effective R&D as function of transportation costs
7 Conclusion

In this paper we have presented and solved a model of spatial competition à la Hotelling (1929), where firms’ absorptive capacity, as function of their investment in R&D and the geographic concentration of firms determine firms’ respective ability to benefit from knowledge created by their competitors. The main message of this paper states that transportation costs do not have any impact either on prices or on internal R&D expenditures, but only influence location choices. Indeed, our analysis of the game reveals that at the optimum, firms choose to locate symmetrically, and spend an equal, fixed and non null amount in internal R&D. We confirm Irmen and Thisse (1998): Firms adopt similar R&D levels and prices at the optimum (and consequently do not differentiate themselves on those criterion), and differentiate themselves on one characteristic only: their location.

A second interesting conclusion is that when transportation costs increase (and consequently price competition is lower), firms maintain their internal R&D expenditures (at the minimum required level) but tend to agglomerate in order to enlarge their market-share. In that case proximity yields to an increase in price competition but also drives up the effective R&D of firms (thanks to spillovers mechanisms). This finally allows prices and profits to remain stable. The principle of minimal differentiation remains valid. On the contrary, when the transportation costs are too low, firms leave the market. As firms have to pay a right to get access to knowledge spillovers (a minimum level of internal R&D), this entry cost requiring in turn a minimum level of profit. The price competition being stronger due to the low transportation costs, firms rather locate out of the market.

The model presented in this study is directly comparable with the one developed by Piga and Poyago-Theotoky (2005) and provides however opposite conclusions. Our analysis of the game allows us to assert that this difference is due to 1) the substitution effect between internal R&D and external spillovers in Piga and Poyago-Theotoky (2005), which motivates firms to decrease their internal R&D when they agglomerate, what is not the case in our model, and 2) their strong assumption to take symmetric locations as given, whereas in the present paper, symmetric locations is not a constraint but a solution of the game.
References


