Platform Competition with Endogenous Multihoming

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Abstract

A model of two-sided market (for credit cards) is introduced and discussed. In this model, agents can join none, one, or more than one platform (multihoming), depending on access prices and the choices made by agents on the opposite market side. Although emerging multihoming patterns are, clearly, one aspect of equilibrium in a two-sided market, this issue has not yet been thoroughly addressed in the literature. This paper provides a general theoretical framework, in which homing partitions are conceived as one aspect of market equilibrium, rather than being set ex-ante, through ad-hoc assumptions. The emergence of a specific equilibrium partition is a consequence of: (1) the structure of costs and benefits, (2) the degree and type of heterogeneity among agents, (3) the intensity of platform competition.

JEL Codes: D85, L10, L15, L89.

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1 Introduction

In two-sided markets, two (or more) parties need access to a common platform, to initiate a transaction or interaction. The capability and willingness to join the platform depend on (1) the number of joining agents on the opposite side and (2) the access price applied to each party. Examples of two-sided markets are: computer operating systems, real estate agencies, scientific journals, payment systems, media, etc..

The number of agents on the opposite market side matters because more agents means more potential interactions, or a better match in searching a partner. In this sense, we can speak of bilateral network externalities.

Access prices on each side matter because agents cannot realize a full pass-through of cost margins. This is due to the existence of membership fees, independent of transaction volumes, or of specific contractual restraints (e.g., non discrimination rules in credit cards). Because of this, market equilibrium is affected by both the aggregate price level, chosen by the platform, and the price structure (Rochet and Tirole (2004)).

Two-sided markets have been the subject of a recent literature, mainly stemmed from the study of credit cards and media industries\(^1\). This literature has initially focused on monopolistic platforms, and on their act of balancing prices “to get both sides on board”. Competition between platforms has been tackled only recently (Armstrong (2004), Rochet and Tirole (2003a), Guthrie and Wright (2003), Schiff (2003), Caillaud and Jullien (2003), Hagiu (2004), Chakravorti and Roson (2004), Manenti and Somma (2004), Gabszewicz and Wauthy (2004)).

One special difficulty of dealing with platform competition is given by the fact that agents can often join more than one platform (multihoming). For example, consumers may carry, and merchants may accept, more than one credit card for payment. Computer users may install a Windows or a Linux operating system

\(^1\)It could be argued that other two-sided markets had been studied in the past (e.g., shopping malls). General principles of two-sided markets do not seems to have been investigated in a general and systematic way, though.
on their PCs, or both. Software developers may write applications for Windows, Linux, or both. People may have one, or more than one, SIM card, of different operators, on their mobile phones. Web pages may be written using a code that allows sophisticated graphical content to be appropriately displayed in one, or multiple, browser environments.

Multihoming involves costs and benefits. Among costs: fixed costs for learning, searching, adapting to the alternative platform; variable transaction costs (possibly different between platforms); plain membership fees. Among benefits: higher acceptance rate, better market penetration, possibility of choice of the preferred platform during a transaction.

Agents should choose between single and multihoming (or, more precisely, on how much to multihome) by comparing costs and benefits. Analyses of multihoming markets, however, are complicated by two elements. First, some of the costs and benefits are endogenously determined in a market equilibrium. For instance, competing platforms may use price instruments to attract customers. In doing so, they do not only affect market shares, but also the extent of multihoming behaviour. Second, customers choices are interdependent. Consider this trivial example: consumers choosing products of different brands. If all brands are offered in two or more shops (multihoming), each consumer need to visit only one shop to have the whole range available (singlehoming). If the shops are located quite close to each other, it may also be possible that brands are sold exclusively in one of the shops (singlehoming), and consumers would then visit more shops (multihoming). Sellers multihome if buyers singlehome, but buyers multihome if sellers singlehome.

Although emerging multihoming patterns are, clearly, one aspect of equilibrium in a two-sided market, this issue has not yet been thoroughly addressed in the literature, mainly because of the need of retaining analytical tractability. Most papers on platform competition have either pre-determined which market side single/multihomes (based on empirical evidence for specific industries), or have adopted specific assumptions (typically, homogeneity in some parameters,
symmetry) that allows one to anticipate which market side will eventually multi-home\(^2\).

The question which side multihomes (possibly both), why and how much, is not a merely theoretical issue. As an example, consider the striking differences that exist between the American and European markets for credit cards. In North America, consumers typically carry several credit cards, although one of them is prevalently used (Rysman (2004)). In Europe, most consumers adopt one credit card, or none, and most merchants accept all major credit cards, or none. Explaining these differences in terms of market competition is a challenging task. Which fundamental characteristics of the two markets may explain this outcome? Are these patterns time-persistent, as one would expect in the presence of network externalities? Is there any role played by market imperfections and barriers to competition? May a shift in policy regime produce an abrupt change in the qualitative characteristics of the market? What are the implications of market integration and increased international competition?

As a further example, consider the penetration of the Linux operating system(s) in the market for personal computers OS. This is a market dominated by the Microsoft Windows family. However, many users have recently started using Linux. Most of them have done that by partitioning the hard disk, thereby retaining both environments. The advantage of increased software availability, compatibility, and flexibility is being weighted against the implicit cost of reducing the hard disk space for Windows native programs. But, what will happen in the future? Will Linux become a serious alternative to Windows, or will it continue living side by side with the dominant standard?

In this paper, we introduce and discuss a model of duopoly competition, with endogenous multihoming, between payment card networks. The case of payment

\(^2\)For example, if agents on one side are all similar, we know that they will end up by making the same choices. In equilibrium, they will either all singlehome on the same platform, or they will all multihome.

A more sophisticated formulation has been adopted in a recent paper by Armstrong and Wright (2004), where conditions for specific homing configurations are derived beforehand and introduced as model assumptions.
cards is taken because the model is derived from Chakravorti and Roson (2004), but most concepts can be readily extended to other two-sided markets. Whereas the latter paper pre-determines which market side singlehomes (the consumers) and which market side - potentially - multihomes (the merchants), the model introduced here allows for endogenous single/multihoming on both sides. To this end, we adopt an approach similar to Hermalin and Katz (2004). Contrary to them, we assume ex-ante which market side (the consumers) has the right to choose the payment instrument when both sides multihome. On the other hand, we consider two aspects that have been neglected in their model: (1) the existence of two-sided network externalities, and (2) the possible existence of fixed costs and benefits.

The paper is structured as follows. In the next section, a general theoretical framework is specified, in which the multihoming pattern stems from the equilibrium of a sequential game, in which platforms choose prices first, and agents select which platform(s) to join afterwards. Since equilibria for these games cannot, in general, be specified as closed form solutions, section three provides some illustrative numerical simulations, shedding light on the implications of various assumptions on the market equilibria and homing configurations. An ending section draws some final remarks.

2 The model structure

2.1 Assumptions and definitions

There are: a set $S$ of consumers (shoppers), a set $M$ of merchants, two payment networks (1 and 2). Every consumer makes one transaction (buys one good) with every merchant$^3$, using cash or one of the two payment instruments. For a payment instrument to be used, both sides must have adopted the corresponding “platform”. When both sides have joined both platforms, the consumer decides which instrument is used.

$^3$This assumption, often adopted in the literature, rules out “business stealing” motivations for adoption of credit cards by merchants.
Except for the right of selecting the network under reciprocal multihoming, the two sides are symmetric. Each agent on each side \((s \in S, m \in M)\) is associated with a vector of (potential) benefits \(b_s = \{B_{s1}, b_{s1}, B_{s2}, b_{s2}\} \in \mathbb{R}^4\) or \(b_m = \{B_{m1}, b_{m1}, B_{m2}, b_{m2}\} \in \mathbb{R}^4\). Benefits \(B_k^i (i = \{1, 2\}, k = \{s, m\})\) express the utility (possibly negative), derived by the mere ownership of a payment instrument (e.g., status), whereas \(b_k^i\) express transaction benefits, obtained every time a transaction is carried out on a specific platform.

Networks apply, to both sides, a membership fee \(P\) (possibly zero or negative) and a transaction fee \(p\). This is a simple form of non-linear pricing which, as we shall see later, allows to price discriminate among different classes of customers, according to their multihoming behaviour. Networks also incur on fixed per-member costs \(C\) and transaction costs \(c\). In short, they select a vector of prices \(p_i = \{P_{si}, p_{si}, P_{mi}, p_{mi}\}\) on the basis of costs \(\{C_{si}, c_{si}, C_{mi}, c_{mi}\}\).

Consumers belong to five categories. First, some consumers do not join any platform, and use only cash. Their utility is normalized to zero \((W_0 = 0)\). Some other consumers carry only card 1, and use it whenever they find a merchant who have joined platform 1. Let us define their utility as (Rochet and Tirole (2004)):

\[
W_1 = (B_{s1} - P_{s1}) + (b_{s1} - p_{s1})(m_1 + m_{12})
\]

where \(m_1\) stands for the number of merchants accepting, in addition to cash, only card 1, and \(m_{12}\) for the number of merchants accepting both payment instruments.

The symmetric definition of utility for consumers joining only platform 2 is:

\[
W_2 = (B_{s2} - P_{s2}) + (b_{s2} - p_{s2})(m_2 + m_{12})
\]

There are also a fourth and a fifth category, including those consumers who carry both cards. Here we make a distinction between those who prefer to use card 1 when a choice is possible, because a merchant has joined both platforms, and those who would rather select card 2. Utility for these groups is defined as:

\[
W_{12.1} = (B_{s1} - P_{s1}) + (b_{s1} - p_{s1})(m_1 + m_{12}) + (B_{s2} - P_{s2}) + (b_{s2} - p_{s2})m_2
\]
\[ W_{12.2} = (B_1^s - P_1^s) + (b_1^s - p_1^s)m_1 + (B_2^s - P_2^s) + (b_2^s - p_2^s)(m_2 + m_{12}) \] (4)

Each consumer belongs to the category in which her utility is highest. Formally, let us define a partition of the set of consumers in the following way:

**Definition 1** A Utility Maximizing Partition (UMP) of the set of consumers is defined as:

\[ H^s(p_1, p_2, G^m) = \{ \gamma_0, \gamma_1, \gamma_2, \gamma_{12.1}, \gamma_{12.2} \} \]

where \( G^m \) is a partition of the set of merchants, determining \( m_1, m_2, m_{12}, \) and:

\[ \gamma_i = \{ s : W_i \geq W_j \quad \forall j \neq i \} \quad i, j \in \{0, 1, 2, 12.1, 12.2\} \]

Let us also define \( n_i = \text{card}(\gamma_i) \) as the number of consumers in each subset.

On the basis of the definition above, it could be possible for a consumer to belong to more than one category, when utilities in two or more groups match. For all practical applications of the model, however, we shall assume that consumers of this type are equally split among the categories for which utility is equal\(^4\).

We adopt a similar framework for the merchant side. The only difference is that here we have four, instead of five, categories, because merchants are assumed not to choose the payment instrument under bilateral multihoming. Again, we can normalize to zero the utility of cash-only merchants: \( V_0 = 0 \). For the remaining three cases, let us define utility as:

\[ V_1 = (B_1^m - P_1^m) + (b_1^m - p_1^m)(n_1 + n_{12.1} + n_{12.2}) \] (5)

\[ V_2 = (B_2^m - P_2^m) + (b_2^m - p_2^m)(n_2 + n_{12.1} + n_{12.2}) \] (6)

\[ V_{12} = (B_1^m - P_1^m) + (b_1^m - p_1^m)(n_1 + n_{12.1}) + (B_2^m - P_2^m) + (b_2^m - p_2^m)(n_2 + n_{12.2}) \] (7)

We can define a UMP for merchants as:

\(^4\)This implies that the intersection between any two subsets is the empty set, whereas the union of all subsets is the entire set of consumers.
**Definition 2** A Utility Maximizing Partition (UMP) of the set of merchants is defined as:

\[ H^m(p_1, p_2, G^s) = \{ \mu_0, \mu_1, \mu_2, \mu_{12} \} \]

where \( G^s \) is a partition of the set of consumers, determining \( n_1, n_2, n_{12.1}, n_{12.2} \), and:

\[ \mu_i = \{ m : V_i \geq V_j \quad \forall j \neq i \} \quad i, j \in \{ 0, 1, 2, 12 \} \]

Let us also define \( m_i = \text{card}(\mu_i) \) as the number of merchants in each subset.

Notice that the partition of consumers can be identified on the basis of a partition of merchants and vice versa. Quite naturally, let us define a configuration in which partitions of the two sets are mutually consistent:

**Definition 3** A Consistent Dual Partition (CDP) is defined as:

\( (H^s(p_1, p_2, H^m), H^m(p_1, p_2, H^s)) \)

As in most coordination games, there can be multiple CDP for given prices. For example, suppose that all agents are homogeneous and platforms apply equal prices (but not too high). There are two possible configurations: in both, only one platform is used to carry out transactions\(^5\). This is because network externalities produce a special type of economies of scale, which may easily bring about corner solutions.

Here, however, we are considering platforms that provide differentiated services, so that if differentiation is sufficiently strong and agents are heterogeneous in terms of benefits, both platform can be active in a CDP. Furthermore, as noted also by Armstrong and Wright (2004), network externalities and differentiation create opposite effects. The higher the degree of differentiation, the more the individual decisions are based on agent-specific parameters, rather than on expectations about other agents’ choices.

\(^5\)The other one could still be joined if membership benefits are high enough.
Notice also that the existence of multiple CDP is linked to the presence of fixed costs and benefits. To see this, suppose that, for one side \( k \) of the market, both \( B^k_1, B^k_2 \), and \( P^k_1, P^k_2 \) are zero. Then, utility of \( k \)-type agents would still depend on the magnitude of the opposite side network, but their decision about joining or not a certain platform would not. Indeed, platform \( i \) would be joined whenever \( b^k_i > p^k_i \). If adoption choices on one side do not depend on the opposite side choices, multiple CDP cannot occur.

Prices are determined by profit-maximizing platforms. Profits for the two platforms are given by:

\[
\Pi_1 = (P^s_1 - C^s_1)(n_1 + n_{12.1} + n_{12.2}) + (P^m_1 - C^m_1)(m_1 + m_{12}) + (p^s_1 + p^m_1 - c_1)[(n_1 + n_{12.1})(m_1 + m_{12}) + n_{12.2}m_1] \tag{8}
\]

\[
\Pi_2 = (P^s_2 - C^s_2)(n_2 + n_{12.1} + n_{12.2}) + (P^m_2 - C^m_2)(m_2 + m_{12}) + (p^s_2 + p^m_2 - c_2)[(n_2 + n_{12.2})(m_2 + m_{12}) + n_{12.1}m_2] \tag{9}
\]

Notice that profits depend on specific partitions of consumer and merchant sets. It is natural, then, to assume that these partitions are determined by the selected prices, and are mutually consistent. More precisely, let us define a game in the following way:

**Definition 4** A Card Multihoming Game (CMG) is defined as a game in which platforms choose prices \( p \) to maximize profits, and demand for platform services is implicitly defined by a CDP associated with the same prices. In a non-cooperative CMG each platforms aims at maximizing profits, while taking the prices of other platforms as given. The equilibrium of the game is a Nash equilibrium. In a cooperative CMG, instead, prices are jointly determined, in order to maximize the sum of profits for all platforms.

When benefits for consumers and merchants, and costs for platforms, are symmetrically distributed, we can speak of a symmetric CMG. A symmetric equilibrium for a symmetric CMG (cooperative or non-cooperative) is the one in which platform prices are equal.
Table 1: Platform interaction types

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<tr>
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<th>$\mu_1$</th>
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<tr>
<td>$\gamma_1$</td>
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<td>$\gamma_{12.1}$</td>
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<td>$\gamma_{12.2}$</td>
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Because of the possible existence of multiple CDP, a CMG can have multiple equilibria. In this case, the issue of equilibria selection could emerge in some practical applications. Criteria for selecting among alternative equilibria are extensively discussed in the literature. For example, one requirement could be that a candidate equilibrium be robust to small deviations, or errors in expectations. Another possibility is to rule out candidate equilibria that are welfare-inferior for all the coordinating agents.

2.2 Profit maximization

Without loss of generality, consider the point of view of platform 1 in the profit maximization problem. Demand for platform 1 stems from consumers and merchants in five groups: $\gamma_1, \gamma_{12.1}, \gamma_{12.2}, \mu_1, \mu_{12}$. However, consumers in $\gamma_1$ and $\gamma_{12.1}$ affect the platform profits in the same way, so we can define a new subset $\gamma_{1+} = \gamma_1 \cup \gamma_{12.1}$, where $n_{1+} = n_1 + n_{12.1}$. As summarized in Table 1, consumers in $\gamma_{1+}$ interact with merchants in $\mu_1$ and $\mu_{12}$, whereas consumers in $\gamma_{12.2}$ interact only with merchants in $\mu_1$. Platform 1 selects a vector of four prices $p_1 = \{P^s_1, p^s_1, P^m_1, p^m_1\}$, to address the four categories of agents.

Within each category, however, there is some redundancy between membership fee and transaction fees. This is because there is no uncertainty, and members of all groups are supposed to know how many interactions will be realized in equilibrium. Since the global price, which is eventually paid, is the sum of membership fee and the product between transaction fee and total number of transactions, utility for each agent could be kept constant if the two fees are changed appropri-
ately, so as to keep the global price constant.

Nonetheless, because of the equality between agent types and price instruments, membership and transaction fees can be fine-tuned, so as to achieve the “right” (profit maximizing) global prices for all the four groups, as the following proposition states:

**Proposition 1** Assume that benefit distributions for merchants and consumers are such that the profit function for platform 1 is concave in prices. Then, profit is maximized when the following four conditions hold:

\[
\begin{align*}
\frac{P_1^s - C_1^s + (m_1 + m_{12})(p_1^s - (c_1 - p_1^m))}{(m_1 + m_{12})p_1^s + P_1^s} + m_1\left(\frac{P_1^m}{(n_{1+} + n_{12,2})}\right) + m_{12}\left(\frac{P_1^m}{n_{1+}}\right) & = \frac{1}{\epsilon_{n_{1+}}} \tag{10} \\
\frac{P_1^m - C_1^m + (n_{1+} + n_{12,2})p_1^m - (c_1 - p_1^s)}{(n_{1+} + n_{12,2})p_1^m + P_1^m} + n_{1+}\left(\frac{P_1^s}{(m_1 + m_{12})}\right) + n_{12,2}\left(\frac{P_1^s}{m_1}\right) & = \frac{1}{\epsilon_{m_1}} \tag{11} \\
\frac{P_1^s - C_1^s + m_1(p_1^s - (c_1 - p_1^m - (P_1^m/(n_{1+} + n_{12,2}))))}{m_1p_1^s + P_1^s} & = \frac{1}{\epsilon_{n_{12,2}}} \tag{12} \\
\frac{P_1^m - C_1^m + n_{1+}(p_1^m - (c_1 - p_1^s - (P_1^s/(m_1 + m_{12})))))}{n_{1+}p_1^m + P_1^m} & = \frac{1}{\epsilon_{m_{12}}} \tag{13}
\end{align*}
\]

where:

\[
\begin{align*}
\epsilon_{n_{1+}} & = -\frac{\partial n_{1+}}{\partial P_1^s} \frac{P_1^s}{n_{1+}} \\
\epsilon_{m_1} & = -\frac{\partial m_1}{\partial P_1^m} \frac{P_1^m}{m_1} \\
\epsilon_{n_{12,2}} & = -\frac{\partial n_{12,2}}{\partial P_1^s} \frac{P_1^s}{n_{12,2}} \\
\epsilon_{m_{12}} & = -\frac{\partial m_{12}}{\partial P_1^m} \frac{P_1^m}{m_{12}}
\end{align*}
\]

\(^6\)Of course, this holds true only if the four sets \(\gamma_1, \gamma_{1+}, \mu_1, \mu_{12}\) are all non-empty. If not, price redundancy would still occur.
Proof. Define the “global prices” faced by the four groups of agents as:

\[ \tilde{P}_{n1+} = P_s + p_s(m_1 + m_{12}) \quad \tilde{P}_{m1} = P_m + p_m(n_1 + n_{12.2}) \]
\[ \tilde{P}_{n12.2} = P_s + p_s m_1 \quad \tilde{P}_{m12} = P_m + p_m n_{1+} \]

and rewrite the profit function as:

\[ \Pi_1 = (\tilde{P}_{n1+} + \tilde{P}_{m1} - c_1)n_{1+}m_1 + (\tilde{P}_{n12.2} + \tilde{P}_{m1} - c_1)n_{12.2}m_{1+} + (\tilde{P}_{n1+} + \tilde{P}_{m12} - c_1)n_{1+}m_{12} - C_s(n_{1+} + n_{12.2}) - C_m(m_1 + m_{12}) \]  

Take partial derivatives of \( \Pi_1 \) w.r.t. \( n_{1+}, n_{12.2}, m_1, m_{12} \), and equalize them to zero. Introduce standard definitions of own-price elasticity, using \( n_{1+}, n_{12.2}, m_1, m_{12} \) as quantities. Next, plug back global prices with membership and transaction fees. Notice that elasticity defined in terms of global price equals elasticity defined in terms of membership fee, e.g.:

\[ \epsilon_{n1+} = -\frac{\partial n_{1+}}{\partial \tilde{P}_s} \frac{P_s}{n_{1+}} = -\frac{\partial n_{1+}}{\partial \tilde{P}_{n1+}} \frac{\tilde{P}_{n1+}}{n_{1+}} \]

Interpretation of first order conditions (10)-(13) is quite simple. They are special versions of the Lerner’s inverse elasticity rule. This rule states that a profit maximizing entity sets prices so that the marginal mark-up (the profit share in the price of the last unit sold) equals the inverse of the own-price demand elasticity. In this case, consumers and merchants should be viewed as quantity units.

Consider the left hand sides of (10)-(13). On the denominator, we found total revenue obtained from an agent in one of the sets \( \gamma_{1+}, \gamma_{12.2}, \mu_1, \mu_{12} \). This includes the fixed fee \( P \) and the transaction fee \( p \) multiplied by the number of interacting agents on the opposite market side.

On the numerator, we have per-member profits. They include three components. First, there is the margin between fixed fee and fixed costs. Second, we have transaction profits. Adding one more agent in a group allows expanding total transactions by a number equal to the size of the interacting parties. Every time
a transaction is carried out, a price \( p \) can be charged, and a transaction cost \( c \) is paid.

However, as stressed by Rochet and Tirole (2004), the relevant cost concept in a two-sided market is the opportunity cost, which should include (as a negative term) the transaction price that can be charged to all members of the opposite side, when a new customer is served. Here, this negative cost component does not only include the direct transaction price \( p \), but also a share of the membership fee \( P \), as it can be seen by defining “per-transaction global prices”:

\[
\frac{\tilde{P}_{n1+}}{(m_1 + m_{12})} = \frac{P_s}{(m_1 + m_{12})} + p_s^{1}, \quad \frac{\tilde{P}_{m1}}{(n_{1+} + n_{12.2})} = \frac{P_m}{(n_{1+} + n_{12.2})} + p_m^{1} \tag{16}
\]

\[
\frac{\tilde{P}_{n12.2}}{m_1} = \frac{P_s}{m_1} + p_s^{1}, \quad \frac{\tilde{P}_{m12}}{n_{1+}} = \frac{P_m}{n_{1+}} + p_m^{1}
\]

Elasticities on the right hand side can take different values, depending also on the competing platforms’ behaviour. In a Bertrand-Nash equilibrium, for example, the elasticity should be computed by changing one platform membership fee, while keeping the prices of the other platform(s) fixed. In a cooperative equilibrium, instead, elasticities should be computed on the basis of simultaneous price changes. Of course, in this latter case, elasticities would be smaller, thereby determining higher profit mark-ups in equilibrium.

Looking at the numerators of (10)-(13), one can see that profits can be raised in four different ways, corresponding to the four different price instruments available. On the other hand, all prices are interdependent. For example, suppose that, starting from an equilibrium state, one elasticity for one type of agent increases. This calls for higher profits on that type of agents, which could be achieved by raising at least one of the four prices appearing on the numerator of corresponding f.o.c. However, once any of these prices are touched, other prices should be also adjusted, to restore equality in the other optimality conditions. Typically, this requires a compensating variation of fixed and variable fees.
Finally, notice that prices determined through (10)-(13) may well be so high that some of the sets $\gamma_{1+}$, $\gamma_{12}$, $\mu_1$, $\mu_{12}$ may be empty. For example, for sufficiently high membership fees, there could be no multihoming consumers or merchants.

3 A numerical simulation of platform competition

To get some insights about the functioning of market competition, and its implications in terms of platform adoption, we present here some results of numerical simulation experiments\(^7\).

We consider two scenarios. In both, production costs for platforms are equal and set to $C_s^1 = C_s^2 = C_m^1 = C_m^2 = 0.5$ and $c_1 = c_2 = 0.05$. The total number of both merchants and consumers is normalized to one. As in Chakravorti and Roson (2004), we consider a Nash CMG game of price competition vs. a cooperative cartel, fixing prices for the two platforms. In addition, we focus on symmetric CDP dual partitions in the identification of the game equilibrium.

We select symmetric equilibria for two reasons. First, when facing equal platform prices, it is reasonable to assume that agents form expectations in which networks are somehow “balanced”. Second, because of the way these equilibria have been numerically determined\(^8\), they must be, at least, “locally stable” in terms of CDP partitions.

In the first case, platforms are differentiated in four dimensions: membership benefits for consumers, membership benefits for merchants, transaction benefits for consumers, transaction benefits for merchants. We assume that all four distributions for the two platforms are uniformly and independently distributed in the $[0, 1]$ interval. In other words, each consumer gets a draw $b^s = \{B_1^s, b_1^s, B_2^s, b_2^s\}$ and each merchant gets a draw $b^m = \{B_1^m, b_1^m, B_2^m, b_2^m\}$, where all components

\(^7\)These experiments have been carried out with the Mathematica software. Original simulation files are freely available from the author.

\(^8\)In practice, this has been obtained by numerical iterations, where UMPs for consumers and merchants have been computed in sequence, starting from an arbitrary partition in which agents were uniformly distributed among the subsets. In this case, since platform prices are equal in equilibrium (because of cost symmetry), the partitions are symmetric as well.
are taken at random, independently, in the $[0, 1]$ segment, with equal probability for all values in the interval.

Given prices, consumers and merchants are allocated in a Dual Consistent Partition, on the basis of which platform profits can be computed. Profit maximization, under the two market structures, gives raise to the equilibria described in Table 2, where prices and sets are displayed for the two cases of cooperative cartel equilibrium (first row) and competitive Nash duopoly (second row).

Because merchants and consumers are very heterogeneous in terms of membership and transaction benefits, we can find some agents in all of the nine categories. Multihoming is more diffused among consumers\(^9\), given the additional advantage of having the right to select the preferred platform, when multihoming occurs on both sides.

Despite the fact that consumers and merchants have identical benefit distributions, we can see that prices are not the same for the two sides. In particular, consumers are charged more per transaction: a fact that may be interpreted as a consequence of their platform selection power under reciprocal multihoming. Indeed, if prices for merchants and consumers would be the same, consumers would achieve higher utility levels, on average. The cartel and, to a lesser extent, the duopolistic platforms succeed in capturing part of this extra potential welfare.

When competition is introduced (row 2), all prices fall and welfare increases for both consumers and merchants. Chakravorti and Roson (2004) demonstrate that this result of welfare gains for both sides\(^10\), due to platform competition, is

\(^9\)To get the total number of multihoming consumers, $n_{12,i}$ has to be doubled.

\(^10\)More precisely, non-negative welfare variations.
Table 3: Own-price elasticities for the four groups (Case A)

<table>
<thead>
<tr>
<th>$\epsilon_{ni+}$</th>
<th>$\epsilon_{mi}$</th>
<th>$\epsilon_{nij,j}$</th>
<th>$\epsilon_{mij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.722120</td>
<td>.692475</td>
<td>.993268</td>
<td>.851222</td>
</tr>
<tr>
<td>.725388</td>
<td>.699961</td>
<td>1.04522</td>
<td>.886929</td>
</tr>
</tbody>
</table>

a general one. Here we can see what this implies in terms of homing partitions, with less agents not joining any platform, and more agents in all other categories.

Table 3 shows the own-price elasticities for the four interacting groups of each platform $(i, j)$, computed by inserting the values of table 2 in the first order conditions.10-13

Let us now consider a second, alternative case. We take the simplifying assumption of fixing all benefits for all agents at 0.5, except for the transaction benefits for the consumers associated with the second platform $(b_{s2})^{11}$, which continue to be uniformly distributed in $[0, 1]$. This means that: (1) all merchants are identical, so they must end up by making the same choices, and (2) consumers are heterogeneous in one dimension (platform-specific transaction benefits). Furthermore, as in the first scenario, platforms are symmetric and set equal prices in equilibrium, both in the cartel and in competition.

Under these conditions, consumers do not multihome. If there are no intrinsic benefits in joining one platform rather than another, a consumer would multihome only if there is a probability that her preferred card is not accepted by some merchants. But this would imply that merchants make different adoption choices, which is impossible here. Therefore, either the market for consumers is equally split between the two platforms, like in a symmetric Hotelling model, or only platform 2 is used by less than a half consumers. This second case cannot emerge under competition, because profits of the first platform would be zero if no con-

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11 Or, alternatively, with the first platform.
12 A similar setting has been analyzed by Armstrong and Wright (2004).
13 That is, by those having sufficiently high transaction benefits associated with this platform.
Table 4: Descriptive variables for collusive and competitive equilibria (Case B)

<table>
<thead>
<tr>
<th>$P^{s}_{i}$</th>
<th>$P^{p}_{i}$</th>
<th>$P^{m}_{i}$</th>
<th>$P^{m}_{i}$</th>
<th>$\Pi_{i}$</th>
<th>$n_{0}$</th>
<th>$n_{i}$</th>
<th>$n_{12,i}$</th>
<th>$m_{0}$</th>
<th>$m_{i}$</th>
<th>$m_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.72</td>
<td>.27</td>
<td>.66</td>
<td>.17</td>
<td>.467</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>.68</td>
<td>.20</td>
<td>.66</td>
<td>.17</td>
<td>.410</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

sumers join platform 1 and no transactions take place on it\(^{14}\).

It turns out that, under the set of parameters considered here, it is better to serve all consumers for the cartel as well. Therefore, all consumers singlehome and half of them adopt each platform. This outcome has strong implications for the merchants. Since the number of consumers on each platform is fixed (0.5), the merchants’ problems of joining the two platforms are *separable* (since utility is additive). As long as the number of consumers stays fixed, each platform is a *monopolist* on the merchant side, even under platform competition. As such, it can extract all merchants’ surplus, and merchants will all multihome.

Table 4 shows the simulation results, using the same format of Table 2. We can see that homing partitions are as expected, and do not change between cartel and duopoly. Remarkably, competition has no effect on the prices faced by merchants, and platforms compete only on the consumer side. Merchant surplus is fully extracted, and merchant are almost indifferent between joining and not joining any of the two platforms.

When these results are compared with those of case A, we can see that the lower degree of heterogeneity among agents in case B is reflected in, on one hand, higher platform profits and, on the other hand, a more significant impact of the introduction of competition in the market.

Because many sets in the homing partitions are empty, some price instruments are redundant, and there is a continuum of market equilibria for the same CDP (so Table 2 shows just one of the many possible equilibria). Any price combination

\(^{14}\)In principle, a platform could still be sold, because of membership benefits. Here, however, membership costs and benefits take the same value (0.5), so there are no profit margins.
satisfying the two relationships $P_m + 0.5 \times p^m = 0.745 = 0.66 + 0.5 \times 0.17$ and $P^s + p^s = 0.99 = 0.72 + 1 \times 0.27$ (for the cartel), or $P^s + p^s = 0.88 = 0.68 + 1 \times 0.20$ (for the competitive duopoly), identifies an equilibrium as well.

4 Concluding remarks

In two-sided markets with multiple platforms, agents can join none, one, or many platforms, depending on prices and adoption choices made by potential partners on the other side. This paper provides a general theoretical framework, in which homing partitions are conceived as one aspect of market equilibrium, rather than being set ex-ante, through ad-hoc assumptions.

The emergence of a specific equilibrium partition is a consequence of: (1) the structure of costs and benefits, (2) the degree and type of heterogeneity among agents, (3) the intensity of platform competition. Relatively high transaction-independent costs, or relatively low transaction-independent benefits, reduce the likelihood of multihoming. Multihoming on one side makes multihoming on the other side less likely. Agent heterogeneity makes coordination problems less severe and equilibrium partitions more stable. Platform competition create a downward pressure on prices, but its implications in terms of multihoming are ambiguous.

As mentioned in the introductory section, real markets are characterized by very diverse homing patterns, even within markets for the same good or service. The analysis conducted so far can help in understanding which factors are at the basis of these differences. Therefore, empirical research could be directed to gauging the relative importance of potential explanatory factors in specific markets. Findings on the determinants of platform adoption would have important policy implications, in several different contexts. For example, understanding why one side singlehomes, and the other side multihomes, could allow forecasting whether or not changes in policy, or technology, will alter key qualitative characteristics of a two-sided market in the future.
References


