Sustaining Collusion in Growing Markets*

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Abstract

The impact of demand growth on the maximal degree of collusion (consistent with equilibrium) is investigated in a Cournot supergame where market growth may trigger future entry and a collusive agreement is enforced by the most profitable ‘grim trigger strategies’ available. It is shown that: (i) delaying entry beyond the first period at which the net present value of the entrant’s expected profits is positive may be optimal; and (ii) the interplay between the intrinsic effect of demand growth and the impact of entry crucially determines the overall effect of demand growth on the maximal degree of sustainable pre-entry collusion.

Keywords: Collusion, Demand Growth and Entry.

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1 Introduction

As pointed out by Ivaldi et al. (2003), previous literature analyzing demand growth effects on collusion\(^1\) has relied on an assumption that is clearly unwarranted. Specifically, it “assumes that the number of market participants remains fixed despite market growth, while in practice, entry may be easier in growing markets” (p. 28).

In markets where entry barriers are prohibitive (e.g. because of needed patents) so that entry does not occur, the standard \textit{intrinsic} pro-collusive effect of demand growth prevails: the expected rise in demand increases the future cost of a deviation, which in turn implies that demand growth facilitates collusion. However, in markets where entry barriers are moderate, market growth is likely to generate entry, a fact which is detrimental to collusion. When this is the case, it seems then important to try and understand whether entry stimulated by market growth can outweigh the intrinsic pro-collusive effect of market growth.

The current paper has taken seriously this argument by considering a simple model in which firms repeatedly set quantities, demand grows steadily at a given rate and, both before and after entry, the collusive agreement is enforced by the most profitable “grim trigger strategies” available (Friedman (1971)).

Within this framework, we start by analyzing firms’ optimal behavior ex-post entry and derive the optimal entry date. It is shown that entry always

\(^1\)See, for instance, Motta (2004, pp. 163-164) and Tirole (1988, p. 250).
occurs in equilibrium if entry sunk costs are moderate. More importantly, and contrary to other supergame models of collusion that examine entry in a context where demand is assumed to be constant over time, the optimal entry time may not correspond to the first period in time at which the net present value (henceforth NPV) of the entrant’s expected profits is positive. Delaying entry beyond this point might be optimal.

We then turn to the analysis of the pre-entry behavior. In particular, we study the relationship between the maximal degree of pre-entry collusion and the market growth rate. This relationship is shown to be in general non-monotone.

The impact of demand growth on collusion occupies an important space in all investigations by the European Commission (EC) assessing ‘joint dominance’ effects of mergers (which refers to the possibility that firms reach a collusive outcome after a merger). Interestingly, however, there is a divergence between the conclusions of the previous literature on the topic - which has mainly focused on the intrinsic pro-collusive effect of demand growth - and opinions expressed by both the EC and the Court of First Instance (CFI) when analyzing merger cases. For both the EC and the CFI, demand growth is often interpreted as a factor hindering collusion. One possible rationale for this divergence is the fact that competition authorities and courts do not assume that the number of market participants is fixed when analyzing the impact of demand growth on collusion. The widely discussed Airtours/First Choice merger is a case in point.\footnote{Airtours/First Choice, Case IV/M. 1524 (Decision of September 22, 1999).} In June 2002, the CFI has overturned
the EC decision in 1999 prohibiting the merger between Airtours and First Choice. The EC had alleged that the merger would have created a situation of joint dominance by Airtours/First Choice and two other competitors in the UK short-haul package holiday market. The CFI concluded that the EC decision was ‘vitiated by a series of errors in assessment’. In particular, the CFI underlined that the EC analysis of demand growth was flawed. The EC relied on a one page extract from a document regarding a study for a major tour operator in finding out that demand growth was ‘quite low’ in the 1990s, would continue to be so, and, therefore, “market growth is not likely to provide a stimulus to competition within the foreseeable future” (paragraph 93 of the EC Decision). Examination of the same document by the CFI showed, however, that the EC reading of that document was inaccurate, given the emphasis of that very same document on the ‘strong growth’ that had taken place.\(^3\) This led the CFI to conclude that “the Commission was not entitled to conclude that the market development was characterized by low growth, which was, in this instance, a factor conducive to the creation of a collective dominant position by the three remaining large tour operators.” (CFI Judgment, paragraph 133). The CFI also observed that there were no significant entry barriers to entry to the market.\(^4\) \(^5\)

\(^3\) The CFI also provided additional evidence from the 1998 British National Travel Survey supporting that the “foreign holiday sector enjoyed strong growth throughout the decade and, thus, also over the recent years” (CFI Judgment, paragraph 131).

\(^4\) See CFI Judgment, paragraph 98.

\(^5\) Ivaldi et al. (2003) point out that another example illustrating the divergence between the conclusions of the literature focusing on the intrinsic pro-collusive effect of demand growth and the opinions expressed by the EC is given by the recent guidelines for market analysis and the assessment of significant market power in electronic communications markets. In fact, in Annex II of the EC Decision 676/2002/EC (OJ L 108, of 24/04/2002) it is stated that “Two or more undertakings can be found in a joint dominant position ...
To the best of our knowledge, the only paper that studies the impact of demand growth on collusion when entry is feasible is Capuano (2002). There exist, however, two major differences between Capuano’s framework and the setting used in this paper. First, while in Capuano’s model entry is assumed to occur as soon as the entrant’s expects a positive NPV of profits, in the current paper entry timing is optimal. Second, and most importantly, the focus of Capuano’s paper is on perfect collusion sustainability (and for the quantity competition case only a partial result is obtained), while the current paper is mainly concerned with the characterization of the maximal degree of collusion consistent with equilibrium (which may be something less than perfect collusion) and how this maximal degree changes with the rate of market growth.\textsuperscript{6,7}

This paper is also related to the strand of literature on collusion in quantity setting games with possible entry.\textsuperscript{8} Research in this area has in-

\textsuperscript{6}For most oligopoly models, there is always an equilibrium with some collusion (as long as the discount factor is positive). So, the issue of whether there is collusion is inherently an equilibrium selection argument. In addition, in reality we are also concerned with the degree of collusion. So the relevant question here is more to try and understand how various factors (such as the market growth rate) impact the degree of collusion rather than whether firms collude.

\textsuperscript{7}For the case of price-setting games with capacity constraints, Brock and Scheinkman (1985) have studied the maximum price sustainable using grim trigger strategies. This best enforceable price was shown to be non-monotone in the number of firms. The intuition is simple. When the number of firms increases there are two opposing forces at work: on the one hand, a larger number of firms gives, at each price, a smaller market share for each colluding firm, but, on the other hand, as the number of firms increases harsher punishments are available since rivals have (in aggregate terms) higher capacity. Hence, the best enforceable collusive price needs not to be a monotonic function of the number of firms. Notice, however, that their model assumes constant demand over time and, therefore, cannot be used to address the question we are interested in, which is the impact of the rate of demand growth on the maximal degree of collusion.

\textsuperscript{8}There is also some important work on collusion in price-setting games with capacity
vestigated the degree of collusion which can be achieved when the number of market participants is endogenously derived from a free-entry condition. Different types of incumbent firms’ responses to entry have been studied. A first possibility is that incumbents use optimal punishment schemes, which have been characterized by Abreu (1986), to make entry unprofitable and support the joint profit maximum (see, for instance, Harrington (1989)).

Harrington (1991) considers the situation in which colluding firms, when facing an entrant, either fully incorporate the new firm in a more inclusive agreement as soon as it enters or forego collusion and trigger to single-shot Cournot equilibrium output levels. Another plausible response to entry has been proposed by Friedman and Thisse (1994). Their paper considers a type of collusion which is not generous to entrants. Entrants’ profits are in the first period after entry a little better than those corresponding to discounted single-shot Cournot equilibrium. Their output then increases.

Sorgard (1996) studies the capacity investment by a new firm into an established market with one incumbent firm whose capacity is fixed. If a collusive equilibrium prevails ex-post entry, then it is shown that capacity limitation may not be optimal for the entrant. The entrant’s optimal capacity in the collusive outcome depends on the punishment path. In particular, if optimal punishment paths are in force, it is shown that the incumbents’ non-collusive outcome always decreases in the entrant’s capacity. Hence, large capacity installation by the entrant results in low profit for the incumbent in the non-collusive outcome, which in turn encourages the incumbent to sustain the collusive outcome.

If the stage game is Bertrand type and firms face symmetric and constant marginal costs, then entry can be deterred by the threat of Nash-reversion. Entry never occurs, unless the entrant anticipates that it will be accommodated in a more inclusive collusive agreement. If, instead, the stage game is Bertrand type but firms face increasing marginal costs and positive fixed costs, then Requate (1994) studies a model in which, besides setting prices, firms have to decide whether to be active and use optimal penal codes to enforce collusion. The number of firms which can collude successfully is determined endogenously. In addition, upper and lower bounds for stationary equilibrium prices are characterized. The lower bound is strictly greater than the average cost. The upper bound, on the other hand, may be strictly smaller than the monopoly price if the number of firms is sufficiently high.
gradually over time until eventually the entrants become full partners in the collusive scheme. However, all these models assume constant demand over time and thus do not model the process by which entry is triggered in the long-run.

The rest of the paper is organized as follows. Section 2 describes the basic model. In Section 3, we analyze firms’ optimal behavior ex-post entry and characterize the optimal entry date. Section 4 will focus on the study of relationship between the maximal degree of pre-entry collusion and the market growth rate. Section 5 discusses whether partial collusion between the two incumbent firms is feasible in this setting. Finally, Section 6 offers some concluding comments.

2 Basic model

Consider an industry in which there are two incumbents and one potential entrant. Firms play an infinite horizon game and in each period of time all active firms (i.e., the two incumbents and the entrant, in case it has already entered) simultaneously choose an output rate. The potential entrant has to decide when to enter the industry (if it enters at all). A one-time entry (sunk) cost $K$, $K \geq 0$, has to be incurred if entry takes place. Active firms offer a homogeneous product and production is assumed to be costless for all firms. The payoff function of a given firm is given by the sum of discounted profits where profits are received at the end of each period and the common discount factor is $\delta \in (0, 1)$.

\footnote{Since this is a unique cost to the potential entrant, the size of $K$ can be interpreted as the ‘height’ of (exogenous) barriers.}
Assume that in each period of time \( t = 0, 1, 2, \ldots \) market demand is given by \( Q_t = (1 - p_t)\mu^t \), where \( p_t \) denotes the market price in period \( t \) and \( \mu > 1 \) is the parameter measuring demand growth. Hence, demand is growing steadily at rate \((\mu - 1)\).\(^{11}\) Assume also that \( \mu\delta < 1 \).

Under this set of assumptions, it is easy to show that the profits in a Cournot-Nash equilibrium for the single-period game played in period \( t \) when there are \( n \) firms in the industry are given by\(^{12}\)

\[
\pi_t^c(n) = \frac{\mu^t}{(n + 1)^2}.
\]  

(1)

The monopoly profit in a given period \( t \) is \( \pi_t^m = \pi_t^c(1) \),

\[
\pi_t^m = \frac{\mu^t}{4}.
\]  

(2)

We will focus on a particular class of subgame perfect Nash equilibria (SPE) of the infinitely repeated game, which we call Most Pro\-\-fitable Trig-\-ger Strategy Equilibrium with Entry (MPTSEE). A MPTSEE is a SPE where active firms and the potential entrant follow the following strategies, respectively. Starting from period 0, active firms will produce in any period \( t \in \{0, 1, 2, \ldots\} \) the best collusive sustainable output consistent with the number of active firms in the market and also with the discount factor \( \delta > 0 \).\(^{13}\) This most collusive output rate will continue to be produced as

\(^{11}\)As pointed out by Tirole (1988), a model of this kind describes the same type of situation discussed by Rotemberg and Saloner (1986). The only differences are that, on the one hand, shocks are perfectly anticipated and there is a trend, on the other.

\(^{12}\)See Appendix A.

\(^{13}\)With a Cournot model, as long as \( \delta > 0 \), there is always an equilibrium where some collusion can be sustained. This continuity offered by the Cournot model allows us to focus the analysis on the characterization of the maximal degree of collusion consistent with equilibrium and which may be something less than perfect collusion. Notice that using
long as no other active firm has deviated from the collusive path. In the event of a deviation, active firms permanently revert to a single-period Nash equilibrium (Friedman (1971)). The potential entrant will enter the industry in the period in which the NPV of its expected post-entry profits is maximal. In case entry occurs, the potential entrant becomes an active firm and should, therefore, follow the strategy of an active firm for the remainder of the horizon.

In what follows, we start by analyzing firms’ optimal behavior ex-post entry and derive the optimal entry date. We then turn to the analysis of the pre-entry behavior. In particular, we will study the relationship between the maximal degree of pre-entry collusion and the parameter $\mu$ measuring market growth. This relationship will be shown to be in general non-monotone.

Bertrand competition would not be appropriate to address the same issue. It is well known that with this alternative mode of competition, as long as the discount factor is above a critical threshold value, then any collusive price can be sustained (even the monopoly price). This is a “knife-edge” configuration - no collusion or full collusion (depending on whether the discount factor is below or above the critical threshold) - which would not allow us to study and characterize the maximal degree of collusion.

It is important to explain at this point why we consider Cournot-Nash reversion while it is well known that, for quantity-setting supergames, Abreu (1986) has characterized a class of more sophisticated and more severe punishments than standard “grim trigger strategies”. First, as pointed out by Harrington (1991, p. 1089) “it is quite natural to think of a punishment strategy as being an industry norm with respect to firm conduct ... Furthermore, once a norm is in place, firms may be hesitant to change it ... Thus, even though the norm might not be the best in some sense (for example, it might not be a most severe punishment strategy), firms might seem choose to maintain it if it seems to work. In light of this interpretation of a punishment strategy, it seems plausible that the grim trigger strategy would be a commonly used norm.” Second, the use of standard trigger strategies has the advantage of requiring simple calculations and also of being easily understood by market participants. We, therefore, restrict attention to this class of simple punishment strategies in this paper and leave the use optimal penal codes for further research.
2.1 Incumbents’ reaction to entry

Before proceeding with the analysis, however, it is important to explain why we consider that the incumbents incorporate the entrant in a more inclusive agreement rather than credibly threaten the entrant to revert to an equilibrium in which the entrant would be minmaxed so as to deter entry.\textsuperscript{15} Three kinds of arguments lead us to believe that the latter type of behavior by the incumbent firms is unlikely in many circumstances. First, as pointed out by Friedman and Thisse (1994), collusion is in many industries a reality incumbent firms have to live with. Therefore, in these situations, “intuition suggests that the incumbent firms might prefer to recontract and include the entrant into a revamped collusive agreement” (p. 272). Second, collusion is illegal and firms in the industry are certainly aware whenever self-enforcing agreements are being implemented. Hence, an entrant which is minmaxed by colluding incumbent firms will have very strong incentives to denounce the existence of the collusive agreement to the antitrust authorities so as to earn a per-period Cournot individual profit rather than its zero minmax payoff.\textsuperscript{16} Lastly, according to Besanko et al. (2004), “accommodated entry is typical in markets with growing demand” (p. 302).

\textsuperscript{15}Minmaxing an entrant consists in treating the entrant as a defector from the collusive agreement (say, because the potential entrant was supposed to produce zero along the equilibrium output path). Since in our setting the minmax payoff is zero independently of the demand level, a firm’s security level is a discounted payoff also equal to zero. Hence, if for some (sufficiently high) values of the discount factor security level punishment can be supported as a SPE, then by credibly threaten the entrant to revert to the equilibrium where it obtains zero profit as a continuation payoff, entry could be prevented.

\textsuperscript{16}Whistle-blowing mechanisms to deter collusion have been studied in detail by Motta and Polo (2003) and Aubert, Rey and Kovacic (2003).
3 Sustaining collusion ex-post entry

Let \( \tilde{q}_t \equiv \mu t \) denote the individual collusive output in period \( t \) when there are \( n \) firms in the market, where \( q \in \left[ \frac{1}{2n}, \frac{1}{n+1} \right] \). Denote by \( \pi_t(q, n) \) the individual collusive profit in period \( t \) when each firm produces \( \tilde{q}_t \), and by \( \pi^d_t(q, n) \) the largest one-shot profit that a firm can make in period \( t \) when \( \tilde{q}_t \) is supposed to be produced by each firm. Some algebra shows that\(^{18}\)

\[
\pi_t(q, n) = (1 - nq) \mu t, \quad (3)
\]

\[
\pi^d_t(q, n) = \left( \frac{1 - (n-1)q}{2} \right)^2 \mu t. \quad (4)
\]

The next proposition identifies and characterizes the maximal degree of collusion (consistent with equilibrium which may be something less than perfect collusion) that can be sustained ex-post entry.

**Proposition 1** Suppose entry occurs at time \( t' \). Then, the best collusive individual quantity that can be sustained in period \( t \geq t' \) as a MPTSEE is given by \( q^*(\mu, \delta) \mu t \), where

\[
q^*(\mu, \delta) = \begin{cases} 
\frac{4 - 3\mu\delta}{10 - 4\mu\delta}, & \text{if } \mu\delta < 4/7 \\
\frac{1}{6}, & \text{if } \mu\delta \geq 4/7
\end{cases}. \quad (5)
\]

Moreover, if \( \mu\delta < 4/7 \), then \( q^*(\mu, \delta) \) decreases with \( \mu \).

**Proof.** For any \( t \geq t' \), \( n = 3 \). So, collusion will be sustainable as a MPTSEE if and only if, for any \( t \in \{t', t' + 1, t' + 2, \ldots, \} \), the following incentive

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\(^{17}\)As shown in Appendix A, the Cournot individual output in period \( t \) when there are \( n \) firms in the market is given by \( q_c(n) = \mu t / (n + 1) \). This implies that \( \tilde{q}_t \) has to be greater or equal than the perfect collusion individual output rate \( \mu t / (2n) \), but at the same time lower than \( q_c(n) \).

\(^{18}\)See Appendix A for details.
compatibility constraint (ICC) holds
\[
\sum_{i=t}^{\infty} \delta^{i-t} \pi_i (q, 3) \geq \pi_t^d (q, 3) + \sum_{i=t+1}^{\infty} \delta^{i-t} \pi_i^c (3).
\] (6)

Making use of eqs. (1), (3) and (4), the previous ICC can be rewritten as
\[
(1 - 3q) q \sum_{i=t}^{\infty} \delta^{i-t} \mu^i \geq \left( \frac{1 - 2q}{2} \right)^2 \mu^t + \frac{1}{16} \sum_{i=t+1}^{\infty} \delta^{i-t} \mu^i,
\] (7)
or, equivalently,
\[
(1 - 3q) q \frac{\mu^t}{1 - \mu \delta} \geq \left( \frac{1 - 2q}{2} \right)^2 \mu^t + \frac{1}{16} \frac{\delta \mu^{t+1}}{1 - \mu \delta}.
\] (8)

Multiplying now both sides of the previous condition by \((1 - \mu \delta) / \mu^t\), the problem becomes stationary (since the \(\mu^t\) terms cancel out in the ICC, each period looks like the first one) and the previous condition is equivalent to
\[
(1 - 3q) q \geq \left( \frac{1 - 2q}{2} \right)^2 (1 - \mu \delta) + \frac{\delta \mu}{16}.
\] (9)

Hence, given \(\delta > 0\) and \(\mu > 1\), period \(t\)'s best individual collusive output is equal to \(q^*(\mu, \delta) \mu^t\), where \(q^*(\mu, \delta)\) is the level of \(q\) for which the previous ICC is binding. Some algebra shows that
\[
q^*(\mu, \delta) = \begin{cases} 
\frac{4 - 3 \mu \delta}{16 - 4 \mu \delta}, & \text{if } \mu \delta < 4/7 \\
\frac{1}{6}, & \text{if } \mu \delta \geq 4/7 
\end{cases}.
\] (10)

In addition, if \(\mu \delta < 4/7\), very simple algebra shows that
\[
\frac{\partial q^*(\mu, \delta)}{\partial \mu} = -\frac{2 \delta}{(4 - \mu \delta)^2} < 0.
\] (11)

This completes the proof.
Corollary 1 If \( K \) is finite, then perfect collusion can be sustained ex-post entry as a MPTSEE if and only if
\[
\delta \geq \frac{4}{7 \mu} \equiv \delta (\mu). \tag{12}
\]

Proof. Since demand grows constantly over time, if \( K \) is finite, then the potential entrant eventually will find entry profitable. Now, since in a perfect collusion scenario the individual output rate in period \( t \) equals \( \mu t / 6 \) when there are three firms in the market, the critical discount factor for perfect collusion to be sustainable ex-post entry as a MPTSEE then follows directly from (5).

Ex-post entry, the number of market participants can no longer be affected by market growth. Hence, perfect collusion will be easier to sustain as a MPTSEE as \( \mu \) increases. The reason is simply that the more the market is growing ex-post entry, the higher is the importance of future profits from collusion relative to the current gain from deviating (this is the so called intrinsic pro-collusive effect of demand growth).

Notice that from (5), one has that, for a given period \( t \) ex-post entry, the individual profit of each firm in the market will be equal to \( \bar{\pi}_t (1/6, 3) \) if \( \mu \delta \geq 4/7 \) and \( \bar{\pi}_t ((4 - 3\mu \delta) / (16 - 4\mu \delta), 3) \) otherwise. Summarizing, individual ex-post entry profits are given by \( \pi_t^e = \Pi \mu^t \), where
\[
\Pi = \begin{cases} 
\frac{(5\mu\delta + 4)(4 - 3\mu \delta)}{16(4 - \mu \delta)^2}, & \text{if } \mu \delta < 4/7 \smallskip \\
\frac{1}{17}, & \text{if } \mu \delta \geq 4/7 
\end{cases} \tag{13}
\]

Armed with the above expression for ex-post entry individual profits, it is now possible to look at the entrants’ optimal behavior. The next Lemma puts forward the first result in this direction.
Lemma 1 Let $t$ denote the first period in time at which the NPV of the entrant’s expected profits is positive. The entrant will optimally choose to enter in period $t + \tau$, $\tau \in \{1, 2, \ldots\}$ rather than in period $t$ if the following condition holds
\[
(1 - (\mu \delta)^\tau) K < \Pi \mu^t (1 + \mu \delta + \ldots + (\mu \delta)^{\tau-1}) < (1 - \delta^\tau) K,
\]
where $\Pi$ is given by (13).

Proof. By definition, $t$ is the first period in time at which the NPV of the entrant’s expected profits is positive. In addition, the entrant’s payoff in each period $t \geq t$ is $\pi_i^{t} = \Pi \mu^t$, where $\Pi$ is given by (13). Hence, the following condition must hold since it just requires that entering in period $t$ yields a positive NPV of profits to the entrant:
\[
\sum_{i = t}^{\infty} \delta^{-i} \Pi \mu^i = \frac{\Pi \mu^t}{1 - \mu \delta} > K,
\]
or, equivalently,
\[
\Pi \mu^t > (1 - \mu \delta) K.
\]

Now, from the point of view of period $t$ money, delaying entry to period $t + \tau$ will be preferred to entering in period $t$ if
\[
\delta^\tau \left( \sum_{i = t+\tau}^{\infty} \delta^{-i} \Pi \mu^i - K \right) > \sum_{i = t}^{\infty} \delta^{-i} \Pi \mu^i - K,
\]
or, equivalently
\[
\delta^\tau \Pi \mu^{t+\tau} - \delta^\tau K > \frac{\Pi \mu^t}{1 - \mu \delta} - K,
\]
or, equivalently
\[
(1 - \delta^\tau) K > \frac{\Pi \mu^t}{1 - \mu \delta} (1 - (\mu \delta)^\tau).
\]
Now, making use of the fact that $1-(\mu\delta)^\tau = (1-(\mu\delta)) \left( 1 + \mu\delta + ... + (\mu\delta)^{\tau-1} \right)$, conditions (16) and (19) can respectively be rewritten as follows

\[
(1 + \mu\delta + ... + (\mu\delta)^{\tau-1}) \Pi\mu^t > (1 - (\mu\delta)^\tau) K, \quad (20)
\]

\[
(1 - \delta^\tau) K > \Pi\mu^t \left( 1 + \mu\delta + ... + (\mu\delta)^{\tau-1} \right). \quad (21)
\]

Hence, conditions (15) and (17) (or, equivalently, (20) and (21)) will simultaneously hold if

\[
(1 - (\mu\delta)^\tau) K < \Pi\mu^t \left( 1 + \mu\delta + ... + (\mu\delta)^{\tau-1} \right) < (1 - \delta^\tau) K.
\]

This completes the proof.

Hence, contrary to standard supergame models of collusion that examine entry in a context where demand is constant, in our setting the optimal entry time may not correspond to the first period in time at which the NPV of the entrant’s payoff is positive. The intuition is simple. Entering in period $t+\tau$ rather than in period $t$ might be more profitable for the entrant. While the delay costs $\tau$ periods of profits, it also delays the payment of the entry costs for $\tau$ periods. We can then have the latter benefit outweighing the former cost, and entry in period $t$ having a positive NPV of post-entry profits, if the condition put forward in the previous Lemma is satisfied.

The next proposition derives the optimal entry time along the equilibrium path.
**Proposition 2**  When collusion is supported by MPTSEE, then the optimal entry time is given by

\[
t_1(\mu, \delta, K, \Pi) = \frac{1}{\ln \mu} \ln \left( \frac{\ln \delta \ K (1 - \delta \mu)}{\ln \delta \mu \Pi} \right)
\]  \hspace{1cm} (22)

where \( \Pi \) is given by eq. (13). In addition, \( t_1(\mu, \delta, K, \Pi) \) decreases in \( \mu \).

**Proof.**  Along the equilibrium path the entrant expects to be accommodated in a more inclusive collusive agreement. In addition, in a given period \( t \) ex-post entry, the entrant’s profits are \( \pi_t^e = \Pi \mu^t \), where \( \Pi \) is given by eq. (13). Hence, the optimal entry time period is found by solving the following optimization problem:

\[
\max_{t \geq 1} \left\{ \delta^t \sum_{i=t}^{\infty} \delta^{i-t} \mu^i \Pi - \delta^t K \right\},
\]  \hspace{1cm} (23)

or, equivalently

\[
\max_{t \geq 1} \left\{ \frac{\delta^t \mu^t \Pi}{1 - \mu \delta} - \delta^t K \right\}.
\]  \hspace{1cm} (24)

The first order condition (FOC) of this maximization problem is

\[
\mu^t = \frac{\ln \delta \ K (1 - \delta \mu)}{\ln(\delta \mu) \ \Pi},
\]  \hspace{1cm} (25)

which in turn implies that

\[
t_1(\mu, \delta, K, \Pi) = \frac{1}{\ln \mu} \ln \left( \frac{\ln \delta \ K (1 - \delta \mu)}{\ln(\delta \mu) \ \Pi} \right).
\]  \hspace{1cm} (26)

Notice that in order for \( t_1(\mu, \delta, K, \Pi) \geq 1 \), one must have that

\[
K \geq \frac{\mu \Pi \ln \delta \mu}{(\ln \delta) (1 - \delta \mu)} \equiv K.
\]  \hspace{1cm} (27)
Making use of (26), one can now carry out a comparative statics exercise to evaluate how the optimal entry time $t_1 (\mu, \delta, K, \Pi)$ is affected by demand growth:

$$\frac{dt_1}{d\mu} (\mu, \delta, K, \Pi) = \frac{\partial t_1}{\partial \mu} + \frac{\partial t_1}{\partial \Pi} \frac{\partial \Pi}{\partial \mu},$$

(28)

where some algebra shows that

$$\frac{\partial t_1}{\partial \mu} = - \left( \ln \left( \frac{K}{\ln (\delta \mu)} \right) \ln (1 - \delta \mu) + (1 - \delta \mu (1 - \ln (\delta \mu))) \ln \mu \right) \frac{\ln (\delta \mu) \ln (\delta \mu)}{\ln (\delta \mu) (\ln \mu)^2 \mu (1 - \delta \mu)}.$$

(29)

which turns out to be negative for all $K \geq K_0$ (see eq. (27)). In addition,

$$\frac{\partial t_1}{\partial \Pi} = - \frac{1}{(\ln \mu) \Pi} < 0,$$

(30)

and, from eq. (13), one has that

$$\frac{\partial \Pi}{\partial \mu} = \left\{ \begin{array}{ll} \delta \frac{(4 - 7\mu \delta)}{(4 - \mu \delta)} > 0, & \text{if } \mu \delta < 4/7 \\ 0, & \text{if } \mu \delta \geq 4/7 \end{array} \right..$$

(31)

Now, from (29)-(31), it can be easily seen that $dt_1 (\mu, \delta, K, \Pi) / d\mu$ is negative.

So, the higher the growth rate is, the sooner will entry occur along the equilibrium path. This effect will play a crucial role in the analysis that follows regarding the extent of collusion which is sustainable pre-entry. An increase in the growth rate has a negative impact on the continuation value of collusion at every period pre-entry which in turn tends to lower the maximal degree of collusion that can be sustained before entry as a MPTSEE.
4 Sustaining collusion pre-entry

In this section, we study how the market growth affects the maximal degree of collusion pre-entry. In order to address this issue, however, we need to determine first the optimal entry time off the equilibrium path, i.e., in case firms play as Cournot oligopolists in every period that follows entry.

**Lemma 2** If firms play as Cournot oligopolists in every stage game that follows entry, then the optimal entry time is given by

\[
t_2(\mu, \delta, K) = \frac{1}{\ln \mu} \ln \left( 16K \left( 1 - \delta \mu \right) \frac{\ln \delta}{\ln \delta \mu} \right), \tag{32}
\]

where \( t_2(\mu, \delta, K) \) decreases in \( \mu \).

**Proof.** If the potential entrant expects that entry will be followed by Cournot competition in every stage game, then its optimal entry time is found by solving the following optimization problem:

\[
\max_{t \geq 1} \left\{ \delta^t \sum_{i=t}^{\infty} \delta^{i-t} \pi_i^c (3) - \delta^t K \right\}. \tag{33}
\]

Making use of (1), the previous maximization problem can be rewritten as:

\[
\max_{t \geq 1} \left\{ \frac{\delta^t}{16} \sum_{i=t}^{\infty} \delta^{i-t} \mu_i^t - \delta^t K \right\}, \tag{34}
\]

or, equivalently

\[
\max_{t \geq 1} \left\{ \frac{\delta^t \mu_i^t}{16 (1 - \mu \delta)} - \delta^t K \right\}. \tag{35}
\]

From the FOC of this problem, some algebra shows that the optimal entry time is given by

\[
t_2(\mu, \delta, K) = \frac{1}{\ln \mu} \ln \left( 16K \left( 1 - \delta \mu \right) \frac{\ln \delta}{\ln \delta \mu} \right), \tag{36}
\]
where we assume that
\[ K \geq \frac{1}{16} \frac{\mu \ln \mu \delta}{(\ln \delta)(1 - \mu \delta)} \equiv \bar{K} \quad (37) \]
in order for \( t_2(\mu, \delta, K) \geq 1 \).

Now, from (36), a simple comparative statics exercise shows that
\[ \frac{\partial t_2}{\partial \mu} = -\left( \ln \left( (\ln \delta) 16K \frac{1-\delta \mu}{\ln \delta \mu} \right) \ln \delta \mu \right) \left( 1 - \delta \mu + (1 - \delta \mu (1 - \ln(\delta \mu))) \ln \mu \right) \ln (\delta \mu) (\ln \mu)^2 \mu (1 - \delta \mu), \]
(38)
which turns out to be negative for all \( K \geq \bar{K} \). ■

Let us now turn to the analysis of the conditions for pre-entry incentive compatibility. It should be stressed that the incumbents’ ICC in the first period of the supergame is not a sufficient condition to ensure pre-entry incentive compatibility. Hence, one has to check the incumbents’ ICC for every time period before entry occurs. Assume that \( \mu \delta < 4/7 \) so that ex-post entry firms will not be able to sustain perfect collusion.\(^{19}\) Now, consider a period \( t \in \{0, 1, \ldots, t_1 - 1\} \). Then, collusion can be sustained in period \( t \) as a MPTSEE if the following ICC holds:\(^{20}\)
\[ \sum_{i=t}^{t_1-1} \delta^{i-t} \pi_i(q, 2) + \sum_{i=t_1}^{\infty} \delta^{i-t} \pi_i \left( \frac{4 - 3\mu \delta}{16 - 4\mu \delta}, 3 \right) \geq \pi^d_i(q, 2) + \sum_{i=t+1}^{t_2-1} \delta^{i-t} \pi^c_i(2) + \sum_{i=t_2}^{\infty} \delta^{i-t} \pi^c_i(3), \]
(39)
where \( q \in [1/4, 1/3] \) and the collusive quantity that each incumbent is supposed to produce in period \( t \) equals \( \bar{q}_t \equiv q \mu^t \) (see Appendix A for the deriva-

\(^{19}\)Since the focus of this section is to understand how market growth affects the maximal degree of collusion pre-entry, this restriction ensures that we concentrate attention in the interesting case where, in case firms observe an increase in the rate of market growth, they anticipate that this increase will affect the maximal level of collusion that they will be able to sustain ex-post entry as a MPTSEE (see Proposition 1).

\(^{20}\)Remember that from (5), one has that for \( \mu \delta < 4/7 \), the best collusive quantity ex-post entry is \( q^*(\mu, \delta) = (4 - 3\mu \delta) / (16 - 4\mu \delta) \).
tion of profits). In addition, \(t_1\) and \(t_2\) are respectively given by eqs. (22) and (32). Now, making use of eqs. (1), (3) and (4), the previous ICC can be rewritten as

\[
(1 - 2q) q \sum_{i=t}^{t_1-1} (\mu^i \delta^{i-t}) + \frac{(5\mu\delta + 4)(4 - 3\mu\delta)}{16 (4 - \mu\delta)^2} \sum_{i=t_1}^{\infty} (\mu^i \delta^{i-t}) \geq (40)
\]

\[
\left(\frac{1 - q}{2}\right)^2 \mu^t + \frac{1}{9} \sum_{i=t_1+1}^{t_2-1} (\mu^i \delta^{i-t}) + \frac{1}{16} \sum_{i=t_2}^{\infty} (\mu^i \delta^{i-t}),
\]

or, equivalently,

\[
(1 - 2q) q \left(\frac{(\mu\delta)^t - (\mu\delta)^{t_1}}{\delta^t (1 - \mu\delta)}\right) + \frac{(5\mu\delta + 4)(4 - 3\mu\delta)}{16 (4 - \mu\delta)^2} \frac{\mu^{t_1} \delta^{t_1-t}}{1 - \mu\delta} \geq (41)
\]

\[
\left(\frac{1 - q}{2}\right)^2 \mu^t + \frac{1}{9} \left(\frac{(\mu\delta)^{t+1} - (\mu\delta)^{t_2}}{\delta^t (1 - \mu\delta)}\right) + \frac{1}{16} \frac{\mu^{t_2} \delta^{t_2-t}}{1 - \mu\delta}.
\]

The previous ICC implicitly defines a set of feasible values of \(q\) that can be sustained in period \(t\) as a MPTSEE. In what follows, we denote the value of \(q\) implicitly defined by eq. (41) when it is a binding constraint as \(q^*_t(\mu, \delta, t_1, t_2)\), where \(t_1\) and \(t_2\) are respectively given by eqs. (22) and (32). This value \(q^*_t(\mu, \delta, t_1, t_2)\) is the best collusive individual (normalized) quantity that can be sustained in period \(t\) as a MPTSEE.\(^{21}\) We now turn to the study of how \(q^*_t(\mu, \delta, t_1, t_2)\) varies with changes in the market growth parameter \(\mu\). As the reader can easily anticipate, because of the complexity of the ICC (41), one cannot obtain a tractable analytical explicit solution for \(q^*_t(\mu, \delta, t_1, t_2)\). This explains why in what follows we revert to a calibration of the model.

\(^{21}\)The best individual collusive quantity is equal to \(q^*_t(\mu, \delta, t_1, t_2)\). Hence, \(q^*_t(\mu, \delta, t_1, t_2)\) is a good indicator of the maximal degree of collusion in each pre-entry period \(t\).
Figure 1: Optimal entry times

Assume that $\delta = 0.3$ and $K = 1$. Making use of (22) and (32), Figure 1 illustrates how the optimal entry times $t_1$ and $t_2$ vary with the market growth rate $\mu$.\footnote{Remember that we are restricting attention to the case in which ex-post entry firms can only imperfectly collude ($\mu \delta < 4/7$). On the other hand, we set $\delta = 0.3$. Hence, the feasible range of values for $\mu$ is $\mu \in (1, 1.9048)$.} This Figure illustrates the results in Proposition 2 and Lemma 2. In addition, it shows that, for the specific example at hand and for every feasible value of $\mu$, there always exist at least four pre-entry periods (periods 0,1,2 and 3). In addition, it can be easily shown that if $\mu < 1.8142$,\footnote{From (22), it is straightforward to show that, for the assumed baseline values for $\delta$ and $K$, $t_1 = 4$ when $\mu = 1.8142$.} then period $t = 4$ is also a pre-entry period.

\[ t_1(\mu, \delta, K, \Pi) \]
\[ t_2(\mu, \delta, K) \]
Figure 2: Pre-entry best collusive individual quantities

Figure 2 illustrates how the best individual collusive (normalized) quantities $q_t(\mu, \delta, t_1, t_2)$ in periods 0, 1, ..., 4 vary with $\mu$.\(^{24}\)

Two important observations should be made regarding this figure. First, for a fixed pre-entry date $t$, the relationship between the maximal degree of pre-entry collusion and the parameter $\mu$ measuring market growth is in general non-monotone. Hence, it is important to identify the different forces at work which justify this behavior. The interplay between the three following forces determines whether the maximal degree of pre-entry collusion increases or decreases with the rate of market growth:

1. An increase in the market growth rate induces a decrease in the best

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\(^{24}\)The same simulation was run with different parameter values and the $q_t^*(\mu, \delta, t_1, t_2)$ curves proved to follow the same pattern as the one presented in Figure 2.
collusive output ex-post entry $q^*(\mu, \delta)$ whenever $\mu \delta < 4/7$ (see Proposition 2). This in turn implies that, for a given entry optimal date $t_1$, an increase in $\mu$ leads to an increase in the most-collusive profits ex-post entry, which tends to increase the maximal degree of pre-entry collusion.

2. Entry, however, nears as the market growth rate increases, which in turn gives rise to two opposite effects on the extent to collusion that can be sustained pre-entry:

(a) On the one hand, $t_2$ decreases in $\mu$. Hence, along the punishment path, the number of periods in which firms earn the triopoly (rather than the duopoly) Cournot profits increases. This means that an increase in the market growth rate induces a decrease in the punishment continuation value. In other words, as entry occurs sooner along the punishment path, the punishment becomes harsher which contributes to an increase in the maximal degree of pre-entry collusion.

(b) On the other hand, also $t_1$ decreases in $\mu$. Hence, as $\mu$ increases, there is an induced decrease in the number of periods in which firms earn the pre-entry most collusive duopoly profit (and thus an increase in the number of periods where firms earn the most collusive triopoly profits). This effect contributes to a decrease in the extent of collusion which can be sustained pre-entry sim-

\footnote{Both $t_1$ and $t_2$ are decreasing in $\mu$ (see Proposition 2 and Lemma 2).}
ply because it lowers the continuation value of collusion at every period pre-entry. This effect is exacerbated when the lag $t_1 - t$ is small.

As a result, $q_\delta(\mu, \delta, t_1, t_2)$ can either decrease or increase in response to changes in the market growth rate, depending on whether forces 1 and 2a jointly dominate force 2b, or otherwise. A one interesting feature in the pattern of the curves presented in the previous figure is that, for intermediate values of $\mu$, force 2b can be sufficiently important to more than compensate the effects of forces forces 1 and 2a, which in turn implies that the extent of collusion that can be sustained pre-entry as a MPTSEE can decrease with the market growth rate.

The other important point that Figure 2 illustrates is that the closer the pre-entry time period $t$ is from the optimal entry time $t_1$, the lower is the degree of (pre-entry) collusion which can be enforced with MPTSEE. The intuition behind this result is simple. The incentives to deviate increase as we approach the end of most attractive part of the collusive path (in which incumbents share the most collusive pre-entry duopoly profits). This effect is especially important for sufficiently high values of $\mu$ simply because the lag $(t_1 - t)$ decreases in $\mu$ (as $t_1$ is an decreasing function of the growth parameter $\mu$).
5 Partial collusion

So far, we have focused attention on the analysis of full collusion, i.e., it has been assumed that all active firms in the market were colluding. An interesting question, however, is to analyze whether partial collusion between the two incumbent firms is feasible in this setting. In particular, suppose now that the two incumbent firms are members of a cartel and should entry occur while the cartel is operating, the entrant becomes a fringe competitor which in every period picks an output that maximizes its own profit.\footnote{The cartel acts as a Stackelberg quantity leader against the Cournot entrant.}

For every period $t$ in which it is active, the entrant plays a one period best-response to the cartel’s aggregate output, i.e., selects its output along the following reaction function:

$$q_{f,t} = \frac{\mu_t - Q_{K,t}}{2},$$

(42)

where $Q_{K,t}$ denotes the cartel output in period $t$.

From (42), the residual demand facing the cartel in period $t$ is:

$$p_t = \frac{\mu_t - Q_{K,t}}{2\mu^t}. \quad (43)$$

Profit maximization for the cartel implies that in each period $t$ it will produce the monopoly output:

$$Q_{K,t} = \frac{\mu^t}{2}. \quad (44)$$

Substituting (44) into (42) and (43), one obtains the fringe firm output and
the cartel price in each period $t$

\[ q_{f,t} = \frac{\mu^t}{4}, \]  
\[ p_t = \frac{1}{4}. \]  

(45)  
(46)

Individual profits of cartel firms and the fringe firm are respectively given by

\[ \pi_{K,t} = \frac{\mu^t}{16}, \]  
\[ \pi_{f,t} = \frac{\mu^t}{16}. \]  

(47)  
(48)

Two notes are in order at this point. First, there is no free-riding problem in this setting since $\pi_{K,t} = \pi_{f,t}$. Second, and most importantly, notice that $\pi_{K,t} = \pi_{f,t} = \pi^c_t$ (3). In other words, by partially colluding, cartel members can earn no higher profit than the Cournot individual profit in each period of time. Partial collusion is therefore unfeasible in this setting.

It is important to underline at this point that this result is consistent with the models by Martin (1990) and Shaffer (1995). It is easy to show that in those models as well, it turns out that if two firms form a cartel and act as Stackelberg leader against a Cournot fringe composed of a single firm, then the cartel is not able to earn a (per-period) profit exceeding the Cournot profit.

\[ \text{This result is in line with Shaffer (1995) who shows that in an industry where } n \text{ firms face a linear demand and linear costs and compete in quantities, a cartel composed of } k < n \text{ firms and acting as a Stackelberg leader with respect to the fringe does not necessarily face the free-rider problem. In particular, in Proposition 1 it is shown that if the cartel is sufficiently small } (k \leq (n + 1)/2), \text{ then each firm in the cartel earns a profit which is no smaller than that of a fringe firm.} \]
6 Conclusion

This paper has explored the relationship between demand growth and collusion in a model where market growth can trigger future entry. This is an issue which has received very little attention in the previous literature on tacit collusion, but is of utmost importance for understanding the relationship between demand growth and firms’ market power in an industry.

Ex-post entry, the number of market participants can no longer be affected by market growth and the standard intrinsic pro-collusive effect of demand growth is shown to prevail: the expected rise in demand increases the future cost of deviation, which in turn implies that an increase in the market growth rate induces an increase in the maximal degree of sustainable post-entry collusion.

Ex-ante entry, however, the relationship between the maximal degree of sustainable pre-entry collusion and market growth is shown to be in general non-monotone. This analysis, therefore, clearly suggests that, as emphasized by Ivaldi et al. (2003), when studying the impact of demand growth on (pre-entry) collusion, it is crucial to try and disentangle the pro-collusive intrinsic effect of demand growth from the impact of entry and other factors affected by market growth so as to assess their relative strengths. By so doing, the current paper sheds some light on the understanding of why the EC usually interprets demand growth as a factor hindering collusion, an interpretation which contrasts with the conclusion of tacit collusion models with growing demand where the possibility of entry is assumed away.
A natural extension of the model developed in this paper would be to assume that demand moves in cycles, as in the post-Rotemberg and Saloner (1986) literature. For the purposes of this paper, however, models of this sort are left for future research.

References


A Profits

In this section we derive the per-period profits earned by each firm in three alternative scenarios: Cournot oligopoly, collusion and one-shot deviation from the collusive norm.

A.1 Cournot

In a Cournot-Nash equilibrium for the single period game played in period $t$ when there are $n$ firms in the industry, a representative firm $i$ chooses its output by solving the following maximization problem.

$$\max_{q_{i,t}} \left\{ \left( 1 - \frac{\sum_{i=1}^{n} q_{j,t}}{\mu^t} \right) q_{i,t} \right\}.$$  \hspace{1cm} (49)

The associated FOC is given by

$$1 - \frac{1}{\mu^t} \left( \sum_{j \neq i}^{n} q_{j,t} + 2q_{i,t} \right) = 0.$$  \hspace{1cm} (50)
By symmetry, \( \forall i \in \{1, \ldots, n\}, q_{i,t} = q_t \). Hence, the individual output rate in period \( t \) is
\[
q^c_t(n) = \frac{\mu^t}{n+1}.
\]
(51)

Now, the industry equilibrium output and equilibrium profits are respectively given by:
\[
Q^c_t(n) = \frac{n}{n+1} \mu^t,
\]
(52)
\[
\pi^c_t(n) = \left( \frac{1}{n+1} \right)^2 \mu^t.
\]
(53)

A.2 Collusion

Assume now that in period \( t \) there are \( n \) firms colluding in the market. Let \( \tilde{q}_t \equiv q \mu^t \) denote the individual collusive output rate, where \( q \in \left[ \frac{1}{2n}, \frac{1}{n+1} \right] \).

Then, it is straightforward to show that industry output rate and individual profits in this period \( t \) are given by:
\[
\tilde{Q}_t = nq \mu^t
\]
(54)
\[
\tilde{\pi}_t(q, n) = \tilde{p}_t \tilde{q}_t = (1 - nq) q \mu^t.
\]
(55)

A.3 Deviation

If a given firm is considering deviating in period \( t \), when each firm is supposed to produce \( \tilde{q}_t \equiv q \mu^t \), then the deviating firm optimal deviation output will result from the following optimization problem:
\[
\max_{q^d_t} \left\{ \left(1 - \frac{(n-1)q \mu^t + q^d_t}{\mu^t} \right) q^d_t \right\}.
\]
(56)
The associated FOC is:

$$1 - \frac{(n - 1)q\mu^t + 2q^d_t}{\mu^t} = 0.$$  \hspace{1cm} (57)

Hence, very simple algebra shows that $q^d_t = q^d_t \mu^t$, where

$$q^d = \frac{1 - (n - 1)q}{2}.$$  \hspace{1cm} (58)

In addition, the industry output rate and the deviator’s individual profit are respectively given by:

$$Q^d_i = \left((n - 1)q + q^d\right)\mu^t = \left(1 + \frac{(n - 1)q}{2}\right)\mu^t,$$  \hspace{1cm} (59)

$$\pi^d_i (q, n) = \left(1 - \frac{(n - 1)q}{2}\right)^2 \mu^t.$$  \hspace{1cm} (60)