

Equilibrium Vertical Foreclosure in the Repeated Game*

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Abstract

This paper analyzes whether vertical foreclosure can emerge as an equilibrium in an infinitely repeated game. In the stage game, a vertically integrated firm gains from foreclosure due to a “raising rival’s costs” effect but foreclosure is not a static Nash equilibrium because of a commitment problem. This paper shows that, in the repeated game, foreclosure is a subgame perfect equilibrium. Regarding the price charged to the nonintegrated firm, equilibrium refinements predict that this price will not be lower than the monopoly price. Further, foreclosure as a collusive equilibrium may be easier to sustain than collusion in the nonintegrated industry.

Keywords: collusion, foreclosure, vertical integration

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1 Introduction

One of the main arguments in the literature on vertical integration is that integration may cause foreclosure because this “raises rival’s costs”. The argument was first put forward by Ordover, Saloner and Salop (1990), henceforth OSS, and is as follows. When a vertically integrated firm forecloses nonintegrated downstream firms, that is, when it withdraws from the input good market, upstream competition becomes weaker. This reduction of competition implies higher input cost for the nonintegrated downstream firms. Since the downstream unit of the integrated firm benefits when rivals’ costs are raised, the integrated firm is better off pursuing such a foreclosure strategy compared to the case where it actively competes.

While OSS has been rather influential in the theoretical industrial organization literature¹, it has also been criticized. Hart and Tirole (1990) and Reiffen (1992) pointed out that the result depends on the assumption that integrated firms can commit not to supply nonintegrated downstream rivals. Without commitment, vertically integrated firms will compete just like the other upstream firms. In the static Nash equilibrium without commitment, nonintegrated downstream firms are not foreclosed and equilibrium prices are the same with and without vertical merger.

This paper investigates whether foreclosure can be an equilibrium of the infinitely repeated game. This possibility has been suggested by Riordan and Salop (1995) and the intuition is straightforward. Repeated interaction (Macauley, 1963) can serve as a commitment device in that it helps the integrated firm establishing a reputation for staying out of nonintegrated markets. Since all upstream firms benefit from vertical foreclosure in the long run, it could be an equilibrium of the repeated game.

The oligopoly model of the stage game in this paper is very similar to the OSS’s model. There is a duopoly both upstream and downstream. Upstream firms charge linear prices to the downstream firms and engage in perfect price competition. At the downstream level, there is differentiated price competition. Whereas OSS consider a static game, we analyze the infinitely repeated game.

The results are that foreclosure is indeed a subgame perfect Nash equilibrium of the infinitely repeated game, provided firms’ discount factor is sufficiently high. In a general model, we show that in fact any individually rational price charged to the nonintegrated downstream firm can be part of a collusive equilibrium. Equilibrium refinements help reducing the set of plausible prices, and they suggest that the nonintegrated firm will be charged a price at least as high as the monopoly price. Using a parametrized

¹Salinger (1988), Hart and Tirole (1990) and OSS are seminal among the new foreclosure theories. Martin, Normann and Snyder (2001) and Rey and Tirole (2003) contain reviews of foreclosure theories.

model, we further show that foreclosure as a collusive equilibrium under vertical integration often requires a lower minimum discount factor than collusion under vertical separation. In other words, vertical integration may facilitate collusion. Finally, we discuss whether downstream collusion can be a more attractive collusive strategy.

Following the critique by Hart and Tirole (1990) and Reiffen (1992), various papers have shown that OSS's foreclosure result can be rigorously derived from game-theoretic models. Ordober, Saloner and Salop (1992) re-establish their result in a descending-price auction. Riordan (1998) analyses backward integration by a dominant firm with a cost advantage. Choi and Yi (2000) and Church and Gandal (2000) show the result if upstream firms can commit to a technology which makes the input incompatible to nonintegrated downstream firms. In Chen (2001) downstream firms strategically choose upstream suppliers, and Riordan and Chen (2003) investigate the connection between vertical integration and exclusive dealing contracts. This paper contributes to this literature by showing equilibrium vertical foreclosure in the infinitely repeated game of the original OSS model.

The next section introduces the general model and section 3 reports the results derived from that model. Section 4 introduces a parametrized model. Whether or not foreclosure facilitates collusion in the parametrized model is analyzed in the section 5 and downstream collusion is considered in section 6. The final section is the conclusion.

2 The Model and Static Nash Equilibrium

Apart from minor differences, the stage game market model is as in OSS. There are two upstream firms and two downstream firms. Call the two upstream firms $U1$ and $U2$, and the two downstream firms $D1$ and $D2$. The integrated firm will be called $U1-D1$.

We start by analyzing the downstream level. Downstream firms pay linear prices for the input which constitute their only cost.² Define c_i as the price per unit firm Di pays. There is differentiated price competition and $Q_i(p_i, p_j)$, $i, j=1, 2, i \neq j$, denotes the demand function of Di . Accordingly, Di 's profits are

$$\pi_{Di} = (p_i - c_i)Q_i(p_i, p_j), \quad i, j = 1, 2, i \neq j. \quad (1)$$

We impose the following assumptions on demand. Demand functions $Q_i(p_i, p_j)$ are twice continuously differentiable with $\partial Q_i / \partial p_i < 0$, $\partial Q_i / \partial p_j > 0$, and $\partial Q_i / \partial p_i - \partial Q_i / \partial p_j < 0$, $i, j = 1, 2, i \neq j$. These

²This can be generalized to more complex downstream cost functions. See OSS.

assumptions ensure downward sloping demand with substitutes goods. Further, we assume that goods are strategic complements, that is, $\partial^2 Q_i / \partial p_i \partial p_j > 0$. A final assumption is that $\partial^2 Q_i / \partial p_i^2 + \partial^2 Q_i / \partial p_i \partial p_j < 0$. These assumptions imply that the upward sloping best-reply functions have a slope of less than one and, hence, they are sufficient to ensure the existence of a unique Nash equilibrium of the stage game.³

Let $p_i^*(c_i, c_j)$, $i, j=1, 2$, $i \neq j$, denote the static Nash equilibrium prices at the D level. In the static Nash equilibrium, the input prices (c_i, c_j) sufficiently describe downstream competition, and we will often use $Q_i^*(c_i, c_j)$ as a shortcut for $Q_i(p_i^*(c_i, c_j), p_j^*(c_j, c_i))$, and $\pi_{Di}^*(c_i, c_j)$ for $\pi_{Di} = (p_i^*(c_i, c_j) - c_i)Q_i^*(c_i, c_j)$.

Given the above assumptions, one can show that raising the cost of a downstream rival is profitable, that is,

$$\frac{\partial \pi_{Di}^*(c_i, c_j)}{\partial c_j} = \frac{\partial \pi_{Di}}{\partial p_j} \frac{\partial p_j^*}{\partial c_j} > 0, \quad i, j = 1, 2, i \neq j. \quad (2)$$

Note that $\partial \pi_{Di} / \partial p_j > 0$ follows from $\partial Q_i / \partial p_j > 0$, and $\partial p_j^* / \partial c_j > 0$ follows from comparative statics of the first order condition $\partial \pi_{Dj} / \partial p_j = 0$.

We now turn to the upstream level. $U1$ and $U2$ have constant marginal cost which we normalize to zero. The upstream firms compete in prices and we assume perfect Bertrand competition between them. The U firm posting the lower price to Di supplies $Q_i^*(c_i, c_j)$ $i, j=1, 2$, $i \neq j$, to this downstream firm and obtains a profit of $c_i Q_i^*(c_i, c_j)$. The high price firm sells nothing to Di and, in the case of a tie, each firm supplies $Q_i^*(c_i, c_j)/2$.

The static Nash equilibrium at the upstream level is as follows. Without vertical integration, competition à la Bertrand implies prices equal to marginal cost for both Di , that is, $c_1 = c_2 = 0$. This static Nash equilibrium is unique. When $U1$ and $D1$ are integrated, the downstream segment of $U1-D1$ is delivered internally at $c_1 = 0$ and the two upstream firms compete for $D2$ only. As emphasized by Hart and Tirole (1990) and Reiffen (1992), the unique static Nash equilibrium has $U1-D1$ and $U2$ charging a price equal to marginal cost. That is, we have $c_1 = c_2 = 0$, just as in the case without integration. $U2$ earns zero profits and $U1-D1$ earns $\pi_{D1}^*(0, 0)$ in the static Nash equilibrium.⁴

3 Collusive Foreclosure in the Repeated Game

In this section, we will analyze under which conditions foreclosure emerges as an equilibrium outcome in the above general model. Collusion at the upstream level with vertical integration will be analyzed

³The assumptions to ensure a unique static Nash equilibrium are merely made to simplify the analysis. Under weaker assumptions, the stage game has multiple equilibria and one would need to distinguish between stable and unstable equilibria (see OSS). Note also that somewhat weaker conditions might be sufficient to guarantee existence and uniqueness (see Vives, 1999).

⁴Also OSS acknowledge that this is the Nash equilibrium without commitment.

in this section—this is the case where foreclosure may occur. Collusion under vertical separation and collusion at the downstream level will be considered below using a parametrized model.

Consider the vertically integrated industry and recall $c_1 = 0$ because $U1$ and $D1$ are integrated. We suppose that firms try to implement the following *collusive foreclosure* strategy where c_2 denotes the collusive input price charged to $D2$. Collusive foreclosure involves $U2$ charging $D2$ an input price $c_2 \geq 0$ and $U1-D1$ not supplying $D2$. (Alternatively, $U1-D1$ could also post a price slightly larger than c_2 as part of the collusion. This alternative implies minor modifications of the results which we discuss at the end of this section.) At the downstream level, static Nash equilibrium prices, $p_1^*(0, c_2)$ and $p_2^*(c_2, 0)$, and outputs, $Q_1^*(0, c_2)$ and $Q_2^*(c_2, 0)$, will be realized.

This notion of foreclosure is just as in the static game but, as a collusive strategy, it is very different from normal oligopoly collusion. When adhering to collusion, $U1-D1$ stays out of the market whereas it enters (at a price smaller than c_2) when defecting. $U2$ is simply a monopolist when collusion is successful but c_2 will generally not be $U2$'s preferred monopoly price. When defecting $U2$, is still a monopolist and then it will surely charge its monopoly price. Many values of c_2 can potentially be part of an equilibrium in the infinitely repeated game, and only careful application of equilibrium selection criteria can help identifying more plausible values of c_2 .

Before solving the repeated game, it is useful to define three particular levels of c_2 . The first is the one just mentioned where $U2$ charges the price which maximizes its own profits. Denote this price by c_2^{mon} and define formally

$$c_2^{mon} := \arg \max_{c_2} c_2 Q_2^*(c_2, 0). \quad (3)$$

The second benchmark is the one that maximizes $U1-D1$ profits if it serves $D2$ in addition to $D1$ as a monopolist. Define this level formally by

$$c_2^{int} := \arg \max_{c_2} \pi_{D1}^*(0, c_2) + c_2 Q_2^*(c_2, 0). \quad (4)$$

Note that $c_2^{int} > c_2^{mon}$ due to $\partial \pi_{D1}^* / \partial c_2 > 0$. This second benchmark will be important when we analyze defection by $U1-D1$. Finally, it is useful to define the level of c_2 where $U2$'s output (and therefore profit) becomes zero, denoted by \bar{c}_2 . Formally

$$\bar{c}_2 := \{Q_2(\bar{c}_2, 0) = 0\}. \quad (5)$$

Note that $\bar{c}_2 > c_2^{mon}$ but c_2^{int} may be smaller or larger than \bar{c}_2 . All three benchmarks are unique.

We now analyze collusion in the infinitely repeated game. Time is indexed from $t = 0, \dots, \infty$. Firms discount future profits with a factor δ , where $0 \leq \delta < 1$. When analyzing the repeated game, denote by π_1^c and π_2^c the profit $U1-D1$ and $U2$ earn respectively when both firms adhere to collusion. Let π_i^d denote the profit when a firm defects, and π_i^p is the average profit per period when a punishment path is triggered.

We look for collusive equilibria that are subgame perfect. In order to prevent defection in any period t , we need

$$\sum_{t=0}^{\infty} \delta^t \pi_i^c \geq \pi_i^d + \sum_{t=1}^{\infty} \delta^t \pi_i^p \quad (6)$$

or

$$\delta \geq \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^p} := \delta_i, \quad (7)$$

where $i = 1, 2$, and δ_i denotes the minimum discount factor required for firm i to adhere to collusion.

Consider the collusive profits, π_i^c , first. Since $c_1 = 0$, $U1-D1$'s collusive profit is $\pi_1^c = \pi_{D1}^*(0, c_2)$, that is, $U1-D1$ does not make any profit at the upstream level in equilibrium. $U2$ makes a collusive profit of $\pi_2^c = c_2 Q_2^*(c_2, 0)$.

Defection from collusive foreclosure yields the following payoffs. If $U2$ defects, it charges c_2^{mon} no matter which c_2 is part of the collusion and earns $\pi_2^d = c_2^{mon} Q_2^*(c_2^{mon}, 0)$. If $U1-D1$ defects, it undercuts $U2$'s price, c_2 , in order to gain the $D2$ business. If $c_2 \leq c_2^{int}$, $U1-D1$ will simply charge a price infinitesimally smaller than c_2 and we get $\pi_1^d = \pi_{D1}^*(0, c_2) + c_2 Q_2^*(c_2, 0) = \pi_1^c + \pi_2^c$. For $c_2 > c_2^{int}$, $U1-D1$ will defect by charging c_2^{int} (as follows from the definition of c_2^{int}) and we obtain $\pi_1^d = \pi_{D1}^*(0, c_2^{int}) + c_2^{int} Q_2^*(c_2^{int}, 0) > \pi_1^c + \pi_2^c$.

Consider now the punishment following a defection. In the unique static Nash equilibrium, $c_2 = 0$ and upstream profits are zero as there is perfect Bertrand competition. These are also the maximin profits and so more severe punishment strategies do not exist. Hence, we adopt simple trigger strategies with Nash reversion here and have $\pi_1^p = \pi_{D1}^*(0, 0)$ and $\pi_2^p = 0$ for $U1-D1$ and $U2$, respectively. Nash reversions are credible threats, so, as long as (7) holds, we have subgame perfect equilibria.

Finally, collusive equilibria must be individually rational, that is, firms must get at least their maximin profits in a collusive equilibrium of the repeated game. Maximin profits, $\pi_{D1}^*(0, 0)$ for $U1-D1$ and zero for $U2$, result when $c_2 = 0$. $U2$ also gets zero profits with $c_2 = \bar{c}_2$. Hence, any positive $c_2 < \bar{c}_2$ implies $\pi_1^c = \pi_{D1}^*(0, c_2) > \pi_{D1}^*(0, 0)$ (from (2)) and $\pi_2^c = c_2 Q_2(c_2, 0) > 0$ and therefore fulfills the individual rationality requirement.

We are now ready to prove

Proposition 1 *Vertical foreclosure is a subgame perfect Nash equilibrium of the repeated game, provided firms' discount factor δ is sufficiently high. Moreover, any individually rational collusive input price $0 < c_2 < \bar{c}_2$ can be sustained with a high enough discount factor.*

Proof. We prove the more general second part by showing that $\delta_i < 1$, $i=1,2$, for all $0 < c_2 < \bar{c}_2$. Now, $\delta_i < 1$ if $\pi_i^d \geq \pi_i^c > \pi_i^p$. First, recall $\pi_1^c = \pi_{D1}^*(0, c_2) > \pi_{D1}^*(0, 0) = \pi_1^p$ and $\pi_2^c = c_2 Q_2^*(c_2, 0) > 0 = \pi_2^p$. This establishes the strict inequalities. Second, we prove $\pi_i^d \geq \pi_i^c$. If $c_2 \leq c_2^{int}$, $U1-D1$ gets $\pi_1^d = \pi_1^c + \pi_2^c > \pi_1^c$ when defecting. If $c_2 > c_2^{int}$, $U1-D1$'s optimal defection is to charge c_2^{int} and we have $\pi_1^d = \pi_{D1}(0, c_2^{int}) + c_2^{int} Q_2^*(c_2^{int}, 0) > \pi_1^c$. To prove $\pi_2^d \geq \pi_2^c$, note that $\pi_2^d = c_2^{mon} Q^*(c_2^{mon}, 0) \geq \pi_2^c$ by definition of c_2^{mon} . ■

Proposition 1 shows that vertical foreclosure can indeed be sustained as a subgame perfect Nash equilibrium of the infinitely repeated game. The result counters the criticism the OSS result received (not being a Nash equilibrium) whenever there is repeated interaction and the discount factor is sufficiently high.

Note that for all $0 < c_2 < \bar{c}_2$ a strictly positive minimum discount factor is required. Define

$$\underline{\delta} := \min_{c_2} \max\{\delta_1(c_2), \delta_2(c_2)\}. \quad (8)$$

$\underline{\delta} > 0$ follows from $\pi_1^d > \pi_1^c > \pi_1^p > 0$ and therefore $\delta_1 > 0$ for all $0 < c_2 < \bar{c}_2$. Similarly $\delta_2 > 0$ unless $c_2 = c_2^{mon}$. Hence $\max\{\delta_1, \delta_2\} > 0$ for all c_2 .

Proposition 1 contains a *Folk Theorem*-like message on the action domain. All individually rational collusive input prices can be an outcome of an equilibrium in the repeated game. This raises the question if equilibrium selection criteria help reducing the set of equilibria.

Proposition 2 *Only collusive equilibria with $c_2 \geq c_2^{mon}$ are Pareto efficient.*

Proof. By definition of c_2^{mon} , $\partial \pi_2^c / \partial c_2 > 0$ if and only if $c_2 < c_2^{mon}$. From (2), $\partial \pi_1^c / \partial c_2 > 0$ for all c_2 . Hence, an input prices $c_2 < c_2^{mon}$ are not Pareto efficient but those in the interval $[c_2^{mon}, \bar{c}_2)$ are. ■

Pareto efficiency (from the firms' point of view) suggests that the collusive prices we may expect to occur will be at least as high as c_2^{mon} . The intuition is that, when c_2 is increased beyond c_2^{mon} , $U1-D1$ gains unambiguously while firm $U2$ loses for either $c_2 < c_2^{mon}$ and $c_2 > c_2^{mon}$. Hence, the bargaining

situation implicit in the repeated game can plausibly lead to equilibrium outcomes with $c_2 \geq c_2^{mon}$. By contrast, in the one-shot game analyzed in OSS (assuming $U1-D1$ can commit not to deliver $D2$), $U2$ is a monopolist in the $D2$ market and would therefore never charge a price other than c_2^{mon} .

The characterization of Pareto efficient equilibria does not indicate whether equilibria with $c_2 \geq c_2^{mon}$ are likely to meet the incentive constraint (7). To answer this question, the following lemma is helpful. It states how minimum discount factors δ_1 and δ_2 as in (7) respond to changes of c_2 .

Lemma *Let $\delta_1(c_2)$ and $\delta_2(c_2)$ denote the minimum discount factors required by $U1-D1$ and $U2$ respectively as functions of the collusive input price c_2 .*

(i) *If $c_2 \leq c_2^{int}$, $\partial\delta_1(c_2)/\partial c_2 \gtrless 0$ if and only if $(\partial\pi_2^c/\partial c_2)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c_2)(\pi_2^c) \gtrless 0$. If $c_2 > c_2^{int}$, $\partial\delta_1(c_2)/\partial c_2 < 0$.*

(ii) *$\partial\delta_2(c_2)/\partial c_2 \gtrless 0$ if and only if $c_2 \gtrless c_2^{mon}$.*

Proof. Consider part (i). If $c_2 \leq c_2^{int}$, $\pi_1^d = \pi_1^c + \pi_2^c$, we get $\delta_1(c_2) = \pi_2^c/(\pi_2^c + \pi_1^c - \pi_1^p)$ and, hence,

$$\frac{\partial\delta_1}{\partial c_2} = \frac{(\partial\pi_2^c/\partial c_2)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c_2)(\pi_2^c)}{(\pi_1^c + \pi_2^c - \pi_1^p)^2}. \quad (9)$$

This yields the condition in part (i) of the lemma. If $c_2 > c_2^{int}$, $\pi_1^d = \pi_{D1}^*(c_2^{int}, 0) + c_2^{int}Q_2^*(c_2^{int}, 0)$,

$$\delta_1 = \frac{\pi_{D1}^*(c_2^{int}, 0) + c_2^{int}Q_2^* - \pi_1^p(c, 0)}{\pi_{D1}^*(c_2^{int}, 0) + c_2^{int}Q_2^* - \pi_1^p} \quad (10)$$

and so

$$\frac{\partial\delta_1}{\partial c_2} = \frac{-\partial\pi_1^c/\partial c_2}{\pi_{D1}^*(c_2^{int}, 0) + c_2^{int}Q_2^* - \pi_1^p} < 0. \quad (11)$$

Then consider part (ii). Note that $\pi_2^d = \pi_2^c(c_2^{mon}, 0)$ is a constant, hence, we get $\partial\delta_2/\partial c_2 = -(\partial\pi_2^c/\partial c_2)/(\pi_2^d)^2$.

Hence, $\partial\delta_2/\partial c_2 \gtrless 0$ if and only if $c_2 \gtrless c_2^{mon}$. ■

The lemma states that δ_1 decreases in c_2 if either a regularity condition is met or if $c_2 > c_2^{int}$ whereas δ_2 has got a global minimum in $c_2 = c_2^{mon}$. The next proposition puts the lemma and proposition 2 together by characterizing extremal equilibria. Extremal equilibria are the Pareto undominated equilibria in the set of subgame perfect Nash equilibria that are feasible given the incentive constraint (7).

Proposition 3 *Extremal subgame perfect Nash equilibria involve $c_2 \geq c_2^{mon}$, provided $(\partial\pi_2^c/\partial c_2)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c_2)(\pi_2^c) < 0$.*

Proof. Consider a subgame perfect Nash equilibrium with $c'_2 < c_2^{mon}$ and assume the condition in the proposition is met. One gets $\partial\delta_1/\partial c_2 < 0$ and $\partial\delta_2/\partial c_2 < 0$ from the lemma. From proposition 2, $\partial\pi_2^c/\partial c_2 > 0$ and $\partial\pi_1^c/\partial c_2 > 0$. This implies that selecting an equilibrium with $c_2 > c'_2$ improves both firms profits and requires a lower minimum discount factor, as long as $c_2 < c_2^{mon}$. Therefore, no equilibrium with $c'_2 < c_2^{mon}$ can be an extremal equilibrium, provided $(\partial\pi_2^c/\partial c_2)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c_2)(\pi_2^c) < 0$. The subgame perfect Nash equilibria with $c'_2 \geq c_2^{mon}$ are not Pareto ranked. Hence, all subgame perfect Nash equilibria with $c_2 \geq c_2^{mon}$ are extremal equilibria. ■

The proposition implies that firms are likely to collude on a price $c_2 \geq c_2^{mon}$ because any c_2 below c_2^{mon} reduces both firms' profits and makes collusion more difficult to sustain for both firms. The condition $(\partial\pi_2^c/\partial c_2)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c_2)(\pi_2^c) < 0$ does not appear to be particularly restrictive. It holds, for example, in the model with linear demand (see below). Note that $\partial\delta_1/\partial c_2 < 0$ if $c_2 \geq c_2^{mon}$ anyway; only if $c_2 < c_2^{mon}$ is the sign of $\partial\delta_1/\partial c_2$ ambiguous. Further $\partial\pi_1^c/\partial c_2 > 0$, so, the condition will be met when $\partial\pi_2^c/\partial c_2$ or $\pi_1^c - \pi_1^p$ are small.

One of the key results in OSS is to show that $U1-D1$ has an incentive to commit to a certain price c_2 to prevent $U2$ and $D2$ from merging. The logic is that, for some values of c_2 , the joint profits of $U2$ and $D2$ can be increased by vertically integrating and thus supplying $D2$ internally at marginal cost. For a parametrized model, OSS show that this price is lower than c_2^{mon} . When $U2$ and $D2$ can merge, this upper bound on c_2 is also relevant in the repeated game. Then the possibility of a $U2-D2$ merger forces firms to collude on a price $c_2 < c_2^{mon}$, that is, a Pareto inferior outcome. This is in contrast to proposition 2. Collusion is generally less likely as a higher δ is required, and less damaging for welfare as a lower c_2 will occur. So, the threat of a $U2-D2$ counter merger is beneficial from a policy perspective.

Figure 1 summarizes the discussion of the general model. It shows minimum discount factors $\delta_1(c_2)$ and $\delta_2(c_2)$ for values of c_2 between zero and \bar{c}_2 . Figure 1 is drawn with the help of the lemma and the following properties of δ_i . It is straightforward to verify that $\lim_{c_2 \rightarrow 0} \delta_2(c_2) = 1$. Further, $\delta_2(c_2^{mon}) = 0$ and $\lim_{c_2 \rightarrow \bar{c}_2} \delta_2(c_2) = 1$ as follows from the definitions of c_2^{mon} and \bar{c}_2 . Regarding δ_1 , note $0 < \delta_1(c_2) < 1$ strictly for all $c_2 < \bar{c}_2$ due to $\pi_1^d > \pi_1^c > \pi_1^p > 0$.⁵ Further $\lim_{c_2 \rightarrow \bar{c}_2} \delta_1(c_2) = 0$ if, as assumed in the figure, $c_2^{int} > \bar{c}_2$.⁶ Finally, the figure is based on the condition in proposition 2, $(\partial\pi_2^c/\partial c_2)(\pi_1^c - \pi_1^p) -$

⁵We cannot directly determine $\delta_1(0)$ since $\delta_1(c_2) = \pi_2^c/(\pi_2^c + \pi_1^c - \pi_1^p)$ but $\pi_2^c(0) = 0$ and $\pi_1^c(0,0) - \pi_1^p = 0$. However, l'Hôpital's rule yields

$$\lim_{c_2 \rightarrow 0} \delta_1(c_2) = \lim_{c_2 \rightarrow 0} \pi_2^c(c_2)' / (\pi_2^c(c_2)' + \pi_1^c(c_2)') < 1$$

from $\pi_2^c(0)' = Q_2$ and $\pi_1^c(0)' > 0$.

⁶This follows from $\pi_2^c(\bar{c}_2) = 0$ and so $\pi_1^d = \pi_1^c > \pi_1^p$. If $c_2^{int} < \bar{c}_2$, $\delta_1(\bar{c}_2) > 0$ follows from $\pi_1^d = \pi_1^c + c_2^{int} Q_2^*(c_2^{int}, 0)$ so

$(\partial\pi_1^c/\partial c_2)(\pi_2^c) < 0$. The figure illustrates $\partial\delta_i(c_2)/\partial c_2 < 0$, $i=1,2$, when $0 < c_2 < c_2^{mon}$, suggesting that foreclosure as a collusive strategy will fail when $\delta < \underline{\delta}$ and will typically involve $c_2 \geq c_2^{mon}$ otherwise.

[Figure 1 about here.]

Finally, we discuss other forms of collusive foreclosure. First, $U1-D1$ may not completely withdraw from the $D2$ market but instead post a price $c_2 + \varepsilon$, ε being small, when colluding. The only thing that would change in this case is that $U2$ could not defect profitably any more when $c_2 < c_2^{mon}$. To see this, note that $U2$ ideally wants to defect by charging the price c_2^{mon} , but since $U1-D1$ charges $c_2 + \varepsilon < c_2^{mon}$, this is not possible. This implies $\pi_2^d = \pi_2^c$ and so $\delta_2 = 0$ if $c_2 < c_2^{mon}$. Everything else and in particular the results in this section remain unchanged. Second, $U1-D1$ and $U2$ could collude by both charging c_2 to $D2$, that is, they could each supply half of $D2$'s output, $Q_2^*(c_2, 0)/2$. In this case, $\pi_2^c = c_2 Q_2^*(c_2, 0)/2$ and so $\delta_2(c_2)$ increases by $c_2 Q_2^*/(2c_2^{mon} Q_2^*)$ if $c_2 > c_2^{mon}$. Even though δ_1 is lower in this case, $\underline{\delta}$ as in (8) will often increase. Hence, while this is a subgame perfect Nash equilibrium of the repeated game, it will typically be more difficult to sustain.

4 A Parametrized Model

In this section, we will develop a parametrized version of the model which is useful to derive further results. The market model is similar to the one in OSS's appendix and has linear demand. Demand is symmetric and the demand intercept is, without loss of generality, normalized to one

$$Q_i(p_i, p_j) = 1 - kp_i + dp_j, \quad i, j, = 1, 2; i \neq j, \quad (12)$$

where $k > d \geq 0$. Products are entirely heterogenous if $d = 0$ while $d \rightarrow k$ would imply perfectly homogenous goods. D_i 's profit is

$$\pi_{D_i} = (1 - kp_i + dp_j)(p_i - c_i), \quad i, j, = 1, 2; i \neq j. \quad (13)$$

Myopic maximization at the downstream level yields Nash equilibrium prices

$$p_i^*(c_i, c_j) = \frac{2k + d + k^2 c_i + k d c_j}{4k^2 - d^2} \quad (14)$$

that $\delta_1 = c_2^{int} Q_2^*/(c_2^{int} Q_2^* + \pi_1^c - \pi_1^p) > 0$.

and equilibrium outputs

$$Q_i^*(c_i, c_j) = k \frac{2k + d - (2k^2 - d^2)c_i + kdc_j}{4k^2 - d^2}. \quad (15)$$

Downstream profits are $\pi_{D_i}^*(c_i, c_j) = (Q_i^*)^2/k$.

Consider now the infinitely repeated game with vertical integration. As above, $U1-D1$ forecloses $D2$ and $U2$ delivers $D2$ at a collusive price $c_2 \geq 0$. The integrated firm delivers its downstream unit at $c_1 = 0$. From (15), $U1-D1$ and $D2$ will sell the following quantities

$$Q_1^*(0, c_2) = k \frac{2k + d + kdc_2}{4k^2 - d^2}, \quad (16)$$

$$Q_2^*(c_2, 0) = k \frac{2k + d - (2k^2 - d^2)c_2}{4k^2 - d^2}. \quad (17)$$

$U1-D1$'s downstream profit is

$$\pi_{D1}^*(0, c_2) = k \left(\frac{2k + d + kdc_2}{4k^2 - d^2} \right)^2 \quad (18)$$

whereas $D2$'s profit is

$$\pi_{D2}^*(c_2, 0) = k \left(\frac{2k + d - (2k^2 - d^2)c_2}{4k^2 - d^2} \right)^2. \quad (19)$$

$U1-D1$ does not make any profit at the upstream level. $U2$ makes a profit of $c_2 Q_2^*(c_2, 0)$ or

$$\pi_{U2}^*(c_2, 0) = c_2 k \frac{2k + d - (2k^2 - d^2)c_2}{4k^2 - d^2}. \quad (20)$$

For $U1-D1$ punishment profits are

$$\pi_1^p = \pi_{D1}^*(0, 0) = \frac{k}{(2k - d)^2} \quad (21)$$

while $\pi_2^p = 0$.

The benchmark prices can easily be obtained as

$$c_2^{mon} = \frac{2k + d}{2(2k^2 - d^2)}. \quad (22)$$

and

$$c_2^{int} = \frac{(2k + d)(4k^2 + 2kd - d^2)}{2(8k^4 - 7k^2d^2 + d^4)}. \quad (23)$$

The third benchmark is

$$\bar{c}_2 = \frac{2k + d}{2k^2 - d^2}. \quad (24)$$

Note $c_2^{mon} = \bar{c}_2/2$ but $c_2^{int} \geq \bar{c}_2$ is ambiguous.

First, consider $U1$ - $D1$'s incentives to collude. If $c_2 < c_2^{int}$, $U1$ - $D1$'s defection profit is the sum of π_{U2} as in (20) and π_{D1}^* , its own equilibrium profit as in (18). If $c_2 \geq c_2^{int}$, $U1$ - $D1$'s defection profit is the sum of $c_2^{int}Q_2^*(c_2^{int}, 0)$ and $\pi_{D1}^*(0, c_2^{int})$. The trigger strategy implies a punishment profit as in (21). Plugging these values into (7), the minimum discount factor required for $U1$ - $D1$ to adhere to collusion is

$$\delta_1 = \begin{cases} \frac{(4k^2 - d^2)(2k + d - c_2(2k^2 - d^2))}{8k^2(k + d) - d^3 - c_2(8k^4 - 7k^2d^2 + d^4)} & \text{if } c_2 < c_2^{int} \\ \frac{(8k^2(k + d) - d^3)c_2^{int} - (8k^4 - 7k^2d^2 + d^4)(c_2^{int})^2 - 2c_2kd(2k + d) - c_2^2k^2d^2}{(8k^2(k + d) - d^3)c_2^{int} - (8k^4 - 7k^2d^2 + d^4)(c_2^{int})^2} & \text{if } c_2 \geq c_2^{int} \end{cases} \quad (25)$$

where c_2^{int} is defined as in (23). In the general model, when $c_2 \leq c_2^{int}$, $\partial\delta_1/\partial c_2 < 0$ if and only if $(\partial\pi_2^c/\partial c_2)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c)(\pi_2^c) < 0$ from the lemma. For the parametrized model, this condition becomes

$$-\frac{(4k^2 + kd - 2d^2)c_2^2k^3d}{(4k^2 - d^2)^2(2k - d)} < 0. \quad (26)$$

When $c_2 > c_2^{int}$, we know $\partial\delta_1/\partial c_2 < 0$ from the lemma.

Now turn to $U2$'s incentives to collude. $U2$'s collusive profit is $\pi_{U2}^*(c_2, 0)$, and $U2$'s defection profit is $\pi_{U2}^*(c_2^{mon}, 0)$. The punishment profit is $\pi_{U2}^*(0, 0) = 0$. Plugging these expressions into (7), one obtains

$$\delta_2 = \left(\frac{d + 2c_2d^2 + 2k - 4c_2k^2}{2k + d} \right)^2. \quad (27)$$

We know the sign of $\partial\delta_2/\partial c_2$ from the lemma.

5 Does Vertical Integration Facilitate Collusion?

The result that vertical foreclosure can emerge as an equilibrium outcome of the repeated game does not necessarily suggest a policy against vertical integration. To make a point against vertical integration, one needs to show that the industry is more prone to collusion with integration than without, that is, one needs to show that vertical integration facilitates collusion. In an infinitely repeated game, an industrial policy can be said to facilitate collusion if the industry requires a lower minimum discount factor than without the policy. To investigate this, we now compare the required minimum discount factors with and without vertical integration.

Without integration, it is straightforward to solve for the minimum discount factor. There are now two independent U firms competing for both downstream firms, $D1$ and $D2$. Suppose firms collude on

arbitrary collusive prices c_1 and c_2 they charge $D1$ and $D2$ respectively. Let π^c denote the sum of profits a firm makes in the two markets and denote the defection profit by π^d . If a firm defects, it will do so in both markets (Bernheim and Whinston, 1990). Hence, $\pi^d = 2\pi^c$. Finally, already simple Nash reversion yield $\pi^p = 0$. From symmetry, π^c , π^d and π^p are the same for both firms. It follows that the collusive prices c_1 and c_2 can be supported as a subgame perfect Nash equilibrium if and only if $\delta \geq 1/2$. This minimum discount factor does not depend on c_1 and c_2 or any functional forms.⁷ Further, the fact that firms collude in two markets here does not affect to propensity to collude (Bernheim and Whinston, 1990).

Using the parametrized model, we now compare the threshold of $1/2$ to the minimum discount factor required for collusion under vertical integration.

Proposition 4 *In the parametrized model, collusive foreclosure with vertical integration requires a lower discount factor than upstream collusion without integration if and only if $d/k > 0.380$, that is, when products are not too heterogenous.*

Proof. We need to show that $\max\{\delta_1(c_2), \delta_2(c_2)\} < 1/2$ for some c_2 . To do this, we solve for $\delta_i(c_2) \leq 1/2$, $i=1,2$, as in (25) and (27), and then search for c_2 values such that both $\delta_i(c_2)$ are below the threshold.

First, we look for solutions to $\delta_1(c_2) \leq 1/2$. As above, we need to distinguish $c_2 \geq c_2^{int}$. Consider $c_2 < c_2^{int}$ and define $\hat{c}_2 := \{c_2 < c_2^{int} | \delta_1(c_2) = 1/2\}$. We obtain a unique solution,

$$\hat{c}_2 = \frac{8k^3 - 4kd^2 - d^3}{8k^4 - 5k^2d^2 + d^4}. \quad (28)$$

From $\partial\delta_1/\partial c_2 < 0$, it follows that $\delta_1(c_2) < 1/2$ for all $c_2 > \hat{c}_2$, provided $c_2 < c_2^{int}$. For such c_2 to exist necessarily requires $\hat{c}_2 < c_2^{int}$. There are two unknown variables, d and k , in this inequality but we can express the solution in terms of d/k , the ratio of the slope parameters indicating the degree of product differentiation (note that $0 \leq d/k < 1$). It turns out that $\hat{c}_2 < c_2^{int}$ if and only if $d/k > 0.541$. Hence, c_2 satisfying $\hat{c}_2 < c_2 < c_2^{int}$ exist if and only if $d/k > 0.541$, and for these c_2 we have $\delta_1(c_2) < 1/2$.

Then consider $c_2 \geq c_2^{int}$ and define $\tilde{c}_2 := \{c_2 \geq c_2^{int} | \delta_1(c_2) = 1/2\}$. Solving $\delta_1(c_2) = 1/2$ for c_2 yields two solutions. Only the positive root is plausible

$$\tilde{c}_2 = \frac{-2(2k + d) + \sqrt{m}}{2kd}, \quad (29)$$

⁷This is true only if c_1 and c_2 are not higher than the monopoly price. But we can discard prices higher than the monopoly price as they both reduce profits and require a higher discount factor.

where $m = 2(8k^2 + 8dk + 2d^2 + c_2^{int}((8k^2(k+d) - d^3) - c_2^{int}(8k^4 - 7k^2d^2 + d^4)))$. Using $\partial\delta_1/\partial c_2 < 0$, we get $\delta_1(c_2) < 1/2$ for all $c_2 > \tilde{c}_2$, provided $c_2 \geq c_2^{int}$. Note that $c_2 > \tilde{c}_2$ is sufficient to get $\delta_1(c_2) < 1/2$ if $\tilde{c}_2 \geq c_2^{int}$. Now, $\tilde{c}_2 \geq c_2^{int}$ if and only if $d/k \leq 0.541$. Hence, c_2 satisfying $c_2 > \tilde{c}_2 \geq c_2^{int}$ exist if and only if $d/k \leq 0.541$ and for these c_2 we again have $\delta_1(c_2) < 1/2$. (There may also be $c_2 \geq c_2^{int} > \tilde{c}_2$ solving $\delta_1(c_2) < 1/2$.)

Second, we look for solutions to $\delta_2(c_2) = 1/2$. It is straightforward to verify that $\delta_2(c_2) \leq 1/2$ if and only if $c_2^{mon}(1 - \sqrt{1/2}) \leq c_2 \leq c_2^{mon}(1 + \sqrt{1/2})$.

We finally establish values of c_2 such that both $\delta_1(c_2) < 1/2$ and $\delta_2(c_2) < 1/2$. To do this, it is useful to express \hat{c}_2 and \tilde{c}_2 in terms of c_2^{mon} and to make the distinction $d/k \lesseqgtr 0.541$ since solutions to $\delta_1(c_2) < 1/2$ depend on the degree of product differentiation.

First, assume $0.541 < d/k < 1$ and consider \hat{c}_2 . If $d/k = 0.541$, we get $\hat{c}_2 = 1.354c_2^{mon}$. If $d \rightarrow k$, $\hat{c}_2 = c_2^{mon}/2$. Further \hat{c}_2/c_2^{mon} monotonically decreases in d . This implies $1.354c_2^{mon} > \hat{c}_2 \geq c_2^{mon}/2$ and therefore $c_2^{mon}(1 - \sqrt{1/2}) < \hat{c}_2 < c_2^{mon}(1 + \sqrt{1/2})$. Hence, there exist c_2 satisfying $\hat{c}_2 < c_2 < c_2^{int} < c_2^{mon}(1 + \sqrt{1/2})$ such that $\max\{\delta_1(c_2), \delta_2(c_2)\} < 1/2$.

Second, assume $0.541 \geq d/k \geq 0$ and consider \tilde{c}_2 . If $d/k = 0.541$, then $\tilde{c}_2 = 1.354c_2^{mon}$, but $\lim_{d \rightarrow 0} \tilde{c}_2 = +\infty$. Because \tilde{c}_2/c_2^{mon} monotonically decreases in d , we need to find the minimum d/k such that $\tilde{c}_2 < c_2^{mon}(1 + \sqrt{1/2})$. This threshold is $d/k > 0.380$. If this condition is met, $c_2^{mon}(1 - \sqrt{1/2}) < \tilde{c}_2 < c_2^{mon}(1 + \sqrt{1/2})$. Hence, if $0.541 \geq d/k > 0.380$, there exist c_2 satisfying $c_2^{int} < \tilde{c}_2 < c_2 < c_2^{mon}(1 + \sqrt{1/2})$ such that $\max\{\delta_1(c_2), \delta_2(c_2)\} < 1/2$.

Since c_2 satisfying $\hat{c}_2 < c_2 < c_2^{int}$ do not exist if $d/k < 0.541$, the condition stated in the proposition, $d/k > 0.380$, is necessary to find c_2 which solve $\max\{\delta_1(c_2), \delta_2(c_2)\} < 1/2$. ■

The proposition shows that the vertical integration facilitates collusion in a probabilistic sense, provided the condition on product differentiation is met. Assume $d/k > 0.380$. For $\delta < \underline{\delta}$, neither the integrated nor the separated industry are collusive and market outcomes are the same. For discount factors between $\underline{\delta}$ and $1/2$, only the integrated industry is collusive and this is the case an active policy would want to prevent. Finally, for $\delta > 1/2$, both the integrated and nonintegrated structure are collusive.⁸ If $d/k < 0.380$, collusive foreclosure requires a higher discount factor than $1/2$ and the result is reversed. In that case, vertical foreclosure is an obstacle to collusion in a probabilistic sense.

⁸Note that if $\delta > 0.5$, both downstream markets are supplied at $c_2 > 0$ under separation, while this is the case only for the $D2$ market with integration ($D1$ is delivered at marginal cost). This suggests the possibility that vertical integration is preferable from a policy point of view if $\delta > 0.5$, although this depends on the specific collusive prices charged by firms.

The proof of the proposition gives a characterization of the range of c_2 values for which the result holds. The interval includes $c_2 > c_2^{mon}$, that is, some of the Pareto efficient extremal equilibria prices.

6 Extending the Model to Downstream Collusion

So far, downstream firms were assumed to charge static equilibrium Nash prices. Using the parametrized model again, we now analyze the integrated industry when downstream collusion is possible and check whether downstream collusion is a more profitable strategy than upstream collusive foreclosure.

If firms' discount factor is such that both upstream and downstream collusion are feasible, it is the integrated firm that will determine at which level collusion will occur. Since $U1-D1$ operates at both levels, it will price competitively at one level of the industry and collude at the other. The reason is that the integrated firm's profit is ultimately affected only by the prices at the downstream level (as it still delivers $D1$ at marginal cost), and $U1-D1$ does not care whether downstream prices are higher because of collusive foreclosure or because of downstream collusion. In any event, $U1-D1$ will prevent collusion at both levels in order to avoid double marginalization.

Will $U1-D1$ prefer collusive foreclosure or downstream collusion? A general answer to this question is difficult as the outcomes of collusive foreclosure and downstream collusion are usually rather different in nature and often impossible to compare. We can, however, compare discount factors when the outcomes of upstream and downstream collusion coincide.

Assume there is downstream collusion by $U1-D1$ and $D2$ and upstream competition, that is, $c_1 = c_2 = 0$. Foreclosure-type collusion yields a unique outcome, determined by $c_1 = 0$ and $c_2 \geq 0$, and implies downstream prices of $p_1^*(0, c_2)$ and $p_2^*(c_2, 0)$ respectively. Now assume that this very outcome occurs as a result of downstream collusion. Downstream firms implement collusive prices, denoted by p_i^c , $i=1,2$, identical to those which would occur with foreclosure, that is, $p_1^c = p_1^*(0, c_2)$ and $p_2^c = p_2^*(c_2, 0)$, even though $c_1 = c_2 = 0$. In other words, downstream firms charge collusive prices as if input costs were $c_1 = 0$ and $c_2 \geq 0$. We obtain

$$p_1^c = p_1^*(0, c_2) = \frac{2k + d + kdc_2}{4k^2 - d^2}, \quad (30)$$

$$p_2^c = p_2^*(c_2, 0) = \frac{2k + d + 2k^2c_2}{4k^2 - d^2}. \quad (31)$$

Collusive quantities and profits are immediate

$$\pi_{D1}^c = k \frac{(2k + d + kdc_2)^2}{(4k^2 - d^2)^2}, \quad (32)$$

$$\pi_{D2}^c = k (2k + d - 2k^2c + d^2c_2) \frac{2k + d + 2k^2c_2}{(4k^2 - d^2)^2}. \quad (33)$$

Note that $\pi_{D2}^c = p_2^c Q_2^*$ and $\pi_{D1}^c = \pi_{D1}^*$ as in (18). $U1-D1$ benefits from this collusion, exactly as much as with collusive foreclosure. Also $D2$ benefits since $\pi_{D2}^c = p_2^c Q_2^* = p_2^*(c_2, 0) Q_2^* > (p_2^*(c_2, 0) - c_2) Q_2^* = \pi_{D2}^*(c_2, 0)$.

When analyzing defection, we need to solve for best replies. Now, $p_1^c = p_1^*(0, c_2)$ is a already best reply to $p_2^c = p_2^*(c_2, 0)$. This implies $\pi_{D1}^d = \pi_{D1}^c$ and hence $\delta_{D1} = 0$ (where δ_{Di} denotes Di 's minimum discount factor) and we can henceforth focus on $D2$. $D2$'s best reply is

$$p_2^d = \frac{1 + dp_1}{2k} = \frac{4k + 2d + c_2 d^2}{2(4k^2 - d^2)}, \quad (34)$$

which implies a defection profit of

$$\pi_2^d = \frac{k}{4} \left(\frac{4k + 2d + c_2 d^2}{4k^2 - d^2} \right)^2. \quad (35)$$

We know the static Nash equilibrium profit is $\pi_{D2}(0, 0) = \pi_{D2}^p = k / (2k - d)^2$ as above. Plugging π_{D2}^c , π_{D2}^d and π_{D2}^p into (7), we obtain

$$\delta_{D2}(c) = \frac{c_2 (4k^2 - d^2)^2}{(8k + c_2 d^2 + 4d) d^2}. \quad (36)$$

Proposition 5 *Consider the parametrized model and downstream collusive prices of $p_1^c = p_1^*(0, c_2)$ and $p_2^c = p_2^*(c_2, 0)$. For a given discount factor, collusive foreclosure at the upstream level yields higher profits if $\delta \geq \max\{\delta_1(c_2^{mon} d^2/k^2), \delta_2(c_2^{mon} d^2/k^2)\}$ whereas downstream collusion yields higher profits if $\delta < \underline{\delta}$.*

Proof. Regarding the first part, note $\delta_{D2}(c_2) \leq 1$ if and only if $c_2 \leq c_2^{mon} d^2/k^2$. That is, collusive prices $c_2 \geq c_2^{mon} d^2/k^2$ cannot be sustained with downstream collusion. By contrast, we know that these prices were an equilibrium of the repeated game with collusive foreclosure. Since higher c_2 raise $U1-D1$'s profit, given any $\delta \geq \max\{\delta_1(c_2^{mon} d^2/k^2), \delta_2(c_2^{mon} d^2/k^2)\}$, collusive foreclosure at the upstream level yields higher profits.

Regarding the second part, note $\delta_{D2}(0) = \delta_{D1} = 0$ and $\partial \delta_{D2}(c_2) / \partial c_2 > 0$. That is, by choosing a sufficiently low c_2 , $D1$ and $D2$ can lower the minimum discount factor arbitrarily close to zero. Above,

we saw that foreclosure-type collusion requires $\underline{\delta} > 0$. Here, for $\delta < \underline{\delta}$, collusion at the D level is feasible and therefore downstream collusion yields higher profits. ■

The proposition shows that upstream collusive foreclosure can be both advantageous and disadvantageous compared to downstream collusion. The advantage is that prices higher than $c_2^{mon} d^2/k^2$ can only be implemented with upstream foreclosure-type collusion. The disadvantage is that any collusive foreclosure requires $\underline{\delta} > 0$ and so it may not be feasible when firms' discount factor is low. With downstream collusion, the minimum discount factor required for collusion can be arbitrarily close to zero since colluding on a low c_2 yields prices close to the static Nash equilibrium where incentives to deviate are small for D firms. The conclusion is that upstream collusive foreclosure can be more profitable than downstream collusion and the possibility of downstream collusion does not make collusive foreclosure redundant.

The proposition only states sufficient conditions. One can show that there exists some $\delta^* \in (\underline{\delta}, \max\{\delta_1(c_2^{mon} d^2/k^2), \delta_2(c_2^{mon} d^2/k^2)\})$ such that collusive foreclosure at the upstream level yields higher profits if and only if $\delta > \delta^*$. However, the computation of δ^* is very cumbersome and the functional forms are not particularly informative.

There are, of course, plausible forms of collusion at the D level other than $p_1^c(0, 0) = p_1^*(0, c_2)$ and $p_2^c(0, 0) = p_2^*(c_2, 0)$. Note, however, that $U1-D1$ can always switch to upstream (foreclosure-type) collusion as an outside option. That is, collusion at the D level should give $U1-D1$ no less than it would get in the foreclosure outcome and so collusion with $p_1^c(0, 0) = p_1^*(0, c_2)$ and $p_2^c(0, 0) = p_2^*(c_2, 0)$ is focal.

7 Conclusions

This paper shows that vertical foreclosure can be sustained as an equilibrium of an infinitely repeated duopoly game with a raising rival's cost effect. The result counters the criticism that foreclosure is not a Nash equilibrium of the static game analyzed by Ordober, Saloner, and Salop (1990). The results also indicate that collusive foreclosure, if successful, can be damaging from a policy perspective as equilibrium selection criteria suggest that non-integrated firms will be charged at least the monopoly price for the input good. The paper further shows that vertical foreclosure can be preferable to downstream collusion. Finally, comparing the industry with and without vertical integration, the paper shows that the minimum discount factor required for collusion is lower with vertical integration unless products are very differentiated. In other words, vertical integration often facilitates collusion.

The result that vertical integration may facilitate collusion has first been obtained by Nocke and

White (2003). Using a model with two-part tariffs, they analyze collusion in vertically related oligopolies. For a variety of settings, they show that vertical integration facilitates collusion at the upstream level compared to the nonintegrated industry. In their model, a vertically integrated firm still competes for nonintegrated downstream firms, so foreclosure does not occur in their paper.

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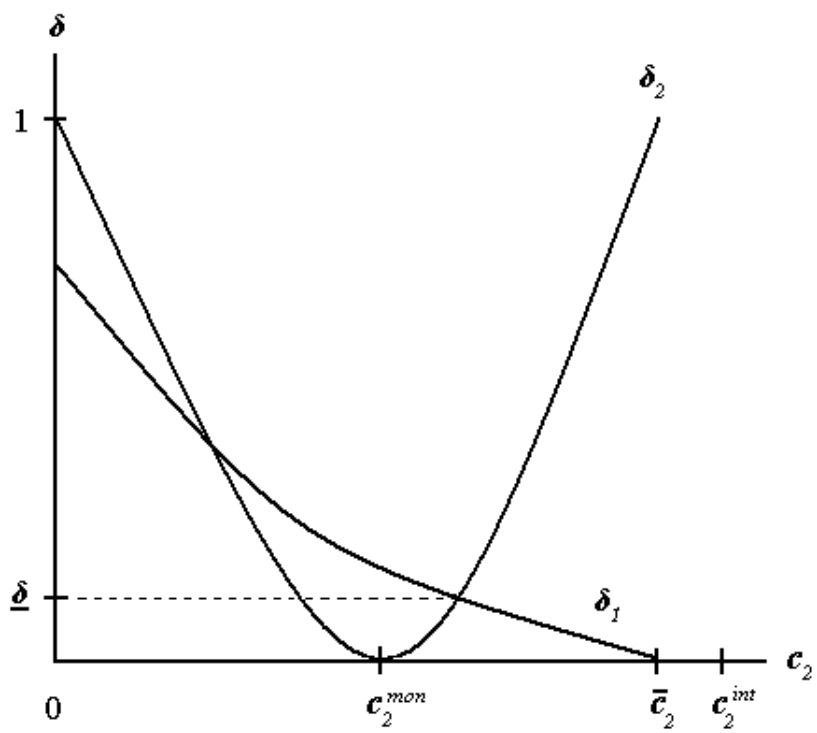


Figure 1. Critical discount factors as functions of c_2 .