**Title:** When Cheaper is Better: Fee Determination in the Market for Equity Mutual Funds

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When Cheaper is Better: Fee Determination in the Market for Equity Mutual Funds

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Abstract

Empirical evidence shows that, in the market for equity mutual funds, quality and fees are negatively correlated. In this paper, we develop an asymmetric information model that explains this apparently anomalous finding. If some investors are relatively insensitive to performance, the model shows that high-quality fund managers may find it optimal to separate themselves by setting low fees. Unable to compete in those terms against high-quality managers, low-quality fund managers set high fees and target only the least performance-sensitive investors.

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1 Introduction

In 2002, U.S. mutual fund assets were worth 6,392 billion dollars, and mutual fund holdings constituted an estimated 17.8 percent of the total financial wealth of U.S. households.\(^1\) The increasing reliance of American investors on mutual funds has raised concerns among industry commentators and regulators alike about the level of fees charged by mutual fund management companies, prompting the General Accounting Office (GAO, 2000) and the Securities and Exchange Commission (SEC, 2000) to conduct reviews of mutual fund fee trends. A pressing question emerges from this debate: How are fees determined in the market for mutual funds? In this paper, we provide a model of the mutual fund market to address this question.

Recent empirical findings underscore the need for a better theoretical understanding of the competitive forces behind mutual fund pricing. Gruber (1996), for instance, documents significant differences in after-fee returns across equity mutual funds. He further shows that funds with lower before-fee returns seem to charge higher fees (see also Carhart, 1997 and Chevalier and Ellison, 1999). Therefore, the evidence suggests not only that fees do not fully adjust so as to equalize differences in before-fee returns, but that, to the contrary, funds of lower quality charge higher fees.

The mutual fund market thus seems to exhibit an inverse price-quality relationship. Explaining how this relationship can emerge from the strategic pricing decisions of mutual fund managers is a challenge for any model of the mutual fund market. In this paper, we show that if some investors are imperfectly informed about their set of investment opportunities, in equilibrium, high-quality funds may charge lower fees. In the context of the mutual fund industry, the existence of these unsophisticated investors has already been postulated by Gruber (1996) as an explanation for why money remains in funds that can be predicted to perform poorly. In the model we propose, we show how the presence of these investors can allow high-quality funds to differentiate themselves by setting lower rather than higher prices. As we discuss in section 4, the complexity associated with the task of selecting an optimal fund

\(^{1}\)Investment Company Institute (2003).
supports the hypothesis that a significant fraction of investors in the mutual fund market will consistently fail to make optimal investment decisions. A report by the SEC shows, among other things, that fewer than one in six fund investors understand that higher fund expenses—which are deducted from the fund’s assets and mostly consist of management fees—can lead to lower returns. This striking finding illustrates the extent to which the fund-picking strategies of a significant fraction of investors may depart from the optimal ones. More recently, a study by Barber et al. (2004) has provided further evidence of investors’ limited understanding of the effects of mutual fund fees.

In our model, funds of different qualities compete for investors’ money, and investors cannot observe fund quality before making their investment decision. Although funds’ past returns could be used as a signal of performance, this signal is, at best, highly noisy. Therefore, the assumption that quality is not observable appears as a reasonable first approximation. The model shows that, if all investors react optimally to differences in expected returns—there are no unsophisticated investors—, there is no price differentiation in equilibrium. Homogeneity in fees, however, is not associated with homogeneity of returns: in equilibrium, good and bad funds coexist, which results in differences in realized after-fee performance across funds.

The presence of unsophisticated investors changes the results greatly. When some investors are relatively insensitive to performance, high-quality funds may find it optimal to differentiate themselves by setting low fees. Low-quality funds are dissuaded by these low fees from competing for the sophisticated segment of the market and focus instead on extracting rents from unsophisticated investors. Therefore, equilibria not only display dispersion in after-fee performance, but also a negative association between fees and before-fee returns, consistent with existing empirical evidence.

Unsophisticated investors are key for this type of equilibrium to exist. If all investors followed an optimal investment strategy, separation of the sort just described could not arise. The existence of these separating equilibria, however, does not follow immediately from the assumption that some investors are unsophisticated. In

\[\text{SEC/OCC (1996).}\]
a standard price competition regime, competition by high-quality funds for the sophisticated segment of the market would drive the profits of these funds to zero. With unsophisticated investors, however, zero profits cannot be part of the equilibrium since it is always possible to extract some rents from them. For a separating equilibrium to exist, something has to stop price-competition from eroding the profits that high-quality funds can obtain from sophisticated investors. In our model, we show that asymmetric information can play that role. It is thus the interaction between asymmetric information and unsophisticated investors what makes possible the existence of equilibria in which low-quality funds charge higher fees.

Although there exists a relatively large theoretical literature, initiated by Bhat-tacharya and Pfleiderer (1985), that aims at characterizing the optimal compensation contract in a delegated portfolio management problem,³ few studies have analyzed fund fees as the outcome of the strategic interaction of competing mutual fund managing companies. Recently, Hortacsu and Syverson (2003) have developed a search model of the market for S&P 500 index funds. In contrast to our paper, however, they analyze a sector in which financial performance differences across funds are relatively small and thus focus on non-portfolio fund differentiation and search frictions as potential sources of fee dispersion. In another recent paper, Berk and Green (2004) have proposed a model of the mutual fund market with no informational asymmetries or search frictions. The goal of their model is to explain why money may rationally follow past good performance even when past performance is not a strong predictor of future performance. The paper is, however, silent about the determinants of the observed distribution of fees. Das and Sundaram (2002) and Metrick and Zeckhauser (1999) have analyzed fee setting in a duopoly context with asymmetric information about fund quality. While Das and Sundaram (2002) compare the performance of two types of incentive schemes for fund managers in such a context, the goal of Metrick and Zeckhauser (1999) is to explain why high- and low-quality producers may charge the same price in certain markets (including the mutual fund market), and is, thus, closely related to ours. We discuss their results in section 3. In another

³See Admati and Pfleiderer (1997) or Palomino and Prat (2003) for more recent contributions.
related paper, Nanda et al. (2000) have developed a model where mutual fund managers of observable quality bear the cost of stochastic investment redemptions, and therefore wish to attract investors who are less likely to experience liquidity needs. In equilibrium, more skilled fund managers impose exit fees and cater investors with lower liquidity needs, while less skilled managers become liquidity providers. Finally, Christoffersen and Musto (2002) have recently explored empirically the effect of investors’ performance sensitivity on fund fees.

Our paper is also related to the more general literature on the role of prices as signals of quality. In this literature, high prices generally signal high quality (Bagwell and Riordan, 1991; Judd and Riordan, 1994, Ellingsen, 1997), although in some contexts involving repeated purchases, it has been shown that low introductory prices can be used as signals of quality (Schmalensee, 1978). This set of work, however, has mostly focused on the case of a single seller of unknown quality. A related strand of research (Chan and Leland, 1982; Cooper and Ross, 1984; Albrecht et al., 2002) has studied price signaling in competitive environments—in which free entry drives profits to zero. Only recently, Fluet and Garella (2002) and Hertzendorf and Overgaard (2001) have studied price and advertising signaling for the case of a duopoly. In their models, high-quality firms charge higher prices in any separating equilibrium.

The rest of the paper is organized as follows: section 2 discusses the available evidence on the relationship between performance and fees; section 3 presents the model, which is extended in section 4 to include unsophisticated investors; and section 5 concludes.

2 Evidence on the Relationship between Performance and Fees

2.1 Mutual Fund Fee Structure

In the market for mutual funds, the prices paid by investors to mutual fund management companies take two forms: periodic fees (operating expenses) and one-time
fees (loads)\textsuperscript{4}. Expenses mostly consist of management fees, but also include 12b-1 (distribution and marketing) fees, custody fees, and administrative fees, as well as operating, legal, and accounting costs. They are computed as a percentage of assets under management—termed the *expense ratio*—and are deducted on a daily basis from the fund’s net assets by the managing company. Fees paid to brokers in the course of the fund’s trading activity are not included in the fund’s expense ratio.

Loads are generally used to pay distributors and they differ from operating expenses in that they are paid by the individual investor as a fraction of the amount invested at the time of purchasing fund shares (*sales charge on purchases*) or redeeming fund shares (*deferred sales charge*). Since fund returns are typically computed from the fund’s net asset value, quoted returns are net-of-expenses, but before loads.

### 2.2 Empirical Evidence

The quality of an actively managed fund is commonly defined as the manager’s ability to deliver returns above those that any investor could obtain following a passive strategy, such as investing in an index fund. Differences in managerial quality could translate into differences in after-expense returns if quality were not fully priced. If higher quality were partly priced, that is, if better funds charged higher expenses, differences in after-expense returns would be smaller than differences in fees. On the other hand, if high-quality funds happened to charge lower fees, differences in after-expense returns would be greater than differences in fees.\textsuperscript{5}

Employing a sample of U.S. non-specialized domestic mutual funds, Gruber (1996) finds evidence of cross-section differences in after-fee performance.\textsuperscript{6} Furthermore, when ranking funds according to after-expense performance, Gruber (1996) finds that performance differences between the best and the worst funds exceed differences

\textsuperscript{4}For a more detailed description of mutual fund fee practices and regulation, we suggest that the reader visit the Online Publications section of the SEC internet site.

\textsuperscript{5}If we let $r_i$ be the before-expense return of fund $i$ and $e_i$ the expenses charged by this fund, then the after-expense return $\tau_i$ is given by $(1 + \tau_i) = (1 - e_i)(1 + r_i) = 1 + r_i - e_i - r_i e_i \approx 1 + r_i - e_i$. Therefore, $\tau_i - \tau_j = r_i - e_i - (r_j - e_j) = (e_j - e_i) + (r_i - r_j)$. It follows that if fund $j$ charges a higher fee ($e_j - e_i > 0$) and $r_i - r_j > 0$, then $r_i - r_j > 0$, that is, fund $j$ must have lower before-expense returns.

\textsuperscript{6}Although a large number of ways to measure performance have been proposed in the literature, Gruber (1996) analyzes three alternative proxies: (i) the fund’s average return relative to the market; (ii) the fund’s average return in excess of the fund’s expected return according to the Capital Asset Pricing Model; and (iii) the fund’s average excess return according to a four-factor model.
in fees. As explained above, this evidence suggests that funds with higher before-expense returns are charging lower expenses: put differently, low-quality funds seem to be more expensive.

Carhart (1997) proposes a different measure of performance. When regressing this performance measure on expense ratios, he estimates that funds with annual expenses of 100 basis points above the average have on average 154 basis points below mean after-expense performance. Again, the effect of fees is to amplify rather than mitigate differences in before-expense performance.

Finally, Chevalier and Ellison (1999), using a measure of performance similar to Carhart’s (1997), report that manager and fund characteristics—such as the portfolio turnover ratio and log of assets—contribute to explaining differences in performance. When controlling for these variables, they provide estimates of the effect on after-expense performance of a 100 basis point reduction in expense ratios that range from 152 to 225 basis points.

Put together, the empirical evidence suggests that there is a significant fraction of differences in mutual fund after-expense performance not explained by observable mutual fund characteristics. Moreover, superior management is not priced through higher expense ratios. In fact, the effect of expenses on the remaining after-expense performance is more than one-to-one, indicating that low-quality funds charge higher fees. Price and quality thus appear to be inversely related in the market for actively managed mutual funds.

3 A Model of Fee Determination in the Market for Mutual Funds

Consider a simple setting in which there is a continuum of investors of mass one who have one dollar to invest, and $N$ mutual fund managers. These managers can be of two types depending on their ability: good ($g$) and bad ($b$). G-managers earn gross expected return $R_g$, and b-managers $R_b$, where $R_g > R_b$, and $R_g > 1$. The ex ante distribution of types is given by the probability $p$ that a manager is good. Once the types are realized, fund managers observe their quality but not the quality of their
rivals and decide what fraction $e$ of the fund’s final asset value to charge to investors. Investors do not observe quality, so they decide where to invest on the basis of the prior distribution $p$ and the fees charged by the different funds.

We assume that all market participants are risk-neutral, and that the only alternative investment is a risk-free asset paying zero interest rate. We also make the assumption that fund performance is independent of the fee charged by the fund, and thus leave aside all moral hazard problems.

The costs of managing the fund are $cw$, where $w$ is the amount of money managed by the fund manager. It is assumed that costs are low enough to make it profitable for $g$-funds to operate if their type is known. The maximum fee a fund of type $k$ can charge if its type is known is such that the net return for investors is equal to one: $R_k(1 - e) = 1$, that is $e = \frac{R_k - 1}{R_k}$. Therefore, this assumption reads:

**Assumption 1** $R_g - c - 1 > 0$.

Note that Assumption 1 can be rewritten as $\frac{R_g - 1}{R_g} > \frac{c}{R_g}$, where the right-hand side of the inequality is the break-even fee for $g$-funds.

We also assume that, given $c$, $b$-funds may find it profitable to operate for some fee less than one hundred percent:

**Assumption 2** $R_b - c > 0$.

We denote by $e_k$ the fee charged by a manager of type $k$ as a proportion of the value of the fund at the end of the period and assume that there are no other fees. Therefore, the amount paid by an investor who invests $w$ dollars in a fund of type $k$ is $we_k R_k$, and payoffs are $w(e_k R_k - c)$ for the manager and $w(1 - e_k)R_k$ for the investor.

The timing of decisions is as follows. First, managers simultaneously set fees. Then investors decide where to invest. We make the assumption that, if several funds have the same net expected returns, investors allocate their wealth among them with equal probability.
3.1 Benchmark Case: Complete Information

Before solving the model, it is instructive to investigate the relationship between fund quality and fees when quality is observed both by competing funds and by investors. It is straightforward to show that, in this case, b-funds will be driven out of the market whenever there are g-funds:

**Proposition 1** With complete information, there do not exist equilibria in which both fund types are active and charge different fees.

**Proof.** First, note that for both types of funds to have a positive market share:

\[(1 - e_b)R_b = (1 - e_g)R_g\]  \hspace{1cm} (1)

Suppose there is only one g-fund. For b-funds to operate \(e_b \geq \frac{c}{R_b} > \frac{c}{R_g}\). Now, for any such \(e_b\), it is profitable for the g-fund to lower \(e_g\) slightly and attract the whole market. Therefore, at the only possible equilibrium, \(e_g = 1 - \frac{R_b-c}{R_g}\), and b-funds do not operate.

If there are several g-funds, the same argument applies for any \(e_g > \frac{c}{R_g}\). The only possible equilibrium fee is \(e_g = \frac{c}{R_g} < \frac{c}{R_b}\), so b-funds remain inactive. ■

Therefore, good and bad funds cannot coexist in equilibrium with complete information: whenever it is profitable for b-funds to operate, it is also profitable for g-funds to lower fees.\(^7\) As a result, b-funds are driven out of the market.

3.2 Asymmetric Information

We first investigate whether there exist equilibria at which g- and b-managers set different fees (separating equilibria) and then turn to equilibria in which both manager types set the same fees (pooling equilibria). We use the Perfect Bayesian Equilibrium (PBE) as our equilibrium concept and focus only on pure-strategy symmetric equilibria (i.e., equilibria in which all funds of the same type have the same equilibrium

\(^7\)In models of vertical differentiation (e.g. Shaked and Sutton, 1982), equilibria in which low- and high-quality producers coexist—with the former charging lower prices—are possible if consumers display differences in their willingness to pay for quality. In the mutual fund industry, however, where the good provided by sellers is end-of-period dollars, one would expect that all consumers have the same willingness to pay for quality: nobody would pay more cents than anybody else for a dollar.
strategies). To limit equilibrium multiplicity, we require investors’ out-of-equilibrium beliefs to satisfy the property that they do not assign positive probability to managers setting fees that are certain to yield them a negative profit. That is, investors cannot assign a positive probability to a fund of type $k$ choosing a fee less than $\frac{c}{R_k}$. Therefore, throughout the paper, by equilibrium we will refer to a pure-strategy Perfect Bayesian Equilibrium satisfying this restriction on investors’ beliefs.

First, note that in a separating equilibrium in which both types are ever active simultaneously, it has to be the case that net returns for investors are equal across types, since otherwise investors would not invest with the two types when both are available. Equality of net returns, in turn, implies that the expected market shares of g- and b-funds also have to be equal, since investors are indifferent between both types. But this implies that, if $e_g > e_b$, it would be optimal for b-managers to imitate g-managers. On the other hand, if $e_g < e_b$, no rational investor that observes both fees would invest with a b-manager. Therefore:

**Proposition 2** If all investors react optimally to differences in expected payoffs, there are no separating equilibria in which both fund-types operate simultaneously.

According to Proposition 2, we should observe no fee dispersion in equilibrium: for any realization of the number of b- and g-funds, all the funds with a positive market share must charge the same fees. Equilibria at which b- and g-funds are active simultaneously and the latter charge higher fees are not possible.

With asymmetric information, however, equilibria depart from the complete information, Bertrand-like outcome. The reason is that any g-fund faces a positive probability of competing only against b-funds. Therefore, there exists a strategy that guarantees positive expected profits to g-funds: setting a fee greater than the break-even fee for g-funds ($\frac{c}{R_g}$) but lower than the break-even fee for b-funds ($\frac{c}{R_b}$). Such a fee guarantees positive profits if the fund is ever able to attract any money and, as long as it is not too high, ensures that the fund would attract investors’ money at least when there are no competing g-funds. The fact that g-funds have a strategy

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8The expectation is taken over the possible realizations of the number of b- and g-funds.
that guarantees positive expected profits immediately implies that, in our model, an equilibrium with zero profits for g-funds is not possible. As the following proposition shows, this implies that there can be no separating equilibria. Proposition 2 ruled out equilibria in which b- and g-funds operate simultaneously while setting different fees. The following proposition strengthens this result. It shows that there are no separating equilibria and, thus, rules out equilibria like the ones obtained with complete information in which g-funds drive b-funds out of the market.

**Proposition 3** If all investors react optimally to differences in expected payoffs, there are no separating equilibria.

**Proof.** First, we prove that g-funds must make positive profits in any equilibrium, for, if they made zero profits, there would always exist profitable deviations: a g-fund setting \( e \in (\frac{c}{R_g}, \frac{c}{R_b}) \) would be identified as being of type g, and Assumption 1 guarantees that, for \( e \) close enough to \( \frac{c}{R_g} \), \( e < \frac{R_g - 1}{R_g} \), so that investors would be willing to invest with the fund setting \( e \), at least when all other funds are of type b. If we let \( w_b \) and \( w_g \) denote the wealth that b- and g-funds expect to obtain from investors if all funds play equilibrium strategies, this argument rules out equilibria with \( w_g = 0 \) or \( e_g \leq \frac{c}{R_g} \).

Next, we show that, in any equilibrium, \( e_g > \frac{c}{R_g} \). Suppose, on the contrary, that \( e_g \leq \frac{c}{R_g} \). Since g-funds must earn positive profits in equilibrium, \( e_g > \frac{c}{R_g} \). Now, consider a fee \( e = e_g - \epsilon \), with \( \epsilon > 0 \), such that \( e > \frac{c}{R_g} \). A fund setting \( e \) would be identified as a g-fund and would capture the whole market with probability one. For \( \epsilon \) small enough, this is a profitable deviation.

Now, suppose that \( e_b = \frac{c}{R_b} \) or \( w_b = 0 \), so b-funds earn zero profits. Since \( e_g > \frac{c}{R_g} \), it would be profitable for them to imitate g-funds. Therefore, in any equilibrium, it must be the case that \( e_b > \frac{c}{R_b} \) and \( w_b > 0 \).

Finally, suppose that a separating equilibrium exists, and consider the following deviation for b-funds: \( e = e_b - \epsilon \) with \( \epsilon > 0 \). The fact that \( w_b > 0 \) implies that, for at least some realizations of the number of b- and g-funds, investors are willing to put their money with funds that charge \( e_b \) and are identified as b-funds. Therefore, they
will prefer to invest with a b-fund charging less than $e_b$, at least for those realizations. This implies that, for any realization of the number of b- and g-funds for which b-funds’ market share is positive if they charge $e_b$, the deviator would obtain the whole market. For $\epsilon$ small enough, this is a profitable deviation.

The logic underlying Proposition 3 is the same that underlies the standard Bertrand outcome: at a separating equilibrium, b-funds cannot earn positive profits, for, otherwise, it would be optimal for a b-fund to slightly undercut $e_b$. A b-fund that deviated in this way would be able to steal the whole market in all instances in which investors would have been willing to invest with funds—identified as b-funds—charging $e_b$. However, an equilibrium in which b-funds earn zero profits is not possible, implying that there are no separating equilibria.

If both fund types set the same fee ($e_p$) in equilibrium, however, investors will believe that any fund setting $e_p$ is of type g with probability $p > 0$. A deviating fund thus runs the risk of being interpreted by investors as being worse than those setting $e_p$ and, therefore, runs the risk of losing all market share even if it sets a fee below $e_p$. The next proposition shows that there can exist equilibria in which both funds set the same fee and obtain positive profits.

**Proposition 4** For some parameter values, there exist (pooling) equilibria in which both types set $e_p \geq \frac{c}{R_b}$.

**Proof.** See appendix.

Figure 1 shows that pooling equilibria can exist for a broad range of reasonable parameter values. In the figure, the region below each curve represents the set of values of $p$ and $c$ for which pooling equilibria can exist for given values of $R_g$ and $R_b$. The figure also shows how increasing $p$ expands the range of values of the other parameters for which pooling equilibria exist. Reducing $c$ or $N$ has a similar effect.⁹

Proposition 4 shows that the presence of asymmetric information can limit competition among funds allowing for equilibria in which both fund types coexist and earn positive profits, an outcome that could not arise under complete information.

⁹In the proof of Proposition 4, the conditions for existence are derived explicitly. From those conditions, it is straightforward to derive this comparative statics result.
The existence of an equilibrium at which funds of different qualities set the same fee has already been proposed by Metrick and Zeckhauser (1999). Their model, however, differs from ours in a number of fundamental dimensions, and, most importantly, the mechanism that allows for a pooling outcome in their paper is different from ours. Metrick and Zeckhauser (1999) study a vertically-differentiated duopoly characterized by sequential price setting (with good funds setting fees–front-end loads–before bad funds) and by investor heterogeneity along two dimensions: on the one hand, different investors value the “good” provided by mutual funds differently; and, on the other hand, some investors can observe quality, while others cannot. In this context, an equilibrium in which both funds set the same price can arise when the qualities are similar enough. The reason is that competition for the investors who can observe quality is strong in this case. As a result, the good fund may find it optimal to set a fee low enough to force bad funds out of the informed segment of the market. It is important to note that, in their model, good funds attract more money than bad funds in a pooling equilibrium, since the latter do not get any money from informed investors. It is also worth noting that, in their model, there are also separating equilibria in which both funds are active and good funds charge higher fees. In our model, these equilibria are not possible.

4 Unsophisticated Investors

To explain why investors hold underperforming funds, Gruber (1996) has proposed that there is a non-negligible proportion of unsophisticated investors who do not react optimally to differences in fund returns. In the U.S., investors can choose from thousands of funds. To pick the optimal fund or set of funds, an investor would, in principle, have to evaluate the expected performance of each fund using all relevant information available. This involves costly search effort, and may even be beyond the capabilities of a significant fraction of investors. Investors could alternatively buy the services of financial advisers, but doing so is also costly, and the quality of the advice received may be hard to evaluate. Therefore, many investors, especially those investing small sums, may not necessarily put their money in the ex ante
optimal funds. These investors may be content to invest in funds (selected because of advertising, advice from acquaintances or brokers, or other reasons) as long as they do not have reasons to believe that they are obvious underperformers.

In this section, we extend the model to incorporate the possibility that a fraction \( \gamma \) of all investors are not able to gather or interpret correctly all available information or to move their money fast enough when differences in expected returns are identified. To reflect the idea that such unsophisticated investors do not perform a full search among all available funds, we assume that each unsophisticated investor is paired with a mutual fund at random. Once paired with a fund, however, unsophisticated investors do not invest blindly: they invest only if the fee charged by the fund is not too high. Instead of proposing a particular model of how boundedly rational investors decide what “too high” means, we denote by \( e_U \) the maximum fee that unsophisticated investors are willing to pay and treat this maximum fee as a parameter. Each fund thus captures \( \frac{\gamma}{N} \) dollars from unsophisticated investors as long as it sets a fee not greater than \( e_U \).

The presence of unsophisticated investors can significantly alter the results in previous sections. Intuitively, it may allow for equilibria in which low- and high-quality funds operate simultaneously and set different fees, if each type of fund serves a different investor segment.

Note that a reasoning similar to Proposition 2 still applies in this case: in equilibrium, b-managers and g-managers cannot both serve the sophisticated market segment and charge different fees. If \( e_g > e_b \), b-managers would mimic g-managers’ pricing strategy. If \( e_g < e_b \), sophisticated investors would not invest in b-funds. Therefore, if the presence of unsophisticated investors allows for the existence of separating equilibria in which both fund types are active simultaneously, sophisticated investors must prefer one type of fund over the other. This implies that expected net returns for b-funds and g-funds are different in equilibrium. One possibility is that b-funds offer a higher net return, which obviously requires that \( e_b < e_g \). However, it is straightforward to show that there cannot exist separating equilibria in which g-funds serve only unsophisticated investors and b-funds serve sophisticated investors.
As argued in the proof of Proposition 3, such equilibria would necessarily imply that b-funds earn zero profits, in which case any b-manager could profitably deviate by setting a higher fee and serving unsophisticated investors only.

We investigate next whether there can exist equilibria in which both fund types are active simultaneously and charge different fees, with b-funds serving only unsophisticated investors as long as there are competing g-funds. The following conditions must hold at this type of equilibrium:

\[
\begin{align*}
    w^U_g (R_g e_g - c) & \geq w^U_b (R_g e_b - c) \quad \text{(Nlg)} \\
    w^U_b (R_b e_b - c) & \geq w^U_g (R_b e_g - c) \quad \text{(Nlb)} \\
    w^U_b (R_b e_b - c) & \geq 0, \quad \text{(Pb)}
\end{align*}
\]

where \( w^U_k \) is the wealth that a fund setting \( e_k \) expects to obtain conditional on all other funds playing the equilibrium strategies. If b-funds serve only unsophisticated investors whenever there are g-funds, \( w^U_g > w^U_b \).

The first two conditions are no-imitation constraints for g- and b-funds, respectively, and the last condition is a participation constraint for b-funds. A participation constraint for g-funds is not necessary, because it is implied by (Nlb) and (Pb). Note that, since \( w^U_g > w^U_b \), condition (Nlb) requires that \( e_g < e_b \): in this type of equilibrium, g-funds must set lower fees.

Fees also have to be low enough to convince both sophisticated and unsophisticated to participate:

\[
\begin{align*}
    e_g & \leq \frac{R_g - 1}{R_g} \quad \text{(2)} \\
    e_b & \leq e_U \quad \text{(3)}
\end{align*}
\]

Finally, it cannot be profitable for b- or g-funds to deviate and set an out-of-equilibrium fee. To evaluate these deviations, we assume that investors’ beliefs are as described in the previous section: any deviation from equilibrium is interpreted as coming from a b-fund unless it yields negative profits for such a fund. The next proposition shows that there are parameter values such that all the above conditions hold simultaneously and there are no profitable out-of-equilibrium deviations:
Proposition 5 For $e_U > \frac{R_b - 1}{R_b}$, there exist separating equilibria with unsophisticated investors at which:

1. $b$-funds serve unsophisticated investors only and charge $e_{b}^{**} = e_U$.

2. $g$-funds charge $e_{g}^{**} > \frac{c}{R_b}$ and serve both sophisticated and unsophisticated investors.

3. $e_{b}^{**} > e_{g}^{**}$.

Proof. See appendix.

Figures 2-4 show that separating equilibria of this sort can exist for reasonable parameter values. The figures graph the minimum and maximum values of $c$ (plotted along the y-axis) for which these equilibria can exist for each possible value of $\gamma$ (plotted along the x-axis) for the case in which $e_U = R - \frac{1}{R_b}$, where $R$ is the unconditional expected gross return. This particular value of $e_U$ would obtain if unsophisticated investors had correct prior beliefs about the distribution of types and did not interpret fees as signals of fund quality.

The model in this section departs from the benchmark complete information model in two dimensions, and it is instructive to see how each of these dimensions contributes to the existence of separating equilibria like the ones described in Proposition 5. First, the existence of unsophisticated investors allows $b$-funds to survive while setting fees that differ from those of $g$-funds. As we saw in the previous section, this would not be possible if all investors held correct beliefs in equilibrium and could move their money freely. In this respect, our model resembles models of price dispersion based on the presence of search costs (like Salop and Stiglitz, 1977), where impediments to search allow firms charging higher prices to obtain positive market shares. Second, the presence of asymmetric information limits the competitive pressure on $g$-funds. If sophisticated investors could observe fund quality, competition among $g$-funds would drive $e_g$ down to $\frac{c}{R_g}$, but such a situation could not be an equilibrium with unsophisticated investors, because $g$-funds can set a higher fee, sell
to unsophisticated investors and make a positive profit. It should thus be emphasized that the existence of unsophisticated investors *alone* cannot generate separating equilibria with both fund types active.

Relaxing the assumption that there are only two fund types would not change the results. If there were several fund types, some of them would sell to unsophisticated investors only and the rest to both sophisticated and unsophisticated ones. Those selling to unsophisticated investors would still charge the maximum possible fee, so that there would be pooling in the low part of the distribution of types. There would also be pooling in the upper part, as, if there was separation, some funds would be charging higher fees than others while still attracting sophisticated investors, which cannot be sustainable in equilibrium.

The proof of Proposition 5 shows that a separating equilibrium exists only if:

$$e_U > \frac{R_b - 1}{R_b},$$

where $\frac{R_b - 1}{R_b}$ is the fee that guarantees the reservation return when investing with a b-fund. Therefore, at this type of equilibrium, some unsophisticated investors (those paired with b-funds) would do better by investing in the reservation asset. A number of realistic—and not exclusive—assumptions about the behavior of unsophisticated investors would yield this result. First, unsophisticated investors may fail to fully understand the equilibrium relationship between fees and gross returns, that is, they may—at least partly—fail to interpret fees as signals of fund quality. Second, they may not account properly for the effect of fees on net returns. In a recent regulatory proposal by the SEC (SEC, 2002) that would require mutual funds to provide a clearer disclosure of the dollar value of the fees paid by investors, one can read: “Despite existing disclosure requirements and educational efforts, the degree to which investors understand mutual fund fees and expenses remains a significant source of concern.” The proposal provides information from a previous report (SEC/OCC, 1996) that found “that fewer than one in five fund investors could give any estimate of expenses for their largest mutual fund and fewer than one in six fund investors un-

10In fact, most studies conclude that the average actively managed mutual fund has historically delivered *below* market returns (see Wermers, 2000, for a recent analysis).
derstood that higher expenses can lead to lower returns.” Similar concerns have been voiced by the General Accounting Office in a report (GAO, 2000) whose principal conclusion was that additional disclosure would help to increase investor awareness and understanding of mutual fund fees. Third, unsophisticated investors could be over-optimistic with respect to fund returns. Finally, unsophisticated investors could be small investors who face returns from the alternative investment lower than those of larger investors, either because of economies of scale in investing or higher interest rates for large investments. According to this last interpretation, unsophisticated investors would be fully rational, yet behave differently because of worse alternative investment opportunities.

5 Discussion

In this paper, we have shown that, in the mutual fund industry, better-quality sellers should not be expected to charge higher prices. Moreover, investors’ limited ability to evaluate fund quality may lead to equilibria in which worse-performing funds charge higher fees. We thus obtain a form of inverse price differentiation consistent with existing evidence on mutual fund performance.

Our analysis suggests several directions for further research. First, in this paper, we have considered a single period, so investors cannot base their decisions on past fund performance. An intertemporal extension of the model would make it possible to investigate the relationship between fees and past performance and their relative role as signals of fund quality. Second, while we have taken fund quality as exogenous, mutual fund management companies may, to some extent, set the quality of the funds they offer through their choice of managers or their expenditure in market analysis. Third, in our model, unsophisticated investors are equally likely to buy from good and bad funds. However, it is more realistic to think that funds may differentiate themselves not only through fees but also through their marketing decisions: in a separating equilibrium, lower-quality funds may not only charge higher fees, but also invest more in their distribution networks or advertising to make sure that they attract a larger proportion of unsophisticated investors.
Although in this paper we have specifically modelled the market for mutual funds, the insights of the model may be generalizable to other markets where quality assessment is costly. The existing empirical evidence on the relationship between quality and price suggests that the correlation between these two variables is typically not strongly positive and that, in a significant number of markets, the correlation is indeed negative (see, for instance, Gerstner, 1985; Tellis and Wernerfelt, 1987; Caves and Greene, 1996). In our model, the negative correlation between price and quality results from the combination of asymmetric information about product quality and the presence of a subset of unsophisticated investors. Future work may further explore, both theoretically and empirically, how these and other factors affect observed price-quality correlations.

Our results indicate that the complexity associated with the evaluation of fund quality may, on the one hand, weaken competition, leading to higher fees, and, on the other hand, lead to a segmented market in which a fraction of investors ends up paying higher fees as well as obtaining lower returns. Whether this state of things could be improved by regulation and the optimal form of this regulation are questions that merit further scrutiny. A possibility, pursued by the SEC, would be to require that funds improve their disclosure of past performance and that they present in their prospectuses gross and net returns separately, thus highlighting the effect of expenses. Results also suggest that some funds may be overcharging investors and, thus, open the question as to whether some form of fee cap could be beneficial in this context.\footnote{In the U.S., NASD, the self-regulatory arm of the securities industry, already imposes limits on the loads that can be charged to mutual fund investors to pay for brokerage services.} Recent judiciary initiatives in this direction, like the settlement reached by a mutual fund company and the New York attorney general in which the former agreed to cut management fees by an average of 20%, highlight the need for more research in this area.\footnote{Brewster (2003).}
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A Appendix.

A.1 Proofs

Proof of Proposition 4.

Let $e_p$ be the pooling fee and $R = pR_g + (1 - p)R_b$ denote the unconditional expectation of gross returns. We will assume that investors’ beliefs are such that, if a fund sets $e \in \left[\frac{c}{R_g}, \frac{c}{R_b}\right)$, it will be believed to be a $g$-fund, while for any $e \geq \frac{c}{R_b}$ other than the equilibrium fee, it will be believed to be a $b$-fund.\(^{13}\)

For investors to be willing to buy from a fund of unknown type:

$$e_p \leq 1 - \frac{1}{R}$$  \hspace{1cm} (PCip)

For $b$-funds to be willing to participate:

$$e_p \geq \frac{c}{R_b}$$  \hspace{1cm} (PCbp)

Let $m_g$ and $m_b$ be the minimum fees that would make it profitable for a $g$- or a $b$-fund, respectively, to deviate.

$$(m_gR_g - c) = \frac{1}{N}(e_pR_g - c)$$

$$(m_bR_b - c) = \frac{1}{N}(e_pR_b - c)$$

It follows that:

$$m_g = \frac{1}{N}e_p + (1 - \frac{1}{N}) \frac{c}{R_g} < \frac{1}{N}e_p + (1 - \frac{1}{N}) \frac{c}{R_b} = m_b,$$

so that if it is not profitable for a $g$-fund to deviate, neither it is for a $b$-fund. Now, let $\underline{e}$ be the maximum fee that would convince investors to shift to a fund believed to be bad, that is:

$$(1 - \underline{e})R_b = (1 - e_p)R,$$

or

$$\underline{e} = 1 - (1 - e_p)\frac{R}{R_b}$$  \hspace{1cm} (4)

\(^{13}\)We select the beliefs most likely to support an equilibrium like the one proposed. If a pooling equilibrium can be supported by these beliefs, it can also be supported by less extreme ones, as long as they assign a sufficiently high probability to a deviator being of low quality.
Therefore, if a fund deviates and sets \( d \equiv \max\{\varepsilon, \frac{c}{R_b}\} \), it will capture the whole market, so, for a g-fund not to be willing to deviate, it has to be the case that:

\[
m_g \geq d,
\]

which also implies that b-funds do not want deviate because \( m_b > m_g \).

**Case 1: \( d = \varepsilon \).** Let us first look at the case in which \( d = \varepsilon \), that is:

\[
1 - (1 - e_p) \frac{R}{R_b} \geq \frac{c}{R_b}, \text{ or }
\]

\[
e_p \geq \hat{e} \equiv \frac{R - (R_b - c)}{R}
\]

(5)

In this case the no-deviation condition for g-funds reads:

\[
m_g = \frac{1}{N} e_p + \left(1 - \frac{1}{N}\right) \frac{c}{R_g} \geq 1 - (1 - e_p) \frac{R}{R_b} = \varepsilon, \text{ or }
\]

\[
e_p \leq \frac{NR_g(R - R_b) + (N - 1)R_b c}{R_g(NR - R_b)}
\]

(6)

For an equilibrium of this sort to exist, thus, conditions (PCip), (PCbp), (5) and (6) have to hold simultaneously.

First note that, given Assumption 2, (5) implies (PCbp). Inspection of the conditions also shows that for (PCip) and (5) to hold simultaneously it is necessary that

\[
R_b - c > 1
\]

(7)

If this condition holds, then it only rests to check that (5) and (6) can hold simultaneously. This requires:

\[
\frac{R - (R_b - c)}{R} < \frac{NR_g(R - R_b) + (N - 1)R_b c}{R_g(NR - R_b)}
\]

(8)

After some algebra, this condition can be shown to be equivalent to:

\[
R_b < p \left( R_b + R_g \left( \frac{R_b - Nc}{(N - 1)c} \right) \right)
\]

(9)

Therefore, if \( R_b > Nc \), a pooling equilibrium will exist for high enough values of \( p \).
Case 2: $d = \frac{c}{R_b}$. In equilibrium, $d = \frac{c}{R_b}$ if and only if:

$$e_p \leq \frac{R - (R_b - c)}{R}$$

(10)

For g-funds not to deviate, we need $m_g \geq d = \frac{c}{R_b}$, i.e.,

$$e_p \geq \frac{c}{R_g} + Nc \frac{R_g - R_b}{R_g R_b},$$

(11)

Thus, for an equilibrium of this sort to exist, conditions (PCip), (PCbp), (10), and (11) must hold. We need to consider two cases:

1. $R_b - c > 1$. In this case, condition (PCip) is implied by (10) and conditions (PCbp) and (10) are always compatible:

$$\frac{R - (R_b - c)}{R} > \frac{c}{R_b} \Leftrightarrow \frac{R}{R_b} - R_b(R_b - c) > cR \Leftrightarrow$$

$$\frac{R}{R_b} - c > 0 \Leftrightarrow R - R_b > 0,$$

which is always true.

It rests to check that conditions (10) and (11) are compatible as well. This will happen if and only if:

$$\frac{c}{R_g} + Nc \frac{R_g - R_b}{R_g R_b} < \frac{R - (R_b - c)}{R}$$

(12)

Rearranging this expression leads to inequality (9), so the same conditions as above guarantee existence of this type of equilibrium.

2. $R_b - c < 1$. Now, condition (10) is implied by (PCip). The latter condition will be consistent with (11) only if:

$$1 - \frac{1}{R} > \frac{c}{R_g} \left(1 + \frac{N R_g}{R_b} - N \right)$$

(13)

For fixed $R_b$ and $R_g$, the supremum of the left-hand side is $1 - \frac{1}{R_g}$ (when $p \to 1$). The infimum of the right-hand side is $\frac{R_b - 1}{R_g} \left(1 + \frac{N R_g}{R_b} - N \right)$ if $R_b > 1$ (when $c \to 1 - R_b$), and 0 if $R_b < 1$ (when $c \to 0$). In the latter case, the above condition will hold. If $R_b > 1$, we must have:

$$\frac{R_g - 1}{R_g} > \frac{R_b - 1}{R_g} \left(1 + \frac{N R_g}{R_b} - N \right)$$

(14)
Rearranging,

\[(R_g - 1)R_b > (R_b - 1)(R_b + NR_g - NR_b) \iff \]
\[R_b < \frac{N}{N - 1}\] (15)

Therefore, if (16) holds and \(\frac{R_b - 1}{R_g} > \frac{c}{R_b}\), then there are pooling equilibria with \(R_b - c < 1\). \(\blacksquare\)

Proof of Proposition 5.

First, notice that \(e_b \leq \frac{R_b - 1}{R_b}\) cannot be an equilibrium fee, as slightly undercutting such \(e_b\) would guarantee the deviating b-fund all the sophisticated market in case there are no g-funds and would only marginally reduce its profits in all other cases. This implies that, in equilibrium \(e_b > \frac{R_b - 1}{R_b}\), so a necessary condition for the existence of a separating equilibrium is \(e_U > \frac{R_b - 1}{R_b}\). In what follows, we assume this to be the case.

Next, notice that, if a separating equilibrium exists, \(e_b^* = e_U\). Any \(e_b \in (\frac{R_b - 1}{R_b}, e_U)\) cannot be an equilibrium, as such a fee will not convince sophisticated investors to invest with a b-fund even if all funds turn out to be of type b, and \(e_U\) yields greater profits from the unsophisticated investors. Since \(e_b \leq \frac{R_b - 1}{R_b}\) cannot be an equilibrium fee either, the only possible equilibrium fee for b-funds is \(e_b^* = e_U > \frac{c}{R_b}\).

Given \(e_b = e_U > \frac{R_b - 1}{R_b}\), \(w_U^b = \frac{\gamma}{N}\), so the participation constraint for b-funds and the no-imitation constraints read:

\[
\frac{\gamma}{N}(R_b e_U - c) \geq 0 \quad \text{(Pb)}
\]

\[
\frac{\gamma}{N}(R_b e_U - c) \geq w_g^U (R_g e_g - c) \quad \text{(NIb)}
\]

\[
w_g^U (R_g e_g - c) \geq \frac{\gamma}{N}(R_g e_U - c), \quad \text{(NIg)}
\]

where \(w_g^U \in (\frac{\gamma}{N}, \frac{\gamma}{N} + (1 - \gamma)]\).

The no-imitation constraints can be rewritten:

\[
e_g \geq \frac{\gamma}{N w_g^U} e_U + (1 - \frac{\gamma}{N w_g^U}) \frac{c}{R_g} = \alpha e_U + (1 - \alpha) \frac{c}{R_g} \quad \text{(NIg’)}
\]

\[
e_g \leq \frac{\gamma}{N w_g^U} e_U + (1 - \frac{\gamma}{N w_g^U}) \frac{c}{R_b} = \alpha e_U + (1 - \alpha) \frac{c}{R_b}, \quad \text{(NIb’)}
\]
where $\alpha \equiv \frac{\gamma}{N w_g}$. Since $w^U_g > \frac{\gamma}{N}$, $\alpha < 1$. Therefore, the incentive constraint (NIb') implies that $e_g < e_U$, which proves part 3.

Let us assume that sophisticated investors’ beliefs are such that if a fund sets $e \in [\frac{c}{R_b}; \frac{c}{R_g})$, it will be believed to be a g-fund, while for any $e \geq \frac{c}{R_b}$ (other than g-funds’ equilibrium fee if $e_g \geq \frac{c}{R_g}$), it will be believed to be a b-fund. This implies that $e_g \in (\frac{c}{R_g}; \frac{c}{R_b}]$ cannot be an equilibrium fee. Since $e_g$ needs to be strictly greater than $\frac{c}{R_g}$, the only possible equilibrium fees satisfy $e_g > \frac{c}{R_b}$.

Let $m_b$ be the minimum fee that would make it profitable for a b-fund to deviate if it captures the whole sophisticated market:

$$
\left(\frac{\gamma}{N} + (1 - \gamma)\right) (R_b m_b - c) = \frac{\gamma}{N} (R_b e_U - c), \text{ i.e.,,}$$

$$m_b = \frac{\gamma}{N - \gamma(N - 1)} e_U + \left(1 - \frac{\gamma}{N - \gamma(N - 1)}\right) \frac{c}{R_b} = \lambda e_U + (1 - \lambda) \frac{c}{R_b}, \quad (17)
$$

where $\lambda \equiv \frac{\gamma}{N - \gamma(N - 1)} < 1$.

Similarly, let $m_g$ be the minimum fee that would make it profitable for a g-fund to deviate if it captures the whole sophisticated market:

$$
\left(\frac{\gamma}{N} + (1 - \gamma)\right) (R_g m_g - c) = w^U_g (R_g e_g - c) \quad (18)
$$

Rearranging:

$$m_g = \frac{N w^U_g}{\gamma + (1 - \gamma) N} e_g + \left(1 - \frac{N w^U_g}{\gamma + (1 - \gamma) N}\right) \frac{c}{R_g} = \phi e_g + (1 - \phi) \frac{c}{R_g}, \quad (19)
$$

where $\phi \equiv \frac{N w^U_g}{\gamma + (1 - \gamma) N} < 1$.

Let $M_b$ ($M_g$) be the be the minimum fee that would make it profitable for a b-fund (g-fund) to deviate and capture the whole sophisticated market only when there are no g-funds:

$$
\frac{\gamma}{N} (R_b e_U - c) = (R_b M_b - c) \left(\frac{\gamma}{N} + (1 - p)^{N-1}(1 - \gamma)\right) \quad (20)
$$

$$w^U_g (R_g e_g - c) = (R_g M_g - c) \left(\frac{\gamma}{N} + (1 - p)^{N-1}(1 - \gamma)\right) \quad (21)
$$
Notice that these inequalities imply $m_g < M_g$ and $m_b < M_b$.

If $e_g > \frac{c}{R_g}$, the maximum fee that a deviating fund can charge while guaranteeing the whole sophisticated-investor market with probability one is $d \equiv \max\{\hat{e}, \frac{c}{R_b}\}$, where

$$
\hat{e} \equiv \frac{e_g R_g - (R_g - R_b)}{R_b}
$$

is defined by $(1 - e_g)R_g = (1 - \hat{e})R_b$.

Similarly, the maximum fee that a deviating fund can charge while guaranteeing the whole sophisticated-investor market in case all other funds are of type $b$ is

$$
D \equiv \max\{\frac{c}{R_b}, \frac{R_b - 1}{R_b}\}
$$

Therefore, the no-deviation conditions for $b$- and $g$-funds are, respectively:

$$
m_b \geq d \quad \text{(NDb)}
$$
$$
m_g \geq d \quad \text{(NDg)}
$$
$$
M_g \geq D \quad \text{(NDg')}\n$$
$$
M_b \geq D \quad \text{(NDg')}\n$$

Finally, $e_g$ has to be such that sophisticated investors are willing to invest with $g$-funds:

$$
e_g \leq \frac{R_g - 1}{R_g},
$$

which will immediately hold as long as $e_U \leq \frac{R_g - 1}{R_g}$, since $e_g < e_U$.

An equilibrium will exist if all the inequality conditions (Pb, NIb’, NIg’, NDb, NDg NDb’, NDg’, and PIg) are satisfied simultaneously.

Given the relatively large number of parameters ($R_g$, $R_b$, $p$, $N$, $c$, $\gamma$) and inequalities, we do not fully characterize the set of equilibria. Instead, we next show existence numerically. Figures 2–4 show parameter regions for which this type of equilibrium exists for the case $e_U = \frac{R}{R-1}$, with $\overline{R} = pR_g + (1 - p)R_b$. 

\[\blacksquare\]
B Figures

Figure 1: Existence of pooling equilibria. $R_b = 1.1; R_g = 1.3$.

Figure 2: Conditions for existence of separating equilibria with unsophisticated investors. $R_b = 1.1; R_g = 1.3; p = 0.5; N=2$. The x-axis displays values of $\gamma$, while $c$ is displayed along the y-axis. The curves represent the minimum and maximum values of $c$ such that a separating equilibrium exists for each $\gamma$. 
Figure 3: Conditions for existence of separating equilibria with unsophisticated investors. $R_b = 1.1; \ R_g = 1.3; \ p = 0.5; \ N=5$. The x-axis displays values of $\gamma$, while $c$ is displayed along the y-axis. The curves represent the minimum and maximum values of $c$ such that a separating equilibrium exists for each $\gamma$.

Figure 4: Conditions for existence of separating equilibria with unsophisticated investors. $R_b = 1.1; \ R_g = 1.3; \ p = 0.5; \ N=10$. The x-axis displays values of $\gamma$, while $c$ is displayed along the y-axis. The curves represent the minimum and maximum values of $c$ such that a separating equilibrium exists for each $\gamma$. 

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