Two-Sided Markets and Price Competition with Multi-homing

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Abstract

We model duopoly competition between two platforms. They operate in a two-sided market where agents are heterogeneous on both sides of the market and are allowed to multihome. Network effects are captured within a vertical differentiation framework. Under single-homing there exists an interior equilibrium where networks exhibit asymmetric sizes and both firms enjoy positive profits. When all agents are allowed to patronize the two platforms, we show that in equilibrium multi-homing takes place on one side of the market only. Moreover, the only equilibrium exhibiting positive profits for both platforms replicates the collusive outcome.

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1 Introduction

This paper proposes a very simple framework to capture network externalities in two-sided markets, and their implications on price competition. In particular, it captures a key-feature of two-sided markets : the prices determine the equilibrium network sizes and, thus, the quality of the services offered by the platforms. Moreover, it allows for a simple treatment of multi-homing behaviour. We model duopoly competition between two platforms which operate in a two-sided market with heterogeneous agents on both sides. Buyers and sellers interact through the platforms, with network effects operating from one market to the other, and *vice-versa*. Buyers are attracted by platforms housing many sellers and, conversely, sellers are drawn to platforms housing many buyers.

A significant number of real-life markets operate under these features. Consider, as specific examples, shopping malls, media markets, credit cards. The larger the number of shoppers attracted in a shopping mall, the higher the willingness of a retailer to locate in that shopping mall. Conversely, the larger the number of shops located in the shopping mall, the higher the willingness of shoppers to pay a visit to it. Similarly, in exhibition centers, the larger the number of visitors, the larger the number of exhibitors who want to participate. In the same manner, the larger the number of exhibitors in an exhibition center, the larger the number of visitors. Another example is provided by the outgrowth of newspapers distributed for free to readers, like *Metro* and *20 Minutes* in France. These newspapers can be viewed as platforms on which readers and advertisers interact. The larger the readership of such a newspaper, the higher the willingness to pay of an advertiser for inserting a commercial in it. Conversely, when readers benefit from a positive externality finding commercials in these newspapers, the larger the number of ads in a newspaper, the higher their willingness to read it. Consider last the platforms consisting of two companies issuing credit cards, and selling them to buyers and sellers as means of payment. Clearly the larger the number of merchants accepting a particular credit card, the larger the willingness of a buyer for holding that credit card. Conversely, the larger the number of potential buyers holding that credit card, the larger the willingness of a merchant to subscribe to its issuer.

All the above examples share two specific features. First, they all display multihoming competition. Indeed, there is a priori no reason why a buyer or a seller should choose to interact through a single platform only, at the exclusion of the other. In the first example above, in which two shopping malls compete for attracting buyers or visitors, there is no reason why one should a priori exclude a particular shopper to visit both shopping malls, and a particular retailer to settle a shop in both malls. Similarly, in the free press example, nothing can prevent readers to read both newspapers, and advertisers to advertise in both of them as well. Finally, in the credit cards example, buyers can choose to hold both credit cards, and sellers to subscribe to both issuers.

A second feature characterising the above examples is that the agents operating in these markets do not value in general the services rendered by each single platform in the same way. In most markets, agents' heterogeneity generally entails valuations of the good exchanged which vary over the agents. This is why, at a given price, not all potential buyers are willing to buy, and not all potential sellers are willing to sell. A similar property of dispersed valuation should also be expected in two-sided markets with heterogeneous agents. In the shopping mall example, valuations of shoppers located at different distances from it, or facing varying tightness in their time constraints, should be different, as the valuations of retailers selling different products in the shopping mall. In the credit cards example, a buyer who is used to make more transactions than another should probably value the holding of a credit card more than the second one. Similarly, the willingness to rent a stand for an exhibitor varies with the cost of loosing a transaction which would be made possible with paying the rental. Yet consider, like in the examples above, the case when the two-sided network effects underlying buyers and sellers interaction entail positive network externalities. Then, beyond the individual valuations'variability which was just evoked, *all buyers are typically more attracted by the platform which houses the larger number of sellers, and all sellers by the platform which houses the larger number of buyers.* In other words, for given networks sizes, the platform housing the larger number of sellers appears, in the eyes of all buyers, as a good of higher quality than the other one. Thus at given networks sizes, the two platforms are vertically differentiated. Similarly, in the other market, the platform housing the larger number of buyers appears, in the eyes of all sellers, of being of higher quality than the other one. In this market, platforms are also vertically differentiated. Of course, their relative qualities vary with the sizes of their networks. Accordingly, any mechanism leading to determine network sizes also endogenously determines the qualities of the platforms in both markets.

Several recent papers deal with the issue of competition in two-sided markets. Rochet and Tirole (2003) in particular offer a precise discussion of the relevant questions pertaining to such markets while Armstrong (2003) provides a detailed analysis of key aspects of platform competition under various setup. A paper very close in spirit to ours is Caillllaud and Jullien (2003). They consider price competition between two platforms which provide intermediation services. A central focus of their paper is indeed the multi-homing issue. To a certain extent the present paper complements theirs. Indeed, they consider a case where agents who make use of the platforms are homogeneous. By contrast, we assume they are heterogeneous. Moreover, they assume that all agents on both sides of the market "participate" (i.e. register to one platform at least) whereas we allow for an endogenous participation in each side of the market (i.e. registrering to no platform is allowed, and is observed in equilibrium). A direct consequence of this last assumption is that the value of participation in the market is endogenous in our model whereas it is exogenous in theirs. On the other hand they allow for more flexible pricing strategies: platforms may jointly charge registration fees (applying ex ante) and transaction fees (applying ex post). This flexibility induces more competition between platforms and a richer set of strategy profiles. In our paper, only registration fees are allowed. All in all, their model best fits the issue of matching two types of agents to form partnerships. Our framework is best viewed as representing a situation where the agents in one side of the market have access to a set of transactions whose size is endogenously determined by the number of affiliated agents in the other side. According to the typology proposed by Evans (2003), our framework better fits into the category of "demand coordinators" or "audience makers", while Caillaud and Julien are more concerned by the category of "market makers".

Although very few papers deal with the multi-homing issue, there are however some noticeable exceptions. We already discussed Caillaud and Jullien (2003). Guthrie and Wright (2003) consider the role of multi-homing in the credit card industry. Yet they focus on interchange fees between credit card companies and bank intermediaries. Very often when multi-homing is considered, it is restricted to one-side of the market (see Armstrong (2004), Hausman, Leonard and Tirole (2003)). In our framework, we allow agents in both sides of the market to multi-home if they wish to do so. In a previous paper we have studied price competition with multi-homing within the framework of a vertically differentiated market *without* network effects (Gabszewicz and Wauthy, 2003). As it will be shown below, the results of this paper are extremely useful for the analysis of price competition with multi-homing in two-sided markets.

Summing up, the key features of the present two-sided market model are that all agents are heterogeneous and allowed to multi-home. Under single-homing there exist *dominant firm equilibria* at which one firm gives the service for free in one side of the market and sets the monopoly price in the other. More interestingly, there also exists an interior equilibrium where networks exhibit asymmetric sizes and both firms enjoy positive profits. When all agents are allowed to patronise the two platforms, we show that there exists a unique multi-homing equilibrium exhibiting positive profits for both platforms. At this unique equilibrium, multi-homing takes place on one side of the market while no-multihoming is observed on the other side, where firms give their product for free to the agents. In section 2, we present the model while section 3 is devoted to equilibrium analysis ; section 4 gathers some final remarks.

2 The Model

There are three types of agents:

- Platforms: they are denoted by i and sell product $i = 1, 2$. Product i is best viewed as a matching device between agents. For the sake of illustration, we shall refer here to the exhibition centers' metaphor. Then one can think of product i as a commercial fair organized at an exhibition center i. Platforms are the organizers of the fairs in exhibition centers. They sell their product in two markets: the visitors' market and the exhibitors' market. The access permit paid by the visitors, as well as the rental fee paid to the platforms by exhibitors, allow visitors and exhibitors to trade if they succeed to match. Platforms maximize sales revenue by setting access prices $p_i \geq 0$ in the visitors' market, and rental fees $\pi_i \geq 0$ in the exhibitors' one.[1]

- Visitors: they are denoted by their type θ. Types are uniformly distributed in the $[0, 1]$ interval. The total number of visitors is normalized to 1. They possibly buy product $i = 1, 2$ according to a utility function $U_i = \theta x_i^e - p_i$, with x_i^e denoting the expectation visitors have about the number of exhibitors. When buying the access permits to *both* exhibition centers, a visitor enjoys a utility $U_i = \theta x_3^e - p_1 - p_2$. Parameter x_3^e depends on the expectation visitors have about the number of exhibitors who exhibit in both centers. Holding no access permit yields a utility level normalized to 0.

- Exhibitors: they are denoted by their type γ. Types are uniformly distributed in the $[0, 1]$ interval. Their total number is normalized to 1. They possibly exhibit in both exhibition centers. When they exhibit in center i, $i = 1, 2$, their utility is measured by $U_i^{'} = \gamma v_i^e - \pi_i$, with v_i^e denoting the expectation they form about the number of visitors in center i. When deciding to exhibit in both centers, an exhibitor enjoys a utility $U_i' = \gamma v_3^e - \pi_1 - \pi_2$. Again, v_3^e depends on the expectation about the total number of visitors in both centers. Refraining from exhibiting in any exhibition center yields a utility level normalized to 0.

Parameters γ and θ are best understood as an indirect measure of the value-added agents in their respective markets derive from realizing a transaction. Exhibitors might be heterogeneous in this respect because of the unit value of the goods they exhibit for sale. Visitors on the

[1]Notice thus that our model corresponds to a "pure membership" type, following the terminology proposed by Rochet and Tirole (2004). Such an assumption is justified in cases where the platform does not benefit from an obvious way to monitor for effective transactions between agents.

other side are likely to be heterogeneous according to the number and the importance of the transactions they wish to perform.

The intuition underlying our model is the following. From the viewpoint of an exhibitor, the willingness to rent a stand in the exhibition fair depends on his own type and on the number of additional sales this exhibitor may expect to realize by accepting to pay the rental fee. This essentially depends on the number of visitors. On the other hand, the willingness to pay for holding some given access permit depends on the visitor's type and on the number of purchases he/she would not miss because there is an exhibitor in the exhibition center he visits. This essentially depends on the number of exhibitors. The market is thus characterized by two-sided network effects. From the viewpoint of one side of the market, say visitors, the two exhibition centers are viewed as selling vertically differentiated products. Products' hierarchy reflects the asymmetry in networks' size. The network effects at work in the present framework can be viewed as defining two parallel vertically differentiated markets where quality in one market is endogeously determined by the size of the network in the other market. In the case where the expectations are that networks of the two platforms are of the same size, we shall assume that if prices are identical, agents are spread evenly accross the two platforms.

3 Equilibrium Analysis

In the above model, products'quality in one market reflects agents expectations relative to the network's size in the other market. We may therefore characterize optimal pricing strategies in each market, conditional on these expectations. To this end, we may rely on the standard analysis of price competition under vertical differentiation (as developed for instance in Wauthy (1996)).We start by considering the monopoly case.

3.1 The Monopoly Case

Define an agent to be *active* if he/she visits, or exhibits in at least one commercial fair. Obviously the number of active agents in each market depends negatively on the product's price and positively on the expected size of the relevant network in the other market. The set of active agents in one market is defined by those types who enjoy a positive surplus when buying the product. For instance in the visitors' market, given some price p and expectation x^e, the set of active visitors is $[\hat{\theta}, 1]$ where $\hat{\theta}$ solves $\theta x^e - p = 0$. Using our specification of agents'preferences, we may derive demand addressed to the platform by the visitors as a function of the expected about the number of exhibitors. Denoting this expected number by x^e, we get

$$D^v(p, x^e) = 1 - \frac{p}{x^e}$$

Regarding demand addressed to the platform by the exhibitors, given an expected number of visitors v^e, we have:

$$D^x(\pi, v^e) = 1 - \frac{\pi}{v^e}.$$

With this specification of demands, we assume that agents hold *passive beliefs*.[2] Individual demands in one market only depend on the expectation about the number of active agents in the other market, and the price quoted in their own market, but not on the price quoted in the other market. A natural way to justify this set-up is to assume that the agents in one market

[2]The importance of this assumption wil be discussed in the last section of the paper.

know the price they are charged but do not know either the price or the preferences of the agents in the other one. Given expectations x^e and v^e, the objective of the monopolist is to maximize the function

$$\max_{p,\pi} pD^v(p,x^e) + \pi D^x(\pi, v^e)$$

Using this objective function, we derive optimal prices, conditional on expectations. The optimal strategies are $p^*(x^e, v^e) = \frac{x^e}{2}$ and $\pi^* = \frac{v^e}{2}$. The corresponding networks'sizes at these prices are $h = m = \frac{1}{2}$ on each market. Therefore, when we require expectations to be fullfilled at equilibrium, we obtain:

Proposition 1 *An optimal strategy for the monopolist is* $p^* = \pi^* = \frac{1}{4}$.

In this equilibrium, the monopolist's payoff is equal to $\frac{1}{4}$. Regarding the participation of visitors and exhibitors to the market, we note that half of the visitors and half of the exhibitors (those with high θ and γ) are active. There also exist two other, symmetric, optimal pricing strategies for the monopolist.

Proposition 2 *An optimal strategy for the monopolist is to give the product free of charge in one market and prices at* $\frac{1}{2}$ *on the other market.*

Under these pricing schedules, all agents in one market, say visitors', hold an access permit they obtain for free while in the other market, the exhibitors' one, half of the market is covered. The monopoly payoff is obviously equal to $\frac{1}{4}$. Propositions 1 and 2 make transparent the mechanism at work in this model. Pricing "low" in one side of the market increases the number of active agents in this market. This makes the other side of the market more attractive, which allows to charge higher prices there. A firm may therefore expect to recoup margins lost in one market by the extra margin it allows for in the other market.

3.2 Duopoly Competition with Single-Homing

We assume now that there exist two platforms which compete in prices. However, in this subsection, we suppose that active agents are not allowed to patronize two platforms, i.e. they have to "single-home".

Let us derive demands addressed to the platforms by the exhibitors. These demands depend on exhibitors' expectations (v_1^e, v_2^e). Assume $v_2^e > v_1^e$; then we get

$$D_1^x(\pi_1, \pi_2) = \frac{\pi_2 v_1^e - \pi_1 v_2^e}{v_1^e(v_2^e - v_1^e)}$$

$$D_2^x(\pi_1, \pi_2) = 1 - \frac{\pi_2 - \pi_1}{v_2^e - v_1^e}.$$

These are the demand functions of a vertical differentiation model with quality products defined exogenously by $v_2^e > v_1^e$. A similar demand specification $D_i^v(p_1, p_2)$ can be defined in the visitors' market given expectations $x_2^e > x_1^e$.

Conditional on expectations v_1^e, v_2^e, $v_2^e > v_1^e$, and $x_1^e, x_{2,}^e x_2^e > x_1^e$, the payoff functions are then derived as

$$p_i D_i^v(p_1, p_2) + \pi_i D_i^x(\pi_1, \pi_2).$$

Formally, we define a Nash equilibrium in the two-sided market duopoly as follows:[3]

[3]This definition essentially extends the definition of Katz and Shapiro (1984) to a context of multsided market.

Definition 1 *A Nash Equilibrium is defined by two quadruples* (p_i^*, π_i^*) *and* (v_i^*, x_i^*) *with* $i = 1, 2$, *such that*

1. *given expectations* $(v_1^*, v_2^*, x_1^*, x_2^*)$, $\left(p_{i,}^* \pi_i^*\right)$*is a best reply against* $\left(p_{j,}^* \pi_j^*\right)$, $i \neq j$, *and vice-versa ;*

2. $D_i^v(p_1^*, p_2^*) = x_i^*$*;* $D_i^x(\pi_1^*, \pi_2^*) = v_i^*$*,* $, i = 1, 2$ *.*

This definition allows firms to deviate simultaneously in the two components of the strategies at their disposal. Obviously, it also implies that, at a Nash equilibrium of the two-sided market, each pair of prices $(p_1^*, p_2^*), (\pi_1^*, \pi_2^*)$ also defines a price equilibrium in its respective market. Notice that, due to the assumption of passive beliefs, when the two pairs $(p_1^*, p_2^*)$and (π_1^*, π_2^*) define each a price equilibrium in the visitors' and exhibitors' market respectively, the pair of strategies $(p_1^*, \pi_1^*), (p_2^*, \pi_2^*)$ must satisfy the first condition in our definition.

We now derive the price equilibrium on the exhibitors' market, conditional on expectations $v_1^e < v_2^e$:

$$\pi_2(v_1^e, v_2^e) = \frac{2v_2^e(v_2^e - v_1^e)}{4v_2^e - v_1^e}$$

$$\pi_1(v_1^e, v_2^e) = \frac{v_1^e(v_2^e - v_1^e)}{4v_2^e - v_1^e},$$

with corresponding demands:

$$D_2^x(v_1^e, v_2^e) = \frac{2v_2^e}{4v_2^e - v_1^e}$$

$$D_1^x(v_1^e, v_2^e) = \frac{v_2^e}{4v_2^e - v_1^e}.$$

Obviously, the symmetry of our model allows us to directly infer the price equilibrium, conditional on expectations, $x_2^e > x_1^e$, on the visitors' market. We obtain

$$D_2^v(x_1^e, x_2^e) = \frac{2x_2^e}{4x_2^e - x_1^e}$$

$$D_1^v(x_1^e, x_2^e) = \frac{x_2^e}{4x_2^e - x_1^e}.$$

Then it remains to solve the model for fullfilled expectations, i.e. condition 2 in the above definition of a Nash equilibrium. This is done by solving the system

$$x_2 = \frac{2D_2^v(x_1, x_2)}{4D_2^v(x_1, x_2) - D_1^v(x_1, x_2)}$$

$$x_1 = \frac{D_2^v(x_1, x_2)}{4D_2^v(x_1, x_2) - D_1^v(x_1, x_2)}.$$

Straighforward computations yield $x_1^* = v_1^* = \frac{2}{7}$ and $x_2^* = v_2^* = \frac{4}{7}$, and corresponding prices $\pi_1^* = p_1^* = \frac{2}{49}$, $\pi_2^* = p_2^* = \frac{8}{49}$.

The presence of heterogeneity on both markets allows for an interior equilibrium where both platforms enjoy strictly positive networks and profits. Notice however that in addition to this equilibrium we also identify the "dominant firm" equilibria where one exhibition center monopolizes the markets by allowing free access on one side and charging the corresponding monopoly price on the other side. Obviously, this equilibrium replicates the monopoly equilibrium of proposition 2. Last, we cannot rule out the pure Bertrand equilibrium where both platforms give their products for free in the two markets. In this equilibrium all visitors and exhibitors are active and the market is shared evenly. Obviously platforms make no profit at this equilibrium.

Proposition 3 *With single-homing only, the set of Nash Equilibria obtains as*

- *the quadruples* ($x_1^* = v_1^* = \frac{2}{7}$, $x_2^* = v_2^* = \frac{4}{7}$) *and* ($\pi_1^* = p_1^* = \frac{2}{49}$, $\pi_2^* = p_2^* = \frac{8}{49}$) *which define the unique (up to permutation) interior equilibrium in which both platforms enjoy positive profits ;*
- *The "dominant firm" equilibria which replicate the outcomes described in proposition (2) ;*
- *and the Bertrand equilibrium* ($v_i^* = x_i^* = \frac{1}{2}$), ($p_i^* = \pi_i^* = 0$), *with* $i = 1, 2$.

3.3 Duopoly Competition with Multi-Homing

Suppose now that exhibitors may opt for exhibiting in both centers, and/or visitors may decide to visit both centers. To what extent does this possibility alter our previous analysis? Intuitively, the willingness to pay for a second purchaser on one side of the market depends on the multi-homing behaviour of the other side. Suppose for instance that most exhibitors attend the two fairs. Then, a visitor's willingness to visit exhibition 1 *in addition to* fair 2 must be almost equal to nil. Indeed, the number of additional transactions that a visitor may realize because he holds two visiting permits is almost zero. On the other hand, if no exhibitor rents a stand simultaneously in the two fairs, then the added-value of visiting a second one is the largest. To put it differently, the added-value of multi-homing in one market depends negatively on the extent of multi-homing expected to take place in the other market. Let us consider the viewpoint of visitors. Given expectations $x_1 < x_2$, we may define the specific value of multi-homing, x_3 say, by the difference between the number of exhibitors renting a stand in one fair at least minus the number of those who exhibit at fair 2. Suppose that no exhibitor rents simultaneously in the two fairs ; then visiting the two allows a visitor to a number of possible transactions equal to $x_2 + x_1$. By contrast, if all exhibitors who rent in fair 1 also rent in fair 2, the added-value of a joint visit as compared to visiting 2 only is nil. In other words, we have $x_3 = x_2$. Obviously, in this last case, we do not expect visitors to multi-home. Thus, the willingness to pay for an additional visit in the visitor's market is negatively related to the expectations about multi-homing behaviour in the exhibitors' market. This basic property of multi-homing decisions in two-sided markets will prove useful in the analysis to follow. Before we proceed to the equilibrium analysis, we propose some definitions aimed at characterizing the various multi-homing structures that can take place within our framework.

Definition 2 *Whenever there exist multihoming agents in each market, we will refer to* parallel multi-homing*;*

Definition 3 *Whenever all active agents in a particular market multihome, we shall speak of* generalized multi-homing*;*

Definition 4 *Whenever all active agents in each market multi-home, we will refer to* global multi-homing*;*

Definition 5 *A Multi-Homing Nash Equilibrium is a Nash equilibrium displaying at least one multi-homing active agent.*

In Gabszewicz and Wauthy (2003) we develop the analysis of duopoly price competition in a traditional vertically differentiated market when the joint purchase of the two variants is allowed. The analysis we develop therein is formally equivalent to the characterization of candidate-price equilibria when agents are allowed to multi-home. In the above paper we do not consider network effects. However, the equilibrium characterization we derive can be applied for describing optimal prices in the present framework, *conditional on expectations of the agents.* Consider for instance the market for visitors. Given their expectations $x_1 < x_2$ we may define the admissible values for x_3: $x_3 \in [x_2, x_2 + x_1]$. Recall that $x_3 = x_2$ can be interpreted as corresponding to the expectation that all exhibitors who rent a stand in fair 1 also do it in fair 2. Whenever $x_3 = x_1 + x_2$, it is expected that no exhibitor rents a stand in both fairs. It is obviously in this last case that the joint purchase option is mostly valued. In Gabszewicz and Wauthy (2003), we characterize the nature of price equilibrium for all admissible values of x_3. We summarize these results in the next two lemmata.

Lemma 1 *Assume $x_2 > x_1$. The open interval $]x_2, x_2 + x_1[$ can be divided into three non-degenerate and connected sub-intervals such that*

1. *In the first sub-interval, there exists a unique price equilibrium with single-homing;*
2. *in the second, two price equilibria may co-exist: one with single-homing and one with multi-homing ; and, finally,*
3. *in the third one, there exists no equilibrium (in pure strategies).*

Proof: See Propositions 1-3 in Gabszewicz and Wauthy (2003, p.823-827)

Lemma 2 *Assume $x_2 > x_1$. When $x_2 = x_3$ the vertical differentiation price equilibrium with single homing prevails. When $x_3 = x_2 + x_1$, there is generalized multihoming in the visitors' market in equilibrium. Platform 1 and 2 set their monopoly prices: $p_i(x_1, x_2) = \frac{x_i}{2}$.*

Proof: The first part of the Lemma is trivially satisfied. In order to prove the second part of the lemma, it is sufficient to notice that, in the case $x_3 = x_2 + x_1$, the utility derived from multi-homing by any visitor with type θ is $\theta(x_1 + x_2) - p_1 - p_2$, which is fully separable in platforms' decisions. Therefore, each platform acts as a monopolist. *QED.*

We are now in a position to state our main results.

Proposition 4 *A configuration displaying generalized multi-homing in one market, and no multi-homing in the other market, is part of a Multi-homing Nash equilibrium. Firms set their monopoly price in the multi-homing market and give their product for free in the other one. Furthermore, this equilibrium is the unique one exhibiting strictly positive profits for both firms.*

Proof: first we prove the first part of the proposition. The characterization of the equilibrium follows immediately from the condition of fullfilled expectations. Suppose that any active visitor actually visits the two fairs. This is optimal for them only to the extent they expect almost no exhibitor to hold a stand in both fairs. Conversely, it is rational for all exhibitors holding a stand in one fair to hold one in the other as well if only they expect almost no visitor to multi-home.

Thus a necessary condition for generalized multi-homing in one market is lack of multi-homing in the other. In other words, global multi-homing cannot be part of an equilibrium because it is not compatible with the fullfilled expectations condition. From Lemma 2 we know that in the market where generalized multi-homing prevails, the platforms set their monopoly prices. Accordingly, the resulting sizes of the networks are identical (and equal to $\frac{1}{2}$) so that, in the other market, products are viewed as homogeneous. Therefore, by a Bertrand-like argument, they are given for free at equilibrium. The proof of the second part ("uniqueness") is developed in 4 steps.

(i) Consider a vector of expectations (v_1, v_2, x_1, x_2) such that $v_3 \in]\max\{v_1, v_2\}, v_1 + v_2[$ and $x_3 \in]\max\{x_1, x_2\}, x_1 + x_2[$. These expectations give rise to two vertically differentiated markets. Applying Lemma (1), three possible situations may arise in each of them: single-homing price equilibrium (which we denote by s), multi-homing price equilibrium (which we denote by m), or no price equilibrium at all. Formally, in order to characterize the equilibria in our two-sided game, we have to consider all possible combinations of these three price equilibrium configurations across the two-sided markets.

(ii) First,we may rule out any configuration where equilibrium s prevails in one of the two markets. Indeed, if equilibrium s prevails, say in the market for visitors, the only expectations compatible with this equilibrium in the market for exhibitors is $v_3 = v_1 + v_2$ which does not belong to the admissible domain $]\max\{v_1, v_2\}, v_1 + v_2[$.

(iii): We also rule out any configuration of expectations which would lead to values x_3 and/or v_3 for which there would exist no price equilibrium. Indeed, our definition of a multi-homing equilibrium requires the existence of a price equilibrium in *both* markets.

(iv): The only case which remains to be considered is a vector of expectations in which in the two markets a multi-homing price equilibrium exists. Using Gabszewicz and Wauthy (2003), we may characterize candidate equilibrium configurations[4]. Assuming $x_2 > x_1$ and defining

$$K = \frac{(x_3 - x_2)(x_2 - x_1) + x_1(x_3 - x_1)}{x_1(x_2 - x_1)(x_3 - x_2)},$$

we obtain:

$$p_1(x_1, x_2) = \frac{3(x_2 - x_1)}{4(x_2 - x_1)K - 1}$$

$$p_2(x_1, x_2) = \frac{(2K(x_2 - x_1) + 1)(x_2 - x_1)}{4(x_2 - x_1)K - 1}$$

and

$$v_1(x_1, x_2) = 1 + \frac{p_2(x_1, x_2)}{x_2 - x_1} - p_1(x_1, x_2)K$$

$$v_2(x_1, x_2) = 1 - \frac{p_2(x_1, x_2) - p_1(x_1, x_2)}{x_2 - x_1}.$$

Replicating the same analysis for the exhibitors'side of the market under the assumption $v_2 > v_1$, we complete the characterization of the candidate equilibrium configuration. It then remains to solve for the fullfilled expectations conditions. Defining $x_3 = x_2 + z$ with $z \in]0, x_1[$, we obtain, as the unique valid solution,

$$\hat{v_1} = \frac{1}{12}(5 - 9z + \sqrt{25 + 30z + 81z^2})$$

[4]We consider the market for visitors, but obviously the symmetric characterization prevails for the exhibitors' market

$$\hat{v_2} = -\frac{1}{6} - \frac{3z}{2} + \frac{1}{6}\sqrt{25 + 30z + 81z^2}.$$

Direct computations indicate however that $\hat{v_1} > \hat{v_2}$, which contradicts our initial assumption. Therefore, there exists no price quadruple which satisfies the fullfilled expectations conditions. We have thus ruled out all possible equilibrium configurations in the relevant interval. *QED*

Notice that other equilibria exist under multi-homing. In particular the dominant firm equilibria still exist. We should however stress the comparison between Proposition (3) and (4). Indeed, it neatly clarifies the main interest of multi-homing from the point of view of platforms, namely relaxing price competition drastically. What is actually surprising in the present case is the fact that the collusive outcome obtains as the *unique* equilibrium of the game where the two firms enjoy positive profits. A comparable equilibrium has been identified in Caillaud and Jullien (2003). Yet, their model assumes that agents on each side are homogeneous and active from the outset. Armstrong (2004) identifies an equilibrium with similar features, with the multi-homing side being "exploited" and the other being targeted "agressively". A key-difference with us is that he *assumes* the homing structure (single-homing on one side, multi-homing in the other) while we obtain it as an equilibrium outcome.

4 Final Remarks

In this paper we have characterized the equilibria in a two-sided market when buyers are attracted by platforms housing many sellers, and sellers by platforms housing many buyers, like in the market for credit cards, shopping malls, newspapers, a.s.o. This analysis has been performed both for the cases of single-homing and multi-homing. To do so, we have proposed a very simple framework which rests crucially on the vertical differentiation structure underlying both sides of the market when agents'expectations are fixed. This structure allows the use of existing results obtained for vertically differentiated markets when no network externalities are present. In the case of single-homing, our analysis has revealed the existence of an asymmetric interior equilibrium which co-habits with the dominant-firm equilibria. In the case when multi-homing is allowed, we have proved that there exists only one multi-homing equilibrium at which both firms make positive profits ; this unique equilibrium mimics the collusive outcome.

Of course, our analysis does not cover all possible cases of competition encountered in multisided-platforms industries. In particular, several issues have not been examined in the above approach, such as the use of more flexible pricing strategies (registration fees combined with transaction fees, for instance). Our assumption of uniform distributions is critical as well. While allowing us to obtain closed form solutions, it also limits the scope of our results. It is clear however that qualitatively, our results would hold for more general distributions, provided they remain reasonable regular. Last, a limitation of our analysis has to do with our formulation of agents' beliefs. We have indeed assumed that agents on one side of the market do not update their anticipations as a function of the information available in the other market. Obviously, this implies that part of the positive externality that characterizes the market as a whole is not internalized in the firms' profits. In our model however, this might be justified by the fact that both sides of the market consists of two different categories of agents which may simply not have at their disposal the information about the other sides.[5] Neverthelees we have considered

[5] Moreover, should the firms try to overcome this by committing to respective network sizes, then they would face a credibility problem because when maximizing their profits on the corresponding demands, they would induce a market outcome where expectations would not be fullfilled.

some cases of "active beliefs". Under a monopoly platform, we obtain a unique optimal outcome where the monopolist covers 2/3 of each of the markets. In a duopoly case without multi-homing and assuming a Cournot-like behaviour, we obtain a unique interior equilibrium. Unfortunately, the multi-homing duopoly case raises untractable issues. All these questions are available for future research. But we feel that the present analysis contributes to our understanding of the basic ingredients underlying competition in two-sided markets.

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