The optimality of ignoring lobbyists

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Abstract

We study a situation in which interest groups compete in an all-pay auction for a political prize. The government, that allocates the prize, may deduce information about which interest group attaches the highest value to it. However, competition for the prize is also socially wasteful. Using mechanisms design techniques, we show that the government optimally trade-offs the costs and the benefits of lobbying by completely ignoring all lobbying activities, and by always assigning the prize to the interest group with the highest ex ante value for it.

Keywords: All-pay auction; Lobbying; Social welfare.

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1 Introduction

Lobbying has become an established practice in modern democracies. Its role in society is an intriguing phenomenon, and it has received a lot of attention from economists. Tullock (1980) views lobbying as an all-pay auction, in which interest groups submit ‘bids’ in order to win a political prize. The prize could for instance be a license to operate in a certain market, a building contract, or the right to organize an important event. Direct bribes, writing research reports, or hiring lobbyists are instances of bids.

The literature that follows Tullock focuses mainly on the social costs of lobbying, which are associated with the fact that the money spent on lobbying cannot be used for other economic activities. Therefore, this branch of the literature devotes much attention to the calculation of total lobbying expenditures by the interest groups (see e.g. Baye et al., 1993, 1996, Amann and Leiniger 1995, 1996, and Krishna and Morgan, 1997). In the case of complete information, with common values, and with a perfectly discriminating contest success function, there is full dissipation of rents. In other words, the expected costs of lobbying are equal to the expected benefits. If the government could give away the object to any interest groups, it would increase the expected interest groups’ utility.

However, only in rare cases are interest groups completely informed about each others’ value for the political prize, and seldom is the value for the prize the same for all. Another stream of work focuses on the social benefits of lobbying which arise when interest groups have the opportunity to separate themselves choosing bids that are contingent on policy
relevant private information. This stream of work views lobbying as a signaling game, in which interest groups submit informative signals to the government (see e.g. Potters and Van Winden, 1992, Lohmann, 1993, and Lagerlöf 1997).

In this study, we combine the above two views by making a trade-off between social costs and social benefits of lobbying. We do so, assuming that interest groups interact according to an all-pay auction and that the government can ignore some of the interest group’s lobbying efforts. In deciding to which extend the government should listen to interest groups, it needs to make a trade-off between the informational benefits lobbying provides, and the social costs. The trade-off turns out to be non-trivial, as both total lobbying expenditures and informational benefits are higher with a higher cap on the interest groups’ ‘bids’ (Matejka et al. 2002 and Gavious et al. 2002).

We will show that the government maximizes expected social welfare when it assigns the political prize to the interest groups with the highest expected value for it. In other words, optimally, the government does not allow the firms to reveal any of the information about their true valuation for the prize. Instead of calculating the interest groups’ expected utility using equilibrium bidding, we will use an indirect approach to prove this result, based on the revelation principle (Myerson, 1981). Our finding implies that the government can optimally base its decision on the allocation of the political prize solely on very limited information, namely the identity of the interest groups’ which has the highest expected value for the prize.

Several papers in the economic literature are related to ours. Some papers focus on
equilibrium behavior in the all-pay auction. For instance, Baye et al. (1996) construct equilibria for the all-pay auction with complete information. Amann and Leininger (1996), and Krishna and Morgan (1997) derive equilibrium bidding in the all-pay auction with incomplete information in models with asymmetric value distributions and affiliated values respectively.

Other papers in the economic literature concentrate on mechanism design. Usually, the mechanism designer is assumed to maximize total bids or total efforts by the players, his instruments being reserve prices, exclusion of players, caps, the mechanism format, and so forth. All standard auctions with the right reserve price maximize the expected revenue for the seller of an indivisible object in the case of symmetric risk neutral bidders (Myerson, 1981 and Riley and Samuelson, 1981). Baye et al. (1996) show that expected total bids may increase when a subset of players is excluded from participation in the all-pay auction in a complete information model. Che and Gale (1998) show that a cap may lead to higher total bids in the all-pay auction. However, Matejka et al. (2002), and Gavious et al. (2002) show that ex ante (i.e., before the social planner knows the values), a tighter cap leads to lower bids. Moreover, the all-pay auction maximizes the expected revenue for the seller of an indivisible object if bidders are risk averse (Matthews, 1983), or budget constrained (Laffont and Robert, 1996). And finally, Boylan (2000) shows that a government official may maximize the sum of the bribes by lobbies by paying money to some lobbies.

Sometimes, like in our model, the mechanism designer has other aims than effort
maximization. For instance, Van Damme (1992) considers fair and efficient mechanisms in a model with incomplete information. He shows that some classical division methods, which turn out to be fair in complete information settings, are not fair anymore in the case of incomplete information, and he constructs mechanisms that do guarantee fair and efficient outcomes. In addition, Maskin (2000) studies mechanisms that are constrained efficient in the case of budget constrained bidders.

The set-up of this paper is as follows. In section 2, we discuss our main result in a simple example. In section 3, we present our general model and Section 4 contains our main result. Finally, section 5 contains some concluding remarks.

2 A simple example

We start with a simple example, which we will generalize in the following sections. Two risk neutral interest groups, labeled 1 and 2, compete for a political prize. Both interest groups submit bids in the all-pay auction. The highest bidder wins the prize and both interest groups pay their bid. If both bidders submit the same bid, each interest groups wins with probability \( \frac{1}{2} \). Assume that interest group \( i \)'s value \( v_i \) for the prize is independently drawn from the uniform distribution on \([0, 1] \). If interest group \( i \) bids \( b_i \) and interest group \( j \) wins the prize, \( i \)'s utility is given by

\[
  u_i(j, b_i) = \begin{cases} 
  v_i - b_i & \text{if } j = i \\
  -b_i & \text{if } j \neq i
  \end{cases}.
\]

The government wishes to maximize the sum of the interest groups' utilities. For simplicity, we assume that the government has a single instrument available to do so, namely it
can introduce a cap $c \geq 0$ on the interest groups’ bids.

In equilibrium, each interest group $i$ submits a bid according to the following bid function (see Matejka et al., 2002 and Gavious et al., 2002 for a derivation):

$$B(v_i) = \begin{cases} \frac{1}{2}v_i^2 & \text{if } v_i < 2c \\ \frac{c}{2} & \text{if } v_i \geq 2c \end{cases}.$$  

Observe that the informational benefits from lobbying are higher with a higher cap: the higher $c$, the smaller the probability that both bidder submit the same bid, so that in turn, the probability that the interest group with the highest value wins the prize. Expected total lobbying expenditures $T(c)$ can be expressed as

$$T(c) = 2 \left\{ \int_0^{2c} \frac{1}{2}v_i^2 dv_i + \int_{2c}^1 cdv_i \right\}$$

$$= \frac{8}{3}c^3 - 4c^2 + 2c.$$  

It is readily verified that $T'(c) = 2(2c - 1)^2 > 0$, so that the expected total lobbying expenditures are increasing in $c$. (Matejka et al., 2002 and Gavious et al., 2002 generalize this finding allowing for any smooth value distribution function.) In other words, the trade-off between the benefits and the costs of lobbying are not trivial.

Now, what is the optimal $c$? Expected utility for interest group $i$ can be expressed as

$$U_i(v_i) = P\{\text{interest group } i \text{ wins}\} v_i - B(v_i)$$

$$= \begin{cases} \frac{1}{2}v_i^2 & \text{if } v_i < 2c \\ \left( \frac{1}{2} + c \right) v_i - c & \text{if } v_i \geq 2c \end{cases}.$$
Therefore, expected total welfare as a function of $c$ is equal to

$$W(c) = E\{v_1\} + E\{v_2\}$$

$$= 2 \left\{ \int_0^{2c} \frac{1}{2} v_i^2 dv_i + \int_{2c}^{1} \left[ v_i \left( \frac{1}{2} + c \right) - c \right] dv_i \right\}$$

$$= -\frac{4}{3} c^3 + 2c^2 - c + \frac{1}{2}.$$

It is then straightforwardly derived that

$$\frac{\partial W(c)}{\partial c} = -(2c - 1)^2 < 0$$

so that welfare is optimized at $c = 0$. In other words, the government optimally introduces a zero cap on lobbying, so that neither interest group exerts any lobbying efforts and the government randomly assigns the prize to one of the interest groups. Analogously, the government optimally commits itself to always allocating the prize to interest group 1.

In the next section, we generalize this result, allowing for (1) any number of interest groups, (2) general, asymmetric value distribution functions (3) the all-pay auction not necessarily being perfectly discriminatory, and (4) the government having more policy instruments available than a cap on the bids.

3 The model

Consider the following lobby game. There is a government which owns a political prize, and $n$ risk neutral interest groups, numbered $1, \ldots, n$. Let

$$N \equiv \{1, \ldots, n\}$$
denote the set of all interest groups. We will use $i$ and $j$ to represent typical interest groups in $N$. Interest groups participate in the all-pay auction, in which they submit bids in order to obtain the prize. We will let $b_i \in B_i$ denote the bid submitted by interest group $i$, where $B_i$ is the set of possible bids for interest group $i$. Let $b \in B \equiv B_1 \times \ldots \times B_n$ be the vector of submitted bids. Typical sets $B_i$ are $B_i = \emptyset$ when interest group $i$ is excluded from the lobby game, $B_i = [0, \infty)$ when no further restriction is placed on interest group $i$’s possible actions, $B_i = [0, c_i]$ when interest group $i$ is not allowed to submit a bid above a cap $c_i$, and $B_i = [r_i, \infty)$ when interest group $i$ should meet a certain minimum bid $r_i$.

The outcome of the lobby game is as follows. Each interest group has to pay its bid. Interest group $i$ wins with probability $q_i(b)$, with $q_i : B \rightarrow [0, 1]$ for all $i$, and $\sum_i q_i(b) \leq 1$ for all $b \in B$. In the contest literature, the functions $q_i$ are referred to as the contest success functions. The appendix contains an overview of commonly studied contest success functions.

Interest groups are expected utility maximizers and have a utility function that is additively separable in money and the prize. We assume that interest groups play according to a Bayesian Nash equilibrium. More specifically, interest group $i$’s utility is given by

$$u_i(b) = v_i q_i(b) - b_i$$

where $v_i$ is interest group $i$’s value of the object. For each $i$, $v_i$ is drawn, independently from all the other private signals, from a distribution function $F_i$. $F_i$ has support on the interval $[\underline{v}_i, \bar{v}_i]$, and continuous density $f_i$ with $f_i(v_i) > 0$, for every $v_i \in [\underline{v}_i, \bar{v}_i]$. Define
the sets

\[ V \equiv [v_1, \bar{v}_1] \times \ldots \times [v_n, \bar{v}_n], \]

and

\[ V_{-i} \equiv \times_{j \neq i} [v_j, \bar{v}_j], \]

with typical elements \( v \equiv (v_1, \ldots, v_n) \), and \( v_{-i} \equiv (v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n) \) respectively. Let

\[ f(v) \equiv \prod_{j \in N} f_j(v_j) \]

be the joint density of \( v \), and

\[ f_{-i}(v_{-i}) \equiv \prod_{j \neq i} f_j(v_j) \]

the joint density of \( v_{-i} \). Let \( EV^{\max} \) be the highest expected value, i.e.,

\[ EV^{\max} \equiv \max_{i \in N} E\{v_i\}. \]

We make the assumption that for each interest group \( i \), \( \frac{1-E_i}{f_i} \) is a strictly decreasing function. Several common distributions, including the uniform distribution, satisfy this condition. Finally, we assume that interest group 1 has the highest ex ante value for the prize, i.e., for all \( i = 2, \ldots, n \),

\[ EV^{\max} = E\{v_1\} \geq E\{v_i\}. \]

4 The optimal mechanism

The government’s problem is to select a mechanism to maximize social welfare among the interest groups. A mechanism is a game specifying a set of allowed bids for each interest
group, the allocation rule, and the payment rule. A feasible mechanism \( \tilde{\mu} \) is a mechanism \( \mu \) including strategies that form a Bayesian Nash equilibrium of \( \mu \). Let \( b^*_1, \ldots, b^*_n \) be the Bayesian Nash equilibrium of \( \mu \), so that

\[
 b^*_i(v_i) \in \arg \max_{b_i \in \mathcal{B}_i} \int_{V_{-i}} u_i(b^*_i(v_1), \ldots, b^*_{i-1}(v_{i-1}), b_i, b^*_i(v_{i+1}), \ldots, b^*_n(v_n)) f_{i-1}(v_{-i}) dv_{-i}
\]

for all \( v_i \) and \( i \).

Let \( SWF(\tilde{\mu}) \) denote social welfare for feasible mechanism \( \tilde{\mu} \). We assume social welfare to be equal to the sum of interest groups’ utility in equilibrium, so that

\[
 SWF(\tilde{\mu}) = \sum_{i \in N} \int_{V} u_i(b^*_i(v_1), \ldots, b^*_n(v_n)) f(v) dv. \tag{1}
\]

A feasible mechanism \( \tilde{\nu} \) is said to be a *socially optimal mechanism* if it maximizes social welfare over all feasible mechanisms. Formally, if \( M \) is the set of all feasible mechanisms, then \( \tilde{\nu} \in M \) is socially optimal if and only if

\[
 SWF(\tilde{\nu}) \geq SWF(\tilde{\mu})
\]

for all \( \tilde{\mu} \in M \).

By the Revelation Principle (Myerson, 1981), we may assume, without loss of generality, that the government only considers feasible direct revelation mechanisms. A feasible direct revelation mechanism is a feasible mechanism, in which each interest group is asked to announce its value, in which it has an incentive to participate (individual rationality) and in which it has an incentive to announce its value honestly (incentive compatibility).
Let \((p, x)\) be a feasible direct revelation mechanism, with

\[
p : V \to [0, 1]^n
\]

having

\[
\sum_j p_j(v) \leq 1,
\]

and

\[
x : V \to \mathbb{R}^n.
\]

We interpret \(p_i(v)\) as the probability that interest group \(i\) wins, and \(x_i(v)\) as the expected payments by \(i\), when \(v\) is announced.

Let

\[
Q_i(v_i) \equiv E_{v_{-i}}\{p_i(v)\}
\]

be the conditional probability that interest group \(i\) wins given its value \(v_i\), and

\[
U_i(p, x, v_i) \equiv v_i Q_i(v_i) - E_{v_{-i}}\{x_i(v)\}
\]

be interest group \(i\)'s interim utility from the feasible direct revelation mechanism. Myerson (1981) shows that individual rationality and incentive compatibility are equivalent to

if \(w_i \leq v_i\) then

\[
Q_i(w_i) \leq Q_i(v_i), \forall v_i, i,
\]

(2)

\[
U_i(p, x, v_i) = U_i(p, x, v_i) + \int_{y_i \leq v_i} Q_i(y_i) dy_i, \forall v_i, i, \text{ and}
\]

(3)

\[
U_i(p, x, v_i) \geq 0, \forall i.
\]

(4)
Consistently with (1), social welfare from \((p, x)\) is given by

\[
SW(p, x) = \sum_{i \in N} \int_{\underline{v}_i}^{\bar{v}_i} U_i(p, x, v_i) f_i(v_i) dv_i.
\]

The following result is the key to our main finding.

**Lemma 1** For each feasible direct revelation mechanism \((p, x)\), \(SW(p, x) \leq EV^{\max}\).

**Proof.** Let \((p, x)\) be a feasible direct revelation mechanism. Then,

\[
SW(p, x) = \sum_{i \in N} \int_{\underline{v}_i}^{\bar{v}_i} \left( U_i(p, x, v_i) + \int_{\underline{v}_i}^{v_i} Q_i(y_i) dy_i \right) f(v_i) dv_i
\]

\[
= \sum_{i \in N} U_i(p, x, v_i) + \int_{\underline{v}_i}^{\bar{v}_i} (1 - F(v_i)) Q_i(v_i) f(v_i) dv_i
\]

\[
\leq \sum_{i \in N} U_i(p, x, v_i) + \int_{\underline{v}_i}^{\bar{v}_i} (1 - F(v_i)) dv_i \cdot \int_{\underline{v}_i}^{\bar{v}_i} Q_i(v_i) f(v_i) dv_i
\]

\[
= \sum_{i \in N} U_i(p, x, v_i) + (E\{v_i\} - v_i) \int_{\underline{v}_i}^{\bar{v}_i} Q_i(v_i) f(v_i) dv_i
\]

\[
\leq \sum_{i \in N} U_i(p, x, v_i) + (EV^{\max} - v_i) \int_{\underline{v}_i}^{\bar{v}_i} Q_i(v_i) f(v_i) dv_i
\]

\[
\leq EV^{\max} \sum_{i \in N} \int_{\underline{v}_i}^{\bar{v}_i} Q_i(v_i) f(v_i) dv_i
\]

\[
= EV^{\max}.
\]

The first equality in the above chain follows with (3), and we get the second equality using integration by parts. The first inequality follows from a theorem from statistics which tells that the expectation of a product is less or equal than the product of the
expectations in case the first term of the product is strictly decreasing in the variable over
which the expectation is taken, and the second term is increasing in this variable. In this
case, \( \frac{1 - F_i(v_i)}{f_i(v_i)} \) is strictly decreasing in \( v_i \) (by assumption), and \( Q_i \) is increasing in \( v_i \) (by
(2)). The other manipulations are straightforward. ■

Now, consider the feasible direct revelation mechanism \((\tilde{p}, \tilde{x})\) with

\[
\tilde{p}_1(v) = 1,
\]

\[
\tilde{p}_i(v) = 0 \text{ for } i = 2, ..., n, \text{ and}
\]

\[
\tilde{x}_i(v) = 0, \forall i \in N.
\]

Observe that \((\tilde{p}, \tilde{x})\) always assigns the prize to the interest group with the highest expected
value, i.e., interest group 1. Moreover, none of the interest groups pays any money. The
expected social welfare generated by \((\tilde{p}, \tilde{x})\) is equal to

\[
SW(\tilde{p}, \tilde{x}) = E\{v_1\} = EV^{\max}.
\]

Then it immediately follows from Lemma 1 that \((\tilde{p}, \tilde{x})\) is socially optimal, as

\[
SW(\tilde{p}, \tilde{x}) \geq SW(p, x)
\]

for all feasible direct revelation mechanisms \((p, x)\).

**Proposition 2** \((\tilde{p}, \tilde{x})\) is socially optimal.
An intuition behind Proposition 2 is the following. Observe in the second equality in the chain (5), that interest group \( i \), if winning the object, adds \( \frac{1-F_i(v_i)}{f_i(v_i)} \) to social welfare. As, by assumption, \( \frac{1-F_i}{f_i} \) is a strictly decreasing function, the government prefers a low type of interest group \( i \) to win more often than a high type. However, (2) requires the probability for interest group \( i \) to win the object to be (weakly) increasing in \( v_i \). Hence, the best the government can do is make the probability that a low type wins equal to the probability that a high type wins. The government can do this by assigning the prize to the interest group with the highest expected value for it, which is interest group 1.

5 Concluding remarks

In this paper, we have constructed an optimal lobbying mechanism in an environment in which lobbying can be best described as an all-pay auction in which interest groups are incompletely informed about each other’s value for the political prize. We have shown that when a government is interested in social welfare among the interest groups (as it is benevolent, or seeks to be re-elected), it optimally ignores all lobbying activities, and has the interest group win the political prize which it expects to have the highest value for it. The costs associated with the lobbying efforts do not outweigh the efficiency gains that result as the government obtains information about which interest group has the highest valuation for the prize. In other words, the best thing the government can do in this environment is to base its decision on its own, limited, information. We conclude by discussing the robustness of this result, and some implications to other economic
5.1 Distribution functions

The analysis was simplified by the assumption that the upside-down hazard rates \( \frac{1-F_i}{f_i} \) are strictly decreasing for each interest group \( i \). Relaxing this assumption may completely change our main finding. When all interest groups draw their value from the same distribution function \( F \), for which the upside-down hazard rate is increasing over its support, allowing for unrestricted lobbying maximizes social welfare. This result follows straightforwardly from McAfee and McMillan’s (1992) finding that colluding bidders bid noncooperatively in the first-price sealed-bid auction if the upside-down hazard rate is increasing.

5.2 Interdependent valuations

Another assumption in our model is that each interest group’s value for the political prize only depends on its own signal, and not on the signals of the other interest groups. Allowing for interdependent valuations actually strengthens our result. Imagine, for instance, that each interest group assigns the same value to the prize, about which all interest groups receive a signal. Then lobbying is not informative at all about which interest group values the price most. In that case, the government optimally assigns the prize to a random interest group, without allowing for any lobbying activities.

Moreover, assume that an interest group’s valuation \( v_i \) is given by
\[ v_i(t_i, t_{-i}) = t_i + \alpha \sum_{j \neq i} t_j \]

with \( \alpha \in (0, 1] \), where \( t_i \) is interest group \( i \)'s private signal, which are independently drawn from potentially different distribution functions. In the limiting case \( \alpha = 1 \), all interest groups attach the same value to the prize. Suppose \( E\{t_1\} > E\{t_i\} \) for all \( i = 2, \ldots, n \). Let \((p, x)\) be an incentive compatible and individually rational mechanism, and define

\[ X_i(t_i) \equiv E_{t_{-i}} \{ x_i(t_i, t_{-i}) \}, \quad \text{and} \]
\[ Q_i(t_i) \equiv E_{t_{-i}} \{ p_i(t_i, t_{-i}) \}. \]

It is readily verified that the lobbying expenditures for interest group \( i \) can be expressed as

\[ X_i(t_i) = X_i(t_i) + \int_{t_i}^{t_1} yQ_i'(y)dy + \alpha E_{t_{-i}} \left\{ \left[ p_i(t_i, t_{-i}) - p_i(t_i, t_{-i}) \right] \sum_{j \neq i} t_j \right\}. \]

(6)

Imagine that the government always assigns the prize to interest group 1 without allowing for any lobbying activities. The additional surplus generated by any other mechanism is at most

\[ (1 - \alpha)E\{t^{\max} - t_1\} \]

where \( t^{\max} \) is the highest signal among \( t_1, \ldots, t_n \). When \( \alpha \) approaches 1, these gains from lobbying tend to 0, while the costs given by (6) are strictly positive and increasing in \( \alpha \), unless \( X_i(t_i) = 0 \), \( Q_i'(t_i) = 0 \) for all \( t_i \), and \( p_i(t_i, t_{-i}) = p_i(t_i, t_{-i}) \) for all \( t_i \) and \( t_{-i} \).

The latter condition only holds true for mechanisms in which the government does not let the allocation of the prize depend on the lobbying efforts by the interest groups.
Always allocating the object to interest group is then optimal, as this interest group has the highest ex ante value for the prize. Therefore, for low enough $\alpha$, the government optimally ignores all lobbying activities, regardless of the distribution functions of the signals $t_i$.

5.3 Divergence of private and social valuations

In this paper, we have assumed that private and social valuations of the political prize are the same. However, this need not be the case. Think for instance about two diffuse groups, such as consumers and firms, lobbying for respectively against more stringent environmental legislation. The members of each interest group may free ride on the lobbying activities of other members, so that in equilibrium, lower lobbying activity may arise compared to the case that both interest groups were well organized. In that case, informative signals may be less costly to society, so that ignoring all lobbying activities may not be socially optimal.

5.4 Auctions

Our result hinges strongly on the idea that all lobbying activities are socially wasteful. This need not be the case. Sometimes the government can sell the political prize in the second-price sealed-bid auction. In this auction, interest groups simultaneously submit sealed bids. The highest bidder wins the prize and pays the government an amount equal to the second highest bid. It is well known that the second-price sealed-bid auction results in an efficient allocation. In our setting, the prize always ends up with the interest group
that attaches the highest value to it. Assuming that any monetary transfer from the winner to the government is welfare neutral, the second-price sealed-bid implements the optimal mechanism. Still, Baye et al. (1993) argue that the justice system in the Western World precludes governments to sell political favors by efficient mechanisms like auctions, so that governments are forced to make use of the wasteful institution of lobbying to acquire information from interest groups.

5.5 Other applications

Our result may have broader interpretations than the one discussed above. One interpretation of our result is that lobbies optimally agree among themselves not to spend money in lobbying even if side-payments are not feasible. Analogously, political parties optimally agree not to spend any effort in political campaigns, and limit the voters’ choice to the set consisting of the strongest parties. Especially in the US, presidential candidates spend huge amounts of money in their political campaigns (which is spent on tv-advertising, tours around the country, fancy internet sites, etc.). Therefore, a political campaign can be seen as an all-pay auction, so that our finding implies that colluding parties agree to spend no money in their campaign, and limit the choice set of the voters.

Finally, consider markets in which competition mainly takes place through advertising (see e.g. Schmalensee, 1976 and Huck et al., 2002). In these markets, the probability that a customer buys a product from a particular firm depends on how much this firm advertises relative to the other firms. Observe the ‘all-pay’ nature of such markets: the
advertising expenditures are sunk before the customer makes his decision. The value of attracting a customer differs from one firm to the next, for instance because one firm is more efficient than the other. Our result implies that when total demand does not depend on the amount of advertising, firms optimally collude by not spending money in advertising. In addition, collusion in terms of advertising may also be welfare improving in situations in which advertising is of no intrinsic or informative value to the customers (Huck et al., 2002 derive similar results in a model with complete information).
A Appendix: Contest success functions

The following examples of contest success functions have been studied in the literature.

The logit form contest success function is characterized by

\[ q_i(b) = \frac{g_i(b_i)}{\sum_{j \in N} g_j(b_j)} \]

where \( g_1, \ldots, g_n \) are strictly increasing functions. Tullock (1980) uses this contest success function, making the special assumption that \( g_i(b_i) = (b_i)^\alpha \) for all \( i \in N \) with \( \alpha > 0 \).

Skaperdas (1996) provides an axiomatic underpinning of this special type of logit form contest success functions, and Fullerton and McAfee (1999) give a further microeconomic support. The difference form contest success function is only defined for the case of two interest groups. The function is given by

\[ q_1(b_1, b_2) = h(\beta b_1 - b_2) \]
\[ q_2(b_1, b_2) = 1 - h(\beta b_1 - b_2) \]

where \( \beta \) is an ability parameter, and \( h \) an strictly increasing function, with \( h(0) = \frac{1}{2} \), and \( h(-y) = 1 - h(y) \). See Baik (1998) for a more extensive discussion on difference form contest success functions. Finally, the perfectly discriminatory contest success function is given by

\[ q_i(b) = \begin{cases} \frac{1}{m} & \text{if } b_i = \max_j b_j \text{ with } m = \#\{k | b_k = \max_j b_j\}, \\ 0 & \text{otherwise.} \end{cases} \]

This contest success function is for instance used by Baye et al. (1996) and Che and Gale (1998).
References


