Vertical integration and product innovation

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Abstract

We study vertical integration and product innovation as interdependent strategic choices of vertically related firms. We consider product differentiation in the downstream market as a strategic device of downstream firms facing a threat of vertical integration and market foreclosure by an upstream monopolist. Our main finding is that, although product differentiation allows to soften product market competition and to avoid market foreclosure, the downstream market may prefer less product differentiation to prevent vertical integration. Therefore, less product innovation can be a possible social cost of a lenient antitrust policy.

Key Words: Vertical Integration; product innovation; market foreclosure; duopoly.

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1. INTRODUCTION

One of the most controversial issues in industrial organization is market foreclosure through vertical integration.\(^1\) Though one major advantage of vertical integration is to eliminate the problem of double marginalization, the major criticism against vertical integration is market foreclosure, which has generated a significant amount of literature to examine the competitive structure of the upstream and downstream industries and welfare.\(^2\) However, surprisingly enough the previous literature on vertical integration has mainly concentrated on the productive activities of the firms, paying less attention to the non-productive activ-

\(^1\)Vertical foreclosure refers to restrictions in supply (demand) of an essential input an integrated firm would apply to its downstream (upstream) competitors, extending in this way its market power in a related market. The negative consequence in terms of welfare would arise from a reduction in consumers’ welfare due to higher prices and lower quantities of the final goods.

\(^2\)Contrary to the benign view of the so-called Chicago school (e.g. Bork (1978)), denying vertical foreclosure as an equilibrium consequence of vertical mergers, subsequent works have proved vertical foreclosure in different models of vertical integration. Salinger (1988) shows that a vertical merger in a successive oligopoly with quantity competition, causes the integrated firm to withdraw from the intermediate good market. The increased concentration raises the intermediate good price and hence the production cost of the integrated firm’s downstream rivals, whose market shares fall. However, due to the avoidance of double marginalisation, the effect on the final good price is ambiguous. Riordan (1998) models a vertical merger between an upstream supplier and a dominant downstream firm. In this model, vertical integration always leads to higher prices for both the intermediate and the final products. Ordover, Saloner and Salop (1990) consider a successive duopoly model in which an integrating downstream firm is required to outbid the rival for the acquisition of an upstream supplier, and the two unintegrated firms (one upstream and one downstream) may react to the threat of foreclosure by vertically integrating as well. Vertical foreclosure arises in equilibrium provided that the increase in the input price following a vertical merger increases the joint-profit of the two unintegrated firms (whilst the individual profit of the unintegrated downstream firm always decreases), what can arise in the case of differentiated product price competition. Finally, in the recent literature following the incomplete contracts approach of Hart and Tirole (1990), market foreclosure is strictly linked to vertical integration, the latter being a means to solve the upstream firm’s commitment problem of not expanding input sales in the downstream market after a contract has been signed with some costumers at monopolistic conditions. See Rey and Tirole (2004) for a comprehensive survey of this literature.
ities of the firms and particularly, for the downstream firms.  

In this paper, we consider product innovation in the downstream market as a strategic device of downstream firms facing a threat of vertical integration and market foreclosure by an upstream monopolist. We examine how horizontal product differentiation in the downstream market affects the incentive for vertical integration and market foreclosure, and how the possibility of vertical integration affects the downstream firms’ incentives to differentiate products.

We use a simple model in which, without vertical integration, an upstream monopolist charges a linear price on the sole input required by two downstream firms in order to produce the final product. In the downstream market, firms compete in quantities. In this setting, vertical integration of the upstream firm with one of the two downstream firms eliminates double marginalisation in a segment of the final product market, giving the integrated firm a competitive advantage over the downstream rival. Moreover, by setting the input price, the integrated firm affects the downstream rival’s costs, and it may choose to foreclose the market (i.e. monopolise the final product market).

The vertical structure of the market (i.e. vertical integration vs no-vertical integration) is endogenously determined by an integration game modeled as a sale-auction between the downstream firms. If the gain from vertical integrating exceeds a fixed integration cost, the upstream firm calls for offers by the downstream firms in order to integrate one of them. Then, competition between the downstream firms in the integration game allows the upstream firm to appropriate more than the full surplus from integration, and reap most of the profit created in the final product market. Therefore, vertical integration is a threat to

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3 One exception is Baake, Ulrich and Norman (2004). They consider capital investment as a non-productive strategic decision of the upstream firm in the Hart and Tirole (1990) model. They show that banning vertical integration has the social cost of a sub-optimal level of capital investment, leading to productive inefficiency in the market. In contrast, vertical integration assures the efficient level of capital investment but output is monopolistically restricted. In this paper, we take a different perspective by giving the downstream firms a non-productive strategic variable (i.e. product innovation) in a vertical integration game.

4 Relative to the recent literature on vertical integration following Hart and Tirole (1990), our model
the downstream firms at the initial stage of the game, when they can invest to differentiate products.

Besides the usual effect of softening competition in the downstream market, product innovation exerts two more effects in our model. Product differentiation eliminates market foreclosure under vertical integration, and it affects the possibility of vertical integration.

The elimination of market foreclosure encourages innovation in the downstream market. However, the trade-off between the benefits from eliminating market foreclosure and softening product market competition, on one hand, and the loss from vertical integration, on the other hand, makes the impact of vertical integration on innovation ambiguous. In fact, we show that whether vertical integration is more likely for higher or lower degrees of product differentiation is ambiguous and depends on the cost of integration. If the cost of integration is very small, vertical integration occurs always. If the cost of integration is moderate, vertical integration occurs for very small and very large degrees of product differentiation. If the cost of integration is sufficiently large, but not large enough to prevent vertical integration, then vertical integration occurs for very large degrees product differentiation. Therefore, while higher product differentiation softens competition in the final goods market, it may also create the threat of vertical integration, which helps the upstream

 relocates upstream firm’s bargaining power from the bargaining game for the essential input to the vertical integration game. In the Hart and Tirole model (ex-post monopolisation variant), the upstream monopolist reaps all the industry profit by making take-or-leave-it offers of non-linear tariffs for the essential input to the downstream firms. Vertical integration (and vertical foreclosure) solves the upstream firm’s commitment problem of not selling more input to other downstream firms at the disadvantage of those who bought the good at monopoly conditions at the first place (without vertical integration, the downstream firms refuse to buy at monopoly conditions in anticipation of the upstream firm’s myopic incentive to sell more ex-post). Therefore, vertical integration consents the upstream firm to realise a higher industry profit (i.e. the monopoly profit), not a higher share in the industry profit at the disadvantage of the downstream firms. In our model, the upstream firm sells the input at a linear price, so that, without vertical integration, the downstream firms gain positive profits. On the other hand, the upstream firm extracts profits from the downstream firms through vertical integration, by putting them in competition to be integrated. Obviously this requires that the downstream firms cannot commit to a collusive behaviour when facing the call for integration offers from the upstream firm.
firm to extract more rent from the downstream firms. As a consequence, there are situations where the downstream market prefers relatively lower degrees of product differentiation to prevent vertical integration. So, instead of market foreclosure, we show a new possible cost of vertical integration, i.e., lower product innovation.

The rest of the paper is organised as follows. In Section 2, we present the model, which consists of a three-stages game with the following timing: innovation stage (first stage), integration stage (second stage), market stage (final stage). In Section 3, we solve the market stage under the two alternative market structures (vertical integration vs. no-vertical integration), and we discuss the effect of product differentiation on market foreclosure. Section 4 analyses the vertical integration game, and shows how the vertical integration outcome depends on product differentiation and integration costs. In Section 5, we study the effects of vertical integration on product innovation. In Section 6 we point out that the possibility of vertical integration can cause the social cost of less product innovation. Finally, Section 7 provides some concluding remarks.

2. THE MODEL

We consider an economy with upstream and downstream markets. In the upstream market, a monopolist (firm U) produces the sole input needed by two downstream firms (firms D₁ and D₂) in order to produce their final products. We assume that the upstream monopolist produces the essential input at zero-costs. The downstream firms share the same production technology, which requires one unit of input in order to produce one unit of final product.

Although the downstream firms use the same production technology, their final product can be differentiated at the outset by investing in R&D. More precisely, on the demand side of the downstream market, the degree of product substitutability perceived by consumers, γ, leads to the inverse demand system:

\[ p_i = a - q_i - \gamma q_j \quad (i, j = 1, 2; \ i \neq j), \]  

(1)
where $\gamma \in [0, 1]$. With $\gamma = 0$, consumers perceive products 1 and 2 as independent goods, while $\gamma = 1$ corresponds to consumers’ perception of perfect substitutes. The perceived degree of product substitutability depends on the downstream firms’ R&D effort. Since our main focus is on the effect exerted by the possibility of vertical integration on the incentive to differentiate products, we set aside any strategic consideration related to R&D competition by assuming that only one firm can invest in R&D. By paying a fixed R&D cost $k$, the innovative firm can reduce the perceived degree of product substitutability from $\gamma = 1$ (perfect substitutes goods) to $\gamma = \tilde{\gamma} \in [0, 1)$. Without the R&D investment, products 1 and 2 are perceived as perfect substitutes by consumers.

After the R&D decision is taken, the vertical integration game takes place. The upstream firm may call for (simultaneous and independent) price-offers by the downstream firms in order to integrate one of them. On the basis of the offers received, the upstream firm decides whether or not to integrate the downstream firm asking for the lowest price (in the case of tie, we assume that both downstream firms have fifty percent probability of merging with the upstream firm). We further assume that vertical integration involves a fixed cost, 

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5The demand side of the downstream market is a simplified version of Singh and Vives (1984). The inverse demand system (1) is generated by the utility function: $U = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) + m$, where $m$ is a numeraire good.

6Whilst this assumption can be justified by only one downstream firm having the capability to alter consumers’ perception of product substitutability, an alternative interpretation of the model is that the downstream firms can cooperate at the R&D stage of the game. Clearly, with cooperation, the joint R&D effort will depend on the joint gain arising from product differentiation, whereas one firm’s R&D effort depends only on its individual gain. However, since the downstream firms turn out to be symmetric in any respect at the innovative stage, the assessment of the effect exerted by vertical integration on product differentiation is qualitatively the same.

7The strategic effects arising from R&D competition for product innovation within the differentiated duopoly model we adopt for the downstream market, are analysed by Lambertini and Rossini (1998). Due to the positive externality exerted by the R&D investment of each firm on the rival’s profit (via product differentiation), a simultaneous R&D game can lead to a prisoner dilemma equilibrium. Therefore, the final products can remain homogeneous even if product differentiation would increase both firms’ profit. The introduction of R&D competition strongly complicates our model, without changing the nature of the effects we focus on, and without affecting qualitatively our main results.
denoted by $E$.$^8$

The outcome of the integration game sets the structure of the markets. If vertical integration does not occur, the upstream monopolist supplies the essential resource to the downstream firms charging a linear price $w_v$.$^9$ The input price acts as the marginal cost of production for both downstream firms, which finally compete à la Cournot in the downstream market. If vertical integration occurs, the downstream market is populated by a vertically integrated firm (firm $V$), and an independent firm (firm $I$). The integrated firm disposes of the essential resource at zero-cost, and sets optimally the price of the input supplied to the rival, $w_v$. Finally, the two firms compete à la Cournot in the downstream market.

Summarising, the model consists of three stages. In stage 1, the R&D decision is taken by the innovative (downstream) firm, and the degree of product differentiation is determined. In stage 2, the vertical integration game takes place, and the market structure is determined. In stage 3, the price of the essential resource is set by firm $U$ (or firm $V$, under vertical integration), and Cournot competition takes place in the downstream market. Production and profits are finally determined. Our solution concept is perfect subgame equilibrium, therefore we solve the game by backward induction starting from the market stage.

$^8$See Hart and Tirole (1990) for the interpretation of the cost of vertical integration. Also similarly to Hart and Tirole (1990), we assume away the possibility of merger between the downstream firms. Hence, if vertical integration occurs, firm $U$ can integrate with one downstream firm only. This can be justified by the cost of integrating both downstream firms being prohibitive.

$^9$Our assumption of linear pricing for the input is similar to Choi (1991), Gerstner and Hess (1995), Colangelo (1995), Economides (1998), Villas-Boas (1998), Tyagi (1999), Rao and Srinivasan (2001) and others. A similar assumption can also be found in the literature on ‘access pricing’ (e.g., Armstrong et al. (1996), Armstrong and Vickers (1998), De Fraja and Price (1999)) and on ‘channel coordination’ (e.g., Gerstner and Hess (1995)). The assumption of linear pricing may be justified by the arguments given by Rao and Srinivasan (2001) in the context of franchising. If the upstream and the downstream firms are in ongoing relationship where the demand and cost conditions vary over time, the uniform pricing of the upstream product is optimal if significant costs are involved in re-writing the contracts between the upstream and downstream firms.
3. THE MARKET STAGE

At the market stage, the degree of product differentiation, $\gamma$, and the market structure (i.e. vertical integration vs. no-vertical integration) are already determined.

We start with the market equilibrium without vertical integration. Given the input price, $w_u$, the downstream firms ($D_1$ and $D_2$) face the same marginal cost. Hence, Cournot competition leads to a symmetric equilibrium in the downstream market, where the downstream firms produce

$$q^D_1(w_u) = q^D_2(w_u) = \frac{a - w_u}{2 + \gamma},$$

and earn profits

$$\pi^D_1(w_u) = \pi^D_2(w_u) = \left[ \frac{a - w_u}{2 + \gamma} \right]^2.$$

The upstream monopolist faces the demand function for the essential input $\frac{2(a - w_u)}{2 + \gamma}$, so that he sets the input price, $w_u$, to maximize

$$\pi^U(w_u) = w_u \frac{2(a - w_u)}{2 + \gamma}.$$

This leads to the input price:

$$w_u^* = \frac{a}{2}. \quad (2)$$

Finally, firms’ equilibrium profits without vertical integration are:

$$\pi^D_{1,2} = \pi^D = \left( \frac{a}{2} \right)^2 \left[ \frac{1}{2 + \gamma} \right]^2, \quad (3)$$

$$\pi^U = \left( \frac{a}{2} \right)^2 \frac{2}{2 + \gamma}. \quad (4)$$

We turn now to the market equilibrium under vertical integration. The integrated firm produces its final product at zero-cost, and charges the linear price $w_v$ on the input sold to the independent firm. Given $w_v$, Cournot competition leads to an asymmetric equilibrium in the downstream market, where the independent firm ($I$) and the integrated firm ($V$) produce, respectively:

$$q^I(w_v) = \frac{a(2 - \gamma) - 2w_v}{4 - \gamma^2}, \quad q^V(w_v) = \frac{a(2 - \gamma) + \gamma w_v}{4 - \gamma^2}. \quad 8$$
The corresponding profits are:

\[ \pi^I(w_v) = \left[ \frac{a(2 - \gamma) - 2w_v}{4 - \gamma^2} \right]^2 \]

for the independent firm, and

\[ \pi^V(w_v) = \left[ \frac{a(2 - \gamma) + \gamma w_v}{4 - \gamma^2} \right]^2 + w_v \frac{a(2 - \gamma) - 2w_v}{4 - \gamma^2} \]

for the integrated firm, where the second term of the integrated firm’s profit comes from its sales of the essential input to the rival. The input price, \( w_v \), is set by the integrated firm to maximise \( \pi^V(w_v) \), leading to:

\[ w_v^* = \frac{a(2 - \gamma)(2\gamma + 4 - \gamma^2)}{2(8 - 3\gamma^2)}. \] (5)

Finally, firms’ equilibrium profits under vertical integration are:

\[ \pi^I = \left( \frac{a}{2} \right)^2 \frac{4(1 - \gamma)}{(8 - 3\gamma^2)} \] (6)

\[ \pi^V = \left( \frac{a}{2} \right)^2 \frac{(2 - \gamma)(6 - \gamma)}{(8 - 3\gamma^2)}. \] (7)

The following lemma makes some useful comparisons of the market outcomes with and without vertical integration.

**Lemma 1** i) Unless \( \gamma = 0 \) or \( \gamma = 1 \), the input price charged to the independent firm under vertical integration is lower than the input price charged to the downstream firms without vertical integration (i.e. \( w_u^* < w_v^* \) for any \( \gamma \in (0, 1) \)). ii) Unless \( \gamma = 0 \), the independent firm earns lower profits under vertical integration than without vertical integration (i.e. \( \pi^I < \pi^D \) for any \( \gamma \in (0, 1) \)). iii) Both the independent firm’s profit \( \pi^I \) (vertical integration) and the downstream firm’s profit \( \pi^D \) (no-vertical integration) increase with product differentiation.

**Proof.** i) Using (2) and (5), we calculate:

\[ w_u^* - w_v^* = \frac{a}{2} \frac{\gamma^2(1 - \gamma)}{(8 - 3\gamma^2)}. \]
From the expression above it follows immediately that \( w^*_u - w^*_v > 0 \) for any \( \gamma \in (0, 1) \), while \( w^*_u - w^*_v = 0 \) for \( \gamma = 0 \) and \( \gamma = 1 \). \(^{10}\)

(ii) From (3) and (6), we find that \( \pi^D \geq \pi^I \) is equivalent to:

\[
\frac{8 - 4\gamma - 4\gamma^2}{8 - 3\gamma^2} \leq 1,
\]

which is strictly satisfied for \( \gamma \in (0, 1] \). Equality clearly holds for \( \gamma = 0 \).

(iii) Differentiating (6) we get:

\[
\frac{\partial \pi^I}{\partial \gamma} = -\frac{32 (1 - \gamma) (8 + 3\gamma^2 - 6\gamma)}{(8 - 3\gamma^2)^3} \left( \frac{a_1}{2} \right)^2,
\]

which is strictly negative for \( \gamma \in [0, 1) \) (it equals zero for \( \gamma = 1 \)). Similarly, from (3) we obtain:

\[
\frac{\partial \pi^D}{\partial \gamma} = -\frac{2}{(2 + \gamma)^2} \left( \frac{a_1}{2} \right)^2,
\]

which is strictly negative for \( \gamma \in [0, 1] \).

3.1. Vertical integration and market foreclosure.—

Before proceeding to the previous stages of the game, we pause here to discuss the effect of product differentiation on the possibility of market foreclosure under vertical integration. Market foreclosure occurs if only the vertically integrated firm is active in the downstream market, i.e., \( q^I(w^*_v) = 0 \).

**Proposition 1** Vertical integration leads to market foreclosure only when products are perfect substitutes (i.e., \( \gamma = 1 \)). In contrast, market foreclosure never occurs under the vertical integration history when products are differentiated (i.e., for any \( \gamma \in (0, 1) \)).

**Proof.** From the expression of \( q^I(w_v) \), the independent firm is inactive in equilibrium iff \( w^*_v \geq \frac{a(2-\gamma)}{2} \). Using equation (5), we find that \( w^*_v < \frac{a(2-\gamma)}{2} \) for any \( \gamma \in [0, 1) \), while \( w^*_v = \frac{a(2-\gamma)}{2} \) for \( \gamma = 1 \). That is, market foreclosure occurs only for \( \gamma = 1 \). In contrast, the independent firm remains active in the market for any \( \gamma \in [0, 1) \). \( ^{10} \)

\(^{10}\)More precisely, while the input price with no-vertical integration is independent of \( \gamma \) (see eq. (2)), it is easy to show that the input price under vertical integration is a U-shaped function of \( \gamma \) in the interval [0, 1].
The interpretation of proposition 1 is as follows. The integrated firm has a strategic incentive to raise the input price charged to the independent firm, since its price and production in the downstream market increase with the rival’s marginal cost. On the other hand, its sales of the essential input decrease. Intuitively, the strategic incentive is stronger the higher the degree of product substitutability (it actually vanishes if products are independent, i.e. for $\gamma = 0$). According to proposition 1, only when products are perfect substitutes the strategic incentive is strong enough to induce the integrated firm to foreclose the market and stop supplying the essential input to the rival.11

Proposition 1 has an interesting implication for the previous stages of the game. Since product differentiation allows to avoid market foreclosure, it also guarantees both downstream firms positive profits under vertical integration. On one hand, the independent firm can assure a positive profit only if products are differentiated. On the other hand, by allowing the independent firm to gain a positive profit, product differentiation helps the downstream firm that vertically integrates to extract a positive profit from vertical integration even if the upstream firm has full bargaining power.

11To see this, suppose that products are perfect substitutes, and that the integrated firm engages in monopoly pricing in the product market while charging the rival the minimum input price such that rival’s production is zero. Notice that, with perfect substitutes, such a critical input price coincides with the monopoly price the integrated firm is charging in the product market (i.e. the monopoly price arising from the demand system (1) with $\gamma = 1$, and zero-marginal cost). Consider now a small decrease in the input price below the critical level. We can distinguish three effects on the integrated firm’s profits. First, the equilibrium price of the final product decreases. However, the negative effect on the integrated firm’s profit is only second order, since the product price falls just below the monopoly price. Second, the independent firm enters the market with a (slightly) positive production, reducing the integrated firm’s production of the final good by a corresponding amount. Third, the integrated firm’s sales of input increase by the same amount of the independent firm’s production of the final good. Therefore, the integrated firm’s sales of input equal the reduction in its sales of the final product, but the price of the input lies below the monopoly price of the final product. This means that the second (negative) effect dominates the third (positive) effect, and the integrated firm’s profit decreases. The basic difference with differentiated products is that, when products are imperfect substitutes, the critical level of the input price lies above the integrated firm’s monopoly price. Then, the second effect is dominated by the third effect, and the integrated firm’s profit increases.
4. THE VERTICAL INTEGRATION GAME

Having solved the final market stage under the two alternative market structures, we are now in the position to examine the incentive for vertical integration. Recall that, at the vertical integration stage, the degree of product differentiation is already determined. We start by noting that there is a positive surplus to gain from vertical integration provided that the integrated firm’s profits, net of the fixed cost of integration, exceed the joint profits of the two firms involved in the merger (i.e. the upstream monopolist and one downstream firm) without vertical integration. Let us denote with $S = \pi^V - (\pi^U + \pi^D)$ the surplus from vertical integration before the integration cost, so that the profitability condition for vertical integration is:

$$S > E$$

(8)

When condition (8) holds, each downstream firm has always an incentive to make a price-offer to be vertically integrated. If firm $D_2$ does not make any offer, it is convenient for firm $D_1$ to make an offer between $\pi^D$ and $\pi^D + (S - E)$. Since a positive surplus is left to the upstream firm, the offer will be accepted, and the bidder will gain a higher profit than under the alternative of not making any offer (without any offer, vertical integration does not occur, and both downstream firms earn $\pi^D$). Alternatively, if firm $D_2$ makes the offer above, then it is convenient for firm $D_1$ to undercut the rival’s offer, since $\pi^D > \pi^I$ (see lemma 1). Furthermore, each downstream firm has always an incentive to undercut any rival’s offer $O_j$ greater than $\pi^I$. By bidding above the rival, a firm ends up being the independent firm under vertical integration, earning $\pi^I$. By matching the rival’s offer, it has equal chances of being independent or integrated, with expected profit $\frac{1}{2}(O_j + \pi^I)$. It is then optimal to bid just below the rival, say $O_j - \epsilon$, which assures to be integrated with a profit $O_j - \epsilon \ (> \frac{1}{2}(O_j + \pi^I) > \pi^I)$. Then, the unique Nash equilibrium pair of offers by the downstream firms is ($\pi^I$, $\pi^I$). The upstream firm is left with more than the full surplus from integration, so that it will certainly call for offers at the outset, and vertical integration occurs. Notice that, due to competition in price-offers to be integrated, the downstream firm that is finally integrated reaps only its outside option under vertical integration (i.e.
the equilibrium profit of the independent firm).

Assume now that condition (8) does not hold. In this case, since the net surplus from integration is negative, the upstream firm rejects any price-offer equal to (or greater than) \( \pi^D \). Then, if the upstream firm calls for offers, the unique (relevant) Nash equilibrium of the game is that both downstream firms make an offer which leaves the upstream firm with negative surplus (any offer above \( \pi^D - (E - S) \) will do), and the upstream firm rejects.\(^{12}\) Then, vertical integration does not occur, and both downstream firms earn profit \( \pi^D \). Anticipating this equilibrium outcome, the upstream firm will not call for offers at the outset.\(^ {13}\)

We have proved:

**Lemma 2** If the net surplus from integration is positive (i.e. \( S > E \)), vertical integration always occurs, and the downstream firm involved in the merger earns the same profit as the independent firm, i.e. \( \pi^I \). If the net surplus from integration is negative (i.e. \( S < E \)), vertical integration never occurs, so that both downstream firms earn profit \( \pi^D \).

In view of the first stage of the model, the following proposition characterises the market structure that arises after the integration stage as a function of the degree of product differentiation and the integration cost level. By lemma 2, this amounts to evaluate the sign of the net surplus from integration, \( S - E \), along the range of product substitutability \( \gamma \in [0, 1] \).

**Proposition 2** a) When the integration cost is small, vertical integration occurs for any degree of product differentiation. b) When the integration cost is moderately high, vertical integration occurs only for large or for small (but not for intermediate) degrees of product differentiation. c) When the integration cost is high (but not prohibitive), vertical integration occurs only for very large degrees of product differentiation.

\(^{12}\) Clearly, neither downstream firm has an incentive to deviate, and make a price-offer low enough to be acceptable by the upstream firm. The deviant would be integrated at a price below \( \pi^D \).

\(^{13}\) A qualification of this result is in order. When condition (8) does not hold, the pair of offers \( (\pi^I, \pi^I) \) still identifies a Nash equilibrium, but firms are playing weakly dominated strategies. We select it away for this reason.
Proof. Using equations (3), (4) and (7), the surplus from integration before the integration cost, $S$, can be written as:

$$S(\gamma) = \left(\frac{a}{2}\right)^2 \frac{8 - \gamma^2 + 2\gamma^3 + \gamma^4}{(8 - 3\gamma^2)(2 + \gamma)^2}$$  \hspace{1cm} (9)

Inspection of (9) suffices to show that $S(\gamma) > 0$ for any $\gamma \in [0,1]$, taking values $S(1) < S(0)$. Furthermore, we prove in Appendix 1 that $S(\gamma)$ is a U-shaped function of $\gamma$ over the interval $[0,1]$, reaching a minimum value for $\gamma \approx 0.61037$ (see Figure 1 below). From the shape of $S(\gamma)$, the proof of proposition 2 is straightforward. Let us denote with $S_m$ the minimum of $S(\gamma)$. Then:

a) if $E < S_m$, vertical integration occurs for any $\gamma \in [0,1]$;

b) if $S_m < E < S(1)$, there must be two critical degrees of product differentiation, say $\gamma_{b_1}$ and $\gamma_{b_2}$ (with $\gamma_{b_1} < \gamma_{b_2}$), such that vertical integration occurs for $\gamma < \gamma_{b_1}$ and $\gamma > \gamma_{b_2}$, whilst it does not occur for $\gamma \in [\gamma_{b_1}, \gamma_{b_2}]$;

c) if $S(1) < E < S(0)$, there must be one critical degree of product differentiation, say $\gamma_c$ ($\gamma_{b_1}$), such that vertical integration occurs only for $\gamma < \gamma_c$.

Finally, if $E \geq S(0)$ vertical integration never occurs (that is, $S(0)$ identifies a threshold level above which the integration cost becomes prohibitive).
that a positive surplus from integration may come from two sources in our model: 1) the avoidance of double marginalisation in one segment of the final product market (the one of the downstream firm that is integrated); 2) the efficiency advantage (i.e. the lower marginal cost in producing the final product) the integrated firm has relative to the independent firm in the downstream market.14

Suppose now that products are independent (i.e. $\gamma = 0$). In this case, only the first source of surplus is active, since the two segments of the downstream market are isolated. As the degree of product differentiation starts decreasing (i.e. $\gamma$ starts increasing from 0), the total demand in the product market starts decreasing as well, since consumers value less any bundle of the two products relative to the numeraire good.15 The fall in the gross surplus from integration is then explained by the lower gain from avoiding double marginalisation in a smaller market, while the second source of surplus (i.e. the efficiency advantage) is still irrelevant (since products are almost independent).16 Only when the degree of product differentiation is sufficiently low, the second source of surplus plays a significant role. Then, the efficiency advantage of the integrated firm allows it to soften the negative effect exerted by an increase of $\gamma$ on the demand for its final product, since consumers tend to substitute the high priced product of the independent firm for the low priced product of the integrated firm. Moreover, the integrated firm benefits from the higher reduction of the rival’s demand while playing the Cournot game in the product market. Hence, the integrated firm has an incentive to increase the rival’s cost (by rising the input price) as the degree of product differentiation further decreases.17 This means that the second source of surplus strengthens

14 The efficiency advantage allows the integrated firm to expand its equilibrium production of the final good at the rival’s expense. Clearly, this also has a negative effect on the component of the integrated firm’s profit related to the sales of the essential input to the independent firm.

15 See footnote 5 for the specification of consumers’ preferences underlying the demand functions of the two differentiated products.

16 This interpretation is also confirmed by the decrease in the input price under vertical integration, $w_e$, as $\gamma$ starts increasing from 0 (see the proof of lemma 1). The integrated firm tries to contrast the decrease of the demand for input by the independent firm (caused by the reduction in the global size of the product market after the increase in $\gamma$), so that its efficiency advantage over the rival decreases.

17 This is confirmed by the U-shaped behaviour of the input price $w_e$ as the degree of product substi-
as the degree of product differentiation decreases. When products are sufficiently close
substitutes, the second source plays a dominant role, inverting the sign of the relationship
between product differentiation and surplus from integration.

5. VERTICAL INTEGRATION AND PRODUCT INNOVATION

In this section, we analyse the effects exerted by vertical integration on the incentive to
differentiate products. As mentioned before, we assume that only one firm, say firm $D_1$,
can invest in R&D aimed at product differentiation.$^{18}$ By paying a fixed R&D cost $k$, the
innovative firm can reduce the perceived degree of product substitutability, $\gamma$, from 1 to
$\hat{\gamma} \in [0,1)$. On the contrary, products are perceived as perfect substitutes ($\gamma = 1$) if firm $D_1$
does not invest.

The degree of product differentiation achievable by investing in R&D, $1 - \hat{\gamma}$, sets the
effectiveness of the R&D technology (so that R&D effectiveness is higher the smaller is $\hat{\gamma}$).
Given the effectiveness of the R&D investment, we measure the incentive to differentiate
products by the highest level of the R&D cost the innovative firm is willing to pay in order
to obtain the associated degree of product differentiation, $\hat{k}(\hat{\gamma})$.

Let $\pi(\gamma)$ be the prospective profit the innovative firm expects to gain at the market stage
as a function of the degree of product substitutability perceived by consumers. We clearly

$^{16}$Recall that, at the innovative stage of the model, the downstream firms share identical profit expecta-
tions under any subsequent evolution of the game (i.e. the independent firm’s profit $\pi_I$ under the vertical
integration history, the downstream firm’s profit $\pi^D$ under the no-vertical integration history). Hence, the
identity of the innovative firm is irrelevant. Moreover, the joint gain the two firms derive from product
differentiation is always twice the individual gain of the innovative firm. Therefore, the joint incentive to
differentiate products if firms $D_1$ and $D_2$ can cooperate in R&D (as measured by the highest joint-R&D
cost they are willing to pay in order to obtain a given degree of product differentiation) is always twice the
innovative firm’s incentive when only one firm can invest with no cooperation. The results of this section
immediately extend to the case of R&D cooperation by simply scaling-up the measure of the incentive to
invest in R&D.
have:

$$\hat{k}(\hat{\gamma}) = \pi(\hat{\gamma}) - \pi(1),$$

(10)

where the relevant profit function $\pi(\gamma)$ depends on the subsequent history of the game associated with any $\gamma$. Building upon proposition 2, we must distinguish four cases according to the level of the integration cost.

**Small integration cost (case (a) of proposition 2).** Vertical integration occurs at the second stage of the game for any degree of product substitutability. Therefore, the vertical integration outcome is independent of both the R&D effectiveness and the investment decision of the innovative firm. Since the innovative firm will end up with the independent firm’s profit under vertical integration (lemma 2), the relevant profit function at the innovative stage coincides with the independent firm’s profit function:

$$\pi(\gamma) = \pi^I(\gamma), \forall \gamma \in [0, 1],$$

where $\pi^I(\gamma)$ is given by equation (6). Notice that, would firm $D_1$ not invest in R&D, products are perceived as perfect substitutes by consumers, and vertical integration leads to market foreclosure (proposition 1), so that $\pi(1) = \pi^I(1) = 0$. Hence, our measure of the incentive to differentiate products becomes:

$$\hat{k}_a(\hat{\gamma}) = \pi^I(\hat{\gamma}), \forall \hat{\gamma} \in [0, 1].$$

(10a)

Since the prospective profit of the innovative firm always coincides with the independent firm’s profit under vertical integration, the incentive towards product differentiation will basically reflect the following three motives: 1) avoiding market foreclosure; 2) softening the competitive pressure of a more efficient firm (i.e. the integrated firm); 3) forcing the integrated firm to charge a lower input price.\(^{19}\)

**Moderately high cost of integration (case (b) of proposition 2).** Vertical integration occurs at the second stage of the game only for large and for small, but not for intermediate, degrees

\(^{19}\)Notice also that $\hat{k}_a(\hat{\gamma})$ always increases with the R&D effectiveness, as $\pi'(\hat{\gamma})$ monotonically decreases with $\hat{\gamma}$ (lemma 1).
of product substitutability. Consequently, the relevant profit function at the innovative stage will jump between the independent firm’s and the downstream firm’s equilibrium profit at both extremes of the interval of product substitutability where vertical integration does not occur. Denoting such an interval with \([\gamma_{b_1}, \gamma_{b_2}]\) (as in figure 1), we have:

\[
\pi(\gamma) = \begin{cases} 
\pi^I(\gamma) & \text{for } \gamma \in (\gamma_{b_2}, 1] \\
\pi^D(\gamma) & \text{for } \gamma \in [\gamma_{b_1}, \gamma_{b_2}] \\
\pi^I(\gamma) & \text{for } \gamma \in [0, \gamma_{b_1})
\end{cases}
\]

where \(\pi^D(\gamma)\) and \(\pi^I(\gamma)\) are given by equations (3) and (6), respectively. Like in the previous case, if the innovative firm does not invest in R&D, then vertical integration and market foreclosure occur at the final stage of the game, so that \(\pi(1) = \pi^I(1) = 0\). Hence, our measure of the incentive to invest in R&D becomes:

\[
\hat{k}_b(\hat{\gamma}) = \begin{cases} 
\pi^I(\hat{\gamma}) & \text{for } \hat{\gamma} \in (\gamma_{b_2}, 1] \\
\pi^D(\hat{\gamma}) & \text{for } \hat{\gamma} \in [\gamma_{b_1}, \gamma_{b_2}] \\
\pi^I(\hat{\gamma}) & \text{for } \hat{\gamma} \in [0, \gamma_{b_1})
\end{cases} \tag{10b}
\]

The outcome of the vertical integration game depends on the investment decision of the innovative firm if the R&D technology allows it to target the interval \([\gamma_{b_1}, \gamma_{b_2}]\). Hence, its incentive towards product differentiation may incorporate the additional motive of preventing vertical integration (recall that, by lemma 1, \(\pi^D > \pi^I\) for any \(\gamma \in (0, 1]\)).

**High integration cost (case (c) of proposition 2).** Vertical integration will occur only for very low degrees of product substitutability (that is, for very high degrees of product differentiation). Therefore, the relevant profit function at the innovative stage jumps from the downstream firm’s to the independent firm’s equilibrium profit at the critical degree of product substitutability below which vertical integration will occur. Denoting the critical degree by \(\gamma_c\) (as in figure 1), we have:

\[
\pi(\gamma) = \begin{cases} 
\pi^D(\gamma) & \text{for } \gamma \in [\gamma_c, 1] \\
\pi^I(\gamma) & \text{for } \gamma \in [0, \gamma_c)
\end{cases}
\]

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Contrary to the previous cases, vertical integration and market foreclosure will not occur in the subsequent stages of the game if the innovative firm decides not to invest in R&D, that is \( \pi(1) = \pi^D(1) > 0 \). On the other hand, vertical integration would follow the decision to invest when the resulting degree of product differentiation is very high. In other words, in the case under examination, not investing in R&D may be the only way to prevent vertical integration at the following stage of the game. Our measure of the incentive to invest in R&D is:

\[
\hat{k}_c(\hat{\gamma}) = \begin{cases} 
\pi^D(\hat{\gamma}) - \pi^D(1) & \text{for } \hat{\gamma} \in [\gamma_c, 1] \\
\pi^I(\gamma) - \pi^D(1) & \text{for } \gamma \in [0, \gamma_c)
\end{cases}
\tag{10c}
\]

Prohibitive integration cost (benchmark case). If the integration cost exceeds \( S(0) \) (i.e. the gross surplus from integration associated with \( \gamma = 0 \)), vertical integration never occurs at the second stage of the game. Therefore, the innovative firm’s profit function coincides with the downstream firm’s profit function,

\[\pi(\gamma) = \pi^D(\gamma), \forall \gamma \in [0, 1],\]

and our measure of the incentive to invest in R&D becomes:

\[
\hat{k}_s(\hat{\gamma}) = \pi^D(\hat{\gamma}) - \pi^D(1), \forall \hat{\gamma} \in [0, 1).
\tag{10s}
\]

In this case, the vertical integration stage of the model is irrelevant for the innovative firm’s incentive to differentiate products, which will only incorporate the usual motive of softening the competitive pressure of a symmetric competitor (i.e. firm \( D_2 \)) in the product market. We use this case as a benchmark to contrast the effects on product differentiation arising from the threat of vertical integration which characterises the previous cases.\(^{20}\)

We start by comparing the case of small integration costs with the benchmark case of prohibitive integration costs.\(^{20}\)

\(^{20}\)As in the case of small integration costs, also with prohibitive integration costs the innovative firm’s incentive to invest, \( \hat{k}_s(\hat{\gamma}) \), always increases with the R&D effectiveness, as \( \pi^D(\hat{\gamma}) \) is monotonically decreasing in \( \hat{\gamma} \) (lemma 1).
**Proposition 3** Unless the R&I effectiveness is very low (i.e. \( \hat{\gamma} \) is very high), the innovative firm’s incentive to invest in R&I is stronger when the integration cost is small (so that vertical integration always occurs) than when the integration cost is prohibitive (so that vertical integration never occurs).

**Proof.** From (10a) and (10*), \( \hat{k}_a(\hat{\gamma}) \geq \hat{k}_s(\hat{\gamma}) \) iff

\[
\pi^I(\hat{\gamma}) \geq \pi^D(\hat{\gamma}) - \pi^D(1).
\]

Using equations (3) and (6), the last inequality reduces to:

\[
16(1 - \hat{\gamma})(6 + 3\hat{\gamma})^2 - \left(5 + \hat{\gamma}(8 - 3\hat{\gamma}^2)\right)^2 \geq 0.
\]

Calculations with Mathematica show that the polynomial on the LHS has only one real root within the admissible range \( \hat{\gamma} \in [0, 1) \), that is \( \hat{\gamma}_a \approx 0.81682 \). Since \( \pi^I(0) \geq \pi^D(0) - \pi^D(1) \) (recall that \( \pi^I(0) = \pi^D(0) \), by lemma 1, and \( \pi^D(1) > 0 \)), it must be:

- \( \hat{k}_a(\hat{\gamma}) > \hat{k}_s(\hat{\gamma}) \) for \( \hat{\gamma} \in [0, \hat{\gamma}_a) \),
- \( \hat{k}_a(\hat{\gamma}) < \hat{k}_s(\hat{\gamma}) \) for \( \hat{\gamma} \in (\hat{\gamma}_a, 1] \).

\[
\hat{k}_a(\hat{\gamma}) \quad \hat{k}_s(\hat{\gamma})
\]

\[
0 \quad \gamma_a \quad 1
\]

Figure 2a
Figure 2a illustrates proposition 3. The intuition is that, when the R&D effectiveness is very low, the gain from softening the competitive pressure of a more efficient competitor (the integrated firm under vertical integration) is smaller than the gain from softening the competitive pressure of a symmetric competitor (the other downstream firm without vertical integration). Although a slight degree of product differentiation allows the independent firm to avoid market foreclosure, the resulting profit is negligible because the efficiency disadvantage relative to the integrated firm remains high when products are poorly differentiated. On the contrary, when the R&D effectiveness is sufficiently high, the gain from softening the competitive pressure of the integrated firm dominates the gain from softening the competitive pressure of a symmetric competitor. The independent firm’s profit is no more negligible when products are sufficiently differentiated, since both the efficiency disadvantage (up to a certain degree of differentiation) and its negative impact on the independent firm’s profit sharply decrease with product differentiation. This allows the incentive to avoid market foreclosure to play a dominant role: under vertical integration, the innovative firm can guarantee a positive profit only by investing in R&D, whilst, with no-vertical integration, a positive profit arises also without investing.

Consider now the case of moderately high cost of integration. Clearly, if the innovative firm cannot target the intermediate degrees of product differentiation where vertical integration does not occur, a comparison with the benchmark case replicates exactly proposition 3. However, when the crucial interval $[\gamma_{b_1}, \gamma_{b_2}]$ can be targeted, the possibility to prevent vertical integration by product differentiation strengthens the innovative firm’s incentive to invest relative to both the benchmark case and the case of small costs of integration.

**Proposition 4** The innovative firm’s incentive to invest in R&D is unambiguously strengthened by the possibility to prevent vertical integration via product differentiation which arises when the integration cost is moderately high and the R&D technology allows to target intermediate degrees of product differentiation.

---

$^{21}$In fact, the independent firm’s profit function, $\pi^I(\gamma)$, is flat at $\gamma = 1$. 

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**Proof.** Assume that $\hat{\gamma} \in [\gamma_{b_1}, \gamma_{b_2}]$. From equations (10b) and (10a) we get:

$$\hat{k}_b(\hat{\gamma}) - \hat{k}_a(\hat{\gamma}) = \pi^D(\hat{\gamma}) - [\pi^D(\hat{\gamma}) - \pi^D(1)] = \pi^D(1) > 0.$$  

This proves that the incentive to invest in R&D is stronger in the case of moderately high costs of integration (where vertical integration and market foreclosure can be prevented only by investing in R&D) than in the benchmark case of prohibitive costs of integration (where vertical integration never occurs).

Similarly, using equations (10b) and (10a), we get:

$$\hat{k}_b(\hat{\gamma}) - \hat{k}_a(\hat{\gamma}) = \pi^D(\hat{\gamma}) - \pi^I(\hat{\gamma}) > 0 \text{ (by lemma 1),}$$

This proves that the incentive to invest in R&D is stronger under the case of moderately high costs of integration (where the R&D investment allows to avoid vertical integration) than under the case of small costs of integration (where vertical integration always occurs).

\[\blacksquare\]

Figure 2b illustrates proposition 4.

![Figure 2b](image-url)
We turn now to the case of high integration costs, where vertical integration occurs only for very high degrees of product differentiation. If the R&D effectiveness is not very high (i.e. if the achievable degree of product substitutability exceeds the critical level $\gamma_c$), the incentive to invest identically coincides with that of the benchmark case. On the contrary, if the R&D effectiveness is very high (i.e. $\hat{\gamma} < \gamma_c$), the possibility to prevent vertical integration by not-differentiating products weakens the innovative firm’s incentive to invest relative to both the benchmark case and the case of small integration costs.

**Proposition 5** The innovative firm’s incentive to invest in R&D is unambiguously weakened by the possibility to prevent vertical integration by not-differentiating products which arises when the integration cost is high and the R&D effectiveness leads to very high degree of product differentiation.

**Proof.** Assume that $\hat{\gamma} < \gamma_c$. From equations (10c) and (10*), we get:

$$\hat{k}_c(\hat{\gamma}) - \hat{k}_a(\hat{\gamma}) = [\pi^I(\hat{\gamma}) - \pi^D(1)] - [\pi^D(\hat{\gamma}) - \pi^D(1)]$$

$$= \pi^I(\hat{\gamma}) - \pi^D(\hat{\gamma}) < 0 \quad \text{(by lemma 1),}$$

This proves that the incentive to invest is lower under in the case of high integration costs (where the R&D investment leads to vertical integration) than under the benchmark case of prohibitive integration cost (where vertical integration never occurs).

Similarly, from equations (10c) and (10a), we have:

$$\hat{k}_c(\hat{\gamma}) - \hat{k}_a(\hat{\gamma}) = [\pi^I(\hat{\gamma}) - \pi^D(1)] - [\pi^I(\hat{\gamma}) - \pi^D(1)] = -\pi^D(1) < 0,$$

This prove that the incentive to invest is lower in the case of high integration costs (where vertical integration can be avoided only by not-investing in R&D) than in the case of small integration costs (where vertical integration always occurs). ■

Figure 2c illustrates proposition 5.
To sum up, a prospective threat of vertical integration may have either positive or negative effects on the downstream firms’ incentive to differentiate products. The nature of the effects crucially depends on the effect of product differentiation on the upstream firm’s incentive to vertically integrate, as well as on the effectiveness of the R&D technology. When vertical integration is an unavoidable outcome because of small integration costs, product differentiation allows to soften the competitive pressure of the integrated firm in the product market. This has a greater value than softening the competitive pressure of a symmetric competitor (what motivates product differentiation in the benchmark case) only if products can be sufficiently differentiated. With higher integration costs, the incentive to differentiate products incorporates the strategic motive of preventing vertical integration. Both strong and weak degrees of product differentiation foster the upstream firm’s incentive to vertically integrate. With strongly differentiated products, the surplus from integration is high since double marginalisation is avoided in a wider market, whilst, with poorly differentiated products, the integrated firm can better exploit its efficiency advantage over the independent firm. Then, the downstream firms have a strategic interest in targeting intermediate degrees of product differentiation, to deter the upstream firm from vertically
integrating. Finally, very high costs of integration impede vertical integration unless products are strongly differentiated. This gives the downstream firms a strategic motive to avoid high degrees of differentiation.

The following two examples further clarify our results.

Example 1.—

Suppose that the R&D investment allows the innovative firm to obtain (exactly) the degree of product substitutability \( \hat{\gamma} = \gamma_{b_1} \) (see figure 2b). Given the "point-to-point" nature of the R&D technology, in equilibrium we will observe either no-differentiation (i.e. \( \gamma = 1 \)) if the innovative firm does not invest in R&D, or the degree of product differentiation \( 1 - \gamma_{b_1} \) (i.e. \( \gamma = \gamma_{b_1} \)) if the innovative firm invests. Assume that the R&D cost, \( k_1 \), is sufficiently high such that \( \hat{k}_a(\gamma_{b_1}) < k_1 < \hat{k}_b(\gamma_{b_1}) \). Then, inspection of figure 2b, immediately reveals that we will observe product differentiation in the downstream market only when moderately high costs of integration give the innovative firm a strategic incentive to invest in R&D in order to deter vertical integration.

\(^{22}\)In this case, the surplus from integration exceeds the integration cost only when the market size is wide because products are strongly differentiated.
Example 2.—

Suppose that the R&D investment allows the innovative firm to reduce the degree of product substitutability up to a minimum level $\hat{\gamma}_2$, with $\hat{\gamma}_2$ slightly lower than $\gamma_c$ (see figure 3). Hence, if the innovative firm decides to invest in R&D, it can select the optimal degree of product differentiation in the range $(0, 1 - \hat{\gamma}_2]$. Let the fixed R&D cost, $k_2$, be sufficiently low such that $k_2 < \hat{k}_c(\gamma_c)$. Clearly, the innovative firm will choose the degree of differentiation to maximize $\hat{k}(\hat{\gamma}) - k_2 = \pi(\hat{\gamma}) - \pi(1) - k_2$. Then, inspection of figure 3 reveals that, whilst products will be differentiated in all cases, the innovative firm will select the maximum degree of differentiation, $1 - \hat{\gamma}_2$, only in the cases of small and prohibitive integration costs. On the contrary, the incentive to avoid vertical integration will lead it to select lower degrees of differentiation in both cases of high and moderately high integration costs, i.e. $1 - \gamma_c$ and $1 - \gamma_{b_1}$, respectively.

![Figure 3. Optimal degrees of product differentiation (example 2) when integration costs are: small (a); moderately high (b); high (c); prohibitive (*).](image-url)
6. A new welfare loss from vertical integration

The previous results suggest that a threat of vertical integration faced by an innovative firm vertically related to a monopolistic supplier may decrease welfare by discouraging socially valuable innovations. The simplest way to show this is to reconsider the example 2 above, where the innovative firm can select the optimal degree of product differentiation up to a maximum level $1 - \gamma_2$. Suppose that the integration cost is high enough to make the innovative firm’s incentive to prevent vertical integration active (i.e. consider either case b) or case c) in figure 3). As we have seen before, the innovative firm will deter vertical integration by choosing a lower degree of product differentiation relative to the benchmark case where the threat of vertical integration is absent. If we re-interpret the benchmark as the case of a severe antitrust policy which bans vertical mergers, we can say that a lenient antitrust policy will cause a lower degree of product differentiation in the case under consideration, whilst the vertical structure of the market is identical in the two policy regimes (i.e. no-vertical integration). On the other hand, it is easy to prove that social welfare, measured by the total surplus generated in the market, is higher when products are more differentiated. Consider first industry profits. Profits in the downstream market increase with product differentiation (see lemma 1 (point iii)). Similarly, inspection of equation (4) suffices to see that also the upstream firm’s profit increases. Hence, industry profits are higher with more differentiation. Consider now the consumer surplus. As shown in Appendix 2, the consumer surplus can be expressed in terms of the equilibrium quantities as: $^{23}$

$$CS = \frac{1}{2} \left[ (q_1^D)^2 + (q_2^D)^2 + 2\gamma q_1^D q_2^D \right].$$

Since $q_1^D = q_2^D = a^{\frac{1}{2\gamma+2}}$ in the symmetric equilibrium without vertical integration, we get:

$$CS = (1 + \gamma) \left( \frac{a}{2 + \frac{1}{2 + \gamma}} \right)^2.$$

$^{23}$More precisely, the expression above gives the consumer surplus as a function of the consumer’s optimal demands of goods $q_1$ and $q_2$ at given prices.
Then, we evaluate:
\[
\frac{\partial CS}{\partial \gamma} = -\left(\frac{a}{2}\right)^2 \frac{\gamma}{(2 + \gamma)^2} < 0,
\]
that is, the consumer surplus increases with product differentiation. Intuitively, consumers’ preference for variety and the increase in equilibrium quantities compound to increase consumers’ welfare even if equilibrium prices increase. Therefore, in our example, a lenient antitrust policy would allow the threat of vertical integration to reduce product differentiation, causing a reduction in total surplus and welfare.

7. CONCLUSIONS

In this paper we have studied vertical integration and product innovation as interdependent strategic choices of vertically related firms. Our main innovation with respect to the previous literature on vertical integration is that we have considered product differentiation as a non-productive strategic decision of the downstream firms, showing its impact on the incentives for vertical integration and market foreclosure.\(^{24}\) Our main innovation relative to the literature on product innovation, is that, besides product market competition, we have accounted for another source of competition capable of affecting product innovation by innovative firms vertically related to a monopolistic supplier, i.e. the threat of vertical integration.\(^{25}\) Due to the downstream firms’ inability to commit to a cooperative behaviour if asked for integration offers, the monopolistic supplier can use vertical integration as a

\(^{24}\)Previous works have analysed the incentives to vertically integrate and foreclose the downstream market when the final products are differentiated (e.g. Ordover, Saloner and Salop (1990), Economides (1994), Colangelo (1995), Hackner (2001)). However, product differentiation is exogenous in all these studies.

\(^{25}\)The literature on product innovation has focused on the effects of product market competition and R&D competition on the incentive to innovate. For instance, Lamberti and Rossini (1998) and Lin and Saggi (2002) focus on R&D and product market competition in a setting similar to ours, where product innovation entails horizontal differentiation in a linear differentiated duopoly. A related literature analyses product differentiation in both the upstream and the downstream market under alternative vertical structures of the industry (e.g. Pepall and Norman (2001), Belleflamme and Toulemonde (2003), Matsushima (2004)). Also this literature essentially concentrates on the relationship between product differentiation and the intensity of product market competition. On the contrary, in this paper we have pointed out a different source of competition that can affect product innovation when the innovative firms depend on an upstream monopolist
means to reap profits in the downstream market. As a consequence, the incentive to differentiate products in the downstream market incorporates the strategic motive of preventing vertical integration. Our main finding has been that, although product differentiation allows to soften product market competition and to avoid market foreclosure under vertical integration, the strategic motive of preventing vertical integration may lead to less innovation in the downstream market. Indeed, the monopolist’s incentive to vertically integrate strengthens with both high and low degrees of product differentiation, so that the downstream firms may find it convenient to refrain from adopting (profit and welfare enhancing) innovations leading to strongly differentiated products. Therefore, instead of market foreclosure, we have shown a new possible welfare loss from vertical integration, i.e. less product innovation.

REFERENCES


APPENDIX

Appendix 1.—

We prove that the surplus form integration before the integration cost, $S$, is a U-shaped function of the degree of product differentiation over the range $\gamma \in [0, 1]$. From equation (9), we calculate:

$$\frac{\partial S(\gamma)}{\partial \gamma} = \left(\frac{a}{2}\right)^2 \frac{2 \left(-64 + 32\gamma + 96\gamma^2 + 40\gamma^3 - \gamma^4 - 3\gamma^5\right)}{(8 - 3\gamma^2)^2 (2 + \gamma)^3}.$$  

Since $(8 - 3\gamma^2)^2 (2 + \gamma)^3 > 0$ for $\gamma \in [0, 1]$, 

$$\text{sign} \left\{ \frac{\partial s(\gamma)}{\partial \gamma} \right\} = \text{sign} \left\{ -64 + 32\gamma + 96\gamma^2 + 40\gamma^3 - \gamma^4 - 3\gamma^5 \right\}.$$  

Using Mathematica, we find that the polynomial on the RHS has an unique real root within the admissible range $[0, 1]$, that is $\gamma_m \simeq 0.61037$. Since $\frac{\partial S(\gamma)}{\partial \gamma}$ is continuous over $[0, 1]$, and takes values $\frac{\partial S(\gamma)}{\partial \gamma} |_{\gamma=0} = -0.25 \left(\frac{a}{2}\right)^2 < 0$ and $\frac{\partial S(\gamma)}{\partial \gamma} |_{\gamma=1} = 0.2963 \left(\frac{a}{2}\right)^2 > 0$, then it must be negative for $\gamma < \gamma_m$ and positive for $\gamma > \gamma_m$. Finally, with simple calculations we get: $S(0) = \frac{1}{4} \left(\frac{a}{2}\right)^2$ and $S(1) = \frac{10}{45} \left(\frac{a}{2}\right)^2$, so that $S(0) > S(1)$.

Appendix 2.—

The representative consumer’s optimisation problem is:

$$\text{Max} \ U = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) + m$$

s.t.  

$$p_1 q_1 + p_2 q_2 + m = I$$

where $I$ is the consumer’s income in units of the numeraire good ($m$).

From the first order conditions $p_i = a - q_i - \gamma q_j$ ($i, j = 1, 2; i \neq j$) and the budget constraint, we get:

$$m = I - a(\widehat{q}_1 + \widehat{q}_2) + (\widehat{q}_1^2 + \widehat{q}_2^2 + 2\gamma \widehat{q}_1 \widehat{q}_2),$$

where $\widehat{q}_i$ denotes the consumer’s optimal demand of good $i$ at given prices.

Substituting for $m$ into the utility function, we get:

$$\widehat{U} = I + \frac{1}{2}(\widehat{q}_1^2 + \widehat{q}_2^2 + 2\gamma \widehat{q}_1 \widehat{q}_2).$$
Finally, the consumer surplus is:

\[ CS = \hat{U} - I = \frac{1}{2}(\tilde{q}_1^2 + \tilde{q}_2^2 + 2\gamma \tilde{q}_1 \tilde{q}_2). \]