Vertical integration and the licensing of innovation with fixed fees or royalties *

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Abstract

In this paper, we analyse a situation where a patent holder is considered as an upstream firm who can license its innovation to some downstream companies that compete on a final market with differentiated products. Licensing contract may be based either on royalty or fixed fee. The patent holder can either be independant or vertically integrated with one of the downstream companies. We show that license based on royalties works better with vertical integration, and that consequently, the patent holder may have some interest to vertically integrate if it enables him to apply royalty based license. The effect of vertical integration on the social surplus can be either positive or negative.

Keywords: Licensing, Innovation, Vertical Integration

JEL: L22, L42, O31, O32

The detailed proof of the results are provided here only with Cournot competition. The appendix with the detailed proof with Bertrand competition can be provided by the author upon request.

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Introduction

It is generally recognized that licensing can represent a large share of the firms’ profit, especially in the technology intensive sectors. This fact is associated with the development of markets for technologies [Arora et al., 2002], where the upstream actors develop some new technologies, and the downstream actors incorporate them into new products or processes in order to improve their competitive position on the final market. In such a setting, a patent holder may choose either to be only an upstream technology provider, or to vertically integrate some downstream activities. With no vertical integrate, the patent holder earn money only from licensing its innovation. With vertical integration, the patent holder can either license its innovation or foreclose its downstream competitor and earn revenue only from its downstream subsidiary. The aim of this paper is to analyse how does such a vertical integration affects the optimal licensing practices of a patent holder and, in turn, to analyse the interest of vertical integration both from the patent holder and the social perspective.

Most of the literature on the licensing of innovation analyses the interest of different payment structure (royalties, fixed fee, auction). Originally, a contradiction appear between the theoretical literature that show that fixed fee or auction are more interesting [Kamien and Tauman, 1986], and the empirical literature that show that royalty based licenses are rather frequent [Rostocker, 1983, Caves et al., 1983]. More recently, several theoretical contributions solved this contradiction by showing that royalty can be prefered to fixed fee with product differentiation [Muto, 1992], vertical integration [Wang, 1998, Wang and Yang, 1999, Wang, 2002, Kamien and Tauman, 2002, Sen and Tauman, 2003], uncertainty or information asymetry on the innovation quality [Bousquet et al., 1998, Gallini and Wright, 1990]. However none of these contribution analyse the effect of vertical integration on the optimal type of contract. On other words, these contributions identify conditions where royalties are prefered to fixed fee, but does analyse whether it occurs more or less likely with vertial integration (compared to the case with no vertical integration).

In this paper, the analysis is based on a simple model with no incertainty and no information asymetry. Competition on the final market occurs between two firms selling one differentiated product each (both Bertrand or Cournot competition are considered). One of these two firms is the subsidiary of the upstream patent holder in the case with vertical integration. Licensing contract may be based either on royalties or fixed fee. This model is similar to the one studied by Muto [1992] with no vertical integration and Wang and Yang [1999] and Wang [2002] with vertical integration.

The crucial result of this paper is that license based on royalties works better with vertical integration. More exactly, if the patent holder can only make royalty based
license, he always has better to vertically integrate one of the downstream firm. Also, if the patent holder can choose between fixed fee or royalty, he prefers royalty more often with vertical integration. This result is based on two mechanisms. First, vertical integration enables a better extraction of the rent let to the independant downstream licensee (with royalty license). More precisely, this licensee face a more competitive environment with vertical integration because its competitor as a preferential access to the innovation (since he is the subsidiary of the patent holder). Second, vertical integration enables to better preserve the industry profit when the royalty are high enough, simply because the double margin is less important (it concerns one non integrated firm instead of two). This second mechanism plays a role here because, with royalty based license, the patent holder has to define a rather high royalty level in order to moderate the profit let to the licensee. Once this result has been established, we can show that the patent holder has an interest for vertical integration when it leads him to choose a royalty based license. Such a choice can lead either to an increase or a decrease of the social surplus.

The analysis is restricted to simple structure of payment for the licensing contract. However, we know that the patent holder can increase its profit by combining fixed fee and royalty through a two-part tariff, instead of choosing only one of these two instruments. However, one can observe that when an innovation is introduced, there might be some lack of information, and complex instrument may be more difficult to implement. This problem can also be addressed by looking at the empirical litterature. Most of these contributions show that the use of royalty is frequent, but few of them gives result on the precise structure of payment (i.e. are royalties combined or not with some fixed fee?). Three articles have to be mentionned:

- Rostocker [1983] analyses the licensing practices of 37 US firms, and show that licensing contract based only on royalty or fixed occur in 52% of the cases, while two-part tariff occur in 39% of the cases.

- Macho-Stadler et al. [1996] analyse a sample of 241 licensing contracts between spanish firms (licensees) and foreign companies (licensor), for a large range of sector at the beginning of the 1990’s. In this sample, more than 80% of the contract are based only on royalty or fixed fee, while only 10% of the contract are based on two-part tariff.

- Jensen and Thursby [2001] study the licensing practices of 62 universities in the US between 1991 and 1995. Their survey shows that most of the university frequenlty combine two types of payment, and in particular fixed fee and royalties.
No clear conclusion can be derived from these empirical results. Hence, the analysis of contract with simple payment structure can hardly be excluded a priori\(^1\).

The paper is organised as follow. The model is presented in the section 1. As mentionned before, the main result of this paper is to show that license based on royalties works better with vertical integration. We first consider the case where the patent holder can only use royalty based license and show that he has then always an interest for vertical integration (section 2). However we know from the literature that fixed fee can be prefered to royalties. Hence, the analysis of the interest for vertical integration is extended to the case where the patent holder can apply fixed fee license only (section 3), or the best type of license between royalty and fixed fee (section 4).

1 The model

The model is first presented in a configuration with no vertical integration. The (minor) modifications made with vertical integration are presented after.

We consider a final market with two differentiated and competing products. Each product is produced and sold by a specific firm \((i = 1 \text{ or } i = 2)\). Two versions of each product can be proposed depending on whether or not it incorporates an innovation. In practice this innovation can be considered as an improvement of one characteristic that increases the interest of the product. It is supposed that each firm can produce and sell only one version of its product. The dummy variable \(\theta_i\) is used to indicate whether or not the product sold by the firm \(i\) incorporate \((\theta_i = 1)\) or not \((\theta_i = 0)\) the innovation. The property right of the innovation is owned by a third actor. This actor will be called either the patent holder or the upstream firm. In constrast, the two firms 1 and 2 will be called the downstream firms.

The inverse demand function on the final market is defined as follow: \(^2\)

\[
p_i = a + \theta_i \delta - bq_i - \lambda bq_j
\]

\(^1\)Note that analysis of the optimal licensing with two-part tariff have also been made with the same model than ours, either with no vertical integration [Erutku and Richelle, 2000] or with vertical integration [Faulí-Oller and Sandonís, 2003]. The interest of vertical integration with two part tariff has been studied recently [Sandonís and Faulí-Oller, 2003]. This last reference analyses the same problem than here, with equivalent model, but with more complex types of contract.

\(^2\)This demand function can be derived from the utility function of a representative consumer defined as follow:

\[
U(q_1, q_2) = (a + \theta_1 \delta)q_1 + (a + \theta_2 \delta)q_2 - \frac{b}{2}(q_1^2 + 2\lambda q_1 q_2 + q_2^2)
\]

See Singh and Vives [1984] for a detailed analysis of the duopoly equilibrium with such a demand function.
\(a\) is the highest propension to pay for the product when it does not incorporate the innovation \((a > 0)\), and \(\delta\) is the additional propension to pay for the product when it incorporates the innovation \((\delta > 0)\). \(b\) reflects the own price elasticity of the demand \((b > 0)\). \(\lambda\) reflects the degree of substituability between the product \((\lambda \in [0, 1])\). The case where \(\lambda = 0\) corresponds to a maximum differentiation of the product: the two products are sold on two different and independant final markets. Conversely, when \(\lambda = 1\) the two products are perfect substitutes.

The innovation patent-protected and can be incorporated in one product if its producer sign a license agreement with the patent holder. Two types of licensing agreement are considered in this paper: (i) a royalty based license where the licensee pays \(w\) to the patent holder for each unit he sells, (ii) a fixed fee based license, where the licensee pays \(F\) to the patent holder whatever the quantity he sells. With both types of agreement, the contracts are supposed to be public. The marginal production cost of the firm \(i\) is \(c + \theta_i w\). Note also that since the upsteam firm only sells a right to access to some intelectual property, its marginal cost is equal to zero.\(^4\)

The interaction between the actors are structured in three stages. First the patent holder decides the type of licensing contract and the corresponding variable \((w\) with royalties, \(F\) with fixed fee). The same contract is proposed to the two downstream firms. Second, each of the downstream firm decides whether it accepts or refuses the license contract (decision variable \(\theta_i\)). Third, competition occurs on the final market with either Cournot or Bertrand competition.

An alternative configuration will be considered, where the patent holder and one of the two downstream firms are vertically integrated. The subscript \(v\) is then used to refer to the vertically integrated firm and the subscript \(s\) is used to refer to the independant downstream firm. The vertically integrated company gives a free access to the innovation to its subsidiary. The three stages presented before are still valid with vertical integration. The only difference is that, at the stage 2, we are concerned with the decision of only one downstream firm (the firm \(s\)), instead of two with no vertical integration.

The choice of whether or not to integrate vertically is done before the three basic stages described before. We suppose here that there is some interest for vertical integration if the joint profit of the patent holder and one licensee is greater when they are both part of the same firm \(v\). Note that, with such an hypothesis, there is no cost associated with vertical integration.

\(^3\)This parameter has a minor influence on the results, and is often supposed to be equal to 1. We keep it here because it does not lead to more complex results.

\(^4\)The cost of the research that leads to the patent is supposed to be already spend, since we are not considering the research stage in this model.
The resolution is made by backward induction. The three basic stages will be solved first and the choice concerning vertical integration will be analysis after.

Note also that the stage 1 resolution indirectly leads the patent holder to choose a certain level of access restriction to its innovation. Access restriction is defined as follow: there is no access restriction when the innovation is incorporated in the two products, a partial access restriction when the innovation is incorporated in only one product, and a complete access restriction when no product incorporate the innovation. By construction, access restriction can only be partial with vertical integration because the downstream subsidiary of \( v \) has free access to the innovation.

2 Analysis with royalty based licenses

The resolution will be presented in details with Cournot competition (section 2.1 to 2.3. The (small) differences that appears with Bertrand will be presented briefly after (section 2.4)

2.1 Stages 3 and 2 subgames

The stages 3 and 2 subgames determine the behavior of the firms on the final market with given licensing contract (i.e. given value of \( w \)).

2.1.1 Stage 3

The stage 3 equilibrium depends on the vertical structure, the incorporation of the innovation in the product (value of \( \theta_i \) or \( \theta_s \)) and the production cost.

Two types of equilibrium are possible at the stage 3: a duopoly equilibrium where the two competitors have positive sales and profit, and a monopoly equilibrium where only one competitor have positive sales and profit. The monopoly equilibrium can appear if only one firm incorporates the innovation and if the innovation is important enough (table 1). \( \delta_M \) is the minimum value of \( \delta \) over which the innovation is drastic\(^5\) \( (\delta_M = (2 - \lambda)(a - c)/\lambda) \). A drastic innovation is a necessary (but not sufficient) condition for having a monopoly equilibrium. \( \delta_M \) is decreasing with \( \lambda \), which means that the condition for having a drastic innovation is less restrictive when the products are close substitutes.

The detailed characteristics of the stage 3 equilibrium are given in the appendix A. With no vertical integration, the gross profit of the firm \( i \) at the stage 3 is \( \pi_i(w, \theta_i, \theta_j) \) when its competitor \( j \) chooses \( \theta_j \) at the stage 2. With vertical integration, the gross

\(^5\)Remind that, following Arrow (1962), an innovation is drastic if the competitor of the innovator cannot make positive profit when the innovator apply the monopoly price.
Table 1: Condition for having the different types of stage 3 equilibrium

<table>
<thead>
<tr>
<th></th>
<th>Duopoly</th>
<th>Monopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td>With no vertical</td>
<td>( \theta_i = \theta_j ) or</td>
<td>( \theta_i \neq \theta_j ) and ( \delta &lt; \delta_M + w )</td>
</tr>
<tr>
<td>integration</td>
<td>( \theta_i \neq \theta_j ) and ( \delta &lt; \delta_M + w )</td>
<td></td>
</tr>
<tr>
<td>With vertical</td>
<td>( \theta_s = 1 ) or</td>
<td>( \theta_s = 0 ) and ( \delta \geq \delta_M )</td>
</tr>
<tr>
<td>integration</td>
<td>( \theta_s = 1 ) and ( \delta &lt; \delta_M )</td>
<td></td>
</tr>
</tbody>
</table>

profit of the firm \( s \) (resp. \( v \)) is \( \pi_s(w, \theta_s) \) (resp. \( \pi_v(w, \theta_s) \)). The notation \( p \) and \( q \) (instead of \( \pi \)) is used to refer to the prices and quantities at the stage 3 equilibrium.

At last, note that the stage 3 equilibrium would be identical with a model where the innovation enables to decrease the production cost from \( c \) to \( c - \delta \), but does not affect the product characteristics. This alternative modelisation is the one used by most of the contribution on licensing [c.f. for example Wang, 2002]. This equivalence for the stage 3 implies that all the other results concerning the previous stages will be also equivalent.

2.1.2 Stage 2

With no vertical integration, and a given choice of the firm \( j \), the firm \( i \) accepts the license its profit increase. Formally, we have:

\[
\pi_i(w, 1, \theta_j) \geq \pi_i(w, 0, \theta_j) \quad \Leftrightarrow \quad w \leq \hat{w}_{NI} \quad \text{with} \quad \hat{w}_{NI} = \delta
\]

Two equilibrium are possible: if \( w \leq \hat{w}_{NI} \) the two firms accept the license because the best response of each firm is to accept the license whatether the choice of its competitor; and conversely, if \( w > \hat{w}_{NI} \) the two firms reject the license because the best response of each firm is to reject the license whatether the choice of its competitor.

Partial access restriction can never appear at the equilibrium with royalties and no vertical integration.

With vertical integration and a non drastic innovation, the firm \( s \) accepts the license if:

\[
\pi_s(w, 1) \geq \pi_s(w, 0) \quad \Leftrightarrow \quad w \leq \hat{w}_I \quad \text{with} \quad \hat{w}_I = \delta
\]

With a drastic innovation the firm \( s \) accepts the license because it earns no profit otherwise.

2.2 Stage 1 subgame

The stage 1, the patent holder choose the royalty level that maximises its profit. From now on, we define the patent holder profit as the difference between the industry profit and the innovation cost.
profit and the downstream and nonintegrated firm. Such a definition enables to better understand the decisions of the patent holder by analysing the effect on these two components.

The industry profit respectively with no vertical integration and with vertical integration is defined as follow:

\[
\Pi_{NI}(w, \theta_1, \theta_2) = w(\theta_1 \cdot q_1(w, \theta_1, \theta_2) + \theta_2 \cdot q_2(w, \theta_2, \theta_1)) + \pi_1(w, \theta_1, \theta_2) + \pi_2(w, \theta_2, \theta_1)
\]

\[
\Pi_I(w, \theta_s) = \pi_v(w, \theta_s) + \pi_s(w, \theta_s)
\]

The patent holder profit is \(\Pi_{NI}(w, \theta_1, \theta_2) - 2\pi_i(w, \theta_1, \theta_2)\) with no vertical integration, and \(\Pi_I(w, \theta_s) - \pi_s(w, \theta_s)\) with vertical integration.

The stage 1 resolution is made through two steps. First, we search for the optimal royalty level with no access restriction and, second, we analyse the interest for access restriction. In between, we will analyse the effect of the royalty level on the industry profit.

### 2.2.1 Stage 1 with no access restriction

Solving the stage 1 with no access restriction corresponds to the search for a local optimal royalty level, such that \(w < \tilde{w}_{NI}\) with no vertical integration and such that \(w < \hat{w}_I\) with vertical integration.

With no vertical integration, we have:

\[
\arg\max_w [\Pi_{NI}(w, 1, 1) - 2\pi_i(w, 1, 1)] = \tilde{w}_{NI} \quad \text{with} \quad \tilde{w}_{NI} = \frac{a - c + \delta}{2}
\]

This royalty level is lower than \(\hat{w}_{NI}\) if \(\delta\) is high enough. Consequently, with no access restriction, the optimal royalty level is:

\[
w^*_{NI} = \begin{cases} 
\tilde{w}_{NI} & \text{if } \delta < \delta_{NI} \text{ with } \delta_{NI} = a - c \\
\hat{w}_{NI} & \text{otherwise}
\end{cases}
\]

(2)

Thereafter, we will say that the optimal royalty level is constrained if \(w^*_{NI} = \hat{w}_{NI}\) and unconstrained if \(w^*_{NI} = \tilde{w}_{NI}\).

The similar resolution can be made with vertical integration. The patent holder chooses an optimal unconstrained royalty level \(w^*_I = \tilde{w}_I\) if \(\delta\) is high enough \((\delta < \delta_I)\), and a constrained level \(w^*_I = \hat{w}_I\) otherwise:

\[
\tilde{w}_I = \frac{8 - 4\lambda^2 + \lambda^3}{8 - 3\lambda^2} \cdot \frac{a - c + \delta}{2} \quad \text{and} \quad \delta_I = \frac{8 - 4\lambda^2 + \lambda^3}{8 - 2\lambda^2 - \lambda^3} \cdot (a - c)
\]

(3)
2.2.2 Industry profit with no access restriction

We first discuss the effect of the royalty level on the industry profit, and then analyse the industry profit when the patent holder choose the optimal unconstrained royalty level.

Lemma 1 With no access restriction the industry profit is a concave function of $w$. Moreover, the industry profit is greater with vertical integration if the royalty level is high enough:

$$\Pi_I(w, 1) > \Pi_{NI}(w, 1, 1) \iff w > w_{NI/I}$$

A qualitative explanation of this result is provided here (see appendix B for a detailed proof). The industry profit is affected by the royalty level through two opposite effects:
A competition relaxing effect. An increase of $w$ leads to an increase of the production cost of the licensees, and to an increase of their prices on the final market. When $w$ is small enough, the prices on the final market are lower than the one that would be practiced by a fully integrated company (i.e. the price that would maximize the industry profit). When $w$ is small, an increase of its value enables the licensees to have prices level closer to the one practiced by the fully integrated company.

A double margin effect [Spengler, 1950]. The royalty causes a negative vertical externality between the licensee and the patent holder. More precisely, the decision taken by the licensee is based on a higher production cost compared to the one of a vertically integrated company ($c + w$ instead of $c$). As a consequence, the licensee’s strategy leads to too low quantities on the final market.

These two effects explain why the industry profit is a concave function of $w$ (figure 1). When $w$ is low enough, the double margin effect is very small, and $w$ is increasing in $w$ because of the competition relaxing effect. Conversely, when the royalty level is very high the double margin effect becomes dominant, and the industry profit is decreasing in $w$. In passing, note that the maximum industry level ($\Pi_{FI}$) can be reached with no vertical integration and an intermediary royalty level

These two effects are more important with no vertical integration because the royalty affects the production cost of two licensees instead of one. When $w = 0$, the industry profit is equal with or without vertical integration because there is no competition relaxing effect and no double margin effect (the formal proof will be given latter with the lemma 6). If $w$ increases from 0, the industry profit increases more rapidly with no vertical integration, because the competition relaxing effect is more important in this case. In other words, there is always a small enough value of $w$ such that the industry profit is greater with no vertical integration. Conversely, when $w$ is very high, the industry profit is higher with vertical integration because the double margin effect is less important in this case. By the continuity of the profit function, we can finally conclude that there is an intermediary level of $w$ such that the industry profit is equal with or without vertical integration.

Lemma 2 The optimal unconstrained royalty level ($\tilde{w}_I$ or $\tilde{w}_{NI}$) is greater than the royalty level that maximises the industry profit.

---

6 $\Pi_{FI}$ is the profit when the patent holder vertically integrates the two downstream companies.

7 Rigourously, one should observe that this result holds only when $\lambda > 0$. Conversely, when $\lambda = 0$, the industry faces two independent downstream markets, each downstream firm being a monopoly on each downstream market. There is no competition relaxing effect, and consequently, the industry profit is always greater with vertical integration for any value of $w$. 

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The main interest of this lemma is to give a simple explanation of the determination of the optimal unconstrained royalty level. The royalty level is a tradeoff between two effects: on the one hand, the patent holder uses the royalty as a way to increase the industry profit through the competition relaxing effect described just before; on the other hand, increasing the royalty level lead to an increase of the licensees’ production cost and, hence, to a decrease of their profit.

Formally, with vertical integration, when \( w = \hat{w}_I \), we have:

\[
\frac{\partial (\Pi_I(w, 1) - \pi_s(w, 1))}{\partial w} = 0 \iff \frac{\partial \Pi_I(w, 1)}{\partial w} = \frac{\partial \pi_s(w, 1)}{\partial w}
\]

Since the profit of the licensee is decreasing with the royalty level, the industry profit is necessarily decreasing when \( w = \hat{w}_I \). In other words, the optimal unconstrained royalty level is such that the marginal gain from reducing the profit of the licensee is equal to the marginal lost of industry profit. Because of the concave form of the profit function, when the industry profit is decreasing, the royalty level is greater than the royalty level that maximises the industry profit. The equivalent argument can be made with no vertical integration.

### 2.2.3 Interest for access restriction

The objective here is to search for the optimal global royalty level at the stage 1. With vertical integration, we compare the profit with \( w_I^* \) (no access restriction) and the profit with \( w > \hat{w}_I \) (partial access restriction). With no vertical integration, we compare the profit with \( w_{NI}^* \) (no access restriction) and the profit with \( w > \hat{w}_{NI} \) (complete access restriction).

**Lemma 3** With royalty based license, there is no interest for access restriction at the equilibrium, with either vertical or no vertical integration.

With vertical integration, this result has already been established in the literature [Wang, 2002, Wang and Yang, 1999] with process innovation. This result is also valid here because the characteristics of the stage 3 equilibrium are similar. Compared to these contributions, some additional comment are made here by analysing the effect of access restriction on the industry and licensee profit.

There is no interest for access restriction when the following condition is checked:

\[
\Pi_I(w_I^*, 1) - \pi_s(w_I^*, 1) > \Pi_I(0, 0) - \pi_s(0, 0)
\]

\( \iff \Pi_I(w_I^*, 1) - \Pi_I(0, 0) > \pi_s(w_I^*, 1) - \pi_s(0, 0) \)

The term on the right hand side of the second inequality is positive (otherwise the firm \( s \) would refuse the licence agreement at the stage 2): access restriction enables to decrease the profit of the independant firm \( s \). However, access restriction leads also to a decrease of the industry profit for two reasons:
• First, the innovation is distributed less widely, and consequently less profit is earned from it. This effect is more important when the product are weak substitute ($\lambda$ close to 0). In such a case the market share of the product sold by $s$ can still be large with access restriction.

• Second, foreclosure prevents the patent from using the competition relaxing effect of the royalty described before (cf. lemma 1). Consequently, access restriction leads to a more intense competition on the final market, and lower industry profit. This effect is more important when the products are close substitutes ($\lambda$ close to 1).

Finally, foreclosure is never interesting with royalties because it leads to a more important decrease of the industry profit, compared to the decrease of the profit let to the firm $s$.

With no vertical integration, the interest for no access restriction is straightforward: partial access restriction cannot appear at the stage 2 equilibrium, and the patent holder earns no profit with complete access restriction. One could wonder however if partial access restriction could not be interesting with a modified version of the model, where partial access restriction can appear at the equilibrium. We can suppose for example that the patent holder can credibly commit to sign a limited number of license agreements at the beginning of the stage 1, before choosing the level of the royalty. Even with this modified version of the model, it can be shown that the lemma 3 still holds.

2.3 The effects of vertical integration

The objective of this section is to show that royalty based license works better with vertical integration, so that there is always an interest for vertical integration. Note first that the comparison needs to be made with different royalty level, since vertical integration affects the optimal royalty level. More precisely, with Cournot competition on the final market, the optimal royalty level with vertical integration is always lower or equal to the level with no vertical integration but the difference between them is minor ($w_{NI}^*/w_I^* \in [0.97, 1]$).

Lemma 4 With no access restriction and royalty based license, vertical integration leads to a decrease the licensee profit:

$$\pi_s(w_I^*, 1) < \pi_i(w_{NI}^*, 1, 1)$$

The intuition behind this result is simple (see appendix C for a detailed proof). Since the two products sold on the final market incorporate the innovation, the difference of the licensee profit is caused by the difference of production cost. With vertical
integration the licensee (firm $s$) faces a competitor with a lower production cost than with no vertical integration, because the competitor has a free access to the innovation in the former case, while he pays some royalties in the last case. Rigourously, one have to observe also that the production cost of the licensee is lower with vertical integration, because of the lower royalty level. However this last effect is negligible compared to the first one.

**Proposition 1** With royalty based licensing contract only, the patent holder has always an interest for vertical integration.

Vertical integration leads to an increase of the social surplus (Cournot specific).

The patent holder has an interest for vertical integration if:

\[
\Pi_I(w_I^*, 1) - \pi_s(w_I^*, 1) > \Pi_{NI}(w_{NI}^*, 1, 1) - 2\pi_i(w_{NI}^*, 1, 1) + \pi_i(w_{NI}^*, 1, 1)
\]

\[
\iff \Pi_I(w_I^*, 1) - \Pi_{NI}(w_{NI}^*, 1, 1) > \pi_s(w_I^*, 1) - \pi_i(w_{NI}^*, 1, 1)
\]

We have just shown (lemma 4), that the term on the right hand side of the second inequality is negative: vertical integration leads to a decrease of the licensee profit because he faces a competitor with lower cost.

Two cases need to be considered depending on the size of the innovation (see appendix D for a detailed proof):

- With large enough innovations ($\delta \geq \delta_{NI}$), the royalty level is unconstrained. We have seen (lemma 2) that the unconstrained optimal royalty level is then high in order to moderate the profit let to the licensee(s). We have seen also (lemma 1) that the industry profit is greater with vertical integration if the royalty levels is high enough. Here, with Cournot competition, we can observed that the optimal unconstrained optimal royalty level is higher than $w_{NI}/I$. Hence, there is an interest for vertical integration because it enables both to increase the industry profit and to decrease the profit let to the licensee.

- With small innovations ($\delta < \delta_{NI}$), the royalty level is constrained at least with no vertical integration. With very small innovation, the industry profit can be lower with vertical innovation, but because of the constraint on the royalty level, the difference of profit let to the licensee is then more important (in relative terms), so that there is still an interest for vertical integration.

Vertical integration leads to an increase of the social surplus because the prices on the final market are lower with vertical integration. Two results explain this property. First, with a given royalty level, the prices are lower with vertical integration ($p_v(w, 1) < p_s(w, 1) < p_i(w, 1, 1)$) because the competition is then more intense (remind that $v$ has free access to the innovation). Secondly, the optimal royalty level is never greater with vertical integration ($w_I^* \leq w_{NI}^*$). Note that this last result is specific to the Cournot competition.
2.4 Analysis with Bertrand competition on the final market

With Bertrand competition, three types of equilibrium appear at the stage 3: a duopoly and a monopoly equilibrium like with Cournot competition, and, inbetween, a constrained monopoly. In the constrained monopoly, only one firm has positive sales, but it is constrained to apply a limit price lower than the monopoly price in order to keep the competitor out of the market.\(^8\)

All the lemma and proposition established with Cournot competition are still valid with Bertrand competition, except one: vertical integration can lead to a surplus decrease when the product are close substitute and the innovation is minor. This result comes from the fact that the maximum royalty level defined from the stage 2 resolution can be much higher with vertical integration (\(\hat{w}_I > \hat{w}_{NI}\)) especially when the product are close substitutes. As a consequence, when the optimal royalty level with no vertical integration is constrained (i.e. when \(\delta\) is small), the optimal royalty can be much higher with vertical integration. Such a high royalty level can leads to higher prices on the final market, and to a surplus decrease.

3 Analysis with fixed fee licenses

The stage 3 resolution made with royalty is valid with fixed fee: the gross profit with fixed fee is equal to the gross profit with royalty when \(w = 0\). All the analysis made in this section is valid both with Cournot and Bertrand competition on the final market.

3.1 Stages 2 and 1 resolution

As in the case with royalties, we first search for the local optimal fixed fee level with a given level of access restriction. We then analyse the interest for access restriction, which coresponds to a search for the global optimal fixed fee.

3.1.1 Fixed fee with given access restriction

At the stage 2, the decision of one potential licensee is based on its net profit, defined as the difference between the gross profit and the fee \(F\).

- With no vertical integration, the downstream firm \(i\) accepts the license in the following cases:
  - If its competitor \(j\) also accepts the license, then we need to have:
    \[
    \pi_i(0, 1, 1) - F \geq \pi_i(0, 0, 1) \quad \iff \quad F \leq F_2 \quad \text{with:} \quad F_2 = \pi_i(0, 1, 1) - \pi_i(0, 0, 1)
    \]

\(^8\)More details on this equilibrium with limit pricing can be found in Muto [1992]
• If \( j \) reject the license, then \( i \) accepts the license if \( F \leq F_1 \) with \( F_1 = \pi_i(0, 1, 0) - \pi_i(0, 0, 0) \)

After observing that \( F_2 \) is always lower than \( F_1 \) (cf. appendix E), we can conclude that three equilibrium are possible at the stage 2:

• If \( F < F_2 < F_1 \), then the best response of \( i \) is to accept the license whatever the choice of its competitor. At the equilibrium both downstream firms accept the license (no access restriction). The net profit of the licensee is then \( \pi_i(0, 0, 1) \).

• If \( F_2 < F < F_1 \), then the best response of \( i \) is to accept the license if its competitor rejects it, and to reject the license if its competitor accepts it. At the equilibrium, only one of the downstream firm accept the license (partial access restriction). Two equilibrium are possible \( ((\theta_1, \theta_2) \in \{(1, 0), (0, 1)\}) \) but they both lead to the same results because the two downstream firms are symetric. The net profit of the licensee is then \( \pi_i(0, 0, 0) \), and the profit of the excluded downstream firm is \( \pi_i(0, 0, 0) \).

• If \( F_2 < F_1 < F \), then the best response of the firm \( i \) is to reject the license whatever the choice of its competitor. At the equilibrium both downstream firms reject the license (complete access restriction).

- With vertical integration, the firm \( s \) accept the license if:

\[
\pi_i(0, 1, 1) - F \geq \pi_i(0, 0, 1) \quad \Leftrightarrow \quad F \leq F_I \quad \text{with:} \quad F_I = \pi_s(0, 1) - \pi_s(0, 0)
\]

The net profit of the licensee is \( \pi_s(0, 0) \) whatever its choice at the period 2. Note that we always have \( F_I = F_2 \). This result will be established in the proof of the lemma 6.

- Synthesis. At the stage 1, the patent holder choose \( F = F_2 = F_I \) if he decides not to restrict the access (with both vertical structure). If he decides to have a partial access restriction, the patent holder choose \( F = F_1 \) with no vertical integration and \( F > F_I \) with vertical integration. The table 2 synthetises the net profit of the firms with the different restriction level and vertical structure. As we did in the previous section with royalty license, the patent holder profit is expressed as the difference between the industry profit and the net profit of the downstream independant firms.

Compared to the royalty license, a new equilibrium with partial access restriction can appear with no vertical integration. This equilibrium exists because there is a range of value for the fee \( (F_2 < F \leq F_1) \) where the best response of each downstream firm is to make the converse choice compared to the competitor.
Table 2: Net profit of the firms with fixed fee based licenses

<table>
<thead>
<tr>
<th></th>
<th>With no vertical integration</th>
<th>With vertical integration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No access restriction</strong> ($F = F_2$ or $F = F_I$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Patent holder</td>
<td>$\Pi_{NI}(0, 1, 1) - 2\pi_i(0, 0, 1)$</td>
<td>$\Pi_I(0, 1) - \pi_s(0, 0)$</td>
</tr>
<tr>
<td>Each licensee</td>
<td>$\pi_i(0, 0, 1)$</td>
<td>$\pi_s(0, 0)$</td>
</tr>
<tr>
<td><strong>Partial access restriction</strong> ($F = F_1$ or $F &gt; F_I$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Patent holder</td>
<td>$\Pi_{NI}(0, 0, 1) - \pi_i(0, 0, 0) - \pi_i(0, 0, 1)$</td>
<td>$\Pi_I(0, 0) - \pi_s(0, 0)$</td>
</tr>
<tr>
<td>Licensee</td>
<td>$\pi_i(0, 0, 0)$</td>
<td></td>
</tr>
<tr>
<td>Excluded firm</td>
<td>$\pi_i(0, 0, 1)$</td>
<td>$\pi_s(0, 0)$</td>
</tr>
</tbody>
</table>

3.1.2 Interest for access restriction

**Lemma 5** With fixed fee license, there is an interest for (partial) access restriction if the innovation is important enough and if the products are close enough substitutes.

The figure 2 illustrates this result. This result has already been established in the literature in the case with vertical integration [Wang, 2002], and can be extended to the case with no vertical integration.

Note first that complete access restriction is excluded from the analysis either because it leads to no profit for the patent holder (with no vertical integration) or because it is not possible by construction (with vertical integration).

With vertical integration, the patent prefers to restrict the access if it leads to an increase of the industry profit:

$$\Pi_I(0, 0) - \pi_s(0, 0) > \Pi_I(0, 1) - \pi_s(0, 0) \iff \Pi_I(0, 0) > \Pi_I(0, 1)$$

With no vertical integration, the condition is different but the increase of the industry profit is still a necessary condition.

Access restriction affects the industry profit through two opposite effects:

- On the one hand, access restriction enables to increase the market power of the firm that distributes exclusively the innovation. Formally, we have $p_v(0, 0) > p_v(0, 1)$ with vertical integration and $\pi_i(0, 1, 0) > \pi_i(0, 1, 1)$ with no vertical integration. The variation of market power caused by access restriction is more important when the product are close substitute because the industry profit with no access restriction is then lower. Moreover, the market power provided by the access restriction is magnified when the innovation is important enough.

- On the other hand, some demand are harder to be satisfied with access restriction. In other terms, the quantity of innovation sold is more important when
Figure 2: Interest of partial access restriction with fixed fee

Cournot competition

Bertrand competition

Access restriction is preferred only with vertical integration

Access restriction is preferred both with and without vertical integration
the innovation is incorporated in two products (non-restricted access) instead of one (partial access restriction). This negative effect is less important when the product are close substitute and when the innovation is important enough.

Finally, with close enough substitute and important enough innovation, the positive effect of access restriction on the industry profit increases while the negative effect decreases.

3.2 Effects of vertical integration

In the figure 2, we can observe that the conditions that lead to access restriction are different depending on whether or not there is vertical integration. This result will now be explained, and we will see that it determines the condition for having an interest for vertical integration.

3.2.1 Effect of vertical integration on the industry profit

Lemma 6 With fixed fee and a given restriction of the access to the innovation, the profit of the industry is identical with or without vertical integration:

\[ \Pi_I(0, \theta) = \Pi_{NI}(0, 1, \theta) = \Pi_{NI}(0, \theta, 1) \]

\[ \theta = 0 \text{ with partial access restriction, and } \theta = 1 \text{ with no access restriction.} \]

The intuition behind this lemma is simple. With a given access restriction, the level of the fixed fee does not affect the production cost of the two downstream companies, and consequently does not affect the equilibrium on the final market. Fixed fee affects only the transfer among firms but not the industry profit.

The profit of \( v \) is equal to the gross profit of one licensee with no vertical integration. Similarly, the gross profit of \( s \) is equal to the gross profit of one downstream firm with no vertical integration, knowing that its competitor is a licensee. Formally we have:

\[ \pi_v(0, \theta) = \pi_i(0, 1, \theta) \quad \text{and} \quad \pi_s(0, \theta) = \pi_i(0, \theta, 1) \]  

(4)

In passing note that one consequence of this result is that \( F_2 = F_I \).

The industry profit are also identical. With no access restriction (\( \theta = 1 \)) we have:

\[ \Pi_{NI}(0, \theta, 1) = \pi_i(0, \theta, 1) + \pi_i(0, 1, \theta) = \pi_s(0, \theta) + \pi_v(0, \theta) = \Pi_I(0, \theta) \]
3.2.2 Effect of vertical integration on the interest for access restriction

**Proposition 2** With fixed fee, if the patent holder prefers to restrict (partially) the access with no vertical integration, then he prefers also to restrict the access with vertical integration. Equivalently, if the patent holder prefers not to restrict the access with vertical integration, then he prefers also not to restrict the access with no vertical integration.

With vertical integration, we have seen before (lemma 5) that the patent holder prefers to restrict the access if and only if it leads to an increase of the industry profit. With no vertical integration, we have:

\[ \Pi_{NI}(0,1) - \pi_i(0,0) - \pi_i(0,1) > \Pi_{NI}(0,1,1) - 2\pi_i(0,0,1) \]
\[ \Leftrightarrow \Pi_{NI}(0,1,0) - \Pi_{NI}(0,1,1) > \pi_i(0,0,0) - \pi_i(0,0,1) \]

The term on the right hand side of the second inequality is positive because the net profit of the licensee is greater with partial access restriction than with no access restriction (\( \pi_i(0,0,0) > \pi_i(0,0,1) \)). Consequently the industry profit needs to be positive and significant in order to have an interest for partial access restriction. Having an increase of the industry profit is a necessary but not sufficient condition in order to prefer access restriction.

By the lemma 6 we know that the variation of the industry profit caused by the partial access restriction is the same with or without vertical integration (\( \Pi_I(0) - \Pi_I(0,1) = \Pi_{NI}(0,1,0) - \Pi_{NI}(0,1,1) \)). As a consequence, as soon as access restriction increases the patent holder profit with no vertical integration, it necessarily means that industry profit increases which is equivalent to say that access restriction increases the patent holder profit with vertical integration. However the converse property is not always true: if partial access restriction increases industry profit, but if this variation is lower than \( \pi_i(0,0,0) - \pi_i(0,0,1) \), the patent holder prefers the partial access restriction with vertical integration and no access restriction with no vertical integration.

**Corollary 1** With fixed fee, there is a set of parameters value for \( \lambda \) and \( \delta \) such that the patent holder has an interest for a partial access restriction with vertical integration but no interest for access restriction with no vertical integration.

The figure 2 gives an illustration if this corollary.
3.2.3 Interest of the patent holder for vertical integration

**Proposition 3** With fixed fee based licensing contract only, there is an interest for vertical integration only if it enables the patent holder to move from a strategy with no access restriction (with no vertical integration) to a strategy with access restriction (with vertical integration).

*Vertical integration leads to a decrease of the social surplus.*

In the zones where the access restriction is the same with the two vertical structure, the joint profit of the patent holder and one licensee is not affected by vertical integration. This result is based on the fact that the equilibrium on the final market are identical in the two cases. With no access restriction in both cases, we have:

\[
\begin{align*}
[\Pi_{NI}(0, 1, 1) - 2\pi_i(0, 0, 1)] + \pi_i(0, 0, 1) &= \Pi_I(0, 1) - \pi_s(0, 0)
\end{align*}
\]

With partial access restriction in both cases, we have:

\[
\begin{align*}
[\Pi_{NI}(0, 1, 0) - \pi_i(0, 0, 1) - \pi_i(0, 0, 0)] + \pi_i(0, 0, 0) &= \Pi_I(0, 0) - \pi_s(0, 0)
\end{align*}
\]

In the zone where there is access restriction with vertical integration and no access restriction with no vertical integration, the profit of the patent holder is greater in the former case. When access restriction is prefered with vertical integration, we have:

\[
\Pi_I(0, 0) - \pi_s(0, 0) > \Pi_I(0, 1) - \pi_s(0, 0)
\]

Using the lemma 6, this expression is equivalent to:

\[
\Pi_I(0, 0) - \pi_s(0, 0) > [\Pi_{NI}(0, 1, 1) - 2\pi_i(0, 0, 1)] + \pi_i(0, 0, 1)
\]

The term on the right hand side is the joint profit of the patent holder and on licensee with no vertical integration and no access restriction. Consequently, vertical integration leads to an increase of the joint profit of the patent holder and one licensee.

With fixed fee, the surplus is maximum with no access restriction because the innovation is then integrated in both products without affecting the marginal cost. When the patent holder prefers to vertically integrate if it enables him to restrict the access. The surplus is then not maximum, while it would be maximum with no vertical integration. Hence, vertical integration leads to a decrease of the social surplus.

---

\(^{9}\text{Remind that there is an interest for vertical integration if it increase the joint profit of the patent holder and one licensee. On the left hand side, the first term in square brackets is the patent holder profit and the second term is the profit of the downstream firm that can be integrated.}\)
4 Analysis with royalties or fixed fee licenses

We consider here that the patent holder choose first whether of not to integrate vertically and then the best type of license between royalties and fixed fee (thereafter, the three basic stages of the model take place). Consequentlty, the best type of license is analyzed first, before the analysis of the interest for vertical integration. All the analysis presented here is valid both with Cournot and Bertrand competition on the final market.

4.1 Interest for fixed fee vs royalties

Lemma 7 With no access restriction, the net profit of the licensees is lower with fixed fee compared to royalties, either with or without vertical integration:

\[ | \pi_s(0, 0) \leq \pi_s(w_I^*, 1) \]
\[ | \pi_i(0, 0, 1) \leq \pi_i(w_{NI}^*, 1, 1) \]

With royalties, the net profit of licensee is equal to the gross profit. With fixed fee, the net profit is defined in the table 2.

With vertical integration and royalties, from the stage 2 resolution we defined the maximum royalty level (\( \hat{w}_I \)) such that \( \pi_s(\hat{w}_I, 1) = \pi_s(0, 0) \). Moreover, knowing that \( w_I^* \leq \hat{w}_I \) and that the profit of the licensee is decreasing in \( w \), we have:

\[ \pi_s(w_I^*, 1) \geq \pi_s(\hat{w}_I, 1) = \pi_s(0, 0) \]

With no vertical integration, we can observe that the net profit of the licensee is lower compared to the case where no one accept the license (\( \pi_i(0, 0, 1) \leq \pi_i(0, 0, 0) \)). We have then:

\[ \pi_i(w_{NI}^*, 1, 1) \geq \pi_i(\hat{w}_{NI}, 1, 1) = \pi_i(0, 0, 0) \geq \pi_i(0, 0, 1) \]

Proposition 4 If the patent holder prefers to apply fixed fee rather than royalties with vertical integration, then he prefers also to apply fixed fee rather than royalties with no vertical integration. Equivalently, if the patent holder prefers to apply royalties rather than fixed fee with no vertical integration, then he prefers also to apply royalties rather than fixed fee with vertical integration.

This property is the indirect consequence of the fact that royalty license works better with vertical integration, while the profit with fixed fee (and no access restriction) are equivalent with or without vertical integration. Interestingly, the proof is based only on the previous lemma and propositions.
We can first observe that this proposition only concerns the cases where there is no access restriction. With fixed fee and vertical integration, access restriction can appear at the stage 1 equilibrium. However, in such cases, the royalties are preferred to the fixed fee because there is never an interest for access restriction with royalties (lemma 3)\(^{10}\). Finally, since the proposition 4 concerns only the cases where fixed fee is preferred to royalties with vertical integration, it will necessarily correspond to cases where there is no access restriction.

We now make the comparisons with no access restriction. We consider the cases where fixed fee is preferred to royalties with vertical integration:

$$\Pi_I(0, 1) - \pi_s(0, 0) > \Pi_I(w_I^*, 1) - \pi_s(w_I^*, 1)$$

Using the proposition 1, we know that the profit with royalties and vertical integration (right hand side) is greater to the joint profit of the patent holder and one licensee with royalty and no vertical integration. Consequently, have:

$$\Pi_I(0, 1) - \pi_s(0, 0) > \Pi_{NI}(w_{NI}^*, 1, 1) - \pi_i(w_{NI}^*, 1, 1)$$

Using the lemma 6, we can replace the terms on the left hand side by their equivalence with no vertical integration. We have then:

$$\Pi_{NI}(0, 1, 1) - \pi_i(0, 0, 1) > \Pi_{NI}(w_{NI}^*, 1, 1) - \pi_i(w_{NI}^*, 1, 1)$$

At last we subtract \(\pi_i(0, 0, 1)\) on both sides and we use the lemma 7, to get:

$$\Pi_{NI}(0, 1, 1) - 2\pi_i(0, 0, 1) > \Pi_{NI}(w_{NI}^*, 1, 1) - \pi_i(w_{NI}^*, 1, 1) - \pi_i(0, 0, 1)$$

This last inequality indicates that royalties are preferred to fixed fee with no vertical integration.

**Corollary 2** There is a set of parameters value for \(\lambda\) and \(\delta\) such that the patent holder applies royalty licensing contract rather than fixed licensing contract with vertical integration, and the converse (fixed fee rather than royalty) without vertical integration.

The figure 3 illustrates this corollary.

\(^{10}\)More precisely, we can observe that, with vertical integration, there is no difference between fixed fee and royalty when there is partial access restriction, since there is no licensing contract. The patent holder earns \(\Pi_I(0, 0) - \pi_s(0, 0)\) in both case. The lemma 3 is equivalent to say that, with vertical integration, the patent holder always prefer to apply royalty (and no access restriction) rather than fixed fee and partial access restriction.
Figure 3: Interest of royalties compared to fixed fee

Cournot competition

Bertrand competition

Royalties is preferred to fixed fee with vertical integration

Royalties is preferred to fixed fee both with and without vertical integration
4.2 Interest for vertical integration

**Proposition 5** If the patent holder can use either fixed fee or royalty based licensing contract, there is a net interest for vertical integration if and only if it leads the patent holder to choose royalty based licensing contract.

The proof of this proposition is based only on the previous lemma and propositions.

We first consider the cases where fixed fee is preferred to royalty with vertical integration. From the lemma 3, we know that there is no access restriction (with vertical integration). From the proposition 4, we know that this case needs only to be compared to a case where fixed fee is preferred to royalty with no vertical integration. From the proposition 2 we know that there is no access restriction with no vertical integration (since there is no access restriction with vertical integration). In summary, the case where fixed fee is preferred to royalties with vertical integration needs only to be compared to the case where fixed fee is preferred to royalties with vertical integration. There is no access restriction in both cases, and from the proposition 3, we can conclude that there is no interest for vertical integration.

We now consider the alternative case where royalty is preferred to fixed fee with vertical integration. Two sub-cases need then to be considered:

- If royalty is also preferred to fixed fee with no vertical integration, the we know from the proposition 4 that there is an interest for vertical integration.

- If fixed fee is preferred to royalty with no vertical integration, then we can also show that there is an interest for vertical integration. This result comes from the fact the profit are equivalent with fixed fee with or without vertical integration, because of the lemma 6.

Suppose first that there is no interest for access restriction with no vertical integration. we have:

\[
\Pi_I(w^*_I, 1) - \pi_s(w^*_I, 1) \geq \Pi_I(0, 1) - \pi_s(0, 0)
\]

\[
\iff \Pi_I(w^*_I, 1) - \pi_s(w^*_I, 1) \geq [\Pi_{NI}(0, 1, 1) - 2\pi_i(0, 0, 1)] + \pi_i(0, 0, 1)
\]

The term on the right hand side of the last inequality is the joint profit of the patent holder (between squared brackets) and the licensee. This joint profit is lower than the profit with vertical integration and royalties.

Similarly, if there is an interest for access restriction with no vertical integration, we have:

\[
\Pi_I(w^*_I, 1) - \pi_s(w^*_I, 1) \geq \Pi_I(0, 0) - \pi_s(0, 0)
\]

\[
\iff \Pi_I(w^*_I, 1) - \pi_s(w^*_I, 1) \geq [\Pi_{NI}(0, 1, 0) - \pi_i(0, 0, 0) - \pi_i(0, 0, 1)] + \pi_i(0, 0, 0)
\]

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The effect of vertical integration on social surplus can be either positive or negative, but it is not possible to define simple rules that can help to distinguish between the different cases. In particular, results are sensitive to the hypothesis on the type of competition (Cournot vs Bertrand).

References


Appendix (Cournot competition)

A  Prices and quantities at the stage 3 equilibrium

- With no vertical integration. At the duopoly equilibrium, the quantities, prices and profit are:

\[
q_i(w, \theta_i, \theta_j) = \frac{(2-\lambda)(a-c) + (2\theta_i - \lambda \theta_j)(\delta - w)}{b(4-\lambda^2)}
\]

\[
p_i(w, \theta_i, \theta_j) = c + \theta_j w + \frac{(2-\lambda)(a-c) + (2\theta_i - \lambda \theta_j)(\delta - w)}{4-\lambda^2}
\]

\[
\pi_i(w, \theta_i, \theta_j) = \frac{((2-\lambda)(a-c) + (2\theta_i - \lambda \theta_j)(\delta - w))^2}{b(4-\lambda^2)^2}
\]

We can check that if i does not incorporate the innovation but its competitor j does it ($\theta_i = 0$ et $\theta_j = 1$), then, at the equilibrium, i supply a positive quantity with a positive margin if $\delta < \delta_M + w$ with $\delta_M = (2-\lambda)(a - c)/\lambda$.

If $\theta_i = \theta_j = 1$, the quantities are positive if $w < a - c + \delta$. This condition will always be fulfilled at the equilibrium because, otherwise, the patent holder who defines $w$, would make no profit.

At the monopoly equilibrium, the quantities and prices are:

\[
q_i(w, 1, 0) = \frac{a - c + \delta - w}{2b} \quad q_i(w, 0, 1) = 0
\]

\[
p_i(w, 1, 0) = c + w + \frac{a - c + \delta - w}{2} \quad p_i(w, 0, 1) = c
\]

\[
\pi_i(w, 1, 0) = \frac{(a - c + \delta - w)^2}{4b} \quad \pi_i(w, 0, 1) = 0
\]

This case appear only if $\delta \geq \delta_M + w$. The quantity is positive if $w < a - c + \delta$, This condition will always be fulfilled at the equilibrium because, otherwise, the patent holder who defines $w$, would make no profit.

- With vertical integration With a duopoly equilibrium, the quantities, prices and profit of the firm $s$ are:

\[
q_s(w, \theta_s) = \frac{(2-\lambda)(a-c) - \lambda \delta + 2\theta_s(\delta - w)}{b(4-\lambda^2)}
\]

\[
p_s(w, \theta_s) = c + \theta_s w + \frac{(2-\lambda)(a-c) - \lambda \delta + 2\theta_s(\delta - w)}{4-\lambda^2}
\]

\[
\pi_s(w, \theta_s) = \frac{((2-\lambda)(a-c) - \lambda \delta + 2\theta_s(\delta - w))^2}{b(4-\lambda^2)^2}
\]

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If $\theta_s = 0$, $s$ sells positive quantities with a positive markup if $\delta < \delta_M$ (otherwise there is a monopoly equilibrium). If $\theta_s = 1$, $s$ sells positive quantities with a positive markup if $w < (a - c + \delta)(2 - \lambda)/2$. This condition is always be fulfilled at the equilibrium.

The quantities, prices and profit of the firm $v$ are:

$$q_v(w, \theta_s) = \frac{(2 - \lambda)(a - c) + 2\delta - \lambda \theta_s(\delta - w)}{b(4 - \lambda^2)}$$

$$p_v(w, \theta_s) = c + \frac{(2 - \lambda)(a - c) + 2\delta - \lambda \theta_s(\delta - w)}{4 - \lambda^2}$$

$$\pi_v(w, \theta_s) = q_v(w, \theta_s)(p_v(w, \theta_s) - c) + q_s(w, \theta_s) \cdot w$$

There is no simple expression of the profit of the firm $v$.

With a monopoly equilibrium, quantity, price and profit are:

$$q_v(0, 0) = \frac{a - c + \delta}{2b}$$

$$q_s(0, 0) = 0$$

$$p_v(0, 0) = c + \frac{a - c + \delta}{2}$$

$$p_s(0, 0) = c$$

$$\pi_v(0, 0) = \frac{(a - c + \delta)^2}{4b}$$

$$\pi_s(0, 0) = 0$$

This equilibrium appears if $\theta_s = 0$ and $\delta \geq \delta_M$.

**B Analysis of industry profit with royalties and no access restriction**

We have only to consider duopoly equilibrium because there is no access restriction. We present here the simplest proof through analytical resolution. A more instructive (but longer) proof can be provided by the author.

Since prices and quantities are linear function of $w$, the profit of the patent holder and the licensees is a quadratic function of $w$, and consequently the industry profit is also a quadratic function of $w$.

With no vertical integration, the industry profit is:

$$\Pi_{NI}(w, 1, 1) = \frac{2}{b(2 + \lambda)^2} \left((a - c + \delta)^2 + (a - c + \delta)\lambda w - (1 + \lambda)w^2\right)$$

With vertical integration, the industry profit is:

$$\Pi_{I}(w, 1) = \frac{2}{b(2 + \lambda)^2} \left((a - c + \delta)^2 + \frac{1}{2}(a - c + \delta)\lambda w - \frac{4 - 3\lambda^2}{2(2 - \lambda^2)}w^2\right)$$
The square terms in the expressions of $\Pi_{NI}(w, 1, 1)$ and $\Pi_I(w, 1)$ are negative, and consequently, the industry profit is concave in $w$.

The industry profit is greater with vertical integration when the following condition is checked:

$$\Pi_I(w, 1) > \Pi_{NI}(w, 1, 1)$$

$$\Leftrightarrow \frac{1}{2}(a - c + \delta)\lambda w - \frac{4 - 3\lambda^2}{2(2 - \lambda)^2}w^2 > (a - c + \delta)\lambda w - (1 + \lambda)w^2$$

$$\Leftrightarrow w > w_{NI/I} \quad \text{with:} \quad w_{NI/I} = \frac{(a - c + \delta)(2 - \lambda)^2}{4 - 3\lambda^2 + 2\lambda^3}$$

C  Effect of vertical integration on the licensee’s profit with royalties

Three cases need to be distinguished. For convenience, we will consider the highest value of $\delta$ first:

- If $\delta > \delta_{NI}$, then $w^*_I = \hat{w}_I$ and $w^*_NI = \hat{w}_{NI}$. After compilation we have:

$$\pi_i(\hat{w}_{NI}, 1, 1) - \pi_s(\hat{w}_I, 1) = \frac{\lambda(4 + \lambda)(16 - 4\lambda + 7\lambda^2)}{4b(2 + \lambda)(8 - 3\lambda^2)^2} \cdot (a - c + \delta)^2 > 0$$

- If $\delta_I < \delta < \delta_{NI}$, then $w^*_I = \hat{w}_I$ and $w^*_NI = \hat{w}_{NI}$. Since the profit of the licensee is decreasing in $w$ and since $\hat{w}_{NI} < \hat{w}_{NI}$, we have $\pi_i(\hat{w}_{NI}, 1, 1) > \pi_i(\hat{w}_{NI}, 1, 1)$. Moreover we showed that $\pi_i(\hat{w}_{NI}, 1, 1) > \pi_s(\hat{w}_I, 1)$ in the previous paragraph. Consequently, we have $\pi_i(\hat{w}_{NI}, 1, 1) > \pi_s(\hat{w}_I, 1)$.

- If $\delta < \delta_I$, then $w^*_I = \hat{w}_I$ and $w^*_NI = \hat{w}_{NI}$. After compilation we have:

$$\pi_i(\hat{w}_{NI}, 1, 1) - \pi_s(\hat{w}_I, 1) = \frac{\delta \lambda(2(2 - \lambda)(a - c) - \lambda \delta)}{b(4 - \lambda^2)}$$

This difference is always positive when $\delta < \delta_I$.

D  Interest of the patent holder for vertical integration with royalties

With royalty license, we have seen that the patent holder have no interest for partial or complete access restriction. Three cases need to be considered depending on whether the royalty level is constrained or not with or without vertical integration.
- If $\delta < \delta_i < \delta_{NI}$, then the royalty level is constrained both with and without vertical integration. The gain from vertical integration positive:

\[
(\Pi_I(\hat{w}_I, 1) - \pi_s(\hat{w}_I, 1)) - (\Pi_{NI}(\hat{w}_{NI}, 1, 1) - \pi_i(\hat{w}_{NI}, 1, 1)) = \frac{\delta(\lambda^2(2 - \lambda)(a - c) + \delta(4 - 4\lambda + \lambda^3))}{b(4 - \lambda^2)^2} > 0
\]

- If $\delta_i < \delta < \delta_{NI}$, then the royalty level is constrained with no vertical integration and unconstrained with vertical integration. In such a case, we can see that there is always an interest for vertical integration because we have the following inequalities:

\[
\Pi_I(\hat{w}_I, 1) - \pi_s(\hat{w}_I, 1) > \Pi_I(\tilde{w}_I, 1) - \pi_s(\tilde{w}_I, 1) > \Pi_{NI}(\hat{w}_{NI}, 1, 1) - \pi_i(\hat{w}_{NI}, 1, 1)
\]

The first part of the inequality comes from the fact that the profit of $v$ is greater when the optimal royalty level is unconstrained rather than constrained. The second part of the inequality was established in the previous paragraph for any value of $\delta$.

- If $\delta_i < \delta_{NI} < \delta$, there is an interest for vertical integration because it leads to an increase of the industry profit and a decrease of the profit let to the licensee. The decrease of the licensee profit comes from the lemma 4.

The increase of the industry profit can be shown has follow:

\[
\Pi_{NI}(\tilde{w}_{NI}, 1, 1) < \Pi_{NI}(\hat{w}_I, 1, 1) < \Pi_I(\hat{w}_I, 1)
\]

- The first inequality is comes from the fact $\tilde{w}_I < \tilde{w}_{NI}$ and that the industry profit is decreasing when the optimal royalty level is unconstrained (lemma 2).

- The second inequality comes from the fact that the unconstrained royalty with vertical is greater than $w_{NI/I}$:

\[
\tilde{w}_I - w_{NI/I} = \frac{(2 - \lambda)(16 - 24\lambda + 14\lambda^3 + \lambda^4 - 2\lambda^5)}{2(8 - 3\lambda^2)(4 - 3\lambda^2 + 2\lambda^3)} \cdot (a - c + \delta) > 0
\]

Remind that if $w > w_{NI/I}$ we have $\Pi_{NI}(w, 1, 1) < \Pi_I(w, 1)$ (lemma 1).

E  Definition and properties of the fixed fee with no vertical integration

Since $w = 0$, the monopoly equilibrium appears if only one firm incorporates the innovation ($\theta_i \neq \theta_j$) and if the innovation is drastic ($\delta \geq \delta_M$).
- **Definition of** $F_2$. Remind that $F_2 = \pi_i(0, 1, 1) - \pi_i(0, 0, 1)$ and the net profit of the licensee is $\pi_i(0, 0, 1)$. With a non-drastic innovation, the patent holder cannot extract all the profit of the licensees ($\pi_i(0, 0, 1)$ is positive). Conversely, with a drastic innovation, the patent holder extracts all the profit of the licensees ($\pi_i(0, 0, 1) = 0$). The detailed expression of $F_2$ is:

$$F_2 = \begin{cases} 
\frac{4\delta((2 - \lambda)(a - c) + \delta(1 - \lambda))}{b(4 - \lambda^2)^2} & \text{if } \delta < \delta_M \\
\frac{(a - c + \delta)^2}{b(2 + \lambda)^2} & \text{otherwise}
\end{cases} \quad (5)$$

- **Definition of** $F_1$. Remind that $F_1 = \pi_i(0, 1, 0) - \pi_i(0, 0, 0)$ and the net profit of the licensee is $\pi_i(0, 0, 0)$. Whatever the type of innovation, the patent holder cannot extract all the profit of the licensees. ($\pi_i(0, 0, 0) > 0$). $\pi_i(0, 1, 0)$ correspond to the duopoly profit if $\delta$ is small enough ($\delta < \delta_M$), and to the monopoly profit otherwise. The detailed expression of $F_1$ is:

$$F_1 = \begin{cases} 
\frac{4\delta((2 - \lambda)(a - c) + \delta)}{b(4 - \lambda^2)^2} & \text{if } \delta < \delta_M \\
\frac{(\lambda(a - c) + (2 + \lambda)\delta)((4 + \lambda)(a - c) + (2 + \lambda)\delta)}{4b(2 + \lambda)^2} & \text{otherwise}
\end{cases} \quad (6)$$

- **Comparison of** $F_2$ and $F_1$. With a non-drastic ($\delta < \delta_M$) innovation we have:

$$F_1 - F_2 = \frac{4\lambda\delta^2}{b(4 - \lambda^2)^2} > 0$$

With a drastic innovation, we have:

$$F_1 - F_2 = \frac{\lambda(4 + \lambda)}{4(2 + \lambda)^2} \delta^2 + \frac{\lambda(4 + \lambda)}{2(2 + \lambda)^2} \cdot (a - c)\delta - \frac{4 - 4\lambda - \lambda^2}{4(2 + \lambda)^2} \cdot (a - c)^2$$

This difference is convex in $\delta$. The lowest root is negative. The difference is positive if $\delta$ is greater than the highest root:

$$F_1 - F_2 > 0 \iff \frac{\delta}{a - c} > -1 + \frac{2}{\lambda \sqrt{4 + \lambda}}$$

This condition is always fulfilled when the innovation is drastic.