Tacit collusion with price matching punishments

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Abstract

We re-examine tacit collusion under a simple punishment rule in which firms match any lower price by their rivals, but otherwise leave their prices unchanged. We provide conditions under which this simple rule sustains collusion and is credible. Provided competition is imperfect, collusion can always be sustained, but never to the point of monopoly. Interestingly, the standard ambiguous relationship between product substitutability and tacit collusion is unambiguous under this new setting. The relationship to existing theories of price matching guarantees, kinked demand curves, and continuous reaction-function equilibria are explained.

Keywords: Tacit collusion, price matching, product differentiation.

JEL : L11, L12, L13, L41

1 Introduction

Ivadi et al. (2003) and Motta (2004, Chapter 4) survey some of the voluminous literature that explores the factors for, and implications of, tacit collusion. Following the formal modeling of tacit collusion by Friedman (1971), the vast majority of this literature is based on firms playing a punishment strategy involving reversion to the one-shot Nash equilibrium following any firm defecting from the collusive agreement (so-called Nash reversion). Based on Abreu (1986, 1988), another literature has focused on more sophisticated optimal punishment strategies, which enable collusion to be sustained under tighter conditions. In both approaches, the punishment chosen is divorced from the crime. A small cut in prices results in the same severe punishment as does a large cut in prices.

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In contrast, Chamberlin, in his seminal article on tacit collusion, seemed to have in mind a punishment strategy in which the severity of the punishment is directly linked to the degree of the crime — firms match price cuts. Chamberlin (1929) wrote “... if either one were to cut below it, he would, by the incursions made upon his rival’s sales, force him at once to follow suit” (p.85) and “If numbers are fairly small, any one seller can be certain that his incursions upon the others by a price cut will be large enough to cause them to follow suit; and therefore no one will cut” (p.89). This paper provides a formal theory of Chamberlin’s idea of a price matching punishment strategy. In our analysis, following the discovery of a lower price by a rival, firms punish the defecting firm in subsequent periods by matching its lower price. Otherwise, they leave their prices unchanged. The question we ask is whether tacit collusion can be sustained under such a price matching punishment strategy, and if so, under what conditions does the strategy define an equilibrium?

Our interest in price matching punishment strategies is partly motivated by anecdotal evidence which is suggestive of such behavior. Scherer (1980) reports, based on interviews with business executives and the testimony recorded in numerous antitrust investigations, that a paramount consideration deterring price cutting was “the belief that cuts will be matched” (p.167). Slade (1990, p.532), in her study of data from various price wars episodes, also reports that for the Vancouver retail gasoline market “the data reveal a high degree of (lagged) price matching during the period of the war.” Levinshtein (1997) studies evidence from the the bromine industry, noting that collusion between Dow Chemical and Bromkonvention involved an explicit reference by each of the firms to a matching punishment rule. Genesove and Mullin (2001) discuss collusion in the U.S. sugar industry, finding considerable evidence that price cuts were subsequently matched.

We model such punishments in an infinitely repeated game setting, keeping as close as possible to the standard framework. An important feature of the price matching punishment strategy is that the punishment a firm faces from defecting is endogenous. An implication of this feature is that under homogenous price competition, a firm can obtain the maximum one-period profit from deviating, while facing an arbitrarily small punishment, by lowering its price by an infinitesimal amount when defecting. As a result, collusion is only sustainable in this case when firms do not discount future profits. In the more realistic setting in which a firm cannot obtain the whole market by lowering its price by an infinitesimal price for a single period, collusive outcomes will be attainable with positive discounting. We model such situations by assuming limited product substitutability.
Tacit collusion will require firms to be more patient to sustain a given collusive outcome under price matching compared to traditional Nash reversion. This reflects the fact that under price matching, a defecting firm can always set the same price as it would in the standard analysis, and face a smaller punishment given rivals simply match its price rather than further undercut it. In addition, the defecting firm can do even better by restricting the amount it deviates from the original agreement, thereby further reducing the severity of the punishment. As a result, firms will be more tempted to defect from any collusive agreement. In particular, the monopoly price is never sustainable when firms discount future profits. Instead, the maximum sustainable collusive price is determined by a simple fixed point condition, which has the property that it is the unique price for which collusion is sustainable as a Nash equilibrium in the case firms match both lower and higher prices.

The simple punishment rule we consider is credible, in that each firm will wish to follow the punishment in the case a firm defects, given its rivals also follow the same rule. In particular, we show that this follows for any collusive outcome that is sustainable under the rule. This reflects the fact that the punishment phase resembles the collusive phase, albeit at a lower collusive price. Given that it is easier to sustain collusion at a lower price, each firm will be willing to stick to the punishment phase if they are willing to stick to the original agreement.

Having established conditions for tacit collusion to emerge, we explore whether price matching punishment strategies change any of the existing results on factors that facilitate tacit collusion. We focus on the relationship between product substitutability and tacit collusion. An extensive literature has studied this question using the standard Nash reversion trigger strategies, finding an ambiguous relationship between the degree of single-period competition and cartel stability when firms compete in prices. Greater product substitutability increases both the gain to cheating on a collusive agreement and the magnitude of the punishments that follow a defection, meaning the total effect is ambiguous. In contrast, in our setting greater product substitutability allows any given gain from cheating to be realized with a smaller reduction in prices, which induces a smaller punishment for the defecting firm. Thus, in contrast to

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1 More generally, the strategies we characterize will define a subgame perfect equilibrium, although this requires some modification of the price matching punishment rule for “extreme” prices which never arise along the equilibrium path.

2 Chang (1991) and Ross (1992) find that increased product differentiation enhances cartel stability in the Hotelling model of spatial competition, while Deneckere (1983), Ross (1992), and Albaek and Lambertini (1998) demonstrate that the critical discount factor to sustain collusion is non-monotone in the degree of product substitutability, using linear demand exogenous differentiation models.
the previous literature, we find that collusion always becomes harder to sustain as firms become less differentiated. In the extreme case with identical firms and products, collusion is not possible at all. This demonstrates that the choice between price matching punishments and Nash reversion (or some other punishment strategy) is not purely one of modeling convenience.

Our work also relates to several other literatures in industrial organization. Price matching as a punishment strategy in tacit collusion is distinct from firms offering price matching guarantees, whereby if a customer receives a better price offer from another seller, the current seller will match that price. These are sometimes also called “meeting-competition” clauses. Unlike our approach, such clauses assume binding commitments on the part of firms. They also only apply to those consumers who make use of them. Kalai and Satterhwaite (1994) and Salop (1986) study how these clauses alter the nature of competition, making firms softer price competitors. On the other hand, Png and Hirshleifer (1987) show that these may work instead as price discrimination devices, since only those willing to spend the time to compare prices will get the lowest price. In either case, the adoption of a price matching policy alters the nature of one-shot price competition, whereas our focus is on what happens in repeated interactions for a given (standard) one-shot game. The two approaches coincide if price matching guarantees imply firms match prices for all consumers, and when in our framework, detection is instantaneous. Then, no firm has an incentive to defect for any collusive price.

Price matching punishments also relate to the old theory of kinked demand in oligopoly, in which rivals match any price decrease but do not match a price increase (Hall and Hitch, 1939, and Sweezy, 1939). The kinked demand theory tries to explain why prices sometimes remain constant under oligopoly. It is argued that firms believe that if they increase prices, no one will follow, while if they lower prices, everyone else will do the same. Our model provides some game theory underpinnings for the kinked demand theory. In our setting, single period demand need not be kinked. Initially, any firm that raises its price loses demand, while any firm which lowers its price gains demand. However, when taking into account demand over more than one period, the fact that a price increase will not be matched, while a price decrease will be matched in the subsequent periods, implies a kink in the demand schedule when measured over multiple periods. Thus, long-run or multi-period demand curves are indeed kinked.

Finally, our analysis relates to a literature on tacit collusion in which firms play continuous reaction functions each period. Continuous reaction functions are distinct from discontinuous strategies, such as Nash reversion, by the fact that small deviations lead to small punishments and large deviations lead to
large punishments. Stanford (1986) considers such strategies in which each firm’s choice of quantity is a function of only the other firm’s choice of quantity in the previous period. He shows that while such strategies can characterize Nash equilibria, the resulting equilibria cannot be subgame perfect. However, Kalai and Stanford (1985) demonstrate that continuous reaction function strategies can be $\varepsilon$-subgame perfect if the reaction time is allowed to become sufficiently small. Slade (1989) finds the same result in a price setting duopoly. Friedman and Samuelson (1990, 1994) restore full subgame perfection under continuous reaction functions by allowing these functions to depend on both firm’s actions in the previous period. These are the same properties characterizing our equilibrium strategies. However, Friedman and Samuelson’s interest is in proving a folk theorem, so that the strategies implicit in their analysis are complex, and do not resemble price matching punishments or price matching more generally. In contrast to these papers, our approach is to see whether the story implicit in the original account of tacit collusion (of matching price cuts but not price increases) allows collusive outcomes to be sustained. Thus, we analyze the properties of a simple, realistic, and arguably focal punishment rule, which involves different strategies from any considered in the literature to date.

The rest of the paper proceeds as follows. Section 2 outlines our basic model of tacit collusion with price matching punishments, and presents some general results on when collusion can be sustained, and the credibility of the price matching punishments. Section 3 re-examines the relationship between product differentiation and tacit collusion in this new framework. Section 4 illustrates (and complements) the general analysis with a specific model of imperfect competition. Finally, Section 5 briefly concludes.

2 A model of tacit collusion

The framework we use to analyze tacit collusion is standard, other than the punishment rule adopted. Consider symmetric duopolists that compete each period in prices for an infinite number of periods. We note the straightforward extension to $n$ firms in the conclusion. Firm $i$ sets a price $p_i$ and faces demand $q_i(p_i, p_j)$ in each period. Single period profits are denoted $\pi_i(p_i, p_j)$, where $\pi_i(p_i, p_j) = p_i q_i(p_i, p_j) - C(q_i)$, and $C$ represents some unspecified cost function. Firms discount the future at the constant discount factor $\delta$. Given symmetry, we denote each firm’s profit in the case in which both firms set the same price $p$ as $\pi(p)$, so $\pi(p) = \pi_i(p, p)$. The common initial price is denoted $p^c$, with the superscript $c$ indicating it is the collusive price. In each subsequent period, either firm can decide to change its price,
We consider the simple trigger strategy in which, if a firm learns that its rival set a lower price than it did in the previous period, it matches that price. Otherwise, the firm keeps its price unchanged. If prices in the previous period are \( p_i \) and \( p_j \), then according to this rule, both firms will set prices in the current period to \( \min (p_i, p_j) \). This price matching punishment strategy is repeated in every period. Initially, we just consider whether it is optimal for each firm to follow this rule given the other firm does. We will find that the rule defines a Nash equilibrium in the supergame for a range of possible (initial) collusive prices \( p^c \). Later we will examine whether these equilibria satisfy subgame perfection.

Consider first the special case with homogenous firms and constant marginal costs (which are normalized to zero), so that a firm gets the entire market demand \( Q(p) \) if it undercuts its rival’s price by the slightest amount. In this setting, if a firm raises its price above \( p^c \), it will get no demand and will obtain zero profits. If both firms stick to the collusive price \( p^c \), they each get

\[
\frac{p^c Q(p^c)}{2 (1 - \delta)},
\]

while if a firm defects, setting a price of \( p^d = p^c - \varepsilon \), it will get profits of

\[
(p^c - \varepsilon) Q(p^c - \varepsilon) + \frac{\delta (p^c - \varepsilon) Q(p^c - \varepsilon)}{2 (1 - \delta)}.
\]

Since \( \varepsilon \) can be chosen to be arbitrarily small, profits in (1) are always less than in (2), unless \( \delta = 1 \). With homogenous Bertrand competition, each firm can make the punishment arbitrarily small by undercutting by a sufficiently small amount. Only if the firms are completely patient, will each be deterred from defecting. Then an infinite stream of small punishments is sufficient to offset the one-off finite gain from defecting.

In more realistic models, where a firm cannot obtain the entire market demand by lowering its price by an infinitesimal amount for a single period, collusive outcomes will be attainable with positive discounting. We model such situations by considering the possibility of limited product substitutability.

Assume price competition in the stage game is well-behaved. Specifically, sufficient (although not necessary) conditions for our results are that in the stage game, the symmetric firms’ profit functions are smooth, best response functions (in prices) are upward sloping (prices are strategic complements), and the best response mapping is a contraction\(^3\). More specifically, we assume for all relevant prices

\[
\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} > 0,
\]

\(^3\)See Section 2.5 in Vives (2000).
and
\[
\frac{\partial^2 \pi_i}{\partial p_i^2} < -\left| \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \right|.
\] (4)

These assumptions ensure each firm’s profit function is concave, and guarantee the existence and uniqueness of the one-shot Nash equilibrium in prices.\footnote{We are implicitly assuming that we can restrict attention to a compact set of prices, which will be true provided profits decrease in a firm’s own price beyond some finite price — say because demand vanishes when prices are set too high. The same is assumed with respect to the collusive profit function \( \pi (p) \).} Typical best response functions are illustrated in figure 1, which identifies several prices of interest. We also assume \( \pi (p) \) is smooth and concave in \( p \), so that
\[
\frac{d^2 \pi}{dp^2} < 0.
\] (5)

The maximum collusive price that we consider is the monopoly price \( p^m \), which is the unique price that maximizes \( \pi (p) \).

**FIGURE 1 ABOUT HERE**

Let \( p^n \) denote the one-shot Nash equilibrium price and \( p^r (p) \) denote the one-shot best response to some common price \( p \). The price \( p^r (p^c) \) is the price a defecting firm sets in the standard approach in which punishments are not tied to the extent of undercutting. We can assume that \( p^c > p^n \), otherwise collusion is not interesting. The contraction property of best responses implies \( p^r (p^c) < p^c \). Starting with both firms setting the collusive price \( p^c \), if a firm sets a higher price \( p \) in any period, given this price will not be matched when the rival plays its price matching strategy, this deviation can never be profitable. Moving further away from one’s best response when the rival does not is never profitable given profit functions are concave. Moreover, given best response functions are upward sloping, we know \( p^r (p^c) > p^r (p^n) \) so that \( p^r (p^c) > p^n \). Thus, as figure 1 illustrates, we have
\[
p^n < p^r (p^c) < p^c \leq p^m.
\] (6)

Now consider a firm deciding on the optimal deviation price under price matching punishments. We restrict attention to a one-off deviation initially, where the firm lowers its price to some level and then keeps its price at this lower level thereafter. We handle more complicated deviations in the proof of Proposition 1, where we show that this restriction to just one-off deviations is sufficient to define the set of Nash equilibria. (Subgame perfection will be considered in Proposition 4.) Define the (one-off)
defection price \( p^d \leq p^c \) to be the price \( p \) that maximizes
\[
\pi_i(p, p^c) + \frac{\delta}{1 - \delta} \pi(p),
\]
which is the present discounted value of deviation profits. This price is the best price a defecting firm can set, if in all subsequent periods it will have this lower price matched by the rival. Facing price matching punishments, firms will not want to deviate by as much as in the standard case, since the punishment they will face is based on the price they deviate to.

**Lemma 1** The optimal deviation price \( p^d \) under price matching punishments exceeds the deviation price \( p^r \) under Nash reversion. That is, \( p^n < p^r(p^c) < p^d(p^c) \leq p^c \leq p^m \).

**Proof.** The first term in (7) is maximized by \( p^r(p^c) \). A small increase in price from \( p^r(p^c) \) therefore has no first order impact on \( \pi_i(p, p^c) \), but implies a first order increase in collusive profits \( \pi(p) \), given that \( p^r(p^c) < p^c \leq p^m \) from (6) and that \( \pi(p) \) is concave in \( p \). The optimal deviation price must therefore exceed \( p^r(p^c) \).

Since the collusive price is also one feasible price for a firm deciding whether to lower its price to maximize profits in (7), we will say that collusion is sustainable with respect to a single price deviation if the optimal deviation price \( p^d \) equals the collusive price. Then the following proposition demonstrates that if some collusive price is sustainable with respect to a single price deviation, this price, and all lower prices, will be sustainable in general. This defines the set of initial collusive prices for which price matching punishment strategies are Nash equilibria.

**Proposition 1** For any given discount factor \( 0 \leq \delta < 1 \), if the initial collusive price \( p^c \) is sustainable with respect to a single price deviation, then the collusive price (or any lower collusive price; that is, between \( p^n \) and \( p^c \)) is also sustainable with respect to any price deviations.

**Proof.** We only need consider price deviations between \( p^n \) and \( p^c \), since a price below \( p^n \) is clearly never optimal, while any increase in price is never optimal given the rival will not follow according to the price matching punishment rule. To prove the proposition, we start by showing that if the collusive price \( p^c \) is sustainable with respect to a single price deviation, then any collusive price between \( p^n \) and \( p^c \) is also sustainable with respect to a single price deviation.

Define
\[
\Delta \pi(p_i, p) = \left[ \frac{\partial \pi_i(p_i, p)}{\partial p_i} + \frac{\delta}{1 - \delta} \frac{d \pi(p_i)}{d p_i} \right]_{p_i=p}.
\]
The fact that $p^c$ is sustainable implies $\Delta \pi(p_i, p^c) \geq 0$, otherwise a firm can lower its price and increase its profit while still satisfying the constraint that $p_i \leq p^c$. Suppose we consider some lower collusive price $p$ such that $p^n \leq p < p^c$. If firm $i$ sets the same or a lower price, its continuing profits are

$$\pi_i(p_i, p) + \frac{\delta}{1 - \delta} \pi(p_i). \tag{8}$$

This is exactly the same problem considered previously for whether a firm wishes to deviate from the collusive price $p^c$, only now the collusive price is the lower price $p$. If we can show that $\Delta \pi(p_i, p) \geq 0$ for any common price $p^n \leq p < p^c$, then firms cannot increase profits with a single price cut, and such prices are also sustainable.

Differentiating $\Delta \pi(p_i, p)$ with respect to $p$ yields

$$\frac{d}{dp} (\Delta \pi(p_i, p)) = \left[ \frac{\partial^2 \pi_i(p_i, p)}{\partial p_i^2} + \frac{\partial^2 \pi_i(p_i, p)}{\partial p_i \partial p} + \frac{\delta d^2 \pi(p_i)}{1 - \delta d^2 p_i} \right]_{p_i = p}.$$

The assumptions (3)–(5) imply the result that $d(\Delta \pi(p_i, p))/dp < 0$. Thus, any collusive price between $p^n$ and $p^c$ is also sustainable with respect to a single price deviation. Moreover, given the inequality is strict, we have also that $\Delta \pi(p_i, p) > 0$ for $p < p^c$. Given the concavity of the profit functions, this implies profit must be strictly lower for any reduction in the price $p_i$ from the common price $p < p^c$.

Using this result, we can also rule out any more complicated price deviations. We can focus on defections that involve some sequence of price decreases. (By the same logic used earlier, price increases are never optimal since the rival will not follow the price increase under our price matching punishment rule. Moreover, we can rule out prices being left unchanged for any period followed by any further decrease in prices, since if any price decrease is profitable, it will be enacted as soon as possible given discounting and the stationarity of the stage game.) Now consider any series of price decreases. If a single price deviation is not profitable, then neither is any finite series of price decreases. This follows from backwards induction. Consider the last of a series of planned price reductions. Then the firm faces the same problem as defined earlier, but for some collusive price below the initial price $p^c$. The result above shows that given $p^c$ is sustainable with respect to a single price deviation, that the lower price is also sustainable with respect to a single price deviation, and so the firm will not want to defect in the last of the sequence of planned price decreases. In fact, its profit will be strictly lower from defecting. Working backwards, the same must be true for the previous price cut, and so on, all the way back to the original collusive price $p^c$, for which profit cannot be increased by defecting.
This logic also implies any infinite series of price cuts cannot be profitable. If an infinite series of price cuts is profitable, then the defecting firm must decrease its price in every period. The defector’s profit is

\[
\max \left[ \pi_i (p_1, p^c) + \delta \pi_i (p_2, p_1) + \delta^2 \pi_i (p_3, p_2) + \ldots \right]
\]

(9)

where \( p_1 \) is the deviation price in period 1, \( p_2 \) is the deviation price in period 2, and so on, and \( p^c \geq p_1 \geq p_2 \geq p_3 \geq \ldots \). The solution will either be an interior solution, with all inequalities holding strictly, or will involve at least one inequality holding with equality. However, from the stationarity of the problem, and given the result above, if one of the inequalities holds with equality at some point, then it must also hold with equality thereafter. This means, as above, we can apply backwards induction to rule out the profitability of defection in the first place. If all inequalities hold strictly, then given the concavity of the profit functions, the defecting firm must earn strictly higher profits from the sequence of price cuts than maintaining its prices at \( p^c \). However, given positive discounting, we can approximate the profit from defecting with an infinite series of price cuts arbitrarily closely by taking a sufficiently long finite sequence of the same price cuts, followed by reversion to price matching thereafter. Given that defecting through any finite sequence of (more than one) price cuts leads to strictly lower profits when the price \( p^c \) is sustainable with respect to a single price deviation, this implies that defecting through an infinite sequence of price deviations cannot be more profitable. ■

Proposition 1 shows that for fairly standard assumptions on the properties of the stage game, firms’ incentive to defect by undercutting will only diminish as the collusive price decreases. Moreover, it rules out more complicated price deviations, showing that if it is not profitable for a firm to defect by lowering its price once, then it cannot be profitable for the firm to lower its price multiple times. This makes the task of checking whether collusion can be sustained as straightforward as for the normal Nash reversion case. We illustrate these results using a linear demand example in Section 4.

Clearly, with no discounting (\( \delta = 1 \)), any price between \( p^n \) and \( p^m \), including \( p^m \), can be sustained as a collusive price under price matching punishments. With no discounting, the punishment from defecting becomes infinitely large, while the gain remains finite. The same result applies for more standard trigger strategies. On the other hand, in contrast to the standard result, the monopoly price is never sustainable as a collusive price under price matching punishments for any positive discounting of future profits. This reflects the fact that starting from the monopoly price, a small price decrease which is matched in subsequent periods has no first order impact on subsequent collusive profits (given collusive profits are...
flat at the monopoly price), but does generate a first order increase in profits for the defection period.

**Proposition 2** The monopoly price $p^m$ is not sustainable under price matching punishments unless $\delta = 1$, while it can always be sustained in the Nash reversion case when $\delta \geq \delta^*$, for some $\delta^*$ strictly less than 1.

**Proof.** The gain from defecting from the monopoly price is

$$\Delta = \max_{p \leq p^m} \left[ \pi_i (p, p^m) - \pi(p^m) + \frac{\delta}{1-\delta} (\pi(p) - \pi(p^m)) \right].$$

When evaluated at $p = p^m$, $\partial \pi_i (p, p^m) / \partial p < 0$ and $d \pi(p) / dp = 0$, so a sufficiently small decrease in price below the monopoly price is always profitable. In contrast, we know that the critical discount factor under the Nash reversion case for $p^m$ to be sustainable is

$$\delta^* = \frac{\pi^r_i - \pi^m_i}{\pi^r_i - \pi^m_i},$$

where $\pi^m_i$, $\pi^r_i$, and $\pi^r_i$ represent firm $i$'s Nash equilibrium profit, share of the monopoly profit, and one-shot deviation profit. Since $\pi^r_i > \pi^m_i > \pi^m_i$, $\delta^*$ is strictly less than 1.

Proposition 2 suggests that collusion is harder to sustain under price matching than under the regular Nash reversion case. If tacit collusion involves price matching punishments, there should always be some scope for a common price increase to increase each firm's profit. This is one testable implication of our theory, which contrasts with the standard approach (where firms use punishments so as to coordinate on the Pareto optimal outcome). We can generalize the result in Proposition 2 by comparing the set of collusive price under price matching for any $0 \leq \delta < 1$, with that under Nash reversion.

**Proposition 3** For any $0 \leq \delta < 1$, the sustainable region of collusive prices in the price matching case is smaller than in the Nash reversion case. That is, collusion is harder to sustain under price matching punishments.

**Proof.** We will prove the result by showing that the set of sustainable prices in the price matching case is a proper subset of the set of sustainable prices in the Nash reversion case for the same discount factor. We first show that for any sustainable price $p^c$ in the price matching case, the price will also be
sustainable under Nash reversion. We have

\[ \pi_i (p^r (p^c), p^c) + \frac{\delta}{1 - \delta} \pi (p^n) < \pi_i (p^r (p^c), p^c) + \frac{\delta}{1 - \delta} \pi (p^r (p^c)) \]  

(10) < \pi_i (p^d (p^c), p^c) + \frac{\delta}{1 - \delta} \pi (p^d (p^c)) \]  

(11) \leq \frac{1}{1 - \delta} \pi (p^c), \]  

(12)

where the inequality in (10) follows from the concavity of \( \pi (p) \) and since \( p^n < p^r (p^c) < p^m \) (Lemma 1), the inequality in (11) follows since \( p^r (p^c) < p^d (p^c) \) (Lemma 1) and \( p^d (p^c) \) is defined to maximize \( \pi_i (p, p^c) + \delta \pi (p) / (1 - \delta) \) subject to \( p \leq p^r \), and the inequality in (12) follows since the collusive price is assumed to be sustainable under price matching. Comparing the left hand side of (10) with the right hand side of (12), implies \( p^c \) is sustainable in the Nash reversion case.

However, the reverse is not true. If \( \delta \geq \delta^* \), then this result follows from Proposition 2. If \( 0 \leq \delta < \delta^* \), then we can find a just sustainable price \( p^* \) in the Nash reversion case. Then we have

\[ \pi_i (p^d (p^*), p^*) + \frac{\delta}{1 - \delta} \pi (p^d (p^*)) > \pi_i (p^r (p^*), p^*) + \frac{\delta}{1 - \delta} \pi (p^r (p^*)) \]  

(13) > \pi_i (p^r (p^*), p^*) + \frac{\delta}{1 - \delta} \pi (p^m) \]  

(14) = \frac{1}{1 - \delta} \pi (p^*) \]  

(15)

where the inequality in (13) follows given \( p^d (p^*) > p^r (p^*) \) (Lemma 1) and that the deviation price under price matching is chosen to maximize \( \pi_i (p, p^*) + \delta \pi (p) / (1 - \delta) \) subject to \( p \leq p^* \), the inequality in (14) follows from the concavity of \( \pi (p) \) and since \( p^m > p^r (p^*) > p^n \), and the equality in (15) follows since \( p^* \) is just sustainable under Nash reversion. Comparing the left hand side of (13) with the right hand side of (15), implies \( p^* \) is not sustainable under price matching punishments.

Tacit collusion will require firms to be more patient to sustain a given collusive outcome under price matching compared to the traditional trigger strategy of reversion to the one-shot Nash equilibrium forever. This reflects the fact that under price matching, a defecting firm can always set the same price as it would in the standard analysis, and face a smaller punishment given rivals simply match its price rather than further undercut it. In addition, the defecting firm can do even better by restricting the amount it deviates from the original agreement, thereby further reducing the severity of its punishment. As a result, firms will be more tempted to defect from any collusive agreement.

This finding provides one answer to the puzzle, noted by Cabral (2000, p. 131), that tacit collusion seems less common than predicted by the standard theory (based on grim punishments):
“In other words, if without repetition we were led to a puzzle (competitive prices even with only two firms), we now have the opposite puzzle, as it were: The model predicts that firms can almost always collude to set monopoly prices. Why don’t firms collude more often in practice?”

If explicit communication about collusive practices is illegal, then firms may find it difficult to coordinate on the Pareto optimal outcome in which they both set the monopoly price enforced by grim strategies. Instead, a focal equilibrium may involve collusion under price matching punishments. Vague statements about matching each other’s lower prices may signal this rule without being considered illegal. Alternatively, if through trial and error, firms arrive at a point where they set equal prices, this price matching may arise naturally. While the theory is silent about the mechanism by which firms arrive at a collusive price, it does make possible that only moderately collusive prices will be sustainable for reasonable discount factors. In Section 4, we give a specific worked example, to illustrate how much lower collusive prices may have to be.

The maximum sustainable collusive price, denoted $p^c$, represents the best the firms can do when the price matching punishment strategies are adopted. It is also the only feasible Nash equilibrium when firms adopt the even simpler strategy of matching each other’s price from the previous period. To see this, note that $p^c$ has the property that the unconstrained maximum of (7) equals $p^c$.\(^5\) At this particular price, neither firm wishes to increase or decrease its price, even if its rival matches the price increase as well as the price decrease. At any higher price, a firm can do better by lowering its price even if its lower price is matched by its rival in subsequent periods. At any lower price, a firm can do better by raising its price given its higher price is matched by its rival in subsequent periods. This suggests $p^c$ has desirable properties. It is not only the Pareto preferred Nash equilibrium under price matching punishments, but it is also the only feasible Nash equilibrium under simple price matching where both price increases and decreases are matched. However, unlike the case with simple price matching, under price matching punishments the punishments are credible.

To show price matching punishments are credible, we also need to show that if a firm does defect, then the rival will wish to pursue the punishment of matching its price, given that the defecting firm also continues thereafter with the price matching strategy. Using our assumptions on price competition in the

\(^5\)Formally, this is the condition $\Delta \pi(p_i, p^c) = 0$. The existence of such a critical collusive price follows given $\Delta \pi(p_i, p^c) > 0$, $\Delta \pi(p_i, p^m) < 0$, and $d(\Delta \pi(p_i, p))/dp < 0$. 
stage game, we can show that if some price can be sustained as a collusive price, then this condition also ensures the punishment is indeed credible. More generally, given the stationarity of our game, subgame perfection requires that for any given prices \( p_1 \) and \( p_2 \) in some period, our proposed strategies define Nash equilibrium in the subgame starting in the subsequent period. There are two additional situations which require modification of our simple strategy. Both relate to prices that will never arise along the equilibrium path and hence do not affect our analysis other than proving subgame perfection.

The first situation occurs if prices in the previous period involve one or both prices being below the one-shot Nash equilibrium prices. Clearly, where the one period best response involves setting higher prices, price matching punishments no longer make sense. In this case, we assume that the firms revert to the Nash equilibrium in the subsequent period. A possible motivation is that moderate price cuts are matched since they still lead to a form of collusion, but more severe price cuts below the one-shot Nash equilibrium end any sense of collusion and lead to the adoption of Nash reversion. The second situation occurs if prices in the previous period involve both firms pricing above the maximum collusive price that is sustainable in our analysis. Clearly, if both firms set prices too high (for instance, above the monopoly price), then continuing to price match is not optimal. Such prices will never arise along any equilibrium path, or in fact, along any deviation from the equilibrium path by a single firm. Nevertheless, to ensure subgame perfection, our specified strategies need to be robust to these prices. We do so by assuming that if in any period both firms price above the maximum sustainable collusive price, they will both revert to the maximum collusive price in the subsequent period. These modifications have the desirable property that prices are still continuous functions of the prices set by the firms in the previous period. We will show that these strategies characterize an equilibrium which is subgame perfect.

**Proposition 4** If the collusive price \( p^c \) is sustainable, the price matching punishment strategy is credible. The modified price matching punishments define a subgame perfect equilibrium.

**Proof.** Consider some sustainable collusive price \( p^c \). Now suppose we arrive in some period where the prices of the two firms in the previous period are given as \( p_1 \) and \( p_2 \). If \( p^n \leq p_2 \leq p_1 \leq p^c \) (the case with \( p_1 \leq p_2 \) follows by a symmetric argument), then firm 1 will not want to set a higher price, given firm 2, according to its price matching punishment strategy, will thereafter keep its price unchanged at \( p_2 \). Similarly, firm 2 will not want to set a higher price, given firm 1, according to its price matching punishment strategy, will thereafter price at \( p_2 \). If firm 1 sets the same or a lower price, its continuing
profits are
\[ \pi_1 (p_1, p_2) + \frac{\delta}{1 - \delta} \pi (p_1). \]

Thus, whether firm 1 will want to match firm 2’s price, or set a lower price, is exactly the same problem as whether \( p_2 \) is a sustainable collusive price or not. Since \( p_2 \leq p^c \) and the price \( p^c \) is assumed to be sustainable, we can apply Proposition 1 to get that firm 1 will want to match firm 2’s price. Moreover, if firm 2 sets the same price, its continuing profits given it expects firm 1 to play its price matching strategy, are

\[ \pi_2 (p_2, p_2) + \frac{\delta}{1 - \delta} \pi (p_2). \]

Thus, whether firm 2 will want to match firm 1’s price, or set a yet lower price, is exactly the same problem as whether \( p_2 \) is a sustainable collusive price or not. Again, we can apply Proposition 1 to get the required result. That is, the proposed price matching punishment strategy is a Nash equilibrium in any such subgame.

If, alternatively, \( p^n \leq p_1 \leq p^c < p_2 \) (the reverse case follows by a symmetric argument), then according to the price matching punishment strategy, firm 2’s decision is exactly as above (as it expects firm 1 will continue to price at \( p_1 \)), as is firm 1’s (since it expects firm 2 to match its price \( p_1 \) in the subsequent period), so our price matching strategy will remain a Nash equilibrium in this subgame. If \( p_1 > p^c \) and \( p_2 > p^c \), then either the minimum of the two prices is still less than the maximum sustainable collusive price \( p^c \), and the same logic as above can be used, or both prices exceed \( p^c \), and price matching is no longer a best response for each firm. According to the strategy defined earlier, in this case firms are assumed to revert to \( p^c \) in the following period, which is optimal for each firm given the other firm also prices at this level and given that they will follow their price matching punishment strategies thereafter. Finally, if, \( p_1 < p^n \) and/or \( p_2 < p^n \), then we assumed firms revert to the one-shot Nash equilibrium prices in the subsequent period. Given price matching punishment strategies, firms will choose the one-shot Nash equilibrium prices in all subsequent periods, which clearly defines a Nash equilibrium in the subgame.

Proposition 4 reflects the fact that the punishment phase resembles a type of collusion itself, albeit at a lower collusive price. Provided it is easier to sustain collusion at a lower price, each firm will be willing to stick to the punishment phase if they are willing to stick to the original agreement.
3 The effects of product substitutability

As mentioned in the introduction, an extensive literature has studied the question of whether product substitutability makes it harder or easier to sustain tacit collusion. A common finding is that the effects of product substitutability are ambiguous and depend on the nature of demand and the type of competition. Even within a specific model of price competition (linear demands and exogenous product differentiation), there can be a non-monotonic relationship between the degree of product substitutability and the critical discount factor that sustains collusion (Deneckere, 1983; Ross, 1992; and Albaek and Lambertini, 1998). This reflects the fact that greater product substitutability increases both the one period gain to defecting on an agreement, and also the subsequent punishment that can be incurred.

In contrast to the ambiguous theoretical results, traditionally, antitrust authorities have tended to view homogeneity as a factor making collusion more likely to occur. For example, Section 2.11 in the “Horizontal Merger Guidelines” issued by the U.S. Department of Justice and the Federal Trade Commission in 1992 states:

“Market conditions may be conducive to or hinder reaching terms of coordination. For example, reaching terms of coordination may be facilitated by product or firm homogeneity and by existing practices among firms, practices not necessarily themselves antitrust violations, such as standardization of pricing or product variables on which firms could compete.”

However, this view may reflect more the ease of reaching an agreement under homogenous conditions (as suggested by Stigler, 1964), than the sustainability of collusion thereafter. In this regard, one has to be careful to distinguish between the symmetry of firms (including the fact they sell like products), which presumably makes it easier for them to reach an agreement in the first place (and in our view makes price matching strategies more likely to arise), with the strength of competition in any given period. For instance, brand loyalty, switching costs, transportation costs, or consumer search may all make single period competition weaker without requiring an asymmetry between firms or that they sell products which are not comparable. Examples of competing gasoline stations, retail banking, or the rivalry between Coke and Pepsi come to mind. To capture such situations, we consider what happens when the degree of product substitutability increases, which can be viewed as an increase in the strength of single period competition.

We model an increase in product substitutability, denoting the original level of substitutability with the
superscript 0 and the new level with the superscript 1, by assuming \( \partial \pi_i^1 (p_i, p_j) / \partial p_i < \partial \pi_i^0 (p_i, p_j) / \partial p_i \) when evaluated at \( p^n < p_i = p_j < p^m \), and \( d\pi^1 (p) / dp \leq d\pi^0 (p) / dp \) for \( p^n < p < p^m \). Note that \( \partial \pi_i^0 (p_i, p_j) / \partial p_i < 0 \) when evaluated at \( p_i = p_j \), given the concavity of firm \( i \)'s profit function and Lemma 1. Increased product substitutability means that firms will capture a greater increase in demand for the same decrease in price, so we should expect that \( \partial \pi_i (p_i, p_j) / \partial p_i \) is bigger in magnitude (or more negative) as a result. Combining these properties, we have

\[
\frac{\partial \pi_i^1 (p_i, p_j)}{\partial p_i} < \frac{\partial \pi_i^0 (p_i, p_j)}{\partial p_i} < 0
\]

for \( p^n < p_i = p_j < p^m \).

Collusive profits should also increase by less (or certainly not by more) when the collusive price is increased under greater product substitutability. We know that \( d\pi (p) / dp > 0 \) for collusive prices that are less than the monopoly price. Greater product substitutability means market demand will decline by more (or at least not decline by less) when the collusive price is increased, and so collusive profits will increase by less (or by no more) as a result. Combining these properties we have

\[
0 < \frac{d\pi^1 (p)}{dp} \leq \frac{d\pi^0 (p)}{dp}
\]

for \( p^n < p < p^m \). For instance, both properties in (16) and (17) are true in the linear demand case examined in Section 4.

In our setting, greater product substitutability allows the gain from cheating to be realized with a smaller reduction in prices, which induces a smaller punishment for the defecting firm. Thus, as discussed in Section 2, in the extreme case with maximal product substitutability, collusion is not possible at all, unless firms do not discount the future. At the other extreme, in the case of no product substitutability, a defection does not make sense since the one-shot Nash equilibrium prices are then already the monopoly prices. Focusing on intermediate cases, we wish to show that collusion always becomes harder to sustain as product substitutability increases.

**Proposition 5** Increased product substitutability makes collusion more difficult to sustain.

**Proof.** Starting from any level of product substitutability, fix a collusive price \( p^c < p^m \) and a discount factor \( \delta \) such that collusion is just sustainable. The existence of such a critical collusive price was shown in Section 3. Now increase the degree of product substitutability, denoting the original outcome with the superscript 0 and the new outcome with the superscript 1.
From the fact that the collusive price was just sustainable initially, we have that the gain from defecting satisfies

\[ \Delta^0 = \max_{p \leq p^c} \left[ \pi^0_i(p, p^c) - \pi^0(p^c) + \frac{\delta}{1 - \delta} \left( \pi^0(p) - \pi^0(p^c) \right) \right] = 0. \]

This implies \( p^d = p^c \), and that the derivative of the expression in square brackets with respect to \( p \) is zero when evaluated at \( p = p^c \) since \( p^c \) is just sustainable. (If it is negative, then the firm can always lower price slightly and obtain higher profits, which means the firm will want to defect; alternatively, if it is positive, then \( p^c + \varepsilon \), where \( \varepsilon \) is sufficiently small, must also be sustainable.) That is, we have

\[
\frac{\partial \pi^0_i(p, p^c)}{\partial p} + \frac{\delta}{1 - \delta} \frac{d\pi^0(p)}{dp} = 0, \tag{18}
\]

when evaluated at \( p = p^c \).

Now consider the gain from defecting under the new demand functions:

\[ \Delta^1 = \max_{p \leq p^c} \left[ \pi^1_i(p, p^c) - \pi^1(p^c) + \frac{\delta}{1 - \delta} \left( \pi^1(p) - \pi^1(p^c) \right) \right]. \]

We wish to show \( \Delta^1 > \Delta^0 = 0 \), so that defection is always profitable. The effect of a change in \( p \) on deviation profits is now

\[
\frac{\partial \pi^1_i(p, p^c)}{\partial p} + \frac{\delta}{1 - \delta} \frac{d\pi^1(p)}{dp}, \tag{19}
\]

Combining the conditions in (16) and (17) evaluated at \( p = p^c \) with (18), implies that the expression in (19) must be negative when evaluated at \( p = p^c \). This means the gain from defecting is positive for a sufficiently small price decrease. \( \blacksquare \)

The economic logic behind Proposition 5 can be explained by comparing the profitability of defection to that under Nash reversion. When a firm lowers its price from the collusive one, its price will be closer to its one-shot best response, and its one-shot profit will increase, and this increase in profit will be greater as products become closer substitutes. This same effect arises under Nash reversion. However, under Nash reversion the punishment also becomes more severe as products become closer substitutes (one-shot Nash equilibrium profits fall as competition becomes more intense). Under price matching, the severity of the punishment is determined by the extent of the price deviation by the defecting firm. Greater product substitutability allows a higher gain from cheating for the same reduction in prices, but no change in the severity of punishment for a given price reduction.\(^6\) In fact, the punishment may

\(^6\)Another way of stating the result is that greater product substitutability allows any given gain from cheating to be realized with a smaller reduction in prices, which induces a smaller punishment for the defecting firm.
actually be less severe under greater product substitutability since collusive profits may decline by less as prices are reduced. For instance, if lowering the collusive price helps stimulate market demand, the positive impact on profits will only be stronger under greater product substitutability.

4 Linear demand example

We illustrate (and complement) our general analysis by considering competition in the stage game being modeled by a standard exogenous differentiation Bertrand setting with linear demands.\(^7\) Inverse demand functions are given by \(p_i = a - b(q_i + \gamma q_j),\) where \(0 < \gamma < 1\) serves as a measure of the degree of product substitutability. The higher \(\gamma\), the higher the degree of product substitutability. Such demand functions can be derived from utility maximization of a representative consumer with quadratic utility functions (see Dixit, 1979 and Singh and Vives, 1984, among many others). Firms have constant marginal costs, \(c\).

When the prices of the two firms are sufficiently close, both firms will have positive demands, and we can easily get firm \(i\)'s demand function in terms of \(p_i\) and \(p_j\) by inverting the inverse demand functions. However, when the prices of the two firms strongly diverge, the high price firm will receive no demand, while the low price firm captures the entire market. Specifically, for any price \(p_j\), when \(0 < p_i \leq (−a(1 − \gamma) + p_j)/\gamma\), firm \(i\) captures the entire market; while its demand becomes zero when \(p_i \geq a(1 − \gamma) + \gamma p_j\). Thus, firm \(i\)'s demand function is

\[
q_i = \begin{cases} \frac{a(1 − \gamma) - p_i + \gamma p_j}{b(1 + \gamma)(1 − \gamma)} & \text{if } -a(1 − \gamma) + p_j < p_i < a(1 − \gamma) + \gamma p_j \\ \frac{a - p_i}{b} & \text{if } 0 < p_i \leq \frac{-a(1 − \gamma) + p_j}{\gamma} \\ 0 & \text{if } p_i \geq a(1 − \gamma) + \gamma p_j. \end{cases}
\]

It is straightforward to check that the monopoly prices, quantities, and profits are \(p^m = (a + c)/2,\) \(q^m = (a - c)/(2b(1 + \gamma))\) and \(\pi^m = (a - c)^2/(4b(1 + \gamma))^2\) respectively. Similarly, the one-shot Nash equilibrium prices, quantities, and profits are \(p^n = (a(1 − \gamma) + c)/(2(1 − \gamma)),\) \(q^n = (a - c)/(b(1 + \gamma)(1 − \gamma))\), and \(\pi^n = (a - c)^2(1 − \gamma)/(b(1 + \gamma)(1 − \gamma))^2\) respectively. For sufficiently low product substitutability \((\gamma < √3 − 1),\) only the demand function in (20) arises for any prices between \(p^n\) and \(p^m\). The properties we assumed in Sections 2 and 3 will then apply given profits are smooth over the relevant range of prices.\(^8\)

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\(^7\)The analysis here has also been replicated for the standard Hotelling model of product differentiation, which is available from the authors at http://profile.nus.edu.sg/fass/ecsjkdw/.

\(^8\)To see this, note that \(\frac{\partial^2 \pi_i}{\partial p_i^2} = -2/(b(1 + \gamma)(1 − \gamma)) < 0\) and \(\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = \gamma/(b(1 + \gamma)(1 − \gamma)) > 0,\)

19
Part of the point of this section is to show that all our results still apply even for higher levels of product substitutability, such that the profit functions are no longer smooth over all prices between \( p^n \) and \( p^m \). This demonstrates that the conditions we assumed to obtain our general results are sufficient but not necessary. It also demonstrates all our results hold for a very standard model of imperfect competition.

We first investigate whether collusion is sustainable under price matching punishments, and therefore whether such a strategy is credible (given Proposition 4). Following Section 2, we start by plotting best response functions. Firm \( i \)'s best response to any price \( p_j \) set by firm \( j \) is as follows:

\[
p_i = \begin{cases} 
\frac{a(1 - \gamma) + \gamma p_j + c}{2} & \text{if } p_j \leq \frac{a(1 - \gamma)(2 + \gamma) + \gamma c}{2 - \gamma^2} \\
\frac{p_j - a(1 - \gamma)}{\gamma} & \text{if } \frac{a(1 - \gamma)(2 + \gamma) + \gamma c}{2 - \gamma^2} < p_j < \frac{a(1 - \gamma) + \gamma c}{2} \\
\frac{a + c}{2} & \text{if } p_j \geq \frac{a(1 - \gamma) + \gamma c}{2}.
\end{cases}
\]  

(23)  

If the rival’s price is low, we get the standard (linear) upward sloping best response functions. If the rival’s price is high, firm \( i \) will set a relatively low price and capture the whole market, while it can do the same thing by setting the monopoly price if the rival sets a very high price. The linear best response function (23) applies for any \( p_j \) between \( p^n \) and \( p^m \) when \( 0 < \gamma \leq \sqrt{3} - 1 \), while for higher \( \gamma \), whether firm \( i \)'s best response is determined by (23) or (24) depends on whether \( p_j \) is closer to \( p^n \) or \( p^m \). Figure 2 plots these best response functions for two cases — when \( \gamma < \sqrt{3} - 1 \) and when \( \gamma > \sqrt{3} - 1 \).

**FIGURE 2 ABOUT HERE**

Now we consider whether a firm, say firm 1, has an incentive to deviate from some collusive price \( p^n < p^c \leq p^m \), given that both firms adopt price matching punishment strategies. When we consider firm 1’s profit if it defects, we need to distinguish three cases: (1) \( 0 < \gamma \leq \sqrt{3} - 1 \); (2) \( \sqrt{3} - 1 < \gamma < 1 \) and \( p^n < p^c \leq \frac{(a(1 - \gamma)(2 + \gamma) + \gamma c)}{(2 - \gamma^2)} \); and (3) \( \sqrt{3} - 1 < \gamma < 1 \) and \( (a(1 - \gamma)(2 + \gamma) + \gamma c) < p^c \leq p^m \).

In the first and second cases, firm 1’s best response is given by (23), and its optimal deviation price will exceed \( (a(1 - \gamma) + \gamma p^c + c)/2 \). In the third case, firm 1’s best response is given by (24), and its optimal deviation price will exceed \( (a(1 - \gamma) + \gamma p^c + c)/2 \). Therefore, whether firm 1’s best response is given by (23) or (24) depends on whether \( p_j \) is closer to \( p^n \) or \( p^m \). Figure 2 plots these best response functions for two cases — when \( \gamma < \sqrt{3} - 1 \) and when \( \gamma > \sqrt{3} - 1 \).

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deviation price will be greater than or equal to \((p^c - a(1 - \gamma)) / \gamma\). Firm 1’s profit if it deviates will be
\[
\left( \frac{a(1 - \gamma) - p_1 + \gamma p^c}{b(1 + \gamma)(1 - \gamma)} \right) (p_1 - c) + \frac{\delta}{1 - \delta} \frac{(a - p_1)(p_1 - c)}{b(1 + \gamma)},
\]
where the first term is firm 1’s current (one period) profit and the second term is the present value of its profit in subsequent periods given price matching. This function also gives the present discounted value of firm 1’s profits if it continues to collude, in which case \(p_1 = p^c\). We also note that \((a(1 - \gamma) + \gamma p^c + c) / 2 > (p^c - a(1 - \gamma)) / \gamma\) when \(\sqrt{3} - 1 < \gamma < 1\) and \(p^n < p^c \leq (a(1 - \gamma)(2 + \gamma) + \gamma c) / (2 - \gamma^2)\) and that \((p^c - a(1 - \gamma)) / \gamma > (a(1 - \gamma) + \gamma p^c + c) / 2\) when \(\sqrt{3} - 1 < \gamma < 1\) and \((a(1 - \gamma)(2 + \gamma) + \gamma c) < p^c \leq p^m\).

Therefore, firm 1 will choose \(p_1\) to maximize (26) subject to
\[
\max (\left( a(1 - \gamma) + \gamma p^c + c \right) / 2, (p^c - a(1 - \gamma)) / \gamma) \leq p_1 \leq p^c.
\]

Solving this constrained maximization problem, we obtain firm 1’s optimal deviation price, which is less than the collusive price when \(p^c > (a(1 - \gamma) + c(1 - \delta \gamma)) / (2 - (1 + \delta)\gamma)\) and equals the collusive price if \(p^c \leq (a(1 - \gamma) + c(1 - \delta \gamma)) / (2 - (1 + \delta) \gamma)\). Thus, under price matching punishments, collusion can be sustained for any collusive price satisfying
\[
p^n \leq p^c \leq \frac{a(1 - \gamma) + c(1 - \delta \gamma)}{2 - (1 + \delta) \gamma},
\]
but cannot be sustained for any higher collusive price. By backwards induction, the same condition implies any finite sequence of price cuts from \(p^c\) will also not be profitable under price matching punishments.\(^9\)

The collusive prices defined by (27) satisfy all the properties derived in Sections 2 and 3. The highest sustainable collusive price defined by (27) is strictly less than the monopoly price for \(\delta < 1\), consistent with Proposition 2. Moreover, consistent with Proposition 5, the highest sustainable collusive price is strictly decreasing as the degree of product substitutability increases. Figure 3 illustrates this relationship for a discount factor of 0.9. In figure 3, parameter values are chosen such that the monopoly price is equal to unity. Therefore, the maximum sustainable collusive price as a proportion of the monopoly price is measured on the vertical axis in figure 3.\(^{10}\) By matching the parameters of our demand system to those in Slade (1989), it is straightforward to check that the maximum sustainable collusive price

\(^9\)In the appendix, we demonstrate that firm 1 cannot do better using an infinite sequence of price cuts given the proposed collusive price satisfies (27).

\(^{10}\)Similarly, the critical discount factor necessary to sustain collusion is defined by \(\delta \geq \delta^c = ((2 - \gamma)p^c - a(1 - \gamma) - c) / (\gamma(p^c - c))\), which is increasing in \(\gamma\) and just equals unity when \(p^c\) is set to the monopoly price, consistent with Propositions 2 and 5.
defined by (27) is identical to the price in equation (12) in her paper, when \( R = 1 \). This confirms that our maximum sustainable collusive price is also the unique Nash equilibrium when firms simply match whatever price their rival sets in the previous period, regardless of whether this involves increasing or decreasing their price. An implication is that the unambiguous relationship between the degree of product substitutability and the ease of collusion that we find, also applies to the Nash equilibrium (or \( \varepsilon \)-subgame perfect equilibrium) characterized by general price matching strategies in Slade (1989).

Finally, Proposition 3 can be examined for this example by comparing the sustainable region of collusive prices under price matching with Nash reversion. Taking into account the possibility of zero demands, the critical discount factor under Nash reversion for any collusive price is a lengthy expression which has been omitted here for brevity.\(^{11}\) It can be shown that for our linear demand model, the set of discount factors and collusive prices sustainable under price matching punishments is a proper subset of those sustainable under Nash reversion, as suggested by Proposition 3. Figure 3 indicates the difference between the two regions can be large. Since with a discount factor of 0.9, the monopoly price can always be sustained under Nash reversion, the vertical axis in figure 3 also indicates the maximum sustainable collusive price under price matching as a proportion of the maximum collusive price under Nash reversion. The figure indicates that tacit collusion may be substantially harder to maintain under price matching, particularly if products are highly substitutable. This provides one explanation for why tacit collusion may be less prevalent than the existing literature predicts. If communication is limited, and price matching is focal, then the conditions for tacit collusion to arise can be considerably harder than those implied by standard grim strategies.

5 Conclusions

It has long been recognized that a wide range of collusive outcomes can be supported when firms compete period after period. Given this “embarrassment of riches”, the literature has focused on equilibria which lead to Pareto optimal outcomes. This has led to the study of how monopoly pricing can be supported by

\(^{11}\)The existing literature (such as Ross, 1992) only contains the critical discount factor where the collusive price is the monopoly price. We have written up the details of the general case, which are available from the authors at http://profile.nus.edu.sg/fass/ecsjkdw/
tacit collusion using grim strategies, optimal punishments, or other more complicated continuous reaction function approaches. Our approach is different. We wished to see whether the simple punishment rule of matching the lowest price set by firms in the previous period that was suggested in the original writings on tacit collusion by Chamberlin (and others), can still sustain collusion in equilibrium. While, monopoly pricing cannot be supported under this strategy, less collusive outcomes can.

We provided conditions under which this simple punishment rule is an equilibrium strategy, and sustains collusion. Most importantly, there must be some degree of imperfect competition, otherwise, a firm can obtain the maximum profit from defection, while facing an arbitrarily small punishment by lowering its price by a sufficiently small amount. Even with imperfect competition, tacit collusion is harder to sustain under price matching punishments than in standard approaches, given punishments are less severe. Unlike the standard ambiguous implications of increased product substitutability, with price matching punishments, increased product substitutability always makes collusion harder to sustain. Greater product substitutability allows any given gain from cheating to be realized with a smaller reduction in prices, which under price matching induces a smaller punishment for the defecting firm.

There are a number of natural extensions of our model. Here we just mention a few. The analysis extends in a straightforward way to \( n \) symmetric firms, given that if a single firm defects by lowering its price below the current collusive price, all the remaining firms match. Under such a strategy, any firm deciding whether to defect assumes all other firms will subsequently match, while any firm deciding whether to match following a defection assumes all the other firms will do so. Then the matching or defecting decisions remain the same as analyzed here, except that the single rival is now replaced by \( n - 1 \) identical rival firms. Since the gains to a single firm that undercuts are likely to be increasing in the number of competing firms, collusion should become harder to sustain as the number of competitors increases.

Another possible extension is to consider tacit collusion with quantity competition assuming quantity matching punishments. One complication with quantity competition is that in the standard Nash-reversion case, a firm’s optimal deviation quantity exceeds the one-shot Nash equilibrium quantity. Provided the discount factor is not very low, this is not a problem here. When its quantity increase will be matched in future periods, a deviating firm will not want to produce more than the one-shot Nash equilibrium output. Thus, the choice of quantities remains between the collusive levels and the one-shot Nash levels, enabling the contraction mapping property to be exploited still. As a result, the same type
of analysis in this paper can still apply in a Cournot setting. Other possible extensions include allowing for asymmetry, capacity constraints, or random shocks.

A limitation of the existing analysis is that it does not explain the setting of prices in the first place. The simplest way to do so is to allow collusive prices to be always set at the maximum sustainable level given price matching punishments. This gives us a predictive theory of price setting that differs from the monopoly prices which arise under standard grim strategies. Alternatively, one could explore price setting when lower prices are matched immediately but higher prices are only matched after some delay.

Another direction is to seek empirical support for (or against) price matching type punishments. A difficulty with testing the simple (non-stochastic) model of tacit collusion studied here, is that defections should never occur in equilibrium. Less direct tests may be able to shed some light on the nature of tacit collusion. For instance, evidence, such as that presented in Domowitz et al. (1987) that firms tacitly collude on prices that are substantially less than monopoly prices even though conditions are such that they should be able to achieve monopoly prices by following grim strategies, can be interpreted as providing some support for our approach. Experimental approaches may provide more direct evidence.

6 References


7 Appendix

This appendix demonstrates that for the linear demand model of Section 4, a firm cannot do better using an infinite sequence of price cuts given the proposed collusive price satisfies (27); that is, given a single price defection is not profitable. Since we have specific functional forms, we can solve for the infinite sequence of price cuts that maximizes a firm’s profit, given its rival passively matches the firm’s price in each subsequent period. Firm 1’s profit at period 1 from defecting is $\pi_1 (p_1, p^c) + \delta \pi_1 (p_2, p_1) + \delta^2 \pi_1 (p_3, p_2) + \ldots$, where (with a slight abuse of notation) $p_1$ is firm 1’s optimal deviation price in period 1, $p_2$ is its optimal deviation price in period 2, and so on. The constraint is $p^c = p_0 \geq p_1 \geq p_2 \geq p_3 \geq \ldots$.

Define $V(p_t) = \max_{p_{t+1}, p_{t+2}, \ldots} \left( \pi_i (p_{t+1}, p_t) + \delta \pi_i (p_{t+2}, p_{t+1}) + \delta^2 \pi_i (p_{t+3}, p_{t+2}) + \ldots \right)$. Then we have the sequence of Euler equations defined by

$$\frac{\partial \pi_i (p_{t+1}, p_t)}{\partial p_{t+1}} + \delta \frac{\partial \pi_i (p_{t+2}, p_{t+1})}{\partial p_{t+1}} = 0$$

(28)
for all $t \geq 0$.

In the linear demand example, $\pi_t(p_{t+1}, p_t) = (a(1 - \gamma) - p_{t+1} + \gamma p_t)(p_{t+1} - c) / (b(1 + \gamma)(1 - \gamma))$, so that (28) becomes the linear difference equation

$$a(1 - \gamma) + c(1 - \delta \gamma) - 2p_{t+1} + \gamma p_t + \delta \gamma p_{t+2} = 0. \quad (29)$$

The solution to this difference equation is

$$p_t = \frac{a(1 - \gamma) + c(1 - \delta \gamma)}{2 - (1 + \delta)\gamma} + A_1 \left(1 + \frac{\sqrt{1 - \delta \gamma^2}}{\delta \gamma}\right)^t + A_2 \left(1 - \frac{\sqrt{1 - \delta \gamma^2}}{\delta \gamma}\right)^t, \quad (30)$$

where $A_1$ and $A_2$ are two constants. Since $\left(1 + \frac{\sqrt{1 - \delta \gamma^2}}{\delta \gamma}\right) / \delta \gamma > 1$, it must be that $A_1 = 0$ since prices are bounded above by $p^c$. We also know that $p_0 = p^c$, so that the solution in (30) becomes

$$p_t = \frac{a(1 - \gamma) + c(1 - \delta \gamma)}{2 - (1 + \delta)\gamma} + \left(p^c - \frac{a(1 - \gamma)}{2 - (1 + \delta)\gamma}\right) \left(1 - \frac{\sqrt{1 - \delta \gamma^2}}{\delta \gamma}\right)^t, \quad (31)$$

where $0 < \left(1 - \frac{\sqrt{1 - \delta \gamma^2}}{\delta \gamma}\right) / \delta \gamma < 1$. This solution implies the constraint $p^c \geq p_t$ is binding if and only if $p^c \leq (a(1 - \gamma) + c(1 - \delta \gamma)) / (2 - (1 + \delta)\gamma)$, which, consistent with Proposition 1, is exactly the same condition we found to rule out the profitability of a single price deviation.
Best response function when $0 < \gamma < \sqrt{3} - 1$

Best response function when $\sqrt{3} - 1 < \gamma < 1$
Figure 3 ($a = 2$, $c = 0$ and $\delta = 0.9$)