Universal Service Obligations in the Postal Sector: Endogenous Quality and Coverage*

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Abstract

The liberalisation of the postal sector makes necessary to analyse the effect of entry in the maintenance of the universal service obligations. This paper examines the consequences of allowing entry in a postal market where there is an incumbent operator regulated with uniform pricing and quality constraints and with a coverage constraint. I consider that the cost of a unit of mail depends on its quality and on the location of the senders. Senders have inelastic demands and differ in their willingness to pay for quality. The incumbent operator and the entrant play a three-stage game, first choosing the villages they will serve, then the quality of the letters and finally the price. Valletti, Hoernig and Barros (2002) have shown that when an incumbent operator is regulated with a uniform pricing and coverage constraint an entrant can strategically maintain a low level of coverage to increase his profits. Here I show that maintaining a low level of coverage the entrant can also increase the differentiation between his services and the service of the incumbent, and as a result he raises its profits even more. I also analyse the consequences of imposing minimum quality and coverage standards on the incumbent and the entrant. Finally, I show that in presence of network externalities firms use the level of coverage as a quality attribute that substitutes the quality of the service.

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1 Introduction

In a liberalised postal market the most important regulatory policy that guarantees the access of all consumers to the postal services is the Universal Service Obligation (USO).¹ This policy requires that the incumbent postal operator provides a basic package of services, in a full geographic area, with a minimum level of quality and at an affordable uniform price. However, the viability of this policy may be threatened if the entrants only serve the profitable mailers, opt for niche markets and select product differentiation strategies. A profound analysis of this type of entry is fundamental to correctly orient public intervention. Governments need to be assured that in introducing competition in the market they will not undermine the provision of the service to all the population.² On the other hand, when designing the USO Governments must be aware of the impact of the USO on competition, and on how the different measures that support the USO interact among them. For example, the regulation of the quality of the service has a direct impact in the coverage of the firms. Therefore, different regulatory scenarios will produce different entry strategies.

In this paper I analyse the interactions of quality and coverage regulations. I show that when an incumbent postal operator is regulated with a uniform pricing constraint an entrant may find profitable to reduce his level of coverage to increase the differentiation between his service and the service of the incumbent and obtain a higher profit. More generally, I show that the specific impact of each measure of the universal service regulation depends on how are regulated the other aspects of the market.

The literature on the universal service in the postal sector has focussed on the effects of implementing different types of regulations. But very few works have taken into account the quality of the service. Crew and Kleindorfer (1998) develop a model that characterises jointly the optimal scope of the reserved area for the postal service and the universal service obligation. The authors argue that quality attributes of mail is a central aspect for the definition of the USO. Moreover, they show their concern that a strict regulation of the USO may jeopardise the benefits of competition. Entry has the potential to improve both static and dynamic efficiency because with entry service and costs innovations is more likely to occur. ³By contrast, under the USO, entry may not provide any of these

¹The first studies about the universal service in network industries have been developed by Crew and Kleindorfer (1998, 2001), Alleman et al. (2000), Laffont and Tirole (2000), Gasmi et al. (2000) and Cremer, Gasmi, Grimaud and Laffont (2001) among others. Rosston and Wimmer (2000) compare the impact of different public programs to provide the universal service in the telecommunications sector.

²Rodriguez and Storer (2000) study different approaches to estimate the cost of the universal service. Choné, Flochel and Perrot (2000) analyse different mechanisms to fund the universal service. On the other hand, Anton, Vander-Weide and Vettas (1998) consider the strategic effects of subsidies to finance the USO.
efficiency gains. As the authors report, in Sweden Citymail provided service innovation, such as "day-certain" delivery. However, it only delivered two days per week.

Crew and Kleindorfer (2000, 2001, 2004 and 2005) argue that a meaningful USO is unlikely to be feasible absent a reserved area or some other method of funding. In Crew and Kleindorfer (2005) the authors claim that "it seems unlikely for most countries that lettermail USO can be supported without a reserved area, unless service standards are relaxed".

Cremer et al. (2001) simulate a postal market where there is an incumbent operator and one entrant. The incumbent is the only that provides single-piece mail and the two firms provide bulk mail. Moreover, while the incumbent faces a uniform pricing constraint the entrant is left unregulated. The authors consider a model where coverage is a quality attribute and assume that the coverage of the firms differs. Taking this into account, they show that the larger is the coverage of the entrant the more attractive is the entrant’s product, and the lower is the price differential at which it can capture a positive market share.

Although the previous papers introduce the concept of quality into the study of the postal sector, the idea of this paper is more related with the literature on vertical product differentiation. In particular, I assume the endogenous nature of quality in the same model than Cremer, De Rycke and Grimaud (1997), which I extent to consider the coverage decision of the firms. In their paper, the authors analyse the welfare impact of quality in market structure. In particular, they analyse whether the case for the existence of a public operator is strengthened or weakened if product choice is endogenous. The authors consider that the firms play a two stage game, first choosing qualities and then prices. An additional interesting specification of their model is that it considers the utility of the addresses. They show that in equilibrium two private operators result in an inefficient provision of quality. The introduction of a public firm changes this result. If one of the firms is a public operator and the budget constraint is not binding, the firms implement the first best.3 If the budget constraint of the public firm is binding, the equilibrium is not efficient but yields a higher level of welfare than a private equilibrium.

This paper is also very related with Valletti, Hoernig and Barros (2002), that study the strategic interaction between uniform pricing and coverage in the context of the telecommunications sector. They show that price competition is critically affected by relative coverage, i.e. the ratio between the entrant’s coverage and the incumbent’s. In particular, they find that the uniform pricing policy distributes the benefits of competition by reducing the prices to customers who will

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3Cremer, Marchand and Thisse (1991) analyse a mixed oligopoly with horizontal product differentiation where firms choose their location and price in a model à la Hotelling. In particular, the authors study if the number of public firms that compete with private ones may affect social welfare.
not be served by the entrant. In spite of this, uniform pricing increases the prices in the duopoly area and can lower the coverage. The authors also show that when the regulator imposes simultaneously a uniform price and a coverage constraint for the incumbent the coverage of the entrant increases, but as a consequence there is a price increase and previously served consumers lose welfare. Indeed, one of the most important contributions of Valletti, Hoernig and Barros (2002) is to show that there is a clear trade-off between larger coverage and higher welfare of served customers, and between the welfare of customers in markets with competition or monopoly. In this paper I show that when firms can endogenously determine the quality of their service the trade-off between coverage and welfare of served consumers may not appear in some circumstances. Finally, the authors demonstrate that uniform pricing constraints create strategic links between otherwise unregulated geographical areas, which can lead to lower coverage chosen by both the incumbent and the entrant. I show that the consideration of quality does not modify this result. Moreover, I explain how the introduction of endogenous qualities may make this strategic link even more powerful.

Finally, Fabra, Escribano and Gagnepain (2004) consider the interaction between quality and coverage in an spirit very similar to this paper. They model competition in a private duopoly where an incumbent and an entrant are differently regulated. In particular, while the incumbent must cover all the population, the entrant is free to operate in the region he prefers. On the other hand, they assume that the entrant can price discriminate depending on the path followed by the piece of the mail. Finally, they consider a public regulator that sets the price to be charged by the incumbent in order to maximize social welfare. As in my model, they consider that the prices of the two firms and the coverage of the entrant are endogenous. However, they consider the quality of the service as an exogenous variable, and they perform their analysis in terms of given quality differences. The game they consider consist of a first stage where, given the qualities offered by the firms, the regulator contracts with the incumbent. The contract specifies the uniform price and a transfer to cover the incumbent’s costs. In the second stage, having observed the price of the incumbent, the entrant chooses his coverage and his prices for each path. One conclusion of this paper is that the optimal coverage of the entrant is larger the higher its quality advantage is. In my paper, on the other hand, I show that the entrant tries to reduce his coverage to increase product differentiation.

The objective of this paper is to analyse the welfare impact of endogenous product selection when an incumbent postal operator is regulated with a uniform pricing and quality constraints and with a coverage constraint. As in the literature on vertical product differentiation, the firms’ equilibrium product strategy is the consequence of two opposing forces. On the one hand, the firms choose the product variety that is most profitable in terms of consumer’s preferences and
costs. This will make the firms choose a similar quality. On the other hand, each firm wants to differentiate his products from those of his competitor because "product differentiation weakens price competition and raises profits". The main contribution of this paper with respect to the previous literature is to show that with endogenous product selection private firms use their level of coverage to increase product differentiation. An entrant can serve more consumers increasing his coverage. But by reducing his coverage he increases the region that the incumbent serves as a monopolist. This increases allow to raise the price of the entrant for two reasons. First, because the price of the incumbent increases with the size of the monopoly area. And second, because the increase in the differences on coverage increases even more the differentiation of the services. The increase of relative coverage allows the firms to increase their product differentiation in an extent that the existence of a duopoly in all the country will not allow. Valletti, Hoernig and Barros (2002) have shown the role of the first effect when there is a incumbent regulated with a uniform price constraint. In their model two firms offering differentiated products could increase relative coverage to raise their profits. Here we show, in addition, that when firms increase relative coverage they also increase product differentiation and raise their profits even more. On the other hand, the increase in the incumbent price due to the increase of relative coverage can be so high that the entrant can end up being a monopolist of the areas he covers.

Finally, I extent the model to introduce network externalities. I show that if consumers have strong preferences for being in a large network the entrant can decide to increase his coverage, even if this reduces product differentiation. However, the most interesting conclusion of this extension is that with network externalities firms can maintain their level of product differentiation spending less in quality. The reason of this is that firms use coverage as a quality attribute that substitute the quality of the service.

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 analyses the first best price, quality and coverage policies that will implement a benevolent regulator. Section 4 considers a private duopoly regulated with some universal service obligations such as uniform pricing. Sections 5 and 6 analyse the impact of imposing certain obligations on the quality and the coverage of the firms. Section 7 extents the model of section 2 to allow the possibility of network externalities. Finally, we summarise our conclusions in section 8.

2 The model

Consider a market with one incumbent postal operator \((i = 1)\) and one entrant \((i = 2)\). The firms serve a continuum of locations \([0, \bar{m}]\). These locations are

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4Moorthy (1988).
considered villages of a country and \( \mu \) represents the size of the country. We assume that all villages have the same number of customers. Moreover, the villages are ordered according to their fixed cost. If firm \( i \) decides to cover the village \( \mu \) he has to pay the fixed cost \( F(\mu) \), where \( F(0) = 0 \), \( F'(\mu) = f(\mu) > 0 \) and \( f'(\mu) > 0 \). This implies that villages have different level of profitability. For simplicity, we also assume that this fixed cost is the same for the two firms.

We assume that the level of coverage of the entrant never is larger than those of the incumbent, \( \mu_2 \leq \mu_1 \). As a result, while some villages are served by a duopoly, the rest are only served by the incumbent. In this situation, it may occur that one client of the entrant sends a letter to an addressee located in the monopoly area. To circumvent this possibility we consider that consumers only send letters to their own villages. Alternatively, we could assume that the firms instead of covering the most profitable villages cover the most profitable paths.\(^5\)

Vertical differentiation is introduced in the model following the work of Cremer, De Rycke and Grimaud (1997).\(^6\) In particular, consider that each village have a continuum of senders with mass 1. The senders have different preferences for the quality of the letters, or more precisely for the frequency of the deliveries. We call \( x \geq 0 \) the quality of a letter. The preference of each sender for the quality is represented by \( \theta \), and this is uniformly distributed in the unit segment \( \theta \in [\underline{\theta}, \bar{\theta}] \), where we assume that \( \underline{\theta} \leq \frac{1}{2} \).\(^7\) On the other hand, we assume that senders have a perfectly inelastic demand that is normalised to one unit. Taking this into account, the surplus of a sender with type \( \theta \) who sends one letter of quality \( x \) at price \( p \) is given by

\[
v_0 + \theta x - p,
\]

where \( v_0 \) is a fixed surplus that all consumers obtain for sending one letter. I assume that \( v_0 \) is large enough to allow all consumers to buy one letter with the prices and qualities of equilibrium.

Each firm \( i \) offers a variant of quality \( x_i \) at price \( p_i \). In what follows I consider that the quality offered by the incumbent is higher than that of the entrant, \( x_1 > x_2 \). This reflects the current situation in the postal sector where the frequency of delivery of the incumbent is greater than those of his new competitors. In spite of this, throughout the text we also comment the consequences of having a competitor that offers a higher quality than the incumbent. Taking this into account, one sender with type \( \theta \), placed in one of the locations served by the

\(^5\)Fabra, Escribano and Gagnepain (2004) model this possibility.

\(^6\)This paper is based on an specification introduced by Mussa and Rosen (1978) and developed in Cremer and Thissen (1994). The two papers consider that preferences are quasi-linear so that the marginal utility of income is constant and is the same for all consumers.

\(^7\)This guarantees that no firm has a negative market share.
duopoly, will be indifferent between the incumbent and the entrant when
\[ \tilde{\theta}x_1 - p_1 = \tilde{\theta}x_2 - p_2. \]

Therefore, the demand of the incumbent postal operator in these locations can be defined as
\[ \bar{\theta} - \tilde{\theta} = \bar{\theta} - \frac{p_1 - p_2}{x_1 - x_2}. \]  

(1)

On the other hand, the demand of the entrant in each of the the duopoly locations will be \( \tilde{\theta} - \theta \). That is, he serves the senders with a lower preference for quality.

For the two firms, the marginal cost of providing a letter is independent of the quantity. However, the cost is quadratic in quality, \( C(x_i) = \frac{c x_i^2}{2} \).

Finally, the dynamic game that play the firms is the following. First, firms decide which villages will serve. Having decided their coverage, in a second stage they choose the quality of the service they will offer to all their consumers. Finally, in a third stage, they set the prices of the letters. The solution concept of the game is the subgame perfect Nash-equilibrium.

3 The first best allocation

This section analyses the presence of a benevolent regulator that chooses the optimal qualities and coverage levels of the incumbent and the entrant to maximise social welfare. The social welfare function of the regulator is the unweighted sum of the consumer surplus and the profit of the firms. This can be written as

\[ W = \mu_2 \left( \int_0^{\bar{\theta}} (\theta x_1 - \frac{c x_1^2}{2})d\theta + \int_0^{\bar{\theta}} (\theta x_2 - \frac{c x_2^2}{2})d\theta \right) \]

\[ + (\mu_1 - \mu_2) \int_0^{\bar{\theta}} (\theta x_1 - \frac{c x_1^2}{2})d\theta - F(\mu_1) - F(\mu_2). \]  

(2)

In this equation, \( \bar{\theta} \) represents the marginal consumers in the villages covered by the duopoly (the segment \([0, \mu_2]\)) that are indifferent between the incumbent and the entrant. The consumers with a higher preference for quality prefer the service of the incumbent, and those with a lower preference prefer the entrant.

The following proposition presents the optimal allocation of qualities and coverage that maximises social welfare.
Proposition 1. Given the relative coverage of the firms, \( K = \frac{\mu_1}{\mu_2} \), the optimal levels of quality of the two firms are

\[
x_1^o = \frac{8\theta + 9K - 3(9K^2 - 8K)^{1/2}}{8c}; \quad x_2^o = \frac{8\theta + 3K - (9K^2 - 8K)^{1/2}}{8c}.
\]

Given that \( \mu_1 = \overline{\mu} = 1 \), the optimal coverage of the entrant satisfies

\[
\mu_2 = \frac{(108cf(\mu_2))^{1/2} - 2^{1/2}}{(64c^2f(\mu_2)^2)^{1/2}}.
\]

The optimal quality allocations in (3) depend on the coverage of the firms. Clearly, only when \( K = 1 \) the optimal qualities are the same than in the standard model of product differentiation developed by Moorthy (1988) and Cremer and Thisse (1994).

\[
x_1^o = \frac{4\theta + 3}{4c}; \quad x_2^o = \frac{4\theta + 1}{4c}.
\]

When the incumbent have a larger coverage than the entrant, \( K > 1 \), the optimal levels quality are lower than with equal coverage, \( K = 1 \). The explanation of this result is that the regulator wants to improve the situation of the consumers in the villages that are only served by the incumbent. As the incumbent offers a uniform high quality across all villages, by reducing the quality of the incumbent the regulator worsens the welfare of the consumers with a high preference for quality in all villages to increase welfare of the consumers with a low preference for quality in the monopoly region. The bigger is the monopoly area the larger is the reduction of quality. On the other hand, if the quality of the incumbent in the duopoly area is reduced it is also optimal to reduce the quality of the entrant.

The determination of the optimal levels of coverage is complex because it is necessary to choose the coverage level of each firm and the optimal difference in coverage between the firms. However, notice that when \( \mu_1 = \overline{\mu} = 1 \), as the incumbent already covers all the country, the regulator only need to determine the coverage of the entrant, \( \mu_2 \). In this case, the optimal coverage is the one stated in proposition. Moreover, if the marginal cost of coverage is sufficiently small \( f(\mu_2) < \frac{1}{2c} \), it is optimal to force the entrant to cover all the country, \( K = 1 \). If the marginal cost of coverage is large, it is not worthy to force the entrant to cover all the country and then \( K < 1 \).

Finally, notice that with the first best prices, \( p_i = \frac{cx_i^2}{2} \), the firms will be making a loss equal to \( F(\mu_i) \) in each village. In this situation, if the regulator cares about the budget equilibrium of the firms she should establish higher prices.
4 Duopoly under Universal Service Obligations

This section develops a model of competition between an incumbent postal operator and an entrant. The incumbent is regulated with a uniform pricing and quality constraints and with a coverage constraint. This implies that he must serve all villages with the same price and the same quality, and that the regulator may regulate his coverage level. As described previously, the firms play a game in three stages. First, they establish their coverage, then the quality of their service and finally they set the prices. To analyse this situation we solve the problem by backwards induction.

4.1 Third stage: determination of prices

In the third stage of the game the firms establish their prices, given that they have previously chosen the frequency of their deliveries and their coverage. We consider that the price of the two firms is subject to a uniform pricing constraint. This implies that they can not discriminate between the consumers of different locations, offering them different qualities and/or different prices. These regulations will have an important impact on the incumbent. Indeed, while the incumbent is a monopoly in the region \([\mu_2, \mu_1]\), he competes as a duopolist in the region \([0, \mu_2]\). If instead of being regulated with a uniform pricing constraint the incumbent was unregulated he would prefer to set different prices in each region.

The problem of the incumbent in the third stage of the game can we written in the following way

\[
\text{maximize} \quad \Pi_1 \{p_1\} = \mu_2(p_1 - \frac{c x_1^2}{2})(\theta - \frac{p_1 - p_2}{x_1 - x_2}) + (\mu_1 - \mu_2)(p_1 - \frac{c x_1^2}{2}) - F(\mu_1). \quad (5)
\]

Here the incumbent has two sources of profits, the profits in the duopoly and the monopoly region, \(\Pi_1 = \pi_1^m + \pi_1^d\). The entrant, however, only obtain profits from the duopoly region.

\[
\text{maximize} \quad \Pi_2 \{p_2\} = \mu_2(p_2 - \frac{c x_2^2}{2})(\frac{p_1 - p_2}{x_1 - x_2} - \theta) - F(\mu_2). \quad (6)
\]

Solving the first-order conditions of these problems and rearranging we obtain the following prices, that depends on the qualities of the firms and on the level of relative coverage \(K = \frac{\mu_1}{\mu_2}\).

\[
p_1 = \frac{1}{3}[(x_1 - x_2)(2\theta - \theta) + \frac{c}{2}(2x_1^2 + x_2^2) + 2(x_1 - x_2)(K - 1)], \quad (7)
\]

\[
p_2 = \frac{1}{3}[(x_1 - x_2)(\theta - 2\theta) + \frac{c}{2}(2x_2^2 + x_1^2) + (x_1 - x_2)(K - 1)], \quad (8)
\]
Observe that as $x_1 > x_2$ and $K \geq 1$, the price of the incumbent is always higher than the price of the entrant. A more careful analysis of these prices allows us write the following Lemma.

**Lemma 1.** In the third stage of the game the prices only depend on the qualities provided by each firm and on the relative coverage $K$. Under uniform pricing constraint $p_1$ and $p_2$ increase with $x_1$, and they decrease with $x_2$ when the marginal cost of quality of the entrant is small. An increase of $x_1$ increases the profits of the incumbent in the monopoly region, $\pi_m^1$, and the profit of the entrant, $\Pi_2$. It can also generate an increase in the profit of the incumbent in the duopoly region, $\pi_d^1$, when this region is large. An increase of $x_2$ produce the opposite effects.

An increase in the incumbent’s quality causes an increase of his price. This is because it increases the product differentiation between the firms and the cost of the quality. Moreover, if the coverage of the entrant is lower than those of the incumbent, $K > 1$, an increase in the incumbent’s quality makes profitable for him to increase in a greater extent his price above the duopoly price. The explanation of this is that when the quality of the incumbent increases the monopoly price increases as well. Given this, if the incumbent has a monopoly area he finds profitable to increase his uniform price to maximise the overall profit. The same type of reasoning explains why the price of the entrant increases with an increase of the quality of the incumbent: (1) the products are more differentiated; (2) the costs of the incumbent increase; and (3) the increase of the incumbent’s price allows the entrant to raise his own price.

An increase in the entrant’s quality may have an opposite impact. When the entrant increases his quality there is a reduction in the product differentiation between the two firms. The effect of this is an increase of competition that reduces the prices. On the other hand, as the reduction of price increases the difference between the price that the incumbent will establish if all the country was a duopoly and the monopoly price, the incumbent prefers to reduce the uniform price. In contrast to these two effects, an increase in the entrant’s quality also produces an increase in his costs. But if this increase is small the overall impact of the increase in the entrant’s quality is a reduction in the prices of the firms.

The increase of quality of the incumbent (and a reduction of quality of the entrant) has a similar impact on the profit of the entrant and on the profit of the incumbent in the monopoly region. An increase in the quality of the incumbent increases the product differentiation between the firms and increases the profit of the entrant. On the other hand, an increase of quality of the incumbent (and a reduction of quality of the entrant, when the cost of the quality is small) allows to increase the price of the incumbent and to set it closer to the monopoly price. This increases the profit of the incumbent in the monopoly region.
Finally, the effect of a change in the quality of the incumbent (or the entrant) in the profit of the incumbent in the duopoly region depends on the level of coverage of the entrant. An increase in the incumbent’s quality (and a reduction in the entrant’s quality) increases the uniform price of the incumbent. As this exacerbates the deviation from the price that will establish a duopolist, the profit of the incumbent in the duopoly region is reduced. At the same time, the increase of quality of the incumbent increases the price of the entrant, and as a consequence increases the profit of the incumbent in the duopoly region. In this situation, if the coverage of the entrant is large enough this later effect will compensate the first, because in this case the incumbent establishes a price closer to the one that will set a duopolist.

4.2 Second stage: determination of qualities

Once the incumbent and the entrant have rolled-out their network they establish the qualities of their services. As we have mentioned before, the election of qualities determines the level of product differentiation between firms and therefore the strength of competition.

Substituting the equations of prices in (7) and (8) in the profit functions in (5) and (6) we obtain:

\[
\text{maximize } \Pi_1 \left\{ x_1 \right\} = \frac{\mu_2(x_1 - x_2)}{9} \left( (2\theta - \theta) + \frac{c}{2}(x_1 + x_2) + 2(K - 1)^2 \right) - F(\mu_1), \quad (9)
\]

\[
\text{maximize } \Pi_2 \left\{ x_2 \right\} = \frac{\mu_2(x_1 - x_2)}{9} \left( (2\theta - 2\theta) + \frac{c}{2}(x_1 + x_2) + (K - 1)^2 \right) - F(\mu_2). \quad (10)
\]

Solving the first order conditions of these problems we obtain the following qualities for each firm:

\[
x_1 = \frac{4\theta + 5K}{4c}; \quad x_2 = \frac{4\theta - K}{4c}. \quad (11)
\]

Substituting these qualities into the equations of prices yields:

\[
p_1 = \frac{16\theta^2 + 40\theta K + 49K^2}{32c}; \quad p_2 = \frac{16\theta^2 - 8\theta K + 25K^2}{32c}. \quad (12)
\]

In order to compare these qualities with the first best qualities in (3) observe that for any given $K > 0$ the quality of the incumbent is always higher than the first best, and the quality of the entrant is always lower. This result reflects the
property demonstrated by Cremer, De Rycke and Grimaud (1997) that private firms tend to choose a higher than optimal degree of product differentiation in order to reduce the intensity of price competition. By increasing product differentiation firms can set higher prices and obtain larger profits.

On the other hand, the analysis of qualities also shows that the degree of product differentiation increases with the level of relative coverage of the firms.

**Proposition 2.** With independence of who provides the highest level of quality, the level of relative coverage of the firms, $K$, increases the degree of product differentiation.

Only when $K = 1$ the private duopoly establish the standard level of quality find by Cremer and Thisse (1994). When the level of relative coverage of the firms increases the quality of the high quality product increases and the quality of the low quality product decreases.

On the other hand, the effect of relative coverage is independent of which firm is offering the product with the high quality. The same result appears when the incumbent offers the high quality product and when the entrant does it. The reason for this is that in the two cases when the difference in coverage increases, the monopoly region of the incumbent is larger and he finds more profitable to increase the product differentiation in order to increase his uniform price. Although this strategy reduces the profit of the incumbent in the duopoly region, these losses are compensated with the higher profit obtained in the monopoly villages. For the entrant it is always profitable to increase the differentiation of the products in order to be able to establish a higher price. This is justified because the reduction of the duopoly region is compensated with higher profits in the remaining duopoly villages.

This result has important implications. It is a well-known result in the literature about vertical differentiation that private markets may provide poor results with regards of service quality. The previous proposition shows that when firms freely decide their coverage their choice of product differentiation is even worse because they have a new mechanism to increase their differentiation. The consequence of this strategy is an increase in the prices of the two firms.

The following proposition further characterises the optimal behaviour of the firms in terms of quality.

**Proposition 3.** For the incumbent and for the entrant the qualities are strategic substitutes,

$$\frac{dx_1}{dx_2} = \frac{dx_2}{dx_1} = \frac{1}{3} > 0.$$  \hspace{1cm} (13)

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For any given level of $K$, if one firm increases his quality the other will do the same. If one firm increases his quality he will attract some consumers of his rival. To neutralise this effect, the best response of the rival is to increase his quality as well. In spite of this, the response of the rival is not proportional. When the quality of the entrant increases the quality of the incumbent increases less than proportionately to maintain his market share. As a result, product differentiation is reduced. On the other hand, when the incumbent increases his quality, the quality of the entrant increases less than proportionately because he benefits with the increase of product differentiation.

4.3 First stage: determination of coverage levels

We finally analyse the coverage decision of the firms in the first stage of the game. After substituting the qualities of the firms in (11) into the profits functions we obtain

$$\text{maximize } \Pi_1 \{\mu_1\} = \frac{\mu_2}{9c} \left(\frac{3K}{2}\right)^3 - F(\mu_1),$$

(14)

$$\text{maximize } \Pi_2 \{\mu_2\} = \frac{\mu_2}{9c} \left(\frac{3K}{2}\right)^3 - F(\mu_2).$$

(15)

Departing from these simplified versions of the profits functions the next proposition characterises the profit maximising prices, qualities and coverage levels of the firms.

**Proposition 4.** A private duopoly where the incumbent offers the high quality service satisfies

$$p_1 = \frac{30\theta^2}{c}; \quad p_2 = \frac{12\theta^2}{c},$$

$$x_1 = \frac{6\theta}{c}; \quad x_2 = 0,$$

$$\frac{18\theta^2}{c} = f(\mu_1); \quad K = 4\theta.$$  

(16)

Regardless of the level of coverage chosen by the incumbent the entrant always establishes a coverage that satisfies $K = 4\theta$. To see this note that the profit-maximising condition of the entrant with respect to coverage shows that he prefers to reduce coverage as much as possible. However, as the quality of the entrant
can not be negative, his minimum level of coverage is $K = 4\theta$. With this level of relative coverage the quality of the entrant is established at $X_2 = 0$. The intuition behind this result is that the entrant finds optimal to reduce as much as possible his level of coverage to increase the differentiation of the products. This strategy weakens price competition and raises profits. For the incumbent the increase of product differentiation is also preferable. However, if the marginal cost of coverage $f(\mu_1)$ is sufficiently high he can choose a level of coverage below $\bar{\mu}$.

On the other hand, it is convenient to explain that the willingness of the entrant to increase as much as possible the degree of product differentiation is exacerbated by our assumption of complete participation. With complete participation all senders buy the service to one firm. This explains the interest of the entrant to increase as much as possible product differentiation to raise the price. However, this strategy could be reversed if senders have a reservation value.

Another interesting consequence of Proposition 4 is that as $\mu_2 < \mu_1$ the entrant obtains a larger profit than the incumbent. Indeed, the profit functions in (14) and (15) show clearly that the only difference between the profits of the two firms is originated by the coverage costs. The fact that the entrant has a higher profit than the incumbent can be better understood if we consider that he can increase his price driven the quality of the incumbent further up. And in our model this can be done reducing coverage. This result suggest that the entrant has more power in determining the quality of the incumbent that vice versa. And this advantage appears regardless of who is providing the high quality, because in the two situations it is the entrant who determines $K$ when he decides his level of coverage.

Finally, the next result shows the best responses of the firms when they establish their level of coverage.

**Proposition 5.** For the incumbent the levels of coverage are strategic substitutes, while for the entrant they are strategic complements

$$ \frac{d\mu_1}{d\mu_2} < 0; \quad \frac{d\mu_2}{d\mu_1} > 0. \quad (17) $$

This proposition is equal to Proposition 1 in Valletti, Hoernig and Barros (2002), who analyse the interaction between a uniform pricing constraint and a coverage constraint in a private duopoly. In our model, the second part of the statement requires a further explanation. The intuition of the first part of the

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9The result that the low quality firm has a high profit is consistent with Moorthy (1988) for the case where firms choose products simultaneously. On the other hand, from Cremer, De Rycke and Grimaud (1997) it can be proof that the two firms obtain the same profit when they have the same level of coverage.
proposition is that when the entrant increases his coverage, the best reaction of
the incumbent is to reduce his own coverage because the large coverage of the
entrant forces him to reduce his price. With a lower uniform price the incumbent
does not find profitable to serve some previously attended high cost villages.

The entrant has a different reaction when incumbent increases his coverage.
An increase in the coverage of the incumbent increases the level of relative cov-
erage, $K$. In this situation, the entrant finds profitable to increase his coverage
in order to reduce $K$ to $4\theta$. The result of this is that the quality of the entrant
maintained in its minimum, but he covers more locations than before and have
more profits.

5 The regulation of quality

In a liberalised postal market the regulator may contemplate the possibility of
modifying the qualities of the firms to increase welfare. As we have seen in
the previous sections, private duopolists choose an inefficient level of product
der differentiation: the low quality firm produces a level of quality too low and the
high quality firm produces a level of quality too high. Having this in mind, in this
section we analyse the impact of regulating the qualities to bring them closer to
the first best allocations. In order to observe the full impact of these policies we
consider that the firms know the extension of the regulatory policy before they
make their coverage choices.

**Proposition 6.** A reduction of the quality of the incumbent maintains constant
the quality of the entrant and reduces the coverage of the incumbent and the
entrant. Compared to the previous situation, the consumers of the incumbent
have a lower quality and a lower price. Some of the consumers that before where
with the entrant now are with the incumbent. As the entrant reduces his coverage
some of the consumers that before were served by the entrant now are served by
the incumbent. The consumers of the entrant have the same quality, but now pay
a lower price.

As we has seen in proposition 3, a reduction in the quality of the incumbent,
$x_1$, reduces the quality of the entrant, $x_2$. However, in equilibrium it is not
possible to reduce $x_2$ anymore. On the other hand, the reduction in the product
der differentiation reduces the prices of the firms and this in turn forces the incumbent
to reduce his coverage. When this happen, in order to maintain $K$ constant the
entrant reduces his coverage as well, but in a lower proportion than the incumbent.

Having in mind all these effects, it is possible to conclude that the main benefit
of a reduction in the quality of the incumbent is the reduction in the prices of the
two firms. However, not all consumers benefit from this situation. First, because
there is a reduction in the coverage of the incumbent and second because some
consumers of the incumbent prefer to pay a higher price in order to have a higher quality.

The next proposition considers the effect of increasing the quality of the entrant to bring it closer to the efficient level. As Cremer, De Rycke and Grimaud (1997) state, the imposition of a minimum quality standard appears to be "the most natural instrument to achieve an increase in average quality". The reason for this is that an increase in the low quality level also brings about an increase in the high quality level. However, when firms can modify their coverage the imposition of a minimum quality does not benefit all consumers.

**Proposition 7.** An increase in the quality of the entrant increases the quality of the incumbent in a lower proportion. The reduction in the product differentiation reduces the prices of the two firms, increases the coverage of the entrant and reduces the coverage of the incumbent. Compared to the previous situation, the consumers of the incumbent and the entrant have a higher quality and pay a lower price. Some of the consumers that before where served by the incumbent in the monopoly region now prefer the entrant, and some villages are not served.

As we have seen in Proposition 3, an increase in one unit of $x_2$ increases $x_1$ by $\frac{1}{3}$. Therefore, it reduces the product differentiation and, in turn, it reduces the prices of the firms.

When the regulator establishes a minimum quality level the entrant is not able to control the product differentiation and as a consequence he is not interested in reducing strategically his coverage. By contrast, the entrant increases his coverage until the marginal revenue of serving an additional village is equal to his marginal cost. On the other hand, by proposition 4 an increase in the coverage of the entrant reduces the coverage of the incumbent. Indeed, with a large coverage of the entrant and with a lower price the incumbent will be willing to reduce his coverage. This reduces the welfare of a group of high cost villages that previously were attended.

To sum up, the establishment of a minimum quality standard for the entrant reduces the coverage of the incumbent. However, it reduces the degree of product differentiation and as a result lowers the prices.

6 The regulation of coverage

A key aspect of the Universal Service policy is the obligation of the incumbent postal operator to serve the non-profitable villages or regions of the country. The

10 Ronnen (1991) and Crampes and Hollander (1994) also analyse the establishment of a minimum quality standard.

11 On the other hand, the increase in the qualities of the firms increase the indifferent consumer. But the reduction of relative coverage reduces it. When the first effect is more important than the second the entrant obtains a higher market share in each village of the duopoly region.
obligation of ubiquity together with the establishment of a uniform price implies that the regulated firm must subsidise the loss-making regions with the profits he obtains in other zones. The viability of this policy, however, is threatened by the possibility of entry into the profitable low-cost regions. Next, we analyse the effect of imposing a minimum coverage for the incumbent and the entrant in order to avoid this situation.

Proposition 8. An increase in the coverage of the incumbent increases the coverage of the entrant and maintains the qualities and the prices of the two firms unaltered. Compared to the previous situation, the consumers of the incumbent and the entrant pay the same and have the same quality. Some consumers that before were served by the incumbent in the monopoly region now are served by the entrant and are better off.

By Proposition 5, when the coverage of the incumbent increases the entrant increases his coverage as well to keep $K$ constant and to maximise product differentiation. However, to maintain $K$ constant the increases of coverage of the entrant needs to be less than proportionally to the increase of the incumbent’s coverage. On the other hand, as $K$ is keep constant, the qualities of the two firms and the prices do not change.

The result of the regulation imposed on the incumbent is very clear. First, the coverage constraint on the incumbent benefits a group of villages that previously where not attended. On the other hand, now a new group of villages are served by the duopoly, and some consumers that before were served by the incumbent now choose the lower quality and price of the entrant.

In contrast to our result, Valletti, Hoernig and Barros (2002) show that when the coverage of the incumbent is increased the price of the incumbent and the entrant increase as well. One explanation for this difference is that in their model when the coverage of the incumbent increases the coverage of the entrant increases less than proportionately and $k$ increases. As a result, the prices of the two firms increase because a larger coverage makes the incumbent more accommodating. In our model, by contrast, the strategy of the entrant is to keep $K$ constant at $4\theta$ to maintain $x_2 = 0$. As a consequence, the prices are kept constant and there are not groups of consumers than can be worse than before.

Finally, the next proposition studies the consequences of imposing a minimum coverage level on the entrant.

Proposition 9. An increase in the coverage of the entrant decreases the coverage of the incumbent and the product differentiation between the two firms. Compared to the previous situation, the consumers of the incumbent and the entrant pay less. Some of the consumers that before were served by the incumbent in the monopoly region now are served by the entrant and are better off. Some consumers that previously were served by the entrant in the duopoly region now are served by the incumbent.
An increase in the coverage of the entrant decreases the relative coverage, $K$. The consequence of this is a reduction in the quality of the incumbent and an increase in the quality of the entrant that reduces the product differentiation. The joint effect of lower product differentiation and lower relative coverage reduces the prices.

The higher coverage of the entrant allows the consumers of some villages that previously were served by the incumbent to use the entrant. On the other hand, the reduction of relative coverage reduces the value of the indifferent consumer in each village and as a consequence some consumers previously served by the entrant move to the incumbent.

To summarise, the overall number of villages served increases with the imposition of a coverage constraint on the incumbent and decreases with the imposition of a minimum coverage on the entrant. In spite of this, the two types of regulations increase the duopoly area. Finally, while the regulation of the incumbent maintains the qualities and the prices unaltered, the regulation over the entrant reduces the qualities and the prices of the two firms.

7 Impact of network externalities

This section extents our model to introduce the presence of network externalities. We assume that senders obtain a higher utility when they belong to a large network because they can send their letters to more addressees. We denote by $b$ the magnitude of the network externality. Taking this into account, one sender with type $\tilde{\theta}$ placed in one of the villages served by the duopoly will be indifferent between the incumbent and the entrant when

$$\tilde{\theta}x_1 + b\mu_1 - p_1 = \tilde{\theta}x_2 + b\mu_2 - p_2$$

(18)

Therefore, the demand of a high quality incumbent in these locations can be defined as

$$\overline{\theta} - \tilde{\theta} = \overline{\theta} - \frac{p_1 - p_2 + b(\mu_2 - \mu_1)}{x_1 - x_2}$$

(19)

On the other hand, the demand of the entrant can be defined as $\tilde{\theta} - \overline{\theta}$. Observe that with this new representation of the demands the larger is the coverage of the incumbent with respect those of the entrant the larger is his demand in each village.

If we incorporate into the model these new demands and we solve the problem of the firms like in the previous sections we find that the profit-maximising qualities of the firms are
\[ x_1 = \frac{4\theta + 5K}{4c} - \frac{2b(K-1)\mu_2}{3K}; \quad x_2 = \frac{4\theta - K}{4c} - \frac{2b(K-1)\mu_2}{3K} \] (20)

Therefore, although the level of product differentiation is the same than before, the presence of the externalities lead the two firms to reduce their qualities. Notice, moreover, that now the firms use their difference in coverage as a mechanism to differentiate their products. With network externalities the firms are able to differentiate their products in the same extent than without externalities, but they need to spend less in quality.

The presence of network externalities increases the price of the incumbent and decreases the price of the entrant. The incumbent increases his price taking advantage of his larger coverage. Moreover, with a higher price he is able to reach more villages. On the other hand, the entrant reduces his price to compensate his lower coverage.

Finally, the next proposition compares the coverage of the entrant with the situation without externalities.

**Proposition 10.** Network externalities increase the coverage of the entrant. Therefore \( K \) is lower than \( 4\theta \). If \( b \) is sufficiently large, the entrant prefers to increase his coverage than to increase product differentiation.

The intuition behind this result is that as the quality of the entrant is lower with network externalities, in order to maintain \( x_2 \) at zero the entrant needs to establish a value for the relative coverage larger than \( 4\theta \). In spite of this, with network externalities it is not sure that the entrant will reduce his quality as much as possible to increase product differentiation. If \( b \) is sufficiently large the entrant will be interested in increasing his coverage in order to attract consumer. When this occurs the entrant establishes a positive quality.

### 8 Conclusions

This paper has analysed the strategic interactions among different regulations that are generally used by Governments to design the universal service obligations in the postal sector. In particular, we have studied the strategic behaviour of the firms when there is endogenous product and coverage selection. In accordance with the previous literature we have shown that one entrant in the postal market may be interested in keeping low his level of coverage to raise profits. On the other hand, we have shown that when the quality of the service is endogenous, the firms are interested in increasing the relative coverage even more to increase product differentiation and to raise further their profits. Finally, we have explained that in presence of network externalities the firms use their level of coverage as a
mechanism to differentiate their product, and that this allows them to reduce the quality of their product.

The analysis of the interaction between coverage and quality may help to understand better the implications of the Universal Service Obligations (USO) in the network industries. In the postal sector, the USO has been traditionally designed to offer a standard service at uniform and affordable rates. However, in the liberalised markets the provision of the service has been affected in several ways by the new participants in the industry. The entrants may reduce the profitability of the incumbent in the low cost regions and may force him to modify the quality of the services that have been offered until present.

In the postal sector there is a clear trade-off between the level of coverage of the operators and the quality they can offer. This implies that some high-cost regions could only be served in a profitable way by reducing the quality of the service. Taking this into account, the entry of new competitors in the sector and the corresponding reduction of revenues of the incumbent operator would call for a reduction of the coverage and the quality of the incumbent if a reserved area is not maintained or if there are not used other mechanisms to finance the operator.

Another solution suggested by Crew and Kleindorfer (1998) to alleviate the problem will be to re-examine the role of the service standards in the sector and to reduce the quality of the service in some circumstances. "For example, outlying areas might receive service three days a week instead of the typical five or six currently. In other areas Saturday service might be eliminated. In the United Kingdom, twice daily deliveries might be eliminated in most areas. Another variable to consider might be to slow delivery. For example, in the case of First Class post in United Kingdom, instead of providing service on the next day, First Class service would be redefined for outgoing areas to mean service on the second business day."\(^{12}\)

Finally, an alternative measure analysed in this paper is to complement the regulation of the incumbent with some quality and coverage regulations on the entrant. This could imply, for example, to impose the coverage of some high-cost regions or to increase the number of delivery days per week.

Appendix

Proof of Proposition 1. Maximizing \( W \) with respect to \( \tilde{\theta} \) we obtain the optimal marginal consumer in each village \( \mu \in [0, \mu_2] \),

\[
\tilde{\theta} = \frac{c(x_1 + x_2)}{2}.
\]

Substituting this expression into the welfare function in (2) and maximising with respect to the two qualities we obtain

\[-\frac{1}{2}\theta^2 + \frac{1}{2}(c\frac{x_1 + x_2}{2})^2 = cx_2 \frac{c(x_1 + x_2)}{2} - cx_2\theta.\]

(22)

\[-\frac{1}{2}\theta^2 + \frac{1}{2}(c\frac{x_1 + x_2}{2})^2 = cx_1 \frac{c(x_1 + x_2)}{2} - cx_1\theta + \frac{\mu_1}{\mu_2}[(\frac{1}{2}\theta^2 - cX_1\theta) - (\frac{1}{2}\theta^2 - cX_2)].\]

Considering that \(K = \frac{\mu_1}{\mu_2}\) and after solving the equations we obtain the first part of the proportion.

Finally, substituting the optimal qualities into the welfare function, assuming that \(\mu_1 = \bar{\mu} = 1\) and maximising with respect to \(\mu_2\) we obtain the optimal coverage of the entrant.

**Proof of Lemma 1.** From the equations of prices we obtain the following results

\[\frac{\partial p_1}{\partial x_1} = \frac{1}{3}[(2\theta - \bar{\theta}) + 2cx_1 + 2(K - 1)] > 0,\]

(24)

\[\frac{\partial p_1}{\partial x_2} = \frac{1}{3}[-(2\theta - \bar{\theta}) - cx_2 + 2(K - 1)],\]

(25)

\[\frac{\partial p_2}{\partial x_2} = \frac{1}{3}[-(\theta - 2\theta) + 2cx_2 - (K - 1)],\]

(26)

\[\frac{\partial p_2}{\partial x_1} = \frac{1}{3}[(\theta - 2\theta) + cx_1 + (K - 1)] > 0.\]

(27)

When the marginal cost of the quality of the entrant is small (when \(cx_2\) is small) then \(\frac{\partial p_1}{\partial x_2}\) and \(\frac{\partial p_2}{\partial x_2}\) are negative.

To analyse how the modification of the quality levels impact the profits of the firms observe that
\[
\begin{align*}
\frac{d\Pi_2}{dx_1} &= \frac{\partial \Pi_2}{\partial p_1} \frac{\partial p_1}{\partial x_1} > 0, \\
\frac{d\Pi_2}{dx_2} &= \frac{\partial \Pi_2}{\partial p_1} \frac{\partial p_1}{\partial x_2} < 0, \\
\frac{d\Pi_m^m}{dx_1} &= \frac{\partial \Pi_m^m}{\partial p_1} \frac{\partial p_1}{\partial x_1} > 0, \\
\frac{d\Pi_m^m}{dx_2} &= \frac{\partial \Pi_m^m}{\partial p_1} \frac{\partial p_1}{\partial x_2} < 0, \\
\frac{d\Pi^d_1}{dx_1} &= \frac{\partial \Pi^d_1}{\partial p_1} \frac{\partial p_1}{\partial x_1} + \frac{\partial \Pi^d_1}{\partial p_2} \frac{\partial p_2}{\partial x_1}, \\
\frac{d\Pi^d_1}{dx_2} &= \frac{\partial \Pi^d_1}{\partial p_1} \frac{\partial p_1}{\partial x_2} + \frac{\partial \Pi^d_1}{\partial p_2} \frac{\partial p_2}{\partial x_2}.
\end{align*}
\]

The sign of equations (33) and (34) depend on the coverage level of the entrant. First, observe that \( \frac{\partial p_1}{\partial x_1} > \frac{\partial p_2}{\partial x_1} \). Taking into account that with the equilibrium price \( \frac{\partial \Pi^d_1}{\partial p_1} > 0 \), we have that \( \frac{d\Pi^d_1}{dx_1} < 0 \). However, when the duopoly area is large it may be that the second term in the right hand-side of equation (33) is bigger than the first because the price is closer to the one that will establish a duopolist, and as a consequence \( \frac{d\Pi^d_1}{dx_1} < 0 \) is small. As a result, an increase in \( x_1 \) may generate an increase in the profit of the incumbent. For the same reason, an increase of \( x_2 \) may generate a decrease in the profit of the incumbent.

**Proof of Proposition 2.** The qualities of the two firms when the incumbent provides the high quality service are defined in equation (11). From them we obtain

\[
\frac{d(x_1 - x_2)}{dK} = \frac{3}{2c}.
\]

On the other hand, the qualities of the two firms when the entrant provides the service with the high quality are

\[
x_2 = \frac{4\theta + 4 + K}{4c}, \quad x_1 = \frac{4\theta + 4 - 5K}{4c}.
\]

In this case, the effect of a variation in relative coverage satisfies

\[
\frac{d(x_2 - x_1)}{dK} = \frac{3}{2c}.
\]
Therefore, regardless of who is providing the high quality service an increase in relative coverage increases product differentiation. ■

Proof of Proposition 3. The first and second order conditions of firm 1 are

\[
\frac{(2\bar{\theta} - \theta)}{2} - cx_1 + K - 1 - \frac{c (x_2^2 - x_1^2)}{4 (x_1 - x_2)} = 0, \\
-\frac{3c}{4} < 0.
\]

(38)

Taking this into account we can write

\[
\frac{dx_1}{dx_2} = -\frac{\frac{c}{x_2}}{\frac{c}{x_1}} > 0.
\]

(39)

The first and second order conditions of firm 2 are

\[
-\frac{(\bar{\theta} - 2\theta)}{2} - cx_2 - \frac{(K - 1)}{2} + \frac{c (x_1^2 - x_2^2)}{4 (x_1 - x_2)} = 0, \\
-\frac{3c}{4} < 0.
\]

(40)

(41)

From them we obtain

\[
\frac{dx_2}{dx_1} = -\frac{\frac{c}{x_1}}{\frac{c}{x_2}} > 0.
\]

(42)

Therefore, the firms react less than proportionately to an increase of the quality of their rival. ■

Proof of Proposition 4. The result is obtained directly from the first order conditions. The first order conditions of \(\pi_2\) with respect to \(\mu_2\) is always negative. ■

Proof of Proposition 5. The first and second order conditions of the incumbent’s profit with respect to \(\mu_1\) are

\[
\frac{1}{2c} (\frac{3K}{2})^2 - f(\mu_1), \\
\frac{3}{2c\mu_2} (\frac{3K}{2}) - f'(\mu_1) \leq 0.
\]

(43)

(44)

The second-order condition is only satisfied when the cost function \(F(\mu_1)\) is sufficiently convex. In this case,
\[
\frac{d\mu_1}{d\mu_2} = -\frac{9\mu_1^2}{4\mu_2^2} - f'(\mu_1) < 0. 
\] (45)

For the second part of the proposition note that the entrant always prefers to reduce \(\mu_2\). When the incumbent increases \(\mu_1\) the entrant can increase \(\mu_2\) to maintain \(K\) constant in \(\frac{\mu_1}{\mu_2} = 4\hat{\theta}\). \(\blacksquare\)

**Proof of Proposition 9.** An increase in \(\mu_2\) decreases the level of relative coverage.

\[
\frac{dK}{d\mu_2} = \frac{1}{\mu_2} \left( \frac{d\mu_1}{d\mu_2} - K \right) < 0. 
\] (46)

The indifferent consumer can be defined as

\[
\tilde{\theta} = \frac{1}{3}[(\theta + \bar{\theta}) + \frac{c}{2}(x_1 + x_2) + (K - 1)] = \theta + \frac{K}{2}. 
\] (47)

Therefore, the value of the indifferent consumer decreases with a reduction on the relative coverage. \(\blacksquare\)
References


