Firms are not all created equal:
Location and Price competition with cost asymmetries

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Abstract
Under Bertrand competition we can obtain either; minimal, maximal, or intermediate horizontal differentiation, or monopolistic competition by introducing varying degrees of firm marginal cost asymmetries. With sufficient asymmetries, we find that the low cost firm wishes to move close to the high cost firm while the high cost firm would prefer to maximally differentiate. This naturally yields Stackleberg behavior where the high cost firm will delay until the low cost firm commits to its location choice. We also find that welfare does not decrease with cost differentiation.

Keywords: asymmetries, horizontal differentiation, Stackleberg

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Introduction

Firms are not all created equal. Indeed there are whole industries (e.g., management consulting) and institutions of higher learning (business schools) which are dedicated to creating and sustaining differences between firms. These firm differences can arise from a myriad of causes ranging from; differing technological opportunities; differing labor conditions; different management qualities; to different timing of entry. The bulk of Industrial Organization theory, however, examines symmetric firms. Recently, there has been some reexamination of the theory to see if this assumption materially affects previously derived results. Relaxing that assumption has also opened up new questions such as which firms are more likely to engage in the behavior in question. Furthermore, by making the theory more realistic we make it more amenable to empirical validation. This study reexamines some of the seminal work in horizontally differentiated price-setting oligopolies.

Hotelling’s (1929) classic “Stability in Competition” paper which introduced horizontal differentiation into a model of Bertrand competition espoused the Principle of Minimum Differentiation. D’Aspremont, Gabsewicz and Thisse’s (1979, hereafter D’AGT) criticise Hotelling’s analysis for lacking a price equilibrium for some location choices. They then derive an example which had price equilibria but resulted in a tendency to maximal differentiation. We find that the results of D’AGT themselves lack stability. We find that their conclusions hinge on firm symmetry in marginal costs. Once slight marginal cost differences between firms are introduced in the example of D’AGT there does not exist a price equilibrium that sustains both firms for any pair of locations.

In this study we construct a modified version of the D’AGT model with firm cost asymmetries and timing considerations which have price and location equilibria. We find parameter regions which support three different types of equilibria. With low cost differences the equilibria are the same as those found by D’AGT, i.e., maximal differentiation. With high cost differences the low cost firm locates in the middle of the market and the high cost firm does not enter, i.e., there is blockading. With intermediate cost differences, the low cost firm would want to minimally differentiate while the high cost firm has a tendency to maximally differentiate.

In this region of intermediate cost differences there is no equilibrium in a game of simultaneous location choice. However, by allowing the delay of entry and location choice, we find that Stackleberg behavior naturally arises and yields unique location equilibria. In this case, the low cost firm would take the role of a Stackleberg leader and choose a location anticipating that the follower maximally differentiates. In this Stackleberg location game there are two types of equilibria, one with lower cost differences where firms maximally differentiate and another with somewhat higher cost differences where the low cost firm chooses an interior point while the high cost firm chooses a boundary point farthest away from its rival. We find cases where the Stackleberg leader must choose its second best location.
At the time when D’AGT developed their theory, the bulk of IO theory focused on models where firms were symmetric. That assumption dramatically simplifies the analysis. More recently, the impact of firm cost asymmetries in a Cournot setting have been explored by Roller, Siebert and Tombak (2005), Salant and Shaffer (2000), and Tombak (2002). One of the findings of Salant and Shaffer – that total welfare increases with cost asymmetries – is also found here in our differentiated Bertrand setting for low cost differentials (where the high cost firm has less than double the marginal cost of the low cost firm). We find, as do Salant and Shaffer, that a greater market share is allocated to the more efficient producer. Furthermore, in our setting, consumers can benefit from the cost differentiation leading to less differentiated products which entail lower transportation costs for consumers.

Asymmetries of different sorts have been examined by Anderson, and De Palma (2001), Marquez (1997), Spulber (1995), Sutton (1986), Thomas (2002), and Waterson (1990) in Bertrand settings. Anderson and De Palma develop and analyze a model of entry where firms produce products that are both vertically and horizontally differentiated of given qualities and with firms having marginal cost differences. They find that high quality goods are overpriced and underproduced and also that there may be excessive entry. They then calibrate the model using data from the yoghurt market. In vertically differentiated products Sutton looks at cost asymmetries and finds that with small cost differences the higher quality product attains greater market share. Spulber examines the situation where rivals’ costs are unknown and there is asymmetric information. Marquez and Thomas study Bertrand competition when firms have different fixed or entry costs. Marquez finds that with relatively small differences is these fixed costs all but the two lowest cost firms are blockaded and that they ignore the potential entrants. Thomas attains that asymmetric entry costs changes many of the results obtained with entry games under symmetric costs. In particular, he finds that the welfare results and the pricing patterns found with entry in symmetric models are more ambiguous when asymmetry is introduced. Waterson analyzes a location problem in a Stackleberg setting where some consumers are easier to serve than others. He finds profits are negatively related, and market share is positively associated, to difficulty of service. Tombak (2004) examines asymmetries of a rather extreme form (where the slopes of the reaction functions differ) in entry games. This present study, to our knowledge, is the first detailed examination of location and price choice under marginal cost asymmetries in a horizontally differentiated Bertrand model.

We address four issues in this study. First, we examine how cost asymmetries affect the price equilibrium. Second, we compute how cost asymmetries affect the degree of horizontal differentiation. Third, we show how cost differentials affect competition. Lastly, we address how those cost differentials affect producers’, consumers’, and total welfare.

The following section describes our horizontally differentiated Bertrand game and how we model marginal cost differences. We then find the price equilibria for given location choices and cost differentials. Subsequently there is an examination of equilibrium location choices and find the regions of cost differentials where those equilibria exist. We
then introduce into the analysis the feature that firms may delay commitment to a location and find Stackleberg equilibrium location choices where simultaneous equilibria did not exist. The equilibrium payoffs and prices are then computed and a welfare analysis is conducted. In the penultimate section we reexamine the Hotelling model with cost differentials. Finally, we summarize and discuss our findings.

The models

Using the notation and basic model of Hotelling and D’AGT say that two sellers A and B of a homogeneous product locate on a line of length 1 at respective distances \( a \) and \( b \) from the ends of this line \((a + b \leq 1; \ a, \ b \geq 0)\). Consumers are evenly distributed on the line, and each consumes a single unit of commodity per unit of time regardless of price. A customer will buy from the seller who quotes the least delivered price, since the product is homogeneous. For the Hotelling model transportation costs are linear \((tx)\). For D’AGT and our subsequent modification the transportation cost will be quadratic \((tx^2)\).

Let \( p_1 \) and \( p_2 \) denote the mill price of A and B, respectively. Our modification to the previous models will be to include constant marginal costs of production and we assume these marginal costs to be different for A and B.\(^2\) We let the average marginal costs of the industry be a multiple \( c \) of the transportation cost rate. Seller A has a marginal cost of production of \((1-r/2)ct\), while seller B has marginal costs of \((1 + r/2)ct\). Thus \( rct \) represents the difference in marginal costs and we can induce a mean preserving spread in those costs by increasing \( r \).

Price Equilibria in the model of D’AGT

D’AGT’s model modified to include differential constant marginal costs of production gives rise to a two-person game where players A and B have strategies \( p_1 \) and \( p_2 \) and identical strategy sets where the prices could range from zero to infinity. The payoff functions for the two players are now given by:

\[
\pi_1(p_1, p_2) = (p_1 - (1 - \frac{r}{2})ct)\left[a + \frac{1}{2}(1-a-b)+\frac{p_2-p_1}{2t(1-a-b)}\right]
\]

\[
\pi_2(p_1, p_2) = (p_2 - t(1-a)^2 - b^2) \leq p_1 \leq p_2 + t[(1-b)^2 - a^2]
\]

\[
= p_1 - (1 - \frac{r}{2})ct \quad \text{if} \quad p_1 < p_2 - t[(1-a)^2 - b^2]
\]

\[
= 0 \quad \text{if} \quad p_1 > p_2 + t[(1-b)^2 - a^2]
\]

\(^2\) Such marginal cost differences can arise, for example, due to one firm having: superior managerial or technical talent; intellectual property; or access to better raw materials. The reasons for such differences are exogeneous to our model.
An important distinction between D’AGT and the above model is the ability of Seller A to monopolize the market. Since $p_2 \geq ct(1+r/2)$ then Seller A would monopolize the market when $p_i < ct(1+\frac{r}{2}) - t[(1-a)^2-b^2]$ . Thus Seller A’s ability to monopolize depends on its proximity to Seller B and the marginal cost differences. For example, at $a+b=1$, in our model, the lower cost firm would charge a price of $p_1 = ct(1+r/2)$ less an arbitrarily small amount, the low cost firm would then capture the entire market and force the higher cost firm from the market and obtain a profit of $ctr$. Clearly, it is not possible for Seller B to monopolize as any undercutting the marginal costs of Seller A would involve loses for Seller B.

**Perhaps insert the graph of profits vs. price a la D’AGT**

Solving for the price equilibrium of the above system of equations (1) for interior solutions yields,

$$
p_1^* = \frac{1}{6} (6 - 4a - 2a^2 - 8b + 2b^2 + 6c - cr) t,
$$

$$
p_2^* = \frac{1}{6} (6 - 8a + 2a^2 - 4b - 2b^2 + 6c + cr)t
$$

Marginal cost differences drive the price of Seller A down while the price of Seller B goes up by an equal amount ($\frac{\partial p_i^*}{\partial r} = \frac{ct}{6}$). These equilibrium prices must always be above the respective marginal costs of the sellers. For any marginal cost differences this condition always holds for the low cost seller. For the high cost firm (Seller B) the condition for equilibrium price to be above its marginal cost is,

$$
3 - 4a + a^2 - 2b - b^2 > rc.
$$

This condition becomes more restrictive with $a$ and $b$. With minimal differentiation ($a = b = \frac{1}{2}$) any marginal cost difference ($r > 0$) would violate the above condition. This also implies that with a sufficiently high marginal cost difference (high $r$), Seller A can position itself so that the equilibrium price of Seller B is lower than its marginal cost. In
particular, when \( rc > \frac{1}{4} \) then it is sufficient for A to locate in the middle of the market \((a = 0.5)\) to drive Seller B out of the market.

Assuming both firms remain in the market, the equilibrium profits after the pricing stage are then

\[
\pi_1^* = \frac{(3 - 2a - a^2 - 4b + b^2 + cr) t}{18(1 - a - b)} \quad \text{and} \quad \pi_2^* = \frac{(3 - 4a + a^2 - 2b - b^2 - cr) t}{18(1 - a - b)}
\]  

(3)

when the conditions in (1) hold for both firms to remain in the market. With marginal cost differences \((cr > 0)\) the consequences of being minimally differentiated are different from those portrayed in D’AGT. In the model with no cost differences the firms made zero rents being in the same location. In this model when \(a+b=1\), Seller A (the firm with lower costs) would price just below its rival’s marginal cost and obtain monopoly rents of \(.crt\). This monopoly rent would be greater than \(\pi_i\) when

\[
cr \geq (1 - a - b)\left((6 - a + b) - 3\sqrt{3} - 2a + 2b\right)
\]  

(4)

Note that the R.H.S. of the above condition is decreasing in both \(a\) and \(b\) in the relevant ranges of those variables. When \(a\) and \(b\) are both zero then the R.H.S. reduces to \(3(2 - \sqrt{3})\). Thus when marginal costs differences become large enough Seller A will have an incentive to undercut Seller B and drive him out of the market regardless of the location choices of A and B. If the sellers are minimally differentiated \((a + b = 1)\) the R.H.S. is equal to zero and the condition above always holds. Consequently, in contrast to D’AGT, with minimally differentiated sellers slight marginal cost differences lead to undercutting and no interior price equilibria for all \(a\) and \(b\).

**Equilibrium Location choices**

Moving forward to the first stage of the game we derive the equilibrium location choices by taking the derivatives of the equilibrium profits (3) with respect to the respective firm location choices,

\[
\frac{\partial \pi_1^*}{\partial a} = -\frac{[(1 - a - b)(3 + a - b) + cr \{1 - a - b\}(1 + 3a + b) - cr]}{18(1 - a - b)^2}, \quad \text{and}
\]

\[
\frac{\partial \pi_2^*}{\partial b} = \frac{[(1 - a - b)(-3 + a - b) + cr \{1 - a - b\}(1 + a + 3b) + cr]}{18(1 - a - b)^2}
\]  

(5)

From the above one can easily check that when \(cr = 0\), the results of D’AGT are obtained, that is, the derivatives are both negative and sellers A and B both have an incentive to maximally differentiate. With marginal cost differences, however, the signs of the derivatives may change.
At \((a,b) = (0,0)\), the critical marginal cost difference \((4)\) becomes \(3(2 - \sqrt{3}) \approx 0.804\).

When \(0 \leq cr < 3(2 - \sqrt{3})\) then both firms prefer to maximally differentiate \((a^*, b^*) = (0, 0)\).

This location choice is a Nash equilibrium since, Seller A, if its competitor locates at \(b=0\) would find it optimal to locate at \(a^* = 0\) so long as \(cr < 3(2 - \sqrt{3})\). Furthermore, for these values of \(a\) and \(cr\) the derivative of seller B’s profits are negative so it will not deviate from \(b = 0\).

For the region of marginal cost differences \(3(2 - \sqrt{3}) < cr < \frac{3}{4}\) then the condition \((4)\) is satisfied for all \(a\) and \(b\). Consequently, seller A would wish to locate wherever seller B locates and thereby takeover the entire market. That is, seller B (the high cost seller) continues to have a tendency to maximally differentiate, while seller A (the low cost seller) wishes to minimally differentiate. If the sellers move simultaneously then seller A tries to guess where seller B would locate and then chooses the same location. Seller B, however, tries to guess where A might locate and chooses a location as far as possible from that location. In this game there is no location equilibrium. Whenever there is a difference in location, Seller A would want to deviate to a location closer to Seller B. Whenever there is no differentiation, Seller B would want to deviate to a location farther from Seller A. Thus for marginal cost differences within a certain range there is no location equilibrium for the simultaneous move game.

When \(cr > \frac{3}{4}\) Seller A continues to have an incentive to monopolize (condition \((4)\) is satisfied). However Seller A can now ensure a monopoly by positioning itself at \(a = 0.5\) (condition \((2)\) is satisfied). Therefore the Nash equilibrium location choice \((a,b)\) is \((0.5,\ NE)\), where NE denotes no entry.

The subgame payoffs for Seller A are illustrated in the following Figure. The payoff for the low values of \(a\) are from \((3)\) and the conditions given in \((1)\) with \(b=0\) implies that this payoff function is valid until \(a = 2 - \sqrt{cr} + 1\). Beyond that value of \(a\), Seller A can undercut the cost of Seller B and obtain the entire market. Seller A’s profits would then be determined by \(p_1 - (1 - \xi)st\) where \(p_1 = p_2 - t[(1 - a)^2 - b^2]\) and the price of Seller B is constrained by its costs \((p_2 = c(1 + \frac{\xi}{2})r\). Taken together this implies that Seller A’s profits are a function of its location as follows, \(\pi_1 = t[c(1 + \frac{\xi}{2}) - (1 - a)^2 + b^2]\) at these higher levels of \(a\). Consequently, comparing the above to the results of D’AGT, marginal cost differences increase the region in which price undercutting may occur. Also, when that undercutting occurs the lower cost firm can obtain positive rents.

As can be seen in Figure 1, when \(cr < 1\), Seller A has two possible optimal locations, both at the extreme points \((a = 0\ or\ a = 1)\). When \(cr < 3(2 - \sqrt{3})\) its optimal location choice is negative so that the boundary solution at \(a=0\) is preferred. When \(cr\) is greater than that value then Seller A’s profits are greater at the other boundary solution \(a=1\).
Figure 1
The payoff for Seller A for $cr < 1$ and given $b=0$

\[
\pi_1 = \begin{cases} 
\pi_1 	ext{ when } cr > \\
\pi_1 	ext{ when } cr < \\
\end{cases} \frac{3(2-\sqrt{3})}{2-\sqrt{cr} + 1} + cr \\
\frac{3(2-\sqrt{3})}{2-\sqrt{cr} + 1} + cr \\
\]

Figure 2
The payoff for Seller A for $1 < cr < 5/4$ and given $b=0$

\[
\pi_1 = \begin{cases} 
\pi_1 \text{ when } cr > \\
\pi_1 \text{ when } cr < \end{cases} \\
\frac{8}{2-\sqrt{cr} + 1} + 1 \\
\frac{8}{2-\sqrt{cr} + 1} + 1 \\
\]
As can be seen from the above illustration of payoffs in Figure 2, Seller A would prefer to locate at \( a=1 \) when \( b=0 \). In other words, \( A \) would prefer to minimally differentiate. That however, is a position Seller B would deviate from and so that cannot be an equilibrium. In order to attain location equilibria we introduce the feature that the timing of a commitment to that location choice may be delayed.

**Timing of Market Entry**

We now turn to the issue of timing of entry into the market to see if location equilibria exist in a Stackelberg framework where it does not for simultaneous location choices. Say that each seller at the moment of market entry must commit to a certain location and then play the price game examined above. We assume that entry, and response to entry, takes place instantaneously. That is, there are no lags during which a leader can make monopoly rents. Furthermore, we assume that firms will not enter unprofitable markets (i.e., that there is a small cost of entry) and that there is perfect information.

It is easily shown that when \( cr \geq \frac{3}{4} \) Seller B will avoid commitment to a location and Seller A locates at \( a = 0.5 \). Seller B will then not enter. In this case, the Stackelberg location equilibrium \((0.5, \text{NE})\) is identical to that of the simultaneous move game.

In the region where \( 3(2-\sqrt{3}) \leq cr < \frac{3}{4} \), if seller B were to move first, wherever it locates seller A moves to the same location and seller B must exit from the market. Consequently, in this case, seller B will avoid entering the market and committing to a location. If seller A moves first, then its location choices range from \( a = 0 \) to \( a = 0.5 \) and its optimal location choice depends on the degree of cost differentiation as we shall see below. Firm B then chooses \( b = 0.3 \) Given the location choices and the profitability in each of the above scenarios, it is clear that seller B would want to delay entry until after firm A has committed to a location. Also, as the game is profitable for seller A and since Seller A knows that Seller B will never enter first, firm A would want to play as soon as possible and thereby commits first to a location. With marginal cost differences, Seller A quite naturally then assumes the role of a Stackelberg leader.

*Proposition 1:* For sufficiently large marginal cost differences \( cr \geq 3(2-\sqrt{3}) \) the location choice becomes a Stackelberg game with the low cost seller being the leader.

In this Stackelberg game of location choice we first derive the optimal strategy of seller B. The profit functions after the price game are as in (3). The first order condition for the optimal location choice of Seller B is that in (4) which is always negative. Clearly, since the follower firm wishes to maximally differentiate its optimal strategy would be:

\[ b = 0.3 \]

We will restrict our analysis to Seller A choosing a location \( a \) between 0 and \( 0.5 \) recognizing that there is a symmetric equilibrium whereby if seller A chooses \( 0.5 < a \leq 1 \) then seller B chooses \( b = 1 \) with seller B being indifferent to \( b=0 \) or \( b=1 \) when \( a = 0.5 \).
If seller A chooses $0 \leq a \leq \frac{1}{2}$ then seller B chooses $b = 0$

Given the above behavior of Seller B, when $3(2-\sqrt{3}) < cr < 1$ then seller A finds it optimal to maximally differentiate as shown from the payoffs in Figure 1. This is because the cost differential is still too small for Seller A to want to engage in the less differentiated price competition with the high cost producer.

As shown in Figure 2, when $1 < cr < \frac{3}{4}$ then seller A finds it optimal to move to an interior location. This optimal interior location depends on the degree of cost asymmetry. This interior location is determined by setting $b=0$ in $\pi_t^*$ of (3) and taking the derivative with respect to $a$. This first order condition yields four possible solutions for $a$, $\frac{1}{3}(1 \pm \sqrt{4 - 3cr})$ and $-1 \pm \sqrt{4 + cr}$. Only the solution $a = \frac{1}{3}(1 - \sqrt{4 - 3cr})$ satisfies second order conditions and the requirements that $a$ be less than $2 - \sqrt{cr} + 1$ and greater than zero for the relevant values of $cr$. In this region of marginal cost differences, the lower cost seller can engage in some degree of price competition and steal some business from the high cost seller. The cost differential in this region, however, is not great enough for the low cost producers to desire undifferentiated price competition.

When $\frac{3}{4} = cr$ then seller A has a sufficient cost differential to make it optimal to monopolize the market it can do so by locating in the middle of the market $a^* = \frac{1}{2}$ and seller B is then blockaded. When $cr > \frac{3}{4}$ then seller A can blockade seller B by locating in a region around the middle of the market. When $cr > 3$ seller A can blockade seller B anywhere in the market and obtains a profit of $crt$ regardless of its location. The point $a^* = \frac{1}{2}$, however, is welfare maximizing as this is the location of a single seller which will yield the greatest utility to all the consumers. That is, due to the quadratic transportation costs of the consumers the mid point would minimize the total transportation costs.

To summarize the above discussion we have the following proposition.

**Proposition 2:** In a Stackelberg game of location choice, the equilibrium locations of sellers A and B ($a^*, b^*$) depend on the marginal cost differences as follows

$$
(a^*, b^*) = \begin{cases} 
(0, 0) & \text{if } 0 \leq cr \leq 1 \\
\left(\frac{1}{3}(1 - \sqrt{4 - 3cr}), 0\right) & \text{if } 1 \leq cr \leq \frac{3}{4} \\
\left(\frac{1}{2}, NE\right) & \text{if } \frac{3}{4} < cr
\end{cases}
$$
The equilibrium location of Seller A for given cost differentials

\[ a = \begin{cases} \frac{1}{6} \left( 1 - \sqrt{4 - 3cr} \right) & \text{if } 1 \leq cr \leq \frac{5}{4} \\ \frac{5}{4} & \text{if } \frac{5}{4} < cr \end{cases} \]

The equilibrium profits are:

\[ \pi^*_1 = \frac{1}{18} (3 + cr)^2 t, \quad \pi^*_2 = \frac{1}{18} (3 - cr)^2 t \quad \text{if } 0 \leq cr \leq 1 \]

\[ \pi^*_1 = \frac{8(4 + 3cr + 2\sqrt{4 - 3cr})^2 t}{243(2 + \sqrt{4 - 3cr})}, \quad \pi^*_2 = \frac{2(6cr - 5(2 + \sqrt{4 - 3cr}))^2 t}{243(2 + \sqrt{4 - 3cr})} \quad \text{if } 1 \leq cr \leq \frac{5}{4} \tag{6} \]

\[ \pi^*_1 = crt, \quad \pi^*_2 = 0 \quad \text{if } \frac{5}{4} < cr \]

The cost differential generally increases the equilibrium profits of Seller A, while decreasing the rents of Seller B.

The equilibrium prices given the equilibrium location choices are then illustrated below.
The price of Seller B is higher as its costs are higher. As the cost differential increases so too does the price differential. Seller A then increases its market share. As $r$ increases beyond 1, Seller A starts to move towards a less differentiated product and thereby engages more aggressively in business stealing. The prices in this region both decrease and there is an increase in the rate of Seller A’s market share increases. Finally, as the cost differential increases to the point where Seller A monopolizes the market, then its price increases at the rate of Seller B’s cost increase. Thus Seller A engages in
monopolistic competition whereby its price tracks Seller B’s costs in order to blockade entry.

The equilibrium profits and the sum of those profits are illustrated in the following figure.

Figure 5
Equilibrium profits and Producers’ Surplus with c=1,t=1
Welfare

In this section we compute the consumers’ surplus which together with the previously derived equilibrium profits make up the total welfare. With an increase in the cost differential, \( r \), the equilibrium prices diverge as shown above. Initially, so long as \( r < 1 \), the equilibrium locations are at \( a = b = 0 \). As the equilibrium price of Seller A goes down, it gains market share. Thus a higher portion of the consumers are enjoying price decreases. The consumers that are switching from Seller B to A are incurring larger transportation costs but clearly those are more than offset by the decrease in price to induce the switching. Thus, with increasing \( r \) the consumers of Seller A (B) are better (worse) off but consumer surplus overall is increasing in this range of \( r \). As \( r > 1 \), then Seller A starts to move towards the center and overall consumers enjoy transportation cost savings. Furthermore, in the range \( 1 < r < 5/4 \), both firms’ equilibrium prices are decreasing so that consumers’ surplus must be increasing at an increasing pace. Finally, when the cost differential shifts competition to monopolistic competition, when \( r > 5/4 \), consumers’ surplus must be decreasing with \( r \) as the location of Seller A does not change while its prices increase with \( r \).

The consumer surplus given a reservation value, \( v \), is defined as,

\[
CS = \int_0^x \left[ v - p_1 - t(x - a)^2 \right] dx + \int_x^1 \left[ v - p_2 - t(x - (1 - b))^2 \right] dx
\]

which after integrating, and substituting the equilibrium prices and locations simplifies to

\[
CS = \begin{cases} 
  v + \frac{t}{36} \left(-39 - 36c + c^2 r^2\right) & \text{if } cr \leq 1 \\
  v - ct + \frac{72c^2 r^2 - 754(2 + \sqrt{4 - 3cr}) + cr(582 + 69\sqrt{4 - 3cr})}{486(2 + \sqrt{4 - 3cr})} & \text{if } 1 < cr \leq \frac{5}{4} \quad (7) \\
  v - (1 + r)ct - \frac{t}{12} & \text{if } cr > \frac{5}{4}
\end{cases}
\]

This consumer surplus is illustrated in the following figure.
We define the total welfare as the sum of consumers’ and producers’ surpluses. We can see from the above systems of equations (6) and (7) that total welfare increases in the range $0 \leq r \leq 1$ as both the consumers and producers surpluses are increasing functions of $r$ in this range. When the cost differential is in the range $1 < r \leq 5/4$, there is a transfer of welfare from producers to consumers. When cost differentials become so big as to sustain monopolistic competition ($r > 5/4$) then there is a transfer of welfare from consumers to producers such that total welfare is constant.
Price Equilibria in the model of Hotelling

The with the linear transportation costs \((tx)\) of the Hotelling model, payoff functions for the two players are now given by:

\[
\pi_1(p_1, p_2) = \begin{cases} 
(p_1 - (1 - \frac{c}{2})pe^{ct}) \left[ \frac{1}{2}(1 + a - b) + \frac{p_2 - p_1}{2t} \right] & \text{if } p_2 + t(1 - a - b) \leq p_1 \leq p_2 - t(1 - a - b), \\
= p_1 - (1 - \frac{c}{2})pe^{ct} & \text{if } p_1 < p_2 - t(1 - a - b), \\
= 0 & \text{if } p_1 > p_2 + t(1 - a - b),
\end{cases}
\]

and

\[
\pi_2(p_1, p_2) = \begin{cases} 
(p_2 - c(1 + \frac{c}{2})pe^{ct}) \left[ 1 - \frac{1}{2}(1 + a - b) + \frac{p_1 - p_2}{2t} \right] & \text{if } p_1 - (1 - a - b)t \leq p_2 \leq p_1 + (1 - a - b)t, \\
= p_2 - c(1 + \frac{c}{2})pe^{ct} & \text{if } p_2 < p_1 - (1 - a - b)t, \\
= 0 & \text{if } p_2 > p_1 + (1 - a - b)t.
\end{cases}
\] (8)

Solving for price equilibria for interior locations yields,

\[
p_1^* = \frac{1}{6} (6 + 2a - 2b + 6c - cr)t,
\]

\[
p_2^* = \frac{1}{6} (6 - 2a + 2b + 6c + cr)t
\]

The equilibrium profits associated with such prices are then

\[
\pi_1^* = \frac{1}{18} (3 + a - b + cr)^2 \quad \text{and} \quad \pi_2^* = \frac{1}{18} (3 - a + b - cr)^2
\]

As Hotelling noted, the derivatives of the above equilibrium profits with respect to location choice are both positive. The above equilibrium prices and profits exist whenever

\[
\left(1 + \frac{a - b + cr}{3}\right)^2 \geq \frac{4}{3}(a + 2b + cr), \quad \text{and} \quad \left(\frac{a - b + cr}{3} - 1\right)^2 \geq \frac{4}{5}(2a + b - cr).
\]

D’AGT find in the absence of cost differences and when \(a=b\) that the locations must be outside the quartiles for price equilibrium in the Hotelling model to exist. It can be
shown that the locations under which price equilibria exist is even more restrictive with
the introduction of differences in marginal costs of production. Since both firms want to
minimally differentiate, no equilibrium with interior solutions exists for the simultaneous
game as D’AGT have observed.

One distinction between the situation modeled here and that of D’AGT is what happens
when \( a + b = 1 \). In the Hotelling model the equilibrium prices are zero and both firms
make zero economic rents. With cost asymmetries, however, we have that the low cost
firm can undercut the high cost firm and make positive profits. As in the above analysis
we again have that the monopolistic competitive profits for the low cost firm will be
greater than duopoly rents with large enough cost asymmetries. The introduction of
timing of entry decisions and commitment to location can restore location and pricing
equilibria for sufficiently large cost asymmetries.

Summary and discussion

In this study we have extended the analysis of horizontally differentiated Bertrand models
to include marginal cost asymmetries. We find that previous results of price equilibria do
not hold when firms are minimally differentiated. At such locations when any cost
asymmetries exist, the lower cost firm can undercut the higher cost firm and capture the
entire market. Furthermore, we find that with sufficient cost asymmetries the incentives
to differentiate change. With such cost differentials the lower cost firm would want to
minimally differentiate while the high cost firm would want to maximally differentiate.
Sufficiently asymmetric costs then lead to no location equilibrium in the simultaneous
location choice game. Introducing an ability for a firm to delay committing to a location
choice results in location equilibria whereby the lower cost firm takes on the role of
Stackleberg leader. This leader, however, must accept its second best location, initially
maximal differentiation, then as cost asymmetries grow, intermediate differentiation.
Finally, when cost asymmetries are large the lower cost firm can monopolize the market
by locating in the center. The low cost firm then behaves in a monopolistically
competitive manner, pricing so that the high cost firm is blockaded.

We then analyze the welfare implications of marginal cost differences. When those
differences are still at a low level but growing, both the Producers’ and Consumers’
surpluses grow. When the cost differences are such that the low cost firm starts to shift
its location towards the high costs firm, then Producers’ surplus decrease, while the
Consumer surplus increases. Total surplus, however, increases in this range. Once
marginal cost differences imply monopolistic competition, then growing cost differences
entail producers surplus growth and consumer surplus declines such that total welfare
remains constant. Thus total welfare increases, or at least, does not decline with mean
preserving cost difference increases. This is a similar result to that found by Salant and
Shaffer in their study of Cournot models. Greater market share is allocated to the more
efficient producer, making the market more efficient.
References


