Price competition in a differentiated products duopoly under network effects

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Abstract

We examine price competition under product-specific network effects, in a duopoly where the products are differentiated horizontally and vertically. When consumers’ expectations are not affected by prices, firms may share the market equally, or one firm (possibly even the low-quality one) may capture the entire market. When product qualities are different, we may also have interior asymmetric equilibria. With expectations affected by prices, firms’ competition becomes more intense and the high quality firm captures a larger market share. Under strong network effects, we characterize a continuum of equilibria and show that a range of prices may be sustained but the higher the prices the smaller the difference between them should be.

Keywords: network effects, product differentiation, product variety, quality, price competition.

JEL classification: L13, D43

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1 Introduction

An extensive literature has explained that network effects (whether direct or indirect) play an important role in many markets.\(^1\) In many of these cases, network effects tend to be product-specific, rather than market-wide. In other words, there are different products in the market and consumers’ value for a product is higher when a larger number of other consumers use that same product. Clearly, in the case of product-specific network effects, the products are not viewed by the consumers as identical – at the very least, they are not fully “compatible” with respect to their network effects and, typically, they also differ in a number of other dimensions. A large number of important cases comes to mind. In the credit card market, users of Visa, MasterCard, American Express or other cards may view these as differentiated (having different varieties or even qualities) and prefer one to the other with respect to their basic characteristics, but they also take into account the network size of each card (since more popular cards are more widely accepted). In the Beta vs. VHS “textbook” case of network effects, the products are differentiated and consumers care both about their inherent characteristics and market share.\(^2\) Similarly with many recent cases of electronics equipment or software. In fact, whenever network effects are product-specific, the issue of product differentiation emerges as an important part of the overall picture. In this paper, we examine price competition between sellers of differentiated products under network effects.

We set up the simplest model that allows us to explore the above discussed issue, a simple static duopoly where suppliers of differentiated products compete in prices. To capture horizontal differentiation, that consumers may have different preferences over the two products, we employ the standard Hotelling “linear-city” model with quadratic transportation costs. We then introduce a network effect, so that given each product’s characteristics and price, each consumer would prefer a product more widely used. In addition, we introduce the possibility of quality differences: in other words, in addition to horizontal, there may also be vertical product differentiation. Thus, we take as given the differentiation between the products (horizontal and/or vertical) and examine how price competition takes place under network effects. Naturally, our focus is on the equilibrium prices and market shares for each firm. In particular, when is it that equilibria have to be symmetric and when can they be asymmetric? Is it possible that products of lower quality obtain a higher market share? How is the equilibrium affected if firms can affect consumers’ expectations? Are these conditions that give rise to a multiplicity of equilibria?

We conduct our analysis under two alternative assumptions concerning the equilibrium con-

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\(^1\)See, e.g., Economides (1996) and Shy (2001) for a review.

\(^2\)See, e.g., Cusumano et al. (1992) and Liebowitz and Margolis (2000).
struction, corresponding to the ability of firms to commit to their product prices. In both cases, consumers’ expectations about firms’ market shares (or, equivalently, network effects) are required to be fulfilled in equilibrium (to be “rational”). First, we examine the case where these expectations cannot be affected by the prices set by firms — technically, the expectations are formed before the prices are set. Our main results, in this case, are as follows. If firms’ products have the same quality (and are differentiated only horizontally), for relatively weak network effects, the only equilibrium is that the firms share the market equally. In contrast, for stronger network effects, there can be three equilibrium configurations: two asymmetric, with one of the two firms capturing the entire market, and one symmetric where the two firms share the market equally. The threshold, above which asymmetric equilibria with only one active firm arise, is determined by comparing the strength of the network effect to the transportation cost (or, equivalently, to the importance of product differentiation). The two possible asymmetric equilibria are extreme (in the sense that one firm captures all the market) and no other asymmetric equilibria (with the market shared unequally) exist. When, in addition, the products differ with respect to their qualities, we find that, if the network effect is relatively weak, the high-quality firm captures a larger share of the market and the low-quality firm a smaller one, depending on the quality difference. If the network effect is relatively strong, either the low-quality firm or the high-quality firm may capture the entire market. Importantly, for the low-quality firm to capture the entire market, it has to be that the quality difference is not too large relative to the network effect.

We then examine the case where consumers’ (rational) expectations can be affected by the prices set by the firms (this should be the case when firms can commit to their prices). We find that when the two products have the same quality, the firms share the market equally. If the network effect is relatively weak, both firms make a positive profit in equilibrium; in contrast, if the network effect is relatively strong, one firm tends to capture the entire market. With weak network effects, the high-quality firm captures a larger market share than its rival, or even the entire demand, depending on the size of the quality difference. If the network effect is strong, the high-quality firm necessarily captures the entire market.

By comparing the various scenarios examined, we see that to obtain an equilibrium where the firms share the market asymmetrically (and both make positive sales), the qualities of the products should not be equal. Also, when expectations are affected by the prices set, the firms tend to compete with greater intensity than in the case when expectations are not affected, leading to lower equilibrium profits. When prices do not affect consumers’ expectations, the low-quality firm may be the one that captures a larger market share. When expectations are affected by prices, the high-quality firm’s position becomes more favorable and, in equilibrium, it
always obtains a larger market share than its low-quality rival. We also contrast our results to the case where the network effects are market-wide (full compatibility). In our model, when the two products are fully compatible, the network effects stop playing any role and this case is equivalent to zero network effects. Comparing this case with the case of incompatible products when the network effects are weak, we observe that full compatibility relaxes competition between firms and allows for higher equilibrium profit.

This paper is related to two distinct literatures, on network effects and on product differentiation. First, our model contributes to the product differentiation literature, by introducing network effects. In particular, we start from a Hotelling (1929) type model, modified as in d’Aspermont et al. (1979) so that transportation costs are quadratic (as well known, this modification brings smoothness to the problem). We take the firms’ “locations” as given and study the implications of network effects. Further, we combine such horizontal aspects with vertical (quality) product differentiation (see e.g. Shaked and Sutton, 1982), following in that regard the analysis in Vettas (1999). So, we can study network effects with product differentiation with respect both to variety and quality. Second, as already mentioned, there is an extensive and important literature on networks. Our paper is close in spirit to Katz and Shapiro (1985) that examine homogeneous oligopoly competition under network effects. In contrast to their analysis of a homogeneous product Cournot oligopoly, the focus of our paper is on price competition with differentiated products, where a number of important new questions emerge. Network effects have been introduced in product differentiation models in Navon et al. (1995). In addition to specific modelling differences between that work and ours, we study a significantly wider set of issues including whether expectations are formed before or after prices, the multiplicity of equilibria (in particular, under strong network effects) and the presence of both horizontal and vertical differentiation.\(^3\)

The remainder of the paper is as follows. In Section 2, we set up the basic model allowing for horizontal differentiation and network effects. In Section 3, we derive the equilibrium prices (and market shares) for arbitrary consumers’ expectations. In Section 4, we examine equilibrium behavior under rational expectations, assuming these cannot be affected by the firms’ prices (firms cannot commit). In Section 5, we derive the equilibrium when consumers’ expectations are

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\(^3\)Related work that examines network effects includes Katz and Shapiro (1986), Farrell and Saloner (1986), Bental and Spiegel (1995), Baake and Boom (2001), Jullien (2001) and Kim (2002), but with emphasis on different aspects of the issue, in particular, on the quality and compatibility decisions. Other such recent related work, including Economides et al. (2004) and Mitchell and Skrzypacz (2004), focuses on the dynamics of market shares, with network effects operating with a time lag – by focusing on the one period case, we emphasize the importance of consumers’ expectations. Laussel et al. (2004) examine a (dynamic) duopoly with congestion effects, which can be thought of as negative network effects.
affected by the prices firms charge (firms can commit). Section 6, compares the full compatibility case to our analysis of product-specific networks. In Section 7 we generalize the model by also allowing for quality differences and derive the equilibrium when we have horizontal and vertical product differentiation. As in the case of only horizontal differentiation, we explore the implications of different assumptions about consumers expectations. Section 8 concludes.

2 The basic model

We consider competition between two firms, A and B, in a simple one-period Hotelling model. Demand is represented by a continuum of consumers with total mass equal to 1. The two firms are located at the two extremes of the [0,1] linear city: firm A is located at point 0 and firm B at point 1. Firms use simple linear pricing and try to maximize their profit. For simplicity, we set their costs equal to zero. Every consumer selects one unit of the product of one of the two firms: the entire market is covered (the underlying assumption here is that the base utility from buying either product is high enough). Every consumer derives utility from the network size of the firm he has chosen. Since these sizes are actually determined after consumers make their choices, consumers choose a product based on expectations about each firm’s network. All consumers have the same expectations since, in equilibrium, these will be required to be fulfilled. Specifically, a consumer located at point \( x \in [0,1] \) will be indifferent between the two products when their “generalized” costs are equal:

\[
p_A + tx^2 - \beta x^e = p_B + t(1-x)^2 - \beta (1-x^e),
\]

(1)

where \( p_i \) is firm \( i \)'s price, \( i = A, B \), \( t > 0 \) is an index of the per unit transportation cost, \( \beta > 0 \) measures the network effect and \( x^e \) is the number of customers expected to purchase firm A’s product.\(^4\) Solving expression (1) for \( x \), we obtain the location of a consumer indifferent between purchasing from firm A or firm B:

\[
x = \frac{p_B - p_A + \beta(2x^e - 1) + t}{2t}.
\]

(2)

If, for a given consumer located at \( x \), the left hand side of (1) is above (respectively, below) its right hand side, that consumer will purchase product B (respectively, A). If the solution in (2) falls within [0, 1], then a consumer indifferent between the two products exists; else, all consumers prefer one of the two products.

\(^4\)In Section 7 we significantly extend the paper by introducing vertical differentiation and allowing the two products to be viewed by the consumers as not only of different varieties but also of different quality.
3 Equilibrium in prices with arbitrary expectations

In this section, we examine firms’ price competition for arbitrary consumers’ expectations about the market shares of each firm (that is, about the network effect). This serves as a first step for the subsequent analysis, where a rationality requirement is imposed on expectations. For the moment, we allow consumers to have any expectations, not necessarily fulfilled in equilibrium, and explore how the firms compete in prices, taking these expectations as given. So let us assume that all consumers expect firm A’s share to be $x^e \in [0, 1]$. Both firms have positive market shares (that is $x \in (0, 1)$) only if $-t - \beta(2x^e - 1) < p_B - p_A < t - \beta(2x^e - 1)$. If $-t - \beta(2x^e - 1) \geq p_B - p_A$, firm B captures the entire market and if $p_B - p_A \geq t - \beta(2x^e - 1)$, firm A captures the entire market. To summarize, firm A’s market share is

$$x = \begin{cases} 0 & \text{if } p_B - p_A \leq -t - \beta(2x^e - 1) \\ \frac{p_B - p_A + \beta(2x^e - 1) + t}{2t} & \text{if } -t - \beta(2x^e - 1) < p_B - p_A < t - \beta(2x^e - 1) \\ 1 & \text{if } p_B - p_A \geq t - \beta(2x^e - 1). \end{cases}$$

As a consequence, firm A maximizes its profit,

$$\pi_A = \begin{cases} 0 & \text{if } p_B - p_A \leq -t - \beta(2x^e - 1) \\ p_A \left(\frac{p_B - p_A + \beta(2x^e - 1) + t}{2t}\right) & \text{if } -t - \beta(2x^e - 1) < p_B - p_A < t - \beta(2x^e - 1) \\ p_A & \text{if } p_B - p_A \geq t - \beta(2x^e - 1), \end{cases}$$

with respect to $p_A$ and firm B maximizes its profit,

$$\pi_B = \begin{cases} p_B & \text{if } p_B - p_A \leq -t - \beta(2x^e - 1) \\ p_B \left(1 - \frac{p_B - p_A + \beta(2x^e - 1) + t}{2t}\right) & \text{if } -t - \beta(2x^e - 1) < p_B - p_A < t - \beta(2x^e - 1) \\ 0 & \text{if } p_B - p_A \geq t - \beta(2x^e - 1), \end{cases}$$

with respect to $p_B$.

We first examine the case where both firms make positive sales, $x \in (0, 1)$. The second-order conditions show that both firms’ profit functions are strictly concave, since $\frac{\partial^2 \pi_A}{\partial p_A^2} = \frac{\partial^2 \pi_B}{\partial p_B^2} = -\frac{1}{t} < 0$. Then, via the first-order conditions, we derive the reaction functions:

$$p_A = R_A(p_B; x^e) = \frac{p_B + \beta(2x^e - 1) + t}{2}$$

and

$$p_B = R_B(p_A; x^e) = \frac{p_A - \beta(2x^e - 1) + t}{2}.$$ 

By solving the system of these reaction functions, we obtain the equilibrium prices (given consumers’ expectations)

$$p_A^*(x^e) = \frac{t + \beta(2x^e - 1)}{3}, \quad (3)$$
an by substituting equations (3) and (4) into (2) we obtain

\( x^*(x^e) = \frac{1}{2} + \frac{\beta(2x^e - 1)}{6t}. \)  

We observe that for \(-3t < \beta(2x^e - 1) < 3t\) both firms have positive market shares. We reach a corner, where firm A captures the entire market, when \(\beta(2x^e - 1) \geq 3t\), and we reach a corner, where firm B captures the entire market, when \(\beta(2x^e - 1) \leq -3t\).

Let us now explore possible corner equilibria. Consider, first, the case where \(\beta(2x^e - 1) \geq 3t\).

If \(p_A = \beta(2x^e - 1) - t\), firm B has zero demand if it charges any non negative price. In this case, firm B achieves its maximum profit, of zero, by charging \(p_B^* = 0\). Next, we need to show that \(p_A = \beta(2x^e - 1) - t\) is a best response for firm A. Firm A is solving \(\max_{p_A} \pi_A \equiv p_A x\), where \(x\) is its market share:

\[
x = \begin{cases} 
\frac{t - p_A + \beta(2x^e - 1)}{2t} & \text{if } p_A \geq \beta(2x^e - 1) - t \\
1 & \text{if } p_A \leq \beta(2x^e - 1) - t.
\end{cases}
\]

As long as \(p_A \leq \beta(2x^e - 1) - t\) and \(x^e > \frac{1}{2}\), the optimal price for firm A is

\( p_A^* = \beta(2x^e - 1) - t, \)  

the highest price that gives the entire market to firm A. If firm A were to lower its price, it would not increase its market share, since firm A already captures the entire market: hence, the lower price would lead to a decreased profit. If \(p_A \geq \beta(2x^e - 1) - t\), firm A shares the market with firm B and its profit decreases. It follows that the best response to \(p_B^* = 0\) is given by expression (6), the maximum price that allows firm A to capture the entire market.

Consider now the case where \(\beta(2x^e - 1) \leq -3t\). If \(p_B = -\beta(2x^e - 1) - t\), firm A has zero demand if it charges a non negative price. In this case, firm A maximizes its profit by charging \(p_A^* = 0\) and obtains zero profit. Next, we show that \(p_B = -\beta(2x^e - 1) - t\) is a best response for firm B when \(p_A = 0\). Firm B solves \(\max_{p_B} \pi_B \equiv p_B(1 - x)\) where \(x\) is

\[
x = \begin{cases} 
\frac{t + p_B + \beta(2x^e - 1)}{2t} & \text{if } p_B \geq -\beta(2x^e - 1) - t \\
0 & \text{if } p_B \leq -\beta(2x^e - 1) - t.
\end{cases}
\]

As long as \(p_B \leq -\beta(2x^e - 1) - t\), the optimal price for firm B is

\( p_B^* = -\beta(2x^e - 1) - t, \)  

that is, the highest price that gives the entire market to firm B. If firm B decreased its price, it would not increase its sales, since firm B already captures the entire market. Hence, such a price decrease is not profitable. If \(p_B \geq -\beta(2x^e - 1) - t\), firm B shares the market with firm A and its
profit function is decreasing. It follows that the best response to \( p^*_A = 0 \) is given by expression (7), the maximum price that allows firm B to capture the entire market. The results from the above analysis can be summarized as follows.

**Proposition 1** Consider competition between firms A and B, given consumers’ expectations that A’s market share is \( x^e \in [0,1] \). The equilibrium prices are

\[
p^*_A(x^e) = \begin{cases} 
\beta(2x^e - 1) - t & \text{if } \beta(2x^e - 1) \geq 3t \\
 t + \frac{\beta(2x^e - 1)}{3} & \text{if } -3t < \beta(2x^e - 1) < 3t \\
 0 & \text{if } \beta(2x^e - 1) \leq -3t 
\end{cases}
\] (8)

and

\[
p^*_B(x^e) = \begin{cases} 
0 & \text{if } \beta(2x^e - 1) \geq 3t \\
 t - \frac{\beta(2x^e - 1)}{3} & \text{if } -3t < \beta(2x^e - 1) < 3t \\
 -\beta(2x^e - 1) - t & \text{if } \beta(2x^e - 1) \leq -3t, 
\end{cases}
\] (9)

while the implied market share for firm A is

\[
x^*(x^e) = \begin{cases} 
1 & \text{if } \beta(2x^e - 1) \geq 3t \\
\frac{1}{2} + \frac{\beta(2x^e - 1)}{6t} & \text{if } -3t < \beta(2x^e - 1) < 3t \\
0 & \text{if } \beta(2x^e - 1) \leq -3t, 
\end{cases}
\] (10)

with firm B’s market share being \( 1 - x^*(x^e) \).

We see that, when consumers expectations are that the market shares are not extreme, both firms will indeed make positive sales. If the expectations are that the market share differences exceed a given threshold, then the firm expected to have a larger market share actually captures the entire market.\(^5\) Naturally, our next step in the analysis is to examine what market shares can be supported in equilibrium when expectations about these shares are fulfilled.

### 4 Equilibrium with rational (fulfilled) expectations not affected by prices

Building on our analysis above, we now proceed to the analysis by requiring that the consumers’ expectations about market shares are rational (fulfilled in equilibrium). We proceed in this Section by assuming that expectations cannot be affected by the prices set by the firms, in other

\(^5\)Since thus far in the analysis expectations are taken as exogenously given, they simple serve to differentiate the two products, increasing the attractiveness of one, relative to the other, as in a model where horizontally differentiated products are also viewed by consumers as of different qualities (see Vettas, 1999).
words that the firms cannot commit, and thus cannot manipulate consumers’ expectations. Instead they have to take these expectations as given.\textsuperscript{6} To study the equilibrium when expectations are rational, we substitute $x^e = x^*$ in expression (10). Simple calculations show that the two corner solutions (the first and the last branch of expression (10)) constitute an equilibrium when $\beta \geq 3t$. It is interesting to examine what happens at the second branch of expression (10) after substituting $x^e = x^*$. In this case, we obtain

$$2(3t - \beta)x = 3t - \beta,$$

which is satisfied either for $\beta = 3t$, or for $x = \frac{1}{2}$. For $\beta = 3t$, relation (11) is satisfied for any market share that consumers may expect, so, in this case, any $x = x^e \in [0, 1]$ is an equilibrium solution. For $x = \frac{1}{2}$, relation (11) is satisfied no matter how strong or weak are the network effects relatively to the transportation cost. More precisely, let us distinguish three cases.

**Case 1:** For relatively weak network effects ($\beta < 3t$), by substituting $x^e = x^*$ into (10) we obtain $x^* = \frac{1}{2}$. We conclude that, when the network effects are weak, the two firms, in equilibrium, can only share the market equally. To obtain the equilibrium prices we substitute $x^e = x^* = \frac{1}{2}$ into (8) and (9) (for the case that corresponds to $\beta < 3t$), and find that

$$p^*_A = p^*_B = t.$$

**Case 2:** For relatively strong network effects ($\beta > 3t$), by substituting $x^e = x^*$ into (10) we find that $x^*$ can take three different values: $x^* = 0$, $x^* = \frac{1}{2}$ or $x^* = 1$. So, if the network effects are strong, either of the two firms can capture the entire market or they may share the market equally. To obtain the equilibrium prices we substitute $x^e = x^*$ and $x^* = 0$, or $x^* = \frac{1}{2}$, or $x^* = 1$ into (8) and (9) (for the cases that correspond to $\beta > 3t$), to obtain

$$p^*_A = \begin{cases} \beta - t & \text{if } x^* = x^e = 1 \\ t & \text{if } x^* = x^e = \frac{1}{2} \\ 0 & \text{if } x^* = x^e = 0 \end{cases}$$

and

$$p^*_B = \begin{cases} 0 & \text{if } x^* = x^e = 1 \\ t & \text{if } x^* = x^e = \frac{1}{2} \\ \beta - t & \text{if } x^* = x^e = 0. \end{cases}$$

**Case 3:** For the borderline case $\beta = 3t$, we substitute $x^e = x^*$ and $\beta = 3t$ into (10) and find that the equation is satisfied for any $x^e = x^* \in (0, 1)$ that is, any expectation about market

\textsuperscript{6}The well known analysis of Katz and Shapiro (1985) also follows, in its greater part, this same assumption that expectations are not affected by the firms' strategic choices – quantities in that analysis.
shares can be fulfilled in equilibrium. To obtain the equilibrium prices we substitute $\beta = 3t$ into (8) and (9):

$$p_A^* = 2tx^e \text{ and } p_B^* = 2t(1 - x^e).$$

We summarize:

**Proposition 2** When consumers’ expectations about market shares are fulfilled in equilibrium (but cannot be affected by prices), there are three possible cases. Case 1: if the network effect is relatively weak ($\beta < 3t$) there is a unique equilibrium, where firms share the market equally ($x^* = \frac{1}{2}$) with the equilibrium prices $p_A^* = p_B^* = t$. Case 2: if the network effect is relatively strong ($\beta > 3t$), there are three possible equilibria with $x^* = 0$, $x^* = \frac{1}{2}$, and $x^* = 1$. Case 3: when $\beta = 3t$, any $x \in [0,1]$ corresponds to an equilibrium with prices $p_A^* = 2tx^e$ and $p_B^* = 2t(1 - x^e)$.

Let us discuss the structure of the equilibria derived above. We have found that when the two firms offer horizontally differentiated products, the equilibrium firm A’s market share is equal to

$$x^* = \begin{cases} 
  \frac{1}{2} & \text{if } \beta < 3t \\
  \text{any } x^* \in [0,1] & \text{if } \beta = 3t \\
  0 \text{ or } \frac{1}{2} \text{ or } 1 & \text{if } \beta > 3t
\end{cases}$$

(see also Figure 1). We see that, for any given network effect $\beta$ or transportation cost $t$, if consumers expect that the two firms will equally share the market, this expectation is fulfilled in equilibrium. For the case where the network effect is weak ($\beta < 3t$), this expectation is the only one that can be fulfilled in equilibrium. When the network effect is strong ($\beta > 3t$), in equilibrium, three expectations concerning market shares can be fulfilled: either one of the two firms captures the entire market, or the two firms share the market equally. No other expectation can be supported in equilibrium. To illustrate why not, suppose, for example, that consumers expect that firm A has share $x^e = 0.75$ when $\beta = 4$ and $t = 1$. Then, by calculating the equilibrium of the model where firms compete in prices (taking this expectation as given) we obtain $p_A = 1 + \frac{4x^0.5}{3} = \frac{5}{3}$, $p_B = 1 - \frac{4x^0.5}{3} = \frac{1}{3}$ with an implied market share equal to
$x = \frac{1}{2} + \frac{40.5}{6} = \frac{5}{6} > 0.75$. Therefore, the expectation $x^e = 0.75$ cannot be supported as a fulfilled expectations’ equilibrium. In general, we find that when $\frac{1}{2} < x^e < 1$, then firms’ competition implies $x > x^e$, while when $0 < x^e < \frac{1}{2}$, then it implies $x < x^e$. Finally, when $\beta = 3t$ all expectation values can be fulfilled in equilibrium. This is the only case where both firms can make positive sales without sharing the market evenly.

5 Equilibrium when expectations are affected by prices

In this section, we examine how prices and market shares are formed when firms can manipulate consumers’ expectations. Now, firms announce their prices and consumers take into consideration the announced prices when forming their expectations. In this scenario, expectations are not treated by the firms as given: instead prices play an important role in the formation of expectations. Thus, the implicit timing of the game is as follows. First, firms announce their prices, knowing that these announcements will affect consumers’ expectations about each firm’s market share. Then, consumers form their expectations taking as given the prices that have been announced and choose which product to purchase.

To construct the equilibrium, we substitute $x^e = x$ in equation (2) to obtain

$$x = \frac{p_B - p_A - \beta + t}{2(t - \beta)}.$$  \hspace{1cm} (12)

This expression determines the location of the indifferent consumer if the prices are $p_A$ and $p_B$ and if all the consumers believe that the market share of firm A is indeed given by expression (12). The idea here is that since firms can affect consumers’ expectations via their prices, these prices should be used when deriving the expected market shares (which, of course, will be equal to the actual market shares). It is essential to observe that, if $t > \beta$, an increase in firm A’s price decreases its market share. On the other hand, if $t < \beta$, an increase in firm A’s price increases its market share.

First, let us assume that $t > \beta$, that is the network effect is relatively weak. Expression (12) shows that $x \in (0, 1)$ only if $\beta - t < p_B - p_A < t - \beta$, assuming that $t > \beta$. If $p_B - p_A \geq t - \beta$, consumers should be affected by the announced prices only if firms commit to them. Whether this is possible depends on the institutional and technological details in each market. If firms could raise their prices after some early stage, then an announcement of a low early price would not be a credible signal interpreted by the consumers as leading to a larger market share.

Katz and Shapiro (1985), following the main body of their analysis, also briefly examine the case where expectations are formed after the firms’ strategic choices have been made. Here, the analysis is different since the firms compete in prices and the implied quantities (that represent the network effect) are only determined in equilibrium and, thus, affected by the strategic choices only indirectly, not “chosen” directly by the firms, as in Katz and Shapiro (1985).
then all consumers choose firm A, while if $\beta - t \geq p_B - p_A$, firm B captures the entire market. To summarize, when consumers form their expectations after observing firms’ prices, expectations are fulfilled, and $t > \beta$, firm A’s market share can be calculated to be:

$$x = \begin{cases} 
0 & \text{if } p_B - p_A \leq \beta - t \\
\frac{p_B - p_A - \beta + t}{2(t - \beta)} & \text{if } \beta - t \leq p_B - p_A \leq t - \beta \\
1 & \text{if } p_B - p_A \geq t - \beta.
\end{cases}$$

It follows that firm A maximizes its profit,

$$\pi_A = \begin{cases} 
0 & \text{if } p_B - p_A \leq \beta - t \\
\frac{p_B - p_A - \beta + t}{2(t - \beta)} & \text{if } \beta - t \leq p_B - p_A \leq t - \beta \\
p_A & \text{if } p_B - p_A \geq t - \beta.
\end{cases}$$

with respect to $p_A$. Similarly, firm B maximizes its profit,

$$\pi_B = \begin{cases} 
p_B & \text{if } p_B - p_A \leq \beta - t \\
p_B(1 - \frac{p_B - p_A - \beta + t}{2(t - \beta)}) & \text{if } \beta - t \leq p_B - p_A \leq t - \beta \\
0 & \text{if } p_B - p_A \geq t - \beta.
\end{cases}$$

with respect to $p_B$.

Note that when both firms have positive market shares, the second-order conditions show that both firms’ profit functions are strictly concave.\footnote{Since $\frac{\partial^2 \pi_A}{\partial p_A^2} = \frac{\partial^2 \pi_B}{\partial p_B^2} = -\frac{1}{t - \beta} < 0$ and we have assumed here that $t > \beta$.} The best response correspondence of firm A is

$$p_A = R(p_B) = \begin{cases} 
p_B - t + \beta & \text{if } p_B \geq 3(t - \beta) \\
\frac{p_B - t + \beta}{2} & \text{if } \beta - t \leq p_B \leq 3(t - \beta) \\
y \text{any price } \geq p_B + t - \beta & \text{if } p_B \leq \beta - t
\end{cases}$$

and is derived as follows. If firm A is to capture the entire market, the maximum price it can charge is $p_A = p_B - t + \beta$. Now assume that firm A shares the market with its rival. Then, its reaction function, derived from the first-order condition from the relevant branch of the profit function, is

$$p_A = R_A(p_B) = \frac{p_B - \beta + t}{2}. \quad (13)$$

Firm A will charge this price as long as $\beta - t \geq p_B - p_A \leq t - \beta$ and, after substituting expression (13) and solving with respect to $p_B$, we obtain $t - \beta \leq p_B \leq 3(t - \beta)$. If $p_B \geq 3(t - \beta)$, firm A finds it more profitable to capture the entire market than to share it with firm B. If, on the other hand, $p_B \leq \beta - t$, firm A cannot compete with firm B because if it tries to share the market it ends up with losses. In this case, firm A prefers to have no customers and charges a price $p_A \geq p_B + t - \beta$.\footnote{Since $\frac{\partial^2 \pi_A}{\partial p_A^2} = \frac{\partial^2 \pi_B}{\partial p_B^2} = -\frac{1}{t - \beta} < 0$ and we have assumed here that $t > \beta$.}
The best response correspondence of firm B,

\[ p_B = R(p_A) = \begin{cases} 
  p_A - t + \beta & \text{if } p_A \geq 3(t - \beta) \\
  \frac{p_A - \beta + t}{2} & \text{if } \beta - t \leq p_A \leq 3(t - \beta) \\
  \text{any price } \geq p_A + t - \beta & \text{if } p_A \leq \beta - t
\end{cases} \]

can be derived in a similar way as for A, where now firm B’s reaction function, if the market is shared, is

\[ p_B = R_B(p_A) = \frac{p_A - \beta + t}{2}. \tag{14} \]

As long as both firms make positive sales, the two best response correspondences of firms A and B intersect when

\[ p_A^* = p_B^* = t - \beta, \]

and, in equilibrium, the indifferent consumer is located at the center of the “city”:

\[ x^* = \frac{1}{2}. \]

To verify that the equilibrium prices are indeed \( p_A^* = p_B^* = t - \beta \), we need to also check that no firm has an incentive to deviate and capture the entire market. Given that \( p_A = t - \beta \), firm B captures the entire market when it charges a price \( p_B \) such that \( t - \beta = p_B + t - \beta \).\(^1\) Solving this equation with respect to \( p_B \), we find that firm B captures the entire market by charging \( p_B = 0 \), and therefore makes zero profit. As firm B’s profit when it shares the market with its rival is \( \pi_B = \frac{t - \beta}{2} > 0 \), it has no incentive to deviate. A symmetric argument applies for firm A. We conclude that, the equilibrium prices are \( p_A^* = p_B^* = t - \beta \) and the two firms share the market equally.

**Proposition 3** When consumers’ expectations about market shares are affected by the firms’ prices, and for \( t > \beta \), there is a unique equilibrium where the two firms share the market equally. The equilibrium prices are \( p_A^* = p_B^* = t - \beta \).

Let us now turn to the case \( t < \beta \) (strong network effects). This case is qualitatively different. In the case we have examined above (\( t > \beta \)) the network effects modify the equilibrium prices relative to the case without network effects. In the case analyzed now (\( t < \beta \)), the network effects play a dominant role in driving the equilibrium behavior: with strong network effects, consumers’ expectations about the firms’ market shares may become self-fulfilled. It becomes

\(^1\)Note that, in this case where consumers’ expectations are affected by firms’ prices, parameters \( t \) and \( \beta \) are multiplied by zero, which is the location of the indifferent consumer when firm B captures the entire market.
then important to study the possible multiplicity of equilibria and their properties. From equation (12), and following similar steps as in the previous case, we find that firm A’s market share is

\[ x = \begin{cases} 
0 & \text{if } p_B - p_A \leq \beta - t \\
\frac{p_B - p_A - (\beta + t)}{2(t - \beta)} & \text{if } t - \beta \leq p_B - p_A \leq \beta - t \\
1 & \text{if } p_B - p_A \geq t - \beta.
\end{cases} \]  

(15)

Again note that the meaning of expression (15) is that this is firm A’s market share when the prices are \( p_A \) and \( p_B \) and when consumers believe that firm A’s market share is indeed given by (15). From (15) it follows that when \( t - \beta \leq p_B - p_A \leq \beta - t \), we can have three different expectations concerning the market shares of the two firms: when consumers observe that the difference of prices belongs to the above stated interval, they may believe that all consumers will choose firm A, or that all consumers will choose firm B, or that \( p_B - p_A - (\beta + t) \in (0, 1) \) consumers will choose firm A and the others will choose firm B. We observe that a firm’s demand may be increasing in its price: this is an implication of the equilibrium requirement that with a consumer indifferent between the two firms, if one firm has a higher price than the other, it must also have a higher share. Otherwise all consumers would prefer the same firm.

As we can see, the expectations concerning the market shares of the two firms play a very important role in the determination of equilibrium prices. Since this is the case, we need to specify how expectations react to a possible change of prices. By looking at expression (15), we observe that for \( t - \beta \leq p_B - p_A \leq \beta - t \) we may end up with different expectations following a change in prices. Analytically, depending on how expectations react we can have “consistent” or “inconsistent” expectations and “continuous” or “discontinuous” expectations. More precisely:

We denote as “consistent” the expectations that react to an increase in the price of a firm in an inverse way to that of a decrease in the price of the same firm. If expectations react towards the same direction to an increase and to a decrease of a price then we call these expectations “inconsistent”.

We denote as “continuous” the expectations that, after a change in prices, remain in the original branch of expectations when this branch can be supported by the new prices. When the original branch of expectations can be supported by the new prices but we make a jump in one of the two other branches of expectations, we have “discontinuous” expectations.

We proceed in determining the equilibrium prices and market shares by assuming specific types of expectations. As we move on in the analysis, we make more restrictive hypothesis concerning the expectations.

**“Inconsistent” and “discontinuous” expectations**

If we let expectations to be “inconsistent” and “discontinuous” we end up with three different cases:
Case 1: For $p_A, p_B \in [0, \beta - t]$, any combination of prices is an equilibrium and the resulting market share of firm A can be $x = 0$, or/and $x = \frac{p_B - p_A - \beta + t}{2(t - \beta)}$ or/and $x = 1$. Let us check whether $0 \leq p_A = p_B \leq \beta - t$ can be supported as an equilibrium with firm A’s market share equal to $x = \frac{1}{2}$. We first examine the behavior of firm A. Since we let expectations to be “discontinuous”, an increase in firm A’s price can lead consumers to believe that firm A will loose all its customers ($x = 0$) and a decrease in its price decreases its market share and its profit. Moreover, for $0 \leq p_A = p_B \leq \beta - t$ firm A will never try to decrease its price so drastically in order to be certain that it will attract the whole market (by charging $p_A = p_B - \beta + t < 0$) since it will make losses. The same reasoning holds for firm B, so we conclude that there are expectations that support $0 \leq p_A = p_B \leq \beta - t$ with $x = \frac{1}{2}$ as an equilibrium. Can the same prices constitute an equilibrium with $x = 1$? Since expectations can be “discontinuous”, an increase in firm A’s price can lead consumers to believe that firm A will lose all its customers ($x = 0$) and a decrease in its price decreases its market share and its profit. As long as firm B is concerned and since expectations can be “inconsistent”, an increase as well as a decrease in firm B’s price may lead consumers to believe that firm B will never attract any customers. Note that for $p_A = p_B = 0$ we do not need to have “inconsistent” expectation in order to have an equilibrium with $x = 1$, since we can not have a decrease in prices. Finally, can the same prices constitute an equilibrium with $x = 0$? Since expectations can be “inconsistent”, an increase as well as a decrease in firm A’s price may lead consumers to believe that firm A will never attract any customers (again, for $p_A = p_B = 0$ we do not need “inconsistent” expectations to have an equilibrium with $x = 1$, since we can not have a decrease in prices). Moreover, since expectations can be “discontinuous”, an increase in firm B’s price can lead consumers to believe that firm B will lose all its customers ($x = 1$) and a decrease in its price decreases its market share and its profit. So, for $0 \leq p_A = p_B \leq \beta - t$ we can have three possible market shares in equilibrium: $x = 0$, $x = \frac{p_B - p_A - \beta + t}{2(t - \beta)}$ and $x = 1$. The same logic holds for any combination of prices $0 \leq p_A, p_B \leq \beta - t$. For the extreme case where one price is zero and the other is $\beta - t$ we end up with two possible market shares in equilibrium: $x = 0$ and $x = 1$.

If at least the highest of the two prices, denoted by $p_i$ where $i = A, B$, is $p_i = (1 + \alpha)(\beta - t)$, where $\alpha$ is a constant $\in (0, 1]$ then in order to reach an equilibrium, the rival’s firm price must be $p_j \geq \frac{1}{2}(\alpha + \sqrt{\alpha^2 + \alpha})(\beta - t)$. To be more analytical, assume $p_B = (1 + \alpha)(\beta - t)$, where $\alpha \in (0, 1]$.\(^{11}\) For $p_A \leq p_B$ firm B can never attract the whole market in equilibrium since firm A has always a profitable deviation to charge $p_A = p_B - \beta + t$ and attract the whole market. In this case, its profit will be $\pi_A = \alpha(\beta - t) > 0$. If firm A shares the market with the rival firm then

\(^{11}\)We have assumed that $p_B \geq p_A$ but we can have a symmetric analysis for $p_A \geq p_B$. The figures that follow represent both cases.
its profit will be \( \pi_A = p_A^{p_A - p_B + \beta - t} \) and after substituting \( p_B \) we find that this profit is greater than \( \alpha(\beta - t) \) if \( p_A \geq \frac{1}{2}(\alpha + \sqrt{\alpha} \sqrt{8 + \alpha})(\beta - t) \geq 0 \). We find that we cannot have an equilibrium with \( p_A < \frac{1}{2}(\alpha + \sqrt{\alpha} \sqrt{8 + \alpha})(\beta - t) \) since firm A, which already captures the entire market has a profitable deviation to increase it price up to \( \frac{1}{2}(\alpha + \sqrt{\alpha} \sqrt{8 + \alpha})(\beta - t) \) and continue to attract the entire market. We now need to check what will be the equilibrium prices and market shares.

We distinguish two cases.

Case 2: For \( p_B = (1 + \alpha)(\beta - t) \), where \( \alpha \in (0, \frac{1}{2}] \), we end up with an equilibrium where \( 0 \leq \frac{1}{2}(\alpha + \sqrt{\alpha} \sqrt{8 + \alpha})(\beta - t) \leq p_A \leq \beta - t \) and the equilibrium market share of firm A is either \( x = 1 \) or \( x = \frac{p_B - p_A - \beta + t}{2(\beta - t)} \). We can have an equilibrium where firm A captures the entire market \( (x = 1) \) because since expectations can be “discontinuous”, an increase in firm A’s price can lead consumers to believe that firm A will lose all its customers \( (x = 0) \) and a decrease in its price decreases its market share and its profit. As long as firm B is concerned and since expectations can be “inconsistent”, an increase as well as a decrease in firm B’s price may lead consumers to believe that firm B will never attract any customers. Firm B can never decrease its price that drastically in order to attract the market for sure since it would have to charge \( p_A - \beta + t < 0 \). Moreover, we can have an equilibrium where firm A captures \( x = \frac{p_B - p_A - \beta + t}{2(\beta - t)} \) because, since expectations can be “discontinuous”, an increase in the price of either firm can lead consumers to believe that the particular firm will lose all its customers and a decrease in the price of either firm decreases its market share and its profit.

Case 3: For \( p_B = (1 + \alpha)(\beta - t) \), where \( \alpha \in \left(\frac{1}{3}, 1\right) \) we end up with an equilibrium for any \( \beta - t \leq \frac{1}{2}(\alpha + \sqrt{\alpha} \sqrt{8 + \alpha})(\beta - t) \leq p_A \leq p_B \leq 2(\beta - t) \) and firms share the market. In this case, firm A’s market share is \( x = \frac{p_B - p_A - \beta + t}{2(\beta - t)} \). Neither firm can capture the entire market since the rival firm has an incentive to charge \( p_i = p_j = \beta + t > 0 \) and make positive profit by attracting the whole market. No firm has an incentive to increase its price because consumers may believe that the particular firm will lose all its customers and a decrease in the price of either firm decreases its market share and its profit.

For the case where at least the highest of the two prices, denoted by \( p_i \) is \( p_i > 2(\beta - t) \) we can not have an equilibrium since the rival firm can always charge \( p_j = p_i - \beta + t \), attract the whole market and make higher profits than before.

Diagrammatically, the equilibrium prices can be shown in Figure 2.

In Figure 2, we can see from the shaded areas which combinations of prices lead to an equilibrium. There are three shaded areas because we distinguish three different cases: Case 1: any combination of \( p_A, p_B \), where \( p_A, p_B \in [0, \beta - t] \), gives us three possible equilibrium market shares \( (x^* = 0, x^* = \frac{p_B - p_A - \beta + t}{2(\beta - t)} \) and \( x^* = 1) \) that can be supported by the expectations (darker shaded area). Case 2: for \( p_i^* = (1 + \alpha)(\beta - t) \) (where \( \alpha \in (0, \frac{1}{3}] \)) we have an equilibrium with
any $0 \leq \frac{1}{2}(\alpha + \sqrt{\alpha\sqrt{8 + \alpha}})(\beta - t) \leq p^*_j \leq \beta - t$ and then either the low price firm captures the entire market or both firms share the market with firm A capturing $x^* = \frac{p_B - p_{A} - \beta + t}{2(t - \beta)}$ (area with vertical lines). Case 3: for $p^*_i = (1 + \alpha)(\beta - t)$ (where $\alpha \in (\frac{1}{3}, 1]$) we have an equilibrium with any $\beta - t \leq \frac{1}{2}(\alpha + \sqrt{\alpha\sqrt{8 + \alpha}})(\beta - t) \leq p^*_i \leq 2(\beta - t)$ and then the two firms can only share the market, with firm A capturing $x^* = \frac{p_B - p_{A} - \beta + t}{2(t - \beta)}$ (white area).

**“Consistent” and “discontinuous” expectations**

If we restrict expectations to be “consistent” but let them be “discontinuous” we end up with the following. For $p_A, p_B \in [0, \beta - t]$, any combination of prices is an equilibrium and the resulting market share of firm A is $x = \frac{p_B - p_{A} - \beta + t}{2(t - \beta)}$, with the exception of $p_A = p_B = 0$ where we end up with three possible equilibrium market shares, where firm A can capture $x = 0, x = \frac{1}{2}$ and $x = 1$. If at least the highest price, denoted by $p_i$ where $i = A, B$, is greater than $\beta - t$ and equal to $p_i = (1 + \alpha)(\beta - t)$ (where $\alpha$ is a constant $\in (0, 1]$) we have an equilibrium only if the rival’s firm price, denoted by $p_j$ where $j = B, A$, is $0 \leq \frac{1}{2}(\alpha + \sqrt{\alpha\sqrt{8 + \alpha}})(\beta - t) \leq p_j \leq p_i \leq 2(\beta - t)$. In this case, the market share of firm A in equilibrium is $x = \frac{p_B - p_{A} - \beta + t}{2(t - \beta)}$. Diagrammatically, the equilibrium prices can be shown in the following figure.

In Figure 3, we can see from that for any combination of $p_A, p_B$, when $p_A, p_B \in [0, \beta - t]$ and for $p^*_i = (1 + \alpha)(\beta - t)$ (where $\alpha$ is a constant $\in (0, 1]$) and $0 \leq \frac{1}{2}(\alpha + \sqrt{\alpha\sqrt{8 + \alpha}})(\beta - t) \leq p^*_j \leq p_i \leq 2(\beta - t)$ we end up with an equilibrium where firm A captures $x^* = \frac{p_B - p_{A} - \beta + t}{2(t - \beta)}$. We have one exception (denoted by the dot in the figure) since we can have an equilibrium with $p^*_A = p^*_B = 0$ with three possible equilibrium market shares for firm A equal to $x^* = 0, x^* = \frac{1}{2}$ and $x^* = 1$.

**“Consistent” and “continuous” expectations**

If we restrict expectations to be “consistent” and “continuous” we can have either firm A
capturing the entire market \((x^* = 1)\) with equilibrium prices \(p_A^* = \beta - t\) and \(p_B^* = 0\), or firm B capturing the entire market \((x^* = 0)\) with equilibrium prices equal with \(p_A^* = 0\) and \(p_B^* = \beta - t\).

More analytically, assume \(0 \leq p_A = p_B\) and let us check if there are expectations that can support \(x = \frac{1}{2}\) as an equilibrium. We first examine the behavior of firm A. Since we let expectations to be “discontinuous”, an increase in firm A’s price can lead consumers to believe that firm A will loose all its customers \((x = 0)\) and a decrease in its price decreases its market share and its profit. For \(0 \leq p_A = p_B \leq 2(\beta - t)\) firm A will never try to decrease its price so drastically in order to be certain that it will attract the whole market (by charging \(p_A = p_B - \beta + t\)) since it will make less profit than when it was sharing the market. The same reasoning holds for firm B so we conclude that there are expectations that support \(0 \leq p_A = p_B \leq 2(\beta - t)\) with \(x = \frac{1}{2}\) as an equilibrium. Can the same prices constitute an equilibrium with \(x = 1\)? Since expectations can be “discontinuous”, an increase in firm A’s price can lead consumers to believe that firm A will loose all its customers \((x = 0)\) and a decrease in its price decreases its market share and its profit. As firm B is concerned and since expectations can be “inconsistent”, an increase as well as a decrease in firm B’s price may lead consumers to believe that firm B will never attract any customers. For \(0 \leq p_A = p_B \leq \beta - t\) firm B will never try to decrease its price so drastically (by charging \(p_B \leq p_A - \beta + t\)) in order to be certain that it will attract the whole market, since it will end up with \(\pi_B \leq 0\).

Let us first examine if we can have an interior solution. If \(p_A = p_B \geq 0\), we find that \(x = \frac{1}{2}\), but do these prices indeed constitute an equilibrium? With \(p_A = p_B \geq 0\), firm A has an incentive to increase its price since it will increase its market share. By increasing its price, firm A increases its profit both directly through its increased price and indirectly through its market share. We conclude that there is no equilibrium where the two firms share the market equally. Similarly, we cannot have any other interior equilibrium where the market is shared unevenly, since each
firm has an incentive to increase its price in order to increase its market share and its profit. We conclude that we cannot have an interior solution.

Can there be a corner solution? Assume $p_A = \beta - t > 0$, $p_B = 0$ and all consumers choose firm A. From expression (15), we observe that all consumers act rationally, given that they expect that everybody will choose firm A. We prove that prices $p_A = \beta - t > 0$ and $p_B = 0$ constitute an equilibrium. Firm A has no incentive to deviate because if it decreases its price it decreases its profit, as it already captures the entire demand. From expression (15), we find that if firm A increases its price, it will lose all its clients and will make zero profit, since then, all consumers will expect that firm B will capture the entire market. We conclude that, when $p_B = 0$, firm A optimally charges $p_A = \beta - t > 0$. Now, let us examine firm B’s behavior. Given $p_A = \beta - t > 0$, firm B clearly has no incentive to decrease its price below zero because it will make losses. If it increases its price and consumers continue to expect that firm A will capture the entire market (based on expression (15)), firm B will not attract any customers and therefore will make again zero profit. We conclude that $p_A^* = \beta - t > 0$ and $p_B^* = 0$ constitute an equilibrium, where firm A captures the entire market. By symmetry, we find that $p_A^* = 0$ and $p_B^* = \beta - t > 0$ also is an equilibrium, where now firm B captures the entire market. Can we have an equilibrium with $p_A = p + \beta - t > 0$ and $p_B > 0$? Firm A has no incentive to deviate since the arguments presented just above still apply. On the other hand, in this case, firm B has an incentive to slightly decrease its price, capture the entire market and make positive profit. We conclude that we cannot have an equilibrium when both firms charge strictly positive prices. We summarize:

**Proposition 4** When $t < \beta$ and consumers’ expectations are affected by firms’ prices, either firm A captures the entire market and the equilibrium prices are $p_A^* = \beta - t$ and $p_B^* = 0$, or firm B captures the entire market and the equilibrium prices are $p_A^* = 0$ and $p_B^* = \beta - t$.

Let us now discuss the structure of the equilibria when expectations are affected by firms’ prices and are “consistent” and “continuous”, and contrast it to the case where prices are ignored in the formation of the expectations.

We have seen that for $t > \beta$ (that is when the network effects are not very strong), in equilibrium, both firms charge prices $p_A^* = p_B^* = t - \beta > 0$. These are lower than the equilibrium prices that firms charge when these prices cannot affect consumers’ expectations, $p_A^* = p_B^* = t$.

The intuition behind this result is that, when firms know that their prices will affect consumer’s expectations, they compete more intensely and are willing to lower their prices to gain, indirectly, through an increase in their network effect (the market share consumers expect for them). Each firm aims at influencing consumers’ expectations in its favor. Given the price of one firm, the other firm tries to undercut its rival’s price in order to strategically alter consumers expectations.
Consumers that observe one firm to have a lower price than its rival, expect that a larger number of other consumers will choose this firm, so the market share of this firm will be larger than its rival’s (and so will be its attractiveness due to the network effect). With both firms acting this way, we end up with lower equilibrium prices compared to the case of unaffected expectations. On the other hand, in the case where \( t < \beta \), we end up only with corner solutions since the network effect is so strong that dominates all other differentiation and drives consumers to believe that a single firm will capture the entire market.

6 Comparison to full compatibility

Thus far, we have examined the case where the network effect operates at the level of each of the two (differentiated) products. Before proceeding to introduce quality differences in the model, it may be useful to briefly compare our results to the case where the network effect operates for the entire market, that is, the products are fully compatible. In our model we operate under the assumption of full market coverage and, as a result, it is easy to see that network effects that are not product specific play no role, they are the same as no network effect.

To see the point that, in this case, network effects play no role, note that the network effect is the same for each consumer, regardless of which product he chooses: \( \beta(x^e + 1 - x^e) = \beta \). Specifically, a consumer located at \( x \in [0, 1] \) is indifferent between the two products when their generalized costs are equal:

\[
p_A + tx^2 - \beta = p_B + t(1 - x)^2 - \beta.
\]

Clearly, as long as full market coverage is not an issue, the network effect (\( \beta \)) plays no role. Thus, this case corresponds to price competition under horizontal product differentiation without network effects and, by the derivations of d’Aspremont et al. (1979), we have:

\textbf{Remark 1} Consider the case where two firms offer horizontally differentiated products. Under full compatibility, the two firms share the market equally by charging \( p_A^* = p_B^* = t \), and make positive profit equal to \( \pi_A^* = \pi_B^* = \frac{t^2}{2} \), exactly as in the case of no network effects.

Contrasting our results above to the case of full compatibility (or no network effects) we see that the introduction of product specific effects may increase competition among the firms, leading to lower profit for both. In particular, in what is perhaps the most relevant case, network effects that are not too strong and expectations affected by prices, Proposition 3 implies that the introduction of product-specific network effects does not affect the way the market is shared in equilibrium, but firms charge lower prices \( (t - \beta \) rather than \( t \) and, as a result, make lower
profit equal to $\pi_A = \pi_B = \frac{t-\beta}{2}$. Then, full compatibility relaxes the competition between the two firms and allows them to enjoy higher profit:

**Remark 2** When the network effect is zero, or equivalently under full compatibility, competition between the two firms is weaker than in the case of product-specific and relatively weak network effects, and both firms enjoy then higher profit.

### 7 Vertically differentiated products

It is interesting to examine how the above derived set of equilibria changes when firms differ, not only with respect to their location, but also with respect to their product qualities. What is the role of the network effect when, in addition to horizontal, there is also vertical differentiation (that is all consumers agree that one of the products has higher inherent quality)? How are the price competition incentives affected? Under what conditions should a low quality firm be expected to gain a larger market share than its rival? We modify the basic model presented in Section 2 by introducing vertical differentiation between the two products. This model of horizontal and vertical differentiation has been explored in Vettas (1999). Here, the analysis is further enriched by also considering network effects. So let us say the product quality of firm A is $q_A$, while the quality of firm B is $q_B$. Without loss of generality we set $q_A = 0$ and $q_B = q \geq 0$, that is, firm B is the “high quality” firm and A the “low quality” firm, while $q \geq 0$ is simply the product quality difference. In this case, a consumer located at point $x \in [0,1]$ will be indifferent between the two products when

$$p_A + tx^2 - \beta x e = p_B + t(1-x)^2 - \beta(1-x e) - q.$$  \hfill (17)

Solving expression (17) for $x$, we obtain the location of the indifferent consumer, when firms are horizontally and vertically differentiated,

$$x = \frac{p_B - p_A + \beta(2xe - 1) + t - q}{2t}. \hfill (18)$$

#### 7.1 Price equilibrium with arbitrary expectations

As we also did in the case without quality difference, let us first examine firms’ competition for arbitrary consumers’ expectations about the market shares (that is, about the network effect). In subsequent steps we require these expectations to be rational. So let us take as given that all consumers expect firm A’s share to be $x^e \in [0,1]$. Both firms have positive market shares (that is $x \in (0,1)$) only if $q - t - \beta(2x^e - 1) < p_B - p_A < q + t - \beta(2x^e - 1)$. If $q - t - \beta(2x^e - 1) \geq p_B - p_A$,
firm B captures the entire market and if \( p_B - p_A \geq q + t - \beta(2x^e - 1) \), firm A captures the entire market. Therefore, firm A’s market share is

\[
x = \begin{cases} 
0 & \text{if } p_B - p_A \leq q - t - \beta(2x^e - 1) \\
\frac{p_B - p_A + \beta(2x^e - 1) + t - q}{2t} & \text{if } q - t - \beta(2x^e - 1) < p_B - p_A < q + t - \beta(2x^e - 1) \\
1 & \text{if } p_B - p_A \geq q + t - \beta(2x^e - 1). 
\end{cases}
\]

As a consequence, firm A maximizes its profit,

\[
\pi_A = \begin{cases} 
0 & \text{if } p_B - p_A \leq q - t - \beta(2x^e - 1) \\
p_A\left(\frac{p_B - p_A + \beta(2x^e - 1) + t - q}{2t}\right) & \text{if } q - t - \beta(2x^e - 1) < p_B - p_A < q + t - \beta(2x^e - 1) \\
p_A & \text{if } p_B - p_A \geq q + t - \beta(2x^e - 1),
\end{cases}
\]

with respect to \( p_A \) and firm B maximizes its profit,

\[
\pi_B = \begin{cases} 
p_B & \text{if } p_B - p_A \leq q - t - \beta(2x^e - 1) \\
p_B\left(1 - \frac{p_B - p_A + \beta(2x^e - 1) + t - q}{2t}\right) & \text{if } q - t - \beta(2x^e - 1) < p_B - p_A < q + t - \beta(2x^e - 1) \\
0 & \text{if } p_B - p_A \geq q + t - \beta(2x^e - 1),
\end{cases}
\]

with respect to \( p_B \).

We first examine the case where \( x \in (0, 1) \). The second-order conditions show that the profit functions are strictly concave, as \( \frac{\partial^2 \pi_A}{\partial p_A^2} = \frac{\partial^2 \pi_B}{\partial p_B^2} = -\frac{1}{t} < 0 \). Then, via the first-order conditions, we derive the reaction functions:

\[
p_A = R_A(p_B; x^e) = \frac{p_B + \beta(2x^e - 1) + t - q}{2}
\]

and

\[
p_B = R_B(p_A; x^e) = \frac{p_A - \beta(2x^e - 1) + t + q}{2}.
\]

By solving the system of the reaction functions, we obtain the equilibrium prices (given the expected market shares):

\[
p_A^*(x^e) = t + \frac{\beta(2x^e - 1) - q}{3} \tag{19}
\]

and

\[
p_B^*(x^e) = t - \frac{\beta(2x^e - 1) - q}{3} \tag{20}
\]

Substituting equations (19) and (20) into (18) we obtain

\[
x^*(x^e) = \frac{1}{2} + \frac{\beta(2x^e - 1) - q}{6t}. \tag{21}
\]

We observe that for \(-3t < \beta(2x^e - 1) - q < 3t\), both firms have positive market shares. We reach a corner, where firm A captures the entire market, when \( \beta(2x^e - 1) - q \geq 3t \). And we reach a corner, where firm B captures the entire market, when \( \beta(2x^e - 1) - q \leq -3t \).

\[\text{In order for } \beta(2x^e - 1) - q \geq 3t \text{ to hold, consumers’ expectations must satisfy } x^e > \frac{1}{2}, \text{ since } q \text{ can only take non negative values.}\]
Let us now explore possible corner solutions. Consider, first, the case where $\beta(2x^e - 1) - q \geq 3t$. Following the same steps as in Section 3 of the paper, but taking into account that the firms are not only horizontally but also vertically differentiated, we obtain that, in equilibrium, $p_A^* = \beta(2x^e - 1) - q - t$, $p_B^* = 0$ and firm A captures the entire market. Consider now the case where $\beta(2x^e - 1) - q \leq -3t$. Similar calculations show that $p_A^* = 0$, $p_B^* = -\beta(2x^e - 1) + q - t$ and firm B captures the entire market. To summarize:

**Proposition 5** Consider competition between firms A and B, given consumers’ expectations that A’s market share is $x^e \in [0, 1]$. The equilibrium prices are

$$
p_A^*(x^e) = \begin{cases} 
\beta(2x^e - 1) - q - t & \text{if } \beta(2x^e - 1) - q \geq 3t \\
\frac{t + \beta(2x^e - 1) - q}{3} & \text{if } -3t < \beta(2x^e - 1) - q < 3t \\
0 & \text{if } \beta(2x^e - 1) - q \leq -3t
\end{cases} \quad (22)
$$

and

$$
p_B^*(x^e) = \begin{cases} 
0 & \text{if } \beta(2x^e - 1) - q \geq 3t \\
\frac{t - \beta(2x^e - 1) - q}{3} & \text{if } -3t < \beta(2x^e - 1) - q < 3t \\
-\beta(2x^e - 1) + q - t & \text{if } \beta(2x^e - 1) - q \leq -3t
\end{cases} \quad (23)
$$

and the implied market share for firm A is

$$
x^*(x^e) = \begin{cases} 
1 & \text{if } \beta(2x^e - 1) - q \geq 3t \\
\frac{1}{2} + \frac{\beta(2x^e - 1) - q}{6t} & \text{if } -3t < \beta(2x^e - 1) - q < 3t \\
0 & \text{if } \beta(2x^e - 1) - q \leq -3t
\end{cases} \quad (24)
$$

with firm B having market share equal to $1 - x^*(x^e)$.

The next step in the analysis is to examine what are the market shares that can be supported in equilibrium when expectations about these are required to be fulfilled.

### 7.2 Equilibrium with rational (fulfilled) expectations not affected by prices

We impose rationality by substituting $x^e = x^*$ into expression (24). We obtain the following cases. **Case 1**: For relatively weak network effects ($\beta < 3t$), we obtain

$$
x^* = \begin{cases} 
0 & \text{if } q \geq 3t - \beta \\
\frac{1}{2} - \frac{q}{2(3t - \beta)} & \text{if } 0 \leq q < 3t - \beta
\end{cases} \quad (25)
$$

As we can see from expression (25), firm B (the high quality firm) has a larger market share than its rival if the quality difference is relatively small ($0 < q < 3t - \beta$), but both firms make positive
sales. If the network effect is relatively strong ($q \geq 3t - \beta$), firm B captures the entire market. To find the equilibrium prices, we substitute $x^e = x^*$ and (25) into (22) and (23) to obtain

$$p^*_A = \begin{cases} 0 & \text{if } q \geq 3t - \beta \text{ and } x^* = 0 \\ t + \frac{tq}{\beta - 3t} & \text{if } 0 \leq q < 3t - \beta \text{ and } x^* = \frac{1}{2} - \frac{q}{2(3t - \beta)} \end{cases}$$

and

$$p^*_B = \begin{cases} q - t + \beta & \text{if } q \geq 3t - \beta \text{ and } x^* = 0 \\ t - \frac{tq}{\beta - 3t} & \text{if } 0 \leq q < 3t - \beta \text{ and } x^* = \frac{1}{2} - \frac{q}{2(3t - \beta)}. \end{cases}$$

Case 2: For relatively strong network effects ($\beta > 3t$), we obtain

$$x^* = \begin{cases} \{0, 1\} & \text{if } 0 \leq q \leq \beta - 3t \\ 0 & \text{if } q \geq \beta - 3t. \end{cases} \tag{26}$$

Thus, when the quality difference between the two firms is relatively small ($0 \leq q \leq \beta - 3t$), one of the firms, either the high quality or the low quality one, captures the entire market. When the quality difference is relatively strong, the only equilibrium is when the high-quality firm captures the entire market.

To find the equilibrium prices, we substitute $x^e = x^*$ and (26) into (22) and (23) to obtain

$$p^*_A = \begin{cases} 0 & \text{if } q \geq 0 \text{ and } x^* = 0 \\ \beta - q - t & \text{if } 0 \leq q \leq \beta - 3t \text{ and } x^* = 1 \end{cases}$$

and

$$p^*_B = \begin{cases} q - t + \beta & \text{if } q \geq 0 \text{ and } x^* = 0 \\ 0 & \text{if } 0 \leq q \leq \beta - 3t \text{ and } x^* = 1. \end{cases}$$

Case 3: For the borderline case $\beta = 3t$, we obtain $x^* = 1$. We observe that when the two products have different qualities, the only expectation that can be fulfilled in equilibrium is when consumers expect that the high quality firm will capture the entire market. To obtain the equilibrium prices we substitute $x^e = x^* = 1$ and $\beta = 3t$ into (22) and (23) to obtain $p^*_A = 0$ and $p^*_B = 2t + q$. To summarize:

**Proposition 6** Suppose consumers have rational (but note affected by the observed prices) expectations about firms’ market shares and the products are horizontally and vertically differentiated. In equilibrium, we have: Case 1: For weak network effects ($\beta < 3t$), the high-quality firm captures a larger market share but both firms have positive market shares if $0 \leq q < 3t - \beta$. If $q \geq 3t - \beta$, the high-quality firm captures the entire market. Case 2: For strong network effects ($\beta > 3t$), either the high or the low-quality firm can capture the entire market if $q \leq \beta - 3t$. If $q \geq \beta - 3t$ the only equilibrium is for the high-quality firm to capture the entire market. Case 3: When $\beta = 3t$, the high-quality firm captures the entire market.
Let us discuss how the presence of a quality difference between the two products has modified our analysis just above, relative to the case of only horizontal differentiation. In this case, in equilibrium, firm A’s (the low quality firm’s) market share is

\[
x^* = \begin{cases} 
\frac{1}{2} + \frac{q}{2\beta - 3t} & \text{if } \beta < 3t \text{ and } q < 3t - \beta \\
0 \text{ or } 1 & \text{if } \beta > 3t \text{ and } q < \beta - 3t \\
0 & \text{if } q \geq \beta - 3t \geq 0.
\end{cases}
\]

It is interesting to observe that under certain conditions, the low-quality firm may capture the entire market. To be more precise, firm A, may end up capturing the entire market, in equilibrium, if the quality difference is small and the network effect is strong. Moreover, we observe that the two firms will never equally share the market, since this expectation cannot be fulfilled in equilibrium. The high-quality firm gets more customers than its rival, but both firms have positive market shares if the network effect is weak and the quality difference is small. If the quality difference is high enough (relative to the network effect), the high quality firm always captures the entire market.

Let us focus on the minimum quality difference that makes the high-quality firm (firm B) capture the entire market (see Figure 4).

![Figure 4: Critical value of quality differentiation in order for the high-quality firm to capture the entire market.](image)

If the quality difference is high enough that \( q > 3t \), the high-quality firm captures the entire market even under no network effects. Firm B can capture the entire market, even if it has the same quality with firm A \( (q = 0) \), when \( \beta \geq 3t \). For \( q \leq 3t \) and \( \beta \leq 3t \) firm B attracts the entire market only if \( q \geq 3t - \beta \), otherwise it shares the market with its rival.
7.3 Equilibrium when expectations are affected by prices

In this section, we examine how prices and market shares are formed when firms can manipulate consumers’ expectations (while, still, of course requiring that expectations are fulfilled in equilibrium). We set \( x^e = x \) in expression (18) to obtain

\[
x = \frac{p_B - p_A - \beta + t - q}{2(t - \beta)}.
\]

(27)

This expression corresponds to the location of the indifferent consumer if the prices are \( p_A \) and \( p_B \) and if all the consumers believe that the market share of firm A is given by expression (27).

It is essential to observe, as we also did in Section 5 for the case \( q = 0 \), that, if \( t > \beta \), an increase in firm A’s price decreases its market share. But, if \( t < \beta \), an increase in firm A’s price increases its market share. Note that \( x \in (0, 1) \) only if \( \beta - t + q < p_B - p_A < t - \beta + q \), assuming that \( t > \beta \).

If \( p_B - p_A \geq t - \beta + q \), then all consumers will choose firm A and if \( \beta - t + q \geq p_B - p_A \), then firm B captures the entire market. To summarize, when consumers form their expectations after observing firms’ prices, and when these expectations are fulfilled, we find that firm A’s market share, when \( t > \beta \), is

\[
x = \begin{cases} 0 & \text{if } p_B - p_A \leq \beta - t + q \\ \frac{p_B - p_A - \beta + t - q}{2(t - \beta)} & \text{if } \beta - t + q \leq p_B - p_A \leq t - \beta + q \\ 1 & \text{if } p_B - p_A \geq t - \beta + q. \end{cases}
\]

We first analyze the case where both firms have positive market shares. Following similar steps as in the case with no quality difference (see Section 5), we find that the best response correspondence of firm A is

\[
p_A = R(p_B) = \begin{cases} p_B - t + \beta - q & \text{if } p_B \geq 3(t - \beta) + q \\ \frac{p_B - \beta + t - q}{2} & \text{if } \beta - t + q \leq p_B \leq 3(t - \beta) + q \\ \text{any price} \geq p_B + t - \beta + q & \text{if } p_B \leq \beta - t + q \end{cases}
\]

and that of firm B is

\[
p_B = R(p_A) = \begin{cases} p_A - t + \beta + q & \text{if } p_A \geq 3(t - \beta) - q \\ \frac{p_A - \beta + t + q}{2} & \text{if } \beta - t - q \leq p_A \leq 3(t - \beta) - q \\ \text{any price} \geq p_A + t - \beta + q & \text{if } p_A \leq \beta - t - q. \end{cases}
\]

The two best response correspondences intersect at

\[
p_A^* = t - \beta - \frac{q}{3} \quad \text{and} \quad p_B^* = t - \beta + \frac{q}{3}.
\]

In equilibrium, the indifferent consumer is located at

\[
x^* = \frac{1}{2} - \frac{q}{6(t - \beta)}.
\]
For $t > \beta$, we observe that $x \in (0, \frac{1}{2}]$ if $0 \leq q < 3(t - \beta)$. When the quality difference is so high that $q \geq 3(t - \beta)$, firm B finds it more profitable to capture the entire market. In this case, firm A charges $p_A^* = 0$ and has no clients and zero profit and firm B charges $p_B^* = \beta + q - t > 0$.

To verify that, if $q \geq 3(t - \beta)$ and $t > \beta$, the equilibrium prices are $p_A^* = 0$ and $p_B^* = \beta + q - t > 0$ and firm B captures the entire market, we need to check whether any of the two firms has an incentive to increase its price in order to share the market with its rival. Given that $p_A = 0$, firm B shares the market with its rival if $t x^2 - \beta x = p_B + t(1 - x)^2 - \beta(1 - x) - q$.\(^\text{13}\)

In order to obtain firms B’s price when it shares the market with its rival, we first solve the equality with respect to $x$ and find

$$x = \frac{q - t + \beta - p_B}{2(\beta - t)}. \quad (28)$$

By replacing expression (28) into $\pi_B = p_B(1 - x)$, and solving the first order condition with respect to $p_B$, we obtain

$$p_B = \frac{q + t - \beta}{2}. \quad (29)$$

We observe that $p_B = \frac{q + t - \beta}{2} > \beta + q - t$, only if $q < 3(t - \beta)$, therefore we examine the case where firm B shares the market with its rival for $q < 3(t - \beta)$.\(^\text{14}\) Replacing expressions (28) and (29) into $\pi_B = p_B(1 - x)$, we find that the maximum profit firm B can make when it shares the market with its rival is $\pi_B = \frac{(q + t - \beta)^2}{8(t - \beta)}$. Comparing this profit with firm B’s profit when it captures the entire market, $\pi_B = q + \beta - t$, we observe that firm B prefers to share the market with its rival when $q < 3(t - \beta)$ and $t > \beta$.

Let us now examine the case where $q \geq 3(t - \beta)$ and $t > \beta$. We proceed by calculating the first derivative of the profit function $\pi_B = p_B(1 - \frac{p_B - p_A - \beta + t - q}{2(t - \beta)})$ with respect to $p_B$, after replacing $p_A = 0$ and $p_B = \beta + q - t$. We obtain $\frac{\partial \pi_B}{\partial p_B} = -\frac{2 - 3(t - \beta)}{2}$ and observe that it becomes negative when $q > 3(t - \beta)$. So, in this case, firm B’s profit function decreases when $p_B \geq \beta + q - t$.

We conclude that, when $\beta > 3t$, $q > 3(t - \beta)$ and $p_A = 0$, firm B maximizes its profit when it captures the entire market by charging $p_B = \beta + q - t$, since, if it increases its price in order to share the market with its rival, it decreases its profit. Similar is the logic that holds for firm A. We conclude that, for $t > \beta$ and $q \geq 3(t - \beta)$, the equilibrium prices are $p_A^* = 0$ and $p_B^* = \beta + q - t > 0$, firm B captures the entire market and no firm has an incentive to deviate.

We summarize as follows.

\(^{13}\)Note that in this case where consumers’ expectations are affected by firms prices, both $t$ and $\beta$ are multiplied by $x$, which is the location of the indifferent consumers when both firms share the market.

\(^{14}\)Firm B already captures the entire market by charging $p_B = \beta + q - t$. It has no incentive to lower its price, since it will continue to capture the entire market, but now with less profit. We only examine the case where firm B increases its price in order to share the market with its rival.
Proposition 7 Suppose consumers’ expectations about market shares are affected by the firms’ prices, and let $t > \beta$. If $0 \leq q \leq 3(t - \beta)$ there is a unique equilibrium where both firms have positive market shares, with the higher quality firm capturing a larger market share than its rival. The equilibrium prices are $p^*_A = t - \beta - \frac{q}{3}$ and $p^*_B = t - \beta + \frac{q}{3}$. If $q \geq 3(t - \beta)$, there is a unique equilibrium where the high-quality firm captures the entire market and the equilibrium prices are $p^*_A = 0$ and $p^*_B = \beta + q - t > 0$.

Let us now turn to the case of strong network effects ($t < \beta$). When $t < \beta$, both firms’ demand functions can be increasing in their own prices. Firm A’s market share, when consumers behave rationally and when $t < \beta$, is

\[
x = \begin{cases} 
0 & \text{if } p_B - p_A \leq \beta - t + q \\
 \frac{p_B - p_A - \beta + t - q}{2(t - \beta)} & \text{if } t - \beta + q \leq p_B - p_A \leq \beta - t + q \\
1 & \text{if } p_B - p_A \geq t - \beta + q
\end{cases}
\] (30)

Note that firm A’s market share is given by expression (30) when the prices are $p_A$ and $p_B$ and when consumers believe that firm A’s market share is given by expression (15). As we can see, when $t - \beta + q \leq p_B - p_A \leq \beta - t + q$, we end up with three different expectations concerning the market shares of the two firms: when consumers observe that the difference of prices belongs to the particular interval, they may believe that all consumers will choose firm A, that all consumers will choose firm B, or that consumers will choose firm A and the rest will choose firm B. We observe that when the price difference belongs to the particular interval, an increase in the price of a firm, increases its market share.

In this case, as in the case of only horizontally differentiated products, we need to specify the different equilibria that emerge depending on the assumptions we make concerning the formation of expectations. Since the analysis is quite similar to the case of strong network effects when the products are only horizontally differentiated, we do not state our analytical results and proceed to the case where we have consistent and continuous expectations. The analytical results can be provided upon request to the authors.

When expectations are consistent and continuous, the network effects are strong ($t < \beta$) and the quality difference is small ($q \leq \beta - t$) we cannot have an interior solution: each firm has an incentive to increase its price in order to increase its market share and its profit. Can we end up with a corner solution? If we set $p_A = 0$ and $p_B = \beta - t + q$, all consumers choose firm B. We need to check whether firm B has an incentive to deviate (obviously, to a higher price, as its market share cannot be increased further). From expression (30), we observe that, if firm B increases its price, it loses all its clients and makes zero profit. We conclude that for $p_A = 0$, firm B optimally charges $p_B = \beta - t + q$. Now, let us check firm A’s behavior. Firm A has no
incentive to decrease its price below zero, since that would lead to a loss. If firm A increases its price and if consumers continue to believe that firm B will capture the entire market, then firm A will not attract any customer and therefore will continue having zero profit. We conclude that \( p_A^* = 0 \) and \( p_B^* = \beta - t + q \) constitute an equilibrium pair of prices and firm B captures the entire market. The same logic holds for \( p_A = \beta - t + q \) and \( p_B = 0 \). Can we have an equilibrium when \( p_A > 0 \) and \( p_B = p_A + \beta - t + q \)? Firm B has no reason to deviate by the same arguments as in the previous case. On the other hand, firm A has an incentive to decrease its price in order to attract the entire market and make positive profit. We conclude that we cannot have an equilibrium when both firms charge strictly positive prices. We conclude:

**Proposition 8** When \( t < \beta \), consumers’ expectations are affected by firms’ prices and the two products have different qualities but \( q \leq \beta - t \), we end up with two equilibria where either firm can capture the entire market. Firm A captures the entire market when \( p_A^* = \beta - t - q \) and \( p_B^* = 0 \) and firm B captures the entire market when \( p_A^* = 0 \) and \( p_B^* = \beta - t + q \).

For the case where the qualities of the two products are quite distinct, that is \( q \geq (\beta - t) \), firm A will never attract the entire market in equilibrium since \( p_A^* = \beta - t - q < 0 \) and firm B always prefers to capture the entire market than to share it with firm A. In this case, when expectations are consistent and continuous we end up with a unique equilibrium where \( p_A^* = 0 \) and \( p_B^* = \beta - t + q \) and firm B, the higher quality firm, attracts the entire market.

**Proposition 9** When \( t < \beta \), consumers’ expectations are affected by firms’ prices and the two product qualities are quite distinct, that is when \( q \geq \beta - t \), there is a unique equilibrium where the high quality firm captures the entire market and the equilibrium prices are \( p_A^* = 0 \) and \( p_B^* = \beta - t + q \).

Some discussion is in order. When firms differ in two dimensions, that is, with respect to location and with respect to quality \( (q > 0) \) and when the network effect is relatively weak \( (t > \beta) \), our results depend on the magnitude of the quality difference between the two firms. The high-quality firm always attracts more customers than its rival and finds it more profitable to capture the entire market if the quality difference is high enough (that is, \( q \geq 3(\beta - t) \)). When both firms have positive market shares, the high-quality firm charges a higher price compared to its rival, and its price increases as the quality difference increases. In the case where the network effect is relatively strong \( (t < \beta) \), either firm can capture the entire market if the quality difference is relatively low (that is \( q \leq t - \beta \)), but we can not have an interior equilibrium. In order to have a unique equilibrium where the high quality firm, firm B, captures the entire market, we need a higher quality difference (that is \( q \geq t - \beta \)), but lower than the quality difference needed in the case of relatively weak network effects.
8 Conclusion

We have studied a simple duopoly model with the following features. The two products are horizontally differentiated (with firms’ locations taken as given), possibly also vertically differentiated (with different qualities) and exhibit network effects. There are two distinct assumptions one can make about rational formation of expectations about equilibrium, essentially depending on the possible commitment of the prices set. We examine both assumptions and show that the results are qualitatively different. When the firms cannot commit to the prices they announce, consumers’ expectations cannot be affected by these. Then, if the two firms are only horizontally differentiated and the network effect is weak, the only equilibrium is that the firms share the market equally; if the network effect is strong, either firm may capture the entire market, or they can share the market equally. When the two products have the same quality we can never have an “interior” asymmetric equilibrium: if both firms make positive sales, then in equilibrium they share the market equally. If the two firms also differ in their quality, we can obtain an asymmetric equilibrium where both firms have positive market shares, but one firm (the high-quality one) attracts more customers. Under certain conditions (small quality difference and strong network effects), in equilibrium, the low-quality firm can be the one that captures the entire market.

When we turn to the case where firms’ prices affect consumers expectations, we observe that firms compete more intensely compared to the previous case. When the network effect is relatively weak and the two products have the same quality, the firms share the market equally (but with lower prices and profits relative to the previous case). If the two products have different qualities, the high-quality firm captures a larger market share; it may capture the entire market if the quality difference is large enough. The high-quality firm always makes a positive profit. When the network effect is strong, we can only obtain corner solutions. When the two firms have different qualities and these qualities are quite distinct, there is a unique equilibrium where the high-quality firm captures the entire demand.

In our model, where consumers have unit demands, full compatibility is equivalent to zero network effects. We find that, generally, the presence of network effects may lead firms to compete more intensively and their profits to be lower than in the case where there are no such effects.

It seems desirable to explore a number of variations of the basic model studied here. These may include the case where the game is dynamic, a richer underlying product differentiation structure (or endogenous determination of product differentiation), or a different specification of the network effect. These are non-trivial extensions and should generate additional insights, thus hopefully will be, along with empirical investigations of differentiated products competition under network effects, the topic of future research.
References


