Title
Intimidating Competitors—Asymmetric Vertical Integration and Downstream Investment in Oligopoly

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1 Introduction

In many industries, vertically integrated firms compete with separated firms. Well-documented examples include the oil industry (Bindemann 1999), the beer industry in the UK (Slade 1998a), the gasoline retail market in Vancouver (Slade 1998b), the U.S. cable television industry (Waterman and Weiss 1996, Chipty 2001), and the Mexican footwear industry (Woodruff 2002). These examples suggest that the coexistence of vertical integration and separation is an important aspect of product market competition in vertically-related industries. Moreover, it is sometimes claimed that there is a close relation between vertical integration and market shares in a given industry. For instance, the European Commission (1999) argued in the highly controversial Airtours/First Choice case that vertical integration was a prerequisite for significant increases of market shares and, conversely, that only firms of sufficient size were able to integrate vertically. While the theoretical basis of the argument was not entirely clear, the UK market for foreign package holidays under consideration appeared to match the Commission’s description. More generally, Adelman (1955) and Chandler (1977) note a positive correlation between firm size and vertical integration.

It is well-known that asymmetric vertical integration may arise endogenously (see, e.g., Ordover et al. 1990, Elberfeld 2002, Jansen 2003). However, there is considerable debate about the robustness of asymmetric integration equilibria. For instance, while Buehler and Schmutzler (2005) argue that there are good reasons why integration decisions tend to be strategic substitutes—so that asymmetric vertical integration may arise even for firms that are symmetric initially—they also emphasize potential countereffects.

In the present paper, we identify a neglected mechanism that helps explain why asymmetric vertical integration seems to be more common in reality than previous theoretical analysis suggests. This mechanism involves the intimidation effect of vertical integration, i.e., the adverse strategic effect on the competitor’s cost-reducing investments. The main contribution of this paper is to show that once one accounts for the intimidation effect of vertical integration, asymmetric integration equilibria are more likely to come about.

We adopt a framework where two downstream firms face at least two up-
stream suppliers. To produce one unit of the final product, downstream firms require one unit of an intermediate good produced by upstream firms. Downstream marginal costs consist of the costs of obtaining the intermediate good plus the costs of transforming the intermediate good into the final product. Initially, all firms are vertically separated. In stage 1, downstream firms decide whether to integrate backwards by acquiring a supplier, thereby getting access to the intermediate good at marginal costs. In stage 2, downstream firms can invest into reducing the costs of transforming the intermediate good into the final product, thereby increasing their transformation efficiency. In stage 3, the wholesale price is determined at which separated downstream firms are supplied. In stage 4, product market competition takes place.

In the core of the paper, we model only stages 1 and 2 explicitly. Stages 3 and 4 are treated in reduced form, using assumptions that are consistent with our analysis of a standard linear Cournot model with $2 \times 2$ firms. In particular, we assume that a firm’s demand and mark-up are higher when it is integrated and when its transformation efficiency is high, whereas demand and mark-up are lower when the competitor is integrated and has high transformation efficiency. Intuitively, this assumption on the effects of integration reflects the presence of *efficiency effects* (reductions in own marginal costs) and *foreclosure effects* (increases in competitor marginal costs). As we shall argue in Section 3.2, these effects have been identified in the literature as plausible consequences of vertical integration, even though they must not necessarily arise either in the real world or in theoretical models. In any case, the presence of efficiency and foreclosure effects is sufficient, but not necessary as a justification of our assumption.

In this setting, we establish our main result—asymmetric integration equilibria are more likely to come about once one accounts for cost-reducing investment—in two steps:

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1Our reduced-form analysis is also consistent with other models of vertically-related industries that consider product market competition explicitly. See, e.g., Ordover et al. (1990), Riordan (1998), and Linnemer (2003).

2For instance, vertical integration will often reduce both demand and supply on the upstream market, with ambiguous effect on the upstream price. As a result, foreclosure effects will not necessarily occur in successive oligopoly models.
(i) In a first step, we demonstrate that a firm’s vertical integration increases its incentives to invest into improving its own efficiency, and decreases the competitor’s incentives to do so. The latter effect on the competitor—the *intimidation effect* of vertical integration—implies that there is a strategic incentive to integrate vertically. Put differently, vertical integration may serve as a top dog strategy (Fudenberg and Tirole 1984) geared towards tapering the competitor’s cost-reducing investments.\(^3\)

(ii) In a second step, we show that the strategic integration incentive is likely to be larger for a firm facing a separated competitor than for a firm facing an integrated competitor due to demand/mark-up complementarities in the product market. Thus, asymmetric equilibria typically involve integrated firms that invest more into efficiency than their separated counterparts and therefore have higher market shares, which is in line with stylized facts.

To put our findings in perspective, note that related papers have appealed to efficiency differences to explain asymmetric vertical integration (Buehler and Schmutzler 2005, and Dufeu 2004). However, the efficiency differences in these paper are exogenous. In the present paper, we argue that both efficiency differences and asymmetric vertical integration arise endogenously even in a symmetric framework when firms decide both on vertical integration and cost-reducing investment.\(^4\) Integrated firms will then have higher market shares than their separated counterparts for two reasons: First, vertical integration itself makes them more efficient. Second, integrated firms face stronger incentives to carry out (further) cost-reducing investments.

Therefore, our paper not only contributes to the literature on asymmetric vertical integration, it also sheds new light on the discussion of endogenous market dominance. A large literature uses dynamic investment models suggesting how such dominance may emerge in a setting where there are initially

---

\(^3\)This result is vaguely related to Colangelo (1995), who finds that vertical integration may pre-empt horizontal mergers.

\(^4\)In another paper, Banerjee and Lin (2003) consider the investment incentives of downstream competitors facing a monopolistic upstream supplier. As downstream firms are assumed to be separated, these authors cannot analyze asymmetric equilibria.
small differences between a leader and a laggard.\textsuperscript{5} This literature does not discuss the role of vertical integration decisions in this context. Our paper shows how asymmetric integration decisions can lead to endogenous market dominance even in a static setting.

The remainder of the paper is organized as follows. In section 2, we discuss a linear Cournot example to set the stage. In section 3, we present the main results for our more general reduced-form model. Section 4 concludes.

2 An Introductory Cournot Example

To motivate our reduced-form analysis below, we modify the Cournot model proposed by Salinger (1988) to allow for endogenous decisions on integration and cost-reducing investment by downstream firms. Note that the example is of purely introductory nature and will be generalized in Section 3.

2.1 The Set-up

Initially, there is full vertical separation with two upstream firms and two downstream firms. To produce one unit of the final product, downstream firms require one unit of the intermediate good provided by upstream firms. Before going into details, we give an overview of the time structure of the game (see Figure 1).

\begin{figure}[h]
\centering
\begin{tabular}{c|c|c|c}
  stage 1 & stage 2 & stage 3 & stage 4 \\
  $V_1, V_2$ & $Y_1, Y_2$ & $w_1, w_2$ & $\Pi_1, \Pi_2$ \\
\end{tabular}
\caption{Time structure of the game}
\end{figure}

**Overview.** In stage 1, downstream firms simultaneously decide whether to integrate backwards by acquiring one of the upstream firms. The decision...
sion of firm \( i = 1,2 \) is summarized in a variable \( V_i \) such that \( V_i = 1 \) if it integrates and \( V_i = 0 \) if it remains separated. In stage 2, downstream firms simultaneously carry out cost-reducing investments \( Y_i \) at cost \( K(Y_i) = kY_i^2 \), thereby determining the efficiency at which the intermediate good is transformed into the final product. In stage 3, any remaining separated upstream firms set wholesale prices or quantities for the downstream market, resulting in marginal costs \( w_i \geq 0 \) for obtaining the intermediate good. In stage 4, downstream firms compete à la Cournot in the product market. We now consider each of the four stages, in turn.

**Stage 4.** In the product market, firms face a linear inverse demand curve \( P(Q) = a - Q \), with \( Q = q_1 + q_2 \) and \( a > 0 \). As will become clear below, the firms’ activities in stages 1, 2 and 3 determine the marginal costs \( c_i \) of downstream firms in stage 4, where they obtain the profits \( \Pi_i = \frac{(a - 2c_i + c_j)^2}{9}, i, j = 1, 2, j \neq i \).

**Stage 3.** Assuming that the marginal cost of producing the input is constant and normalized to zero, \( w_i \) is the marginal cost of producing the intermediate good for an integrated firm, and the equilibrium upstream price faced by a separated downstream firm. Depending on the integration decisions \( V = (V_1, V_2) \), this upstream price is either set directly by a single upstream firm or results from upstream competition between separated upstream firms in stage 3.\(^6\) Obviously, stage 3 is irrelevant if both firms are vertically integrated: If \( V = (1, 1) \), the costs of obtaining the input are given exogenously as \( w_i = 0, i = 1, 2 \), by assumption. However, if \( V = (1, 0) \) or \( V = (0, 1) \), the remaining separated upstream firm sets the monopoly price for the separated downstream firm; only for the integrated firm is \( w_i = 0 \). If \( V = (0, 0) \), separated upstream firms compete à la Cournot.

**Stage 2.** In the investment stage, both firms initially have identical transformation costs \( \bar{t} \). Denoting the efficiency improvement for the subgame \( \bar{V} \) as \( Y_i(\bar{V}) \), ex post transformation costs are \( t_i = \bar{t} - Y_i(\bar{V}) \), and firm \( i \)'s marginal costs are thus given by \( c_i = w_i + t_i = w_i + \bar{t} - Y_i(\bar{V}) \).

**Stage 1.** In the integration stage, downstream firms can acquire an up-

\(^6\)Like much of the related literature, we are thus abstracting in this example from the possibility that integrated firms also sell the intermediate input on the wholesale market. Our more general approach in Section 3 allows for such a possibility.
stream firm at fixed cost $F$. Let $w_i(V, Y)$ denote the equilibrium choice of $w_i$ in subgame $(V, Y)$ and the resulting level of $c_i$ as $c_i(V, Y)$. Thus, downstream firms choose $V_i \in \{0, 1\}$ so as to maximize $\Pi_i(c_1(V, Y), c_2(V, Y)) - V_i F$.

We now proceed to the subgame perfect equilibrium of the game.

### 2.2 Subgame Perfect Equilibrium

In the first three stages, the subgame perfect equilibrium gives rise to integration decisions $V_i$, efficiency levels $Y_i(V)$ and input prices $w_i(V, Y)$. As to stage 4, we use $Q_i(V, Y)$ to denote downstream outputs for arbitrary integration vectors $V$ and efficiency levels $Y$, assuming that marginal downstream costs are $c_i(V, Y) = w_i(V, Y) + \ell - Y_i(V)$. Similarly, we write the equilibrium mark-ups and profits of downstream firms as $M_i(V, Y)$ and $\Pi_i(V, Y)$, respectively.

Table 1 describes the subgame equilibrium for the three market configurations $V = (0, 0)$, $V = (1, 0)$ and $V = (1, 1)$ and the associated reference configurations where firms are not allowed to invest (i.e. $Y = (Y_1, Y_2) = (0, 0)$ by assumption). Equilibrium quantities are given as functions of the efficiency levels $Y$ (and of $k$ where appropriate). Throughout, $\alpha \equiv a - \ell$ is a measure of market size. Table 2 summarizes the results for the special case $k = 1$.

We now highlight the implications of Table 1 that are particularly important for our more general analysis below.

<Tables 1 and 2 around here>

### 2.2.1 Investments Under Asymmetric Vertical Structure

First, we compare the investments of integrated and separated firms in the same market. Thus, we consider the asymmetric vertical market structure $V = (1, 0)$. Figure 2a) depicts the optimal investment levels $Y_1$ and $Y_2$ as functions of the cost parameter $k$, fixing $\alpha = 1$. Figure 2b) shows the resulting market shares $s_1$ and $s_2 = 1 - s_1$, respectively. Clearly, the integrated firm 1 invests more and has a higher market share than the separated firm 2.
To put the result into perspective, consider output decisions when firms are unable to invest into cost reduction (or equivalently, \(k \to \infty\)). Figure 2b) indicates that even when firms cannot invest into cost reduction, the market share of the integrated firm is higher than that of the separated firm, i.e. \(s_1(Y_1 = 0, Y_2 = 0) > 0.5\). This reflects the simple fact that the integrated firm has lower marginal costs than the separated firm due to the elimination of a mark-up at the upstream level. However, this is not the end of the story: If firms can invest into cost reduction, the gap between the two firms widens, since the integrated firm invests more than the separated firm (see Figure 2b)). Why the integrated firm invests more than the separated firm is not quite as obvious. In Section 3, we will show that the intuition for this result becomes clearer in a more general model. We summarize our results for \(V = (1,0)\) as follows:

**Observation 1 (fixed structure)** *In the linear Cournot example, the integrated firm has higher output, mark-up and market share than the separated firm, even if efficiency levels are exogenous and identical. If investment levels are endogenous, the integrated firm invests more and the differences in outputs, mark-ups and market shares increase.*

**2.2.2 Investments when Vertical Structure Changes**

In Observation 1, we considered the investment behavior of integrated and separated firms for a fixed asymmetric vertical market structure. This comparison was natural to understand the relation between integration and market share. However, it is also important to understand how changes in vertical structure affect the firms’ investment behavior. For instance, if, in a setting with initially separated firms, a vertical merger is prohibited, does the prohibition affect the investment decisions of the firms? Table 2 indicates that starting from \(V = (0,0)\), firm 1’s vertical integration increases own investment and decreases firm 2’s investment.\(^7\) Starting from \(V = (1,0)\), firm

\(^7\)More specifically, \(Y_1(1,0) = 0.363\alpha > Y_1(0,0) = 0.108\alpha\) and \(Y_2(1,0) = 0.040\alpha < Y_2(0,0) = 0.108\alpha\).
2's integration has similar effects on investments. The adverse effect on the competitor's investment is what we call the *intimidation effect* of vertical integration. We summarize these findings as follows:

**Observation 2 (changing structure)** *In the linear Cournot example, a firm's vertical integration increases its own investments and decreases the competitor's investments (intimidation effect).*

### 2.2.3 Asymmetric Integration Equilibria

Next, we consider the subgame perfect equilibrium of the entire game. In particular, we show that the integration equilibrium is always asymmetric for suitable values of $F$. To this end, reconsider Table 2. For $k = 1$, this table lists the equilibrium profits for the subgames starting in stage 2, net of investment costs, $\Pi_i^*(V) = \Pi_i(V, Y(V))$. It is straightforward to calculate firm $i$'s integration incentive $\Delta_i^\pi$, which is defined as the profit differential resulting from integration if the competitor's integration status is $V$. The table shows that

\[
\Delta_i^\pi(0) \equiv \Pi_i^1(1,0) - \Pi_i^1(0,0) \approx 0.233\alpha^2; \\
\Delta_i^\pi(1) \equiv \Pi_i^1(1,1) - \Pi_i^1(0,1) \approx 0.089\alpha^2.
\]

Thus, we immediately obtain our first result for the linear Cournot example.

**Result 1** *In the linear Cournot game with vertical integration and cost-reducing downstream investment, the subgame perfect equilibrium involves asymmetric vertical integration for suitable levels of fixed acquisition costs ($0.233\alpha^2 \geq F \geq 0.089\alpha^2$).*

We now show that cost-reducing investments are important for explaining asymmetric vertical integration. To this end, we consider a setting where firms are not allowed to invest into cost-reduction. We define

\[
\Delta_i^{NJ}(V) \equiv \Pi_i(1, V; 0) - \Pi_i(0, V; 0),
\]
and similarly for $\Delta_{2}^{NI}(V_1)$, where the superscript indicates “No Investment”. Straightforward calculations show that

$$\Delta_{i}^{NI}(0) \approx 0.146\alpha^2 < \Delta_{i}^*(0); \quad \Delta_{i}^{NI}(1) \approx 0.083\alpha^2 < \Delta_{i}^*(1).$$

Therefore, asymmetric integration is also possible without the investment stage; however, the relevant interval is much smaller ($0.146\alpha^2 \geq F \geq 0.083\alpha^2$): Compared with a game without investments, integration incentives are higher if firms can make cost-reducing investments, and this effect is much more pronounced when competitors are separated. We thus obtain the second result for the Cournot example.

**Result 2** In the linear Cournot game with vertical integration and cost-reducing downstream investment, the subgame perfect equilibrium involves asymmetric vertical integration for a larger range of parameters than for a reference game without investment.

Result 2 is our main result for the Cournot example. It suggests that the empirical fact of asymmetric integration is easier to explain in a model which takes the effects of vertical integration on investment decisions into account than in a setting without investment. This result relates to the intimidation effect of integration identified in Observation 2. Apparently, a firm that faces a separated competitor gains more from influencing investment decisions by vertical integration than a firm that faces an integrated competitor. Thereby, the strategic substitutes property of integration decisions—which holds that a competitor’s decision to integrate decreases own integration incentives—becomes more pronounced.

We now proceed to a more general framework to improve our understanding of this mechanism.

### 3 A Reduced Form Model

In this section, we show how the results from Section 2 generalize beyond the linear Cournot model and clarify the intuition behind these results.
3.1 Assumptions

Rather than explicitly considering all four stages of the game (as in the Cournot example), we now focus on stages 1 and 2; stages 3 and 4 are treated in reduced form. We still assume that, initially, there are two identical separated downstream firms. However, we now allow that there are \( s \geq 2 \) identical separated upstream firms.

Recall that in stage 1, downstream firms simultaneously decide whether to take over one of the upstream suppliers, \( (V_i = 1 \text{ if they do, } V_i = 0 \text{ otherwise, } i = 1,2) \). We suppose that the acquisition costs are given by a constant \( F > 0 \). In stage 2, firms choose cost-reducing investments. We now assume that reducing costs by \( Y_i \) involves investment costs of \( K_i (Y_i) \), which are increasing in \( Y_i \). To model stages 3 and 4 in reduced form, we make the following assumptions.

**Assumption 1** The firms’ decisions in stages 1 and 2 result in unique input prices, \( w_i (V, Y), i = 1,2 \). Therefore, marginal costs are

\[
c_i (V, Y) = w_i (V, Y) + t - Y_i.
\]

For given marginal costs \( c_i, i = 1,2 \), downstream competition must satisfy the next assumption.

**Assumption 2** For every cost vector \( c = (c_1, c_2) \), there exists a unique product market equilibrium resulting in outputs \( q_i (c) \), prices \( p_i (c) \), mark-ups \( m_i (c) = p_i (c) - c_i \), and profits \( \pi_i (c) \), respectively, such that

\[
\pi_i (c) = q_i (c) \cdot m_i (c).
\]

The functions \( q_i (c) \), \( m_i (c) \) and thus \( \pi_i (c) \) are non-increasing in \( c_i \) and non-decreasing in \( c_j \).

Assumption 2 holds for many standard oligopoly models. Using Assumption 2, equilibrium product market profits \( \Pi_i \) as well as mark-ups \( M_i \) and outputs \( Q_i \) are functions of the firms’ vertical structures and efficiency levels:
Notation 1 (equilibrium quantities) For $i = 1, 2, j \neq i$, we denote downstream profits, mark-ups and quantities, respectively, as

\[
\begin{align*}
\Pi_i (V, Y) &= \pi_i (c_1 (V, Y), c_2 (V, Y)); \\
M_i (V, Y) &= m_i (c_1 (V, Y), c_2 (V, Y)); \\
Q_i (V, Y) &= q_i (c_1 (V, Y), c_2 (V, Y)).
\end{align*}
\]

(1) \hspace{1cm} (2) \hspace{1cm} (3)

We require that profits satisfy the following symmetry condition, which obviously holds in the Cournot case.

Assumption 3 Product market profits are exchangeable, i.e. for all $V', V'' \in \{0, 1\}$ and $Y', Y'' \in [0, \infty)$,

\[\Pi_1 (V', V'', Y', Y'') = \Pi_2 (V'', V', Y'', Y').\]

Our next assumption is crucial. It states relations between vertical structure and outputs and mark-ups, respectively, which are satisfied in the linear Cournot model.

Assumption 4 The firms’ mark-ups and outputs satisfy the following conditions:

(i) $M_1 (1, 0; Y) > M_1 (0, 1; Y); Q_1 (1, 0; Y) > Q_1 (0, 1; Y)$

(ii) $M_2 (0, 1; Y) > M_2 (1, 0; Y); Q_2 (0, 1; Y) > Q_2 (1, 0; Y)$

Assumption 4 compares the mark-ups and outputs of an integrated firm facing a separated competitor with those of a separated firm facing an integrated competitor. Part (i) can be justified by reference to (2) and (3), which show that an increase in $V_1$ and a decrease in $V_2$ will affect $Q_1$ and $M_1$ via $c_1$ and $c_2$. More specifically, starting from $V = (0, 1)$, suppose firm 1 integrates, resulting in $V = (1, 1)$. Our earlier discussion suggests that this reduces $c_1$ by eliminating the upstream mark-up or benefiting from technical efficiencies. As firm 2 is integrated and therefore supplies itself with the intermediate good, there is no effect on firm 2’s cost. The only effect of moving from $V = (0, 1)$ to $V = (1, 1)$ on firm 1’s mark-up and output is
thus a reduction of firm 1’s marginal cost, which should be unambiguously positive. Next, compare $V = (1, 1)$ and $V = (1, 0)$. Arguing as before, the costs of firm 1 should be unaffected and firm 2’s costs should increase. If firm 1’s mark-up and output are increasing in the competitor’s costs, they should therefore increase. Combining the two steps, we get part (i) of Assumption 4. The argument for part (ii) is analogous.

Our last assumption concerns the effect of higher efficiency on own and competitor mark-up and output, respectively.

**Assumption 5** $M_i$ and $Q_i$ are both increasing in $Y_i$ and decreasing in $Y_j$.

Assumption 5 states that an increase in own efficiency increases own mark-up and output, and conversely for an increase in competitor efficiency. The first part of the assumption is natural if higher efficiency does not only decrease own costs, but also (weakly) increases the wholesale price, so that competitor costs increase. If, however, higher efficiency decreases the wholesale price, competitor cost reductions could, in principle, outweigh the positive effect of higher efficiency on own costs. The second part of the assumption can be justified with similar arguments.8

### 3.2 Relation to the Literature

Our reduced form set-up is motivated by various more specific models. In particular, our crucial Assumption 4 is consistent with several models of successive oligopolies, in which efficiency and/or foreclosure effects of vertical integration have been identified.

For example, Salinger (1988) treats vertical integration decisions as exogenous. He considers a fixed-proportion linear Cournot model with arbitrary numbers of homogeneous firms. It turns out that integration always causes an efficiency effect as it eliminates successive mark-ups. Whether integration also generates a foreclosure effect depends on parameter values.9 Neverthe-

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8 We use a similar set-up in Buchler and Schnutzler (2004). There, however, efficiency differences are exogenous as there is no investment stage.
9 More specifically, vertical integration increases the wholesale price if more than half of the firms producing the intermediate product are vertically integrated.
less, Assumption 4 is always satisfied in Salinger’s model.\(^\text{10}\)

Ordover et al. (1990) examine a fixed-proportion model with endogenous integration decisions. Two upstream firms produce a homogenous input good and compete in prices. In one version of their model, two downstream firms produce differentiated products and compete in prices. Their model thus rules out an efficiency effect of integration when the competitor is not integrated.\(^\text{11}\) Similarly, there is a foreclosure effect only when the competitor is not integrated.

Hart and Tirole (1990) analyze different variants of a model where two upstream firms produce a homogenous input good and compete in prices, whereas two downstream firms engage in Cournot competition.\(^\text{12}\) Non-linear contracts between upstream and downstream firms are allowed. In the so-called ex post monopolization variant of the model, the more efficient upstream firm integrates with one of the downstream firms and slightly undercuts the less efficient upstream firm to supply the other downstream firm. This results in an efficiency effect of integration.\(^\text{13}\) Integration has no foreclosure effect, as it does not raise the downstream rival’s costs.

Chen (2001) examines a fixed-proportion model where two or more upstream firms produce a homogenous input good and may have different marginal costs, and two downstream firms produce differentiated final products and compete in prices. This author also identifies an efficiency effect and, depending on parameters, a foreclosure effect.

Summing up, previous literature suggests that vertical integration is likely to help gaining competitive advantage by cutting own costs or by raising rivals’ costs.

\(^{10}\) Analogous statements hold for the modified Cournot model analyzed by Gaudet and van Long (1996), where integrated firms purchase from the upstream market to increase rivals costs, and integration decisions are endogenous.

\(^{11}\) Because of Bertrand competition upstream, there is an upstream (monopoly) mark-up only when there is an integrated competitor.

\(^{12}\) These variants differ with respect to the bargaining power pertaining to upstream and downstream firms.

\(^{13}\) With non-linear upstream prices and homogeneous suppliers, integration may have no effect whatsoever on downstream marginal costs (Rey and Tirole, forthcoming). Yet, even in this case, there is the standard textbook argument (e.g. Besanko et al. 2000, 173) that post-integration costs might be lower because of economies of scope between upstream and downstream production.
3.3 Properties of Profit Functions

We now derive useful properties of the profit function \( \Pi_i (V, Y) \). More specifically, in Lemma 1 we show how investment incentives \( \partial \Pi_i / \partial Y_i \) depend on \( V \) and \( Y \).

**Lemma 1 (profit function properties)** Suppose Assumptions 1-5 are satisfied.

(i) Suppose that, in addition,
\[
\frac{\partial Q_1}{\partial Y_1}(1,0;Y) \geq \frac{\partial Q_1}{\partial Y_1}(0,1;Y); \quad \frac{\partial M_1}{\partial Y_1}(1,0;Y) \geq \frac{\partial M_1}{\partial Y_1}(0,1;Y). \quad (4)
\]

Then
\[
\frac{\partial \Pi_1}{\partial Y_1}(1,0;Y) > \frac{\partial \Pi_1}{\partial Y_1}(0,1;Y). \quad (5)
\]

(ii) Suppose that, in addition,
\[
\frac{\partial^2 Q_1}{\partial Y_1 \partial Y_2} \leq 0; \quad \frac{\partial^2 M_1}{\partial Y_1 \partial Y_2} \leq 0. \quad (6)
\]

Then
\[
\partial \Pi_i / \partial Y_i \text{ is non-increasing in } Y_j \text{ for } j \neq i, \ i,j = 1,2 \quad (7)
\]

(iii) Suppose that, in addition,
\[
\frac{\partial Q_i}{\partial Y_i}, \frac{\partial M_i}{\partial Y_i} \text{ are non-decreasing in } V_i \text{ and non-increasing in } V_j. \quad (8)
\]

Then
\[
\frac{\partial \Pi_i}{\partial Y_i} \text{ is non-decreasing in } V_i \text{ and non-increasing in } V_j. \quad (9)
\]

**Proof.** See Appendix. ■

Let us consider each part of Lemma 1, in turn.

Part (i) gives conditions for investment incentives to be higher for an integrated firm facing a separated competitor than for a separated firm facing
an integrated competitor. The intuition relies on Assumption 4 which says that both mark-up and demand are higher for an integrated firm facing a separated competitor than for a separated firm facing an integrated competitor. Since higher mark-up means that demand increases resulting from greater efficiency are more valuable, and higher demand means that mark-up increases are more valuable, the benefits from vertical integration and cost-reducing investment tend to reinforce each other. Put differently, there are demand/mark-up complementarities in product market competition. As a result, an integrated firm with high mark-up typically finds it more beneficial to invest into cost reduction than a separated competitor.

There is, however, a potential countervailing effect: The size of the demand and mark-up increases associated with higher efficiency could, in principle, decrease with vertical integration. The additional conditions on $\partial Q_i/\partial Y_i$ and $\partial M_i/\partial Y_i$ exclude this possibility. We think these conditions are fairly natural, as higher efficiency tends to increase the input price (due to higher demand), which, in turn, reduces the output and mark-up increases resulting from higher transformation efficiency for a separated firm. This effect is clearly absent for an integrated firm, at least if it does not buy from the input market. Further note that the conditions on $\partial Q_i/\partial Y_i$ and $\partial M_i/\partial Y_i$ are stronger than necessary. All we require is that the demand/mark-up complementarities in the product market dominate over any negative effect of vertical integration on $\partial Q_i/\partial Y_i$ and $\partial M_i/\partial Y_i$.

Part (ii) gives conditions for investment incentives to decrease when competitors become more efficient. Intuitively, the changes of variables under consideration (increases in $Y_i$ and decreases in $Y_j$, respectively) lead to increases in firm $i$’s demand and mark-up which are mutually reinforcing. Again, there might be countereffects of $Y_i$ and $Y_j$ on $\partial Q_i/\partial Y_i$ and $\partial M_i/\partial Y_j$ that could upset the results. The additional conditions on the second partial derivatives of $Q_i$ and $M_i$ exclude this possibility.14

Part (iii) is closely related to part (i). Note that (8) is more restrictive than (4). Using this more restrictive condition leads to a stronger implication: (9) implies (5), but not vice versa.

14In the linear Cournot example, these additional conditions are satisfied (see Table 1).
With these properties of the profit functions in place, we now proceed to study the firms’ investment and integration decisions in the second and the first stage of the game, respectively.

3.4 Analyzing Cost-Reducing Investments

Consider the second stage of the game in which firms take simultaneous decisions on cost-reducing investment. The first result generalizes Observation 1, showing how the investments of integrated firms differ from those of separated competitors.

**Proposition 1 (fixed structure)** Consider the subgame starting in stage 2. Suppose that, in addition to Assumptions 1-5, (4) and (6) are satisfied. Then, the integrated firm invests **more** into cost reduction than the separated firm, i.e. if $V_k = 1, V_\ell = 0$, then $Y_k > Y_\ell$.

**Proof.** See Appendix. ■

Theorem 1 states that, if the firms differ only with respect to their vertical integration status, the integrated firm will invest more into cost reduction than the separated firm. Intuitively, by Assumption 4, the integrated firm has higher equilibrium demand and mark-up than its competitor. The demand/mark-up complementarity therefore implies that the integrated firm has higher incentives to invest than the separated competitor, as reflected in (5). In addition, condition (7) implies that the higher investment of the integrated firm and the lower investment of the competitor are mutually reinforcing. Thus, integrated firms should invest more than separated firms.

To sum up, at least in a set-up where integrated downstream firms are not active as suppliers on the upstream market, the observation that integrated firms tend to invest more into cost reduction than separated firms should be expected to hold quite generally, as it only requires fairly natural assumptions on product market competition. Proposition 1 is consistent with the observation that integrated firms tend to have high market shares: Not only does integration have a direct efficiency effect which works towards higher market
shares, but integrated firms also tend to invest more into cost reduction.\textsuperscript{15}

We now generalize Observation 2 and study the effect of a firm’s vertical integration on both firms’ investments. This involves a comparison of the firms’ investments under different market structures, whereas Proposition 1 was based on a comparison of investments within a given asymmetric vertical market structure.

**Proposition 2 (changing structure)** Suppose that, in addition to Assumptions 1-5, conditions (6) and (8) hold. Then the equilibrium level of $Y_i$ is non-decreasing in $V_i$ and non-increasing in $V_j$.

**Proof.** Apply Lemma 1 and Milgrom and Roberts (1990, Theorem 5).

Intuitively, by (9), if one firm integrates, this increases its own incentive to invest, and it decreases the competitor’s incentive to invest (intimidation effect). By (7), these two effects are mutually reinforcing.

As argued in the justification of (6), there may be countereffects arising because the size of the output and mark-up increases from integration may depend on the efficiency levels as well. Also, as already noted, the required condition (8) is less general than the corresponding condition (4) for Proposition 1: Condition (8) implies condition (4), but the converse statement is not true.\textsuperscript{16} Nevertheless, Proposition 2 indicates that under fairly natural assumptions based on demand/mark-up complementarities in the product market, the finding that a firm’s vertical integration increases own investment and reduces competitor investment generalizes beyond the linear Cournot model.

### 3.5 Analyzing Integration Decisions

We now consider integration decisions in stage 1. To this end, we introduce the following notation:

\textsuperscript{15}Proposition 1 generalizes to more than two firms. The proof uses similar techniques as the special case of two firms.

\textsuperscript{16}Also, generalization of Proposition 2 to more than two firms requires additional conditions.
**Notation 2** For $f = \Pi, Q, M,$

(i) let

$$f_i^*(V) \equiv f_i(V, Y(V)), \quad i = 1, 2,$$

describe equilibrium profits, outputs and mark-ups, respectively, as functions of integration decisions (evaluated at the equilibrium choices of cost-reducing investment).

(ii) let

$$\Delta f_1^*(V_2) \equiv f_1^*(1, V_2) - f_1^*(0, V_2); \quad \Delta f_2^*(V_1) \equiv f_2^*(V_1, 1) - f_2^*(V_1, 0)$$

denote the effect of vertical integration on equilibrium profits, outputs and mark-ups, respectively, taking induced effects on efficiency levels into account.

We introduce the following additional assumption.

**Assumption 6** The firms’ equilibrium **mark-ups** and **outputs** satisfy the following conditions:

(i) $M_i^*(V)$ and $Q_i^*(V)$ are non-decreasing in $V_i$.

(ii) $M_i^*(V)$ and $Q_i^*(V)$ are non-increasing in $V_j$.

Assumption 6 holds in our linear Cournot example (see Table 2 for the special case $k = 1$). To understand why the assumption is plausible, recall that $V$ affects mark-ups and outputs both directly and indirectly, that is, via $Y(V)$. Roughly speaking, the direct effects already lend some plausibility to these assumptions, but the indirect effects reinforce them.

More specifically, consider condition (i): If $V_i$ decreases $c_i$, this should support a direct positive effect on $M_i^*(V)$ and $Q_i^*(V)$. Also, $V_i$ affects $c_j$ by the impact on the wholesale price. If the wholesale price increases, the marginal costs of a non-integrated firm $j$ go up, and the wholesale price effect
will thus reinforce the direct efficiency effect.\footnote{Whether such vertical foreclosure emerges in equilibrium depends on subtle details of the specific model under consideration: For instance, foreclosure will typically occur for low numbers of upstream suppliers (Salinger 1988) or high costs of switching suppliers (Chen 2001).} Importantly, there also is an indirect strategic intimidation effect of integration: As argued before, vertical integration tends to reduce a competitor’s incentive to invest into efficiency improvement, which yields an indirect effect strengthening the direct positive effect of own integration on output and mark-up.

Condition (ii) can be justified in a similar way: First, as firm j’s integration reduces its marginal costs, firm j becomes a stronger downstream competitor, which reduces $M^*_i(V)$ and $Q^*_i(V)$ by way of downstream interaction. Second, after integration, the integrated upstream firm may have an incentive to curtail supply to the input market, thereby raising the cost of a non-integrated rival by way of upstream interaction. This is the foreclosure effect of vertical integration. Again, crucially, these direct effects are reinforced by the indirect intimidation effect of vertical integration.

It is important to note that Assumption 6 holds in our linear Cournot model, even though the direct effect of $V_j$ on $M^*_i(V)$ and $Q^*_i(V)$ will typically have the wrong sign for given $Y$. This illustrates that the indirect effect of integration on mark-up and output over investments derived in Proposition 2 may well dominate the direct effect. Put differently, cost-reducing downstream investment makes the existence of demand/mark-up complementarities in the product market more likely than without investment.

Our next result gives conditions under which integration decisions are strategic substitutes. We shall argue that these conditions are more likely to come about than in a setting without downstream investment.

**Proposition 3 (strategic substitutes)** Suppose Assumptions 1-6 hold. Further assume that the following statement holds for $i, j = 1, 2, i \neq j$:

\[ \Delta Q^*_i(V_j) \text{ and } \Delta M^*_i(V_j) \text{ are non-increasing in } V_j. \]  

(10)
Then vertical integration decisions are strategic substitutes, i.e.

\[ \Delta \Pi^*_{i} (0) \geq \Delta \Pi^*_{i} (1) \quad \text{for} \quad i = 1, 2. \] 

(11)

**Proof.** By exchangeability, it suffices to consider firm 1’s incentive to integrate, \( \Delta \Pi^*_{1} (V_2) \). Rewriting this profit differential yields

\[ \Delta \Pi^*_{i} (V_2) = Q^*_i (1, V_2) \cdot \Delta M^*_i (V_2) + M^*_i (0, V_2) \cdot \Delta Q^*_i (V_2). \] 

(12)

By Assumption 6(i), both \( \Delta M^*_i (V_2) \) and \( \Delta Q^*_i (V_2) \) are non-negative. By Assumption 6(ii), \( Q^*_i (1, V_2) \) and \( M^*_i (0, V_2) \) are both non-increasing in \( V_2 \). By (10), \( \Delta M^*_i (V_2) \) and \( \Delta Q^*_i (V_2) \) are non-increasing in \( V_2 \). Thus (12) is non-increasing in \( V_2 \).

The intuition for Proposition 3 is as follows: By Assumption 6, vertical integration by firm \( i \) increases firm \( i \)'s demand and mark-up, whereas vertical integration by firm \( j \) decreases these quantities. Now, as firm \( j \)'s integration reduces the mark-up \( M^*_i \), the positive effect of a given output increase \( \Delta Q^*_i \) on firm \( i \)'s profits is smaller when firm \( j \) is integrated. Similarly, the positive effect of a given mark-up increase \( \Delta M^*_i \) on firm \( i \)'s profits is smaller when firm \( j \) is integrated, because firm \( j \)'s integration reduces firm \( i \)'s demand \( Q^*_i \). Thus, if the size of \( \Delta Q^*_i \) and \( \Delta M^*_i \) are independent of the competitor’s vertical structure, integration decisions are strategic substitutes. A fortiori, when the integration of firm \( j \) reduces the increases \( \Delta Q^*_i \) and \( \Delta M^*_i \) resulting from own integration, as required by (10), the strategic substitutes property holds.

That is, demand/mark-up complementarities in the product market, which are present in many oligopoly models, are likely to make vertical integration decisions strategic substitutes. However, condition (10) may be violated when firm \( j \)'s integration gives rise to a strong foreclosure effect, as firm \( i \) will then have relatively high (low) costs when it is separated (integrated, respectively). Thus, firm \( i \)'s cost reduction from own integration is likely to be higher when firm \( j \) is integrated. That is, other things being equal, output and mark-up increases \( \Delta Q^*_i \) and \( \Delta M^*_i \) will be higher when firm \( j \)
is integrated. As a result, in spite of demand/mark-up complementarities, the strategic substitutes property may be violated when the foreclosure effect is strong.\textsuperscript{18} In spite of the qualification that (10) may be violated in general, it does, for instance, hold in the Cournot example discussed in Section 3. Therefore, in this particular model vertical-integration decisions are strategic substitutes.

Proposition 3 immediately implies that there may be asymmetric equilibria where one firm integrates and the other remains separated, even if firms start out identically and face the same exogenous integration costs.\textsuperscript{19}

**Proposition 4 (asymmetric equilibria)** Suppose the conditions of Proposition 3 are satisfied, i.e. vertical integration decisions are strategic substitutes. Then, for suitable values of $F$, there exist equilibria where exactly one firm integrates ($\Delta\Pi^*_i (0) \geq F \geq \Delta\Pi^*_i (1)$).

**Proof.** If integration decisions are strategic substitutes, we must have $\Delta\Pi^*_i (0) \geq F \geq \Delta\Pi^*_i (1)$ for suitable values of $F$. \hfill \Box

Intuitively, Proposition 4 states that, if vertical integration decisions are strategic substitutes, then there must be levels of acquisition costs where integration is profitable for only one of the firms.

Importantly, while results that are analogous to Propositions 3 and 4 can also be formulated for the case without cost-reducing investments (Buehler and Schmutzler 2005), the strategic effects of integration on investment lend additional support to the underlying monotonicity Assumption 6. The intuition for the strategic substitutes property relies on the monotonicity of $M^*$ and $Q^*$ in the integration variables. As we argued, this is likely to be more pronounced than the corresponding property for $M_i$ and $Q_i$, which would justify strategic substitutes. Therefore, the observation from the Cournot example that asymmetric integration arises for a larger parameter interval when cost reductions are considered is not a coincidence.

\textsuperscript{18}In Ordover et al. (1990), a similar effect occurs: If the competitor’s integration raises the upstream price excessively, the separated firm has an incentive to integrate.

\textsuperscript{19}Proposition 4 can be extended to sequential integration decisions. For instance, if $\Delta\Pi^*_i (0) \geq F \geq \Delta\Pi^*_i (1)$ holds, any subgame perfect equilibrium of a sequential vertical integration game must be asymmetric.
While Propositions 3 and 4 together give a very intuitive rationale why asymmetric vertical integration equilibria might emerge, there are two caveats. First, we have already seen that the strategic substitutes condition may be violated if there is a strong foreclosure effect. If integration decisions were strategic complements rather than substitutes, vertical integration by firm \(j\) would render vertical integration more profitable for firm \(i\) (i.e. \(\Delta \Pi^*_i (1; Y) > \Delta \Pi^*_i (0; Y)\)). As a result, only symmetric equilibria could exist: Either both firms would integrate or none.\(^{20}\) Second, even if integration decisions are strategic substitutes, symmetric equilibria will still arise when \(F\) is so high that \(F > \Delta \Pi^*_i (0)\): Then, no firm will integrate. Similarly, when \(F\) is so low that \(F < \Delta \Pi^*_i (1)\), all firms integrate.

### 4 Conclusions

We have argued in this paper that once one accounts for endogenous cost-reducing investment, asymmetric integration equilibria are more likely to come about than previous literature suggests.

The underlying mechanism is straightforward. As a firm’s vertical integration increases own investment and decreases competitor investment, there is a strategic effect of vertical integration: Firms integrate partly to intimidate their competitor, i.e., reduce the competitor’s incentives to invest into cost-reduction. This strategic effect is clearly absent when firms cannot make cost-reducing investments. Since the strategic effect is typically stronger when facing a separated rather than an integrated competitor, asymmetric equilibria involve integrated firms that invest more than separated firms and thus have higher market shares, which appears to be in line with stylized facts.

\(^{20}\)There is an interesting relation between Proposition 4 and the familiar Chicago school argument that strategic vertical integration cannot generate competitive harm, because non-integrated firms can always counter integration by vertically integrating themselves so as to assure input supply at competitive prices (see e.g. Bork 1978). Proposition 4 indicates that the argument is somewhat misleading, as countering vertical integration by own integration (“bandwagoning”) may be unprofitable even when the conditions are favorable, i.e. when firms are symmetric initially and face the same costs of acquiring an upstream firm.
5 Appendix

5.1 Proof of Lemma 1

(i) Differentiating firm 1’s profit function yields
\[ \frac{\partial \Pi_1}{\partial Y_1} = \frac{\partial Q_1}{\partial Y_1} M_1 + \frac{\partial M_1}{\partial Y_1} Q_1. \]

Using Assumption 5, all terms on the r.h.s. of this equation are positive, and both \( M_1 (1, 0; Y) > M_1 (0, 1; Y) \) and \( Q_1 (1, 0; Y) > Q_1 (0, 1; Y) \). (5) thus follows immediately from
\[ \frac{\partial Q_1}{\partial Y_1} (1, 0; Y) \geq \frac{\partial Q_1}{\partial Y_1} (0, 1; Y) \text{ and } \frac{\partial M_1}{\partial Y_1} (1, 0; (Y)) \geq \frac{\partial M_1}{\partial Y_1} (0, 1; Y). \]

(ii) Using
\[ \frac{\partial^2 Q_1}{\partial Y_1 \partial Y_2} \leq 0, \quad \frac{\partial^2 M_1}{\partial Y_1 \partial Y_2} \leq 0, \]
arguments similar to those used in the proof of (i) show that \( \frac{\partial^2 \Pi_1}{\partial Y_1 \partial Y_2} \leq 0 \). (7) thus follows immediately.

(iii) is analogous.

5.2 Proof of Proposition 1

(i) By (5) and exchangeability (Assumption 3), we have
\[ \frac{\partial \Pi_2}{\partial Y_2} (0, 1; Y) = \frac{\partial \Pi_1}{\partial Y_1} (1, 0; Y) > \frac{\partial \Pi_1}{\partial Y_1} (0, 1; Y) = \frac{\partial \Pi_2}{\partial Y_2} (1, 0; Y). \] (13)

Now, define \( \theta = (V_1, V_2) \) and \( x_1 = Y_1, x_2 = -Y_2 \). Thus, consider the game with objective function
\[ f_i (x_1, x_2; \theta) = \Pi_i (\theta; x_1, -x_2) - K_i (|x_i|). \]
By (7), this game is supermodular.\(^{21}\) By (5), changing \(\theta\) from \((0,1)\) to \((1,0)\) increases \(\partial \Pi_1 / \partial Y_1\) and reduces \(\partial \Pi_2 / \partial Y_2\). In other words, \(\Pi_i\) has increasing differences in \((\theta,x)\). Thus, the result follows from Theorem 5 in Milgrom and Roberts (1990).

References


\(^{21}\)See Milgrom and Roberts (1990) for the concept of supermodular games.


Figure 2: Investments and market shares in the linear Cournot model ($\alpha = 1$).
Table 1: The linear Cournot example

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<th>( V = (1, 1) )</th>
<th>( V = (0, 0) )</th>
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Table 2: The linear Cournot example with $k = 1$

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