The Allocation of Authority under Limited Liability

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Abstract

If a project-oriented decision in a joint project is non-contractible, we consider authority as the right to undertake this decision. The impact of this decision influences the success probability of the project as well as the private costs of all parties. The decision-maker exerts an externality on the other parties. Overall surplus is shared according to generalized Nash bargaining. Under limited liability, some inefficiencies arise which might be enlarged if bargaining results in an inefficient allocation of authority. We analyze how this is related to the agents’ cost shares and bargaining power.

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Key words: Authority, Decision Rights, Generalized Nash Bargaining, Incomplete Contracts

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1 Introduction

When several persons or institutions undertake a joint project, a project-oriented decision might influence not only the decision-maker but all involved parties. Such a decision can change the expected output of the project as well as the private costs of all participants. Who should and who will make such a decision in a world of incomplete contracts?

This question arises in very different applications. For example, two scientists working on a joint research project have to decide what kind of experiment to carry out. If they conduct a complex experiment, their workload and therefore their costs increase while the expected outcome is influenced as well. They are able to find more relevant results than within a simpler framework. Similar ideas apply to two firms forming a research joint venture. As another example, consider two firms or two departments within a firm working on a new product. While one firm resp. department designs the product itself, the other one is working on a marketing strategy. Decisions about the quality of the product or the included features have an impact on both parties. A high-quality or complex product is more difficult to develop and more difficult to explain to the customers. On the other hand, such a quality-decision also influences the expected outcome of the sales. All these examples have in common that a certain project-oriented decision influences the costs of all parties as well as the expected outcome independent of who makes the decision.

If the project-oriented decision is contractible, the involved parties will specify it in a contract. But in a lot of situations, contracts a necessarily incomplete. Hart (1995) and Salanié (1997) give several reasons for the existence of incomplete contracts. A variable might be non-contractible because it is unobservable to a third party or at least unverifiable in court. Even if all contractual partners observe this variable, they cannot enforce the contract in a trial. In addition, it might be too costly to specify everything in detail in a contract. As a result, a contract remains incomplete. Hart and Moore (1999) give a more formal foundation of the incompleteness of contracts. Instead of specifying the complete project in a contract, one can determine who has the right to make the remaining decisions. This concept of the allocation of authority is used for example in Aghion and Bolton (1992), Bester (2002) or Grossman and Hart (1986). The authority - that is, the decision right - might be enforced via ownership as in Hart and Moore (1990). This property rights approach usually assumes a decision that is not describable at the contracting stage but verifiable at the bargaining stage. In our model, the project-oriented decision remains unverifiable ex post and only project
output can be contracted upon. The complexity of a scientific experiment or the detailed quality properties of a new product are too hard to identify for a third party, especially a court. Another approach for enforcing an allocation of authority is constituting informal authority via repeated interaction as described in Baker, Gibbons, and Murphy (1999).

All parties incur costs from undertaking the project. These costs might be disutility from work, caused from effort to spend. But the decision analyzed here is not an effort choice in the usual sense. An effort choice as modelled in Aghion and Tirole (1997) and other papers influences the decision-makers costs only. In the applications considered here, the decision determines the costs of all parties involved. It can be considered as a kind of externality the decision-maker exerts on the other parties. Bester (2002) analyzes a principal-agent-model of externalities and the allocation of authority in a firm. His model includes in addition effort incentives and deals with asymmetric information.

In our model, information is completely symmetric. Instead of a principal-agent model, we consider the broader approach of two agents who share the expected overall surplus according to generalized Nash bargaining. In most of the mentioned applications, it is reasonable to assume some differences in bargaining power while a principal-agent model seems to constitute too much asymmetry between the parties. Imposing limited liability constraints creates a new trade-off between rent extraction and surplus maximization. The resulting inefficiencies depend on the proportion of each agent’s cost share and bargaining power. Those inefficiencies are increased further, if bargaining allocates authority inefficiently. We examine the relationship between the efficiency of the bargaining result and the asymmetry between the agents. It turns out that asymmetries in the agents’ cost shares and in their bargaining power play a role for the efficiency of the allocation of authority.

The rest of the paper is structured as follows: Section 2 describes a formal model of the allocation of authority in a joint project. The benchmark case of contractible project choice is analyzed in section 3. Section 4 studies non-contractible project choice, while in section 5 the allocation of authority under limited liability is examined. A brief summary of the results and conclusions as well as some open research questions are given in section 6.
2 The Model

This section describes a formal model of the allocation of authority under limited liability. Some possible applications illustrate the assumptions.

Two agents $i = 1, 2$ jointly undertake a project. They negotiate a contract in order to specify their partnership. The agents could be, for example, two researchers carrying out an experiment or writing a paper together. They might as well be representatives of two institutions forming a research joint venture or two firms working together on a new product. The agents might be very different in skills and positions. For example, one could be a scientist developing a new product while the other one is creating the marketing strategy. Maybe one agent owns a machine and the other one knows how to use it. The examples are enhanced further below.

The project can either succeed or fail. The success is random and the project output is a random variable which can take two values: In case of success, the project generates a positive output $X > 0$. Instead, if the project fails, no output is generated and the random variable takes the value 0. Once the project has been undertaken, the realized output is observable and therefore contractible. The success probability is given by the project characteristic $d$ which could describe, for example, the size of the project or the level of complexity. The set of possible projects is $\mathcal{D} = (0, 1)$. The results of the paper generalize to the case of $\mathcal{D} \subseteq (0, 1)$, if some additional assumptions on the other parameters ensure to avoid corner solutions. Depending on the interpretation of $d$, the meaning of this structure is that a large project is more likely to succeed than a small one or a sophisticated project is more likely to succeed than a simple one. Within the research application, conducting a difficult experiment or writing a complex paper increases the probability of finding a publisher. A new product is more likely to succeed on the market if it has a high quality.

If project $d$ is undertaken, each agent incurs private costs which are independent of the project output and the allocation of authority. Agent $i$’s cost function is $c_i d^2$ with $c_i > 0$. Without loss of generality agent 1 is assumed to have a smaller cost parameter than agent 2, that is $c_1 \leq c_2$. To avoid corner solutions, $c_1$ and $c_2$ have to fulfill some further restrictions ensuring that the decision-maker’s favorite project is feasible. A sufficient condition

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1The projects $d = 0$ and $d = 1$ can be excluded because they turn out to create no surplus.
for all relevant projects to be in \( D = (0, 1) \) is \( 2c_1 + c_2 > X \). This condition can be relaxed in several situations.

The costs may be non-monetary costs like disutility from effort to spent on the project. A complex or large project might force both agents to work harder and increase their private costs, no matter who has chosen the project. Even though the costs might describe disutility from work, the action \( d \) should not be interpreted as an effort choice in the usual way. An effort choice typically influences only the decision-maker’s costs but not the other party’s costs (e.g. \cite{AghionTirole1997}). But here, the choice of \( d \) influences not only the costs of the decision-maker but of the other agent as well. Similar to \cite{Bester2002} the decision-maker exerts an externality on the other agent’s costs.

For example, when \( d \) represents the quality of a new product, a higher \( d \) implies more difficult and therefore more costly work for the product designer as well as for the marketing specialist - the latter has to explain a more complex product to the customers. A complex software is harder to develop and its features are harder to communicate to the customers. The chosen level of complexity - no matter who has made the choice - influences the private costs of both the software developer and the marketing specialist.

The contract between the agents contains a payment scheme \((w_h, w_l)\) which determines - conditioning on the verifiable project output- how project output is split between the agents, possibly combined with additional transfer payments. If the project succeeds, output \( X \) is realized and agent 1 receives \( X - w_h \) while agent 2 receives \( w_h \). If the project fails, no output is generated, agent 1 receives \(-w_l\) and agent 2 receives \( w_l \). To put it differently, one can consider \( w_l \) as a transfer independent of success while \( w_h - w_l \) and \( X - w_h + w_l \) are payments received in case of success only.

The agents are risk-neutral and their payoffs are composed of their expected benefits and their private costs. With probability \( d \), the project succeeds and agent 1 receives \( X - w_h \), while with probability \( 1 - d \), she gets \(-w_l\). Subtracting the private costs \( c_1d^2 \) results in the payoff function \( U_1 \).

Analogously, agent 2’s payoff \( U_2 \) is derived so that

\[
U_1(w_h, w_l, d) = d(X - w_h + w_l) - w_l - c_1d^2 \\
U_2(w_h, w_l, d) = d(w_h - w_l) + w_l - c_2d^2 .
\]  

\footnote{The restriction has to be generalized to ensure the relevant projects to lie in \( D \) in case of \( D \subset (0, 1) \).}
Each agent’s outside option gives a zero payoff. The overall expected surplus $U_1 + U_2$ is a function of the project $d$ only. A project $d$ is called (first-best) efficient if and only if it maximizes $U_1 + U_2$.

The agents share the available expected surplus $U_1 + U_2$ according to the generalized Nash bargaining solution. Let $\alpha \in [0,1]$ indicate agent 1’s exogenously given bargaining power and $(1-\alpha)$ the bargaining power of agent 2. The agents sign a contract which maximizes

$$B(w_h, w_l, d) = U_1^\alpha(w_h, w_l, d) U_2^{1-\alpha}(w_h, w_l, d),$$

subject to $d \in D$ and the individual rationality constraints (participation constraints) $U_1, U_2 \geq 0$. Such a contract is called optimal. In case of $\alpha = 0$ (resp. $\alpha = 1$), the generalized Nash bargaining results in a principal-agent-model with agent 2 (resp. agent 1) as a principal who makes a take-it-or-leave-it offer.

The following sections compare contractible versus non-contractible project choice as well as limited liability versus unlimited liability. The focus of the model is on non-contractible project choice under limited liability.

Project $d$ might be non-contractible because it is unobservable to a third party or it is observable, but not verifiable in court so that a contract conditioning upon $d$ is not enforceable in a trial. In other cases, it might simply be too costly to specify $d$ completely in a contract. Hart (1995) and Salanié (1997) give more detailed explanations for the existence of incomplete contracts. Note that non-contractible project choice implies non-contractible private costs since there is an one-to-one relationship between costs and project characteristic.

While the project $d$ itself is non-contractible, the allocation of authority is contractible. One possible way to enforce authority is via ownership of or access to certain assets. This approach to allocating authority is considered e.g. in Aghion and Bolton (1992), Bester (2002), or Grossman and Hart (1986). Instead of specifying a certain project, the contract specifies the allocation of authority denoted by $r$. If $r = 1$, agent 1 receives authority over the project choice, $r = 2$ gives the decision right to agent 2. After the contract has been signed, the decision-maker chooses the project to be undertaken, i.e. the project characteristic $d$. In the research application, it seems a reasonable

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\[\text{Nash (1950)}\] introduced this concept for the case of equal bargaining power. It can easily be generalized by dropping this symmetry assumption in order to allow for agents with different bargaining power.
assumption that a third-party is not able to identify the level of complexity of a research project. If a new product is introduced, it might be too costly to specify in a contract the quality if it is determined by too many details.

If \( d \) is contractible, a contract specifies a payment scheme and a project so that the maximization of \( B \) is over \((w_h, w_l, d)\). If \( d \) is non-contractible, a contract consists of a payment scheme and the allocation of authority \( r \) so that maximization is over \((w_h, w_l, r)\). The decision-maker will choose a project which maximizes her own payoff. Both agents anticipate this behavior during the negotiation. Mathematically, the optimization of \( B \) is restricted by the additional constraint \( d \in \arg\max_{d' \in D} U_r(w_h, w_l, d') \). If the project fails, no output is generated, agent 1 receives \(-w_l\) and agent 2 receives \(w_l\).

As long as the set of possible payment schemes is unrestricted, a commitment problem might arise so that the contract is not renegotiation-proof. If \( w_l \neq 0 \) and the project fails, one agent has to pay a transfer to the other one. If \( w_h \notin [0, X] \) and the project succeeds, one agent does not receive any share of output but has to pay a transfer to the other agent. Those transfers are not possible if the agent is wealth-constrained. Apart from that, in a lot of situations it seems unreasonable that the agents can commit to undertake these transfer payments. After the realization of output, private costs are sunk. The agent can credibly threat to break up the contract and walk away instead of paying. This forces the other agent to accept renegotiations. Since both agents anticipate this possibility, they restrict the possible payment schemes from the beginning by the limited liability constraints \( w_l = 0 \) and \( 0 \leq w_h \leq X \). If these constraints are fulfilled, there is no incentive to walk away after the project is done. Both agents stick to the contract in order to receive their predetermined share of project output.

The timing of the contract game is as follows: In the initial stage, the agents bargain and sign the contract. In the second stage, the decision-maker chooses a project. The project is undertaken, private costs occur and project output is realized. The payment scheme is executed.

3 Contractible Project Choice

Consider the benchmark case of contractible project choice. A contract consists of a payment scheme \((w_h, w_l)\) and a project \(d \in \mathcal{D}\). This section shows that the first-best efficient project which maximizes overall expected surplus
$U_1 + U_2$ is always implemented, even if limited liability constraints are required. Lemma 1 characterizes this first-best solution. Proposition 1 shows the implementation of the first-best efficient project under unlimited as well as limited liability.

**Lemma 1** When the project characteristic $d$ is contractible, the overall surplus $U_1 + U_2$ is maximized if and only if the project

$$d_e := \frac{X}{2(c_1 + c_2)} \in \mathcal{D}$$

is implemented. The resulting surplus is

$$U_1 + U_2 = \frac{X^2}{4(c_1 + c_2)} > 0.$$  

Proof: see Appendix A.

For any given project $d$, the overall surplus is independent of the payment scheme. Accordingly the surplus maximizing project $d_e$ is also independent of the payment scheme. The larger the possible project output $X$ and the smaller the cost parameters $c_1$ and $c_2$, the larger is the efficient project as well as the maximum reachable surplus. The trade-off between maximizing success probability and minimizing costs is solved in favor of high success probability if success pays a lot and costs do not increase too fast in project characteristic.

Now consider the optimal contracts. According to section 2, a contract is optimal if it solves $\max_{w_h, w_l, d} B(w_h, w_l, d)$, that is

$$\max_{w_h, w_l, d} U_1^\alpha(w_h, w_l, d) U_2^{1-\alpha}(w_h, w_l, d),$$

subject to $d \in \mathcal{D}$ and the participation constraints $U_1, U_2 \geq 0$. Under limited liability, the additional constraints $w_l = 0$ and $0 \leq w_h \leq X$ are imposed. Proposition 1 states the results of the maximization.

**Proposition 1 (Contractible Project Choice)** Suppose project choice is contractible. Each optimal contract implements the efficient project $d_e$. The agents share the resulting surplus according to their bargaining power, that is $U_1 = \alpha(U_1 + U_2)$ and $U_2 = (1-\alpha)(U_1 + U_2)$. These results still hold true if limited liability constraints are imposed.

Proof: see Appendix A.
An explicit construction of the optimal contracts can be found in the proof, see Appendix A. Under unlimited liability, there are infinitely many optimal contracts. Under limited liability, this multiplicity of contracts boils down to one unique optimal contract, efficiency is preserved. This unique optimal contract under limited liability implements a payment scheme with $w_l = 0$ and $w_h = X/2 \left( \frac{c_2}{c_1 + c_2} + 1 - \alpha \right)$. (6)

If the project succeeds, the agents share half of the project output $X$ according to bargaining power, the other half according to cost share. E.g. agent 2 who has bargaining power $1 - \alpha$ and cost share $c_2/(c_1 + c_2)$ receives $w_h$ given in (6). Overall expected surplus is shared according to bargaining power.

In case of contractible project choice, there are no inefficiencies even if limited liability is required.

4 Non-Contractible Project Choice

From now on, the project choice is non-contractible and only the allocation of authority can be contracted upon. A contract consists of a payment scheme $(w_h, w_l)$ and an allocation of authority $r \in \{1, 2\}$. Agent $r$ is the decision-maker who receives the right to choose the project $d \in D$ once the contract is signed. Throughout this section, assume unlimited liability so that $w_l, w_h \in \mathbb{R}$.

Each optimal contract solves

$$\max_{w_h, w_l, r} U_1^\alpha(w_h, w_l, d) U_2^{1-\alpha}(w_h, w_l, d)$$

subject to $U_1, U_2 \geq 0$ and

$$d \in \arg\max_{d' \in D} U_r(w_h, w_l, d')$$.

Once the contract is signed, the decision-maker will choose a project which maximizes her own payoff. Both agents anticipate this fact during the bargaining process, which is reflected by (8).

In this setting, the efficient project $d_e$ is always implemented, even though the project choice is non-contractible. The agents share the expected surplus according to their bargaining power. These results are stated in Proposition 2.
Proposition 2 (Non-Contractible Project Choice) Suppose project choice is non-contractible. Each optimal contract implements the efficient project \( d_e \). There are exactly two optimal contracts, one gives the authority to agent 1 while the other one gives it to agent 2. The agents share the resulting surplus according to their bargaining power, that is \( U_1 = \alpha(U_1 + U_2) \) and \( U_2 = (1 - \alpha)(U_1 + U_2) \).

Proof: see Appendix A.

Proposition 2 shows that there is no inefficiency even though the project choice is non-contractible. An explicit construction of the two optimal contracts can be found in the proof, see Appendix A. Both optimal contracts implement the efficient project and use the same payment scheme. Even if the decision right is exogenously fixed, the efficient project is implemented and payoffs are independent of the allocation of authority. Since efficiency is reached, the optimal payment scheme in case of non-contractible project choice is necessarily one of the optimal payment schemes in case of contractible project choice.

If the project is successful, each agent receives a share of the project output according to cost share: agent 1 earns \( X - w_h + w_l = c_1/(c_1 + c_2) \) \( X \) and agent 2 gets \( w_h - w_l = c_2/(c_1 + c_2) \) \( X \) in addition to the payoffs in case of failure. Different from the case of contractible project choice, this extra benefit is independent of bargaining power.

Apart from the extra benefit and independent of success, there is a transfer

\[
\begin{align*}
    w_l &= \frac{X^2}{4(c_1 + c_2)} \left( \frac{c_1}{c_1 + c_2} - \alpha \right) \\
    &= \frac{X^2}{4(c_1 + c_2)} \left( 1 - \alpha - \frac{c_2}{c_1 + c_2} \right). \tag{9}
\end{align*}
\]

The agent who has a bargaining power larger than the cost share receives a payment. If \( \alpha < c_1/(c_1 + c_2) \), it is \( w_l > 0 \) and agent 1 pays a transfer to agent 2, while in case of \( \alpha > c_1/(c_1 + c_2) \) it is the other way around. The optimal payment scheme fulfills the limited liability constraints if and only if \( \alpha = c_1/(c_1 + c_2) \), that is bargaining power equals cost share.

In case of non-contractible project choice, there are no inefficiencies. But for any bargaining power \( \alpha \neq c_1/(c_1 + c_2) \), limited liability is not fulfilled and imposing limited liability is expected to create some inefficiencies.
5 Non-Contractible Project Choice under Limited Liability

Consider the case of non-contractible project choice under limited liability constraints \( w_l = 0 \) and \( 0 \leq w_h \leq X \). The agents simply share the project output and do not perform any additional transfer payments. Imposing limited liability constraints usually creates a new trade-off between efficiency and rent extraction. Under limited liability, it is no longer possible to maximize overall surplus and extract a rent share according to bargaining power at the same time. As soon as bargaining power does not equal cost share, the first-best efficient project is no longer implemented. The inefficiency is smaller if the decision right is given to the agent with the smaller cost share, but the unique bargaining solution does not necessarily implement this allocation of authority. Subsection 5.1 analyzes the case of an exogenously given allocation of authority with a focus on the efficiency of the two possible allocations. Subsection 5.2 describes the allocation of authority in case of bargaining and studies the impact of the agents’ bargaining power and cost shares.

5.1 Exogenous Allocation of Authority

Use \( w_l = 0 \) to simplify the payoff functions to

\[
U_1 = d(X - w_h) - c_1d^2, \\
U_2 = dw_h - c_2d^2.
\]

(10)

Each optimal contract solves

\[
\max_{w_h,d} U_1^\alpha(w_h,d) U_2^{1-\alpha}(w_h,d)
\]

subject to \( U_1, U_2 \geq 0, 0 \leq w_h \leq X \) and

\[
d \in \arg\max_{d \in D} U_r(w_h,d')
\]

(12)

**Proposition 3 (Agent 1 decides)** Consider the case of non-contractible project choice under limited liability. Let agent 1 be the decision-maker so that \( r = 1 \). The optimal contract implements project

\[
d_1 := \frac{(1 + \alpha)X}{2(2c_1 + c_2)}
\]

which is the efficient project \( d_e \) if and only if \( \alpha = c_1/(c_1 + c_2) \).

Proof: see Appendix A.
In case of \( r = 1 \), the efficient project \( d_e \) is implemented if and only if each agent’s bargaining power equals her cost share. If the decision-maker is too powerful in terms of bargaining power, that is \( \alpha > c_1/(c_1 + c_2) \), she implements an inefficiently large project \( d_1 > d_e \). The trade-off between minimizing private costs and maximizing success probability is solved in favor of a large success probability since the decision-maker can extract a large share of the output in case of success. Instead, if \( \alpha < c_1/(c_1 + c_2) \), it does not pay out to choose a large and therefore costly project since the extra rent from success is small.

**Proposition 4 (Agent 2 decides)** Consider the case of non-contractible project choice and limited liability. Let agent 2 be the decision-maker, that is \( r = 2 \). The optimal contract implements the project

\[
d_2 := (2 - \alpha)X \frac{1}{2(c_1 + 2c_2)}
\]

which is the efficient project \( d_e \) if and only if \( \alpha = c_1/(c_1 + c_2) \). Proof: see Appendix A.

Exactly as if \( r = 1 \), the first-best efficient project \( d_e \) is implemented if and only if \( \alpha = c_1/(c_1 + c_2) \). Again, a decision-maker who is too powerful, that is \( 1 - \alpha > c_2/(c_1 + c_2) \) resp. \( \alpha < c_1/(c_1 + c_2) \), implements an inefficiently large project since she can extract a large share of output in case of success. If instead \( \alpha > c_1/(c_1 + c_2) \), the decision-maker implements an inefficiently small project.

As soon as \( \alpha \neq c_1/(c_1 + c_2) \), the first-best efficient project is no longer implemented. Giving the decision right to the agent with bargaining power larger than cost share will lead to a project “too large” while giving it to the other agent will result in a project “too small” compared to the efficient one.

The following proposition describes the allocation of authority which maximizes overall surplus in case of non-contractible project choice. Even though usually not implementing the first-best efficient project, such an allocation of authority is called efficient.

**Proposition 5 (Efficient Allocation of Authority)** In case of non-contractible project choice under limited liability, overall surplus is maximized if and only if either \( \alpha = c_1/(c_1 + c_2) \) or the agent with the larger cost share receives the decision right. Since \( c_1 \leq c_2 \) by assumption, agent 2 should be the decision-maker.

Proof: see Appendix A.
The proof of Proposition 5 shows

\[ |d_1 - d_e| = d_e \frac{\alpha(c_1 + c_2) - c_1}{2c_1 + c_2} \]
\[ |d_2 - d_e| = d_e \frac{\alpha(c_1 + c_2) - c_1}{c_1 + 2c_2} \]  \hspace{1cm} (15)

As already mentioned, the decision-maker’s project choice is distorted away from the first-best efficient project because of a disproportion between bargaining power and cost share. This is reflected by \( |\alpha(c_1 + c_2) - c_1| \) which is independent of the allocation of authority. If any other source of distortion is eliminated by assuming agents who differ only in bargaining power, that is assuming \( c_1 = c_2 \), both possible allocations of authority result in the same amount of distortion \( |d_1 - d_e| = |d_2 - d_e| \). What one agent chooses “too much” is exactly what the other one chooses “too little”. If \( c_1 \neq c_2 \), there is an additional source of distortion measured by the terms \( 2c_1 + c_2 \) resp. \( c_1 + 2c_2 \). When choosing the project, the decision-maker cares about her own costs, but puts inefficiently small weight to the other agents costs. This effect is less drastic, if authority is given to the agent with the larger cost share.

5.2 Allocation of Authority via Bargaining

As a result of the bargaining process, the allocation of authority under limited liability might be inefficient. This is analyzed in the following Proposition.

**Proposition 6 (Allocation of Authority)** Consider the case of non-contractible project choice under limited liability. Generalized Nash Bargaining allocates authority efficiently if and only if \( \alpha = c_1/(c_1+c_2) \) or \( c_1 = c_2 \) or

\[ \phi(k, \alpha) \geq 0 \] \hspace{1cm} (16)

with

\[ \phi(k, \alpha) := (2k + 1)^{1+\alpha} (2 - \alpha)^{2-\alpha} \alpha^\alpha \]
\[ - (k + 2)^{2-\alpha} k^\alpha (1 + \alpha)^{1+\alpha} (1 - \alpha)^{1-\alpha} \] \hspace{1cm} (17)

and \( k := c_1/c_2 \).

Proof: see Appendix A.

In case of \( \alpha = c_1/(c_1 + c_2) \) or \( c_1 = c_2 \), the allocation of authority does not influence efficiency. In any other case, it is efficient to allocate authority to agent 2 which is the result of the bargaining process if (16) holds.
Figure 1 shows a contour plot of $\phi(k, \alpha) \geq 0$.

Under unlimited liability, each agent receives a fixed share of the surplus determined by her bargaining power. The agents’ common goal is to maximize the surplus.

Under limited liability, there is a trade-off between surplus maximization and rent extraction. If the agents have similar cost shares, the allocation of authority does not have much influence on the surplus. In case of $k = 1$, that is $c_1 = c_2$, the overall expected surplus is even independent of the allocation of authority. Rent extraction becomes important. For each agent, getting the decision right increases the payoff. If agent 1’s bargaining power is large enough, she manages to receive authority. The closer the cost shares the smaller is the necessary bargaining power. If $c_1 = c_2$, agent 1 gets authority if $\alpha \geq 1/2$. On the other hand, if the cost shares are very different so that $k$ is small, the allocation of authority has an important influence on the surplus and rent extraction becomes less important. Both agents benefit from the increased efficiency reached by giving authority to agent 2. Even if agent 1 has a large bargaining power, she does not use it for getting authority.
Only if the agents are very similar concerning their cost parameters but differ much in bargaining power, authority is allocated inefficiently.

6 Conclusion

In this paper, we have developed a simple model of the allocation of authority in a joint project. A decision undertaken by one of the agents has an impact on all agents involved so that some kind of externality is exerted. As long as the decision is contractible, efficiency is reached even if limited liability constraints are imposed. If the decision is non-contractible, the decision right is contractually assigned to one of the agents. The first-best efficient project is still implemented. Imposing limited liability constraints now creates a trade off between rent extraction and surplus maximization which induces some inefficiency. This is independent of the allocation of authority, but an inefficient project is implemented. Generalized Nash bargaining might - in addition - allocate authority in a way that increases the inefficiency further. A very powerful agent can manage to receive the decision right even though this allocation of authority decreases the overall expected surplus compared to the alternative allocation of authority. The less the agents differ in cost share, the smaller is the necessary bargaining power. If on the other hand the agents are very different in cost shares, even a very powerful agent might not use her bargaining power for getting authority. Both agents benefit from the increased surplus reached by an efficient allocation of authority.

The approach has a wide variety of applications, ranging from two-person projects to institutional relationships. Co-authorship might be modelled as well as research joint ventures or other cooperations inside or between firms. Possible extensions of the model include the division of tasks in multi-task projects, the introduction of effort incentives for the involved agents and third-party involvement with collusion.
References


Appendix

A Proofs

Proof of Lemma 1:
The surplus $U_1 + U_2$ is a function of $d$ independent of the payment scheme. Maximizing $U_1 + U_2$ by solving the first order condition yields the project $d_e$ and the surplus $U_1 + U_2 = (U_1 + U_2)(d_e) = X^2/[4(c_1 + c_2)]$. Since $0 < X < c_1 + 2c_2 < 2(c_1 + c_2)$, we have $d_e \in (0,1) = D$ and $U_1 + U_2 > 0$. ■

Proof of Proposition 1:
Each optimal contract solves $\max_{w_h, w_l, d} B(w_h, w_l, d)$, that is

$$\max_{w_h, w_l, d} U_1^\alpha(w_h, w_l, d) U_2^{1-\alpha}(w_h, w_l, d)$$

subject to $d \in D$ and the participation constraints $U_1, U_2 \geq 0$. The first order conditions (suppressing the arguments for notational purposes) are given by

$$\alpha U_1^{\alpha-1} U_2^{1-\alpha} \frac{\partial U_1}{\partial d} + (1 - \alpha) U_1^\alpha U_2^{-\alpha} \frac{\partial U_2}{\partial d} = 0$$
$$\alpha U_1^{\alpha-1} U_2^{1-\alpha} \frac{\partial U_1}{\partial w_l} + (1 - \alpha) U_1^\alpha U_2^{-\alpha} \frac{\partial U_2}{\partial w_l} = 0$$
$$\alpha U_1^{\alpha-1} U_2^{1-\alpha} \frac{\partial U_1}{\partial w_h} + (1 - \alpha) U_1^\alpha U_2^{-\alpha} \frac{\partial U_2}{\partial w_h} = 0$$

(19)

For the moment, assume there is at least one contract with $B > 0$ fulfilling the constraints so that each optimal contract satisfies $B > 0$ as well. One of the following three cases must apply: It is either $U_1, U_2 > 0$ or $U_1 > 0, U_2 = 0, \alpha = 1$ or $U_1 = 0, U_2 > 0, \alpha = 0$. The last two cases imply $B = U_1 + U_2$ and Lemma 1 gives $d = d_e \in D$. Furthermore, it follows that $\alpha U_2 = (1 - \alpha) U_1$ and therefore $U_1 = \alpha (U_1 + U_2)$ and $U_2 = (1 - \alpha) (U_1 + U_2)$.

If $U_1, U_2 > 0$ holds, then $U_1^{\alpha-1} U_2^{-\alpha} > 0$ holds as well. The system (19) can therefore be rewritten as

$$\alpha U_2 \frac{\partial U_1}{\partial d} + (1 - \alpha) U_1 \frac{\partial U_2}{\partial d} = 0$$
$$\alpha U_2 \frac{\partial U_1}{\partial w_l} + (1 - \alpha) U_1 \frac{\partial U_2}{\partial w_l} = 0$$
$$\alpha U_2 \frac{\partial U_1}{\partial w_h} + (1 - \alpha) U_1 \frac{\partial U_2}{\partial w_h} = 0$$

(20)
Since $\partial U_1/\partial w_l = -\partial U_2/\partial w_1 \neq 0$ and $\partial U_1/\partial w_h = -\partial U_2/\partial w_h \neq 0$, the second and third equation are fulfilled if and only if $\alpha U_2 = (1 - \alpha)U_1$. This yields the sharing rule $U_1 = \alpha(U_1 + U_2)$ resp. $U_2 = (1 - \alpha)(U_1 + U_2)$ as stated in the Proposition.

From $U_1, U_2 > 0$ it follows that $\alpha U_2 = (1 - \alpha)U_1 > 0$ and $\alpha \in (0, 1)$. Plugging $\alpha U_2 = (1 - \alpha)U_1$ into the first equation gives $\partial U_1/\partial d + \partial U_2/\partial d = 0$ which is the first order condition for maximizing $U_1 + U_2$ in $d$. Lemma 1 again implies $d = d_e \in D$.

Given $d = d_e$, $\alpha U_2 = (1 - \alpha)U_1$ results in

$$w_h = X/2 \left( \frac{c_2}{c_1 + c_2} + 1 - \alpha \right) + w_l \left( 1 - \frac{2(c_1 + c_2)}{X} \right). \quad (21)$$

Since $\alpha U_2 = (1 - \alpha)U_1$ leads to $U_1 = \alpha(U_1 + U_2) \geq 0$ with equality if and only if $\alpha = 0$ and $U_2 = (1 - \alpha)(U_1 + U_2) \geq 0$ with equality if and only if $\alpha = 1$, the ad-hoc-assumption of $B > 0$ is justified. A contract is optimal if and only if $d = d_e$ and (21) is satisfied.

Now impose the limited liability constraints $w_l = 0$ and $0 \leq w_h \leq X$. The contract with $d = d_e$, $w_l = 0$ and

$$w_h = X/2 \left( \frac{c_2}{c_1 + c_2} + 1 - \alpha \right) \quad (22)$$
meets the limited liability constraints and is optimal (even without requiring limited liability) since the payment scheme fulfills (21). To see the uniqueness of the optimal contract under limited liability, note that any other contract either hurts the limited liability constraints or equation (21). In the latter case, the contract results in a smaller $B$ and is not optimal.

Proof of Proposition 2:
A contract is optimal if and only if it solves

$$\max_{w_h, w_l, r} U_1^\alpha(w_h, w_l, d) U_2^{1-\alpha}(w_h, w_l, d) \quad (23)$$
subject to $U_1, U_2 \geq 0$ and

$$d \in \argmax_{d \in D} U_r(w_h, w_l, d') \quad . \quad (24)$$

To find the optimal contract(s), calculate the optimal contract(s) given $r = 1$ as well as the optimal contract(s) given $r = 2$. Among these contracts, find the optimal one(s) by evaluating the bargaining function.
Let \( r = 1 \) so that agent 1 is the decision-maker. Once the contract is signed, she will choose project \( d^* \) maximizing \( U_1 \) over \( D \) given \( w_h, w_l \), that is
\[
d^* = \frac{X - w_h + w_l}{2c_1}
\] if this expression is an element of \( D \). Assume for the moment that \( d^* \in D \) and note that \( d^* \) is the unique maximum of \( U_1(w_h, w_l, d) \). Both agents anticipate the later implementation of \( d^* \) during the bargaining process. Plugging in \( d^* \) gives
\[
U_1 = \left( \frac{X - w_h + w_l}{4c_1} \right)^2 - w_l \]
\[
U_2 = \frac{X - w_h + w_l}{2c_1}(w_h - w_l) + w_l - \frac{c_2(X - w_h + w_l)^2}{4c_1^2}.
\] (26)
The first order conditions for maximizing \( B \) are
\[
\alpha U_1^{1-\alpha} U_2^{1-\alpha} \frac{\partial U_1}{\partial w_l} + (1 - \alpha) U_1^{\alpha} U_2^{-\alpha} \frac{\partial U_2}{\partial w_l} = 0
\]
\[
\alpha U_1^{1-\alpha} U_2^{1-\alpha} \frac{\partial U_1}{\partial w_h} + (1 - \alpha) U_1^{\alpha} U_2^{-\alpha} \frac{\partial U_2}{\partial w_h} = 0.
\] (27)
From now on, the procedure is similar to the proof of Proposition 1. Making the ad-hoc assumption of \( B > 0 \) for the optimal contract(s), there are again three possible cases: \( U_1, U_2 > 0 \) or \( U_1 > 0, U_2 = 0, \alpha = 1 \) or \( U_1 = 0, U_2 > 0, \alpha = 0 \). The latter two cases imply \( B = U_1 + U_2 \). Some straightforward calculations show that the maximum of \( B = U_1 + U_2 \) is reached in \( w_h - w_l = c_2/(c_1 + c_2) X \). Using (26), it follows \( \alpha U_2 = (1 - \alpha) U_1 \) so that \( U_1 = \alpha(U_1 + U_2) \) and \( U_2 = (1 - \alpha)(U_1 + U_2) \).
If \( U_1, U_2 > 0 \), then \( U_1^{\alpha-1} U_2^{-\alpha} > 0 \). Dividing the first order conditions through \( U_1^{\alpha-1} U_2^{-\alpha} \) results in
\[
\alpha U_2 \frac{\partial U_1}{\partial w_l} + (1 - \alpha) U_1 \frac{\partial U_2}{\partial w_l} = 0, \]
\[
\alpha U_2 \frac{\partial U_1}{\partial w_h} + (1 - \alpha) U_1 \frac{\partial U_2}{\partial w_h} = 0.
\] (28)
Since \( \partial U_1/\partial w_l = -1 - \partial U_1/\partial w_h \) and \( \partial U_2/\partial w_l = 1 - \partial U_2/\partial w_h \), the conditions in case of \( U_1, U_2 > 0 \) are equivalent to
\[
\alpha U_2 = (1 - \alpha) U_1, \]
\[
0 = \frac{\partial U_1}{\partial w_l} + \frac{\partial U_2}{\partial w_1}.
\] (29)
The first equation implies \( \alpha \in (0, 1) \). Rearranging the second equation again leads to

\[
w_h - w_l = \frac{c_2}{c_1 + c_2} X
\]  

(30)

which now holds for all three cases. Using (25) gives

\[
d^* = d_e.
\]  

(31)

The assumption \( d^* \in D \) is justified. According to Lemma 1, the implementation of the project results in a surplus \( U_1 + U_2 = X^2/[4(c_1 + c_2)] \). Combined with \( \alpha U_2 = (1 - \alpha)U_1 \), it follows that \( U_1 \geq 0 \) with equality if and only if \( \alpha = 0 \) and \( U_2 \geq 0 \) with equality if and only if \( \alpha = 1 \). The ad-hoc assumption \( B > 0 \) is justified. Some straightforward calculations using these results together with (26) and (30) give the unique payment scheme

\[
w_l = \frac{X^2}{4(c_1 + c_2)^2} \left[-\alpha c_2 + (1 - \alpha)c_1\right]
\]  

\[
w_h = \frac{X^2}{4(c_1 + c_2)^2} \left[-\alpha c_2 + (1 - \alpha)c_1\right] + \frac{c_2}{c_1 + c_2} X
\]  

(32)

To summarize, the unique optimal contract given \( r = 1 \) is described by (32).

Now assume \( r = 2 \) and proceed analogue to the case \( r = 1 \). Note that the two problems are symmetric by \( \alpha \leftrightarrow (1 - \alpha) \), \( c_1 \leftrightarrow c_2 \), \( w_l \leftrightarrow -w_l \) and \( w_h \leftrightarrow X - w_h \). The decision-maker agent 2 implements

\[
d^{**} = \frac{w_h - w_l}{2c_2}
\]  

(33)

once the contract is signed. Maximizing the bargaining function \( B \) results in exactly the same payment scheme and project as if \( r = 1 \). The payment scheme is determined by (32) and the project \( d_e \) is implemented.

To summarize, there are two contracts which are candidates for an optimal contract, one with \( r = 1 \) and one with \( r = 2 \). Both contracts implement project \( d_e \) and payment scheme (32). Hence the payoffs \( U_1 \) and \( U_2 \) as well as the bargaining function \( B \) take the same values in both cases. The two candidates turn out to be the optimal contracts.

\( \blacksquare \)
Proof of Proposition 3:

The optimal contract solves

$$\max_{w_h} U_1^\alpha(w_h, d)U_2^{1-\alpha}(w_h, d)$$  \hspace{1cm} (34)

subject to $0 \leq w_h \leq X$, $U_1, U_2 \geq 0$, and $d \in \arg\max_{d' \in D} U_1(w_h, d')$. Once the contract is signed, the decision-maker agent 1 chooses a project in order to maximize $U_1$. This is the project

$$d_1 := \frac{X - w_h}{2c_1}$$  \hspace{1cm} (35)

as long as it is an element of $D$. For the moment, assume $d_1 \in D$. Plugging $d_1$ in yields to

$$U_1 = \frac{(X - w_h)^2}{4c_1}$$
$$U_2 = \frac{(X - w_h)w_h}{2c_1} - \frac{c_2(X - w_h)^2}{4c_1^2}$$  \hspace{1cm} (36)

and

$$\frac{\partial U_1}{\partial w_h} = -\frac{X - w_h}{2c_1}$$
$$\frac{\partial U_2}{\partial w_h} = \frac{c_1 + c_2}{2c_1^2}(X - w_h) - \frac{w_h}{2c_1} \hspace{1cm} (37)$$

The remaining proof is similar to the proof of Proposition 2. Assume that there is at least one contract fulfilling $B > 0$ and the required constraints so that each optimal contract satisfies $B > 0$ as well. Since $U_1 = 0$ implies $w_h = X$ which in turn implies $U_2 = 0$ and finally $B = 0$, it is necessarily $U_1 > 0$ and $w_h < X$. There are only two possible cases, namely $U_1, U_2 > 0$ or $U_1 > 0, U_2 = 0, \alpha = 1$.

Consider $U_1, U_2 > 0$. Dividing $\partial B/\partial w_h = 0$ through $U_1^{\alpha-1}U_2^{-\alpha} > 0$ leads to the first order condition

$$\alpha U_2 \frac{\partial U_1}{\partial w_h} + (1 - \alpha)U_1 \frac{\partial U_2}{\partial w_h} = 0$$  \hspace{1cm} (38)

which by (36) and (37) is

$$\alpha \left[ \frac{(X - w_h)^2 w_h}{4c_1^2} + \frac{c_2(X - w_h)^3}{8c_1^2} \right] +$$
$$\left(1 - \alpha \right) \left[ \frac{(c_1 + c_2)(X - w_h)^3}{8c_1^3} - \frac{(X - w_h)^2 w_h}{8c_1^2} \right] = 0 \hspace{1cm} (39)$$
Since \( w_h = X \) is already ruled out, the unique solution of (39) is
\[
  w_h = \frac{c_2 + (1 - \alpha)c_1}{2c_1 + c_2} \cdot X .
\] (40)

Solving \( U_2 = 0 \) for \( w_h \) gives exactly the same payment for the case \( U_1 > 0, U_2 = 0, \alpha = 1 \).

This payment fulfills the limited liability constraints and leads to
\[
d_1 = \frac{(1 + \alpha)X}{2(2c_1 + c_2)}
\] (41)

which is in \( D \) since \( 1 + \alpha \leq 2 \) and \( 2c_1 + c_2 > X \) by assumption. Using (36), (40) and (41) gives
\[
  U_1 = \frac{(1 + \alpha)^2 X^2 c_1}{4(2c_1 + c_2)^2} \geq 0
\] (42)

and
\[
  U_2 = \frac{(1 - \alpha)(1 + \alpha)X^2}{4(2c_1 + c_2)} \geq 0
\] (43)

with equality if and only if \( \alpha = 1 \). The ad-hoc assumption \( B > 0 \) is justified.

\[\blacksquare\]

**Proof of Proposition 4:**
The proof is completely analogue to Proposition 3 simply replace \( w_h \mapsto X - w_h, \alpha \mapsto 1 - \alpha, c_1 \mapsto c_2 \) and note that the agents changed their roles.

The decision-maker agent 2 chooses the project \( d_2 = w_h/(2c_2) \) which turns out to be
\[
d_2 = \frac{(2 - \alpha)X}{2(c_1 + 2c_2)}
\] (44)

in the end. It is \( d_2 \in D \) since \( 2 - \alpha \leq 2 \) and \( c_1 + 2c_2 \geq 2c_1 + c_2 > X \) by assumption. The payoffs are
\[
  U_1 = \frac{\alpha(2 - \alpha)X^2}{4(c_1 + 2c_2)} \geq 0
\] (45)
\[
  U_2 = \frac{(2 - \alpha)^2 X^2 c_2}{4(c_1 + 2c_2)^2} \geq 0
\] (46)

with equality if and only if \( \alpha = 0 \) so that \( B > 0 \). \[\blacksquare\]
Proof of Proposition 5:
Note that $U_1 + U_2 = dX - (c_1 + c_2)d^2$ is a parabola open below with its maximum in $d_e$. To put it differently, $U_1 + U_2$ is strictly decreasing in $|d - d_e|$. Agent 2 should receive authority if and only if $|d_1 - d_e| \geq |d_2 - d_e|$. It is

$$|d_1 - d_e| = \frac{(1 + \bar{\alpha})X}{2(2c_1 + c_2)} - \frac{X}{2(c_1 + c_2)}$$

$$= \frac{X[\alpha c_2 - (1 - \bar{\alpha})c_1]}{2(c_1 + c_2)(c_1 + c_2)}$$

$$= d_e \frac{\alpha(c_1 + c_2) - c_1}{2c_1 + c_2}$$  \hspace{1cm} (47)$$
and

$$|d_2 - d_e| = \frac{(2 - \bar{\alpha})X}{2(c_1 + 2c_2)} - \frac{X}{2(c_1 + c_2)}$$

$$= \frac{X[\alpha c_2 - (1 - \bar{\alpha})c_1]}{2(c_1 + 2c_2)(c_1 + c_2)}$$

$$= d_e \frac{\alpha(c_1 + c_2) - c_1}{c_1 + 2c_2}$$ \hspace{1cm} (48)$$

If $\alpha = c_1/(c_1 + c_2)$, it is $|d_1 - d_e| = |d_2 - d_e| = 0$ and the allocation of authority does not influence efficiency. If $\alpha \neq c_1/(c_1 + c_2)$, it is

$$|d_1 - d_e| \geq |d_2 - d_e| \iff \frac{1}{2c_1 + c_2} \geq \frac{1}{c_1 + 2c_2} \iff c_1 \leq c_2.$$  \hspace{1cm} (49)$$

Proof of Proposition 6:
Consider $r = 1$. Using $U_1$ and $U_2$ from the proof of Proposition 3, the value of the bargaining function is calculated to be

$$B = \frac{X^2}{4} \left( \frac{1}{2c_1 + c_2} \right)^{1+\alpha} c_1^{\alpha} (1 + \alpha)^{1+\alpha} (1 - \alpha)^{1-\alpha} =: B_1.$$  \hspace{1cm} (50)$$

If $r = 2$, the proof of Proposition 4 shows

$$B = \frac{X^2}{4} \left( \frac{1}{c_1 + 2c_2} \right)^{2-\alpha} c_2^{-\alpha} (2 - \alpha)^{2-\alpha} \alpha =: B_2.$$  \hspace{1cm} (51)$$

If $\alpha = c_1/(c_1 + c_2)$, allocation of authority does not influence payment scheme, implemented project, payoffs or efficiency. In case of $c_1 = c_2$, allocation of authority depends on bargaining power but does not influence efficiency since
both allocations result in the same amount of distortion. If \( \alpha \neq c_1/(c_1 + c_2) \) and \( c_1 < c_2 \), efficiency is increased if authority is given to agent 2. This is the outcome of the bargaining if and only if \( B_2 \geq B_1 \). Define \( k := c_1/c_2 \) so that

\[
B_1 = \frac{X^2}{4} (2k + 1)^{-1-\alpha} k^\alpha c_2^{-1} (1 + \alpha)^{1+\alpha} (1 - \alpha)^{1-\alpha} 
\]

(52)

and

\[
B_2 = \frac{X^2}{4} (k + 2)^{-2+\alpha} c_2^{-1} (2 - \alpha)^{2-\alpha} \alpha^\alpha .
\]

(53)

It follows

\[
B_2 \geq B_1 \iff \phi(k, \alpha) \geq 0
\]

(54)

with

\[
\phi(k, \alpha) := (2k + 1)^{1+\alpha} (2 - \alpha)^{2-\alpha} \alpha^\alpha \\
- (k + 2)^{2-\alpha} k^\alpha (1 + \alpha)^{1+\alpha} (1 - \alpha)^{1-\alpha} .
\]

(55)