Dynamic Price Competition
with Consumer Switching Costs
in Vertically Related Markets

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Abstract
An important feature of some industries is the coexistence between vertical integration and consumer switching costs. The aim of this paper is to uncover how these elements interact in determining prices as well as the endogenous vertical structure. More specifically, we consider an overlapping-generations model of price competition between two retailers that sell a homogenous good to consumers facing switching costs. Retailers have to satisfy their patronized consumers’ demand by acquiring inputs in the upstream market. We study Markovian pricing strategies such that, for a given vertical configuration, retail pricing decisions depend on retailers’ customer bases, and upstream market outcomes depend on retailers’ downstream market shares. Last, we endogenize the industry vertical structure and find that the level of switching costs determines whether all firms or only some of them will integrate.

Keywords: Vertical integration, Switching costs, Markov Perfect Equilibrium, Foreclosure.

JEL classification numbers: L13, L22.

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1 Introduction

An important feature of several industries is the coexistence between vertical integration and consumer switching costs. This is the case of the oil industry (Blass and Carlton 1999 and Borenstein 1991), the electricity and gas industries (Giulietti et al. 2003), the cable television industry (Chipty 2001), or the automobile industry (Monteverde and Teece 1982). While the previous literature has extensively analyzed the effects on pricing behavior of both elements in isolation,\(^1\) little attention has been devoted to study their combined effects.\(^2\) The aim of this paper is to uncover how vertical integration and switching costs interact in determining prices as well as the endogenous vertical structure.

For this purpose, we consider an industry composed of two upstream producers and two downstream retailers, and allow for all conceivable market configurations: full integration, no integration and partial integration. In every period, there is a continuum of identical consumers who enter the market. Each consumer lives for two periods, so that at every period one half of the population is composed of new consumers and the other half is composed of old consumers. Consumers aim at minimizing their current expenditures. The new consumers buy from the retailer offering the lowest price so that one of the two retailers serves all the new consumers. The old consumers face exogenous and common switching costs if they switch retailer. Hence, they will not switch unless the retailer they are committed to is undercut by more than the switching cost. At the end of every period, the number of new consumers served by a retailer creates its customer base for the following period. In every period, retailers have to buy the inputs needed to satisfy their patronized consumers’ demand in an upstream market. We study Markovian equilibria where the pricing strategies in the downstream market are functions of the current state only, which is given by each firm’s consumer base. Furthermore, as long as there is one integrated firm, pricing decisions at the retail level not only affect current and future (through the effect of switching costs) retail profits, but they also affect current profits and prices at the upstream level.

The electricity sector in Great Britain provides an interesting example of an industry where vertical integration (between generation and retailing) and switching costs co-exist. The com-

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\(^1\)See Rey and Tirole (2003) and Farrell and Shapiro (2004) for recent surveys on vertical integration and switching costs, respectively.

\(^2\)Valetti (2004) is an exception.
plete liberalization of electricity retail choice and tariffs that took place in March 2001 has led to important changes in the industry structure. First, the generation segment has gone through a process of de-concentration and the degree of vertical integration has increased substantially. Second, a dual structure has emerged in the domestic supply market, where there is a clear dichotomy between the residential and non-residential segments (residential consumers being primarily households, while the non-residential segment is formed by businesses and public administration). For non-residential consumers, prices have fallen in line with wholesale prices, price differentials among different suppliers are almost non-existent, and the degree of concentration is much lower as compared to the residential segment (Ofgem 2004). By contrast, the residential segment is characterized by persistent price differences, and regional incumbents have managed to maintain market shares above 50% (with the highest regional market share standing at 83%, Ofgem). Given that electricity is essentially a homogeneous good, the persistence of price differentials can only be explained by the existence of switching costs (real and perceived) for residential consumers (Giuletti et al. 2003). In addition, the retail segment has witnessed a process of rapid vertical and horizontal concentration. While the number of incumbents (14) was high and entry did occur (depending on the source, as many as 30 retail suppliers co-existed at one point), the industry quickly evolved towards a concentrated structure. The residential customer segment is currently dominated by five large incumbents that supply more than 80% of residential customers. The only large scale entrant that is currently active is British Gas (Centrica), itself a dominant incumbent in the gas market. The five large incumbents as well as British Gas are vertically integrated, and their market shares in the downstream residential segment is larger than their shares in the upstream segment (generation). Last, with exception of niche players, non-vertically integrated firms have experienced serious difficulties, particularly those players without a portfolio of residential customers. The UK’s largest electricity generator, British Energy (15.8% of installed capacity) recently went into receivership, and a government bail out amounting to 2.1 billion pounds turned out to be necessary. AES DRAX has also experienced financial difficulties; both these generators lack a residential customer base (British Energy initially acquired Swalec, but sold after two years. A similar pattern emerges for some B2B suppliers that do not sell to residential customers (for instance, Maverick Energy went bankrupt in June 2003). The intrinsic value of a residential customer base (market share) is illustrated by the following: while it is estimated that capturing a new customer costs about
60 pounds, recent mergers (PowerGen/TXU, London Electricity/SEABOARD) yield an implicit value of about 280-310 pounds per customer.

The paper proceeds as follows. In Section 2 we describe the model, and in Sections 3 to 5 we characterize the upstream and downstream market equilibria for all vertical industry configurations: Full Integration (Section 3), No Integration (Section 4) and Partial Integration (Section 5). Section 6 analyzes integration decisions and Section 7 concludes.

2 The Model

We consider two vertically related markets: the upstream and the downstream market. In the upstream market there are two producers, $U_A$ and $U_B$, who compete to produce an identical intermediate good, facing constant marginal costs $c \geq 0$ up to their symmetric capacities $k$. In the downstream market, there are two potential retailers, $D_A$ and $D_B$, who compete to sell an homogenous good which they produce by transforming the intermediate good on a one-to-one basis at constant marginal costs (i.e. delivery and customer services) normalized to zero. Thus, the retailers’ only cost is the price at which they have to buy the good in the upstream market.

We analyze a dynamic game that evolves as follows (Figure 1 depicts the timing of the game).

Integration Decisions

In period $t = 0$, firms take simultaneous integration decisions. In particular, we assume that producer $U_i$ and retailer $D_i$ decide whether to vertically integrate or to remain separated, $i = A, B$. We assume that an integration takes place whenever the gains from vertical integration exceed its costs $C \geq 0$. Furthermore, integration decisions are irreversible.

Hence, at the end of period $t = 0$, one of either four possible vertical structures may arise: both producers and retailers are vertically integrated (Full Integration, $FI$), all firms remain separated (No Integration, $NI$), or only producer $U_i$ and retailer $D_i$ are vertically integrated (Partial Integration by firms $i$, $PI_i$). For convenience, we let $I_i$ reflect firms $i$’s state of vertical integration, where

$$I_i = \begin{cases} 
0 & \text{if } D_i \text{ is separated from } U_i, \\
1 & \text{if } D_i \text{ is integrated with } U_i, 
\end{cases} \quad i = A, B \quad (1)$$

Integration decisions are irreversible.
Consumer Dynamics

At the beginning of every period $t \in \{1, 2, \ldots \}$, there is a continuum of final consumers $[0, \frac{1}{2}]$ entering the downstream market. Each consumer lives for two periods and has an inelastic demand, with reservation price $v > c$. Consumers face exogenous and common switching costs, $s \geq 0$. Hence, at any given period, there are two generations of consumers co-existing in the market: locked-in consumers who would have to pay the switching cost $s$ to switch retailer, and new uncommitted consumers. It is assumed that both types of consumers behave so as to minimize their current payments. This implies that locked-in consumers do not change retailer unless the one they are committed to is undercut by more than the switching cost $s$ (we assume that if both retailers charge the same price, the new consumers purchase from the one with no locked-in consumers whereas the old consumers do not switch).

In every period $t \in \{1, 2, \ldots \}$, each consumer demands $\theta$ units of the good. Given that at any time there is a mass one of co-existing consumers, $\theta$ also denotes total demand in a given period. We assume that $\theta \in (k, 2k)$, i.e. demand exceeds the capacity of a single producer but there is excess capacity overall.\(^3\)

Downstream Market

Knowing the entire history of the game up to period $t - 1$, retailers set prices, $p_i \in [0, v]$, simultaneously and non-cooperatively, in every period $t \in \{1, 2, \ldots \}$.

Retail pricing decisions lead to several subgames, depending on each retailer’s market share in the current period and on the industry’s vertical structure. The different subgames are of the form $(I_A \alpha_{At}, I_B \alpha_{Bt})$, where for $i = A, B$, $I_i$ is defined in (1), and $\alpha_{it}$ denotes retailer’s $D_i$ downstream market share in the current period $t$. Given that consumers in a generation are all identical, it follows that a retailer either serves all or none of the uncommitted customers, so that $\alpha_{it} \in \{0, \frac{1}{2}, 1\}$, $i = A, B$ and $\alpha_{At} + \alpha_{Bt} = 1$.

Retail pricing decisions also determine the dynamics of the game. Retailer $D_i$’s customer base equals one if the old consumers in period $t$ were already patronizing retailer $D_i$ in $t - 1$, and it equals zero otherwise, $i = A, B$.

Upstream Market

In the upstream market, the demand side is made of the downstream retailers, who have to

\(^3\)The alternative cases, either $\theta \leq k$ or $\theta \leq 2k$, seem not relevant as wholesale prices would equal marginal cost or the price cap, respectively, independently of downstream market shares and the industry vertical configuration.
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buy as many units of the good as demanded by their patronized final consumers. Hence, given that all consumers are identical, retailer $D_i$ has to demand $\theta$ units of the good times the number of consumers it serves, i.e. $\alpha_i \theta$, $i = A, B$. The supply side is made of the upstream producers, who compete on the basis of bids made to the auctioneer.

Producers compete by simultaneously and independently submitting a bid which specifies the minimum price at which it is willing to produce with the whole of its capacity. Bids cannot exceed the ‘market reserve price’ $P$, possibly determined by regulation. We will assume that the market reserve price does not exceed consumers’ reservation price, $P \leq v$. On the basis of the bid profile, the auctioneer calls producers into operation. If producers submit different bids, the lower-bidder produces up to capacity $k$, and the higher-bidder satisfies the residual demand, $\theta - k$. Ties are broken randomly (symmetrically for both producers). The price received by a producer for any positive quantity despatched by the auctioneer is equal to the highest accepted bid in the auction (uniform-price auction). Hence, the wholesale price $w_t$ is also equal to the highest accepted bid.

Producers’ profits will depend on the vertical structure and the allocation of downstream market shares, and retailers’ profits will depend on their retail price, the wholesale price, and downstream market shares. Accordingly, it will be convenient to use the following notation: for $i = A, B$, let $\pi^U(I_i \alpha_{it}, I_j \alpha_{jt})$ and $\pi^D(p_{it}, w_t; \alpha_{it}, \alpha_{jt})$ respectively denote producer $U_i$’s and retailer $D_i$’s profits in period $t$.

Firms, which are assumed to be risk-neutral, aim at maximizing their net present values; their common discount factor is denoted $\delta \in (0, 1)$. Since all the aspects of the game are common knowledge, we solve the game by backward induction to characterize the symmetric stationary Markovian Perfect Equilibrium of the repeated game.

The analysis is simplified by the fact that in the upstream market, only short-term incentives are relevant. That is, given that the upstream pricing decisions do not affect the state of the system (i.e. retailers’ customer bases), we do not need to take into account any intertemporal considerations at this stage. It thus suffices to study the one-shot game to characterize the equilibrium bidding strategies in the upstream market in every possible subgame (i.e. depending

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4The game in which the good is traded among the merging companies at transfer prices, and each firm has to acquire (sell) its excess demand (supply) to the rival would yield the same results as the ones presented here as long as the seller has all the bargaining power.
on the allocation of downstream market shares).

In the downstream market however, retailers condition their pricing strategies on the state of the system, i.e. each retailer’s customer base. We analyze the model by using dynamic programming. Since we are considering Markov equilibria only, we can let \( V_{1D}^D (I_i, I_j) \) and \( V_{0D}^D (I_i, I_j) \) denote the present discounted value (at the beginning of a period) for retailer \( D_i \) when it either has a customer base or none and the industry vertical structure is \((I_i, I_j)\). Similarly, let \( V_{1U}^U (I_i, I_j) \) and \( V_{0U}^U (I_i, I_j) \) denote the present discounted value for producer \( U_i \) when retailer \( D_i \) either has a customer base or none. Last, we write \( V_{1U+D}^U (I_i, I_j) \) and \( V_{0U+D}^U (I_i, I_j) \) to denote the sum of the present discounted values of \( U_i \) and \( D_i \).

We proceed by characterizing the equilibria for each vertical structure separately: Full Integration (Section 4), No Integration (Section 5) and Partial Integration (Section 6). Last, Section 7 endogenizes the industry vertical structure.
3 Full Integration

In this section we characterize the equilibrium in the downstream and upstream markets when both pairs of firms are vertically integrated. We proceed by backward induction by first characterizing the Nash equilibrium in the upstream market, for all possible allocations of downstream market shares.

**Proposition 1 (Equilibrium in the Upstream Market under Full Integration)** Suppose $I_A = I_B = 1$.

(i) Suppose that downstream market shares are symmetric, i.e. $\alpha_A = \alpha_B = \frac{1}{2}$. The unique pure-strategy equilibrium is $b_A = b_B = c$. Producers make zero profits and the wholesale price equals $c$.

(ii) Suppose that downstream market shares are asymmetric, i.e. $\alpha_A = 1 > \alpha_B = 0$. All pure-strategy equilibria are given by bid profiles satisfying $b_A \leq \frac{\theta - k}{k} [P - c] + c$ and $b_B = P$. Producer $U_A$ makes profits $[P - c] k$ and producer $U_B$ makes profits $[P - c] [\theta - k]$. The wholesale price equals $P$.

**Proof.** It can be extended from the proof of Proposition 1 in Fabra, von der Fehr and Harbord (2004).

First, when firms’ downstream market shares are symmetric, there cannot exist an equilibrium in which firms’ bids are asymmetric, given that the firm with the high bid would stand at a net-buying position and hence, it would have incentives to bid slightly below its rival. Furthermore, there cannot exist an equilibrium in which firms bid the same price above marginal costs, as each of them would be better off by marginally decreasing its bid in order to increase its production with only a slight decrease in the profits made by its downstream subsidiary. Therefore, drives wholesale prices down to marginal costs and the upstream producers make zero profits.\(^5\)

\(^5\)Strictly speaking, any bid profile such that both firms tie at any price not greater than marginal cost constitutes an equilibrium, as symmetric downstream and upstream market shares, wholesale prices imply a pure transfer between the upstream and downstream subsidiaries. Without loss of generality, we do not consider equilibria with prices below marginal costs.
When firms’ downstream market shares are asymmetric, the firm with the larger (smaller) downstream market share stands at a net buying (selling) position.\textsuperscript{6} This asymmetry confers the firm with a large downstream market share the commitment to bid more aggressively, given that its profits are decreasing in the wholesale price. This induces the rival to maximize its profits over the residual demand by bidding at the market reserve price.

Proposition 1 implies that, when downstream market shares are asymmetric, the profits of the upstream producer integrated with the retailer that monopolizes the downstream market are larger than those of its rival. We will refer to this phenomenon as the \textit{upstream effect}: having a larger downstream market share confers the producer with the commitment to bid low, thereby increasing its production and forcing the rival firm to set a high market price. This effect will turn out to be crucial in the analysis of downstream competition, as it strengthens the integrated firms’ incentives to fight for market share.

The overall profitability of an integrated firm also depends on the retail price at which customers are served. With the help of Figure 2, let us compare the profits made by the integrated firms as a function of retail prices, for given downstream market shares. Consider first the case of asymmetric downstream market shares, e.g. $\alpha_A = 1 > \alpha_B = 0$. The profits made by firm $A$, $\pi^U_A(1, 0) + \pi^D_A(p, P; 1, 0)$, are increasing in the retail price $p$, whereas those of firm $B$, $\pi^U_B(0, 1)$, are independent of $p$ as it serves no customers. If the retail price equals $P$, retailer $D_A$ makes no profits given that the wholesale price with asymmetric downstream market shares also equals $P$. However, producer $U_A$ makes profits $\pi^U_A(1, 0)$ which, by the upstream effect, exceed the profits made by its rival, $\pi^U_B(0, 1)$. As the retail price is reduced below $P$, retailer $D_A$ incurs in losses, thereby reducing the gap between the profits made by firms $A$ and $B$. If the retail price is low enough (below $\hat{p}$), retailer $D_A$’s losses reduce firm $A$’s profits below the constant profits made by its rival.

When downstream market shares are symmetric, producers make no profits, and retailers profits, $\pi^D(p, c; \frac{1}{2}, \frac{1}{2})$, are increasing in the retail price $p$. Given that the profits made by the whole industry are independent of the allocation of downstream market shares, it follows that the profits made by each firm when downstream market shares are symmetric, are just the

\textsuperscript{6}Note that this is true independently of which firm has the low price. Trivially, the firm with no downstream market share stands at a net sell position both when it sells $k$ or $(\theta - k)$; the firm that monopolizes the downstream market demands $\theta$ units of the good in the upstream market, in which it sells at most $k < \theta$. 
average between firms’ profits when downstream market shares are asymmetric, i.e.

\[ \pi^D \left( p, c; \frac{1}{2}, \frac{1}{2} \right) = \frac{1}{2} \left[ \pi^U \left( 1, 0 \right) + \pi^D \left( p, P; 1, 0 \right) + \pi^U \left( 0, 1 \right) \right] \]

Hence, for retail prices above \( \hat{p} \), \( \pi^D \left( p, P; 1, 0 \right) + \pi^U \left( 0, 1 \right) > \pi^U \left( 1, 0 \right) \), whereas for retail prices below \( \hat{p} \), \( \pi^D \left( p, P; 1, 0 \right) + \pi^U \left( 0, 1 \right) < \pi^D \left( p, c; \frac{1}{2}, \frac{1}{2} \right) \). 

As shown in the following Lemma (see also Figure 2), the upstream effect may not be strong enough so as to compensate for any potential losses in the downstream market if stealing market share comes at the cost of reducing retail prices below a certain threshold.

**Lemma 1** There exists a price \( \hat{p} \in (c, P) \) that verifies the following property:

\[ \pi^U_i \left( 1, 0 \right) + \pi^D_i \left( p, P; 1, 0 \right) \geq \pi^D_i \left( p, c; \frac{1}{2}, \frac{1}{2} \right) \geq \pi^U_i \left( 0, 1 \right) \quad \text{if} \quad p \geq \hat{p} \]

\[ \pi^U_i \left( 1, 0 \right) + \pi^D_i \left( p, P; 1, 0 \right) < \pi^D_i \left( p, c; \frac{1}{2}, \frac{1}{2} \right) < \pi^U_i \left( 0, 1 \right) \quad \text{if} \quad p < \hat{p} \]

**Proof.** See the Appendix. ■

The critical retail price \( \hat{p} \) will play a central role in the analysis of downstream competition, which is developed next.

A retailer’s pricing strategy affects its profits and those of its upstream subsidiary, in different ways. First, for a given price of its rival, a retailer’s pricing decision has a direct impact on its current downstream profits as it determines its downstream market share and the price that consumers will pay for the service (downstream effect). Second, retail prices determine whether the new customers will contract with one retailer or another, therefore affecting the following period’s customer bases and hence firms’ continuation values (consumer-base effect). These effects are the same as in a standard game with no integration. However, vertical integration adds new insights into the problem, given that the allocation of downstream market shares determines the upstream market outcomes: by Proposition 1, it has an impact on the wholesale price (wholesale price effect) and hence, on the retailer’s profits, as well as an impact on the profits made by its upstream subsidiary (upstream effect).

The characterization of the equilibria in the downstream market results from the interplay between the effects identified above.
Figure 2: The Profits of the Vertically Integrated Firms as a Function of Retail Prices

**Proposition 2 (Equilibrium in the Downstream Market under Full Integration)** Suppose $s > 0$, $\delta \in (0, 1)$, and $I_A = I_B = 1$.

There does not exist a pure-strategy MPE. There exists a symmetric stationary MPE in which retailers name prices according to the function $F(p)$, which selects $F_0(p; 1, 1)$ if the retailer has no customer base and $F_1(p; 1, 1)$ otherwise, where

(i) If $s \geq [v - \hat{p}] \frac{1 + \delta}{2 + \delta}$,

$$F_1(p; 1, 1) = 1 - \frac{[1 + \delta] \frac{v - \hat{p}}{p - \hat{p} + \delta}}{2 + \delta} \quad \text{for} \quad p \in \left[ \frac{(1 + \delta) \hat{p} + v}{2 + \delta}, v \right]$$

$$F_0(p; 1, 1) = 2F_1(p; 1, 1) \quad \text{for} \quad p \in \left[ \frac{(1 + \delta) \hat{p} + v}{2 + \delta}, v \right]$$

$$F_0(p; 1, 1) = F_1(p; 1, 1) = 1 \quad \text{for} \quad p = v$$
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(ii) If \( s \leq [v - \hat{p}]^{1+\delta} \),

\[
F_1(p; 1, 1) = 1 - \frac{s}{p - \hat{p} + s^{1+\delta}} \quad \text{for} \quad p \in \left[\hat{p} + \frac{s}{1+\delta}, \hat{p} + s^{2+\delta}\right]
\]

\[
F_0(p; 1, 1) = 2F_1(p; 1, 1) \quad \text{for} \quad p \in \left[\hat{p} + \frac{s}{1+\delta}, \hat{p} + s^{2+\delta}\right]
\]

\[
F_0(p; 1, 1) = F_1(p; 1, 1) = 1 \quad \text{for} \quad p = \hat{p} + s^{2+\delta}
\]

Proof. See the Appendix for details.

The fight for market share induces firms to undercut each other. However, the fact that it is unprofitable sell below \( \hat{p} \) (Lemma 1) sets a lower bound below which firms will not want to further undercut each other. This two elements destroy any candidate pure strategy equilibrium. In equilibrium, retailers will be randomizing their prices over a range which lies strictly above \( \hat{p} \) and at or below the reservation price, \( v \). The upper bound of the support of the mixed strategy does not exceed the lower bound by more than the switching cost \( s \).

The mixed-strategy equilibrium of the retailer with no customer, \( F_0(p; 1, 1) \), and that of the retailer with a positive customer base, \( F_1(p; 1, 1) \), must satisfy the following system of dynamic equations, for \( p \) in the support of the mixed strategy equilibrium:

\[
V_{0}^{U+D}(1, 1) = F_1(p; 1, 1) \left[ \pi^U(0, 1) + \delta V_0^{U+D}(1, 1) \right]
+ [1 - F_1(p; 1, 1)] \left[ \pi^D(p, c; \frac{1}{2}, \frac{1}{2}) + \delta V_1^{U+D}(1, 1) \right]
\]

\[
V_1^{U+D}(1, 1) = F_0(p; 1, 1) \left[ \pi^D(p, c; \frac{1}{2}, \frac{1}{2}) + \delta V_0^{U+D}(1, 1) \right]
+ [1 - F_0(p; 1, 1)] \left[ \pi^U(1, 0) + \pi^D(p, P; 1, 0) + V_1^{U+D}(1, 1) \right]
\]

The first equation represents the firm’s current and future expected profits when its retailer has no customer base. Given that the price charged by the retailer with a large customer base
never exceeds the price of its rival by more than the switching cost \( s \), it follows that the retailer with no customer base will serve the new consumers only when it prices below its rival. Therefore, with probability \( F_1 (p; 1, 1) \), the retailer does not serve any consumers, but makes positive profits in the upstream market, \( \pi^U (0, 1) \). The system reverts to the same state in the following period, with continuation value \( V_0^{U+D} (1, 1) \). With probability \( 1 - F_1 (p; 1, 1) \), the retailer with no customer base only serves the new consumers. As this results in symmetric downstream market shares, the equilibrium wholesale price equals marginal costs (Proposition 1), and the firm only makes current profits from the downstream market, \( \pi^D (p, c; \frac{1}{2}, \frac{1}{2}) \). Furthermore, the firm creates a customer base for the following period, and hence has continuation value \( V_1^{U+D} (1, 1) \).

Similarly, the second equation represents the firm’s current and future expected profits when its retailer has a positive customer base. With probability \( F_0 (p; 1, 1) \), the retailer only serves its patronized consumers. Since this results in equal market shares, wholesale prices are driven down to marginal cost, and the firm only makes profits in the downstream market, \( \pi^D (p, c; \frac{1}{2}, \frac{1}{2}) \). The firm loses its customer base for the next period, so its continuation value is \( V_0^{U+D} (1, 1) \). With probability \( 1 - F_0 (p; 1, 1) \), the retailer also serves the new consumers, thus making downstream profits \( \pi^D (p, P; 1, 0) \). As this results in asymmetric downstream market shares, the upstream producer makes positive profits \( \pi^U (1, 0) \), and the system reverts to the same state with continuation value \( V_1^{U+D} (1, 1) \).

Retailers’ pricing incentives depend on the net gain from marginally raising their price. The net gain can be decomposed into two elements. First, for given market shares, the gain from the resulting increase in the price. Second, for a given price, the loss from reducing the chance of having the low price. These marginal gains and losses are reflected in the following differential equations, which must be satisfied for every price \( p \) on the interior of the support of the mixed strategies:
\[
\frac{\theta}{2} [1 - F_1 (p; 1, 1)] = \left[ \pi^D (p, c; 1, 1) - \pi^U (0, 1) \right] f_1 (p; 1, 1) \\
+ \delta \left[ V^{U+D}_1 (1, 1) - V^{U+D}_0 (1, 1) \right] f_1 (p; 1, 1)
\]

\[
\theta [1 - F_0 (p; 1, 1)] + \frac{\theta}{2} F_0 (p; 1, 1) = \left[ \pi^U (1, 0) + \pi^D (p, P; 1, 0) - \pi^D (p, c; 1, 1) \right] f_0 (p; 1, 1) \\
+ \delta \left[ V^{U+D}_1 (1, 1) - V^{U+D}_0 (1, 1) \right] f_0 (p; 1, 1)
\]

(2)

where \( f_z (p; 1, 1) \) denotes \( F'_z (p; 1, 1) \), \( z = 1, 0 \).

The comparison of the left hand sides of the differential equations shows that, for given market shares, the gain from marginally increasing the price is twice as large for the retailer with the large customer base as it is for the retailer with no customer base. First, consider the event that the rival prices above, which occurs with probability \( [1 - F_0 (p; 1, 1)] \) for the retailer with a positive customer base, and with probability \( [1 - F_1 (p; 1, 1)] \) for the one with no customer base. For the former, the gain from increasing the price applies to the demand of all consumers, \( \theta \), whereas for the latter, it only applies to the demand of the new ones, \( \frac{\theta}{2} \). Second, consider the event that the rival prices below, which occurs with probability \( F_0 (p; 1, 1) \) for the retailer with a positive customer base, and with probability \( F_1 (p; 1, 1) \) for the one with no customer base. For the former, the gain from increasing the price applies to the demand of its patronized consumers. The latter has no gain as it does not serve any customers.

Also, by comparing the right hand sides of the equations, it follows that, for a given price, the loss from reducing the chance of having the low price is equal for both retailers. Consider first the retailer with no customer base. The costs from being undercut derive from the loss in retail revenues from serving the new customers at a wholesale price \( c \), \( \pi^D (p, c; 1, 1) \), minus the gains of the upstream producer when its downstream subsidiary serves no customers, \( \pi^U (0, 1) \) (when it has the low price, the upstream producer makes no profits). Consider now the retailer with a positive customer base. Its costs from being undercut can be decomposed into the loss in upstream profits, \( \pi^U (1, 0) \) (when it has the high price, the upstream producer makes no profits), and the difference in downstream profits from serving all customers at a wholesale price \( P \), \( \pi^D (p, P; 1, 0) \), rather than just serving its patronized customers at a wholesale price \( c \), \( \pi^D (p, c; 1, 1) \). Since the profits made by the whole industry are independent of the allocation.
of downstream market shares, it follows that
\[
\pi^U(1,0) + \pi^D(p,P;1,0) - \pi^D(p,c;1/2,1/2) = \pi^D(p,c;1/2,1/2) - \pi^U(0,1)
\]
i.e. the loss from reducing the chance of having the low price is equal for both retailers.\(^7\)

On the interior of the mixed strategy equilibrium, the net gain from raising the price must be zero. From our previous remarks, it follows that the retailer with a positive customer base must place higher probability on higher prices for the net gains to be zero for both retailers. In other words, the probability distribution \(F_1(p;1,1)\) first-order stochastically dominates \(F_0(p;1,1)\), i.e. \(F_1(p;1,1) \leq F_0(p;1,1)\) for all \(p\) in the support. In particular, since, for a given market share, the large retailer gains twice as much as the small retailer from marginal increases in the price but, for a given price, both lose the same, the large retailer must put twice as much weight on higher prices than the small retailer. This implies that the probability that the retailer with a large customer base prices above its rival is \(3/4\).\(^8\)

In equilibrium, the retailer with the large customer base charges higher retail prices on average and retains a larger downstream market share. However, when it monopolizes the downstream market, the wholesale price is also high. Thus, depending on parameter values, the retailer with a larger customer base may end-up worse off than the retailer with no customer base (whenever the latter sells a positive quantity, the wholesale price equals marginal costs). Nevertheless, the fact that firms are integrated also implies that the larger market share enjoyed by the retailer with a large customer base allows its upstream subsidiary to increase its profits over the ones made by the producer integrated with the small retailer. Overall, the integrated firm with a positive customer base makes more profits.\(^9\)

### 4 No Integration

Consider now the case in which all firms stand alone. Again, we proceed by back induction by first characterizing the Nash equilibrium in the upstream market. The fact that all firms stand

\(^7\)Using Lemma 1, these expressions are equal to \(|p - P|\).\(^6\).

\(^8\)It puts mass 1/2 on the upper bound, and on the interior of the support, it prices above its rival with probability 1/2. Hence, from Proposition 1, the wholesale price equals marginal costs \(c\) with probability 1/4 and it equals the market reserve price \(P\) with probability 1/4.

\(^9\)See the appendix for details on how this expression is derived.
alone implies that the equilibrium in the upstream market is independent of the allocation of downstream market shares.

**Proposition 3** *(Equilibrium in the Upstream Market under No Integration)* Suppose $I_A = I_B = 0$.

All pure-strategy equilibria are given by bid profiles satisfying $b_i \leq \frac{\theta - k}{k} [P - c] + c$ and $b_j = P$, so that producer $U_i$ makes profits $\theta [P - c]$ and producer $U_j$ makes profits $[\theta - k][P - c]$, $i, j = A, B, i \neq j$. The wholesale price equals $P$.

**Proof.** See Proof of Proposition 1 in Fabra, von der Fehr and Harbord (2004).

Under No Integration, the equilibria in the upstream market are independent of the allocation of downstream market shares. In pure-strategies, there are two sets of price-equivalent equilibria, that differ on who bids low and sells at capacity, and who maximizes its residual demand by bidding at the market reserve price $P$. There also exists a continuum of equilibria in mixed strategies, that differ on who plays a mass point at the price-cap and with what probability.

However, since the mixed strategy equilibria are Pareto dominated from the view point of producers by any of the pure strategy equilibria, in what follows we will assume that firms play the symmetric correlated equilibrium among the two pure strategy equilibria.\(^\text{10}\)

Under No Integration, a retailer’s pricing strategy affects its current downstream market profits and its consumer base, thereby determining its future stream of profits *(downstream and customer base effects)*. However, the upstream and wholesale price effects identified in the case of Full Integration do not play any role, given that retailers do not internalize the effect of their pricing decisions on the upstream profits, and given that the wholesale price is independent of the allocation of downstream market shares. Thus, the characterization of the equilibrium in the downstream market under No Integration results from the interplay between the downstream and customer base effects only.

**Proposition 4** *(Equilibrium in the Downstream Market under No Integration)* Suppose $s > 0$, $\delta \in (0, 1)$, and $I_A = I_B = 0$.

\(^{10}\)If we allowed for mixed strategy pricing, equilibrium wholesale prices would be lower, and this would just reinforce our results.
There does not exist a pure-strategy MPE. The structure of the symmetric stationary MPE is similar to the one under FI, with \( \hat{p} \) replaced by \( P \) in all expressions.

For similar reasons as under Full Integration, a pure-strategy equilibrium does not exist. The mixed-strategy equilibrium of the retailer with no customer, \( F_0 (p; 0, 0) \), and that of the retailer with a positive customer base, \( F_1 (p; 0, 0) \), must satisfy the following system of dynamic equations:

\[
V_0^D (0, 0) = F_1 (p; 0, 0) \delta V_0^D (0, 0) + [1 - F_1 (p; 0, 0)] \left[ \pi^D (p, P; \frac{1}{2}, \frac{1}{2}) + \delta V_1^D (0, 0) \right]
\]

\[
V_1^D (0, 0) = F_0 (p; 0, 0) \left[ \pi^D (p, P; \frac{1}{2}, \frac{1}{2}) + \delta V_0^D (0, 0) \right]
\]

\[
+ [1 - F_0 (p; 0, 0)] \left[ \pi^D (p, P; 1, 0) + \delta V_1^D (0, 0) \right]
\]

The interpretation of these equations is similar to the one given under Full Integration, with the already mentioned exceptions: retailers do not internalize the profits made by the producers; and the wholesale price, \( P \), is independent of the allocation of downstream market shares.

On the interior of the support of the mixed strategies, strategies must satisfy the following system of differential equations:

\[
\frac{\theta}{2} \left[ 1 - F_1 (p; 0, 0) \right] = \left[ \pi^D (p, P; \frac{1}{2}, \frac{1}{2}) + \delta \left[ V_1^D (0, 0) - V_0^D (0, 0) \right] \right] f_1 (p; 0, 0)
\]

\[
\theta \left[ 1 - F_0 (p; 0, 0) \right] + \frac{\theta}{2} F_0 (p; 0, 0) = \left[ \pi^D (p, P; 1, 0) - \pi^D (p, P; \frac{1}{2}, \frac{1}{2}) \right] f_0 (p; 0, 0)
\]

\[
\delta \left[ V_1^D (0, 0) - V_0^D (0, 0) \right] f_0 (p; 0, 0)
\]

where \( f_z (p; 0, 0) \) denotes \( F_z^\prime (p; 0, 0) \), \( z = 1, 0 \).

From the comparison of the differential equations under FI and NI, (2) and (3), it is clear that, for given market shares, the gain from marginally increasing the price is the same under both vertical structures and the same for both retailers. However, the costs from reducing the chance of having the low price are lower under NI.\(^{11} \) This derives from the fact that the upstream

\(^{11} \) In detail, it equals \(| p - P | \frac{\theta}{2} \) under NI and \(| p - \hat{p} | \frac{\theta}{2} \) under FI.
effect no longer plays any role. Therefore, at the mixed strategy equilibrium under \( NI \), retailers must put higher weight on higher prices than under \( FI \). In other words, the probability distribution \( F_z (p; 0, 0) \) first-order stochastically dominates \( F_z (p; 1, 1) \), i.e. \( F_z (p; 0, 0) \leq F_z (p; 1, 1) \) for \( z = 1, 0 \) and \( p \) in the support. Last, this implies that the expected retail prices must also be larger under \( NI \), that the profitability of the whole industry is increased, and that consumer surplus decreased as compared to \( FI \).

5 Partial Integration

Last, consider the case in which only one pair of firms is vertically integrated.

**Proposition 5 (Equilibrium in the Upstream Market under Partial Integration)** Suppose \( I_i = 1, I_j = 0, i, j = A, B, i \neq j \).

(i) If \( \alpha_i = 0 \), all pure-strategy equilibria are given by bid profiles satisfying \( b_i \leq \frac{\theta - k}{k} [P - c] + c \) and \( b_j = P \), so that producer \( U_i \) makes profits \( k[P - c] \) and producer \( U_j \) makes profits \( [\theta - k] [P - c] \). The wholesale price equals \( P \).

(ii) If \( \alpha_i > 0 \), all pure-strategy equilibria are given by bid profiles satisfying \( b_i \leq \frac{\theta - k}{k} [P - c] + c \) and \( b_j = P \). Producer \( U_i \) makes profits \( [P - c] k \) and producer \( U_j \) makes profits \( [P - c] [\theta - k] \). The wholesale price equals \( P \).

Trivially, if the integrated retailer has no downstream market share, the equilibrium in the upstream is the same as under \( NI \). However, as long as it has a positive market share (independently of whether it is \( \frac{1}{2} \) or 1), its profits are larger than those of the stand-alone producer, for the same reasons as under \( FI \). Last, note that as under \( NI \), the wholesale price is \( P \), regardless of the allocation of downstream market shares.

Under Partial Integration, a retailer’s pricing strategy affects its current downstream market profits and its consumer base, thereby determining its future stream of profits (downstream and customer base effects), just as in the previous cases. The differences will derive from two facts. First, the upstream effect will affect firms in different ways, depending on their state of integration and customer basis. And second, by Proposition 5, the wholesale price effect disappears.
Proposition 6 (Equilibrium in the Downstream Market under Partial Integration) Suppose $s > 0$, $\delta \in (0, 1)$, and $I_A \neq I_B$.

There does not exist a pure-strategy MPE. There exists an asymmetric stationary MPE in which retailers name prices according to the function $F(p)$, which selects $F_0(p; I_i, I_j)$ if retailer $D_i$ has no customer base and $F_1(p; I_i, I_j)$ otherwise.

(i) The retailer with no customer base behaves as under NI, independently of whether it is integrated or not, i.e. $F_0(p; 0, 1) = F_0(p; 1, 0) = F_0(p; 0, 0)$.

(ii) The retailer with a positive customer base behaves as under NI only when it is integrated, i.e. $F_1(p; 1, 0) = F_1(p; 0, 0)$. Otherwise, it prices according to following function:\footnote{Note that the difference between $F_1(p; 0, 0)$ and $F_1(p; 0, 1)$ is that $[P - \tilde{\bar{p}}]$ appears on the numerator and denominator of $F_1(p; 0, 1)$.}

\begin{itemize}
  \item For $s \geq [v - P] \frac{1 + \delta}{2 + \delta}$,
    \begin{align*}
      F_1(p; 0, 1) &= 1 - \frac{(1 + \delta) (v - P) + [P - \tilde{\bar{p}}]}{(p - P) + \delta \frac{(v - P)}{1 + \delta} + [P - \tilde{\bar{p}}]} \quad \text{for } p \in \left(\frac{(1 + \delta)P + v}{2 + \delta}, v\right) \\
      F_1(p; 0, 1) &= 1 \quad \text{for } p = v
    \end{align*}
  
  \item For $s \leq [v - P] \frac{1 + \delta}{2 + \delta}$,
    \begin{align*}
      F_1(p; 0, 1) &= 1 - \frac{s + [P - \tilde{\bar{p}}]}{p - P + s \frac{1 + \delta}{1 + \delta} + [P - \tilde{\bar{p}}]} \quad \text{for } p \in \left[P + \frac{s}{1 + \delta}, P + s \frac{2 + \delta}{1 + \delta}\right] \\
      F_1(p; 0, 1) &= 1 \quad \text{for } p = v
    \end{align*}
\end{itemize}

Proof. See the Appendix. $\blacksquare$

First, consider those periods in which the integrated retailer has the large customer base. The following system of dynamic equations must be satisfied for $p$ in the support of the mixed strategy equilibrium,

\begin{align*}
  V_0^D(0, 1) &= F_1(p; 1, 0) \delta V_0^D(0, 1) + [1 - F_1(p; 1, 0)] \left[\pi^D(p, P; \frac{1}{2}, \frac{1}{2}) + \delta V_1^D(0, 1)\right] \\
  V_1^{U+D}(1, 0) &= F_0(p; 0, 1) \left[\pi^D(p, P; \frac{1}{2}, \frac{1}{2}) + \pi^U(\frac{1}{2}, 0) + \delta V_0^{U+D}(1, 0)\right] \\
  &\quad + [1 - F_0(p; 0, 1)] \left[\pi^D(p, P; 1, 0) + \pi^U(1, 0) + \delta V_1^{U+D}(0, 0)\right]
\end{align*}
The retailer with no customer base only makes current profits $\pi_D(p, P; \frac{1}{2}, \frac{1}{2})$ when the rival prices above, and none otherwise. Hence, when it marginally increases its price, it reduces the chances of earning $\pi_D(p, P; \frac{1}{2}, \frac{1}{2})$, just as under NI. The retailer with a positive customer base makes current profits $\pi_D(p, P; \frac{1}{2}, \frac{1}{2})$ if it is undercut, and $\pi_D(p, P; 1, 0)$ otherwise. However, its pricing strategy does not affect the profits made by its upstream subsidiary, as these are independent of whether the retailer has the low or the high price (by Proposition 5, $\pi_U(\alpha_i, 0)$ is independent of $\alpha_i$ as long as it is positive). Paradoxically, given that the integrated producer enjoys the advantage from being integrated independently of the downstream market shares, integration does not affect the integrated retailer’s pricing behavior. Hence, the equilibrium pricing strategies in the downstream market are the same as under NI.

Second, consider those periods in which the integrated retailer has no customer base. The following system of dynamic equations must be satisfied for $p$ in the support of the mixed strategy equilibrium,

$$V_1^D(0, 1) = F_0(p; 1, 0) \left[ \pi_D(p, P; \frac{1}{2}, \frac{1}{2}) + \delta V_0^D(0, 1) \right]$$

$$+ [1 - F_0(p; 1, 0)] \left[ \pi_D(p, P; 1, 0) + \delta V_1^D(0, 1) \right]$$

$$V_0^{U+D}(1, 0) = F_1(p; 0, 1) \left[ \pi_U(0, 0) + \delta V_0^{U+D}(1, 0) \right]$$

$$+ [1 - F_1(p; 0, 1)] \left[ \pi_D(p, P; \frac{1}{2}, \frac{1}{2}) + \pi_U(\frac{1}{2}, 0) + \delta V_1^{U+D}(1, 0) \right]$$

When the retailer with a positive customer base is undercut, its downstream profits are reduced from $\pi_D(p, P; 1, 0)$ to $\pi_D(p, P; \frac{1}{2}, \frac{1}{2})$. Hence, the costs of marginally increasing its price equal $\pi_D(p, P; \frac{1}{2}, \frac{1}{2})$, and these are the same as under NI. When retailer with no customer base is undercut, it does not serve any downstream consumers but makes profits $\pi_U(0, 0)$ in the upstream market. However, when it prices below its rival, not only it earns positive downstream profits $\pi_D(p, P; \frac{1}{2}, \frac{1}{2})$, but also it increases its profits in the upstream market to $\pi_U(\frac{1}{2}, 0)$. Hence, given that it only enjoys the advantage from being integrated when it has the low price, the upstream effect reappears. In particular, given that the net gain from raising the price must
be zero, a marginal increase in the price of the retailer with no customer base must reduce the probability of having the low price, \( F_1 (p; 0, 1) \), more under PI as under NI. It follows that \( F_1 (p; 0, 1) \) must place higher probability on higher prices under PI as under NI for the first order condition to be satisfied. That is, the probability distribution \( F_1 (p; 0, 1) \) first-order stochastically dominates \( F_1 (p; 0, 0) \) (by our previous results, also \( F_1 (p; 1, 1) \)) for all \( p \) in the support of the mixed strategies.

In consequence, the expected equilibrium retail prices under PI are strictly larger than those under NI (as with some positive probability the stand alone retailer will have a positive customer base, and hence it will price less aggressively as under NI). From our previous results, retail prices under PI are also strictly higher than under FI. In other words, the worst scenario for competition arises when only one pair of firms is vertically integrated. Retail prices would be reduced if either the integrated firm is forced to divest, or on contrary, if the non-integrated retailer and producer are allowed to integrate. The question that remains to be analyzed is whether firms would be willing to vertically integrate, or, on the contrary, if integration by one firm preempts subsequent integrations.

\section{Integration Decisions}

In this section we analyze firms’ incentives to integrate in period \( t = 0 \). We will use the following notation to characterize firms’ incentives to integrate vertically. For \( z = 0, 1 \), define:

\[
G_z(0) \equiv V_z^{U+D} (1, 0) - V_z^{U+D} (0, 0)
\]

\[
G_z(1) \equiv V_z^{U+D} (1, 1) - V_z^{U+D} (0, 1)
\]

In words, \( G_0(0) \) and \( G_0(1) \) represent the gains from vertical integration between the retailer with no customer base and a producer either when the rival firms stand alone, or when they are vertical integrated, respectively. \( G_1(0) \) and \( G_1(1) \) are symmetrically defined.

\textbf{Lemma 2 (Gains from Vertical Integration)}

(i) The gains from vertical integration when the rivals stand alone are the same for both pairs of firms, independently of whether the retailer that integrates has a customer base or none, i.e. \( G_0(0) = G_1(0) \).

(ii) For both pairs of firms, the gains from vertical integration are larger when the rivals stand alone than when they are vertically integrated, i.e. \( G_0(1) < G_0(0) \) and \( G_1(1) < G_1(0) \).
(iii) The gains from vertical integration when the rivals are vertically integrated are larger for the pair composed of the retailer with no customer base, i.e. $G_1(1) < G_0(1)$.

Using this result, we can now characterize the equilibrium vertical configuration as a function of the fixed costs of integration, $C$, when firms’ integration decisions are simultaneous.

**Proposition 7** (Equilibrium Vertical Structure) Suppose that integration decisions are simultaneous.

(i) For $C > G_0(0)$, the equilibrium is No Integration.

(ii) For $G_0(1) < C < G_0(0)$, the equilibrium is Partial Integration by either pair of firms.

(iii) For $G_1(1) < C < G_0(1)$, the equilibrium is Partial Integration by the retailer with no customer base.

(iv) For $C < G_1(1)$, the equilibrium is Full Integration.

**Remark 1** (Equilibrium selection) The retailer with no customer base is more likely to integrate vertically than its rival: whenever there is an equilibrium where only the retailer with a customer base integrates, there is also an equilibrium in which the retailer with no customer base integrates, while the contrary is not true.

**Remark 2** (The effect of Switching costs $s$). On the one hand, the gains from integration when the rivals stand alone are independent of the value of the switching cost $s$. On the other hand, the gains from integration when the rivals are integrated are increasing in $s$, and $G_1(1) < G_0(1) = 0$, when switching costs are sufficiently low. Hence, for $C = 0$, the equilibrium vertical configuration is Partial Integration when switching costs are low, and it is Full Integration whenever switching costs are large.

## 7 Conclusions

We have analyzed a dynamic game of price competition in two vertically related markets. The existence of switching costs at the retail level introduces intertemporal links across periods, as they create firms’ customer basis for future periods. Furthermore, the vertical links between the two markets add new insights into the analysis of competition with consumer switching costs, given that the retailers’ pricing decisions affect upstream pricing decisions, and hence the value of firms’ customer basis.
The analysis helps to identify the interplay between the industry vertical configuration, the
downstream market shares, and the bidding behavior in the upstream market. An integrated
producer with a large share in the retail market stands at a net buying position (i.e. its producer
sells less than its retailer’s demand). Hence, he is committed to bid aggressively and is therefore
able to increase its market share and its upstream market profits. The rival firm, which stands
at a net selling position, maximizes his profits over the residual demand by bidding high, thereby
setting a high market price at the expense of selling below capacity. If both producers are verti-
cally integrated and have equal downstream market shares, the upstream market is competitive
for all demand realizations. Under No Integration and under Partial Integration the equilibrium
wholesale price is always equal to the market reserve price, independently of the allocation of
downstream market shares.

In the downstream market, pure strategy Markov Perfect Equilibria do not exist. In the
mixed strategy equilibrium, retailers strike a balance between opposing effects: on the one
hand, charging a higher price results in higher downstream profits in the event that the rival
prices below; on the other hand, it reduces the chance of capturing the new customers who would
create its customer base for future periods. Integration affects retailers’ bidding incentives, as
the fight for market share in affected by what we have referred to as the upstream and wholesale
price effects: an integrated producer increases its profits when its downstream subsidiary retains
a large market share, and the allocation of downstream market shares determines producers’
bidding incentives and hence, the wholesale equilibrium price faced by retailers.

The characterization of the upstream and downstream equilibria allows to rank the expected
equilibrium wholesale and retail prices (and thus, consumer surplus) as well as the value of the
customer base across all vertical configurations. On the one hand, we have found that Full
Integration leads both to the lowest wholesale prices (below the market reserve price) and to the
lowest expected retail prices. On the other hand, Partial Integration leads to the highest retail
prices and it is therefore the most anti-competitive scenario. Both the market structure and the
customer bases affect retailers’ pricing behavior in significant way. Both retailers charge higher
prices when they stand alone than when they are vertically integrated, and the retailer with the
large customer base charges higher prices than the retailer with the small customer base.

Last, we have endogenized the industry vertical configuration. When the rival firms stand
alone, both retailers have equal positive gains from integration. These gains result from the
upstream effect enjoyed through integration, and are independent of the value of switching costs. The gains from integration when the rivals are integrated are lower than when they stand alone, and are also lower for the retailer with a positive customer base (as both under Partial Integration and Full Integration he is enjoys the same Upstream Effect). Hence, depending on parameter values, the equilibrium vertical configuration may be Partial Integration. This outcome is more likely the lower switching costs are, given that the gains from vertical integration when the rivals are integrated are decreasing in the level of switching costs.

8 Appendix

8.1 Proof of Lemma 1

By Proposition 1,

\[
\pi^U(1, 0) + \pi^D(p, P; 1, 0) = [P - c] k + [p - P] \theta,
\]

\[
\pi^U(0, 1) = [P - c] [\theta - k],
\]

\[
\pi^D(p, c; 1, 2) = [p - c] \frac{\theta}{2}.
\]

Hence,

\[
\pi^U(1, 0) + \pi^D(p, P; 1, 0) = \pi^U(0, 1) = \pi^D(p, c; 1, 2) \iff p = \hat{p} = 2 \frac{\theta - k}{\theta} P + \left(1 - 2 \frac{\theta - k}{\theta}\right) c.
\]

The proof of the Lemma concludes by noting that \(\pi^U(1, 0) + \pi^D(p, P; 1, 0)\) is increasing in \(p\), that \(\pi^U(0, 1)\) is independent of \(p\), and that total industry profits are independent of the allocation of downstream market shares. Hence,

\[
\pi^D(p, c; 1, 2) = \frac{\pi^U(1, 0) + \pi^D(p, P; 1, 0) + \pi^U(0, 1)}{2},
\]

implying that for prices \(p\) such that \(\pi^D(p, P; 1, 0) + \pi^U(0, 1)\) is above (below) \(\pi^U(1, 0)\), then \(\pi^D(p, c; 1, 2)\) is also above (below) \(\pi^U(1, 0)\), and vice-versa.

8.2 Characterization of the Equilibria in the Downstream Market

For \(z = 1, 0\) and \(I \in \{(1, 1), (0, 0), (1, 0), (0, 1)\}\), let \(\sup F_z(p; I) = \left[p_z(I), p_z(I)\right]\), \(z = 1, 0\), and assume that
(A1): \( p_z (I) = p(I) \), and \( p_0 (I) \) is named with zero probability.

(A2): \( p_z (I) = p(I) \), and both retailers name \( p(I) \) with zero probability.

(A3): \( p(I) \leq \min \{ p(I) + s, v \} \);

(A4): \( p(1,0) = p(0,1) \) and \( p(1,0) = p(0,1) \); and finally

(A5): \( V_1 (I) \geq V_0 (I) \).

Standard arguments imply that there is no \( p \in \sup F_z (p; I) \) such that \( p \) is named with positive probability, \( z = 0, 1 \), that the interior of the support is connected, and that \( p(I) = \min \{ p(I) + s, v \} \).

8.2.1 Proof of Proposition 2: Downstream Equilibrium under FI

The solution to the differential equations in (2) is given by:

\[
F_1 (p; 1, 1) = 1 - \frac{C_1 (1, 1)}{\pi^D (p, c; \frac{1}{2}, \frac{1}{2}) - \pi^U (0, 1) + \delta \left[ V_1^{U+D} (1, 1) - V_0^{U+D} (1, 1) \right]}
\]

\[
F_0 (p; 1, 1) = 2 - \frac{C_0 (1, 1)}{\pi^U (1, 0) + \pi^D (p, P; 1, 0) - \pi^D (p, c; \frac{1}{2}, \frac{1}{2}) + \delta \left[ V_1^{U+D} (1, 1) - V_0^{U+D} (1, 1) \right]}
\]

where \( C_z (1, 1), z = 1, 0 \) are constants of integration.

Since the boundary conditions imply \( F_z (p(1, 1); 1, 1) = 0, z = 0, 1 \), and \( F_0 (p(1,1); 1, 1) = 1 \), explicit solutions for \( p(1,1), p(1,1), F_0 (p; 1, 1) \), and \( F_1 (p; 1, 1) \), and the continuation values, \( V_1^{U+D} (1, 1) \) and \( V_0^{U+D} (1, 1) \), can be readily computed. Note that the expressions \( F_1 (p; 1, 1) \) and \( F_0 (p; 1, 1) \) provided in the statement of Proposition 2 are proper probability measures satisfying \((A1) - (A3)\).

Firms’ continuation values as a function of their customer base are,

\[
V_1^{U+D} (1, 1) = \frac{1}{1 - \delta} \left[ \pi^D (p(1,1), P; 1, 0) + \pi^U (1, 0) \right]
\]

(4)

\[
V_0^{U+D} (1, 1) = \pi^D (p(1,1), c; \frac{1}{2}, \frac{1}{2}) + \delta V_1^{U+D} (1, 1)
\]

(5)

\[
= \pi^D (p(1,1), c; \frac{1}{2}, \frac{1}{2}) + \frac{\delta}{1 - \delta} \left[ \pi^D (p(1,1), P; 1, 0) + \pi^U (1, 0) \right]
\]
8.2.2 Proof of Proposition 4: Downstream Equilibrium under NI

The solution to the differential equations in (3) is given by:

\[
F_1 (p; 0, 0) = 1 - \frac{C_1 (0, 0)}{\pi^D (p, P; 1, 1) + \delta [V_{1D} (0, 0) - V_{0D} (0, 0)]}
\]

\[
F_0 (p; 0, 0) = 2 - \frac{C_0 (0, 0)}{\pi^D (p, P; 1, 1) + \delta [V_{1D} (0, 0) - V_{0D} (0, 0)]}
\]

where \(C_z (0, 0), z = 1, 0\) are constants of integration.

Since the boundary conditions imply \(F_z (p (0, 0); 0, 0) = 0\), for \(z = 0, 1\), and \(F_0 (\overline{p} (0, 0); 0, 0) = 1\), explicit solutions for \(p (0, 0), \overline{p} (0, 0), F_0 (p; 0, 0)\) and \(F_1 (p; 0, 0)\), and the continuation values, \(V_{1D} (0, 0)\) and \(V_{0D} (0, 0)\), can be computed. Note that the expressions \(F_1 (p; 0, 0)\) and \(F_0 (p; 0, 0)\) provided in the statement of Proposition 2 with \(\hat{p}\) replaced by \(P\) are proper probability measures satisfying \((A1) - (A3)\).

Retailers’ continuation values as a function of their customer base are,

\[
V_{1D} (0, 0) = \frac{1}{1 - \delta} \pi^D (p (0, 0), P; 1, 0)
\]

\[
V_{0D} (0, 0) = \pi^D (p (0, 0), P; 1, 1) + \delta V_{1D} (0, 0)
\]

Last, both producers make equal profits. For \(z = 0, 1\),

\[
V_{zU} (0, 0) = \frac{\pi^U (0, 0)}{1 - \delta}
\]

\[
= \frac{\theta P - c}{2 (1 - \delta)} > 0
\]

Hence,

\[
V_{1U+D} (0, 0) = \frac{1}{1 - \delta} [\pi^D (p (0, 0), P; 1, 1) + \pi^U (0, 0)]
\]

\[
V_{1U+D} (0, 0) = \pi^D (p (0, 0), P; 1, 1) + \pi^U (0, 0) - \frac{\delta}{1 - \delta} [\pi^D (p (0, 0), P; 1, 1) + \pi^U (0, 0)]
\]
8.2.3 Proof of Proposition 6: Downstream Equilibrium under PI

On \( p \in \left[ p(0,1), \bar{p}(0,1) \right] = \left[ p(1,0), \bar{p}(1,0) \right] \), strategies must satisfy the following differential equations:

\[
[1 - F_1 (p; 1, 0)] \frac{\theta}{2} = f_1 (p; 1, 0) \left[ \pi_A^D \left( p, P; \frac{1}{2}, \frac{1}{2} \right) + \delta \left[ V_1^D (0, 1) - V_0^D (0, 1) \right] \right]
\]

\[
[1 - F_0 (p; 0, 1)] \frac{\theta}{2} = f_0 (p; 0, 1) \left[ \pi_A^D \left( p, P; \frac{1}{2}, \frac{1}{2} \right) + \delta \left[ V_1^{U+D} (1, 0) - V_0^{U+D} (1, 0) \right] \right]
\]

\[
[1 - F_1 (p; 0, 1)] \frac{\theta}{2} = f_1 (p; 0, 1) \left[ \pi_A^D \left( p, P; \frac{1}{2}, \frac{1}{2} \right) + \pi_U^U \left( \frac{1}{2}, 0 \right) - \pi_A^U (0, 0) \right]
\]

\[
\frac{f_1 (p; 0, 1) \delta \left[ V_1^{U+D} (1, 0) - V_0^{U+D} (1, 0) \right] + f_0 (p; 0, 1) \delta \left[ V_1^D (0, 1) - V_0^D (0, 1) \right]}
\]

where \( f_z (p; \mathbf{I}) = F_z' (p; \mathbf{I}) \), \( z = 1, 0 \) and \( \mathbf{I} \in \{(1,0), (0,1)\} \). The above expressions have solutions:

\[
F_1 (p; 1, 0) = 1 - \frac{C_1 (1, 0)}{\pi_A^D \left( p, P; \frac{1}{2}, \frac{1}{2} \right) + \delta \left[ V_1^D (0, 1) - V_0^D (0, 1) \right]}
\]

\[
F_0 (p; 0, 1) = 2 - \frac{C_0 (0, 1)}{\pi_A^D \left( p, P; \frac{1}{2}, \frac{1}{2} \right) + \delta \left[ V_1^{U+D} (1, 0) - V_0^{U+D} (1, 0) \right]}
\]

\[
F_1 (p; 0, 1) = 1 - \frac{C_1 (0, 1)}{\pi_A^D \left( p, P; \frac{1}{2}, \frac{1}{2} \right) + \pi_U^U \left( \frac{1}{2}, 0 \right) - \pi_A^U (0, 0) + \delta \left[ V_1^{U+D} (1, 0) - V_0^{U+D} (1, 0) \right]}
\]

\[
F_0 (p; 0, 1) = 2 - \frac{C_0 (1, 0)}{\pi_A^D \left( p, P; \frac{1}{2}, \frac{1}{2} \right) + \delta \left[ V_1^D (0, 1) - V_0^D (0, 1) \right]}
\]

where \( C_z (\mathbf{I}) \), for \( z = 1, 0 \) and \( \mathbf{I} \in \{(1,0), (0,1)\} \), are constants of integration.

Since the boundary conditions imply \( F_z \left( p(\mathbf{I}); \mathbf{I} \right) = 0 \), \( z = 0, 1, \) and \( F_0 \left( \bar{p}(\mathbf{I}); \mathbf{I} \right) = 1 \), explicit solutions for \( p(\mathbf{I}), \bar{p}(\mathbf{I}), F_0 (p; \mathbf{I}), F_1 (p; \mathbf{I}), V_1 (\mathbf{I}) \) and \( V_0 (\mathbf{I}) \), for \( \mathbf{I} \in \{(1,0), (0,1)\} \), can be readily computed. Note that \( F_1 (p; \mathbf{I}) \) and \( F_0 (p; \mathbf{I}) \) are proper probability measures satisfying \((A1) - (A4)\) and also that \( V_1 (\mathbf{I}) \) and \( V_0 (\mathbf{I}) \) satisfy \((A5) \), \( \mathbf{I} \in \{(1,0), (0,1)\} \).
\[ V_{1}^{U+D}(1,0) = \frac{1}{1-\delta} \left[ \pi^{D} (p(1,0), P; 1,0) + \pi^{U} (1,0) \right] \] (8)

\[ V_{0}^{U+D}(1,0) = \pi^{D} \left( p(0,1), P; \frac{1}{2}, \frac{1}{2} \right) + \pi^{U} \left( \frac{1}{2}, 0 \right) + \delta V_{1}^{U+D}(1,0) \] (9)

Also note that
\[ V_{1}^{D}(0,1) > V_{0}^{U}(0,1) = \frac{\pi^{U}(0,1)}{1-\delta} \]

So that,
\[ V_{1}^{U+D}(0,1) > \frac{1}{1-\delta} \left[ \pi^{D} (p(0,1), P; 1,0) + \pi^{U} (0,1) \right] \] (10)

\[ V_{0}^{U+D}(0,1) = \pi^{D} \left( p(1,0), P; \frac{1}{2}, \frac{1}{2} \right) + \frac{\delta}{1-\delta} \pi^{D} \left( p(0,1), P; 1,0 \right) + \frac{1}{1-\delta} \pi^{U} (0,1) \] (11)

8.3 Proof of Lemma 2: The Gains from Vertical Integration

(i) The gains from vertical integration when the rivals stand alone.

Using (8) and (6),
\[ G_{1}(0) = V_{1}^{U+D}(1,0) - V_{1}^{U+D}(0,0) \]
\[ = \frac{1}{1-\delta} \left[ \pi^{U}(1,0) - \pi^{U}(0,0) \right] \]

Using (9) and (7),
\[ G_{0}(0) = V_{0}^{U+D}(1,0) - V_{0}^{U+D}(0,0) \]
\[ = \pi^{U}(1,0) - \pi^{U}(0,0) + \delta G_{1}(0) \]
\[ = G_{1}(0) \]

Further, note that \( G_{z}(0), z = 0, 1, \) is positive and independent of \( s, \)
\[ G_{1}(0) = \frac{\pi^{U}(1,0) - \pi^{U}(0,0)}{1-\delta} \]
\[ = \frac{1}{1-\delta} \frac{\pi^{U}(1,0) - \pi^{U}(0,1)}{2} \]
\[ = \frac{P - \hat{p} \theta}{1-\delta 2} > 0 \]
(ii) The gains from vertical integration when the rivals are vertically integrated.

\[ G_0 (1) = V_0^{U+D} (1, 1) - V_0^{U+D} (0, 1) \]
\[ G_1 (1) = V_1^{U+D} (1, 1) - V_1^{U+D} (0, 1) \]

First, it will be useful to note that

\[ p (0, 1) - p (1, 1) \in \left[ [P - \hat{p}] \frac{1 + \delta}{2 + \delta}, P - \hat{p} \right] \]

and that it continuous and decreasing in \( s \in \left[ [v - P] \frac{1 + \delta}{2 + \delta}, [v - \hat{p}] \frac{1 + \delta}{2 + \delta} \right] \).

\[ p (0, 1) - p (1, 1) = \begin{cases} P - \hat{p} & \text{if } s \leq [v - P] \frac{1 + \delta}{2 + \delta} \\ \frac{(1+\delta)P + v}{2 + \delta} - \hat{p} - s \frac{1 + \delta}{2 + \delta} & \text{if } [v - P] \frac{1 + \delta}{2 + \delta} \leq s \leq [v - \hat{p}] \frac{1 + \delta}{2 + \delta} \\ P - \hat{p} - \frac{1 + \delta}{2 + \delta} & \text{if } s \geq [v - \hat{p}] \frac{1 + \delta}{2 + \delta} \end{cases} \]

Using (5) and (11), and some algebra,

\[ G_0 (1) = \left[ [P - \hat{p}] - [p (1, 0) - p (1, 1)] \right] \theta \left[ \frac{1}{2} + \frac{\delta}{1 - \delta} \right] \]

Note that if \( s \leq [v - P] \frac{1 + \delta}{2 + \delta} \), \( G_0 (1) = 0 \). Otherwise, \( G_0 (1) > 0 \) and it is increasing in \( s \) up to \( s = [v - \hat{p}] \frac{1 + \delta}{2 + \delta} \).

Comparing the gains from vertical integration when the rivals stand alone and when they are vertically integrated,

\[ G_0 (0) - G_0 (1) = -\delta \frac{P - \hat{p}}{1 - \delta} + \left[ [p (1, 0) - p (1, 1)] \right] \theta \left[ \frac{1}{2} + \frac{\delta}{1 - \delta} \right] \]

If \( s \geq [v - \hat{p}] \frac{1 + \delta}{2 + \delta} \),

\[ G_0 (0) - G_0 (1) = \theta \frac{[P - \hat{p}]}{2(2 + \delta)(1 - \delta)} = \frac{G_0 (0)}{2 + \delta} > 0 \]

Otherwise, if \( s < [v - \hat{p}] \frac{1 + \delta}{2 + \delta} \), given that \( G_0 (1) \) is increasing in \( s \),

\[ G_0 (0) - G_0 (1) > \frac{G_0 (0)}{2 + \delta} > 0 \]

Let us verify the same result for the retailer with a positive customer base.

\[ G_1 (1) < \frac{1}{1 - \delta} \left[ \pi^D \left( p (1, 1), P; 1, 0 \right) - \pi^D \left( p (0, 1), P; 1, 0 \right) + \pi^U \left( 1, 0 \right) - \pi^U \left( 0, 1 \right) \right] \]
\[ < \frac{\theta}{1 - \delta} \left[ [P - \hat{p}] - [p (1, 0) - p (1, 1)] \right] \]
Note that if $s \leq [v - P] \frac{1 + \delta}{2 + \delta}$, $G_1 (1) < 0$.

Comparing the gains from vertical integration across retailers when the rivals are vertically integrated,

$$G_0 (1) - G_1 (1) > \left( \frac{1 + \delta}{2 + \delta} - \frac{\theta}{1 + \delta} \right) \left( [P - \hat{p}] - [p(1, 0) - p(1, 1)] \right)$$

$$> -\frac{\theta}{2} \left( [P - \hat{p}] - [p(1, 0) - p(1, 1)] \right)$$

If $s < [v - \hat{p}] \frac{1 + \delta}{2 + \delta}$, given that $G_0 (1)$ is increasing in $s$,

$$G_0 (0) - G_0 (1) > \frac{G_0 (0)}{2 + \delta} > 0$$

References


