Advertising and Prices as Signals of Quality: 
Competing Against a Renown Brand 

Francesca Barigozzi and Paolo G. Garella  
University of Bologna  

Martin Peitz  
International University in Germany  

March 2005  
Preliminary draft  

Abstract 

We analyze the problem faced by a firm that has to signal quality of its product in a market where it competes against a rival selling a branded good. Firms know qualities perfectly, while consumers only know the quality of the branded good. While the literature has stressed the role of prices as signals, absent rivalry and multiple signal senders, we show that in our context advertising can be a more powerful signal than price, while price signals may fail. The model we use is just one step away from monopoly and yet it obtains drastically different results. Since advertising is typically an oligopoly phenomenon, this suggests that prudence should be exerted in attempting to generalize conclusions derived from one-firm models about the role of prices and advertising as signals.  

Keywords: quality, prices and advertising, signaling, competition.  

1 Introduction 

A well known explanation of advertising as a rational phenomenon is based on Nelson’s (1976) idea that advertising cost works as a device to signal high
quality of a brand producing experience goods. The signaling motive filled a gap in understanding a controversial economic phenomenon, namely the apparently wasteful advertising campaigns. The argument for informative advertising is based on the idea that the cost, and not the content, of an ad is what really matters. However, as analytically shown by Milgrom and Roberts (1986), advertising is neither necessary to signal quality nor cheaper than alternative signalling devices such as a high price - unless repeat purchase is assumed. Thus, in the pure monopoly case, a price signal with no advertising always provides the best signaling strategy for the high quality monopolist. In this sense, economic theory has proved that, in the monopoly case, the role of advertising as a signal of product quality is limited in scope. Moreover, Nelson’s conjecture suggests a positive correlation between quality and advertising expenditures, which empirical tests have not been able to decisively support.

More recently Fluet and Garella (2001) showed that things are different in the duopoly case. The role for advertising signaling is here more important. In fact, when there is price rivalry, advertising may be necessary to signal quality. In their model two firms, whose product quality is unknown to consumers, compete on the same market. None of these two firms has an established reputation; therefore, consumers are confronted with the problem of interpreting the price-advertising strategies of both firms at the same time. The authors show that, even if signaling through price alone prevails for sufficient inter-brand quality differentials, joint price and advertising signals prevail when the quality differential is small.

For what briefly stated above, the extent of competition in the market is crucial in explaining the role of advertising and price as signals of product quality. Building on this consideration, our model describes an intermediate setting between the monopoly case in Milgrom and Roberts (1986) and the symmetric duopoly analyzed in Fluet and Garella (2001). We consider two firms competing in a market, where one firm has an established reputation such that the quality of its product is known to consumers. We show that price signaling emerges as an equilibrium under certain conditions on the parameters. Furthermore, and more importantly, when these conditions are not met, signalling with price and advertising may arise thus proving that price and advertising together are a more powerful and effective signal of

---

1The idea is that consumers who bought the product once and experienced high quality, will probably buy the product also in the future.
quality than price alone. Indeed, even though the model we use is just one step away from monopoly, it obtains drastically different results on the role of advertising in signaling quality.

In our model, firm 1 and firm 2 compete in price in a market for a differentiated good. The quality of firm 1’s product is known to be high, while the quality characterizing firm 2’s product is unknown to consumers but is observable by the two firms.\footnote{One may think of firm 2 as of a firm that has the opportunity to upgrade its product above the level that consumers have recognized so far. Alternatively firm 2 can be considered an entrant in a market where firm 1 is the incumbent.} Such a quality can be either high or low and the two firms compete in price. The profits of firm 1 are decreasing while the profits of firm 2 are increasing in the quality of firm 2 as perceived by consumers. Thus, a low quality firm 2 has interest in mimicking high quality and, vice versa, firm 1 has always incentive to signal a low quality of firm 2. Prices of both firms have the potential to signal quality to consumers and, in addition, firm 2 can also use dissipative advertising.\footnote{Here firm 1’s quality is known. Since the aim of the paper is to verify whether and when price and advertising are an efficient way to signal high quality, we do not consider the case where firm 1 uses advertising as well. In fact, in this model, advertising from firm 1 would correspond to a sort of counter-advertising.} We also introduce a pre-play communication stage to make the analysis of separating Bayesian Equilibria simpler: firm 2 must announce its type before the two prices are set. Such a claim is completely costless and can be thought of as a cheap-talk message sent to consumers. Consumers observe the firms’ strategies (the claim by firm 2, the two prices and, eventually, advertising by firm 2), update their priors on firm 2’s quality and make their purchase decisions. Moreover, firm 1 can use a limit price which reduces the signaling possibility of firm 2: firm 2 cannot signal high quality with a price larger than a (endogenous) threshold value because firm 1 would react setting a (low) price which attracts all the demand.

Focusing on the separating Bayesian Equilibria in which consumers are able to identify the quality of firm 2 and in the case price alone is used, we show that consumers update their priors on quality only on the base of firm 1’s price when quality is low and only on the base of firm 2’s price when quality is high. Thus, an interesting feature of the separating equilibria of the game is that consumers’ inference of firm 2’s quality depends only on one price at each time. Analyzing necessary conditions for the separating equilibria we characterize cases where signaling with price alone and signaling with price
and advertising are respectively admissible. Then, we derive and compare the profits of the two firms in both cases.

The structure of the paper is as follows. In section 2 the model is presented and the equilibrium under full information is derived. In section 3 price signaling is analyzed. First, a general characterization of separating equilibria is provided and conditions for their existence are discussed. Second, necessary conditions for separation are considered and calculated in a particular (and treatable) case with pre-play communication. In section 4 advertising is introduced. The same analysis of separating equilibria performed in section 3 is here undertaken with price and advertising signaling. Finally, section 5 concludes.

2 The model

In a market for a differentiated product a firm, called firm 2, is faced with a renown brand rival, firm 1. One may think of firm 2 as of a firm that has the opportunity to upgrade its product above the level that consumers have recognized so far. This situation can be represented by assuming that the quality level of firm 1 is high, denoted $\theta_H$, and common knowledge, while the quality of firm 2 is known only to the firms. Such a quality can be high, namely $\theta_H$, or low, namely $\theta_L$. The prior distribution of the random variable $\theta$, representing the quality of firm 2 is common knowledge too. This assumption, about firms being perfectly informed, is a simplified version of the assumption that firms have better information than consumers.

Summarizing the notation, $\theta_1 = \theta_H$ is the quality of the incumbent (common knowledge), while $\theta_2 \in \{\theta_L, \theta_H\}$ is the quality of the entrant (known only to the two firms). Furthermore $\Delta \theta = \theta_H - \theta_L$ denotes the quality difference. We assume that it is common knowledge that the entrant’s quality results from a random mechanism and it is high with probability $\alpha$ or low with probability $1 - \alpha$. The unit cost of low quality is zero and the unit cost of high quality is $c > 0$, as better quality is obtained with a more sophisticated production process. We write therefore $C(q_2, \theta_L) = c(\theta_L)q_2 = 0$ and $C(q_2, \theta_H) = c(\theta_H)q_2 = cq_2^\prime$, where $q_2$ is the quantity produced.

Consumers have addresses in $[0, 1]$ and are uniformly distributed, with mass $N$, as in the Hotelling linear city model of horizontal differentiation. However products here also have a vertical quality dimension. Firm 1 is located at $x = 0$ and firm-2 at $x = 1$. “Transportation costs” $t$ are linear.
Firms learn the value of quality $\theta_2$ and compete in prices. A consumer buys either one or zero units of the good.

Let $x$ and $z$ be two points in $[0, 1]$, then let $d(x, z)$ represent the euclidean distance between points $x$ and $z$. The full information utility of consumer with address $x$ in $[0, 1]$ of buying a unit of good $i = 1, 2$ is represented as follows:

$$u_i(x, \theta_i) = v + \theta_i - t d(x, i - 1) - p_i,$$  \hspace{1cm} (1)

Where the parameter $v > 0$ is assumed to be so high as to guarantee that all consumers buy at a price equilibrium.

### 2.1 The solution under full information

We first summarize the features of a Nash equilibrium under full information when $\theta_2 = \theta_L$. The indifferent consumer’s address is

$$\hat{x}(p_1, p_2) = \frac{1}{2} + \frac{(p_2 - p_1) + \Delta \theta}{2t},$$  \hspace{1cm} (2)

when it is in $(0, 1)$. If $\hat{x}(p_1, p_2) \geq 1$ then all market is served by firm 1, and when $\hat{x}(p_1, p_2) \leq 0$ then all market is served by firm 2. If we exclude these two cases, the conditional demand functions to the two firms under duopoly are given by:

$$D_1(p_1, p_2, \theta_L) = \frac{t + (p_2 - p_1) + \Delta \theta}{2t} N,$$

$$D_2(p_1, p_2, \theta_L) = \frac{t - (p_2 - p_1) - \Delta \theta}{2t} N.$$

The maximization problem for firm 1 is:

$$\max_{p_1} \quad (p_1 - c) \frac{t + (p_2 - p_1) + \Delta \theta}{2t} N$$  \hspace{1cm} (3)

This provides the best reply function for firm 1 at the second stage.

$$p_1(p_2, \theta_L) = \frac{p_2 + \Delta \theta + t + c}{2}.$$  \hspace{1cm} (4)

The maximization problem for firm 2 is:

$$\max_{p_2} \quad p_2 \frac{t - (p_2 - p_1) - \Delta \theta}{2t} N$$  \hspace{1cm} (5)
This provides the best reply function for firm 2 at the second stage.

\[ p_2(p_1, \theta_L) = \frac{p_1 - \Delta \theta + t}{2}. \] (6)

Equilibrium prices under full information are:

\[ p_1^*(\theta_L) = \frac{3t + \Delta \theta + 2c}{3} \text{ and } p_2^*(\theta_L) = \frac{3t - \Delta \theta + c}{3}. \] (7)

Accordingly, \( p_1^*(\theta_L) - p_2^*(\theta_L) > 0 \). When both firms can survive at equilibrium, the equilibrium demand functions are:

\[ D_1(\theta_L) = N \frac{3t + (\Delta \theta + c)}{6t} \text{ and } D_2(\theta_L) = N \frac{3t - (\Delta \theta + c)}{6t} \]

The equilibrium profits are:

\[ \pi_1(\theta_L) = \frac{N}{18t} ((3t + \Delta \theta)^2 - c^2), \text{ and } \pi_2(\theta_L) = \pi_2^L = \frac{N}{18t} ((3t - \Delta \theta)^2 - c^2) \] (8)

The condition for existence of full information equilibrium with non-negative profits for the low quality entrant is:

**Assumption 1:** either \( 3t \geq \Delta \theta + c \) or \( 3t \leq \Delta \theta - c \)

Notice that, when assumption 1 is verified, also the incumbent profits are non-negative. When \( \theta_2 = \theta_H \), by contrast, firms have identical qualities and the best-reply functions are

\[ p_i(p_j, \theta_H) = \frac{p_j + t + c}{2} \] (9)

The full information equilibrium when the entrant is high quality, and therefore costs are symmetric and equal to \( c \), is characterized as follows:

\[ p_1^*(\theta_H) = p_2^*(\theta_H) = t + c, \] (10)

and

\[ \pi_1(\theta_H) = \pi_2(\theta_H) = \frac{N}{2} t \] (11)
2.1.1 Limit price

Suppose that, when $\theta_2 = \theta_L$, firm 1 decides whether to set a limit price which enable it to take all the market. A limit price is such that all consumers prefer to buy from firm 1:

$$v + \theta_H - tx - p_1 \geq v + \theta_L - t(1-x) - p_2 \quad \forall x$$

for $x = 1$ this implies:

$$v + \theta_H - t - p_1 \geq v + \theta_L - p_2$$

or

$$p^L_1(\theta_L) = \Delta \theta + p_2 - t$$

When firm 1 sets price $p^L_1(\theta_L)$, its profits are:

$$\Pi_1(p^L_1, \theta_L) = N(\Delta \theta + p_2 - t - c)$$

Thus, when $\theta_2 = \theta_L$, firm 1 compares its profits under the limit price $p^L_1(\theta_L)$ with its profits under the best reply price $p_1(p_2, \theta_L) = p_2 + \frac{\Delta \theta + t + c}{2}$. The limit price is used if: $p_2 > 3t + c - \Delta \theta$.

In the same way, the limit price for $\theta_2 = \theta_H$ is used if $p_2 > 3t + c$. By inspection it is clear that a Nash price for a low quality firm 2 is always low enough so as not to trigger the use of limit price by firm 1.

2.2 Price signaling

As a foreword, the aim of the analysis is restricted to find a case where the use of advertising at a separating equilibrium is of help for firm 2; either because it increases its profit with respect to a signal composed of a distorted price and zero advertising expenditures, or because it expands the region for signaling, or both. We shall not analyze the separating equilibria of the game, therefore, for all possible parameters configurations, as this is possibly a formidable task.

To simplify our analysis of the necessary conditions for separation, we assume that a pre-play communication occurs: firm 2 must announce its type to consumers before the two prices are set. Firm 2 claim is completely costless.

The new sequence of moves can be summarized as follows:
Stage 1. Nature chooses the type of firm 2. Firms learn firm 2’s product quality.

Stage 2. Firm 2 must claim $\theta'$ with either $\theta' = \theta_H$ or $\theta' = \theta_L$.

Stage 3: Firms simultaneously choose prices.

Stage 4: Consumers observe firm 2 claim and the two prices and form beliefs about the probability of firm 2 being of high quality, denoted $\mu(\theta', p_1, p_2) \equiv \Pr(\theta_2 = \theta_H | \theta', p_1, p_2)$.

Stage 5: Consumers make purchasing decisions.

Note that at stage 2 the obligation to make a claim could be removed and a no-claim option could be introduced to enlarge the strategy space of firm 2.

We shall use the solution concept of Perfect Bayesian Equilibrium (PBE). A PBE of the game is defined as a strategy profile such that, given beliefs and equilibrium strategies of the other players, profits are maximized and consumers’ beliefs are updated according to Bayes’ rule. An equilibrium is said to be separating when firms play pure strategies and consumers beliefs at equilibrium allow them to separate the low from the high quality.

Letting $\hat{p}_i(\theta_s)$ be the pure strategy played by firm $i$ given quality $\theta_s$, for $i = 1, 2$ and $s = L, H$, at a separating equilibrium the vectors $\{\hat{p}_1(\theta_H), \hat{p}_2(\theta_H)\}$ and $\{\hat{p}_1(\theta_L), \hat{p}_2(\theta_L)\}$ must be such that $(\hat{p}_1(\theta_H), \hat{p}_2(\theta_H)) \neq (\hat{p}_1(\theta_L), \hat{p}_2(\theta_L))$. Furthermore, the belief system must assign probabilities $\mu(\hat{p}_1(\theta_H), \hat{p}_2(\theta_H)) = 1$ and $\mu(\hat{p}_1(\theta_L), \hat{p}_2(\theta_L)) = 0$.

Notice that the two firms’ profits depend on the two prices and on consumers’ posterior $\mu(p_1, p_2)$ about firm 2’s quality: $\Pi_i(p_1, p_2; \mu)$.

As we show below, in the model we use, any belief system in which consumers make their assessment of the quality of firm 2 depend also upon the price of firm 1 when $\theta_2 = \theta_H$ and also upon the price of firm 2 when $\theta_2 = \theta_L$ will likely fail.

**Lemma 1.** (i) At any separating equilibrium, when the state is $\theta_H$, along the equilibrium path, firm 1, whose quality is common knowledge, will use the price which maximizes its profit given both $\hat{p}_2(\theta_H)$ and $\mu = 1$. (ii) At any separating equilibrium, when the state is $\theta_L$, along the equilibrium path, firm 2 will use the price which maximizes its profit given both $\hat{p}_1(\theta_L)$ and $\mu = 0$. 

8
Proof In the model firms’ profits are unambiguously affected by consumers’ perception of the entrant’s quality. In particular, for any price vector \((p_1, p_2)\), the profit function for firm 1, \(\Pi_1 (p_1, p_2, \mu)\) is decreasing, while \(\Pi_2 (p_1, p_2, \mu)\) is increasing in \(\mu\).

Suppose \(\theta_2 = \theta_H\) and separation occurs with \((\hat{p}_1(\theta_H), \hat{p}_2(\theta_H))\) and profits \(\Pi_1 (\hat{p}_1(\theta_H), \hat{p}_2(\theta_H), 1)\). Suppose \(\hat{p}_1(\theta_H)\) is such that \(\hat{p}_1(\theta_H) \neq p_1(p_2(\theta_H)) \equiv \arg \max \Pi_1 (p_1, \hat{p}_2(\theta_H), 1)\). Then, firm 1 can use \(p_1 = p_1(p_2(\theta_H))\) in place of \(\hat{p}_1(\theta_H)\) and get profits strictly larger than \(\Pi_1 (\hat{p}_1(\theta_H), \hat{p}_2(\theta_H), 1)\). But this destroys the equilibrium. This proves part (i).

Consider now part (ii). Suppose that when \(\theta_2 = \theta_L\), separation requires prices to equal \((\hat{p}_1(\theta_L), \hat{p}_2(\theta_L))\), with profit for firm 2 equal to \(\Pi_1 (\hat{p}_1(\theta_L), \hat{p}_2(\theta_L), 0)\). Then, by a similar reasoning to that conducive to prove point (i) above, firm 2 will use its best reply at an equilibrium. Q.E.D.

A different issue concerns the out-of-equilibrium beliefs. Suppose \(\theta_2 = \theta_H\) and separation occurs with \((\hat{p}_1(\theta_H), \hat{p}_2(\theta_H))\) and profits \(\Pi_1 (\hat{p}_1(\theta_H), \hat{p}_2(\theta_H), 1)\). Suppose \(\hat{p}_1(\theta_H)\) is such that \(\hat{p}_1(\theta_H) = p_1(p_2(\theta_H)) \equiv \arg \max \Pi_1 (p_1, \hat{p}_2(\theta_H), 1)\), as according to Lemma 1. Then, a necessary condition for a deviation by firm 1 to be profitable is that it modifies beliefs such that \(\mu (p_1, \hat{p}_2(\theta_H)) < 1\).

A belief system that sustains separation and relying upon the price of firm 1 and that of firm 2, and not on that of firm 2 only, therefore, would require that such deviations be not profitable. Any such system, however, could be replaced by a simpler system where \(\mu (p_1, \hat{p}_2(\theta_H)) = 1\) for all \(p_1\) (not relying on information provided by the incumbent). In fact, given part (i) of the Lemma, \(\hat{p}_1(\theta_H)\) is firm 1’s full information best reply to \(\hat{p}_2(\theta_H)\); the no deviation by firm 1 condition then is simpler to be met if \(\mu (p_1, \hat{p}_2(\theta_H)) = 1\).

In a sense, any belief system relying upon the price pair \((p_1, p_2)\) in state \(\theta_H\) makes redundant use of information. This allows us to restrain our discussion to separating equilibria with belief systems such that in state \(\theta_H\) one has \(\mu (p_1, \hat{p}_2(\theta_H)) = 1\) for all \(p_1\). This is tantamount saying that the burden of separation is on firm 2 when \(\theta_2 = \theta_H\).

Similarly, one can show that, when \(\theta_2 = \theta_L\), any separating equilibrium with a belief system such that \(\mu (\hat{p}_1(\theta_L), p_2) = 0\) for all \(p_2\) can be used to replace any other belief system supporing separation, given part (ii) of Lemma 1. This allows us to restrain our discussion to separating equilibria with belief systems such that in state \(\theta_L\) one has \(\mu (\hat{p}_1(\theta_L), p_2) = 0\) for all \(p_2\). This is tantamount saying that the burden of separation is on firm 1 when \(\theta_2 = \theta_L\).

As it was specified before, the aim of this model is to verify whether and
how advertising signaling can make firm 2 better off when price is already used as a signal of the firm’s quality. Firm 1 signaling role when the state is \( \theta_2 = \theta_L \) is of a lower concern for us. For this reason in the following we shall consider only separating equilibria in which, when firm 2 is characterized by low quality, firm 1 uses its full-information price.

**Definition 1: restricted separating PBE.** The separating PBE we consider are such that when \( \theta_2 = \theta_L \), then \( \hat{p}_1(\theta_L) = p_1^*(\theta_L) \) and \( \hat{p}_2(\theta_L) = p_2^*(\theta_L) \).

Definition 1 describes a subset of the separating equilibria considered in Corollary 1. In the separating PBE we are going to analyze, firm 1 uses its full information price when quality of firm 2 is low, and, as a consequence, firm 2 also plays the full-information price when its quality is low. Our restriction about the behavior of firm 1, therefore, only implies that firm 1 uses \( \hat{p}_1(\theta_L) = p_1^*(\theta_L) \) instead of any other generic price when the state is \( \theta_L \). No other loss of generality is implied, while consumers’ beliefs at equilibrium appear to be much more natural than they would be without this restriction.\(^4\)

As for the obligation to make a claim for firm 2, notice that if a no-claim option was available too, a no-claim behavior would be interpreted as a low quality claim, so, for simplicity, let us proceed under the description of the game as given above. In accordance to this last argument, we assume that \( \mu(\theta' = \theta_H, p_1, p_2) = 0 \) : a low-quality claim is always taken as true by consumers, whatever the prices are. Of course, one can construct separating equilibria where \( \mu(\theta' = \theta_H, p_1, p_2) \) is not identically zero, but, similarly to the arguments used in the proof of Lemma 1 above, in these separating equilibria consumers would make use of price information where they could instead make use of information about \( \theta' \) only. Of course, the pricing by firms could result in different equilibrium behavior, with different beliefs, but it is hard to justify why consumers would coordinate on more complicated beliefs than those with \( \mu(\theta' = \theta_H, p_1, p_2) = 0 \). In any case there is one circumstance where the equilibrium beliefs must be such that \( \mu(\theta' = \theta_H, p_1, p_2) = 0 \) for all \((p_1, p_2)\) pairs. This is when a low quality firm makes zero profit at a full information equilibrium. Then, by forward induction, any equilibrium requiring prices to differ from the full information prices is destroyed: why

\(^4\)A simple way to rationalize the use of the full information prices when \( \theta_2 = \theta_L \), is by relating these prices to experience by consumers in the duopoly market before the possible claim by firm 2 that its product be upgraded.
should firm 2 ever wanted to choose $\theta' = \theta_2$ when this is not true, if by this it gets zero profits?

Claim: Any weak Bayesian Perfect equilibrium with a belief system such that $\mu(\theta' = \theta, p_1, p_2) > 0$ for at least one $(p_1, p_2)$ price pair can be eliminated by the use of forward induction (it is not Kohlberg-Mertens-stable), provided the payoff of a high quality entrant $\Pi_2(\theta, \hat{p}_1(\theta), \hat{p}_2(\theta))$ is larger than that of a low quality entrant, namely $\Pi_2(\theta, \hat{p}_1(\theta), \hat{p}_2(\theta)) > \Pi_2(\theta, \hat{p}_1(\theta), \hat{p}_2(\theta))$.

Proof: Intuitive, by extension of the argument for the case $\Pi_2(\theta, \hat{p}_1(\theta), \hat{p}_2(\theta)) = 0$; to be proved formally.

According to definition 1, at a separating equilibrium where pre-play communication is added, when $\theta_2 = \theta_H$:

- $\theta' (\theta_H) = \theta_H$ and the incumbent uses its strategy denoted $p_1(\hat{p}_2(\theta_H)) = \hat{p}_1(\theta_H) = \frac{p_2 + t + c}{2}$, provided $\hat{p}_2(\theta_H) < 3t + c$ (see lemma 1 and the definition of limit price in subsection 2.1.1).

- the separating constraint for a high-quality firm 2 is satisfied. That is, firm 2 when $\theta_2 = \theta_H$ prefers to use the $\hat{p}_2(\theta_H)$, equilibrium price of a high quality, rather than to price according to its best reply as a low quality against $\hat{p}_1(\theta_H)$. Namely, it must prefer to play $\hat{p}_2(\theta_H)$ rather than $p_2^d$, where $p_2^d = \frac{p_1 + t + c + \Delta \theta}{2}$ is the price that maximizes the profits function $(p_2 - c) \left[ \frac{1}{2} - \frac{p_2 - \hat{p}_1(\theta_H) - \Delta \theta}{2t} \right] N$. The condition can thus be rewritten as:

$$\hat{p}_2(\theta_H) - c \left( \frac{1}{2} - \frac{\hat{p}_2(\theta_H) - \hat{p}_1(\theta_H)}{2t} \right) N \geq (p_2^d - c) \left( \frac{1}{2} - \frac{p_2^d - \hat{p}_1(\theta_H) - \Delta \theta}{2t} \right) N$$

By substituting $\hat{p}_1(\theta_H)$ and $p_2^d$, in we find:

$$\left( \frac{13t - p_2 + c}{4} \right) (p_2 - c) \geq \frac{1}{32} (p_2 + 3t - c + 2\Delta \theta) \frac{3t + p_2 - c - 6\Delta \theta}{t}$$

(12)

when $\theta_2 = \theta_L$:
• If $\theta' (\theta_L) = \theta_H$, the incumbent uses its strategy denoted $p_1(\hat{p}_2(\theta_H)) = \hat{p}_1(\theta_H)$, provided $\hat{p}_2(\theta_H) < 3t + c − \Delta \theta$ (see lemma 1 and the definition of limit price in subsection 2.1.1).

• Since $\mu(\theta' = \theta_H, p_1, p_2) = 0$, to analyze the no-mimicking condition for low quality firm 2 we must consider a false claim in stage 2. In other words, when $\theta_2 = \theta_L$ and firm 2 cheats, it must be $\theta'(\theta_L) = \theta_H$. The no-mimicking condition implies that price $\hat{p}_2(\theta_H)$ must be such that

\[
\hat{p}_2(\theta_H) \left( \frac{t - (\hat{p}_2(\theta_H) - \hat{p}_1(\theta_H))}{2t} \right) N \leq \frac{N}{18t} ( (3t - \Delta \theta)^2 - c^2 ) \quad (13)
\]

Inequality (13) can be rewritten as:

\[
\hat{p}_2(\theta_H) \left( \frac{t - (\hat{p}_2(\theta_H) - \hat{p}_2(\theta_H) + t + c)}{2t} \right) N \leq \frac{N}{18t} ( (3t - \Delta \theta)^2 - c^2 )
\]

Solving the previous inequality we find that $\hat{p}_2(\theta_H)$ must be either higher than $\frac{3}{2}t + \frac{1}{2}c + \frac{1}{6}\sqrt{B}$ or lower than $\frac{3}{2}t + \frac{1}{2}c - \frac{1}{6}\sqrt{B}$ where

\[
B = 9t^2 + 54tc + 17c^2 + 48t\Delta \theta - 8(\Delta \theta)^2
\]

Therefore, signaling through a high price implies $\hat{p}_2(\theta_H) > \frac{3}{2}t + \frac{1}{2}c + \frac{1}{6}\sqrt{B}$.

Recalling the limit price and rearranging, the following lemma holds:

**Lemma 3**: if $\sqrt{B} < 9t + 3c - 6\Delta \theta$, signaling is possible with a high price.

Signaling through a low price, on the other hand, implies $\hat{p}_2(\theta_H) < \frac{3}{2}t + \frac{1}{2}c - \frac{1}{6}\sqrt{B}$, this is impossible if $\hat{p}_2(\theta_H)$ does not cover the high quality cost $c : \frac{3}{2}t + \frac{1}{2}c - \frac{1}{6}\sqrt{B} < c$. Rearranging:

**Lemma 4**: if $\sqrt{B} < 9t - 3c$, signaling is possible with a low price.

In other words, if the two inequalities:

\[
\sqrt{B} > 9t + 3c - 6\Delta \theta \quad (14)
\]
\[
\sqrt{B} > 9t - 3c \quad (15)
\]

hold simultaneously, no price signaling is viable. Thus, when the LHS of (14) and (15) is higher than $\max[(9t - 3c), (9t + 3c - 6\Delta \theta)]$ no signaling with price alone is possible.
Proposition 1 if $\Delta \theta > c$ holds, then (15) is sufficient to eliminate price signaling. Viceversa, if $\Delta \theta < c$, then (14) is sufficient to eliminate price signaling.

Proposition 1 shows conditions on the parameters of the model such that firm 2 cannot signal its quality with price alone. We will prove in the next section that, when advertising is used as a signal together with price, the conditions for signaling are less stringent than when advertising is not used.

3 Price and advertising signaling

In this section we analyze how price signaling changes when dissipative advertising is used together with price. Applying again lemma 1, firm 1 will not use any advertising expenditure at a separating equilibrium when $\theta_2 = \theta_H$. Only firm 2 eventually uses a positive amount of advertising expenditure when its quality is high. As before (see definition 1) we are interested in separating equilibria where firm 1 always uses its best reply function and firm 2 plays the full-information price when its quality is low:

Definition 2: restricted separating PBE with advertising. The separating PBE with advertising we consider are such that:

- when $\theta_2 = \theta_H$, then $\hat{p}_{1}^{A}(\theta_H) \in \arg \max_{p_1} \Pi_1(p_1, \hat{p}_{2}^{A}(\theta_H); \theta_H)$ and $\hat{A}_2 > 0$
- when $\theta_2 = \theta_L$, then $\hat{p}_{1}^{A}(\theta_L) = p_{1}^{*}(\theta_L)$ and $\hat{p}_{2}^{A}(\theta_L) = p_{2}^{*}(\theta_L)$.

3.0.1 Existence of a separating equilibrium with prices and advertising

Given that only firm 2 possibly uses advertising expenditure at any separating equilibrium, in the following $\hat{A}$ will indicate $\hat{A}_2$. Condition (13) can be rewritten as:

$$
\hat{p}_{1}^{A}(\theta_H) \left( t - \left( \hat{p}_{1}^{A}(\theta_H) - \hat{p}_{1}^{A}(\theta_H) + t + c \right) \right) - A \leq \frac{N}{18t} \left( (3t - \Delta \theta)^2 - c^2 \right)
$$
It can be checked that price $\hat{p}^A_1(\theta_H)$ must be such that either $\hat{p}^A_1(\theta_H) < \frac{1}{18N}(27Nt + 9Nc - 3\sqrt{B_A})$ or $\hat{p}^A_1(\theta_H) > \frac{1}{18N}(27Nt + 9Nc + 3\sqrt{B_A})$, where

$$B_A = 9N^2t^2 + 54N^2tc + 17N^2c^2 + 48N^2t\Delta\theta - 8N^2(\Delta\theta)^2 - 144NAt$$

Rearranging the previous inequalities we find that if $\hat{p}^A_1(\theta_H) < \frac{3t+c-\frac{1}{6}\sqrt{B'_A}}{2}$, signaling is eventually possible with a low price and costly advertising. If $\hat{p}^A_1(\theta_H) > \frac{3t+c+\frac{1}{6}\sqrt{B'_A}}{2}$, signaling is eventually possible with a high price and costly advertising; where

$$B'_A = 9t^2 + 54tc + 17c^2 + 48t\Delta\theta - 8(\Delta\theta)^2 - 144At$$

Now we must complete our conditions by considering the limit price eventually set by firm 1 and the cost of high quality. Rearranging the previous expressions, the following lemmas hold:

**Lemma 5**: signaling is possible with a high price and costly advertising if $\sqrt{B'_A} < 9t + 3c - 6\Delta\theta$.

**Lemma 6**: signaling is possible with a low price and costly advertising if $\sqrt{B'_A} < 9t - 3c$.

As before, if the two inequalities:

$$\sqrt{B'_A} > 9t + 3c - 6\Delta\theta$$
$$\sqrt{B'_A} > 9t - 3c$$

hold simultaneously, no price signaling with dissipative advertising is viable. Moreover:

**Proposition 2**: if $\Delta\theta > c$ holds, then (16) is sufficient to eliminate signaling with price and dissipative advertising. Viceversa, if $\Delta\theta < c$, then (17) is sufficient to eliminate signaling with price and dissipative advertising.

By comparing the two expressions $B$ and $B'_A$ we can state the following proposition:

**Proposition 3**: a small amount of dissipative advertising, joined with a low price, enlarges the region of parameters where signaling is possible.

The previous proposition proves our main result: adding advertising allows signals of quality which are impossible with price alone.
3.1 Comparative versus generic dissipative advertising

Profits in case generic advertising is used are

\[(p^+(A) - c) \left( \frac{1}{4} 3t - p^+(T) + c \right) - A\]

When comparative advertising is used, let \( T = A + \mu F \), where \( F \) is a fine for cheating incurred when a court intervenes and \( \mu \) is a probability of being find lying when effectively lying. Suppose the same probability is zero for an entrant when truth-telling.

Condition for separation with comparative advertising is

\[\frac{1}{4} p_2 (3t - p_2 + c) \frac{N}{t} - T \leq \frac{N}{18t} (3t - \Delta)^2 - c^2\]

**Proposition**: \( p^+(T) < p^+(A) \).
Proof: \( p^+(x) \) is decreasing in its argument.

**Corollary**: The high quality entrant can get a higher market share, and a higher profit, by using comparative rather than generic advertising as a signal of quality.
Proof: Equilibrium demand is

\[\frac{1}{4} 3t - p^+(x) + c \]

therefore, since \( p^+(T) < p^+(A) \) the result on market share follows. As for profits, the cost to a high quality entrant of comparative advertising is only \( A \), so that its profits at a separating equilibrium are

\[(p^+(T) - c) \left( \frac{1}{4} 3t - p^+(T) + c \right) - A.\]

Since both, \( p^+(T) \) and \( p^+(A) \) are distorted upwards, a high quality entrant finds them larger than its best reply against its opponent’s price (given beliefs that both firms are high quality). Therefore, the least distorted price, \( p^+(T) \) also allows higher profits to the high quality entrant.
4 Conclusion

The results obtained for the use of advertising contrast with the pure monopoly case where, unless advertising has a direct effect on demand (and therefore is not purely dissipative), a price signal with zero advertising always provides the best signalling strategy for the high quality monopolist. The framework in which we have cast our analysis is the one that more closely resembles the framework with a single informed monopolist: (i) only two firms are on the market, (ii) only the quality of one of the two is chosen by Nature and it is private information of the firms. The distinctive features of the model, therefore are that price interaction occurs even under full information, and that two instead of just one player has private information.

While the idea that prices reveal the quality of firms is often given for granted, we have shown that price signaling is possible only if some restrictions on the parameters are met. When these restrictions are not met there are cases where adding advertising allows for an otherwise impossible signal of quality (see also Fluet and Garella (2001). In general, as it appears from our discussion, in a duopoly the construction of equilibria based upon price signals is a complex matter even for the professional theorist. One must in particular elaborate excessively upon the effects of the belief system upon the firms’ incentives to use out-of-equilibrium strategies. This also implies that consumers must be highly sophisticated, meaning by this that the required level of understanding and of ability to predict becomes dramatically higher than that required in the monopoly case. Indeed, one may wonder whether another type of justification for the use of advertising, rather than prices, as signals in a situation of duopoly, be that of complexity. If prices are not affecting beliefs, one would obtain much easier separating conditions, based only upon advertising. This argument, however, relies upon personal judgements and is only discussed here informally.

References

