Bidding for the Unemployed: An Application of Mechanism Design to Welfare-to-Work Programs*

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Abstract

This paper applies the theory of mechanism design to welfare-to-work programs. When procuring welfare-to-work projects to employment service providers, governments face the problems of adverse selection (the winning provider is not the most efficient one) and moral hazard (the winning provider shirks in its effort to reintegrate unemployed people). We compare three auctions with the socially optimal mechanism and show that two of these auctions approximate the optimal mechanism if the number of providers is large. Moreover, both auctions are preferable to the optimal mechanism if the government is able to attract only one additional bidder.

Keywords: Auctions; Incentive contracts; Welfare-to-work programs

JEL classification: D44; D82; J68

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1 Introduction

In several countries, the government procures welfare-to-work programs as a part of their active labor market policy. In these procurements, the government allocates welfare-to-work projects to employment service providers. A welfare-to-work project typically consists of a number of unemployed people, and the winning provider is rewarded on the basis of the number of these people that find a job within a specified period of time. The procurements give flesh and blood to Demsetz’ (1968) idea of competition ‘for’ the market.

In the Netherlands in 2002, about 160,000 unemployed people were matched to employment service providers (1% of the total population and 2% of the labor force). The Dutch government transferred roughly 800 million euros to the providers, which comprises around 0.2% of Dutch GDP, and more than 10% of total expenditures on active labor market policy. A natural question that arises is: how can governments optimally spend this money? We will answer this question in this paper.

Before we do this, we should be clear about the governments’ targets in the procurements. We assume that the success of a procurement depends on the following three factors: (1) the number of people that find a job, (2) the payments made from the government to the employment service provider, and (3) the costs born by the employment service provider. The first factor is important as the more people that are employed, the higher production, and hence the higher social welfare. Moreover, unemployment benefits decrease, so that the government has to raise less distortionary taxes. The same holds true for the second factor: lower payments to the provider imply lower distortionary taxes. Finally, the higher the provider’s costs, the lower welfare.

In reaching their targets, governments may be confronted with two types of economic problems: adverse selection and moral hazard. Adverse selection occurs if the procurement does

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not select the ‘best’ employment service provider, i.e., the provider that, relative to all other providers, is able to help the unemployed people back to work in the most cost efficient way. Moral hazard may occur if the winner of the procurement has no incentive to put much effort in the welfare-to-work project.

Most governments that procure welfare-to-work programs use a beauty contest. Providers submit an offer that contains a bid on several pre-specified dimensions. In the Netherlands, some of these dimensions are well-defined (such as the price for a successful placement), others are rather vague (such as ‘experience’). The government signs a contract with the provider submitting the ‘best’ bid. This contract specifies how the government rewards the firm, which is usually both input and output based: the government partly covers the cost of the provider, and in addition rewards the provider for each successful placement. However, it is not clear whether beauty contests are indeed optimal. At least the administrative burden is usually high for these mechanisms: it is time consuming and costly for firms to write an offer and for the government to study and compare these offers.

In this paper, we study mechanisms that are simpler than beauty contests, auctions, and compare these with a socially optimal mechanism. In an auction, several providers submit a bid for a welfare-to-work project, and the winner is chosen according to a well-defined and anonymous allocation rule which is universal in the sense that it does not depend on the details of the project. Several economists claim that auctions perform better than beauty contests as they are more transparent, are less prone to favoritism, and give rise to less administrative burden for both the bidders and the procurer. In this setting, a fixed number of employment service providers submit a bid on a project. The providers differ with respect to their efficiency level, which is private information to each provider, and about which its competitors and the government are incompletely informed. The winner of the project produces ‘output’, i.e., reduces the total amount of social benefits.

We examine three auction types. Two of these auctions are based on the beauty contests

\[3\] Krishna (2002).

\[4\] See e.g. Binmore and Klemperer (2002).
that we observe in practice. Usually, there are two objective dimensions in these beauty contests on which bidders submits bids: (1) the price per successful placement, and (2) the expected success rate, i.e., the fraction of people in the welfare-to-work project that finds a job. The first auction is the lowest-reward auction. In this auction, the provider that submits the lowest price per successful placement wins the project. Imagine that the winner submits a price equal to \( b \). Then the government rewards the provider with \( b \) for every unit of ‘output’, i.e., for each unit of savings in the unemployment benefits. We find that this auction solves the adverse selection problem in the sense that the most efficient provider is selected. However, a strong moral hazard problem remains as the winning providers end up in a ‘race to the bottom’ in the sense that equilibrium output tends to zero when the number of bidders increases. This finding may place some doubt on the usefulness of having the price per successful placement as one of the dimensions in a beauty contest.

Second, we focus on the highest-output auction. This auction rewards the project to the provider that promises the highest output. The government pays the winner a reward \( r (e - b) \), where \( r \) is a constant, \( e \) the actual output by the employment service provider, and \( b \) its promise in the auction. A negative reward is interpreted as a fine.

The third auction type is the constant-reward auction, which OECD (2001) proposes as an alternative way to procure welfare-to-work projects. The government sells the project to the highest bidder, for instance in the second-price sealed-bid auction (the constant-reward second-price auction\(^5\)). The winner is paid a fixed reward for each unit of its output. OECD (2001) argues that this auction is optimal, provided that the government awards the winner of this auction for each placement the marginal social value \( \sigma \). However, the OECD’s claim is based on McMillan (1992), who relies on the assumption of complete information regarding the efficiency of the provider, and who ignores the positive impact of a decrease in unemployment benefits on government finances.

We compare the three auctions with a socially optimal mechanism. This mechanism is an

\(^5\)We will see later that the constant-reward first-price auction is strategically equivalent to the highest-output auction.
incentive compatible and individually rational direct revelation mechanism with the following properties. First, the government selects the most efficient provider, provided that its efficiency level exceeds a threshold level. The winning provider then establishes an output level that is below the full-information optimum. The three auctions are not optimal. However, if the number of bidding providers tends to infinity, if the tax distortion approaches zero, or if the social welfare from a unit of output increases, the constant-rewards second-price auction and the highest-output auction approach the optimal outcome.

In addition, under a technical assumption on the distribution function, we show that regardless of how the government organizes the procurement with \( n \) providers, it is better to invite an \((n+1)\)-th provider and hold the highest-output auction or the constant-reward second-price auction. The two auctions have the advantage over the socially optimal mechanism that (1) they are ‘detail free’: the government does not have to acquire information about the number of bidders and the distribution of the signals and (2) they are less demanding with respect to the government’s commitment. This result supports the OECD’s claim that the constant-reward auction is a good alternative to the beauty contests that are used in practice, albeit with a fixed reward that is equal to the marginal social value of each successful placement adjusted for the tax distortion.

This paper is organized as follows. In the next section, we discuss the literature that is related to our paper. In Section 3, we describe our model. Section 4 deals with the three auctions and in Section 5, we construct the optimal mechanism and compare its outcome to the outcomes of these auctions. In Section 6, we show that the highest-output auction and the constant-reward second-price auction dominates the socially optimal mechanism if the government is able to attract only one additional provider. Finally, Section 7 includes some concluding remarks.
2 Related Literature

The following papers in the economic literature are related to ours. First of all, there is a well established literature on welfare-to-work programs. See Heckman et al. (1999), Martin and Grubb (2001), Kluve and Schmidt (2002), and Boone and Van Ours (2004) for overviews. This literature mainly concentrates on the effect of specific programs, such as financial stimuli, training and skill development, and work support subsidies. The question how to select and how to give incentives to intermediaries, such as employment service providers, has been virtually ignored.\footnote{Two notable exceptions can be found in the work of James Heckman and Al Roth. Heckman et al. (1996) investigate the incentives for case workers in job training programs. Al Roth studies algorithms that match workers and employers. Some of these algorithms have been applied in the US to match young doctors with hospitals. See Roth (2002) and the references contained therein.} The main goal of this paper is to fill this gap in the literature.

In addition, there is a substantial literature on the optimal design of auctions. Myerson (1981) and Riley and Samuelson (1981) show that the seller has an incentive to screen out the bidders with the lowest types, for instance by setting a reserve price below which he accepts no bids. Bulow and Klemperer (1996) argue that the value of the bargaining power of the seller is small relative to the value of attracting more bidders. For overviews of auction theory, see Klemperer (1999) and Krishna (2002).

Also the literature on incentive contracts is related to our paper. One of the main results is that an incompletely informed principal asks an agent to establish less output than with complete information unless the agent has the highest possible efficiency level. See Laffont and Tirole (1993) and Laffont and Martimort (2002) for overviews.

McAfee and McMillan (1986, 1987) and Laffont and Tirole (1987, 1993) build a bridge between auction theory and incentive theory. Laffont and Tirole (1987, 1993) study a model in which the government auctions an indivisible project to one of several risk neutral firms. The government has to incentivize the selected firm to reduce the costs of the project. McAfee and McMillan (1986) investigate a similar setting, assuming risk averse bidders. The optimal contract in their model is usually an incentive contract, i.e., a contract that shares the risks.
among the government and the winning bidder. McAfee and McMillan (1987) is the most closely related to our paper. The most substantial difference between their paper and ours, is that McAfee and McMillan maximizes the principal’s profit, whereas we maximize social welfare, taking into account the costs incurred by the winning agent. Qualitatively, we derive the same results as McAfee and McMillan (1987) with respect to the optimal mechanism: the optimal mechanism screens out all providers below a fixed threshold, and the winning provider’s output is lower than the full-information optimum.7

Finally, several papers in the field of industrial organization study intermediation.8 In contrast to our paper, these papers focus on situations in which customers are free to choose their intermediary. In our paper, we assume that unemployment people do not have this freedom. Instead, the government matches them to an intermediary, i.e., the employment service provider that wins the procurement. A practical reason for this may be that these people are probably not very well able, or lack the incentives, to select the most appropriate provider.

3 The model

A risk neutral government wishes to procure a welfare-to-work project. We assume that $n$ risk neutral employment service providers participate in the procurement. Each provider $i$, $i = 1, ..., n$, when winning the project, is able to produce output $e_i$ at the cost $C_i(e_i, \alpha_i)$ where $\alpha_i \in [0, 1]$ is provider $i$’s efficiency level. The output level $e_i$ is observable, or the relevant output is the sum of $e_i$ and a disturbance term with mean 0. The latter is irrelevant as by assumption, both the government and the providers are risk neutral. In the specific context of welfare-to-work programs, we interpret output as the savings on social benefits when people in

7Che (1993) studies ‘scoring auctions’ that implement the optimal mechanism. In a scoring auction, bidders submit bids on several dimensions, and the auctioneer determines the winner using a scoring rule that is a function of these bids. In practice, scoring auctions have been used to allocate electricity reserve supply, highway construction projects, and software development contracts. See e.g. Wilson (2002) and Asker and Cantillon (2003).

8See e.g. Armstrong (2004), Caillaud and Jullien (2003), and Rochet and Tirole (2003, 2004).
the project find a job. $C_i$ is increasing and convex, and $C_i(0, \alpha_i) = 0$. An implication of these assumption is that the per unit costs of establishing savings on social benefits are increasing.

The providers draw the $\alpha_i$'s independently from the same distribution with a cumulative distribution function $F$ on the interval $[0, 1]$ with differentiable density function $f$. $F$ is common knowledge. We assume that

$$\alpha_i - \frac{1 - F(\alpha_i)}{f(\alpha_i)} \text{ is strictly increasing in } \alpha_i,$$

which holds true for several standard distributions, including the uniform and the exponential distributions. Provider $i$ has the utility function

$$U_i = t_i - C_i$$

where $t_i$ is the monetary transfer that it receives from the government.

Let $S$ denote the net social welfare of the project. We follow Laffont and Tirole (1987) in that the social cost of one unit of money is $1 + \lambda$, where $\lambda \geq 0$. Ballard et al. (1985) estimate deadweight losses of raising taxes to lie between 17 and 56 cents for every extra $1 raised, so that in practice, $\lambda \approx 0.37$. Net social welfare is given by

$$S = (\sigma + \lambda)e_i - (1 + \lambda)t_i + t_i - C_i(e_i, \alpha_i)$$

$$= (\sigma + \lambda)e_i - \lambda U_i - (1 + \lambda)C_i(e_i, \alpha_i)$$

where $i$ is the provider the government has selected for the welfare-to-work program. The parameter $\sigma$ represents the marginal social value $\sigma$ of each unit of the provider’s output, i.e., the savings on social benefits, net from a decrease in tax distortions. $\sigma$ includes all effects on the economy associated with people finding a job, and may be positively related to increased production, a decrease in criminality, and diminishing intergenerational welfare dependency, and negatively related to, for instance, the placement of one person detracting from the employment chances of others (see e.g. Calmfors, 1994). In other words, the model captures the possibility that the micro-economic effect of a person finding a job may be quite different than

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9 Jacobs et al. (2004) show in an optimal taxation model that under certain conditions, $\lambda = 0$. 

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the macro-economic effect, as stressed by e.g. Heckman et al. (1999). Throughout the paper, we assume that \( \sigma > 0 \).\(^{10}\)

An optimal mechanism maximizes \( S \) under the restriction that the providers play a Bayesian Nash equilibrium, and that it satisfies a participation constraint (each participating provider should at least receive zero expected utility). For the sake of simplicity, we assume

\[
C_i(e_i, \alpha_i) = \tilde{\sigma} \left( \frac{1}{2} e_i^2 + e_i - \alpha_i e_i \right)
\]

where

\[
\tilde{\sigma} \equiv \frac{\sigma + \lambda}{1 + \lambda}
\]

is the adjusted marginal social value of each successful placement. We choose this specific cost function so that by construction, in the first-best optimum, i.e., the optimum under complete information, the winning provider produces output equal to \( \alpha_i \).\(^{11}\) In addition, note that \( C''_i(e_i) = \tilde{\sigma} > 0 \). In other words, the marginal costs of output is strictly increasing in output. The reason for this is not diseconomies of scale, but that some persons are easier to place than others. If economies of scale played an important role, the government would have a good reason to split up the program in smaller programs, and have several providers do the job.

Let us be more specific on the properties of the first-best optimum. First of all, the government selects the most efficient provider, i.e., the provider with the highest type \( \alpha_i \), as this provider has the lowest \( C_i \) for a given output level. Secondly, the government induces this provider to produce output \( \alpha_i \). Finally, the government exactly covers the costs \( C_i \). This first-best optimum cannot be reached in our setting with incomplete information: the government has to pay informational rents to the provider. We will see that in the optimal mechanism under

\(^{10}\)From Mirrlees’ (1971) analysis of optimal taxation, we may deduce that under some circumstances, \( \sigma = 0 \). In his model, it is optimal that the persons with the lowest abilities remain unemployed.

\(^{11}\)Of course, we lose some generality by making this assumption. However, the main results still hold true if we allow for a more general cost function

\[
C_i(e_i, \alpha_i) = \theta \left( \frac{1}{2} e_i^2 + e_i - \alpha_i e_i \right)
\]

where \( \theta > 0 \).
incomplete information, the government ‘pays’ these informational rents by (1) only selecting
the most efficient provider if its type exceeds a threshold level, (2) inducing an output level
which is lower than \( \alpha_i \), and (3) covering more than the costs the provider actually incurs.

4 Auctions

We consider three auctions the government may use to allocate the welfare-to-work project to
one of the employment service providers. Each auction is a two-stage game. In the first stage,
the government procures the project. In the second stage, the winning provider chooses its
output level and the government rewards the provider depending on its output choice. In the
remainder of this paper, we will with a slight abuse of terminology refer to the entire two-stage
game as an auction.

4.1 The lowest-reward auction

We first consider the lowest-reward auction. In this auction, the provider that submits the
lowest reward, wins the project. Imagine that the winner submits a reward level equal to \( b \).
Then the government rewards the provider with \( b \) for every unit of output. The following
proposition characterizes equilibrium bidding for this auction. The proof of this proposition
(and the following) are relegated to the appendix.

Proposition 1 Consider the following bidding function and output level function.

\[
B(\alpha) = \tilde{\sigma} \left[ 1 - \alpha + \int_0^\alpha \left( \frac{F(y)}{F(\alpha)} \right)^{\frac{n-1}{2}} dy \right], \quad \text{and} \\
L(b, \alpha) = \frac{b}{\tilde{\sigma}} + \alpha - 1,
\]

where \( b \) is the bid of the winner. \( B \) and \( L \) constitute a symmetric Bayesian Nash equilibrium
of the lowest-reward auction. \( B \) is strictly decreasing in \( \alpha \) and \( n \).

From Proposition 1, we can draw the following conclusions. First of all, as \( B \) is strictly
decreasing in \( \alpha \), the auction always rewards the project to the most efficient provider. In
other words, this auction completely solves the adverse selection problem. Second, the provider chooses its output at the level \( L \) at which the marginal benefits of output \( (b) \) are equal to the marginal costs \( \left( \frac{\partial C_i(L, \alpha_i)}{\partial L} = \tilde{\sigma} [L + 1 - \alpha_i] \right) \). Third, given that the winner has efficiency level \( \alpha \), equilibrium output is given by

\[
L(B(\alpha), \alpha) = \int_0^\alpha \left( \frac{F(y)}{F(\alpha)} \right)^{\frac{n-1}{2}} dy \leq \alpha.
\]

Note that \( L(B(\alpha), \alpha) \) is below the first-best socially optimal level of output \( \alpha \) and is decreasing in \( n \). In fact, a ‘race to the bottom’ emerges in the sense that the output converges to zero when the number of providers tends to infinity: more competition strengthens the moral hazard problem. This follows from the following trade-off. The more providers, the more efficient is the most efficient provider. However, the larger the number of providers in the auction, the more aggressively they bid. The winner, having submitted the lowest reward, then has little incentives to put much effort in the project as the marginal benefits from its output are very low.

### 4.2 The highest-output auction

In order to avoid the moral hazard problem that is imminent in the lowest-reward auction, the government may pay the winning provider a constant reward for each unit of its output equal to the adjusted level of social welfare \( \tilde{\sigma} \). The highest-output auction implements a mechanism with this property. This auction rewards the project to the provider that promises the highest output level. The government pays the winner the following amount of money, depending on its actual output level \( e \) and the output level \( b \) it promised in the auction:

\[
t(e, b) = \tilde{\sigma} (e - b).
\]

We provide the equilibrium properties of this mechanism in the next proposition.

**Proposition 2** Let

\[
B(\alpha) = \frac{1}{2} \alpha^2 - F(\alpha)^{-n+1} \int_0^\alpha y F(y)^{n-1} \, dy
\]

\[
L(b, \alpha) = \alpha
\]
with $B$ and $L$ a bidding function and an output function respectively. $B$ and $L$ constitute a symmetric Bayesian Nash equilibrium of the highest-output auction. $B$ is strictly increasing in $\alpha$ and $B$ is strictly increasing in $n$.

This proposition suggests that the highest-output auction is better than the lowest-reward auction. Both auctions are efficient in the sense that the project is always rewarded to the provider with the highest efficiency parameter. However, the output by the winning provider is higher in the highest-output auction, which is exactly equal to the output level in the optimum under complete information. In fact, as we will show later, if $\lambda = 0$, $\sigma \to \infty$, or $n \to \infty$, the highest-output auction is optimal.

4.3 The constant-reward second-price auction

Alternatively, the government may use the constant-reward auction. OECD (2001) proposes this auction as an alternative to the beauty contests that are usually used in welfare-to-work programs. We focus on the constant-reward second-price auction. In this auction, the project is allocated in the second-price sealed-bid auction: the winner is the highest bidding provider, which has to pay the bid of the second highest bidder to the government. The winner is then rewarded $\tilde{\sigma}$ monetary units per unit of output. Note that the constant reward first-price auctions is strategically equivalent to the highest-output auction. A bid $b$ in this auction is equivalent to a bid $b/\tilde{\sigma}$ in the highest-output auction.

Proposition 3 provides the equilibrium properties of the constant-reward second-price auction.

**Proposition 3** Consider the following bidding function and output level function.

$$B(\alpha) = \frac{1}{2}\alpha^2, \text{ and}$$

$$L(b, \alpha) = \alpha,$$

$B$ and $L$ constitute a symmetric Bayesian Nash equilibrium of the constant-reward auction. $B$ is constant in $n$ and strictly increasing in $\alpha$. 

The constant-reward second-price auction turns out to have the same properties as the highest-output auction. Both auctions are equally efficient and share the same expected output levels, expected payments, and expected social welfare. This result follows from a ‘revenue equivalence theorem’ which states that providers obtain the same expected payment from all mechanisms which allocate the project to the same provider and in which the winner provides the same output level, provided that the utility of the lowest type equals zero. We prove this result in the appendix in the proof of Proposition 4. Moreover, as we will show in the next section, if $\lambda = 0$, or $\sigma \to \infty$, or $n \to \infty$, the constant-reward second-price auction is optimal.

5 The socially optimal mechanism

What is the socially optimal mechanism, i.e., the mechanism that maximizes (3)? We use the techniques developed by Laffont and Tirole (1987) and McAfee and McMillan (1987) to answer this question. According to Myerson (1981), we may, without loss of generality, restrict our attention to incentive compatible and individually rational direct revelation mechanisms. Let

$$\tilde{\alpha} = (\tilde{\alpha}_1, \ldots, \tilde{\alpha}_n)$$

be the vector of announcements by provider 1, ..., $n$ respectively. We consider mechanisms $M = (x_i, e_i, t_i)_{i=1,...,n}$ that induce a truth-telling Bayesian Nash equilibrium, where, given the announcement $\tilde{\alpha}$, $x_i(\tilde{\alpha})$ is the probability that provider $i$ wins the contract, and, given that provider $i$ wins the contract, $e_i(\tilde{\alpha})$ is its output and $t_i(\tilde{\alpha})$ is the monetary transfer it receives from the government.

**Proposition 4** The optimal mechanism $M^* = (x^*_i, e^*_i, t^*_i)_{i=1,...,n}$ has the following properties:

$$x^*_i(\alpha) = \begin{cases} 1 & \text{if } \alpha_i > \max_{j \neq i} \alpha_j \text{ and } \alpha_i \geq \alpha \\ 0 & \text{otherwise} \end{cases}$$

$$e^*_i(\alpha) = e^*(\alpha_i) = \alpha_i - \frac{\lambda}{\sigma + \lambda} \frac{1 - F(\alpha_i)}{f(\alpha_i)}, \text{ and}$$

$$t^*_i(\alpha) = C_i(e^*(\alpha_i), \alpha_i) + \int_{\alpha}^{\alpha_i} e^*(y) F(y)^{n-1} dy$$
where $\alpha$ is the unique solution to $y$ in $y = \frac{\lambda}{\sigma + \lambda} \frac{1-F(y)}{f(y)}$.

The optimal mechanism $M^*$ has the property that the government selects the most efficient provider, provided that its efficiency level exceeds $\alpha > 0$. This provider produces output according to $e^*_i$, and $t^*_i$ determines the payments it receives from the government. Observe that the desired output level $e^*_i(\alpha)$ and $\alpha$ do not depend on the number of bidding providers.

Three types of inefficiency arise from this mechanism. First, since $e^*_i(\alpha) < \alpha_i$ for all $\alpha_i < 1$, the provider’s output is lower than in the full-information optimum. Second, the government will not contract with any provider whose efficiency level is below $\alpha$, whereas in the full-information world, the government would contract with any provider. The latter is analogous to a reserve price in an optimal auction (see e.g. Myerson, 1981). Third, as $t^*_i(\alpha) > C_i(e^*_i(\alpha), \alpha_i)$ for $\alpha_i > 0$, the government covers more than the costs that are actually born by the winning provider, which is inefficient as government finances are socially costly. These types of inefficiency give the government the opportunity to capture some of the informational rents that arise because of incomplete information.

**Corollary 5** Both the highest-output auction and the constant-reward second-price auction are optimal if (1) $\lambda = 0$, or (2) $\sigma \to \infty$, or (3) $n \to \infty$.

Corollary 5 follows immediate from the expressions for $e^*_i$ and $\alpha$. Recall that the highest-output auction and the constant-reward second-price auction, always select the provider $i$ with the highest efficiency level, and induce it to choose output $\alpha_i$. If $\lambda = 0$ or $\sigma \to \infty$, then $e^*(\alpha_i) = \alpha_i$ and $\alpha = 0$, so that both auctions are optimal. Moreover, as both $e^*$ and $\alpha$ do not depend on $n$, the only effect of increasing $n$ is to change the distribution of the efficiency level of the selected provider. When $n$ increases, the probability that the highest type exceeds $\alpha$ equals one. In addition, $e^*(\alpha_i)$ tends to 1 for $\alpha_i$ approaching 1. As the highest type approaches 1 for $n$ tending to infinity, the two auctions induce output to be equal to 1, so that both are optimal for large $n$. In the next section, we show in a simple setting that the highest-output auction and the constant-reward second-price auction perform better than the socially optimal mechanism if the government is able to only attract one additional bidder in these auctions.
Figures 1 and 2 contain a plot of expected output and expected social welfare respectively arising from the three simple mechanisms when $\sigma = 1$, $\lambda = 1$, and $F$ is the uniform distribution on the interval $[0, 1]$. LRA denote the lowest-reward auction, CRA the constant-reward second-price auction, and HOA the highest output auction.

From these figures, we observe the following. First of all, CRA and HOA are equivalent in the sense that both induce the same output from the winner of the auction and that the two mechanisms generate the same level of social welfare. Second, the outcomes of LRA deviate quite dramatically from the outcomes of the socially optimal mechanism: an increase in the number of providers may result in a decrease in the expected output from the winning firm. The positive effect on output arising from more bidders increasing the expected efficiency turns out to be diminished by the negative effect that more bidders decrease the per unit payment.

Third, the output under CRA and HOA are the same as in first-best optimum. Fourth, the output in the optimal mechanism is lower than the output in a first-best world. As we have seen before, the reason for this is that the government has to pay an informational rent, so that it induces less output than in the complete information optimum. Fifth, the expected output level and social welfare arising from both CRA and HOA converge to the social optimum. Sixth, the deviation from these two mechanisms from the social optimal is small even for a small number of bidders. Just a bit of competition (three bidders) is sufficient for CRA and HOA to perform well (reaching at least 95% of the maximum level of social welfare). And finally, the optimal mechanism converges to the first best. The intuition behind this observation is that for large $n$, the government can exploit competition between the providers, which reduces the level of informational rents it has to pay.
6 Auctions versus optimal mechanisms

In this section, we further explore the properties of the socially optimal mechanism in comparison to the auction formats that we discussed in section 3. More specifically, we compare the socially optimal mechanism to the highest-output auction and the constant-reward second-price auction with one additional bidder. The analysis follows up on Bulow and Klemperer (1996), who compare revenue maximizing auctions with an English auction with one additional bidder.

We do so in a simplified setting in which the providers draw their efficiency parameter from the distribution $F$ on the interval $[0, 1]$:

$$F(y) = 1 - (1 - y)^\zeta$$

where $\zeta \in (0, 1]$. Note that $\zeta = 1$ corresponds to the uniform distribution on the interval $[0, 1]$.

The inverse hazard rate $h$ is given by

$$h(\alpha_i) \equiv \frac{1 - F(\alpha_i)}{f(\alpha_i)} = \frac{1 - \alpha_i}{\zeta}$$

so that the function $\alpha_i - h(\alpha_i)$ is strictly increasing in $\alpha_i$.

It is readily verified that the socially optimal mechanism can be implemented in the following two-stage game. In the first stage, the government auctions the project in a second-price auction. In the second stage, if the winning provider’s output level is $e$, he receives a reward equal to

$$\frac{\tilde{\sigma}}{1 + \gamma \left(1 + \frac{1}{2} \gamma e\right)} e$$

where

$$\gamma = \frac{\lambda}{\zeta (\sigma + \lambda)}.$$

Compared to the highest-output auction and the constant-reward second-price, the socially optimal mechanism has two disadvantages. First, it is not ‘detail free’ as it is context dependent in the sense that ‘the rules of the game’ depend on the distribution of the signals, i.e., on $\zeta$.\footnote{Of course, the auctions do depend on $\lambda$ and $\sigma$, as the marginal reward for each unit of output in these auction is equal to $\tilde{\sigma} \equiv \frac{\sigma + \lambda}{1 + \lambda}$, so that the government still needs to acquire some information.}
The standpoint that the auction designer should restrict his attention to mechanisms that do not depend on the details of the environment is sometimes called ‘the Wilson doctrine’, named after Robert Wilson, who is a well known advocate of this point of view.\textsuperscript{13} Second, the socially optimal mechanism is demanding with respect to the government’s commitment: in contrast to the auctions, the optimal mechanism requires the government to demand an ex post suboptimal level of output from the winning provider, and not even to assign a welfare enhancing project at all when the efficiency parameter of all providers turns out to be below $\alpha$.

We will now show that the two auctions perform better than the optimal mechanism if the government is able to attract just one additional bidder in the auctions.

**Lemma 6** Let $\alpha^{(1)} = \max_i \alpha_i$. Expected social welfare from the socially optimal mechanism is

$$\frac{\sigma + \lambda}{2} E \left\{ \left( \alpha^{(1)} - \frac{\lambda}{\sigma + \lambda} h(\alpha^{(1)}) \right)^2 \middle| \alpha^{(1)} \geq \alpha \right\}.$$

**Lemma 7** Let $\alpha^{(1)} = \max_i \alpha_i$. Expected social welfare from the auctions is

$$\frac{\sigma + \lambda}{2} E \left\{ \alpha^{(1)} \left[ \alpha^{(1)} - \frac{2\lambda}{\sigma + \lambda} h(\alpha^{(1)}) \right] \right\}.$$

From lemmas 6 and 7, it becomes clear that with a fixed number of bidders, the socially optimal mechanism dominates the auctions. First,

$$\left( \alpha^{(1)} - \frac{\lambda}{\sigma + \lambda} h(\alpha^{(1)}) \right)^2 \geq \alpha^{(1)} \left[ \alpha^{(1)} - \frac{2\lambda}{\sigma + \lambda} h(\alpha^{(1)}) \right]$$

and second, the function

$$\alpha^{(1)} \left[ \alpha^{(1)} - \frac{2\lambda}{\sigma + \lambda} h(\alpha^{(1)}) \right]$$

is negative for $\alpha^{(1)} < \alpha$. The question is whether an additional bidder in the auctions compensates for these disadvantages. This is indeed the case for the above class of distribution functions.

\textsuperscript{13}Krishna (2002), p. 75.
Proposition 8 Let $F(y) = 1 - (1 - y)^\zeta$ with $\zeta \in (0, 1]$. Expected social welfare from the highest-output auction and the constant-reward second-price auction with $n+1$ providers exceeds expected social welfare from the socially optimal mechanism with $n$ providers.

Proposition 8 shows that regardless of how the government organizes the procurement with $n$ providers, it is better to invite an $(n+1)$-th provider and hold the highest-output auction or the constant-reward second-price auction.

7 Conclusion

In this paper, we have applied mechanism design theory to welfare-to-work programs. When procuring welfare-to-work projects to employment service providers, governments face the problems of adverse selection (the winning provider is not the most efficient one) and moral hazard (the winning provider shirks in its effort to reintegrate unemployed people). We have compared the optimal mechanism with three auctions (the lowest-reward auction, the highest-output auction, and the constant-reward second-price auction). All three auctions ‘solve’ the adverse selection problem as it is always the most efficient provider that wins. However, the moral hazard problem is strongly present in the lowest-reward auction. If the number of providers is large, a ‘race-to-the-bottom’ will emerge, i.e. the winning bid converges to zero. As a consequence, the winner has little incentives to put much effort in the project as the marginal benefits from its effort are very low.

In contrast, we have shown that in the highest-output auction and the constant-reward second-price auction, the moral hazard problem is solved as the winning provider’s output is exactly equal to the optimal output in the full-information world. Moreover, both auctions approximate the socially optimal mechanism if the number of providers $n$ is large, the tax distortion parameter $\lambda$ low, or social welfare per unit of output $\sigma$ high. In addition, we have shown that it is more profitable for the government to attract one more firm in these auctions than to gather sufficient information for the design of the socially optimal mechanism. In contrast to the auctions, the government needs this information as the optimal mechanism is
context dependent, i.e., ‘the rules of the game’ depend on \( n \) and the distribution \( F \) of the efficiency levels. An interpretation of this result is that attracting just one more bidder more than makes up for any decrease in bargaining power the government may use in a beauty contest.

There are several interesting subjects for future research. First of all, as far as we know, auctions are rarely used in the practice of welfare-to-work programs. In some countries, beauty contests are used (e.g. in the Netherlands), and in others, the government (partly) awards contracts on the basis of the reputation the employment service providers gained in the past (e.g. Job Network in Australia). The question that arises is whether there are circumstances in which beauty contests or reputation mechanisms outperform auctions. Secondly, from a theoretical point of view, it may be interesting to explore to which extend our ‘auctions versus optimal mechanisms’ result generalizes to other settings. Finally, the effect of the winner’s curse and risk aversion among the employment service providers may be interesting topics for further research.
A Proofs of Propositions

A.1 Proof of Proposition 1

We construct the equilibrium using backward induction, first deriving the output level by the winning firm, and then deriving the bids in the auction. The winner solves

$$\max_e be - \tilde{\sigma} \left( \frac{1}{2} e^2 + e - \alpha e \right)$$

where \(b\) is its bid in the auction and \(\alpha\) its efficiency level. Straightforward calculations yield the equilibrium output level

$$L(b, \alpha) = \frac{b}{\sigma} + \alpha - 1.$$ 

To derive equilibrium bidding in the auction, we suppose that in equilibrium, all providers use the same bid function \(B\). By a standard argument, this bid function must be strictly increasing and continuous. Let \(U(\alpha, \beta)\) be the utility for a provider with efficiency level \(\alpha\) who behaves as if having signal \(\beta\), whereas the other bidders play according to the equilibrium bid function. Then

$$U(\alpha, \beta) = F(\beta)^{n-1} \left[ \frac{1}{2} \tilde{\sigma} \left( \frac{B(\beta)}{\sigma} + \alpha - 1 \right)^2 \right]. \quad (5)$$

The first term in the RHS of (5) refers to the probability that a provider announcing \(\beta\) wins the auction, and the second term refers to its expected profit when winning. A necessary equilibrium condition is that

$$\frac{\partial U(\alpha, \beta)}{\partial \beta} = 0$$

at \(\beta = \alpha\), which results in the following differential equation:

$$\frac{dF(\alpha)^{n-1}}{d\alpha} \left[ \frac{B(\alpha)}{\sigma} - 1 \right] + \alpha \frac{dF(\alpha)^{n-1}}{d\alpha} = 0.$$

The bidding function

$$B(\alpha) = \tilde{\sigma} \left( 1 - F(\alpha)^{\frac{n-1}{2}} \int_0^\alpha yF(y)^{\frac{n-1}{2}} dy \right)$$

= \tilde{\sigma} \left( 1 - \alpha + F(\alpha)^{\frac{n-1}{2}} \int_0^\alpha F(y)^{\frac{n-1}{2}} dy \right)$$
is a solution. Let $Y^{(n)}$ be a stochastic variable with distribution function $F(\cdot)^{\frac{n-1}{2}}$. Note that $B$ can be rewritten as

$$B(\alpha) = \tilde{\sigma} \left( 1 - E(Y^{(n)}|Y^{(n)} \leq \alpha) \right)$$

so that it is readily observed that $B$ is strictly decreasing in $\alpha$. Moreover, as $Y^{(n+1)}$ strictly first-order stochastically dominates $Y^{(n)}$, $B$ is strictly decreasing in $n$.

### A.2 Proof of Proposition 2

We construct the equilibrium using backward induction, first deriving the output level by the winning firm, and then deriving the bids in the auction. The winner solves

$$\max_e \tilde{\sigma} (e - b) - \tilde{\sigma} \left( \frac{1}{2} e^2 - e + \alpha e \right).$$

Straightforward calculations yield the equilibrium output level

$$L(b, \alpha) = \alpha.$$

To derive equilibrium bidding in the auction, we suppose that in equilibrium, all providers use the same bid function $B$. By a standard argument, this function must be strictly increasing and continuous. Let $U(\alpha, \beta)$ be the utility for a provider with efficiency level $\alpha$ who behaves as if having efficiency level $\beta$, whereas the other bidders play according to the equilibrium bid function. Then

$$U(\alpha, \beta) = F(\beta)^{\frac{n-1}{2}} \left[ \frac{1}{2} \tilde{\sigma} \alpha^2 - \tilde{\sigma} B(\beta) \right].$$

The first term in the RHS of (5) refers to the probability that a provider announcing $\beta$ wins the auction, and the second term refers to its expected profit when winning. A necessary equilibrium condition is that

$$\frac{\partial U(\alpha, \beta)}{\partial \beta} = 0$$

at $\beta = \alpha$, which results in the following differential equation:

$$- \frac{dF(\alpha)^{n-1} B(\alpha)}{d\alpha} + \frac{1}{2} \alpha^2 \frac{dF(\alpha)^{n-1}}{d\alpha} = 0$$
with boundary condition

\[ B(0) = 0. \]

The bidding function

\[ B(\alpha) = \frac{1}{2} \alpha^2 - F(\alpha)^{-n+1} \int_0^\alpha y dF(y)^{n-1} \]

is a solution.

Let \( Z^{(n)} \) be a stochastic variable with distribution function \( F(\cdot)^{n-1} \). Note that \( B \) can be rewritten as

\[ B(\alpha) = \frac{1}{2} \mathbb{E}(Z^{(n)}|Z^{(n)} \leq \alpha) \]

so that it is readily observed that \( B \) is strictly increasing in \( \alpha \). Moreover, as \( Z^{(n+1)} \) strictly first-order stochastically dominates \( Z^{(n)} \), \( B \) is strictly increasing in \( n \).

### A.3 Proof of Proposition 3

We construct the equilibrium using backward induction, first deriving the output level by the winning firm, and then deriving the bids in the auction. The winner solves

\[ \max e \tilde{\sigma} e - \tilde{\sigma} \left( \frac{1}{2} e^2 - e + \alpha e \right). \]

Straightforward calculations yield the equilibrium output level

\[ L(b, \alpha) = \alpha. \]

In the auction, each provider has a dominant strategy, which is to submit a bid \( B \) equal to its profits in the second stage, i.e.

\[ B(\alpha) = \frac{1}{2} \alpha^2. \]

These dominant strategies constitute a Bayesian Nash equilibrium. It is readily observed that \( B \) is constant in \( n \) and strictly increasing in \( \alpha \).
A.4 Proof of Proposition 4

A.4.1 Providers’ bidding behavior

If all providers bid truthfully, provider $i$’s interim utility (i.e., its expected utility given its efficiency parameter $\alpha_i$) is equal to

$$U_i(\alpha_i) = E_{\alpha_{-i}}[t_i(\alpha) - x_i(\alpha)\{\varphi(e_i(\alpha)) - \alpha_i e_i(\alpha)\}]$$

(7)

where

$$\varphi(e) = \frac{1}{2}e^2 + e.$$ 

Let $U_i(\alpha_i, \tilde{\alpha}_i)$ be provider $i$’s utility when it has efficiency parameter $\alpha_i$, it announces $\tilde{\alpha}_i$, and all other providers truthfully reveal their type. Then

$$U_i(\alpha_i, \tilde{\alpha}_i) = E_{\alpha_{-i}}[t_i(\alpha_{-i}, \tilde{\alpha}_i) - x_i(\alpha_{-i}, \tilde{\alpha}_i)\varphi(e_i(\alpha_{-i}, \tilde{\alpha}_i))] + \alpha_i E_{\alpha_{-i}}[e_i(\alpha_{-i}, \tilde{\alpha}_i)x_i(\alpha_{-i}, \tilde{\alpha}_i)].$$

(8)

Incentive compatibility requires that the firms optimally announce their own type, so that the first-order condition is that at $\tilde{\alpha}_i = \alpha_i$

$$\frac{\partial U_i(\alpha_i, \tilde{\alpha}_i)}{\partial \tilde{\alpha}_i} = 0.$$ 

(9)

Below, we show that for the optimal mechanism, the second-order condition holds as well. From (7) and (8), it immediately follows (using the envelope theorem) that

$$\frac{dU_i(\alpha_i)}{d\alpha_i} = \frac{\partial U_i(\alpha_i, \tilde{\alpha}_i)}{\partial \tilde{\alpha}_i} \bigg|_{\tilde{\alpha}_i = \alpha_i} + E_{\alpha_{-i}}[e_i(\alpha)x_i(\alpha)]$$

(10)

$$= E_{\alpha_{-i}}[e_i(\alpha)x_i(\alpha)].$$

The participation constraint then immediately reduces to

$$U_i(0) \geq 0.$$ 

(11)
A.4.2 The government’s problem

To maximize social welfare given (10) and (11), we apply the Pontryagin principle. The government solves

\[
\max_{(x_i, e_i, U_i)} E_{\alpha} \sum_i \left\{ x_i(\alpha) \left[ (\sigma + \lambda)e_i(\alpha) - (1 + \lambda)C_i(e_i(\alpha), \alpha_i) \right] - \lambda U_i(\alpha_i) \right\}
\]

s.t. \[\dot{U}_i(\alpha_i) = E_{\alpha_{-i}}[e_i(\alpha)x_i(\alpha)] \quad \text{for all } i, \]
\[U_i(0) \geq 0 \quad \text{for all } i.\]

We take three steps to solve this problem. First, it can be shown that \(e_i(\alpha)\) only depends on \(\alpha_i\). We do not prove this formally, but the intuition is that \(e_i\) is a stochastic scheme when it depends on announcements other than \(\alpha_i\). As the firm’s cost function is convex, there is a deterministic scheme that only depends on \(\alpha_i\) which strictly improves the objective function of the government. (See Laffont and Tirole (1987) and McAfee and McMillan (1987) for a formal proof.)

The second step is to keep the \(x_i\)'s fixed, and solve the government’s problem. Let

\[X_i(\alpha_i) \equiv E_{\alpha_{-i}}[x_i(\alpha)].\]

For given \(X_i(\alpha_i)\), the government’s problem can be decomposed into the following \(n\) independent maximization programs:

\[
\max_{e_i(U_i)} \int_0^1 \left\{ X_i(\alpha_i) \left[ (\sigma + \lambda)e_i(\alpha_i) - (1 + \lambda)C_i(e_i(\alpha_i), \alpha_i) \right] - \lambda U_i(\alpha_i) \right\} dF(\alpha_i)
\]

s.t. \[\dot{U}_i(\alpha_i) = e_i(\alpha_i)X_i(\alpha_i),\]
\[U_i(0) \geq 0.\]

The third step is to realize that these programs amount to dynamic optimization programs where \(U_i\) is the state variable and \(e_i\) the control variable. The Hamiltonian \(H_i\) of each program is given by

\[H_i(\alpha_i, e_i, U_i, \mu_i) = \left\{ X_i(\alpha_i) \left[ (\sigma + \lambda)e_i - (1 + \lambda)C_i(e_i, \alpha_i) \right] - \lambda U_i \right\} f(\alpha_i) + \mu_i e_i X_i(\alpha_i).\]
Using the Pontryagin principle, we obtain the first-order condition of the program:

$$\dot{\mu}_i(\alpha_i) = \lambda f(\alpha_i)$$

$$\mu_i(\alpha_i) = \left[ -(\sigma + \lambda) + (1 + \lambda) \frac{\partial C_i(e_i^*(\alpha), \alpha_i)}{\partial e_i} \right] f(\alpha_i)$$

$$\mu_i(1) = 0$$

Let $\tilde{\alpha}$ be the unique solution to $y = \frac{\lambda \cdot 1 - F(y)}{\sigma + \lambda}$ w.r.t. $y$. Substituting $C_i(e_i, \alpha_i) = \frac{\sigma + \lambda}{1 + \lambda} \left( \frac{1}{2} e_i^2 + e_i - \alpha_i e_i \right)$ together with some straightforward calculations yields

$$e_i^*(\alpha) = e^*(\alpha_i) \equiv \begin{cases} \alpha_i - \frac{\lambda \cdot 1 - F(\alpha_i)}{\sigma + \lambda} f(\alpha_i) & \text{if } \alpha_i \geq \tilde{\alpha} \\ 0 & \text{if } \alpha_i < \tilde{\alpha} \end{cases}. \quad (12)$$

Given these output levels, the government’s problem is reduced to

$$\max \chi_i(\cdot) \sum_i \int_0^1 X_i(\alpha_i) \left[ (\sigma + \lambda) e^*(\alpha_i) - (1 + \lambda) C_i(e^*(\alpha_i), \alpha_i) \right] dF(\alpha_i)$$

$$- \sum_i \int_0^\alpha \int_0^\alpha e^*(y) X_i(y) dy dF(\alpha_i)$$

which is equivalent to

$$\max \chi_i(\cdot) \frac{1}{2} (1 + \lambda) \sum_i \int_0^1 X_i(\alpha_i) e^*(\alpha_i)^2 dF(\alpha_i). \quad (13)$$

From (13) it is straightforward to see to which firm $i$ the project should be allocated. The government’s expected utility is proportional to the sum of the firms’ winning probability times the square of the optimal output. By (1), $e^*(\alpha_i)$ is strictly increasing in $\alpha_i$ for $\alpha_i \geq \tilde{\alpha}$. Therefore, it is optimal for the government to maximize the winning probability of the firm with the highest $\alpha_i$, i.e., to always allocate the project to the most efficient provider. The payment by the winning bidder follows directly from (2).

### A.4.3 Second order condition

We finish the proof by showing that the second order condition $\text{sign} \left( \frac{\partial U_i(\alpha, \tilde{\alpha})}{\partial \tilde{\alpha}} \right) = \text{sign}(\alpha - \tilde{\alpha})$ holds true. Observe that $\frac{\partial U_i(\alpha, \tilde{\alpha})}{\partial \tilde{\alpha}}$ can be written as:

$$\frac{\partial U_i(\alpha, \tilde{\alpha})}{\partial \tilde{\alpha}} = \partial E_{\alpha_{-i}}[t_i(\alpha_{-i}, \tilde{\alpha}) - x_i(\alpha_{-i}, \tilde{\alpha}) \varphi(e_i(\alpha_{-i}, \tilde{\alpha}))] + \alpha \partial E_{\alpha_{-i}}[e_i(\alpha_{-i}, \tilde{\alpha}) x_i(\alpha_{-i}, \tilde{\alpha})]$$

$$= (\alpha - \tilde{\alpha}) \partial E_{\alpha_{-i}}[e_i(\alpha_{-i}, \tilde{\alpha}) x_i(\alpha_{-i}, \tilde{\alpha})] \quad (14)$$
The second equality follows from the observation that at $\beta = \tilde{\alpha}$,

$$\frac{\partial U_i(\beta, \tilde{\alpha})}{\partial \tilde{\alpha}} = 0.$$ 

Furthermore,

$$E_{\alpha-i}[e_i(\alpha_{-i}, \tilde{\alpha})x_i(\alpha_{-i}, \tilde{\alpha})] = E_{\alpha-i}\left[\tilde{\alpha} - \frac{\lambda}{1 + \lambda} \frac{1 - F(\tilde{\alpha})}{f(\tilde{\alpha})} + \frac{\sigma - 1}{1 + \lambda} \tilde{\alpha} \geq \max_{j \neq i} \alpha_j\right]$$

$$= F(\tilde{\alpha})^{n-1}\left(\tilde{\alpha} - \frac{\lambda}{1 + \lambda} \frac{1 - F(\tilde{\alpha})}{f(\tilde{\alpha})} + \frac{\sigma - 1}{1 + \lambda}\right).$$

By (1), $E_{\alpha-i}[e_i(\alpha_{-i}, \tilde{\alpha})x_i(\alpha_{-i}, \tilde{\alpha})]$ is strictly increasing in $\tilde{\alpha}$, so that by the differentiability of $f$, $\frac{\partial E_{\alpha-i}[e_i(\alpha_{-i}, \tilde{\alpha})x_i(\alpha_{-i}, \tilde{\alpha})]}{\partial \tilde{\alpha}} > 0$. From (14), it then immediately follows that the second order condition is satisfied.

### A.5 Proof of Lemmas 6 and 7

In both the optimal mechanism and the auctions, provider $i$’s output only depends on its own signal $\alpha_i$. Social welfare $S(M)$ of an incentive compatible and individually rational mechanism $M = (x_i, e_i, t_i)_{i=1,...,n}$ for which $e_i$ only depends on $\alpha_i$ equals

$$S(M) = \sum_i \int_0^1 X_i(\alpha_i) \left[(\sigma + \lambda)e_i(\alpha_i) - (1 + \lambda)C_i(e_i(\alpha_i), \alpha_i) - \lambda U_i(\alpha_i)\right] dF(\alpha_i)$$

$$= \sum_i \int_0^1 X_i(\alpha_i) \left[(\sigma + \lambda)e_i(\alpha_i) - (1 + \lambda)C_i(e_i(\alpha_i), \alpha_i)\right] dF(\alpha_i)$$

$$- \lambda \sum_i \int_0^{\alpha_i} \int_0^{e_i} c(y)X_i(y)dydF(\alpha_i)$$

$$= \sum_i \int_0^1 X_i(\alpha_i) \left[(\sigma + \lambda)e_i(\alpha_i) - (1 + \lambda)C_i(e_i(\alpha_i), \alpha_i) - \lambda e_i(\alpha_i)h(\alpha_i)\right] dF(\alpha_i)$$

with $X_i(\alpha_i) \equiv E_{\alpha-i}[x_i(\alpha)]$. The first equality follows from the first order condition $\dot{U}_i(\alpha_i) = e_i(\alpha_i)X_i(\alpha_i)$ and the second by applying integration by parts to the second integral. The auctions have equilibrium output $e_i(\alpha_i) = \alpha_i$, so that social welfare from the auctions is given
by
\[
\sum_i \int_0^1 X_i(\alpha_i) \left[(\sigma + \lambda)\alpha_i - (1 + \lambda)C_i(\alpha_i, \alpha_i) - \lambda \alpha_i h(\alpha_i)\right] dF(\alpha_i)
= \frac{\sigma + \lambda}{2} \sum_i \int_0^1 X_i(\alpha_i) \alpha_i \left[\alpha_i - \frac{2\lambda}{\sigma + \lambda} h(\alpha_i)\right] dF(\alpha_i)
= \frac{\sigma + \lambda}{2} E \left\{ \alpha^{(1)} \left[\alpha^{(1)} - \frac{2\lambda}{\sigma + \lambda} h(\alpha^{(1)})\right]\right\}.
\]

The first equality in the chain is derived by substituting (4), while the second follows from the fact that it is always the provider with the highest \( \alpha_i \) that wins the project. Analogously, social welfare from the optimal mechanism equals
\[
\frac{\sigma + \lambda}{2} E \left\{ e^*(\alpha^{(1)})^2 \left| \alpha^{(1)} \geq \alpha \right. \right\}
= \frac{\sigma + \lambda}{2} E \left\{ \alpha^{(1)} \left[\alpha^{(1)} - \frac{2\lambda}{\sigma + \lambda} h(\alpha^{(1)})\right]^2 \left| \alpha^{(1)} \geq \alpha \right. \right\}.
\]

### A.6 Proof of Proposition 8

Let us fix the efficiency level \( \hat{\alpha} \) of the highest of providers \( i = 1, \ldots, n \). Then social welfare of the optimal mechanism with these \( n \) providers is given by
\[
S^*_n(\hat{\alpha}) = \frac{\sigma + \lambda}{2} \max \left\{ e^*(\hat{\alpha}), 0 \right\}^2
\]
while social welfare of two auctions with \( n + 1 \) providers equals
\[
S_{n+1}(\hat{\alpha}) = \frac{\sigma + \lambda}{2} E \max \{ \Phi(\hat{\alpha}), \Phi(\alpha_{n+1}) \}
\]
where
\[
\Phi(\alpha) = \alpha \left[ \alpha - \frac{2\lambda}{\sigma + \lambda} h(\alpha) \right].
\]

Suppose that \( \hat{\alpha} < \alpha \). Then social welfare of the optimal mechanism with \( n \) providers is zero. The expected social welfare of the auctions is
\[
\frac{\sigma + \lambda}{2} E \max \{ \Phi(\hat{\alpha}), \Phi(\alpha_{n+1}) \}.\]
Now,
\[
E \{ \Phi(\alpha_{n+1}) \} = E \left\{ \alpha_{n+1} - \frac{2\lambda}{\sigma + \lambda} h(\alpha_{n+1}) \right\} \\
= E \left\{ \alpha_{n+1} \left[ \alpha_{n+1} - 2h(\alpha_{n+1}) + \frac{2\sigma}{\sigma + \lambda} h(\alpha_{n+1}) \right] \right\} \\
= E \left\{ \frac{2\sigma \alpha_{n+1}}{1 + \lambda} h(\alpha_{n+1}) \right\} \\
> 0.
\]
As \( E \{ \Phi(\alpha_{n+1}) \} \) is strictly larger than zero, \( E \max \{ \Phi(\hat{\alpha}), \Phi(\alpha_{n+1}) \} \) is strictly larger than zero as well, so that for \( \hat{\alpha} < \alpha \), the auctions yield more revenue.

What remains is the situation that \( \hat{\alpha} \geq \alpha \). Social welfare from the optimal mechanism is \( \frac{1}{2}(\sigma + \lambda)e^* (\hat{\alpha})^2 \), while the auctions yield on average
\[
\frac{1}{2}(\sigma + \lambda) \left\{ F(\hat{\alpha}) \Phi(\hat{\alpha}) + \int_{\hat{\alpha}}^{1} \Phi(\alpha_{n+1}) dF(\alpha_{n+1}) \right\}.
\]
The first term refers to the situation that \( \alpha_{n+1} \leq \hat{\alpha} \), so that provider \( n+1 \) does not win the auction. This event takes place with probability \( F(\hat{\alpha}) \). The second term refers to the situation that \( \alpha_{n+1} > \hat{\alpha} \), so that provider \( n+1 \) wins the auction, and contributes \( \Phi(\alpha_{n+1}) \) to social welfare.

Then, with \( \psi = \frac{\lambda}{\sigma + \lambda} \in (0, 1) \),
\[
S^*_n(\hat{\alpha}) \leq S_{n+1}(\hat{\alpha}) \\
\iff e^* (\hat{\alpha})^2 \leq F(\hat{\alpha}) \Phi(\hat{\alpha}) + \int_{\hat{\alpha}}^{1} \Phi(y) dF(y) \\
\iff [\hat{\alpha} - \psi h (\hat{\alpha})]^2 \leq F(\hat{\alpha})\hat{\alpha} [\hat{\alpha} - 2\psi h (\hat{\alpha})] + \int_{\hat{\alpha}}^{1} y [y - 2\psi h(y)] dy \\
\iff [\hat{\alpha} - \psi h (\hat{\alpha})]^2 \leq F(\hat{\alpha})\hat{\alpha} [\hat{\alpha} - 2\psi h (\hat{\alpha})] + [1 - F (\hat{\alpha})] \hat{\alpha}^2 + \int_{\hat{\alpha}}^{1} 2y (1 - \psi) h (y) dy \\
\iff \psi h (\hat{\alpha}) [\psi h (\hat{\alpha}) - 2\hat{\alpha} (1 - F (\hat{\alpha}))] \leq \int_{\hat{\alpha}}^{1} 2y (1 - \psi) h (y) dy.
It is now easily observed that a sufficient condition for $S_n^*(\hat{\alpha}) \leq S_{n+1}(\hat{\alpha})$ to hold is that $\psi h(\hat{\alpha}) - 2\hat{\alpha} (1 - F(\hat{\alpha})) \leq 0$, or, equivalently, $\hat{\alpha} \geq \frac{\psi}{2f(\alpha)}$.

As $f(\hat{\alpha}) = \zeta(1 - \hat{\alpha})^{-1}$ is increasing in $\hat{\alpha}$, a sufficient condition for this inequality to hold is that

$$\alpha \geq \frac{\psi}{2f(\alpha)} = \frac{\psi}{2\zeta(1 - \alpha)^{1-\zeta}}. \tag{15}$$

Recall that $\alpha$ is the unique solution to

$$y = \psi \frac{1 - F(y)}{f(y)} = \frac{\psi}{\zeta}(1 - y),$$

so that

$$\alpha = \frac{\psi}{\psi + \zeta}.$$  

Equation (15) then reduces to

$$\frac{\psi}{\psi + \zeta} \geq \frac{\psi}{2\zeta} \left( \frac{\zeta}{\psi + \zeta} \right)^{1-\zeta}$$

which is equivalent to

$$g(\zeta) \equiv \left( \frac{\zeta}{\psi + \zeta} \right)^{\zeta} \geq \frac{1}{2}.$$

Note that $g(1) = \frac{1}{2}$. Moreover, it is readily shown that

$$\lim_{y \to \infty} \frac{d \log g(y)}{dy} = 0$$

and

$$\frac{d^2 \log g(y)}{dy^2} = \frac{\psi^2}{y (\psi + y)^2} > 0$$

so that $g$ is strictly decreasing and hence $g(\zeta) \geq \frac{1}{2}$ for all $\zeta \in (0, 1]$. This completes the proof.
Figure 1
Figure 2
B Literature


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