Does it Pay to Innovate First? A Dynamic Duopoly with R&D Spillovers

Gianluca Femminis§ and Gianmaria Martini+

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§ Università Cattolica di Milano, Istituto di Teoria Economica e Metodi Quantitativi
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Department of Management and Information Technology, University of Bergamo, Italy

Abstract

We analyze a dynamic duopoly where firms have in each period the possibility to make a once-and-for-all R&D investment. The latter generates a cost saving innovation to the innovative firm and a spillover over the R&D investment cost of the non-innovative firm. We show, differently from D’Aspremont and Jacquemin [1988] where firms have an incentive to innovate immediately, that the spillover may induce a war of attrition equilibrium, where both firms would like the rival to innovate first. Last, by comparing the non-cooperative regime with the RJV case, we show that R&D cooperation may increase welfare even if the spillover is relatively small.

JEL classification: O33, L13, L41

Keywords: R&D, spillovers, war of attrition, RJV.

*Correspondence to: G. Femminis, Università Cattolica di Milano, Largo Gemelli 1, 20123 Milano, email: gianluca.femminis@unicatt.it
1 Introduction

There is a general consensus among economists and antitrust practitioners that Research Joint Ventures (RJV’s) yield a welfare improvement when R&D spillovers are sufficiently high (D’Aspremont and Jacquemin [1988]). Under these circumstances price–maker firms are induced to underinvest in R&D, since the business stealing effect due to the process innovation is small in comparison to the cost savings generated, at no cost, to rivals. In this case a R&D cooperative stage may (partially) correct this inefficiency, and so RJV’s deserve an antitrust block exemption.\(^1\)

However, this insight has been reached using two–stage models, where firms first decide the level of R&D investment and then compete in the final market. A relevant feature of this setting is that firms take their R&D decisions in a static framework. Hence they are “forced” to invest immediately in R&D, since they cannot delay it (if so R&D benefits are lost forever). Dynamics is instead an important factor in many R&D processes: innovations not producible before become feasible at a given period. Furthermore, the process of costs reduction of an innovation keeps going, since in many industries producing an innovation becomes cheaper as time goes. Hence an essential feature of R&D activities is the possibility to postpone (strategically) the investment, when the firm believes that the trade–off between R&D cost and benefit, in a competitive framework, will be greater in the future. Our aim is to explore this strategic decision within a dynamic duopoly framework with R&D spillovers.

\(^1\)See Grossman and Shapiro [1986] for a discussion of the antitrust issues involved in a R&D joint venture.
We show the following results: First, firms may be engaged either in a war of attrition (Tirole [1988]) in the R&D stage or in an equilibrium with R&D diffusion. Second, the war of attrition equilibrium is unique if R&D spillovers are high; if instead the spillovers are low both equilibria emerge. Last, RJV’s are welfare improving even when the spillovers are low (differently from D’Aspremont and Jacquemin [1988]).

A war of attrition equilibrium arises because each firm would like the other firm to be the first innovator. The intuition is that, on the one hand, R&D costs are sufficiently low that both firms want to be involved in the R&D process; on the other hand, the spillover effect is sufficiently high to make each firm to prefer the decrease in its R&D cost due to the information leaked out from the innovative firm. The equilibrium with R&D diffusion prevails instead when R&D costs are sufficiently high, so that the innovation is profitable only for one firm. Under these circumstances an innovation leader emerges within the industry, while the other firm is an innovation follower. When both equilibria emerge, the spillover is the crucial parameter for equilibrium selection: the R&D with diffusion equilibrium is chosen when the spillover is very low.

Two driving forces characterize the settings where the above equilibria arise: (1) R&D spillovers and (2) the R&D’s costs reduction process. Spillovers are due to some information that the innovator cannot lock-in within its firm, a stylized fact quite common in case of process or organizational innovation. R&D spillovers make the imitators’ R&D costs lower than that of “the innovator”. Hence spillovers matter only after that some firm has innovated, not before. The R&D’s costs reduction process is instead relevant
as long as the innovation cost becomes not prohibitively high. This implies that R&D costs for both firms will be lower in future periods even without an innovation leader. Under these circumstances rival firms face a trade-off: being the first to innovate by paying a relatively high R&D cost and getting the benefits of the business stealing effect due to lower production costs; or being the innovation follower by paying the costs of an efficiency gap for a limited temporal interval and getting the benefits of lower R&D costs due to both the R&D costs reduction process and the spillovers.

A possible consequence of such a trade-off is that the innovation is introduced much later than when it begins to be realizable. Under these circumstances an RJV might improve welfare because it allows firms to anticipate the introduction of the innovation.

In contrast with previous static contributions, we take into account that R&D spillovers produce a delayed reduction in the rival firms’ R&D costs and not an instantaneous decrease in their production costs. Reverse engineering is a good example of this: through this practice a firm can learn a new production process without incurring in the same investment costs made by the innovator (clearly we consider a framework, rather common in case of process or organizational innovations, where the innovator cannot fully protect its R&D output). Hence the first innovation yields a lower R&D costs for the imitators, i.e. the R&D activity performed within a firm may

\footnote{In many industries a certain type of process innovation, or an innovative way to organize the internal production, might become, at a given period, realizable. This means that thanks to the general advances in pure research or to the innovations obtained in another industry, the firms operating in the industry understand that an innovation might be realized, i.e. the R&D costs necessary to obtain that innovation are no longer prohibitive.}
spread some knowledge in the market. The latter may reduce the costs of R&D investment for the follower firms in the innovation process.

We investigate an infinite horizon game where firms, in each period, are involved in a two-stage interaction: first they decide whether to invest in R&D or not and then compete à la Cournot. The paper compares two competitive regimes: the non-cooperative solution, where firms compete both at the R&D stage and at the quantity stage; the RJV regime, where firms form a unique R&D department and share the benefits of innovation by jointly setting when to invest in R&D and then compete in the Cournot stage. This comparison highlights the benefits of the RJV solution with respect to the non-cooperative regime, and ranks them in welfare terms. We adopt, as in Suzumura [1992], as welfare criterion a second-best welfare function, where the oligopolistic competition in the second stage quantity game lies beyond the regulatory power of the non omnipotent government, i.e. firms cannot be forced to produce at marginal costs pricing while may rationally play the Cournot–Nash solution.

**Related work** Our paper is related with several contributions on welfare effect of RJV’s. Ordover and Willig [1985] present an interesting dynamic model where two firms decides when to enter a new product in the market, in presence of a market leader. They compare the non-cooperative solution with the RJV outcome and show that RJV’s should be judged by antitrust authorities under the rule of reason approach. They have no spillover effect. D’Aspremont and Jacquemin’s [1988] seminal contribution gave rise to several extensions: Suzumura [1992] generalized their results, Salant and Shaffer [1998] show the existence of optimal asymmetric strategies in the R&D
stage and that RJV’s may raise welfare even when there are no spillovers, Motta [1992, 1996] presents a model which enlarges the D’Aspremont and Jacquemin’s contribution to the product innovation case in a vertical differentiation context. Again, he displays that RJV’s increase the amount of research activity within the economy and so are welfare enhancing. Poyago-Theotoky [1999] investigates a model where firms can choose the level of the spillover, and shows that under the RJV solution they select a full disclosure strategy.

The paper proceeds as follows. In Section 2 we present the model. In Section 3 we study non-cooperative solution, where firms compete at both stages. In Section 4 we identify the equilibrium under the war of attrition equilibrium. Concluding comments in Section 5 end the paper. Some analytical details are provided in the Appendix.

2 The model

We consider an industry composed by two firms, 1 and 2, selling an homogeneous good and competing à la Cournot. Market demand is linear and equal to: \( P = a - bQ \), where \( P \) is the market clearing price and \( Q = q_1 + q_2 \) is the total quantity supplied. Each firm has a unit cost of production \( c \); if the firm undertakes an R&D activity it obtains a process innovation which, in turn, reduces the unit production cost by an amount \( x \), with \( x \ll c \). Hence firm \( i \)’s \( (i = 1, 2) \) post-innovation production cost is \( C(q_i) = (c - x)q_i \). The decision to undertake R&D involves an investment cost to the innovative firm, that will be modeled in Section 3.

Firms compete with an infinite horizon, and discount future profits at the
common rate $r$. In each period $t$ firm $i$ decides whether to invest in R&D or not. If both firms do not invest in period $t$, the outcomes in the Cournot subgame at $t$ coincide with those of the pre–innovation stage. If we define $A = a - c$, they are the following: the equilibrium individual output is

$$q_i^{00} = \frac{A}{3b}$$

where $\{00\}$ indicates that both firms do not innovate at $t$ and so they are the same as in the status quo. The individual profit at $t$ is

$$\pi_i^{00} = \frac{(A)^2}{9b}$$

Given that the market clearing price is $P^{00} = \frac{a+2c}{3}$, welfare (computed à la Marshall) is equal to:

$$W^{00} = \frac{4}{9} \frac{(A)^2}{b}$$

Expression (1) identify our welfare criterion, i.e. a second–best welfare function, since, as in Suzumura [1992], we assume that any rational firm will never select, in a free market economy, to produce at the marginal cost pricing level. Hence (1) is the reference welfare if both firms do not innovate at time $t$.

If instead only firm $i$ innovates at $t$ ($i = 1, 2$, $i \neq j$), it benefits of an efficiency advantage. In this case the individual quantity in the Cournot subgame are:

$$q_i^{10} = \frac{A + 2x}{3b}, \quad q_j^{01} = \frac{A - x}{3b}$$
where \( \{10\} \) indicates that firm \( i \) has invested in R&D and innovated while firm \( j \) has not. The above quantities show that the more efficient firm has a higher market share (with respect to both the non-innovative firm’s share and to its pre-innovation stage share); the corresponding price is \( p^{10} = \frac{a+2c-x}{3} \) (the unilateral innovation yields a lower price). The two individual profits at \( t \) under this industrial configuration are:

\[
\pi_{i}^{10} = \frac{(A + 2x)^2}{9b}, \quad \pi_{j}^{01} = \frac{(A - x)^2}{9b}
\]

which implies that the innovative firm gets a higher profit that the non-innovative firm. Last second-best welfare under these circumstances is:

\[
W^{10} = \frac{8A(A + x) + 11x^2}{18b} \tag{2}
\]

Last, we need to compute the outcomes in the Cournot subgame if both firms innovate at period \( t \). In this case the individual output is equal to:

\[
q_{i}^{11} = \frac{A + x}{3b}
\]

where \( \{11\} \) indicates that both firms have innovated. It is evident that both firms produce an higher quantity w.r.t. the status quo because they are more efficient. This implies that the market clearing price is lower than the pre-innovation one and \( t \) is equal to \( p^{11} = \frac{a+2c-2x}{3} \), while individual profits are higher, and correspond to:

\[
\pi_{i}^{11} = \frac{(A + x)^2}{9b}
\]

Society does benefit from the innovation adoption since welfare is also greater than the pre-innovation level, since we have:
\[ W^{11} = \frac{4(A + x)^2}{9b} \]  

(3)

In each period of the infinite horizon game one of the three above industrial configuration arises, with the following restriction: if firm \( i \) has decided to invest in R&D at period \( t \), it will benefit from the process innovation thereafter, and so in each period its profit might be either \( \pi_i^{10} \), if the other firm has not innovated yet (the latter gets \( \pi_j^{01} \)), or \( \pi_i^{11} \), if its rival has also innovated.

We will investigate two market regimes: the non-cooperative regime, where firms compete both in the R&D and in the market stage, and the RJV regime, where firms jointly select the period to innovate and then compete in the market stage. The welfare levels shown in (1)–(3) are per-period (instant) welfare. The R&D investment decisions clearly depend upon the discounted stream of profits generated by firms’ action in each period. Hence to rank the two regimes in welfare terms, we will consider the discounted stream (from the first period to \( \infty \)) of instant welfare, according to firms’ R&D decisions.

3 The non-cooperative solution

3.1 Innovation costs

For the first firm introducing the innovation, the innovation cost evolves over time according to the following equation:

\[ C_1(t) = \gamma x \exp(-\rho(t - t_0)), \text{ when } t \geq t_0. \]  

(4)

Hence, we are assuming that the innovation becomes technically feasible
at time $t_0$ at a cost ($\gamma x$), which then decreases at the constant rate $\rho$ thanks to advances in pure research and to the availability of new results obtained in related fields. Of course, this form of technical progress is exogenous to single firms. Alternatively, one might think that $\rho$ stylizes a market dimension effect related to the increase in aggregate income: an increase in profits per period lowers the innovation cost in relative terms. We assume that the initial cost is very high (immediately introduce the technical assumption?)

As for the second firm introducing the innovation, the cost evolution is described by the following equation:

$$C_2(t) = \gamma x \exp(-\rho(t - t_0))[\theta + (1 - \theta) \exp(-\mu(t - t_1))], \text{ when } t > t_1. \quad (5)$$

where $t_1$ is the calendar time when the first firm has introduced the innovation, $\theta \in [0, 1]$ and $\mu \in [0, \infty)$. Equation (5) describes a situation where the innovation may be only partially appropriable: whenever $\theta < 1$, and $\mu > 0$, the second comer enjoys a reduction in cost when imitates its competitor at any $t > T_1$. Notice that $\mu$ is a parameter capturing the "imitation ease": the higher $\mu$, the higher the speed in cost reduction for the second entrant. The ease of imitation depends upon many features: the nature of the innovation, the working of the legal system, the imitation effort set up by the potential imitator(s) and so on. We shall limit our attention to situations where $\mu$ is exogenous (justify this??). $\theta$ represent the extent of the cost reduction when all the relevant information have leaked out of the innovating firm. In what follows we will largely rely on the following simplified version of the cost function:
\[ \tilde{C}_2(t) = \theta \gamma x \exp(-\rho(t - t_0)). \] (6)

Notice that \( \tilde{C}_2(t) = \lim_{\mu \to \infty} C_2(t) \). What we are assuming is that the cost reduction is immediate: as soon as the first firm invests, the cost for the second is reduced to a share \( \theta \); the cost then goes on decreasing due to the exogenous technical progress. Clearly, this assumption stylizes an extreme form of spillover and - admittedly - it is adopted because it allows many explicit results. Equation (5) will be used in the numerical part of the paper.

### 3.2 Second firm

Following Fudenberg and Tirole (1985) and Weeds (2002) we start characterizing the optimal strategy for the second entrant. When the first firm has already sunk the innovation cost, the payoff at time \( t > t_1 \) for the second firm, when it invests at \( t_2 \), is:

\[
V_2(t, t_2) = \int_t^{t_2} \frac{(A - x)^2}{9b} \exp(-r(\tau - t))d\tau + \\
+ \int_{t_2}^{\infty} \frac{(A + x)^2}{9b} \exp(-r(\tau - t))d\tau - \tilde{C}_2(t_2) \exp(-r(t_2 - t)).
\] (7)

The solution of problem (7) yields the second comer’s optimal strategy, which is to enter at:

\[ t_2 = T_2 \equiv t_0 - \frac{1}{\rho} \ln \left( \frac{4A}{9b\gamma(r + \rho)\theta} \right). \] (8)

Comparative statics: an increase in \( A \) or a decrease in \( b \) induce an expansion in per period profit and hence anticipates the decision to innovate;
an increase in $\gamma$ and $r$ delays the investment decision, since the innovation is more costly or the future is more heavily discounted. The technical progress parameter $\rho$ plays an important role: its increases induce a relevant delay in investment since the firm rationally decides to wait in order to grasp the advantage of a faster reduction in innovation costs. Notice also that the higher is $\theta$ (the lower is the spillover), the farther away in the future is the adoption date, since the second comer’s cost are higher.

Substituting the optimal adoption time (equation (8)) into (7), we obtain the maximum value for the second comer, computed at the generic time $t > t_1$:

$$V_2(t, T_2) = \left( \frac{A - x}{gbr} \right)^2 + \exp(-r(t_0 - t)) \left( \frac{4A}{9b\gamma(r + \rho)\theta} \right)^{r/\rho} \left( \frac{4Ax\rho}{9br(r + \rho)} \right)$$ \hspace{1cm} (9)

When $t \leq t_1$, the maximum value for the second firm depends also on the investment date for the first firm: hence, it is introduced as $V_2(t, t_1, T_2) = \frac{A^2}{gbr}[1 - \exp(-r(t_1 - t))] + \exp(-r(t_1 - t))V_2(t_1, T_2)$, which is the value of being the second entrant, computed at time $t$ when the first innovators sinks the cost at $t_1$ and the second entrant invests at the optimal date $T_2$.

### 3.3 First firm

We now characterize the behavior for the first entrant. This firm maximizes its payoff (at a generic time $t$) by choosing $t_1$, knowing that the second will enter at $T_2$:
\[ V_1(t, t_1, T_2) = \int_t^{t_1} \frac{A^2}{9b} \exp(-r(\tau - t))d\tau + \int_{t_1}^{T_2} \frac{(A + 2x)^2}{9b} \exp(-r(\tau - t))d\tau \\
+ \int_{T_2}^\infty \frac{(A + x)^2}{9b} \exp(-r(\tau - t))d\tau - C_1(t_1) \exp(-r(t_1 - t)) \]

(10)

Solving problem (10), the first adopter obtains its profit-maximizing strategy, which is to enter at:

\[ t_1 = T_1^* \equiv t_0 - \frac{1}{\rho} \ln \left( \frac{4(A + x)}{9b\gamma(r + \rho)} \right) \]

(11)

The comparative statics is similar to the one described for \( T_2 \), but for two aspects. First, the spillover parameter \( \theta \) plays no role. To understand this point, notice that the profit-maximizing adoption date \( T_1^* \) is obtained balancing three forces. An increase in \( T_1^* \) lengthens the period in which the instantaneous profits are at the pre-innovation level (which is, the first integral in equation (10)), and reduces the present value of the innovation cost. These two element concur in increasing \( V_1(t, T_1^*, T_2) \). However, the increase in \( T_1^* \) decreases in the time span in which the instantaneous profit is at its highest, since the firm is the unique innovator (the second integral in equation (10)), which induces a reduction in \( V_1(t, T_1^*, T_2) \). The spillover level is not relevant in determining the quantitative importance of any of these three effects. Second, \( T_1^* \) positively depends also on the per unit cost reduction \( (x) \). This is simply due to the fact that lower costs increase profits both directly and through an increase in the quantity traded on the market.

\(^3\)The dependence of the adoption time \( T_2 \) from \( x \) did not come un in equation (8)
Substituting the profit-maximizing adoption date $T_1^*$ and $T_2$ (from (8)) into (10), we obtain the maximum value for the preempting firm, computed at time $t \leq T_2$:

$$V_1(t, T_1^*, T_2) = \frac{A^2}{9br} + \exp(-r(t_0 - t))$$

$$ + \left[ \frac{4(A + x)}{9b\gamma(r + \rho)} \right]^{r/\rho} \frac{4(A + x) x \rho}{9br(r + \rho)} - \left( \frac{4A}{9b\gamma(r + \rho) \theta} \right)^{r/\rho} \frac{(2A + 3x) x}{9br} \tag{12}$$

The entries at times $T_1^*, T_2$ can be an equilibrium only for some specific particular values.

**Proposition 1** When $\theta < \bar{\theta} \equiv \frac{A}{A+x} \left( 1 + \frac{4\rho}{(6A+3x)\rho+(2A-x)\theta} \right)^{\rho/r}$, the entry equilibrium is: $\min\{T_1, t_0\}, T_2$, where $T_1$ is such that, at $t_1 = T_1 < T_1^*$, $V_1(t, t_1, T_2) = V_2(t, t_1, T_2)$.

Proof: Consider that: $V_1(t_1, t_1, T_2) = \frac{A^2}{9br}[1 - \exp(-r(t_1 - t))] + \exp(-r(t_1 - t))V_1(t_1, t_1, T_2)$ and that $V_2(t_1, t_1, T_2) = \frac{A^2}{9br}[1 - \exp(-r(t_1 - t))] + \exp(-r(t_1 - t))V_2(t_1, T_2)$. Hence, if $V_1(T_1^*, T_1^*, T_2) > V_2(T_1^*, T_2)$, then also $V_1(t, T_1^*, T_2) > V_2(t, T_1^*, T_2)$. From (12) and (9), respectively, one immediately obtains:

$$V_1(T_1^*, T_1^*, T_2) = \frac{A^2}{9br} + \frac{4(A + x) x \rho}{9br(r + \rho)} - \left( \frac{A}{(A + x) \theta} \right)^{r/\rho} \frac{(2A + 3x) x}{9br},$$

and

because the second firm, by adopting, moves from an instantaneous profit level of $\frac{(A-x)^2}{9b}$ to a profit level of $\frac{(A+x)^2}{9b}$. 14
\[ V_2(T_1^*, T_2) = \frac{(A - x)^2}{9br} + \left( \frac{A}{(A + x)\theta} \right)^{r/\rho} \left( \frac{4Ax\rho}{9br(r + \rho)} \right). \]

Simple calculations show that \( V_1(T_1^*, T_1^*, T_2) > V_2(T_1^*, T_2) \) when \( \theta < \bar{\theta} \). We now follow the argument developed in Fudenberg and Tirole (1985): when \( V_1(T_1^*, T_1^*, T_2) > V_2(T_1^*, T_2) \), it is in each firm’s interest to adopt at time \( T_1^* \) if the other firm does not adopt at that time. But if a firm knows that the other will adopt at time \( T_1^* \), it is in its interest to preempt at time \( T_1^* - dt \). By backward induction, at any time \( t_1 \) such that \( V_1(t_1, t_1, T_2) > V_2(t_1, T_2) \), any firm wants to invest and anticipate the other to avoid be preempted later on. Since \( V_1(t_1, t_1, T_2) > V_2(t_1, T_2) \) implies \( V_1(t_1, t_1, T_2) > V_2(t, t_1, T_2) \), we now show that these two function must cross. Notice that \( \frac{\partial V_1(t_1, t_1, T_2)}{\partial t_1} > 0 \) for \( t_1 < T_1^* \); moreover, for \( t_1 < T_1^* \), \( \frac{\partial^2 V_1(t_1, t_1, T_2)}{(\partial t_1)^2} > 0 \). Notice, also that \( \lim_{t_1 \to T_2} V_1(t_1, t_1, T_2) < \lim_{t_1 \to T_2} V_2(t_1, T_2) \) for \( \theta \in [0, 1) \).\(^4\) Since \( \frac{\partial V_1(t_1, T_2)}{\partial t_1} > 0 \) and \( \frac{\partial^2 V_1(t_1, T_2)}{(\partial t_1)^2} > 0 \) for \( t_1 < T_2 \), we can represent \( V_1(t, t_1, T_2) \) and \( V_2(t, t_1, T_2) \) as in Figure 1.

In words, when \( t_0 > T_1 \), which is, when the initial cost is low enough, one of the two firms invests as soon as the innovation becomes technically feasible, while the other waits to invests at time \( T_2 \). On the other hand, when the initial cost is high, one of the two firms waits to invest at \( T_1 \), and the other waits to invests at time \( T_2 \).

Corollary 1. When \( \theta < \bar{\theta}, T_1 \leq T_2 \) (By inspection of equations (11) and (8)).

Corollary 2. When \( \theta > \bar{\theta} \), no firm firm has interest to preempt the other.

\(^4\)The economic intuition for this point is interesting: when the second firms enjoys a positive spillover and it invests at "approximately" the same time the first comer invests, then it intertemporal profits must be higher.
and invest at $t_1 < T_2$.

Notice that, this case could not arise in Fudenberg and Tirole (1985) and in Weeds (2002). In fact, it is the presence of a spillover which induces the possibility that the second entrants enjoys profits that are higher than the ones grasped by the first innovator.

4 War of attrition

We have just shown that, in presence of a positive spillover, the equilibrium characterized by the "diffusion" of the innovation may not exist. As we remarked above, the diffusion equilibrium can turn out into a situation where the "first mover" obtains a profit that is lower that the one obtained by the follower. Hence, being the first innovator may be costly in the sense that this action implies the dismissal of the possibility to obtain later higher profit as a second entrant. This situation closely resemble a classic war of attrition (as described e.g. in Tirole, (1988)). In that model, two firms produce a non-differentiated goods and the presence of a fixed cost induces losses. However, every oligopolist stays in the market - incurring in some loss - if it knows that there is a sufficiently high probability that its rival drops out of the market, because in this case it will grasp higher future profits. In this framework, any firm decide to postpone its innovation decision - giving up some profits - if it reckons that the probability that its rival decide to innovate is sufficiently high, because in this case it will grasp the higher profits accruing to the second innovator.

Hence, we model the equilibrium as a war of attrition.
Proposition 2  (a) When $t < T_2$, ; (b) when $t \geq T_2$, every firm innovates first with probability $(1 - \exp(-qdt))$ where $q$ can be approximated to:

$$q = \begin{cases} 
0 & \text{for } t < \hat{T} \equiv t_0 - \frac{1}{\rho} \ln \left( \frac{2A+x}{9b\gamma(r+\rho)} \right) \\
\frac{1}{1-\theta} \left[ \frac{2A+x}{9b\gamma} \exp(\rho(t-t_0)) - (r+\rho) \right] & \text{for } t \geq \hat{T} \equiv t_0 - \frac{1}{\rho} \ln \left( \frac{2A+x}{9b\gamma(r+\rho)} \right)
\end{cases}$$

(b). Suppose that firm $\alpha$ faces a probability $(1 - \exp(-qdt))$ of investment on behalf of firm $\beta$. At any instant of time $t$, the value of firm $\alpha$, if it decides not to invest in the innovative process is:

$$V_{\alpha NI}(t) = (1 - \exp(-qdt)) \left\{ \frac{(A-x)^2}{9b} dt + \exp(-r dt) \left[ \frac{(A+x)^2}{9br} - \theta \gamma x \exp(-\rho(t + dt - t_0)) \right] \right\} + \exp(-qdt) \left\{ \frac{A^2}{9b} dt + \exp(-r dt) [V_\alpha(t + dt)] \right\}$$

(13)

The second line in equation (13) express the fact that - if firm $\alpha$ does not invests while the other does - it obtains a low profits for the time span $dt$; then it invests, sinking the cost at time $t + dt$ and its per period profit increase due to the cost reduction and to the increase in production. This term, which is grouped in the big square brackets, is discounted back to time $t$. The third line in equation (13) is given by the fact that - when nobody invests - the two firms obtain the pre-innovation profits for the time span $dt$; then firm $\alpha$ has again the option to invest, the value of which is discounted back to time $t + dt$.

If firm $\alpha$ decides to invest at time $t$, it must consider that it will be followed by its competitor at time $t + dt$. Hence, its maximum value is:
\[ V^I_\alpha(t) = \frac{(A + 2x)^2}{9b} dt - \theta \gamma x \exp(-\rho(t - t_0)) + \exp(-rdt) \frac{(A + x)^2}{9br} \] (14)

The first addendum represents the profits obtained being the unique innovator for a time span of length \( dt \), the second is the cost of the innovation while the third represents the discounted stream of profits obtained from time \( t + dt \) (when firm \( \beta \) sinks the innovation costs) onwards.

Notice that we have ignored the probability that firm \( \beta \) invests at time \( t \), too. Since \( (1 - \exp(-qdt)) \simeq -qdt \), it involves \((dt)^2\) terms, which can be ignored.

Notice that - if no firm has invested at time \( t \) - the value of introducing the innovation at time \( t + dt \) for firm \( \alpha \) is:

\[ V^I_\alpha(t + dt) = \frac{(A + 2x)^2}{9b} dt - \theta \gamma x \exp(-\rho(t + dt - t_0)) + \exp(-rdt) \frac{(A + x)^2}{9br} \]

Since \( \exp(-\rho dt) \simeq 1 - \rho dt \), the expression above can be written as:

\[ V^I_\alpha(t + dt) = V^I_\alpha(t) + \theta \gamma x \rho dt \] (15)

Firm \( \beta \) must set - at each instant of time - the probability \((1 - \exp(-qdt))\) to keep firm \( \alpha \) indifferent between investing and non investing. Hence:

\[ V^I_\alpha(t) = V^{NI}_\alpha(t) \] and \[ V^I_\alpha(t + dt) = V^{NI}_\alpha(t + dt) = V_\alpha(t + dt) \]

Substituting expressions (14) and (15) into (13) we obtain the probability intensity of innovation for firm \( \beta \), which is \( q \), and the value for firm \( \alpha \). These computations are performed using the approximation \( \exp(-x dt) \simeq 1 - x dt \), and dropping \((dt)^2\), and complete the proof for part (b).
5 Summary and Conclusions

We have analyzed firms’ decision on when to invest in R&D and introduce a costs reduction innovation in a dynamic duopoly with R&D spillovers and a R&D costs reduction process. This settings enriches the analysis on the impact of spillovers in the decentralized market and highlights the welfare improvements due to firms’ coordination in the R&D activity (RJV’s).

We have shown that two equilibria in the R&D activity emerge: (1) war of attrition and (2) R&D diffusion. Both equilibria are dominated, in welfare terms, if firms can be engaged in a RJV’s. The latter is true even if R&D spillovers are low, differently from D’Aspremont and Jacquemin [1988]. The intuition is that under the decentralized equilibrium R&D spillovers play a major role in firms’ strategic interdependence and induce them to delay the period of the innovation. The latter is always true unless the spillover effect is very low. Hence our policy implication is that antitrust block exemptions concerning rival firms engaged in only R&D cooperation should be in general acknowledged, unless R&D spillovers are negligible. The authority should only tests for the presence of R&D spillovers and not measure their levels.

This paper has only analyzed process or organizational innovations. The dynamic analysis of the introduction of product innovation is left for future research.
References


APPENDIX A