Title:
“How much efficiency gains and price reductions to put as ingredients into an efficiency defense? ‘Quanto Basta’”

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Potential efficiency gains due to a merger can be used by competition authorities to judge upon proposed mergers. In a world where agents’ efforts, observable or unobservable, affect the success of a production cost reducing project that may be conducted as a stand-alone firm or in a merger, we characterize the merger decision and the type of errors a competition authority may make when it relies on an efficiency defense. In addition, we show that the occurrence of either type of errors is always smaller under the unobservable efforts assumption, than under the observable efforts one.

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JEL Codes: D23, D82, L12, L23, L41

Young Economists Award: I would like to be considered (I got my PhD in March 2003)
How much efficiency gains and price reductions to put as ingredients into an efficiency defense? ’Quanto Basta’*

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Abstract

Potential efficiency gains due to a merger can be used by competition authorities to judge upon proposed mergers. In a world where agents’ efforts, observable or unobservable, affect the success of a production cost reducing project that may be conducted as a stand-alone firm or in a merger, we characterize the merger decision and the type of errors a competition authority may make when it relies on an efficiency defense. In addition, we show that the occurrence of either type of errors is always smaller under the unobservable efforts assumption, than under the observable efforts one.

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1 Introduction

In recent years, a wave of horizontal mergers has occurred giving rise to more concentrated market structures in many industries. Historically, such a higher concentration has been viewed as detrimental to welfare as it often leads to a reduction of competitiveness in the respective market. However, mergers usually do not only have an impact on the output

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market side but also on the production processes, the organizational structures, the relations to suppliers of intermediate inputs or even on the financial resources of the firms through a relaxation of credit constraints. The effects of mergers can be better understood when looking at what drives the decisions of firms on whether to merge or not. On the one side, we may think that potential synergies, efficiency gains, reduction of internal organizational costs or the increase of market power are considered positively by merging parties. On the other side, concerns about the future division of control rights, the loss in control over the actions of the management, conflicts about the sharing of potential profits may be the responsible for potential merger not to come to existence.

In our paper, with their merger decision, the merging parties trade off between potential synergies, the coordination in giving incentives within the firm(s), as well as the increase of market power on the one hand and the loss in control over the actions of the management on the other one. Potential synergies and the coordination of the incentive schemes within the firm(s) are responsible for lower expected unit production costs and, thus, are beneficial for social welfare.

Therefore, not only from a private, but also from a social point of view, it is far from clear whether a merger is beneficial. There may exist, thus, a need for competition authorities to discriminate between mergers that are harmful to the society’s objective functions, or standards, either the consumers’ surplus or the social welfare, from the ones that instead may enhance either of them.

In their decisions on horizontal merger cases, competition authorities may accept a so called efficiency defense, according to which they can allow mergers on the basis of "merger specific, substantial efficiency gains that are likely to be passed on to consumers via price reductions¹" (Farrell and Shapiro, 1990 and 2001, provide an excellent assessment of this efficiency defense analyzing efficiency gains in horizontal mergers). The US Antitrust Law allows explicitly for this defense, and, even though the European Competition Law does not explicitly account for it, it is not incompatible with the use of it. The rationale behind allowing for an efficiency defense is that mergers usually do not only have an impact on the competitiveness of the market for the products sold by the parties but also on their production processes and their organizational structures. This paper takes this rationale literally and models the effects of an efficiency defense if the efficiency gains are endogenously

¹See e.g. the FTC 1992 Horizontal Merger Guidelines, section 4.
determined through incentives to innovate with and without the merger.

Merging parties have the burden of proving whether efficiencies can be reached according to the US Antitrust law, while in Europe the competition authorities are responsible for collecting and processing the relevant information to justify their decision upon a merger. This is a difficult task for both, the merging parties and the competition authorities. The merging parties very often are not able to produce the hard evidence for potential efficiency gains, and, on the other side, the competition authority cannot judge perfectly upon other possible configurations as the information that it possesses comes often from the merging parties themselves. There exist attempts to measure the efficiency gains that a merger would lead to. However, the efficiency gains considered in these studies are especially the ones associated with decreases in the production costs, transaction costs, or with the internalization of costs that cancel out within the merged entity eventually. To our knowledge, no attempt exists to qualify the costs parties may have to incur for compensating the managerial effort which has to be exerted for the realization of the mentioned efficiencies gains a merger may lead to, whenever this effort is crucial in determining their magnitude.

In this paper we allow for efficiency gains that may come from the exploiting synergies on the management efforts which are responsible for productive efficiency. This is done by considering firms willing to pursue a production cost reducing project to be conducted by agents, either in a stand-alone situation or in a merger. Agents are able to affect the success of the project, therefore, to affect the realized costs in the industry and their efforts are either observable or unobservable.

We determine the private decision to merge as a function of the interaction between these agents’ efforts when they work together in a merger, as compared to the stand-alone situation, as well as the parameters of the industry, such as the initial costs firms face, and the potential for cost reduction. We show that mergers happen either when the potential for cost reduction is low, i.e. due to pure market power considerations, or when managerial efforts are close substitutes, or alternatively slight duplicates, i.e. when synergies induced by implemented contracts are present.

Once the private decision has been obtained, we match it with its impact on both standards, the consumers’ surplus and the social welfare. The reason for exploring both of them is that, contrary to what it is often argued\(^2\), results of our analysis suggest that the one

\(^2\)Motta (2004) argues that "[...] Article 2.1 of the Merger Regulation accepts in principle an efficiency defense "provided that it is to consumers’ advantage". These provisions might indicate that consumer welfare
standard may be positively affected by a given merger while the other is not, or vice versa. We will discuss more in detail the motivation behind this choice as well as the impact of privately taken decisions over either standards in the section devoted to the policy analysis.

Result will show that an efficiency defense based on potential efficiency gains due to a merger is generally too lax a requirement — some bad mergers would be accepted — for the observable efforts case, and, for both informational assumptions, when the initial costs are not too high. Requiring substantial efficiency gains may reduce this distortion for these mentioned cases. However, requiring substantial efficiency gains for either the unobservable case when initial costs are high, or for a larger range of costs when the standard used is the social welfare, may be too strict: some good mergers would be refused. As a general result, efficiency gains guarantee an enhancement in both consumers’ surplus and social welfare more often when efforts are not observable. In this case, the efficiency due to the merger can replace can be reached without an explicit requirement of a reduction in expected prices. Otherwise, the two requirements may be used alternatively, or in the right proportions, adjusting them ‘quanto basta’.

Our work is mainly related to three different strands of literature. First, it is connected to the literature on research joint ventures and R&D cooperation in the spirit of Kamien, Muller, and Zang (1992) and d’Aspremont and Jacquemin (1988). As in Kamien et al., we also characterize a project as a reduction in the associated production costs of a good, even though in our model this project does not necessarily come from a joint research and development: when firms decide to stay alone, they can conduct this project separately as well. Second, our work is in line with a recent literature on the endogenous formation of partnerships for specific projects, as e.g. the work of Espinosa and Macho-Stadler (2003). However, our function for the probability of success of the project includes a parameter that captures how the managers/agents work together, i.e. the degree of substitutability of their efforts in the ”production” of the success of the project. With this approach, contrary to what has been considered in the literature on moral hazard, we depart from the standard literature on joint projects allowing for efforts to be substitutes, duplicates or complements. This allows us to consider more than one specific degree of complementarity between agents’ efforts as it has been extensively studied for example in the team production literature; and

is among the ultimate objectives of competition law. However, I am not aware of any statement of the ECJ on this point, nor of any (Commission or Court’s) decision where reliance on either standard has made a difference in practice.
also does not limit us to cases where agents’ efforts have to duplicate necessarily in a very specific way as it is the case in the banking literature when it looks for example at the incentives for external monitoring to be assigned to one or more banks.

We furthermore differ from the existing literature on endogenous partnership formation in our way of using a principal-agent-framework as opposed to a double-sided moral hazard one. A third related strand of literature is the principal-agent literature. We use Rogerson’s (1985) first order approach to the ”standard” agency models with hidden action. Given a configuration, agents take a non-observable action from a continuous interval which influences the expected payoff of the project. Principals write contracts on the realization of the payoff and reward agents accordingly. In the case of joint projects with multiple agents, we use the multi-agent single principal framework common to the moral hazard in teams literature. Important work on incentives and team production includes Alchian and Demsetz (1972) and Holmström (1982). A closely related work from that literature is Itoh’s (1991) paper on endogenous team production. He shows that giving incentives to help, i.e. inducing team production, is optimal if own effort and helping effort are complementary. Contrary to his approach, we do not model complementarity/substitutability coming from the form of the agents’ disutility of providing effort. In our model instead, efforts are substitutes to a varying degree in the probability of success they induce. As a result, it is not optimal anymore to induce team work if efforts are complementary since it is too expensive to induce a high effort of both agents.

In the way we allow for different degrees of substitutability between agents’ efforts we adopt the modelling used in Fabrizi and Lippert (2005). In that paper we analyze the organizational choice of entrepreneurs pursuing a product innovation project, either alone or jointly, in the presence of moral hazard behavior on their agents’ side. Contrary to the product innovation approach considered there, in the present paper, we instead allow for a process innovation and market power considerations are made possible as a consequence so that, in addition, we are able to provide insights on the policy implications of mergers.

In this paper, even though we allow for the management to be responsible of the possible synergies that may arise though merger, the management is not able to decide directly whether a merger is going to take place or not. When the management is allowed to enter actively this type of decisions, additional constraints to the ones we will explore in this analysis will have to be considered. This is done by Lippert (2005) where the management
proposes the mergers at the first place and possesses superior information on the synergies and, therefore, the profitability that a merger can bring about.

The rest of the paper is organized as follows. In section 2, we introduce the model, section 3 is devoted to the private decision of firms about the merger, without and with moral hazard, section 4 considers the policy analysis, and section 5 concludes.

2 The model

We consider a situation where a good with demand \( Q = 1 - P \) is exchanged in the economy. This good is initially produced by two firms \( i = 1, 2 \) at a unit production cost equal to \( c \leq 1 \). Each firm employs one agent. We assume that there exists a project that in the case of success leads to a lower unit production cost \( \beta c \), with \( \beta \in [0, 1] \). The probability of success of the project thus measures the capacity of a given firm to reduce its unit production costs.

The project can be conducted by each firm alone — we will refer to this case as a stand-alone situation, \((S)\) — or together with the other firm by merging. If each firm conducts the project alone, then each project will be conducted by one agent. However, if firms decide to merge, the agents previously employed by each firm will work jointly in the project — we will refer to this case as a merger, \((M)\).

In our model, the agent(s) affect the probability of success or failure of the project they conduct through their chosen effort. Depending on the assumptions on the observability of efforts, this may result in a moral hazard problem. We assume both: The agents exert either an observable or a non observable, therefore not-contractible, effort \( e \) which is a continuous choice from the interval \([0, 1]\). Exerting this effort \( e \) implies a disutility for the agent that is equal to \( c_i (e_i) = \frac{1}{2} e_i^2 \). Agents receive a transfer \( t_i \) from their respective firm (stand-alone case) or from the merged entity (joint project). They are risk neutral and their utility is additively separable between effort and money, \( U_i = u_i (t_i) - c_i (e_i) = t_i - \frac{1}{2} e_i^2 \). However, we assume that agents have limited liability so that for any state of nature they have to receive a non-negative transfer.

In the next subsections we will describe the possible configurations that these decisions may lead to.

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\(^3\)We normalize the level of the initial cost such that it varies between zero and one. It is positive, but it cannot exceed the willingness to pay for the good to be exchanged in the economy in order to allow for non-negative gross profits.
2.1 Pre-merger

This is the case where two separate firms $i = 1, 2$, competing à la Bertrand, decide to undertake each the production cost reducing project on their own. Each firm employing one agent, the success probability of each project undertaken will be defined as $p_i(S) = e_i$, i.e. is equal to the effort exerted by agent $i$. We assume the probabilities of success in the two firms to be independent.

Given the Bertrand competition assumption, a firm receives a non-zero gross profit only if its own project succeeds while the one of the other firm does not. Given this assumption, the efficiency on the production side of the economy will be given by:

$$E(S) = p_1(S) + p_2(S) - p_1(S)p_2(S)$$

The success of one firm may lead to either a drastic or a non-drastic innovation.

It is drastic when the monopoly price associated to it is lower than the initial unit production costs, i.e.:

$$\frac{1 + \beta c}{2} \leq c \quad \Leftrightarrow \quad \beta \leq \frac{2c - 1}{c}$$

If it is, the succeeding firm can charge the monopoly price while the other one exits because at that price it cannot recover its costs.

If it is non-drastic, the succeeding firm can charge a price at most equal to the cost of the rival firm. In this case, we assume the unsuccessful firm not to make any sales$^4$. Firms may face a drastic or non-drastic innovation, depending on the combinations of the parameters $\beta$ and $c$. As these parameters can take values between zero and one, it is straightforward to show that any innovation is non-drastic for $c \in \left[0, \frac{1}{3}\right]$, it might be either drastic or not depending on the combination of $\beta$ and $c$ for $c \in \left[\frac{1}{3}, \frac{1}{2}\right]$, it is certainly drastic for $c \in \left[\frac{1}{2}, 1\right]$.

If both firms have either a high production cost, i.e. both projects fail, or a low production cost, i.e. both projects succeed, they both charge a price equal to their marginal costs and make zero gross profits.

As discussed before, the probabilities of success of each firm are independent of each other, so that for each of the four different states of nature (failure of one firm while the other does not, and, vice versa, failure of both, success of both of them) we can associate the following

$^4$This assumption is the limit case for the successful firm pricing slightly below the rival’s high cost which would induce it to exit the market.
probabilities: \( \Pr(\beta_c, c) = e_1 (1 - e_2), \Pr(c, \beta_c) = (1 - e_1) e_2, \Pr(c, c) = (1 - e_1)(1 - e_2), \) and \( \Pr(\beta_c, \beta_c) = e_1 e_2. \)

Given these assumptions, w.l.o.g. we can write the profit of firm 1, gross of the transfer to be paid to its agent, as follows:

\[
\pi_1(\cdot, \cdot) = \begin{cases}
\pi_1(\beta_c, c) = \frac{(1 - \beta_c)^2}{4 c (1 - c) (1 - \beta)} & \text{if } \beta \leq \frac{2c-1}{c} \\
\pi_1(\beta_c, \beta_c) = 0 & \text{otherwise}
\end{cases}
\]

In the same way, we can summarize the prices for the different realizations of the unit production costs as:

\[
P(\cdot, \cdot) = \begin{cases}
P(\beta_c, c) = P(c, \beta_c) = \frac{1 + \beta_c}{c} & \text{if } \beta \leq \frac{2c-1}{c} \\
P(\beta_c, \beta_c) = \beta c & \text{otherwise}
\end{cases}
\]

Let us characterize now also the corresponding levels of consumers’ surplus, \( CS(\cdot, \cdot) \), associated with each possible state of nature:

\[
CS(\cdot, \cdot) = \begin{cases}
CS(\beta_c, c) = CS(c, \beta_c) = \frac{(1 - \beta_c)^2}{8 (1-c)^2} & \text{if } \beta \leq \frac{2c-1}{c} \\
CS(\beta_c, \beta_c) = \frac{1-\beta_c}{2} & \text{otherwise}
\end{cases}
\]

As for the prices and profits that prevail in the economy when one firm succeeds in reducing its costs while the other does not, for this case as well two different levels of consumers’ surplus are induced depending on whether the succeeding firm may charge a monopolistic price or not. In addition, when both firms succeed in reducing their production costs they will go on competing à la Bertrand, they will propose a final price equal to the reduced production cost, \( \beta_c \), which explains the second expression the consumers’ surplus may take. Finally, when both firms fail in reducing their production costs they will go on competing à la Bertrand, but this time they will set their final price equal to the initial production cost, \( c \), leading to the last expression listed above in the consumers’ surplus matrix.

Note that in the stand-alone situation, a process innovation never makes consumers worse off: it always leads to a (at least weak) consumers’ surplus increase as compared to
the situation without the innovation.

As already introduced, the consumers’ surplus will be used in our analysis as one possible objective function the competition authority cares about. The alternative objective function that we will consider will be the social welfare. In this case, the consumers’ surplus will enter the expression of the social welfare that we will describe in detail later on when introducing the policy analysis.

We have described the profits, the prices, the consumers’ surplus and the social welfare associated with each of the states of nature. What is missing is to characterize the transfers the agents get from their respective principal. Transfers enter agents’ utility in the way we have described above.

We will consider two cases regarding the ability to write contracts contingent on agents’ actions. In one case, contracts can be written contingent on the agents’ exerted efforts and the firms just pay transfers which compensate the agents for their disutility of exerting the effort:

\[ t_i = \frac{1}{2} e_i^2. \]

In the other case, contracts cannot be made contingent on the agents’ efforts but only on the realized costs, either \( c \) or \( \beta c \). From standard principal-agent theory, we know that it is optimal for the bonus paid to an agent not to be a function of the cost realizations of the other firm. Therefore, contracts will only be such that agent \( i \) will receive a positive bonus, \( b_i \), in case firm \( i \) succeeds in reducing its costs, or a transfer equal to zero, in case firm \( i \) fails in reducing them instead, no matter whether the other firm succeeded or not. The limited liability of the agents we have assumed, means that firms cannot offer any contract that might pay a negative wage to their agent for a given realization. Thus, the transfers to be paid to the agent in the stand-alone situation are:

\[
t_1 = \begin{cases} 
  b_1 & \text{if } (\beta c, c) \text{ or } (\beta c, \beta c) \\
  0 & \text{otherwise}.
\end{cases}
\]

### 2.2 Merger

When firms merge we consider that the previously employed agents undertake the project jointly, so that its success probability becomes a function of both agents’ efforts, i.e.:

\[
p(M) = \left( e_1^{1-\varepsilon} + e_2^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \text{ with } \varepsilon \in [\varepsilon_1, \varepsilon].
\]
The parameter $\varepsilon$ captures the degree of substitutability between agents’ efforts. As discussed in the introduction, different degrees of substitutability will be considered: agents’ efforts can be either perfect substitutes, or slight complements, or duplicates. This is done, in order to account for possible synergies between agents’ efforts, as agents work together in the merged entity. We characterize the probability of success using values of the parameter $\varepsilon$ that are bounded below and above. The upper bound is necessary to rule out cases where agents’ efforts would be too complementary, because otherwise the assumption that the project could have been undertaken separately by each firm would not be consistent anymore. The lower bound is imposed to guarantee that the second order conditions of the maximization problems we will consider are satisfied. In addition, we will always check for the conditions such that this measure of probability is well defined, i.e. never exceeds the value of one\(^5\).

Pure synergies, which are merger specific, are the ones that are able to generate, for the same level of inputs, a higher output. In our case, we will have merger specific synergies whenever the induced efficiency by a merger, $E(M)$, is higher as compared to the expected one induced by a stand-alone situation, $E(S)$, i.e. if:

$$E(M) \simeq p(M) = \left(e_1^{1-\varepsilon} + e_2^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \geq e_1 + e_2 - e_1e_2 = E(S).$$

A sufficient condition for having synergies through a merger is that the parameter $\varepsilon$ takes values between zero and one. This is a sufficient but not necessary condition as lower values than zero might produce some merger specific synergies for special combinations of high levels of both agents’ efforts. In addition, it can be shown that higher values than one never produce any synergy.

However, given that in our model we will solve for the optimal contracts that induce a certain level of agents’ efforts, ex-post the inputs (optimal efforts) in a stand-alone situation may take different values from the ones in a merger situation. Because of this, we will need to distinguish between ex-ante synergies and ex-post ones: the first ones refer to the definition just given and the second ones refer to the same condition where the optimal levels of efforts for each situation have been replaced instead.

Once we have defined the probability of success induced by a merger, we can concentrate

\(^5\)We use the same definition of the probability of success here as in Fabrizi and Lippert (2005), where agents work together in a joint research project. Thus, for further comments and details about the chosen measure of the probability of success used here we refer to that work.
as before on the profits, prices, consumers’ surplus, social welfare and the transfers agents receive which are associated with it.

When merging, firms enjoy monopoly profits on the product market, no matter whether the merged entity succeeds in reducing its production costs or not. Only the magnitude of the profits will be affected by the success or failure of this project. Thus the profit enjoyed by the merged entity, before making the transfers to their agent(s) is:

\[
\pi(\cdot) = \begin{cases} 
\pi(\beta c) = \frac{(1-\beta c)^2}{4} \\
\pi(c) = \frac{(1-c)^{3}}{4} 
\end{cases}
\]

i.e. either high or low, depending on whether the merger respectively succeeds or not in reducing its unit production costs.

As we consider mergers among equals, we assume that, once merged, merging parties share equally the overall monopolistic profit. Thus, assuming an equal sharing of the realized profits of a merger, we implicitly disregard the way merging parties can commit to such a sharing.

Let us write now the corresponding prices prevailing in the economy once the merger is formed:

\[
P(\cdot) = \begin{cases} 
P(\beta c) = \frac{1+\beta c}{4} \\
P(c) = \frac{1+c}{4} 
\end{cases}
\]

The realizations of the consumers’ surplus, \(CS(\cdot,\cdot)\), associated with these states of nature are then the following:

\[
CS(\cdot) = \begin{cases} 
CS(\beta c) = \frac{(1-\beta c)^2}{8} \\
CS(c) = \frac{(1-c)^{3}}{8} 
\end{cases}
\]

As before, the consumers’ surplus, as well as the social welfare that we will discuss in detail when looking at the policy analysis, will be alternatively used in our analysis as the objective functions the competition authority cares about.

Let us finally characterize the transfers the agents get within a merger. As before, we will need to distinguish whether the efforts are observable or unobservable in order to let the transfer to the agents be contingent or not on their respective exerted effort. If contracts can be written contingent on the agents’ exerted efforts both agents receive the same transfer equal to:
\[ t_i = \frac{1}{2} e_i^2. \]

Otherwise, contracts will give the same bonus to each agent such that:
\[ t = \begin{cases} 
  b & \text{if success}, \\
  0 & \text{otherwise.} 
\end{cases} \]

As before, limited liability on the agents’ side is assumed so that the minimum transfer agents can get even when there is no success is non-negative.

3 Private decision

3.1 No Moral Hazard

The goal of this section is to isolate the market power effects from the ones that would be driven by pure moral hazard behavior merging parties have to face when deciding about merging or not and which type of contract to propose to the agent(s). In this section we characterize the maximization problems firms face in a world without moral hazard.

3.1.1 Stand-alone, (S)

This is the case where two firms decide each to let their respective agent conduct the project alone. The success probability of this project is therefore \( p_i (S) = e_i \), with \( i = 1, 2 \).

For notational simplicity, we will refer to the gross profits \( \pi (\cdot, \cdot) \) as defined above when describing the pre-merger case: they are a function of each state of nature and also depend on the combination of the parameters that lead to a drastic or a non-drastic success, \( \beta \) and \( c \). The advantage of using this notation is that it encompasses both the drastic and non-drastic situation we need to account for.

Given this, we can, w.l.o.g., write the maximization problem firm 1 faces:

\[
\max_{t_1} \Pi_1 (S) \equiv \max_{t_1} \left[ e_1 (1 - e_2) \pi (\beta c, c) - t_1 \right]
\]

\[
s.t. \quad t_1 - \frac{1}{2} e_1^2 \geq 0, \quad (IR)
\]

where \( \Pi_1 (S) \) is the expected net profit of firm 1.

Given the observability of the effort, each firm can extract any potential rent from the agent, so that the individual rationality constraint, \( (IR) \), is binding. Therefore, the maximization problem becomes simply:
\[
\max_{e_1} \Pi_1 (S) \equiv \max_{e_1} \left[ e_1 (1 - e_2) \pi (\beta c, c) - \frac{1}{2} e_1^2 \right].
\]

The Nash equilibrium of this problem each firm faces is:

\[
e^o_1 (S) = e^o_2 (S) = e^o (S) = p^o (S) = \frac{\pi (\beta c, c)}{\pi (\beta c, c) + 1} = \left\{ \begin{array}{ll}
\frac{(1-\beta c)^2}{(1-\beta c)^2 + 4} & \text{if } \beta \leq \frac{2c-1}{c} \\
\frac{c(1-c)(1-\beta)}{c(1-c)(1-\beta) + 1} & \text{otherwise}
\end{array} \right.
\]

where the superscript \(o\) stands for "observability of effort". The level of the optimal effort coincides here with the measure of the probability of success.

The optimal expected profit for each firm is therefore:

\[
\Pi^o_i (S) = \Pi^o (S) = \frac{1}{2} \left( \frac{\pi (\beta c, c)}{\pi (\beta c, c) + 1} \right)^2.
\]

### 3.1.2 Merger, \((M)\)

In this case, the merged entity solve for the following problem:

\[
\max_t \Pi (M) \equiv \max_t \left[ p (M) \pi (\beta c) + (1 - p (M)) \pi (c) - 2t \right]
\]

\[
s.t. \quad t - \frac{1}{2} e_i^2 \geq 0 \quad \forall i \quad (IR)
\]

\[
e_i \leq 1
\]

\[
p (M) = (e_1^{1-\varepsilon} + e_2^{1-\varepsilon}) \frac{c}{1-\varepsilon} \leq 1,
\]

At this stage we solve for the unconstrained problem, i.e. taking the constraint on the probability of success as not binding. This way we get the potential unconstrained solution of the maximization problem. If this solution does not exceed the value of one, given the combination of the parameters of the model, then it will be used for the following analyses; otherwise, the maximum value of one will be considered instead\(^6\).

This is why we can rewrite the previous problem as follows:

\[
\max_{e_1, e_2} \Pi (M) \equiv \max_{e_1, e_2} \left[ (e_1^{1-\varepsilon} + e_2^{1-\varepsilon}) \frac{c}{1-\varepsilon} \pi (\beta c) + \left( 1 - \left( e_1^{1-\varepsilon} + e_2^{1-\varepsilon} \right) \frac{1}{1-\varepsilon} \right) \pi (c) - \frac{1}{2} \left( e_1^2 + e_2^2 \right) \right].
\]

\(^6\)This is possible as we have restricted the values of the parameter \(\varepsilon\) such that the problem to be solved is a well behaved one.
The first order conditions associated with this problem are:

\[
\frac{\partial \Pi(M)}{\partial e_1} = e_1^{1-\epsilon} \left( e_1^{1-\epsilon} + e_2^{1-\epsilon} \right) \frac{\epsilon}{1-\epsilon} \left( \pi(\beta c) - \pi(c) \right) - e_1 = 0, \\
\frac{\partial \Pi(M)}{\partial e_2} = e_2^{1-\epsilon} \left( e_1^{1-\epsilon} + e_2^{1-\epsilon} \right) \frac{\epsilon}{1-\epsilon} \left( \pi(\beta c) - \pi(c) \right) - e_2 = 0.
\]

They tell us that the solution to the problem should be such that:

\[
e_1^o = e_2^o = e^o(M).
\]

We should remember that the constraint over the probability of success not to exceed the value of one is necessary and sufficient to guarantee that each agent’s effort will not exceed the value of one as long as \( e^o(M) \leq 2 - \frac{1}{1+\epsilon} \). Implementing a higher \( e^o(M) \) would not lead to a higher probability of success (and, therefore, to higher expected revenues), but to increased costs as the agents would eventually have to be compensated for these extra efforts. Using this property, and the one just found above, we can solve for the system and derive the following result:

\[
e^o(M) = \min \left\{ 2^{\frac{1-\epsilon}{1+\epsilon}} \left( \pi(\beta c) - \pi(c) \right), 2^{-\frac{1}{1+\epsilon}} \right\}.
\]

We now proceed in characterizing the measure of the probability of success associated with this case. Remember that when two agents are kept in the merger, the success probability is a function of both agents’ efforts so that:

\[
p^o(M) = \min \left\{ 2^{\frac{1+\epsilon}{1+\epsilon}} \left( \pi(\beta c) - \pi(c) \right), 1 \right\}.
\]

The expected profit each merging party can enjoy in a merger is then:

\[
\frac{\Pi^o(M)}{2} = \frac{1}{2} \left( p^o(M) \pi(\beta c) + (1 - p^o(M)) \pi(c) - (e^o(M))^2 \right).
\]

### 3.1.3 Merger decision under no MH

Firms are willing to merge any time the share of the expected merged entity’s profit they may enjoy is higher than the one they expect in a stand-alone situation. Therefore, to characterize under which conditions firms would propose a merger, we simply need to solve for the following inequality:
\[
\frac{\Pi^o(M)}{2} > \Pi^o(S) \\
\Leftrightarrow \\
\frac{1}{2} \left( p^o(M) \pi (\beta c) + (1 - p^o(M)) \pi (c) - (e^o(M))^2 \right) > \frac{1}{2} \left( \frac{\pi (\beta c, c)}{\pi (\beta c, c) + 1} \right)^2
\]

The expression referred to the merger’s profit is a function of three different parameters, \(\beta, c,\) and \(\varepsilon.\) Given the complexity of the expression it is difficult to solve analytically for the values of these parameters for which the inequality might hold. We therefore use graphical representations in order to show for which combination of the relevant parameters firms are willing to merge.

To this purpose, we will fix different values of the initial cost economy faces and we let the other parameters vary freely. This will allow us to characterize qualitatively the decision to merge which will be shown to be taken more often, the lower the initial level of costs. This result can be seen in figures 1 and 2.

Figure 1: Private merger decision for \(c = 0.99\) under no moral hazard.
In the vertical axis different values of $\beta \in [0, 1]$ are considered: low values of $\beta$ imply high relative cost savings, while high levels of it imply the opposite. In the horizontal axis the parameter $\varepsilon$ is represented only for a restricted range such that $\varepsilon \in [-5, 5]$. The reason to restrict the attention to this interval is for pure descriptive reasons. Extending the range would not add any information on the firms’ merger decision as the lines of indifference will continue asymptotically. In addition, in the analysis that will follow the interesting results will come in the interval where $\varepsilon \in [-1, 1]$, so that we will even concentrate on this range to show under which circumstances a competition authority’s decision may induce a type I or type II error when relying on an efficiency defense.

Let us describe these graphs.

Firms are always willing to merge, no matter which is the value of $\beta$, as long as $\varepsilon \in [0, 1]$. This result was partially expected as we know that pure synergies occur in that range. However, ex-post synergies might justify the decision to merge in ranges where pure synergies do not arise. This is the case, for values of $\varepsilon$ above one where mergers are still privately preferred as long as potential cost savings are not too high. On the other way around, when $\varepsilon$ is negative we might still expect pure synergies for special combinations of $e_i$. Results show us that the ex-post synergies driven by different levels of implemented efforts, levels of transfers corresponding to these efforts levels, and degrees of market power, between the stand-alone and the merger case are responsible for the decision of merging even when pure synergies are not present anymore.
As soon as the initial level of cost decreases, the area for which firms are willing to merge increases. In particular, for \( c < 0.6 \) firms are always willing to merge no matter which values \( \beta \) and \( \epsilon \) may take: when the stake for staying alone is not big enough, renouncing to the chance of being the one succeeding in the market alone is not so costly anymore, the enhanced efficiency and the reduced cost of compensating for the efforts induced by the merger justify going for the merger.

3.2 Moral Hazard

In this section, we will repeat the analysis made for the observable efforts case considering the efforts as unobservable instead. This is done in order to be able to compare results obtained in the absence of moral hazard with the ones obtained introducing it into the model and to discuss them.

3.2.1 Stand-alone, \((S)\)

As before, we can write the maximization problem each firm solves when choosing to conduct the project alone. In this case, however, we need to remember that, given the moral hazard assumption, the contract firms propose to their respective agent has to be incentive compatible. Agents will receive a transfer higher than the one they would have enjoyed if their effort was observable.

Thus, w.l.o.g. we can now write the profit maximization of firm 1 as follows:

\[
\max_{b_1} \Pi_1 (S) \equiv \max_{b_1} [e_1 (1 - e_2) \pi (\beta c, c) - e_1 b_1] \\
\text{s.t. } e_1 b_1 - \frac{1}{2} e_1^2 \geq 0 \quad (IR) \\
\quad e_1 = \arg \max_{e_1} \left[ e_1 b_1 - \frac{1}{2} e_1^2 \right]. \quad (IC)
\]

Solving for the Nash equilibrium, we get:

\[
b_1 = b_2 = b(S) = e^u(S) = p^u(S) = \frac{\pi(\beta c, c)}{\pi(\beta c, c) + 2} = \begin{cases} \frac{(1-\beta c)^2}{(1-\beta c)^2 + 8} & \text{if } \beta \leq \frac{2c-1}{c} \\ \frac{e(1-c)(1-\beta)}{c(1-c)(1-\beta)+2} & \text{otherwise} \end{cases},
\]

where the superscript \( u \) stands for "unobservability of effort".

As a consequence, the expected profit each firm can enjoy in this case corresponds to:
\[ \Pi_1 (S) = \Pi_2 (S) = \Pi^u (S) = \frac{1}{2} \left( \frac{\pi (\beta c, c)}{\pi (\beta c, c) + 2} \right)^2. \]

### 3.2.2 Merger, \((M)\)

In this case, remember that the probability of success in reducing production costs is given by:

\[ p(M) \equiv \left( e_1^{1-\varepsilon} + e_2^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \]

The merged entity pays the same bonus \(b\) in case of success to both agents. The non-observability of agents’ efforts, makes it impossible to separate the contribution one agent has given to the project from the contribution of the other agent. The merged entity faces again the \((IC)\) constraint due to this non observability of agents’ efforts. The \((IC)\) constraint here will result from the Nash equilibrium of each agent’s utility maximization problem. The \((IC)\) - that will be a function of the bonus - can be obtained this way as each agent maximizes his utility considering that his chosen level of effort will not lead to a probability exceeding one. The merged entity will have already internalized the feasibility constraint over the probability of success when it chooses the level of the bonuses. This means that the merged entity will never propose a level of the bonus that may induce each agent to choose an effort that may lead to a probability exceeding the level of one.

Therefore, the merged entity solves for the following problem:

\[
\max_b \Pi (M) = \max_b \left[ p(M) \pi (\beta c) + (1 - p(M)) \pi (c) - 2p(M)b \right]
\]

\[
s.t. \quad p(M)b - \frac{1}{2}e_i^2 \geq 0 \quad \forall i \quad \text{(IR)}
\]

\[
e_i = \arg \max_{e_i} \left[ p(M)b - \frac{1}{2}e_i^2 \right] \quad \forall i \quad \text{(IC)}
\]

\[
p(M) = \left( e_1^{1-\varepsilon} + e_2^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \leq 1
\]

The solution to this maximization problem gives:
\[ b(M) = \min \left\{ \frac{1}{4} (\pi (\beta c) - \pi (c)), 2^{-\frac{1+\varepsilon}{4}} \right\} \]
\[ e^u(M) = 2^{1-\varepsilon} b(M) \]
\[ p^u(M) = \min \left\{ 2^{\frac{1+\varepsilon}{4}} (\pi (\beta c) - \pi (c)), 1 \right\} \]

The merged entity’s expected profit associated to this optimal bonus and probability of success is:

\[ \frac{\Pi^u(M)}{2} = \frac{1}{2} [p^u(M) (\pi (\beta c) - 2b) + (1 - p^u(M)) \pi (c)] . \]

### 3.2.3 Merger decision under MH

As before, we know that a merger will be proposed as long as:

\[ \frac{\Pi^u(M)}{2} > \Pi^u(S) \]

\[ \Downarrow \]

\[ \frac{1}{2} \left( p^u(M) \pi (\beta c) + (1 - p^u(M)) \pi (c) - (e^u(M))^2 \right) > \frac{1}{2} \left( \frac{\pi (\beta c, c)}{\pi (\beta c, c) + 2} \right)^2 \]

We have again the same type of complexity as before, so to solve for this inequality we will proceed with graphical representations in order to describe the combination of the parameters of the model that will drive the merger decisions of firms. Different values of the initial cost will be fixed here as well, while the other parameters will be let free to vary.

Figures 3 and 4 show results of the merger decision under moral hazard, for a comparable range of the parameter \( \varepsilon \) as in the case of observable efforts.
As before, in the vertical axis different values of $\beta \in [0, 1]$ are considered. Again, low values of $\beta$ imply high relative cost savings, and high levels of it imply the opposite. The horizontal axis accounts again for different values of the parameter $\varepsilon$ in the range $[-5, 5]$, even though, as discussed already, we will restrict the policy analysis only to the range $\varepsilon \in [-1, 1]$ instead.

Let us describe the firms’ decision about the merger.
First thing to be noticed is that for high initial costs firms are not anymore always willing to merge in the range where \( \varepsilon \in [0, 1] \). Instead, this might still be the case, as long as the potential for cost savings is relatively low. This means that pure synergies are not anymore the driving forces for a merger decision as long as the stake for being alone is high enough. However, as soon as the initial cost is lower a similar behavior can be expected by firms: the lower the initial cost the more often they are willing to merge. For \( c < 0.69 \) firms are always willing to merge no matter which values \( \beta \) and \( \varepsilon \) may take. The same comments of before apply here as well.

### 3.3 Comparing private decisions on mergers

Now that we have characterized the private decision on mergers under the assumption of observable and unobservable agents’ efforts, we can compare these decisions with each other. To do so, we combine the graphs referred respectively to \( c = 0.99 \) and \( c = 0.79 \) for both the observable and unobservable case, obtaining figures 5 and 6. In these figures some profits appear with the superscript \( j = o, u \). This is done as, when combining figures 3 and 4, there exists regions in which the decisions about merging stay the same for both assumptions on observability of efforts. Instead, we highlight the differences in the behavior of the merging parties as results of this comparison show that there exist areas where the decision taken under observable efforts does not coincide with the one taken in the presence of moral hazard.

We have two different types of these areas. Areas (a) are the ones where under observable efforts firms are not willing to merge, but they are instead under moral hazard. In area (b), arising for values of \( \varepsilon \in [-1, 1] \), the opposite occurs: firms are willing to merge for observable efforts, but not for unobservable ones.
4 Policy analysis

Now that we have determined under which conditions firms are willing to merge, we need to verify the impact of their decisions on both the consumers’ surplus and the social welfare depending on which of them is taken as the objective function by a competition authority. We study the impacts on both, because competition authorities often seem to care about
consumers’ surplus, even though economists often consider the overall social welfare impact to judge a merger instead.

Several justifications why a merger authority should care about consumers’ surplus and not about overall social welfare have been given.

One reason is that, in the decisions a national competition authority has to make on mergers, only consumers’ surplus should matter given that the profits accruing from a merger might be enjoyed elsewhere than in the domestic market. This is certainly the case for mergers occurring between firms the respective ownership of which is based abroad indeed. However, this is not a general case as many mergers may occur between firms selling their products in a given domestic market while one of them at least can be held domestically. In these alternative cases, it is not obvious anymore why the profits of the merging firms should be disregarded.

Another reason that has been put forward to privilege the consumers’ surplus as an objective by the competition authorities is that consumers are less likely to coordinate in order to voice their concerns. The argument implies that they are in a weaker position than merging parties might be. However, it is not clear why, even if this was the case, this should represent a justification for considering only consumers’ surplus. What we mean with this, is that if consumers cannot protect themselves from the potentially adverse effects of a merger it does not have to be that the competition authority has to take a biased point of view favoring them.

One last argument in favor of adopting consumers’ surplus that is often advocated is that it does not matter whether a competition authority adopts a consumers’ surplus standard instead of a social welfare one, as the first represents anyhow a good approximation for the second. However, as results will show, it is not clear whether this is always true. Apart from overstating the role of consumers as opposed to firms, consumers’ surplus also neglects another component which might be crucial in a merger situation: namely the cost of efforts associated with the undertaken project. The integration efforts that enable the firms to create synergies comes at a cost. It is therefore important as well to compare it with the cost separate firms will have to encounter when developing projects alone. In our model, the cost of effort can be interpreted as the cost of development of a given technology that comes to existence. Such a development cost is not a fixed cost though, as it is often modeled in R&D literature, as we have made it become endogenous through the optimal contracts.
implemented before and after the merger.

On the other hand, a tendency of considering a broader objective than the consumers’ surplus has been observed as well. As Motta (2004) points out:

"which contributes to improving the production or distribution of goods or to promoting technical or economic progress, while allowing consumers a fair share of the resulting benefit". Furthermore, Article 2.1 of the Merger Regulation accepts in principle an efficiency defense "provided that it is to consumers' advantage". These provisions might indicate that consumer welfare is among the ultimate objectives of competition law. However, I am not aware of any statement of the ECJ on this point, nor of any (Commission or Court’s) decision where reliance on either standard has made a difference in practice.

In other jurisdictions, such as Canada, Australia, New Zealand, competition authorities seem instead to lean towards a total welfare standard (Lyons, 2003:3).

We would have restricted our analysis to one of the two standards, either the consumers’ surplus or the social welfare, if considering either of them would have led to similar results in the characterization of the type of errors a competition authority commits when it relies on an efficiency defense.

In the following subsections, we will define two measures as proxies of either the enhanced consumers’ surplus or the social welfare, namely the expected efficiency gains, and the reduction in expected prices.

The way proxies are determined is always disputable. For example, any time prices are weighted using a different weight than the exchanged quantities associated to each of them, the measure of the expected price reductions differs from the enhanced consumers’ surplus. Therefore, using such a measure as a 'correct' approximation for the consumers’ surplus would lead to some errors. When using the expected efficiency gains, we may converge to the real value of the enhanced consumers’ surplus or social welfare. Again, the precision of the approximation may differ as a function of the different parameters of the model and also whether a substantial expected efficiency gains is required or not.

However, as it will become clear, it is possible to characterize the type of errors that can be committed when relying on either proxy.
4.1 Objective functions: expected welfare and consumers’ surplus

Before we turn to evaluating of the mentioned proxies, let us describe which would be the ideal decisions if the objective functions expected social welfare and expected consumers’ surplus were to be followed directly.

Irrespective of the observability of efforts, in the stand-alone case, there are four states of nature implying three different realizations of the consumers’ surplus: both firms succeed in reducing their costs, leading to $CS_j(\beta c, \beta c)$, no firm succeeds leading to $CS_j(c, c)$ or either one firm succeeds while the other one fails, in which case we will have either $CS_j(\beta c, c)$ or $CS_j(c, \beta c)$, respectively, where $CS_j(\beta c, c) = CS_j(c, \beta c)$. Weighted with the corresponding probability, we obtain the expected consumers’ surplus:

$$CS_j(S) = (p_j^i(S))^2 CS_j(\beta c, \beta c) + (1 - p_j^i(S))^2 CS_j(c, c) + 2p_j^i(S)(1 - p_j^i(S)) CS_j(\beta c, c),$$

where the superscript $j$ stands for $o$ or $u$, the observability or unobservability of the agents’ efforts.

In addition to the expected consumers’ surplus, the expected social welfare also includes the expected gross profits, as well as the agents’ disutility of exerting an effort. Taking this into account, we can write for the expected social welfare in the pre-merger situation:

$$W_j^i(S) = 2p_j^i(S)(1 - p_j^i(S)) (\pi(\beta c, c) + CS(\beta c, c)) + (p_j^i(S))^2 CS(\beta c, \beta c) +$$

$$(1 - p_j^i(S))^2 CS(c, c) - 2\left(\frac{1}{2}(p_j^i(S))^2\right).$$

If firms merge, there are only two states of nature, either the merged firm succeeds or it does not, leading to the respective realizations of the consumers’ surplus, $CS_j^i(\beta c)$ and $CS_j^i(c)$, respectively. Again, weighted with their probability, we obtain the expression of the expected consumers’ surplus:

$$CS_j^i(M) = p_j^i(M) CS_j^i(\beta c) + (1 - p_j^i(M)) CS_j^i(c),$$

where we use the superscript $j$ as above.

As for the pre-merger case, additionally to the expected consumers’ surplus, the expected social welfare includes the merged entity’s expected gross profit, and the agents’ disutility of exerting an effort, leading to:

$$W_j^i(M) = p_j^i(M) (\pi(\beta c) + CS_j^i(\beta c)) + (1 - p_j^i(M)) (\pi(c) + CS_j^i(c)) - 2\left(\frac{1}{2}(c_j^i(M))^2\right).$$
Figure 7 represents the impact of a merger on social welfare and consumer’s surplus if efforts are observable and unobservable, respectively, and for high initial unit production costs, $c = 0.99$. The $CS - CS$ and $W - W$ lines represent the combinations of $\beta$ and $\varepsilon$ for which a competition authority, pursuing alternatively the consumers’ surplus or the welfare standard, would be indifferent between the stand-alone situation and the merger. In addition to these two lines, we again depict the lines of indifference between merging and staying alone for the firms, labelled $\Pi - \Pi$.

In both worlds, the firms decide to stand-alone to the lower left of the $\Pi - \Pi$ line and the welfare (consumers’ surplus), if firms merge, is higher than otherwise to the lower right of the $W - W$ line (the $CS - CS$ line). We see that a consumers’ surplus standard is stricter than a welfare standard in both worlds, that is also when the agents that are responsible for reducing unit production costs are able to capture an information rent.

However, the $CS - CS$ and $W - W$ lines are closer to each other when efforts are unobservable. That means that adopting either standard would make less of a difference in a world where agents are subject to moral hazard. On top of this, the $\Pi - \Pi$ line also approaches the two lines mentioned so that the area of a potential conflict, i.e. the area where privately preferred mergers are undesired from either a welfare or a consumers’ surplus point
of view, is reduced. This is particularly striking for high initial unit production costs and not too low potential for their reduction, i.e. for $\beta < 0.8$.

Furthermore, in the unobservable efforts case with high initial costs, the $W - W$ line crosses the $\Pi - \Pi$ line. This means that there exist welfare increasing potential mergers which would not be proposed by the merging parties. In this area, the expected welfare is increasing, and both, expected net profits and expected consumers’ surplus, are falling. This means that there has to be another component of the expected welfare that is increasing. This component is the agents’ informational rent, due to the moral hazard assumption we have made.

When the level of the initial cost is not too high, the gap between the $\Pi - \Pi$, $W - W$ and $CS$ lines is in general bigger than for higher initial costs. This is due to a large extent to the increased profitability of a merger as compared to the stand-alone situation when costs are not initially high. Opposite to the high initial costs case, i.e. when firms are more willing to go for a stand-alone project – as this guarantees them to be the sole beneficiary of a high monopoly profit should they succeed as the only one – lower initial costs make the perspective to merge be more attractive.

![Figure 8: The firms’ merger decision and its social welfare and consumers’ surplus impact under no moral hazard (left) and under moral hazard (right) for $c = 0.79$](image)
4.2 Proxies for the society’s objective functions

Once the distinction between the consumers’ surplus and the social welfare has been made, we can describe the consequences in the characterization of the type of errors a competition authority may commit when relying on an efficiency defense on the basis of the proxies suggested above.

As for the consumers’ surplus and for the social welfare, we can use the common superscript \( j = o, u \) for each of the components of both the expected efficiency and prices in order to account for the observable and the unobservable efforts cases. This is possible as the nature of each component stays the same through these two environments: the only thing changing is their respective level as each of them has been obtained solving for different optimization problems.

4.2.1 Expected efficiency gains

Let us concentrate first on the composition of the elements that enter the expected efficiency measure. In the stand-alone case, the expected efficiency takes the following form:

\[
E^j(S) = \left( p^j(S) \right)^2 + 2p^j(S)(1 - p^j(S)),
\]

whereas in the merger case:

\[
E^j(M) = p^j(M).
\]

These expressions play the equivalent role of the measure of the overall efficiency on the production side of the economy, respectively when two firms face a Bertrand competition on the market, and when the two firms merge: they give the probability of producing the good with the low production costs \( \beta c \) in the economy. In both these cases we will talk about ex-post efficiency, as we are considering that when writing the expressions the way we are doing that the privately optimal induced probabilities of success have been replaced into the expressions of the ex-ante measures of efficiency.

The differences between the post-merger efficiencies and the pre-merger ones, both, under the no-moral hazard and the moral hazard assumptions, will determine, when positive, whether a merger leads to expected efficiency gains. If the following inequality holds, we will talk about efficiency gains due to a merger, either when moral hazard is not an issue, or when it is instead:
\[ E^j(M) - E^j(S) > 0 \]

The interest of the analysis is to combine the results of these inequalities, one for each of the two regimes considered, alternatively with the consumers’ surplus and the social welfare. As argued in the previous section, in our analysis these two objective functions will be taken respectively as a standard by a competition authority when it has to judge upon a proposed merger.

In order to perform the necessary comparisons, we repeat the same type of analyses as the ones we made when considering the merger decision of firms, or when we explained the relationships between the consumers’ surplus and the social welfare as possible objective functions chosen by a competition authority. Thus, once more, in order to make the relevant graphical comparisons, we will fix different values of the initial costs, and we will let the parameters \( \beta \) and \( \varepsilon \) free to vary\(^7\).

Let us take the consumers’ surplus as the reference objective function first. Efficiency comparisons for this objective function can be described using figures 9 and 10, where the \( E - E \) line depicts, in both, the curve where the difference between the pre and the post merger expected efficiency is zero. Comparisons are made for the observable and the unobservable efforts cases, and for two levels of the initial costs, a high and a moderate one. Given that when taking lower levels of the initial costs, the \( \Pi - \Pi \) curve moves gradually to the left lower corner, and the \( CS - CS \) as well as the \( E - E \) line move to the right lower corner, restricting our attention to only two levels of the initial costs is enough to describe the main results that can be obtained when performing this type of comparisons. Note that for any value of the initial cost \( c \), the \( E - E \) line always stays above the \( CS - CS \) one.

Expected efficiency gains due to a merger only occur to the right of the \( E - E \) line, i.e. both, in regions II and IV. However, while region II is not a problematic one as consumers’ surplus increases too, in region IV consumers’ surplus would decrease due to a permitted merger. Therefore, if an efficiency defense of all the proposed mergers leading to an expected efficiency gain was accepted then type I errors would be made for mergers falling in region IV: bad mergers would be accepted. Notice that the occurrence of the type I error is reduced, when facing a moral hazard behavior from the agents’ side: area IV shrinks and the \( E - E \) line gets closer to the \( CS - CS \) line.

\(^7\)Remember that the relevant comparisons will be made only in the range where \( \varepsilon [-1, 1] \).
Notice also that requiring a significant efficiency gain due to a merger, may help reducing the occurrence of type I error when the society does not face a moral hazard behavior from the agents’ side. The relevant indifference curve would be to the right of the $E - E$ line so that area IV would shrink as a result. However, the same recommendation is not a valid one anymore when facing a moral hazard behavior instead. The relevant indifference curve for the application of this recommendation would lie to the lower right of the $E - E$ line. Thus, using this new indifference curve another type of error would arise: a type II error, i.e. some good mergers would be refused instead. This is true at least when the level of the initial costs that the economy faces is high (see figure 9).

Figure 9: Efficiency gains: under consumers’ surplus objective function, for both, observable (left) and unobservable (right) efforts, for $c = 0.99$. 
We can now repeat the same type of analysis made for the consumers’ surplus, taking the social welfare as the reference objective function a competition authority may care about. Doing so, we obtain figures 11 and 12. As for the case where the consumers’ surplus is the objective function chosen by the competition authority, we can characterize an area where type I errors are made when accepting an efficiency defense based on an enhanced efficiency induced by a merger. This area is labelled again as area IV, given that it has the same characteristics as before.

It is easy to check that, if social welfare is taken as a standard, the recommendation of having significant efficiency gains due to a merger may lead to type II errors both, in a world with and without moral hazard. Figure 11 shows that in particular, these errors would be made systematically when the industry faces high initial costs: good mergers would be more often rejected than if only enhanced efficiency due to a merger were required instead.

Furthermore, under the moral hazard assumption, for values of $\beta$ below 0.8 the $\Pi - \Pi$, $E - E$ and $W - W$ lines get closer to each other, up to the point that the $E - E$ crosses the $W - W$ line and the latter one crosses the $\Pi - \Pi$ line instead. This tells us two different things. First, that whenever the potential for cost reduction is not very low, even asking for an enhanced efficiency induced by a merger in order to approve it may be too demanding, in the sense that some mergers that would have been desirable under the social welfare standard would be rejected. These mergers are the ones falling in area V. Second, due to the crossing
between the $W - W$ and the $\Pi - \Pi$ lines, an area VI appears where there exist mergers that would be socially desirable which are not even privately proposed. These two areas, V and VI, disappear for lower levels of the initial costs as shown in figure 12, thus making the requirement of enhanced, or even substantially enhanced, efficiency due to a merger be justified.

Figure 11: Efficiency gains: under social welfare objective function, both, observable (left) and unobservable (right) efforts, for $c = 0.99$.

Figure 12: Efficiency gains: under social welfare objective function, both, observable (left) and unobservable (right) efforts, for $c = 0.79$. 

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To summarize:

**Proposition 1** If a competition authority adheres to a \((CS)\) standard and relies on expected efficiency gains for judging proposed mergers:

(i) Under no MH, type I errors are systematically committed, i.e. bad mergers are accepted, unless significant expected efficiency gains are required;

(ii) Under MH, type I errors are committed as well, but they occur less often than under the no MH assumption; for high initial costs the requirement of significant expected efficiency gains would lead to type II errors in addition, i.e. some good mergers would be rejected.

**Proposition 2** If a competition authority adheres to a \((W)\) standard and relies on an expected efficiency gains for judging proposed mergers:

(i) Under no MH, type I errors are systematically committed, i.e. bad mergers are accepted, but they occur less often than when using a \((CS)\) standard. For high initial costs, the significant expected efficiency gains requirement would lead to type II errors in addition, i.e. some good mergers would be rejected;

(ii) Under MH, type I errors are committed for smaller regions than under the \((CS)\) standard, but type II errors can be committed in addition unless the potential for cost savings is low, i.e. \(\beta > 0.8\). For high and moderate initial cost asking for substantial efficiency gains would lead to refuse some good mergers, i.e. type II errors are committed in addition.

### 4.2.2 Expected price reductions

We move now to the comparisons concerning possible price reductions due to a merger. In order to do so, we characterize the expected price under the stand-alone and the merger cases respectively, replacing the relevant optimal levels of the probabilities of success obtained under each regime. Thus, the induced expected prices of a stand-alone situation are:

\[
E^j [P(\cdot, \cdot)] = 2p^j(S)(1 - p^j(S))P(\beta c, c) + (p^j(S))^2 P(\beta c, \beta c) + (1 - p^j(S))^2
\]

where \(j = o, u\) accounts, as before, for both the observable and the unobservable efforts cases.

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The induced expected prices by a merger are:

\[ E^{i}[P(\cdot,\cdot)] = p^i(M)P(\beta c) + (1 - p^i(M))P(c). \]

When the following is true, a merger induces expected price reductions, either under observable or unobservable efforts:

\[ E^{i}[P(\cdot,\cdot)] - E^{j}[P(\cdot)] > 0 \]

As before, we want to determine the impact over alternatively the consumers’ surplus and the social welfare of relying on reductions in expected prices when deciding upon proposed mergers. We use here graphical representations in the same way as before to comment results of these comparisons.

Figures 13 and 14 show the comparisons when consumers’ surplus is taken as a standard. In both figures, the \( P - P \) line depicts the frontier between the mergers that do not lead to expected price reductions – the ones to the left of this line – and the mergers that would lead to reductions of the expected prices instead – the ones to the right of it.

For high levels of the initial costs, the \( P - P \) line intersects the \( CS - CS \) one both in a world with and without moral hazard. The crossing results in the occurrence of two different areas, areas IV and V. The requirement of a reduction in the expected prices induced by a merger would provoke a type I error for mergers falling in area IV: bad mergers would be accepted, or a type II error for mergers falling in area V: good mergers would be rejected instead. The reduction of the expected price is too lax a requirement in certain cases, and a too strict one in others. However, under the moral hazard assumption the occurrence of type I errors in particular is sensibly reduced when compared to the one under the no moral hazard assumption. For lower values of the initial costs, the \( P - P \) line always lies below the \( CS - CS \) line, implying that only type II errors would eventually be made in these cases by requiring a reduction in the expected price induced by a merger in order to approve it: all the bad mergers are refused, together with some good ones.

When we perform the parallel analysis using the social welfare as a standard we get figures 15 and 16. We can conclude that requiring a reduction in the expected price induced by a merger always leads only to type I errors, as the \( P - P \) line always lies below the \( W - W \) line no matter which is the initial level of costs considered, or whether or not we are facing a moral hazard problem in the industry. The type I error occurs again in the area labelled V.
As has been pointed out in section 4.1, under moral hazard and high initial costs, we have an area VI, given by the crossing between the $W - W$ and the $\Pi - \Pi$ lines, where welfare enhancing mergers are not proposed by the parties.

Figure 13: Expected Price: under consumers’ surplus objective function, both, observable (left) and unobservable (right) efforts, for $c = 0.99$.

Figure 14: Expected Price: under social welfare objective function, both, observable (left) and unobservable (right) efforts, for $c = 0.99$. 

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To summarize:

**Proposition 3** If a competition authority adheres to a (CS) standard and relies on reductions in the expected prices for judging proposed mergers:

(i) Under no MH, type I and II errors are systematically made for high initial costs: for low $\beta$ some bad mergers are accepted and for higher $\beta$ some good ones rejected. For
lower initial costs, only type II errors are made. The requirement is either too lax or too strict.

(ii) Under MH, type I errors are committed as well, but to a lower extent than under the no MH assumption and only for high initial costs. In addition, type II errors are made regardless of the level of the initial costs, i.e. the requirement is overall too strict.

Proposition 4 If a competition authority adheres to a \((W)\) standard and relies on reductions in the expected prices for judging proposed mergers: under no MH and MH assumptions, type II errors are systematically committed, i.e. some good mergers are rejected regardless of the level of the initial costs; however, under the MH assumption the occurrence of this type of errors is sensibly reduced as compared to the no MH one: the requirement is better calibrated under the MH assumption.

4.3 Policy implications

Up to now, our analysis has focussed on the positive side of evaluating mergers using simple proxies such as expected efficiency gains or reductions in the expected prices.

We have argued that a proxy leads to errors as it cannot perfectly substitute for the real objective function. In this paper we have provided a characterization of the type of errors that might be committed when using these proxies, under two different regimes, i.e. with and without moral hazard. This characterization has been made in terms of parameters such as the potential for innovation, \(\beta\), the type of the joint project, \(\varepsilon\), and the level of the initial costs of the industry, \(c\). Implicitly, we thereby also provided suggestions about which combination of these parameters do not lead to commit systematically errors of either type I or II.

At this point, we would like to go back to the criticism related to the use of proxies, such as expected efficiency gains and price reductions, as substitutes for either the consumers’ surplus or the social welfare. If the use of these proxies helps giving some precise recommendation about which mergers to accept and which ones to refuse, then they may be used whenever they may allow to save on information to be collected and processed.

If a measure for these proxies could be constructed using the partial information that a competition authority may have access to, on the basis of the results obtained, we could conclude which is the potential danger in adopting one proxy instead of another as a function of the characteristics of the industry the merger is concerned with.
As an example, whenever the industry faces a moral hazard behavior from the agents’ side, the potential for innovation is not too low, and the initial costs are high, then the expected efficiency gains would be a good statistic for the ‘real’ objective function pursued by a competition authority when deciding whether a proposed merger should be rejected or not. In this particular case, the collection of data to construct a precise measure for that proxy may result to be not as essential to judge upon the likelihood for the merger to lead to an improvement from both the social welfare and the consumers’ surplus point of views. We know that in this case both consumers’ surplus and social welfare induced by a proposed merger would be enhanced.

Whenever the initial costs are lower, no matter whether we face moral hazard or not, expected efficiency gains should be proved instead. In this case, the collection of data to construct an as precise measure of the proxy as possible may be crucial in deciding upon the merger. This is particularly true as long as the standard is the $(CS)$. Furthermore, the lower the initial costs the more substantial the expected efficiency gains should be. Again, doing so, information on the demand side can be saved.

These are only some general implications that are directly coming from the results we have obtained in our analysis. Depending on the real objective of the competition authority and the parameters characterizing the industry, one or another proxy, or different combinations of them, could be alternatively preferred and constructed to adapt them and use them ‘quanto basta’ in order to fit real merger cases decisions.

5 Conclusions

The wave of horizontal mergers that has occurred in this last decade, has opened a debate about the effects of mergers in their respective markets. On the one hand, worries have been voiced about the potential increase in market power, concentration, therefore decreased degree of competitiveness in these markets due to mergers. On the other hand, arguments in favor of mergers have been put forward, stressing their potential virtues, as their ability to create efficiency gains, coming from reduced production costs, transaction costs, or internal organization costs.

We have built a model where merger decisions and their potential efficiency gains are endogenous. The channel we propose and analyze has been formed by interactions of agents’ efforts when they are devoted to a joint project within a merger as compared to when each
agent has to conduct the same project alone. We have shown this way that mergers happen
either when the potential for cost reduction is low, i.e. out of pure market power consid-
erations, or when managerial efforts are close substitutes, or alternatively slight duplicates,
when ex-post synergies, i.e. induced by implemented contracts, are present. Synergies are the
results of both the interaction between these agents’ efforts and the implemented contracts
that drive the profitability of mergers, and determine the impact on consumers’ surplus and
social welfare.

Results tell us that an efficiency defense based on potential efficiency gains due to a
merger is generally too lax a requirement — some bad mergers would be accepted — for the
observable efforts case, and, for both informational assumptions, when the initial costs are
not too high. In these cases, requiring substantial efficiency gains may reduce this distortion.
Results also show that requiring the same for either the unobservable case when initial costs
are high, or for a larger range of costs when the standard used is the social welfare, may be
too strict: some good mergers would be refused. In any case when efforts are unobservable,
this requirement is more aligned to either type of standards.

We have also shown that the alternative requirement of comparing prices before and after
the merger, which would require to have additional information processed from the demand
side, turns out not to be necessary. The measure of the expected efficiency gains can be
precise enough when adjusted in the right way: using it ‘quanto basta’.

References

[1] Alchian and Demsetz (1972), ”Production, Information Costs, and Economic Organi-


