Abstract: This paper develops the theory of price discrimination in small-number oligopoly, and tests the theory with data on airline prices charged by Air New Zealand and Qantas. In a linear model, price discrimination does not change the average price paid by consumers. The extent of price discrimination diminishes as the number of competitors increases.

Key Words: Price discrimination, oligopoly, airline pricing

JEL numbers: D43 L93 L41
1. Introduction

“Price discrimination is as common in the market place as it is rare in economics textbooks” (Louis Phlips)

The above quotation from Louis Phlips\(^1\) is at least half correct. It certainly is true that price discrimination – being the charging of different prices to different customers of similar products where the price differential is not fully justified by any differences in the costs of supplying the customers – is almost pervasive. Indeed, it is quite hard to come up with a long list of markets in which all customers do pay the same price, as noted in the next section.

However, it is not fair to say that textbooks largely ignore price discrimination. Nearly all microeconomics texts have a section on discrimination, even if they do not emphasize or even note how potentially subversive the practice is for conventional (single-) price theory. But what is true is that price discrimination is invariably covered in the chapter or chapters on monopoly pricing. In particular, the topic is entirely missing from the oligopoly section of the text, even though, if the practice really is common across different market structures, these must and indeed do include oligopolistic structures.

And the disjointedness of price discrimination and oligopoly theory carries across to the journal literature. Some attention has been paid to price discrimination in monopolistically competitive markets, but there is very little indeed on the implications for price discrimination of small-number competition.

This paper focuses on a standard form of price discrimination – when a service is sold at different prices to different groups according to their willingness to pay – and asks what difference it makes whether the service is provided by a monopolist, by duopolists, or by any larger number of identical suppliers. The motivation for the research was a recent competition case involving two quite large international airlines, Qantas and Air New Zealand. These carriers sought permission from the antitrust authorities in Australian and New Zealand to form a cartel covering all their operations within, from and to New Zealand – markets generating several billion dollars of revenues annually.

After nearly two years of hearings and appeals the airlines eventually withdrew their application, after the decision of the NZ Commerce Commission to not authorise it was upheld by the NZ High Court, in July 2004. The Australian Competition and Consumer Commission had also turned down their cartel, but on appeal to that country’s Competition Tribunal the decision was overturned. However, as this case necessarily involves commerce between the two jurisdictions, each has in effect a veto power, and the airlines are now continuing to compete independently on the basis of the High Court’s ruling.

The author of the present paper was involved as an expert witness in the case. I opposed it, on the unsurprising grounds that what would amount to monopolisation or close to that of these markets would generate a substantial and harmful lessening of competition. My arguments were made – as were others on both sides – on the basis of theoretical and empirical modelling of airline competition in the standard context of single-price (average fare) price theory and econometrics. However, submissions and testimony of both lay and expert witnesses frequently referred to price

\(^1\) From Phlips’ contribution on Price Discrimination for the *New Palgrave*. 
discrimination (known in the industry as ‘yield management’), and assertions or predictions were made as to the impact of discriminatory practices on competition and pricing. For example, it was claimed that part of the competitive impact of low-cost carriers, with their simpler one-way fare structures, was to make unviable the whole traditional structure of airline price discrimination, which had been based for twenty years on the use of advance-purchase and return fare Saturdays stayover restrictions as instruments for sorting leisure from business travellers and enabling the latter group to be charged higher fares. It was also argued that the prevalence of price discrimination in airline matters could mean that reducing the number of independent carriers would not have the same harmful effects on allocative efficiency as is predicted by standard single-price oligopoly models.\(^2\)

These and other hypotheses and assertions might be partially contradictory, but in any case it became clear from searching that the extant literature did not deal with them. So, in the present paper we will explore the theory and practice of oligopolistic price discrimination. The theory uses quite simple linear models, extending the standard Cournot-Nash oligopoly model to the case where the sellers have two rather than one prices at which they sell their product. The empirical section explores pricing and price discrimination in a subset of the markets supplied by Air New Zealand and Qantas, using quoted prices from the airlines’ websites.

The main results are as follows. First, in the single-seller case, we find that increasing the number of prices (ie, increasing the segmentation of the market according to differences in willingness to pay) makes no difference to the average price earned by a profit-maximising monopolist. This is a simple result, which is perhaps obvious, at least in hindsight, but it does not appear to have been noted before.

Second, we find that this result applies to oligopoly, at least when there are just two prices charged (I have not yet extended the oligopoly case to a finer degree of discrimination). That is, the Nash Equilibrium price that emerges from the standard single-price analysis equals the share-weighted average of the prices charged by a price discriminating oligopoly, in Nash equilibrium. The corollary of this result, of course, is that bringing in price discrimination does not necessarily require us to alter our predictions of the effects on competition of a structural change such as a cartel or merger, although it is true that the deadweight triangle losses must be smaller.

Third, the theory predicts that price discrimination as measured by the gap between highest and lowest price offered diminishes as the number of competitors increases. Again this is probably what intuition should lead us to expect (because we know that in the limit of many competitors all prices must be close to marginal cost), but it is interesting to have it confirmed in a model.

The empirical work tests these predictions. Using variations over time in the lowest price offered for a particular flight, we find the following. First, even in the absence of advance purchase or return ticket stayover restrictions, there is quite a lot of price discrimination. On average, the lowest fare offered by the airline for a particular flight over an eight week period preceding the flight has a range of about 80%.\(^3\)

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\(^2\) Discrimination enables more of the potential surplus in a market to be realised – that is, there is less restriction of output. In the case of perfect (1\(^\text{st}\) degree) price discrimination, a monopolist supplies all potential viable customers -- there is no restriction of output at all.

\(^3\) That is, the highest low fare is, on average, 80% greater than the lowest low fare.
Second, in this price discriminating environment competition still matters. After controlling for some other factors affecting price, it seems that markets in which Air New Zealand and Qantas compete with each other on fairly equal terms have airfares around 25% lower than markets in which Air New Zealand has a monopoly. And, third, the theoretical prediction of less discrimination with more competition appears also to be born out: the difference between highest- and lowest-low fare is significantly larger on the monopoly routes.

2. A brief history of pricing and price discrimination

One of the most pervasive and important technologies of the modern world is surely the humble price tag. The invention, or at least the innovation of this has been attributed to a nineteenth century Philadelphia retailer named John Wanamaker, who considered the then ubiquitous system of haggling ‘inefficient and discourteous’. Mr Wanamaker assigned and showed fixed prices to all the merchandise in his store, and started a transaction technology revolution which quickly spread in all the cities of Europe and the United States.

It is easy in hindsight to see how essential this innovation was on the path of continued capitalist development. It links back to the Enlightenment ideas of the individual’s right to participate on equal terms with all other individuals in society and the economy: the principle of ‘one dollar one vote’ which actually preceded full implementation of the political suffrage principle of ‘one (wo)man one vote’. It was an absolutely necessary condition for the development of what we would now call a ‘price system’ generating signals to guide the efficient allocation and reallocation of resources, and of the Marshallian price theory that assumed a stable demand curve on which a single point would be observed. And it saved time, which was increasingly valuable. The technology of haggling was static, so that the opportunity cost of engaging in this activity increased as economic growth raised the value of time spent actually producing goods and services.

Yet, even in highly developed economies, price discrimination --charging different people different prices for the same good or service -- has stubbornly refused to disappear. And it is not hard to see why. The problem with single- or ‘take-it-or-leave-it’ (TOL) pricing is that leaves surplus on the table. Even a monopolist who ‘owns’ a demand curve and sets the ‘profit-maximising’ price on that curve is giving almost everyone who pays that price a ‘deal’ in that they would have paid more, and is missing out entirely on the business of consumers who would be willing to pay more than cost of production but less than the TOL price.

If demand is linear and marginal cost constant, then even the sharpest monopolist will extract just one half of the potential consumer surplus with a TOL price. It is hardly surprising then that monopolists, or more generally any seller with some control over price, will in reality often search for instruments which allow partitioning of their market by the willingness to pay of their customers, with the keenest consumers charged the highest price.

The difficulties in implementing a price discrimination policy come, of course, in identifying the different types of consumer, preventing the keeners from consuming at the lower price, and preventing those who do pay the lower price or prices from setting up in business themselves as arbitrages. In practice, these problems can never be perfectly solved (so that what is called ‘1st degree price discrimination, under

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which the monopolist captures all the surplus, will never be observed), but it does turn out to be generally quite easy to improve on TOL pricing, in essence by getting the higher-value consumers to self-select, from the imposition of some restriction or cost on access to lower prices that many or most higher value individuals can’t be bothered incurring.

As a result, when one looks around, it is actually quite difficult to find real-world examples of markets in which everybody does in fact pay the same price, even without bringing in imperfect information (and search costs), at least if one is willing to stretch the definition of price discrimination to include different prices charged for the same item by different sellers, such as supermarkets and convenience stores.\(^5\)

Quite typically, when the good or service does have a price tag, there are consumers who pay this amount and there are other consumers who pay less. Probably the most common class of devices for sorting out who pays more and who less rely on a statistical link between willingness to pay and valuation of time, generated by the relationship between both these characteristics and the consumer’s income.

Thus, there are people who pay less than the sticker price for their groceries because they are willing to put time into collecting and sorting from the awkward paper coupon books that supermarkets distribute through the mail, and there are those of us who cannot be bothered doing this. If you care enough to sift through the Auckland Yellow Pages, you can get a 25% discount with Super Taxis Ltd, ‘by mentioning Yellow Pages ad.’ There is an obvious similarity here with the persistence of price dispersions for similar goods across sellers, with the higher-price outlets getting their custom from those who are unwilling to devote time to search.

But income (or time valuation) is not the only sorting mechanism. For many items, especially services and ‘big ticket’ consumer durables, a lower price is literally there for the asking: ‘What is your best price on this?’; ‘Can I have the corporate rate, please?’; ‘Could you sharpen your pencil on this quote?’ Such may be especially prevalent in New Zealand, a country which as a result of a ‘neoliberal’ policy revolution, never reversed, in the 1980s, shows signs of reverting to a more primitive form of market capitalism.

Whilst the willingness to ask for a lower price – and risk rebuff -- may in some cases be linked with ability to pay (income), it might more often be associated with the *chutzpah* possessed, in particular, by confident middle class males. If so, then the welfare implications of price discrimination are far from clear, even without bringing in the transaction cost implications, which may be significant.\(^6\)

Be this as it may, it seems reasonable to propose that price discrimination is a widely observed phenomenon of modern market life, which deserves our analytical attention. Such indeed it has received, but to a remarkable extent this has focussed on the monopoly case. In particular, the literature on price discrimination and the literature on small-number competition (oligopoly) are almost disjoint -- almost without exception oligopoly modelling proceeds on the assumption that if the product is homogeneous, then it is sold at a unique price.

\(^5\) Examples of same-price selling may be restricted to goods whose manufacturer for some reason prints the retail price on the packaging. Examples are locally published newspapers and magazines. However, these products are usually also available on subscription at a unit price well below the newsstand retail price -- too far below, I expect, to be justifiable by costs of supply differences.

\(^6\) Hazledine (2001) finds that the proportion of the NZ workforce engaged in transaction activities rather than in actually producing goods and services has risen markedly over the past two decades to around half the labour force -- more than in Australia.
The present paper explores the implications of price discrimination for oligopoly modelling. The focus is on the airline industry, which especially since the 1978 U.S. deregulation has been an interesting and highly visible source of ingenious schemes for capturing the surplus under the travelling public’s demand curves. The industry has also generated a number of important antitrust cases involving requests for authorisation of mergers and cartels, including the recently concluded Air New Zealand/Qantas case, which have been modelled by economists, including the present author, as though the firms set single prices, even though the pervasiveness of price discrimination in airline fare setting is undeniable.

How much does that matter? To the extent that price discrimination is successful, it will ameliorate the traditional allocative inefficiency of market power, which of course is generated by the restriction of output below competitive levels. So, for merger analysis, it is important to know how much price discrimination we can expect from oligopolies, and the extent to which this changes as the structure of the oligopoly changes.

3. Pricing in Airline Markets

The purchase of an airline ticket is a quite expensive transaction, usually costing hundreds if not thousands of dollars. The willingness to pay for a ticket on a given route and flight date evidently can vary substantially between customers, in particular according to whether they are or are not bearing the costs of the ticket themselves, and according to the urgency and unpredictability of the decision to fly. The flight itself is a non-durable service, so that there is limited opportunity for arbitraging of ‘used’ tickets, unlike the situation for other ‘big ticket’ consumer purchases, such as home appliances.

For all these reasons, it has long been particularly tempting for airlines to devote resources to identifying customers with different willingness to pay and charging them different prices. In general, such can be achieved in one of two ways: by discriminating explicitly between customers with different observable characteristics, or by attaching conditions to tickets offered at different prices which encourage customers to sort themselves by their willingness to pay.

Explicit discrimination between customers is usually effected by choosing an observable characteristic that is empirically associated with income and thus willingness to pay: ‘student’ and ‘senior’ discounts; discounts for children. In the case of age-related conditions, the price differences can be very substantial. Airlines, for example, fly infants (not occupying a seat) free, and usually offer large discounts to sub-teenage children.

Analytically, student and senior discounts correspond to the standard textbook presentation of (3rd degree) price discrimination – the market is split into ‘high’ and ‘low’ elasticity submarkets, and a different price charged in each. So long as there is no linkage between the submarkets, there is no need for any extension of our existing oligopoly theory: competition in the student sub-market can be analysed independently from competition in the rest of the market.

Discounts for children are somewhat different, since often the services are consumed by the child accompanied by an adult. Thus, we really have here a family

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7 It might not improve overall cost efficiency. If profits attract entry in an industry with scale economies, much of the consumer surplus generated by price discrimination could be dissipated in an overcrowded industry with higher than necessary unit production costs.
market, rather than sub-markets which can be analysed independently, and the pricing problem seems more like 2nd degree price discrimination -- ie, non-linear pricing such that the average price falls with the quantity sold per transaction. There may be an oligopoly angle to 2nd degree price discrimination, but it will not be pursued in the present paper. I note that child discounts on airfares appear to be a quite standard and unchanging feature in airline markets, not used much or at all as instruments for competing for market share.

The competitive action in airline pricing appears to be focused on exploiting the possibilities of the second form of price discrimination, where surplus is extracted by attaching conditions to tickets, rather than by charging different customers different prices for the same ticket. The most famous and widely used such condition is the Saturday-night-stayover return fare restriction on cheap fares, introduced in the 1980s by American Airlines (who also invented Frequent Flier Programs). The motivation for this is, first, that business travellers are able and willing in general to pay higher prices for a flight than discretionary leisure travellers, and, second, that business travellers are unlikely, mostly, to be willing to stay away from home for a weekend, given that most business is conducted on weekdays.

Coupled with advance-purchase restrictions, the Saturday night stayover condition has been a brilliant marketing instrument for airlines around the world. It is not perfect, however. The conditions come at a cost: they make cheap tickets less attractive to high-value business travellers, which is good (for the airlines), but they must also reduce their value to leisure travellers, not all of whom wish to stay over a Saturday night, or even to purchase a return ticket. And the advance-purchase (APEX) requirement limits the airlines’ ability to vary prices in response to unforeseen fluctuations in demand as the flight date approaches.

These considerations may have increased the vulnerability of the traditional or full-service or ‘legacy’ air carriers to competition beginning in the 1980s and intensifying in the 1990s from ‘low –cost carriers’ (LCCs), of which the most successful are Southwest Airlines in the U.S., Ryanair and EasyJet in Europe, and Virgin Blue in Australian. These operators have innovated a business model based on supplying a no-frills product at low cost (made possible by non-unionised labour and the use of modern planes with low operating costs which can be exploited by the high utilisation rates achieved by these carriers, with their fast turn-around times). The LCCs sell simple point-to-point (ie, one-way) tickets which obviously do not require any Saturday night stayovers. Nor do they impose APEX requirements, though, depending on sales, fares offered will tend to increase as the flight date approaches.

Possibly in response to the threat of LCC competition, Air New Zealand in December 2002 introduced the first major innovation to legacy airlines’ pricing instruments since the Saturday Night Stay-over innovation two decades before. In their ‘Express Fare’ system, all Air New Zealand’s tickets are one-way tickets and are priced from twelve ‘price points’ (nine for trans-Tasman routes) in three fare classes – four price points in each class. At any time, for any flight, up to three prices will be on offer (on Air NZ’s website) at once -- one from each class. The fare classes differ modestly according to ‘add-ons’: fares from the lowest class do not attract frequent flier or status points; fares from the highest class can be more freely changed after purchase.8

8 Note that the new system does not make Air New Zealand in effect just another LCC. Unlike LCCs with their basic point-to-point itineraries, Air NZ operates -- and makes much of in its marketing – a full network of routes. The product is also differentiated by its Frequent Flier program, and by membership in a global airline alliance (Star Alliance for Air New Zealand; Oneworld for Qantas).
The lowest available fare is supplied in a ‘bucket’, containing a limited number of seats. When this bucket is full, it is removed from the market, and a new, higher-price bucket is offered. Often, but not always, by the eve of the flight date, no fares are available from the lowest price class, so that last-minute customers must purchase from the middle or sometimes the highest fare class.

Thus, what the airlines call ‘yield management’ involves choosing a rising path of prices over time to exploit the relationship between willingness to pay and (in)ability to commit to taking a flight well in advance. Note that the system of price points allows the airline to change ‘price’ without changing prices -- the actual average fare or yield achieved by a flight depends on the distribution of tickets sold over time at the different price points, which can be varied with no adjustment to the posted price structure.

Compared with the previous APEX-with-stayover system, the Express fares do make flying more attractive in general, especially to leisure travellers who might otherwise drive or not travel at all. Unfortunately (for the airlines) it also must to an extent ‘cannibalise’ the high-value business travel market, by offering without significant restrictions fares which might be much less than the willingness to pay.9

In any case, the Air NZ innovation was quickly copied by its main rival on domestic and trans-Tasman routes, the Australian carrier Qantas, and it is now well established in these markets. It has, in essence, now been adopted by other carriers, including Air Canada and, most recently, Delta Airlines. It represents a system of price discrimination and oligopolistic competition which will be explored in this paper, first theoretically and then with data on prices charged by Air New Zealand and Qantas on a set of their routes.

There is a small but interesting literature on price discrimination in airline markets. Using US databases of samples of tickets actually purchased, Borenstein and Rose (1994) and Stavins (2001) found that price dispersion -- the distribution of prices paid for a particular flight – is increasing in the number of airlines flying the route, consistent with theoretical predictions of Gale (1993) and Dana (1998). Note that this appears counter-intuitive. We know that a monopolist will wish to price discriminate if they can get away with it. And we believe that in a highly competitive industry, competition between many sellers drives profit margins close to zero, leaving no room for price discrimination even if it is technically feasible. So why would the path for intermediate market structures not be monotonic -- that is, price discrimination diminishing as the number of sellers increases? The present paper develops a model in which discrimination in terms of the spread between prices does diminish as the number of sellers increases and supports . We do not at this stage examine the reasons for the apparent conflict between these results and those in the price dispersion literature.

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9 Denying Frequent Flier and status points to tickets purchased from the lowest fare class is likely to limit their attractiveness to business travellers, especially those who are not in the end paying for their own ticket.
4. Modelling Price Discrimination in Oligopoly

We model airline pricing for a ‘flight’, meaning a plane or planes flying a particular route at a particular time on a particular date, for which potential purchasers of a (single) ticket can be ranked according to their willingness to pay:

\[ MV = 1 - Q \]

where \( MV \) is the maximum willingness to pay of the \( Q \)th keenest customer. The ranking of valuations is assumed to be linear, for simplicity, and units are chosen to give a unit intercept and slope. All airlines competing for these customers are identical in the minds of the customers, and the airlines know the form of (1). Costs of supplying a seat on this flight are assumed to be constant and equal across carriers.

**Monopoly Case**

First we examine the monopoly case. In the standard single-price situation (figure 1), a profit-maximising monopolist will, of course set price

\[ P = \frac{(1 + c)}{2} \]

and will supply output

\[ Q = \frac{(1 - c)}{2} \]

Now we bring in price discrimination of the type currently observed in air travel markets. We assume that the monopolist not only knows the shape of (1), but can generate a limited number of partitions or ‘fences’ between customers such that, if a range of prices is offered, the monopoly carrier can prevent anyone who would willingly pay a higher price, rather than not fly at all, from purchasing a ticket at a lower price. The most plausible way of justifying this assumption is probably to assume that willingness to pay is perfectly (inversely) correlated with time before the flight that the customer finds out that they are interested in buying a ticket. Then, the airline offers first a ‘bucket’ of lowest-fare tickets. When that bucket is full, the fare is withdrawn from the market and subsequent purchasers have to pay a higher price until that bucket is full, whereupon a yet higher-price bucket is substituted, until all profitable seats are sold.

We will take the number of buckets as exogenous, determined by transaction costs which are not here modelled. Suppose first that the monopolist can sell at just two prices. The number of seats made available at ‘low’ and ‘high’ prices (the sizes of the buckets) will determine the actual level of those prices.

Demand at the higher price is

\[ p'' = 1 - q'' \]
Where \(q^H\) is the number of consumers paying the higher price \(p^H\), and demand at the lower price satisfies:

(5) \[ p^L = 1 - Q = 1 - q^H - q^L \]

Profit maximisation requires maximising:

(6) \[
\pi = \left( p^H - c \right) q^H + \left( p^H - c \right) q^H \\
= \left( 1 - q^H - q^L - c \right) q^H + \left( 1 - q^H - c \right) q^H
\]

The partial derivatives are:

(7) \[
\frac{\partial \pi}{\partial q^L} = 1 - q^H - 2q^L - c
\]

(8) \[
\frac{\partial \pi}{\partial q^H} = 1 - q^L - 2q^H - c
\]

Equating to zero and rearranging gives:

\[ q^H = 1 - 2q^L - c \]

Substituting:

\[ q^L = 1 - 2[1 - 2q^L - c] - c \]
\[ = -1 + 4q^L + c \]

Which solves for revenue-maximising

(9) \[ q^L = (1 - c)/3 \]

and

(10) \[ q^H = (1 - c)/3 \]

With, of course:

(11) \[ p^H = (2 + c)/3 \]

(12) \[ p^L = (1 + 2c)/3 \]

So, whereas a single-price monopolist splits the market into two and supplies one half of it, the two-price monopolist splits it into thirds and supplies two thirds (Fig. 2).

Note that the average price paid under price-discrimination is

(13) \[ P^{av} = 0.5(2 + c)/3 + 0.5(1 + 2c)/3 \]
\[= (1 + c)/2\]

-- which is the same as for the single-price case!

These results generalise to any number of fare buckets. A monopolist offering \( k \) different prices will supply \( k/(1 + k) \) of the total potential viable market, with \( (1 - c)/(1 + k) \) sold at each price (constant-sized buckets) and the average price paid remains unchanged at \((1+c)/2\). Note that in the limit of perfect (1st degree) price discrimination with \( k \) very large and all surplus generated (and appropriated by the seller) it is obvious that the average price is \((1+c)/2\).

What if the demand curve is non-linear? Then the fare buckets will not in general be equal-sized. I have not at this stage of the research established whether the linear-demand result of invariant average price paid carries across to any or all non-linear demand specifications.

**Oligopoly**

Note first that for a demand curve like equation (1) and constant and equal marginal costs, the (symmetric) \( n \)-firm Cournot-Nash equilibrium in the single-price case is:

\[
\begin{align*}
(14) & \quad q = (1 - c)/(n + 1) \quad \text{(output per firm)} \\
(15) & \quad p = 1 - Q = 1 - nq = (1 + nc)/(n + 1) \quad \text{(market price)}
\end{align*}
\]

For the duopoly case, we can expand the demand curves (2) and (3):

\[
\begin{align*}
(16) & \quad p^H = 1 - q_i^H - q_j^H \\
(17) & \quad p^L = 1 - q_i^H - q_j^H - q_i^L - q_j^L
\end{align*}
\]

For airlines \( i \) and \( j \). Airline \( i \)’s profit is:

\[
\begin{align*}
(18) & \quad \pi_i = \left( p^H - c \right) q_i^H + \left( p^L - c \right) q_i^L \\
& \quad = \left[ 1 - c - q_j^H - q_j^L \right] q_i^H + \left[ 1 - c - q_i^H - q_j^H - q_i^L - q_j^L \right] q_i^L
\end{align*}
\]

\[
\begin{align*}
(19) & \quad \frac{\partial \pi_i}{\partial q_i^H} = 1 - c - 2q_i^H - q_j^H - q_i^L \\
(20) & \quad \frac{\partial \pi_i}{\partial q_i^L} = 1 - c - q_i^H - q_j^H - 2q_i^L - q_j^L
\end{align*}
\]
Equating to zero and rearranging:

\[ q_i^T = 1 - c - 2q_{ii}^H - q_{ji}^H \]  
(21)

\[ 1 - c - 2q_i^T - q_i^H - q_{ji}^H - q_j^H = 0 \]  
(22)

\[
1 - c - 2\left[1 - c - 2q_i^H - q_{ji}^H\right] - q_{ii}^H - q_{ji}^H - q_j^H = 0 \\
1 - 2 + 3q_j^H + 2q_j^H - q_{ji}^H - q_j^H = 0
\]

\[ q_i^H = \left[1 - c - q_{j}^H + q_j^T\right]/3 \]  
(23)

Substituting (23) into (21):

\[ q_i^T = 1 - 2\left[1 - c - q_j^H + q_j^T\right]/3 - q_j^H - c \]

\[ = 1 - 2/3 - 1/3q_j^H - 2/3q_j^T - c/3 \]

\[ = \left[1 - c - q_j^H - 2q_j^T\right]/3 \]

i’s total output is determined as:

\[ q_i = q_i^H + q_j^T = \left[1 - 2c - q_j^H + q_j^T + 1 - q_j^H - 2q_j^T\right]/3 \]  
(24)

\[ = \left[2 - 2c - 2q_j^H - q_j^T\right]/3 \]

Solve for the (symmetric) Nash equilibrium by removing the \( i \) and \( j \) subscripts in (23) and (24):

\[ q^H = \left[1 - c - q^H + q^T\right]/3 \]  
(26)
\[(27) \quad q^L = \left[1 - c - q^H - 2q^L \right]/3 \]

Rearranging, etc:

\[4q^H = 1 - c + q^L, \text{ so } q^L = 4q^H - (1 - c)\]

\[5q^L = 1 - q^H - c\]

\[20q^H - 5(1 - c) = 1 - c - q^H\]

so:

\[(28) \quad q^H = 2(1 - c)/7\]

\[(29) \quad q^L = (1 - c)/7\]

Each airline offers twice as many high price tickets as low price tickets.

The prices will be:

\[(30) \quad p^H = 1 - 2q^H = 1 - 4(1 - c)/7\]

\[= (3 + 4c)/7\]

\[p^L = 1 - 2q^H - 2q^L\]

\[= 1 - 4(1 - c)/7 - 2(1 - c)/7\]

\[= (1 + 6c)/7\]

We can calculate the average price charged:

\[(32) \quad p^{av} = \frac{2}{3} (3 + 4c)/7 + \frac{1}{3} (1 + 6c)/7\]

\[= (1 + 2c)/3\]

which is the same as the single-price duopoly Nash-equilibrium price from substituting \( n=2 \) in (15). That is, the result that monopoly price discrimination does not affect average fares paid (though of course more fares are sold) carries through to the duopoly case, at least when there are just two fare buckets (prices). As with
monopoly, we do not yet know whether this holds for non-linear demand specifications.

Now we generalise the number of firms, to \( n \).

Reaction functions (23) and (24) will satisfy, in Nash-equilibrium:

\[
q'' = \left[ 1 - c - (n - 1)q'' + (n - 1)q' \right] / 3
\]

\[
q' = \left[ 1 - c - (n - 1)q'' - 2(n - 1)q' \right] / 3
\]

Re-arranging, etc:

\[
3q'' + (n - 1)q'' = 1 - c + (n - 1)q'
\]

\[
(n + 2)q'' = 1 - c + (n - 1)q'
\]

\[
q' = \left[ (n + 2)/(n - 1) \right] q'' - (1 - c)/(n - 1)
\]

\[
3q' + 2(n - 1)q' = 1 - c - (n - 1)q''
\]

\[
(2n + 1)q' = 1 - c - (n - 1)q''
\]

\[
q' = (1 - c)/(2n + 1) - [(n - 1)/(2n + 1)] q''
\]
Equating (35) and (36):

\[
\left[\frac{(n+2)}{(n-1)}\right]q^\prime\prime - \frac{(1-c)}{(n-1)}
\]

\[= \frac{(1-c)}{(2n+1)} - \left[\frac{(n-1)}{(2n+1)}\right]q^\prime\prime\]

\[
q^\prime\prime \left[\frac{(n+2)}{(n-1)} + \frac{(n-1)}{(2n+1)}\right] = \frac{(1-c)}{(2n+1)} + \frac{(1-c)}{(n-1)}
\]

or

\[q^\prime\prime \left[\frac{(n+2)(2n+1)}{(2n+1)} + (n-1)^2\right] = [(n-1) + (2n+1)](1-c)\]

which simplifies to

(37) \[q^\prime\prime = n(1-c)/\left[n^2 + n + 1\right]\]

And so:

(38) \[q^L = (1-c)/\left[n^2 + n + 1\right]\]

Total output:

(39) \[Q = n(q^\prime\prime + q^L) = (n^2 + n)(1-c)/\left[n^2 + n + 1\right]\]

And prices:

(40) \[p^\prime\prime = 1 - nq^\prime\prime\]

\[= \frac{(n+1 + n^2 + c)}{\left[n^2 + n + 1\right]}\]

(41) \[p^L = 1 - nq^\prime\prime - nq^L\]
Note that the average revenue or price per seat is:

$$p_{av} = \frac{n}{n+1} \left[ \frac{n+1+n^2c}{n^2+n+1} \right] + \frac{1}{n+1} \left[ \frac{nc+1+n^2c}{n^2+n+1} \right]$$

$$= \frac{(1+nc)}{(n+1)}$$

which is the same as the single-price Cournot-Nash price (equation (15)).

Finally, we calculate the differential between high and low price:

$$p^H - p^L = n(1-c)/[n^2+n+1]$$

which is decreasing in $n$, as expected. As competition increases, the price distribution is squeezed, approaching zero for the large-number case.
5. Empirical analysis

Our theoretical analysis, showed that, up to a first order (linear) approximation, and with other simplifications such as cost symmetry and homogeneous products, we can expect the following of a price-discriminating airline market:

- average fares charged will fall as number of competitors increases, just as in single-price models
- the differential between highest and lowest price will fall as the number of competitors increases

Motivated by the Air New Zealand/Qantas cartel case, I set out to test these predictions on a sample of data on prices offered by these airlines on some of their routes within New Zealand and across the Tasman Sea between NZ and Australia.

Much empirical U.S. empirical research into airline pricing has used (10%) samples of prices actually paid for the tickets of passengers travelling on given flights. These are indeed very high quality data for econometric work, although they (apparently) do not allow distinction between ‘horizontal’ and ‘dynamic’ discrimination, because they do not record when the ticket was purchased (and what other prices, if any, were available at purchase date).

In any case, we do not have such data in New Zealand. But we now can, under the new ‘Express’ fare system of Air New Zealand, and its Qantas equivalent ‘Red e-Deals’, observe on the airlines’ websites the prices offered on any day for a given future flight, and thus how these prices change as the flight date approaches. Unfortunately, we cannot know how ‘deep’ are the fare buckets -- that is, how many tickets were sold at each quoted fare before it was taken off the market and replaced with a higher fare.

The database has the following characteristics:

- the unit of observation is the ‘flight’, being a particular flight number flown on a particular date
- about one hundred different flight numbers were observed from nine routes
  - eight internal NZ routes
  - one trans-Tasman (Auckland-Sydney)
- the flights were Wednesday (and Saturday) flights
- the Wednesday flights began November 17 2004, and ended January 5, 2005 (ie 8 flight dates)
• each flight was observed every Tuesday up to nine weeks before the flight date, and several times during the week before the flight date, with the last observation on prices the day before the flight date

• we observed the (up to) three prices offered simultaneously, in the three different fare classes

• There are 4 possible fare levels within each class on the domestic routes; 3 on the trans-Tasman routes. These ‘price points’ changed very little over the observation period

The total number of data points observed is 743. However, for the econometric results shown below, the set is restricted to 378 observations by excluding the last three of the eight weeks of flight dates, and by excluding the Qantas fares. That is, the (preliminary) results shown below will attempt to explain levels and differences in Air New Zealand’s lowest posted fares for Wednesday flights from November 17 to December 15, 2004.\(^\text{10}\)

Variables used in the regressions are defined as:

LNPWAVK: \( \log [ \text{WEIGHTED AVERAGE FARE PRICE} \ \text{PER KILOMETRE}] \)

LNDIST: \( \log [ \text{NON-STOP ROUTE DISTANCE}] \)

HHI: \( \text{HERFINDAHL/HIRSCHMAN INDEX BASED ON NUMBER OF DAILY FLIGHTS BY AIR NZ AND QANTAS ON A ROUTE} \)

SOLD: \( \text{DUMMY = 1, IF FLIGHT SOLD OUT BY FLIGHT DATE} \)

PEAK: \( \text{DUMMY = 1, IF FLIGHT APPEARS TO BE A ‘PEAK-TIME’ BUSINESS FLIGHT} \)

PDIFF: \( \text{RATIO OF HIGHEST TO LOWEST LOW PRICE OFFERED ON AIRLINE WEBSITE IN THE EIGHT WEEKS BEFORE THE FLIGHT} \)

Just two equations are shown here. One has as its dependent variable the log of the weighted average price per kilometre of flight distance. This is the average of the lowest price observed (usually eight weeks before flight date), the highest price observed in the last week before flight date, and the price offered 15 days before the flight date. The log of route distance is used as a proxy for the costs of supplying a seat on the flight. The Herfindahl/Hirschman index (the sum of the squared market shares of sellers) is a standard measure of market structure. Here it takes the value 1 for the four routes on which Air New Zealand flies alone. It would take the value 0.5

\(^{10}\)The last three flights dates (December 22 and 29; January 5) clearly had much less business travel and much more leisure travel than the earlier weeks.
in a symmetric duopoly, but none of the five routes on which Qantas also flies are quite symmetric, and the value of HHI on these routes varies from 0.755 to 0.375.\footnote{The minimum HHI value for a duopoly is 0.5, in the symmetric case. In the Auckland-Sydney route, however, Air New Zealand and Qantas face some competition from 'fifth freedom' carriers (notably Emirates), which I estimate reduces the HHI in this market to 0.375.}

Some flights disappear from the website before the flight date and are assumed to have been sold out. To the extent that the airline’s yield management system was able to take advantage of the excess of demand over supply, it should have captured higher fares than on a flight with normal load factors, \textit{ceteris paribus}. Some flights, for example those leaving Auckland for Wellington between 700 and 830 am, are clearly peak period for business travellers, and are expected to be priced higher.

The ordinary least squares regression equation calculated by the EViews package for average lowest fare offered is:

\[
\text{LNPWAVK} = 6.09 - 0.48 \text{LNDIST} + 0.44 \text{HHI} + 0.35 \text{PEAK} + 0.18 \text{SOLD}.
\]

The \( R^2 \) of this regression is 0.64. The numbers in ( ) are coefficient t-statistics and the numbers in [ ] are sample means of the variables HHI, PEAK and SOLD.

The model appears to be a success. The price of a ticket increases less than proportionately with the distance of the flight, which is absolutely to be expected, given that many flight costs are not distance dependent (e.g., transaction costs, airport handling costs, airport charges, marketing). Of main interest, of course, is the coefficient on the HHI. This tells us that in markets on which it has a monopoly, Air New Zealand sets its lowest fares at levels around 25-30% higher than on routes, such as Auckland-Sydney and Auckland-Wellington, on which it shares the market with Qantas on roughly equal terms.\footnote{\( \exp(0.44) = 1.55 \)}

If we go back to the basic Cournot-Nash pricing equation (15):

\[
P = \frac{(1+nc)}{(n+1)}
\]

and solve this for costs, \( c \) such that the monopoly price (\( n=1 \)) is 1.25 times the duopoly price (\( n=2 \)), we get an implied value of \( c = 0.25 \), and prices 0.5 and 0.625 for the duopoly and monopoly cases. The price elasticities of demand at these prices are -1 and -1\( \frac{2}{3} \). This is a range of price elasticities which is very consistent with the elasticities for airline travel found in econometric demand analyses, which tend to find values around -1 or somewhat larger, in absolute values (Gillen \textit{et al}, 2003).

Now we model PDIFF, which is a measure of the extent of dynamic price discrimination, being the ratio of the highest observed low-price to the lowest observed low-price offered by Air New Zealand on its website for a particular flight on a particular date, observed up to eight weeks before the flight date. The OLS regression equation is:
PDIFF = 1.54 (12) [1.82]
+ 0.51 HHI (3.1)
+ 0.29 PEAK (2.6)
- 0.0045PWAVK (-1.5) [30.7]

The $R^2$ of this regression is only 0.036, which shows that we have some way to go before we can explain price discrimination with the same success as the average level of fares, but again the coefficient of the index of seller concentration, HHI, is significant and with the expected sign. The coefficient and the mean value of PDIFF imply that on monopoly routes the lowest fare offered by Air New Zealand just about doubles over the eight weeks up to the flight date, whereas on duopoly routes the ratio of highest to lowest is about 1.75 -- the highest fare is about 75% more expensive than the lowest fare.

This result is consistent with our qualitative theoretical prediction that the dispersion of prices will be compressed as competition increases. Quantitatively, it may be consistent with equation (43) giving the difference between high and low price as a function of costs and the number of competitors, but because this is derived for the simplest case of just two price steps, and the empirical data generally reveal Air New Zealand to be using more than two steps, a meaningful comparison is not possible.

These econometric results are not, I hope, preliminary, but they are of course incomplete. Much further analysis is needed, including modelling the ‘high’ and ‘low’ prices separately, and extending the sample to cover Qantas’s pricing, and pricing behaviour over the Christmas vacation period.

6. Implications and conclusions

This paper has shown theory and empirics on price discrimination in small-number oligopoly. The theory suggests that if our interest -- for example for antitrust purposes -- is in predicting the impact of structural changes in a market on the average prices paid by consumers, then explicitly modelling price discrimination may not be necessary. This is because, to a first order linear approximation, it appears that increasing the number of price points (the range of prices offered for a good or service) does not affect the average price as predicted by a standard Cournot-Nash model of oligopolistic competition. The analysis also showed, as expected, that the extent of price dispersion reduces as the number of competitors increases. Price discrimination involves less restriction of output below perfectly competitive levels for all market structures, and the deadweight loss of output restriction when a merger or cartel reduces the number of independent competitors would be predicted to be less when the existence of price discrimination is taken into account. Thus, whilst price discrimination may not much affect antitrust based on consumer surplus considerations (because average price paid is not affected), it could affect antitrust jurisdictions in which a total surplus or efficiency test is used.
Clearly the work reported above needs extending, most urgently by generalising the analysis of price discriminating oligopoly to allow multiple price steps. This will not be analytically difficult. Much harder, because we may not yet have the theoretical tools to deal with it, is to endogenise the number of price steps (the fineness of discrimination). In essence, the question here is: what are the transaction costs that prevent the seller from charging every consumer a price equal to their willingness to pay.

References


Gillen, David and Hazledine, Tim (2003)


Figure 1: Monopoly case: single price
Figure 2: Two-price monopoly
Figure 3: Duopoly with two-price discrimination

\[ p^H = \frac{3 + 4c}{7} \]

\[ p^L = \frac{1 + 6c}{7} \]